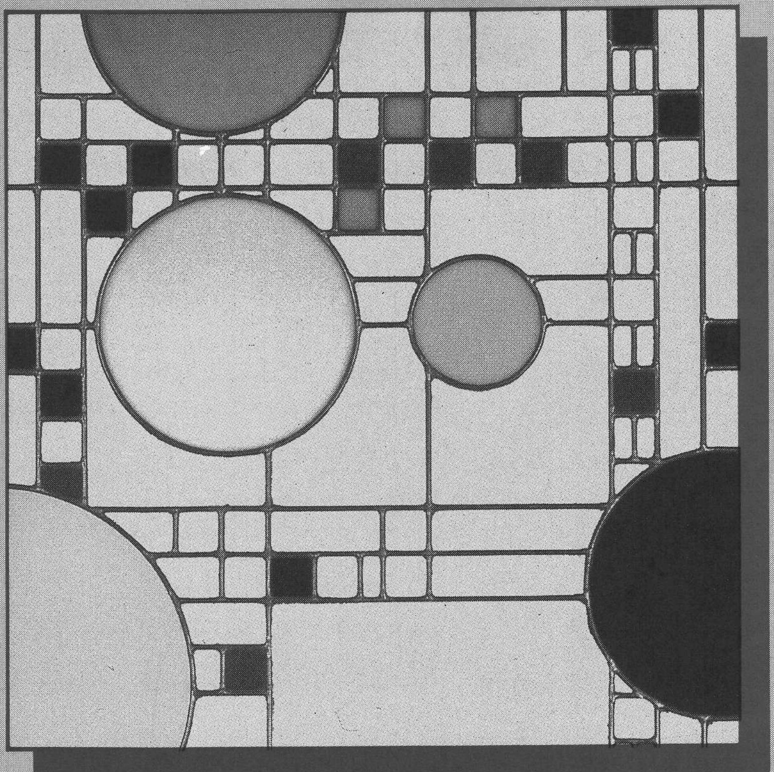
3

Exponents, Roots, and Factorizations of Whole Numbers



After completing this chapter, you should

Section 3.1 Exponents and Roots
- understand and be able to read exponential notation
- understand the concept of root and be able to read root notation
- be able to use a calculator having the $\boxed{y^x}$ key to determine a root

Section 3.2 Grouping Symbols and the Order of Operations
- understand the use of grouping symbols
- understand and be able to use the order of operations
- use the calculator to determine the value of a numerical expression

Section 3.3 Prime Factorization of Natural Numbers
- be able to determine the factors of a whole number
- be able to distinguish between prime and composite numbers
- be familiar with the fundamental principle of arithmetic
- be able to find the prime factorization of a whole number

Section 3.4 The Greatest Common Factor
- be able to find the greatest common factor of two or more whole numbers

Section 3.5 The Least Common Multiple
- be able to find the least common multiple of two or more whole numbers

3.1 Exponents and Roots

Section Overview

- EXPONENTIAL NOTATION
- READING EXPONENTIAL NOTATION
- ROOTS
- READING ROOT NOTATION
- CALCULATORS

EXPONENTIAL NOTATION

Exponential Notation

We have noted that multiplication is a description of repeated addition. **Exponential notation** is a description of repeated multiplication.

Suppose we have the repeated multiplication

$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Exponent

The factor 8 is repeated 5 times. Exponential notation uses a *superscript* for the number of times the factor is repeated. The superscript is placed on the repeated factor, 8^5, in this case. The superscript is called an **exponent.**

The Function of an Exponent

An **exponent** records the number of identical factors that are repeated in a multiplication.

☆ SAMPLE SET A

Write the following multiplication using exponents.

1. $3 \cdot 3$. Since the factor 3 appears 2 times, we record this as

3^2

2. $62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62$. Since the factor 62 appears 9 times, we record this as

62^9

Expand (write without exponents) each number.

3. 12^4. The exponent 4 is recording 4 factors of 12 in a multiplication. Thus,

$12^4 = 12 \cdot 12 \cdot 12 \cdot 12$

4. 706^3. The exponent 3 is recording 3 factors of 706 in a multiplication. Thus,

$706^3 = 706 \cdot 706 \cdot 706$

★ PRACTICE SET A

Write the following using exponents.

1. $37 \cdot 37$ **2.** $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$ **3.** $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Write each number without exponents.

4. 85^3 **5.** 4^7 **6.** $1{,}739^2$

READING EXPONENTIAL NOTATION

In a number such as 8^5,

Base
Exponent, Power

8 is called the **base.**
5 is called the **exponent,** or **power.**
8^5 is read as "eight to the fifth power," or more simply as "eight to the fifth," or "the fifth power of eight."

Squared

When a whole number is raised to the second power, it is said to be **squared.** The number 5^2 can be read as

5 to the second power, or
5 to the second, or
5 squared.

Cubed

When a whole number is raised to the third power, it is said to be **cubed.** The number 5^3 can be read as

5 to the third power, or
5 to the third, or
5 cubed.

When a whole number is raised to the power of 4 or higher, we simply say that that number is raised to that particular power. The number 5^8 can be read as

5 to the eighth power, or just
5 to the eighth.

ROOTS

In the English language, the word "root" can mean a source of something. In mathematical terms, the word "root" is used to indicate that one number is the source of another number through repeated multiplication.

We know that $49 = 7^2$, that is, $49 = 7 \cdot 7$. Through repeated multiplication, 7 is the source of 49. Thus, 7 is a root of 49. Since two 7's must be multiplied together to produce 49, the 7 is called the second or **square root** of 49.

Square Root

We know that $8 = 2^3$, that is, $8 = 2 \cdot 2 \cdot 2$. Through repeated multiplication, 2 is the source of 8. Thus, 2 is a root of 8. Since three 2's must be multiplied together to produce 8, 2 is called the third or **cube root** of 8.

Cube Root

We can continue this way to see such roots as fourth roots, fifth roots, sixth roots, and so on.

READING ROOT NOTATION

There is a symbol used to indicate roots of a number. It is called the radical sign $\sqrt[n]{\quad}$

The Radical Sign $\sqrt[n]{\quad}$

The symbol $\sqrt[n]{\quad}$ is called a **radical sign** and indicates the *n*th root of a number.

We discuss *particular roots* using the radical sign as follows:

Square Root

$\sqrt[2]{\text{number}}$ indicates the **square root** of the number under the radical sign. It is customary to drop the 2 in the radical sign when discussing square roots. The symbol $\sqrt{\quad}$ is understood to be the square root radical sign.

$\sqrt{49} = 7$ since $7 \cdot 7 = 7^2 = 49$

Cube Root

$\sqrt[3]{\text{number}}$ indicates the **cube root** of the number under the radical sign.

$\sqrt[3]{8} = 2$ since $2 \cdot 2 \cdot 2 = 2^3 = 8$

Fourth Root

$\sqrt[4]{\text{number}}$ indicates the **fourth root** of the number under the radical sign.

$\sqrt[4]{81} = 3$ since $3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$

In an expression such as $\sqrt[5]{32}$,

Radical Sign — $\sqrt{}$ is called the **radical sign.**

Index — 5 is called the **index.** (The index describes the indicated root.)

Radicand — 32 is called the **radicand.**

Radical — $\sqrt[5]{32}$ is called a **radical** (or radical expression).

☆ SAMPLE SET B

Find each root.

1. $\sqrt{25}$. To determine the square root of 25, we ask, "What whole number squared equals 25?" From our experience with multiplication, we know this number to be 5. Thus,

   $\sqrt{25} = 5$

   Check: $5 \cdot 5 = 5^2 = 25$.

2. $\sqrt[5]{32}$. To determine the fifth root of 32, we ask, "What whole number raised to the fifth power equals 32?" This number is 2.

   $\sqrt[5]{32} = 2$

   Check: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$.

★ PRACTICE SET B

Find the following roots using only a knowledge of multiplication.

1. $\sqrt{64}$ 2. $\sqrt{100}$ 3. $\sqrt[3]{64}$ 4. $\sqrt[6]{64}$

❑ CALCULATORS

Calculators with the $\boxed{\sqrt{x}}$, $\boxed{y^x}$, and $\boxed{1/x}$ keys can be used to find or approximate roots.

☆ SAMPLE SET C

1. Use the calculator to find $\sqrt{121}$.

		Display Reads
Type	121	121
Press	$\boxed{\sqrt{x}}$	11

2. Find $\sqrt[7]{2187}$.

		Display Reads
Type	2187	2187
Press	$\boxed{y^x}$	2187
Type	7	7
Press	$\boxed{1/x}$	.14285714
Press	$\boxed{=}$	3

$\sqrt[7]{2187} = 3$. (Which means that $3^7 = 2187$.)

★ **PRACTICE SET C**

Use a calculator to find the following roots.

1. $\sqrt[3]{729}$
2. $\sqrt[4]{8503056}$
3. $\sqrt{53361}$
4. $\sqrt[12]{16777216}$

Answers to Practice Sets are on p. 101.

Section 3.1 EXERCISES

For problems 1–9, write the expressions using exponential notation.

1. $4 \cdot 4$
2. $12 \cdot 12$
3. $9 \cdot 9 \cdot 9 \cdot 9$
4. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot$
5. $826 \cdot 826 \cdot 826$
6. $3{,}021 \cdot 3{,}021 \cdot 3{,}021 \cdot 3{,}021 \cdot 3{,}021$
7. $\underbrace{6 \cdot 6 \cdot \cdots \cdot 6}_{\text{85 factors of 6}}$
8. $\underbrace{2 \cdot 2 \cdot \cdots \cdot 2}_{\text{112 factors of 2}}$
9. $\underbrace{1 \cdot 1 \cdot \cdots \cdot 1}_{\text{3,008 factors of 1}}$

For problems 10–15, expand the terms. (Do not find the actual value.)

10. 5^3
11. 7^4
12. 15^2
13. 117^5
14. 61^6
15. 30^2

For problems 16–45, determine the value of each of the powers. Use a calculator to check each result.

16. 3^2
17. 4^2
18. 1^2
19. 10^2
20. 11^2
21. 12^2
22. 13^2
23. 15^2
24. 1^4
25. 3^4
26. 7^3
27. 10^3

28. 100^2

29. 8^3

30. 5^5

31. 9^3

32. 6^2

33. 7^1

34. 1^{28}

35. 2^7

36. 0^5

37. 8^4

38. 5^8

39. 6^9

40. 25^3

41. 42^2

42. 31^3

43. 15^5

44. 2^{20}

45. 816^2

For problems 46–63, find the roots (using your knowledge of multiplication). Use a calculator to check each result.

46. $\sqrt{9}$

47. $\sqrt{16}$

48. $\sqrt{36}$

49. $\sqrt{64}$

50. $\sqrt{121}$

51. $\sqrt{144}$

52. $\sqrt{169}$

53. $\sqrt{225}$

54. $\sqrt[3]{27}$

55. $\sqrt[5]{32}$

56. $\sqrt[4]{256}$

57. $\sqrt[3]{216}$

58. $\sqrt[7]{1}$

59. $\sqrt{400}$

60. $\sqrt{900}$

61. $\sqrt{10{,}000}$

62. $\sqrt{324}$

63. $\sqrt{3{,}600}$

For problems 64–75, use a calculator with the keys $\boxed{\sqrt{x}}$, $\boxed{y^x}$, and $\boxed{1/x}$ to find each of the values.

64. $\sqrt{676}$

65. $\sqrt{1{,}156}$

66. $\sqrt{46{,}225}$

67. $\sqrt{17{,}288{,}964}$

68. $\sqrt[3]{3{,}375}$

69. $\sqrt[4]{331{,}776}$

70. $\sqrt[8]{5{,}764{,}801}$

71. $\sqrt[12]{16{,}777{,}216}$

72. $\sqrt[8]{16{,}777{,}216}$

73. $\sqrt[10]{9{,}765{,}625}$

74. $\sqrt[4]{160{,}000}$

75. $\sqrt[3]{531{,}441}$

EXERCISES FOR REVIEW

(1.6) **76.** Use the numbers 3, 8, and 9 to illustrate the associative property of addition.

(2.1) **77.** In the multiplication $8 \cdot 4 = 32$, specify the name given to the numbers 8 and 4.

(2.2) **78.** Does the quotient $15 \div 0$ exist? If so, what is it?

(2.2) **79.** Does the quotient $0 \div 15$ exist? If so, what is it?

(2.5) **80.** Use the numbers 4 and 7 to illustrate the commutative property of multiplication.

★ ANSWERS TO PRACTICE SETS (3.1)

A. **1.** 37^2 **2.** 16^5 **3.** 9^{10} **4.** $85 \cdot 85 \cdot 85$ **5.** $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **6.** $1739 \cdot 1739$

B. **1.** 8 **2.** 10 **3.** 4 **4.** 2

C. **1.** 9 **2.** 54 **3.** 231 **4.** 4

3.2 Grouping Symbols and the Order of Operations

Section Overview

- ❑ GROUPING SYMBOLS
- ❑ MULTIPLE GROUPING SYMBOLS
- ❑ THE ORDER OF OPERATIONS
- ❑ CALCULATORS

❑ GROUPING SYMBOLS

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are the following:

( )
[ ]
{ }
—

Parentheses: ( )
Brackets: [ ]
Braces: { }
Bar: —

In a computation in which more than one operation is involved, grouping symbols indicate which operation to perform first. If possible, we perform operations *inside* grouping symbols first.

☆ SAMPLE SET A

If possible, determine the value of each of the following.

1. $9 + (3 \cdot 8)$

Since 3 and 8 are within parentheses, they are to be combined first.

$$9 + (3 \cdot 8) = 9 + 24$$
$$= 33$$

Continued

Thus,

$9 + (3 \cdot 8) = 33$

2. $(10 \div 0) \cdot 6$

Since $10 \div 0$ is undefined, this operation is meaningless, and we attach no value to it. We write, "undefined."

★ PRACTICE SET A

If possible, determine the value of each of the following.

1. $16 - (3 \cdot 2)$ **2.** $5 + (7 \cdot 9)$ **3.** $(4 + 8) \cdot 2$ **4.** $28 \div (18 - 11)$

5. $(33 \div 3) - 11$ **6.** $4 + (0 \div 0)$

❑ MULTIPLE GROUPING SYMBOLS

When a set of grouping symbols occurs *inside* another set of grouping symbols, we perform the operations within the innermost set first.

☆ SAMPLE SET B

Determine the value of each of the following.

1. $2 + (8 \cdot 3) - (5 + 6)$

Combine 8 and 3 first, then combine 5 and 6.

$2 + 24 - 11$ **Now combine left to right.**
$26 - 11$
15

2. $10 + [30 - (2 \cdot 9)]$

Combine 2 and 9 since they occur in the innermost set of parentheses.

$10 + [30 - 18]$ **Now combine 30 and 18.**
$10 + 12$
22

★ PRACTICE SET B

Determine the value of each of the following.

1. $(17 + 8) + (9 + 20)$ **2.** $(55 - 6) - (13 \cdot 2)$ **3.** $23 + (12 \div 4) - (11 \cdot 2)$

4. $86 + [14 \div (10 - 8)]$ **5.** $31 + \{9 + [1 + (35 - 2)]\}$ **6.** $\{6 - [24 \div (4 \cdot 2)]\}^3$

☐ THE ORDER OF OPERATIONS

Sometimes there are no grouping symbols indicating which operations to perform first. For example, suppose we wish to find the value of $3 + 5 \cdot 2$. We could do *either* of two things:

1. Add 3 and 5, then multiply this sum by 2.

$$\begin{aligned} 3 + 5 \cdot 2 &= 8 \cdot 2 \\ &= 16 \end{aligned}$$

2. Multiply 5 and 2, then add 3 to this product.

$$\begin{aligned} 3 + 5 \cdot 2 &= 3 + 10 \\ &= 13 \end{aligned}$$

We now have two values for one number. To determine the correct value, we must use the *accepted order of operations.*

Order of Operations

1. Perform all operations *inside* grouping symbols, beginning with the innermost set, in the order 2, 3, 4 described below.
2. Perform all exponential and root operations.
3. Perform all multiplications and divisions, moving left to right.
4. Perform all additions and subtractions, moving left to right.

☆ SAMPLE SET C

Determine the value of each of the following.

1. $21 + 3 \cdot 12.$ **Multiply first.**
$21 + 36$ **Add.**
57

2. $(15 - 8) + 5 \cdot (6 + 4).$ **Simplify inside parentheses first.**
$7 + 5 \cdot 10$ **Multiply.**
$7 + 50$ **Add.**
57

3. $63 - (4 + 6 \cdot 3) + 76 - 4.$ **Simplify first within the parentheses by multiplying, then adding.**

$63 - (4 + 18) + 76 - 4$
$63 - 22 + 76 - 4$ **Now perform the additions and subtractions, moving left to right.**
$41 + 76 - 4$ **Add 41 and 76: $41 + 76 = 117$.**
$117 - 4$ **Subtract 4 from 117: $117 - 4 = 113$.**
113

Continued

4. $7 \cdot 6 - 4^2 + 1^5$.
$7 \cdot 6 - 16 + 1$
$42 - 16 + 1$
$26 + 1$
27

Evaluate the exponential forms, moving left to right.
Multiply 7 and 6: $7 \cdot 6 = 42$.
Subtract 16 from 42: $42 - 16 = 26$.
Add 26 and 1: $26 + 1 = 27$.

5. $6 \cdot (3^2 + 2^2) + 4^2$.

Evaluate the exponential forms in the parentheses: $3^2 = 9$ and $2^2 = 4$.

$6 \cdot (9 + 4) + 4^2$
$6 \cdot (13) + 4^2$
$6 \cdot (13) + 16$
$78 + 16$
94

Add the 9 and 4 in the parentheses: $9 + 4 = 13$.
Evaluate the exponential form: $4^2 = 16$.
Multiply 6 and 13: $6 \cdot 13 = 78$.
Add 78 and 16: $78 + 16 = 94$.

6. $\dfrac{6^2 + 2^2}{4^2 + 6 \cdot 2^2} + \dfrac{1^3 + 8^2}{10^2 - 19 \cdot 5}$.

Recall that the bar is a grouping symbol. The fraction $\dfrac{6^2 + 2^2}{4^2 + 6 \cdot 2^2}$ is equivalent to $(6^2 + 2^2) \div (4^2 + 6 \cdot 2^2)$.

$\dfrac{36 + 4}{16 + 6 \cdot 4} + \dfrac{1 + 64}{100 - 19 \cdot 5}$

$\dfrac{36 + 4}{16 + 24} + \dfrac{1 + 64}{100 - 95}$

$\dfrac{40}{40} + \dfrac{65}{5}$

$1 + 13$
14

★ PRACTICE SET C

Determine the value of each of the following.

1. $8 + (32 - 7)$

2. $(34 + 18 - 2 \cdot 3) + 11$

3. $8(10) + 4(2 + 3) - (20 + 3 \cdot 15 + 40 - 5)$

4. $5 \cdot 8 + 4^2 - 2^2$

5. $4(6^2 - 3^3) \div (4^2 - 4)$

6. $(8 + 9 \cdot 3) \div 7 + 5 \cdot (8 \div 4 + 7 + 3 \cdot 5)$

7. $\dfrac{3^3 + 2^3}{6^2 - 29} + 5\left(\dfrac{8^2 + 2^4}{7^2 - 3^2}\right) \div \dfrac{8 \cdot 3 + 1^8}{2^3 - 3}$

❑ CALCULATORS

Using a calculator is helpful for simplifying computations that involve large numbers.

☆ SAMPLE SET D

Use a calculator to determine each value.

1. $9{,}842 + 56 \cdot 85$

	Key		Display Reads
Perform the multiplication first.	Type	56	56
	Press	$\boxed{\times}$	56
	Type	85	85
Now perform the addition.	Press	$\boxed{+}$	4760
	Type	9842	9842
	Press	$\boxed{=}$	14602

The display now reads 14,602.

2. $42(27 + 18) + 105(810 \div 18)$

Operate inside the parentheses.	Type	27	27
	Press	$\boxed{+}$	27
	Type	18	18
	Press	$\boxed{=}$	45
Multiply by 42.	Press	$\boxed{\times}$	45
	Type	42	42
	Press	$\boxed{=}$	1890
Place this result into memory by pressing the memory key.			
Now operate in the other parentheses.	Type	810	810
	Press	$\boxed{\div}$	810
	Type	18	18
	Press	$\boxed{=}$	45
Now multiply by 105.	Press	$\boxed{\times}$	45
	Type	105	105
	Press	$\boxed{=}$	4725
We are now ready to add these two quantities together.	Press	$\boxed{+}$	4725
Press the memory recall key.			1890
	Press	$\boxed{=}$	6615

Thus, $42(27 + 18) + 105(810 \div 18) = 6{,}615$.

Continued

3. $16^4 + 37^3$

Nonscientific Calculators

Key		Display Reads
Type	16	16
Press	$\boxed{\times}$	16
Type	16	16
Press	$\boxed{\times}$	256
Type	16	16
Press	$\boxed{\times}$	4096
Type	16	16
Press	$\boxed{=}$	65536
Press the memory key		
Type	37	37
Press	$\boxed{\times}$	37
Type	37	37
Press	$\boxed{\times}$	1369
Type	37	37
Press	$\boxed{\times}$	50653
Press	$\boxed{+}$	50653
Press memory recall key		65536
Press	$\boxed{=}$	116189

Calculators with $\boxed{y^x}$ Key

Key		Display Reads
Type	16	16
Press	$\boxed{y^x}$	16
Type	4	4
Press	$\boxed{=}$	4096
Press	$\boxed{+}$	4096
Type	37	37
Press	$\boxed{y^x}$	37
Type	3	3
Press	$\boxed{=}$	116189

Thus, $16^4 + 37^3 = 116{,}189$.

We can certainly see that the more powerful calculator simplifies computations.

4. Nonscientific calculators are unable to handle calculations involving very large numbers.

$85612 \cdot 21065$

Key		Display Reads
Type	85612	85612
Press	$\boxed{\times}$	85162
Type	21065	21065
Press	$\boxed{=}$	

This number is too big for the display of some calculators and we'll probably get some kind of error message. On some scientific calculators such large numbers are coped with by placing them in a form called "scientific notation." Others can do the multiplication directly. (1803416780)

★ **PRACTICE SET D**

Use a calculator to find each value.

1. $9{,}285 + 86(49)$

2. $55(84 - 26) + 120(512 - 488)$

3. $106^3 - 17^4$

4. $6{,}053^3$

Answers to Practice Sets are on p. 110.

Section 3.2 EXERCISES

For problems 1–43, find each value. Check each result with a calculator.

1. $2 + 3 \cdot (8)$

2. $18 + 7 \cdot (4 - 1)$

3. $3 + 8 \cdot (6 - 2) + 11$

4. $1 - 5 \cdot (8 - 8)$

5. $37 - 1 \cdot 6^2$

6. $98 \div 2 \div 7^2$

7. $(4^2 - 2 \cdot 4) - 2^3$

8. $\sqrt{9} + 14$

9. $\sqrt{100} + \sqrt{81} - 4^2$

10. $\sqrt[3]{8} + 8 - 2 \cdot 5$

11. $\sqrt[4]{16} - 1 + 5^2$

12. $61 - 22 + 4[3 \cdot (10) + 11]$

13. $121 - 4 \cdot [(4) \cdot (5) - 12] + \frac{16}{2}$

14. $\frac{(1+16)-3}{7} + 5 \cdot (12)$

15. $\frac{8 \cdot (6+20)}{8} + \frac{3 \cdot (6+16)}{22}$

16. $10 \cdot [8 + 2 \cdot (6+7)]$

17. $21 \div 7 \div 3$

18. $10^2 \cdot 3 \div 5^2 \cdot 3 - 2 \cdot 3$

19. $85 \div 5 \cdot 5 - 85$

20. $\frac{51}{17} + 7 - 2 \cdot 5 \cdot \left(\frac{12}{3}\right)$

21. $2^2 \cdot 3 + 2^3 \cdot (6-2) - (3+17) + 11(6)$

22. $26 - 2 \cdot \left\{\frac{6+20}{13}\right\}$

23. $2 \cdot \{(7+7) + 6 \cdot [4 \cdot (8+2)]\}$

24. $0 + 10(0) + 15 \cdot \{4 \cdot 3 + 1\}$

25. $18 + \frac{7+2}{9}$

26. $(4+7) \cdot (8-3)$

27. $(6+8) \cdot (5+2-4)$

28. $(21-3) \cdot (6-1) \cdot (7) + 4(6+3)$

29. $(10 + 5) \cdot (10 + 5) - 4 \cdot (60 - 4)$

30. $6 \cdot \{2 \cdot 8 + 3\} - (5) \cdot (2) + \frac{8}{4} +$
$(1 + 8) \cdot (1 + 11)$

31. $2^5 + 3 \cdot (8 + 1)$

32. $3^4 + 2^4 \cdot (1 + 5)$

33. $1^6 + 0^8 + 5^2 \cdot (2 + 8)^3$

34. $(7) \cdot (16) - 3^4 + 2^2 \cdot (1^7 + 3^2)$

35. $\frac{2^3 - 7}{5^2}$

36. $\frac{(1 + 6)^2 + 2}{3 \cdot 6 + 1}$

37. $\frac{6^2 - 1}{2^3 - 3} + \frac{4^3 + 2 \cdot 3}{2 \cdot 5}$

38. $\frac{5(8^2 - 9 \cdot 6)}{2^5 - 7} + \frac{7^2 - 4^2}{2^4 - 5}$

39. $\frac{(2 + 1)^3 + 2^3 + 1^{10}}{6^2} - \frac{15^2 - [2 \cdot 5]^2}{5 \cdot 5^2}$

40. $\frac{6^3 - 2 \cdot 10^2}{2^2} + \frac{18(2^3 + 7^2)}{2(19) - 3^3}$

41. $2 \cdot \{6 + [10^2 - 6\sqrt{25}]\}$

42. $181 - 3 \cdot \{2\sqrt{36} + 3\sqrt[3]{64}\}$

43. $\dfrac{2 \cdot (\sqrt{81} - \sqrt[3]{125})}{4^2 - 10 + 2^2}$

EXERCISES FOR REVIEW

(1.6) **44.** The fact that

0 + any whole number = that particular whole number

is an example of which property of addition?

(2.1) **45.** Find the product. $4{,}271 \times 630$.

(2.2) **46.** In the statement $27 \div 3 = 9$, what name is given to the result 9?

(2.6) **47.** What number is the multiplicative identity?

(2.6) **48.** Find the value of 2^4.

★ ANSWERS TO PRACTICE SETS (3.2)

A. **1.** 10 **2.** 68 **3.** 24 **4.** 4 **5.** 0 **6.** not possible (indeterminant)

B. **1.** 54 **2.** 23 **3.** 4 **4.** 93 **5.** 74 **6.** 27

C. **1.** 33 **2.** 57 **3.** 0 **4.** 52 **5.** 3 **6.** 125 **7.** 7

D. **1.** 13,499 **2.** 6,070 **3.** 1,107,495
4. This number is too big for a nonscientific calculator. A scientific calculator will probably give you $2.217747109 \times 10^{11}$.

3.3 Prime Factorization of Natural Numbers

Section Overview

- ❑ FACTORS
- ❑ DETERMINING THE FACTORS OF A WHOLE NUMBER
- ❑ PRIME AND COMPOSITE NUMBERS
- ❑ THE FUNDAMENTAL PRINCIPLE OF ARITHMETIC
- ❑ THE PRIME FACTORIZATION OF A NATURAL NUMBER

❑ FACTORS

From observations made in the process of multiplication, we have seen that

(factor) · (factor) = product

Factors
Product

The two numbers being multiplied are the **factors** and the result of the multiplication is the **product.** Now, using our knowledge of division, we can see that a first number is a factor of a second number if the first number divides into the second number a whole number of times (without a remainder).

One Number as a Factor of Another

A first number is a **factor** of a second number if the first number divides into the second number a whole number of times (without a remainder).

We show this in the following examples:

3 is a factor of 27, since $27 \div 3 = 9$, or $3 \cdot 9 = 27$.
7 is a factor of 56, since $56 \div 7 = 8$, or $7 \cdot 8 = 56$.
4 is *not* a factor of 10, since $10 \div 4 = 2$ R2. (There is a remainder.)

DETERMINING THE FACTORS OF A WHOLE NUMBER

We can use the tests for divisibility from Section 2.4 to determine *all* the factors of a whole number.

SAMPLE SET A

Find all the factors of 24.

Try 1: $24 \div 1 = 24$ **1 and 24 are factors.**
Try 2: 24 is even, so 24 is divisible by 2.

$24 \div 2 = 12$ **2 and 12 are factors.**

Try 3: $2 + 4 = 6$ and 6 is divisible by 3, so 24 is divisible by 3.

$24 \div 3 = 8$ **3 and 8 are factors**

Try 4: $24 \div 4 = 6$ **4 and 6 are factors.**
Try 5: $24 \div 5 = 4$ R4 **5 is *not* a factor.**

The next number to try is 6, but we already have that 6 is a factor. Once we come upon a factor that we already have discovered, we can stop.
All the whole number factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

PRACTICE SET A

Find all the factors of each of the following numbers.

1. 6 **2.** 12 **3.** 18 **4.** 5 **5.** 10 **6.** 33 **7.** 19

PRIME AND COMPOSITE NUMBERS

Notice that the only factors of 7 are 1 and 7 itself, and that the only factors of 3 are 1 and 3 itself. However, the number 8 has the factors 1, 2, 4, and 8, and the number 10 has the factors 1, 2, 5, and 10. Thus, we can see that a whole number can have only *two* factors (itself and 1) and another whole number can have *several* factors.

We can use this observation to make a useful classification for whole numbers: prime numbers and composite numbers.

PRIME NUMBERS

Prime Number

A whole number (greater than one) whose only factors are itself and 1 is called a **prime number.**

The Number 1 is *Not* a Prime Number

The first seven prime numbers are 2, 3, 5, 7, 11, 13, and 17. Notice that the whole number 1 is *not* considered to be a prime number, and the whole number 2 is the *first prime* and the *only even prime* number.

COMPOSITE NUMBERS

Composite Number

A whole number composed of factors other than itself and 1 is called a **composite number.** Composite numbers are not prime numbers.

Some composite numbers are 4, 6, 8, 9, 10, 12, and 15.

☆ SAMPLE SET B

Determine which whole numbers are prime and which are composite.

1. 39. Since 3 divides into 39, the number 39 is composite: $39 \div 3 = 13$.
2. 47. A few division trials will assure us that 47 is only divisible by 1 and 47. Therefore, 47 is prime.

★ PRACTICE SET B

Determine which of the following whole numbers are prime and which are composite.

1. 3 **2.** 16 **3.** 21 **4.** 35 **5.** 47 **6.** 29

7. 101 **8.** 51

❑ THE FUNDAMENTAL PRINCIPLE OF ARITHMETIC

Prime numbers are very useful in the study of mathematics. We will see how they are used in subsequent sections. We now state the Fundamental Principle of Arithmetic.

Fundamental Principle of Arithmetic

Except for the order of the factors, every natural number other than 1 can be factored in one and only one way as a product of prime numbers.

Prime Factorization

When a number is factored so that all its factors are prime numbers, the factorization is called the **prime factorization** of the number.

The technique of prime factorization is illustrated in the following three examples.

1. $10 = 2 \cdot 5$. Both 2 and 5 are primes. Therefore, $2 \cdot 5$ is the prime factorization of 10.
2. 11. The number 11 is a prime number. Prime factorization applies *only* to composite numbers. Thus, 11 has *no* prime factorization.
3. $60 = 2 \cdot 30$. The number 30 is not prime: $30 = 2 \cdot 15$.

   $60 = 2 \cdot 2 \cdot 15$

   The number 15 is not prime: $15 = 3 \cdot 5$.

   $60 = 2 \cdot 2 \cdot 3 \cdot 5$

   We'll use exponents.

   $60 = 2^2 \cdot 3 \cdot 5$

   The numbers 2, 3, and 5 are each prime. Therefore, $2^2 \cdot 3 \cdot 5$ is the prime factorization of 60.

❑ THE PRIME FACTORIZATION OF A NATURAL NUMBER

The following method provides a way of finding the prime factorization of a natural number.

The Method of Finding the Prime Factorization of a Natural Number

1. Divide the number repeatedly by the smallest prime number that will divide into it a whole number of times (without a remainder).
2. When the prime number used in step 1 no longer divides into the given number without a remainder, repeat the division process with the next largest prime that divides the given number.
3. Continue this process until the quotient is smaller than the divisor.
4. The prime factorization of the given number is the *product* of all these prime divisors. If the number has no prime divisors, it is a prime number.

We may be able to use some of the tests for divisibility we studied in Section 2.4 to help find the primes that divide the given number.

☆ SAMPLE SET C

1. Find the prime factorization of 60.

Since the last digit of 60 is 0, which is even, 60 is divisible by 2. We will repeatedly divide by 2 until we no longer can. We shall divide as follows:

2 | 60	
2 | 30	**30 is divisible by 2 again.**
3 | 15	**15 is not divisible by 2, but it is divisible by 3, the next prime.**
5 | 5	**5 is not divisible by 3, but it is divisible by 5, the next prime.**
1	

The quotient 1 is finally smaller than the divisor 5, and the prime factorization of 60 is the product of these prime divisors.

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

We use exponents when possible.

$60 = 2^2 \cdot 3 \cdot 5$

Continued

2. Find the prime factorization of 441.

441 is not divisible by 2 since its last digit is not divisible by 2.
441 is divisible by 3 since $4 + 4 + 1 = 9$ and 9 is divisible by 3.

$$\begin{array}{r|l} 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

147 is divisible by 3 ($1 + 4 + 7 = 12$).
49 is not divisible by 3, nor is it divisible by 5. It is divisible by 7.

The quotient 1 is finally smaller than the divisor 7, and the prime factorization of 441 is the product of these prime divisors.

$441 = 3 \cdot 3 \cdot 7 \cdot 7$

Use exponents.

$441 = 3^2 \cdot 7^2$

3. Find the prime factorization of 31.

31 is not divisible by 2. **Its last digit is not even.**

$31 \div 2 = 15$ R1

The quotient, 15, is larger than the divisor, 2. Continue.

31 is not divisible by 3. **The digits $3 + 1 = 4$, and 4 is not divisible by 3.**

$31 \div 3 = 10$ R1

The quotient, 10, is larger than the divisor, 3. Continue.

31 is not divisible by 5. **The last digit of 31 is not 0 or 5.**

$31 \div 5 = 6$ R1

The quotient, 6, is larger than the divisor, 5. Continue.

31 is not divisible by 7. **Divide by 7.**

$31 \div 7 = 4$ R1

The quotient, 4, is smaller than the divisor, 7. We can stop the process and conclude that 31 is a prime number.

The number 31 is a prime number.

★ PRACTICE SET C

Find the prime factorization of each whole number.

1. 22 **2.** 40 **3.** 48 **4.** 63 **5.** 945 **6.** 1,617

7. 17

8. 61

Answers to Practice Sets are on p. 117.

Section 3.3 EXERCISES

For problems 1–10, determine the missing factor(s).

1. $14 = 7 \cdot$ ________

2. $20 = 4 \cdot$ ________

3. $36 = 9 \cdot$ ________

4. $42 = 21 \cdot$ ________

5. $44 = 4 \cdot$ ________

6. $38 = 2 \cdot$ ________

7. $18 = 3 \cdot$ ________ $\cdot$ ________

8. $28 = 2 \cdot$ ________ $\cdot$ ________

9. $300 = 2 \cdot 5 \cdot$ ________ $\cdot$ ________

10. $840 = 2 \cdot$ ________ $\cdot$ ________ $\cdot$ ________

For problems 11–20, find all the factors of each of the numbers.

11. 16

12. 22

13. 56

14. 105

15. 220

16. 15

17. 32

18. 80

19. 142

20. 218

For problems 21–40, determine which of the whole numbers are prime and which are composite.

21. 23

22. 25

23. 27

24. 2

25. 3

26. 5

27. 7

28. 9

29. 11

30. 34

31. 55

32. 63

33. 1,044

34. 924

35. 339

36. 103

37. 209

38. 667

39. 4,575

40. 119

For problems 41–50, find the prime factorization of each of the whole numbers.

41. 26

42. 38

43. 54

44. 62

45. 56

46. 176

47. 480

48. 819

49. 2,025

50. 148,225

EXERCISES FOR REVIEW

(1.3) **51.** Round 26,584 to the nearest ten.

(1.5) **52.** How much bigger is 106 than 79?

(2.2) **53.** True or false? Zero divided by any nonzero whole number is zero.

(2.3) **54.** Find the quotient. $10{,}584 \div 126$.

(3.2) **55.** Find the value of $\sqrt{121} - \sqrt{81} + 6^2 \div 3$.

★ ANSWERS TO PRACTICE SETS (3.3)

A. **1.** 1, 2, 3, 6 **2.** 1, 2, 3, 4, 6, 12 **3.** 1, 2, 3, 6, 9, 18 **4.** 1, 5 **5.** 1, 2, 5, 10 **6.** 1, 3, 11, 33 **7.** 1, 19

B. **1.** prime **2.** composite **3.** composite **4.** composite **5.** prime **6.** prime **7.** prime **8.** composite

C. **1.** $22 = 2 \cdot 11$ **2.** $40 = 2^3 \cdot 5$ **3.** $48 = 2^4 \cdot 3$ **4.** $63 = 3^2 \cdot 7$ **5.** $945 = 3^3 \cdot 5 \cdot 7$ **6.** $1617 = 3 \cdot 7^2 \cdot 11$ **7.** 17 is prime **8.** 61 is prime

3.4 The Greatest Common Factor

Section Overview

- ❑ THE GREATEST COMMON FACTOR (GCF)
- ❑ A METHOD FOR DETERMINING THE GREATEST COMMON FACTOR

❑ THE GREATEST COMMON FACTOR (GCF)

Using the method we studied in Section 3.3, we could obtain the prime factorizations of 30 and 42.

$30 = 2 \cdot 3 \cdot 5$
$42 = 2 \cdot 3 \cdot 7$

Common Factor

We notice that 2 appears as a factor in both numbers, that is, 2 is a **common factor** of 30 and 42. We also notice that 3 appears as a factor in both numbers. Three is also a common factor of 30 and 42.

Greatest Common Factor (GCF)

When considering two or more numbers, it is often useful to know if there is a largest common factor of the numbers, and if so, what that number is. The largest common factor of two or more whole numbers is called the **greatest common factor,** and is abbreviated by **GCF.** The greatest common factor of a collection of whole numbers is useful in working with fractions (which we will do in Chapter 4).

❑ A METHOD FOR DETERMINING THE GREATEST COMMON FACTOR

A straightforward method for determining the GCF of two or more whole numbers makes use of both the prime factorization of the numbers and exponents.

Finding the GCF

To find the **greatest common factor (GCF)** of two or more whole numbers:

1. Write the prime factorization of each number, using exponents on repeated factors.
2. Write each base that is common to each of the numbers.
3. To each base listed in step 2, attach the *smallest exponent* that appears on it in either of the prime factorizations.
4. The GCF is the product of the numbers found in step 3.

☆ SAMPLE SET A

Find the GCF of the following numbers.

1. 12 and 18

1. $12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
   $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$
2. The common bases are 2 and 3.
3. The *smallest exponents* appearing on 2 and 3 in the prime factorizations are, respectively, 1 and 1 (2^1 and 3^1), or 2 and 3.
4. The GCF is the product of these numbers.

   $2 \cdot 3 = 6$

The GCF of 30 and 42 is 6 because 6 is the largest number that divides both 30 and 42 without a remainder.

2. 18, 60, and 72

1. $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$
   $60 = 2 \cdot 30 = 2 \cdot 2 \cdot 15 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$
   $72 = 2 \cdot 36 = 2 \cdot 2 \cdot 18 = 2 \cdot 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$
2. The common bases are 2 and 3.
3. The smallest exponents appearing on 2 and 3 in the prime factorizations are, respectively, 1 and 1:

   2^1 from 18.
   3^1 from 60.
4. The GCF is the product of these numbers.

   GCF is $2 \cdot 3 = 6$

Thus, 6 is the largest number that divides 18, 60, and 72 without a remainder.

3. 700, 1,880, and 6,160

1. $$\begin{aligned} 700 &= 2 \cdot 350 = 2 \cdot 2 \cdot 175 = 2 \cdot 2 \cdot 5 \cdot 35 \\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \\ &= 2^2 \cdot 5^2 \cdot 7 \\ 1{,}880 &= 2 \cdot 940 = 2 \cdot 2 \cdot 470 = 2 \cdot 2 \cdot 2 \cdot 235 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 47 \\ &= 2^3 \cdot 5 \cdot 47 \\ 6{,}160 &= 2 \cdot 3{,}080 = 2 \cdot 2 \cdot 1{,}540 = 2 \cdot 2 \cdot 2 \cdot 770 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 385 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 77 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \cdot 11 \\ &= 2^4 \cdot 5 \cdot 7 \cdot 11 \end{aligned}$$
2. The common bases are 2 and 5
3. The smallest exponents appearing on 2 and 5 in the prime factorizations are, respectively, 2 and 1.

   2^2 from 700.
   5^1 from either 1,880 or 6,160.
4. The GCF is the product of these numbers.

   GCF is $2^2 \cdot 5 = 4 \cdot 5 = 20$

Thus, 20 is the largest number that divides 700, 1,880, and 6,160 without a remainder.

★ PRACTICE SET A

Find the GCF of the following numbers.

1. 24 and 36
2. 48 and 72
3. 50 and 140
4. 21 and 225
5. 450, 600, and 540

Answers to the Practice Set are on p. 121.

Section 3.4 EXERCISES

For problems 1–27, find the greatest common factor (GCF) of the numbers.

1. 6 and 8
2. 5 and 10
3. 8 and 12
4. 9 and 12
5. 20 and 24
6. 35 and 175
7. 25 and 45
8. 45 and 189
9. 66 and 165
10. 264 and 132
11. 99 and 135
12. 65 and 15
13. 33 and 77
14. 245 and 80
15. 351 and 165

16. 60, 140, and 100

17. 147, 343, and 231

18. 24, 30, and 45

19. 175, 225, and 400

20. 210, 630, and 182

21. 14, 44, and 616

22. 1,617, 735, and 429

23. 1,573, 4,862, and 3,553

24. 3,672, 68, and 920

25. 7, 2,401, 343, 16, and 807

26. 500, 77, and 39

27. 441, 275, and 221

EXERCISES FOR REVIEW

(2.1) **28.** Find the product. $2,753 \times 4,006$.

(2.3) **29.** Find the quotient. $954 \div 18$.

(2.4) **30.** Specify which of the digits 2, 3, or 4 divide into 9,462.

(3.1) **31.** Write $8 \times 8 \times 8 \times 8 \times 8 \times 8$ using exponents.

(3.3) **32.** Find the prime factorization of 378.

★ **ANSWERS TO PRACTICE SET (3.4)**

A. **1.** 12 **2.** 24 **3.** 10 **4.** 3 **5.** 30

3.5 The Least Common Multiple

Section Overview

- ☐ MULTIPLES
- ☐ COMMON MULTIPLES
- ☐ THE LEAST COMMON MULTIPLE (LCM)
- ☐ FINDING THE LEAST COMMON MULTIPLE

☐ MULTIPLES

When a whole number is multiplied by other whole numbers, with the exception of zero, the resulting products are called **multiples** of the given whole number. Note that any whole number is a multiple of itself.

☆ **SAMPLE SET A**

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 8	MULTIPLES OF 10
$2 \times 1 = 2$	$3 \times 1 = 3$	$8 \times 1 = 8$	$10 \times 1 = 10$
$2 \times 2 = 4$	$3 \times 2 = 6$	$8 \times 2 = 16$	$10 \times 2 = 20$
$2 \times 3 = 6$	$3 \times 3 = 9$	$8 \times 3 = 24$	$10 \times 3 = 30$
$2 \times 4 = 8$	$3 \times 4 = 12$	$8 \times 4 = 32$	$10 \times 4 = 40$
$2 \times 5 = 10$	$3 \times 5 = 15$	$8 \times 5 = 40$	$10 \times 5 = 50$
⋮	⋮	⋮	⋮

★ **PRACTICE SET A**

Find the first five multiples of the following numbers.

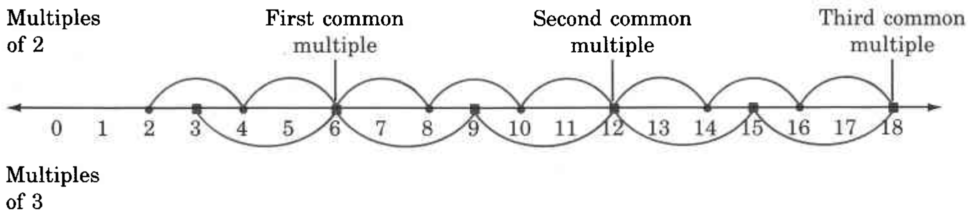
1. 4 **2.** 5 **3.** 6 **4.** 7 **5.** 9

☐ COMMON MULTIPLES

There will be times when we are given two or more whole numbers and we will need to know if there are any multiples that are common to each of them. If there are, we will need to know what they are. For example, some of the multiples that are common to 2 and 3 are 6, 12, and 18.

☆ SAMPLE SET B

We can visualize common multiples using the number line.



Notice that the common multiples can be divided by *both* whole numbers.

★ PRACTICE SET B

Find the first five common multiples of the following numbers.

1. 2 and 4 **2.** 3 and 4 **3.** 2 and 5 **4.** 3 and 6 **5.** 4 and 5

☐ THE LEAST COMMON MULTIPLE (LCM)

Notice that in our number line visualization of common multiples (above), the first common multiple is also the smallest, or **least common multiple,** abbreviated by **LCM.**

Least Common Multiple

The **least common multiple,** LCM, of two or more whole numbers is the smallest whole number that each of the given numbers will divide into without a remainder.

The least common multiple will be extremely useful in working with fractions (Chapter 4).

☐ FINDING THE LEAST COMMON MULTIPLE

Finding the LCM

To find the LCM of two or more numbers:

1. Write the prime factorization of each number, using exponents on repeated factors.
2. Write each base that appears in each of the prime factorizations.
3. To each base, attach the *largest exponent* that appears on it in the prime factorizations.
4. The LCM is the product of the numbers found in step 3.

There are some major differences between using the processes for obtaining the GCF and the LCM that we must note carefully:

The Difference Between the Processes for Obtaining the GCF and the LCM

1. Notice the difference between step 2 for the LCM and step 2 for the GCF. For the GCF, we use only the bases that are *common* in the prime factorizations, whereas for the LCM, we use *each* base that appears in the prime factorizations.
2. Notice the difference between step 3 for the LCM and step 3 for the GCF. For the GCF, we attach the *smallest* exponents to the common bases, whereas for the LCM, we attach the *largest* exponents to the bases.

☆ SAMPLE SET C

Find the LCM of the following numbers.

1. 9 and 12

1. $9 = 3 \cdot 3 = 3^2$
   $12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
2. The bases that appear in the prime factorizations are 2 and 3.
3. The *largest exponents* appearing on 2 and 3 in the prime factorizations are, respectively, 2 and 2:

   2^2 from 12.
   3^2 from 9.
4. The LCM is the product of these numbers.

   $\text{LCM} = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$

Thus, 36 is the smallest number that both 9 and 12 divide into without remainders.

2. 90 and 630

1. $90 = 2 \cdot 45 = 2 \cdot 3 \cdot 15 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$
   $630 = 2 \cdot 315 = 2 \cdot 3 \cdot 105 = 2 \cdot 3 \cdot 3 \cdot 35 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$
   $= 2 \cdot 3^2 \cdot 5 \cdot 7$
2. The bases that appear in the prime factorizations are 2, 3, 5, and 7.
3. The largest exponents that appear on 2, 3, 5, and 7 are, respectively, 1, 2, 1, and 1:

   2^1 from either 90 or 630.
   3^2 from either 90 or 630.
   5^1 from either 90 or 630.
   7^1 from 630.
4. The LCM is the product of these numbers.

   $\text{LCM} = 2 \cdot 3^2 \cdot 5 \cdot 7 = 2 \cdot 9 \cdot 5 \cdot 7 = 630$

Thus, 630 is the smallest number that both 90 and 630 divide into with no remainders.

3. 33, 110, and 484

1. $33 = 3 \cdot 11$
   $110 = 2 \cdot 55 = 2 \cdot 5 \cdot 11$
   $484 = 2 \cdot 242 = 2 \cdot 2 \cdot 121 = 2 \cdot 2 \cdot 11 \cdot 11 = 2^2 \cdot 11^2$.
2. The bases that appear in the prime factorizations are 2, 3, 5, and 11.
3. The largest exponents that appear on 2, 3, 5, and 11 are, respectively, 2, 1, 1, and 2:

   2^2 from 484.
   3^1 from 33.
   5^1 from 110
   11^2 from 484.

Continued

4. The LCM is the product of these numbers.

$$\begin{aligned} \text{LCM} &= 2^2 \cdot 3 \cdot 5 \cdot 11^2 \\ &= 4 \cdot 3 \cdot 5 \cdot 121 \\ &= 7260 \end{aligned}$$

Thus, 7260 is the smallest number that 33, 110, and 484 divide into without remainders.

★ PRACTICE SET C

Find the LCM of the following numbers.

1. 20 and 54

2. 14 and 28

3. 6 and 63

4. 28, 40, and 98

5. 16, 27, 125, and 363

Answers to Practice Sets are on p. 126.

Section 3.5 EXERCISES

For problems 1–45, find the least common multiple of the numbers.

1. 8 and 12

2. 6 and 15

3. 8 and 10

4. 10 and 14

5. 4 and 6

6. 6 and 12

7. 9 and 18

8. 6 and 8

9. 5 and 6

10. 7 and 8

11. 3 and 4

12. 2 and 9

13. 7 and 9

14. 28 and 36

15. 24 and 36

16. 28 and 42

17. 240 and 360

18. 162 and 270

19. 20 and 24

20. 25 and 30

21. 24 and 54

22. 16 and 24

23. 36 and 48

24. 24 and 40

25. 15 and 21

26. 50 and 140

27. 7, 11, and 33

28. 8, 10, and 15

29. 18, 21, and 42

30. 4, 5, and 21

31. 45, 63, and 98

32. 15, 25, and 40

33. 12, 16, and 20

34. 84 and 96

35. 48 and 54

36. 12, 16, and 24

37. 12, 16, 24, and 36

38. 6, 9, 12, and 18

39. 8, 14, 28, and 32

40. 18, 80, 108, and 490

41. 22, 27, 130, and 225

42. 38, 92, 115, and 189

43. 8 and 8

44. 12, 12, and 12

45. 3, 9, 12, and 3

EXERCISES FOR REVIEW

(1.3) **46.** Round 434,892 to the nearest ten thousand.

(1.5) **47.** How much bigger is 14,061 than 7,509?

(2.3) **48.** Find the quotient. 22,428 ÷ 14.

(3.1) **49.** Expand 84^3. Do not find the value.

(3.4) **50.** Find the greatest common factor of 48 and 72.

★ ANSWERS TO PRACTICE SETS (3.5)

A. **1.** 4, 8, 12, 16, 20 **2.** 5, 10, 15, 20, 25 **3.** 6, 12, 18, 24, 30 **4.** 7, 14, 21, 28, 35 **5.** 9, 18, 27, 36, 45

B. **1.** 4, 8, 12, 16, 20 **2.** 12, 24, 36, 48, 60 **3.** 10, 20, 30, 40, 50 **4.** 6, 12, 18, 24, 30 **5.** 20, 40, 60, 80, 100

C. **1.** 540 **2.** 28 **3.** 126 **4.** 1,960 **5.** 6,534,000

Chapter 3 SUMMARY OF KEY CONCEPTS

Exponential Notation **(3.1)**	*Exponential notation* is a description of repeated multiplication.
Exponent **(3.1)**	An *exponent* records the number of identical factors repeated in a multiplication.
	In a number such as 7^3.
Base Exponent Power **(3.1)**	7 is called the *base.* 3 is called the *exponent,* or power. 7^3 is read "seven to the third power," or "seven cubed."
Squared Cubed **(3.1)**	A number raised to the second power is often called *squared.* A number raised to the third power is often called *cubed.*
Root **(3.1)**	In mathematics, the word *root* is used to indicate that, through repeated multiplication, one number is the source of another number.
The Radical Sign $\sqrt{\ }$ **(3.1)**	The symbol $\sqrt{\quad}$ is called a *radical sign* and indicates the square root of a number. The symbol $\sqrt[n]{\quad}$ represents the *n*th root.
Radical Index Radicand **(3.1)**	An expression such as $\sqrt[4]{16}$ is called a *radical* and 4 is called the *index.* The number 16 is called the *radicand.*
Grouping Symbols **(3.2)**	Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are Parentheses: ( ) Brackets: [ ] Braces: { } Bar: ___
Order of Operations **(3.2)**	1. Perform all operations inside grouping symbols, beginning with the innermost set, in the order of 2, 3, and 4 below. 2. Perform all exponential and root operations, moving left to right. 3. Perform all multiplications and division, moving left to right. 4. Perform all additions and subtractions, moving left to right.
One Number as the Factor of Another **(3.3)**	A first number is a factor of a second number if the first number divides into the second number a whole number of times.
Prime Number **(3.3)**	A whole number greater than one whose only factors are itself and 1 is called a *prime number.* The whole number 1 is not a prime number. The whole number 2 is the first prime number and the only even prime number.
Composite Number **(3.3)**	A whole number greater than one that is composed of factors other than itself and 1 is called a *composite number.*
Fundamental Principle of Arithmetic **(3.3)**	Except for the order of factors, every whole number other than 1 can be written in one and only one way as a product of prime numbers.
Prime Factorization **(3.3)**	The prime factorization of 45 is $3 \cdot 3 \cdot 5$. The numbers that occur in this factorization of 45 are each prime.
Determining the Prime Factorization of a Whole Number **(3.3)**	There is a simple method, based on division by prime numbers, that produces the prime factorization of a whole number. For example, we determine the prime factorization of 132 as follows.

$$\begin{array}{l} 2\,\underline{|132} \\ \quad 2\,\underline{|66} \\ \qquad 3\,\underline{|33} \\ \qquad\quad 11 \end{array}$$

The prime factorization of 132 is $2 \cdot 2 \cdot 3 \cdot 11 = 2^2 \cdot 3 \cdot 11$.

Common Factor (3.4)

A factor that occurs in each number of a group of numbers is called a *common factor.*

3 is a common factor to the group 18, 6, and 45

Greatest Common Factor (GCF) (3.4)

The largest common factor of a group of whole numbers is called the *greatest common factor.* For example, to find the greatest common factor of 12 and 20,

1. Write the prime factorization of each number.

   $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
   $60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$

2. Write each base that is common to each of the numbers:

   2 and 3

3. The smallest exponent appearing on 2 is 2.
   The smallest exponent appearing on 3 is 1.
4. The GCF of 12 and 60 is the product of the numbers 2^2 and 3.

   $2^2 \cdot 3 = 4 \cdot 3 = 12$

Thus, 12 is the largest number that divides both 12 and 60 without a remainder.

Finding the GCF (3.4)

There is a simple method, based on prime factorization, that determines the GCF of a group of whole numbers.

Multiple (3.5)

When a whole number is multiplied by all other whole numbers, with the exception of zero, the resulting individual products are called *multiples* of that whole number. Some multiples of 7 are 7, 14, 21, and 28.

Common Multiples (3.5)

Multiples that are common to a group of whole numbers are called *common multiples.* Some common multiples of 6 and 9 are 18, 36, and 54.

The LCM (3.5)

The *least common multiple* (LCM) of a group of whole numbers is the smallest whole number that each of the given whole numbers divides into without a remainder. The least common multiple of 9 and 6 is 18.

Finding the LCM (3.5)

There is a simple method, based on prime factorization, that determines the LCM of a group of whole numbers. For example, the least common multiple of 28 and 72 is found in the following way.

1. Write the prime factorization of each number.

   $28 = 2 \cdot 2 \cdot 7 = 2^2 \cdot 7$
   $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

2. Write each base that appears in each of the prime factorizations, 2, 3, and 7.
3. To each of the bases listed in step 2, attach the *largest* exponent that appears on it in the prime factorization.

   2^3, 3^2, and 7

4. The LCM is the product of the numbers found in step 3.

   $2^3 \cdot 3^2 \cdot 7 = 8 \cdot 9 \cdot 7 = 504$

Thus, 504 is the smallest number that both 28 and 72 will divide into without a remainder.

The Difference Between the GCF and the LCM (3.5)

The GCF of two or more whole numbers is the largest number that divides into each of the given whole numbers.
The LCM of two or more whole numbers is the smallest whole number that each of the given numbers divides into without a remainder.

EXERCISE SUPPLEMENT

Section 3.1

For problems 1–25, determine the value of each power and root.

1. 3^3
2. 4^3
3. 0^5
4. 1^4
5. 12^2
6. 7^2
7. 8^2
8. 11^2
9. 2^5
10. 3^4
11. 15^2
12. 20^2
13. 25^2
14. $\sqrt{36}$
15. $\sqrt{225}$
16. $\sqrt[3]{64}$
17. $\sqrt[4]{16}$
18. $\sqrt{0}$
19. $\sqrt[3]{1}$
20. $\sqrt[3]{216}$
21. $\sqrt{144}$
22. $\sqrt{196}$
23. $\sqrt{1}$
24. $\sqrt[4]{0}$
25. $\sqrt[6]{64}$

Section 3.2

For problems 26–45, use the order of operations to determine each value.

26. $2^3 - 2 \cdot 4$
27. $5^2 - 10 \cdot 2 - 5$
28. $\sqrt{81} - 3^2 + 6 \cdot 2$
29. $15^2 + 5^2 \cdot 2^2$
30. $3 \cdot (2^2 + 3^2)$
31. $64 \cdot (3^2 - 2^3)$
32. $\dfrac{5^2+1}{13} + \dfrac{3^3+1}{14}$
33. $\dfrac{6^2-1}{5 \cdot 7} - \dfrac{49+7}{2 \cdot 7}$
34. $\dfrac{2 \cdot [3 + 5(2^2+1)]}{5 \cdot 2^3 - 3^2}$
35. $\dfrac{3^2 \cdot [2^5 - 1^4(2^3+25)]}{2 \cdot 5^2 + 5 + 2}$
36. $\dfrac{(5^2 - 2^3) - 2 \cdot 7}{2^2 - 1} + 5 \cdot \left[\dfrac{3^2-3}{2} + 1\right]$
37. $(8-3)^2 + (2+3^2)^2$
38. $3^2 \cdot (4^2 + \sqrt{25}) + 2^3 \cdot (\sqrt{81} - 3^2)$
39. $\sqrt{16+9}$
40. $\sqrt{16} + \sqrt{9}$
41. Compare the results of problems 39 and 40. What might we conclude?
42. $\sqrt{18 \cdot 2}$
43. $\sqrt{6 \cdot 6}$
44. $\sqrt{7 \cdot 7}$
45. $\sqrt{8 \cdot 8}$
46. An ________ records the number of identical factors that are repeated in a multiplication.

Section 3.3

For problems 47–53, find all the factors of each number.

47. 18
48. 24
49. 11
50. 12
51. 51
52. 25
53. 2
54. What number is the smallest prime number?

For problems 55–64, write each number as a product of prime factors.

55. 55
56. 20
57. 80
58. 284
59. 700
60. 845
61. 1,614
62. 921
63. 29
64. 37

Section 3.4

For problems 65–75, find the greatest common factor of each collection of numbers.

65. 5 and 15
66. 6 and 14
67. 10 and 15
68. 6, 8, and 12
69. 18 and 24
70. 42 and 54
71. 40 and 60
72. 18, 48, and 72
73. 147, 189, and 315
74. 64, 72, and 108
75. 275, 297, and 539

Section 3.5

For problems 76–86, find the least common multiple of each collection of numbers.

76. 5 and 15
77. 6 and 14
78. 10 and 15
79. 36 and 90

80. 42 and 54

81. 8, 12, and 20

82. 40, 50, and 180

83. 135, 147, and 324

84. 108, 144, and 324

85. 5, 18, 25, and 30

86. 12, 15, 18, and 20

87. Find all divisors of 24.

88. Find all factors of 24.

89. Write all divisors of $2^3 \cdot 5^2 \cdot 7$.

90. Write all divisors of $6 \cdot 8^2 \cdot 10^3$.

91. Does 7 divide $5^3 \cdot 6^4 \cdot 7^2 \cdot 8^5$?

92. Does 13 divide $8^3 \cdot 10^2 \cdot 11^4 \cdot 13^2 \cdot 15$?

Chapter 3 PROFICIENCY EXAM

1. ______

1. **(3.1)** In the number 8^5, write the names used for the number 8 and the number 5.

2. **(3.1)** Write using exponents.

$12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

2. ______

3. **(3.1)** Expand 9^4.

3. ______

For problems 4–15, determine the value of each expression.

4. **(3.2)** 4^3

4. ______

5. **(3.2)** 1^5

5. ______

6. **(3.2)** 0^3

6. ______

7. **(3.2)** 2^6

7. ______

8. ______

8. **(3.2)** $\sqrt{49}$

9. ______

9. **(3.2)** $\sqrt[3]{27}$

10. ____________

10. **(3.2)** $\sqrt[8]{1}$

11. ____________

11. **(3.2)** $16 + 2 \cdot (8 - 6)$

12. ____________

12. **(3.2)** $5^3 - \sqrt{100} + 8 \cdot 2 - 20 \div 5$

13. ____________

13. **(3.2)** $3 \cdot \dfrac{8^2 - 2 \cdot 3^2}{5^2 - 2} \cdot \dfrac{6^3 - 4 \cdot 5^2}{29}$

14. ____________

14. **(3.2)** $\dfrac{20 + 2^4}{2^3 \cdot 2 - 5 \cdot 2} + \dfrac{5 \cdot 7 - \sqrt{81}}{7 + 3 \cdot 2}$

15. ____________

15. **(3.2)** $[(8 - 3)^2 + (33 - 4\sqrt{49})] - 2[(10 - 3^2) + 9] - 5$

16. ____________

For problems 16–20, find the prime factorization of each whole number. If the number is prime, write "prime."

16. **(3.3)** 18

17. ____________

17. **(3.3)** 68

18. ______________

18. **(3.3)** 142

19. ______________

19. **(3.3)** 151

20. **(3.3)** 468

20. ______________

21. ______________

For problems 21 and 22, find the greatest common factor.

21. **(3.4)** 200 and 36

22. ______________

22. **(3.4)** 900 and 135

23. ______________

23. **(3.4)** Write all the factors of 36.

24. ______________

24. **(3.4)** Write all the divisors of 18.

25. ______________

25. **(3.4)** Does 7 divide into $5^2 \cdot 6^3 \cdot 7^4 \cdot 8$? Explain.

26. ______________

26. **(3.4)** Is 3 a factor of $2^6 \cdot 3^2 \cdot 5^3 \cdot 4^6$? Explain.

27. ______________

27. **(3.4)** Does 13 divide into $11^3 \cdot 12^4 \cdot 15^2$? Explain.

28. ______________

For problems 28 and 29, find the least common multiple.

28. **(3.5)** 432 and 180

29. ______________

29. **(3.5)** 28, 40, and 95