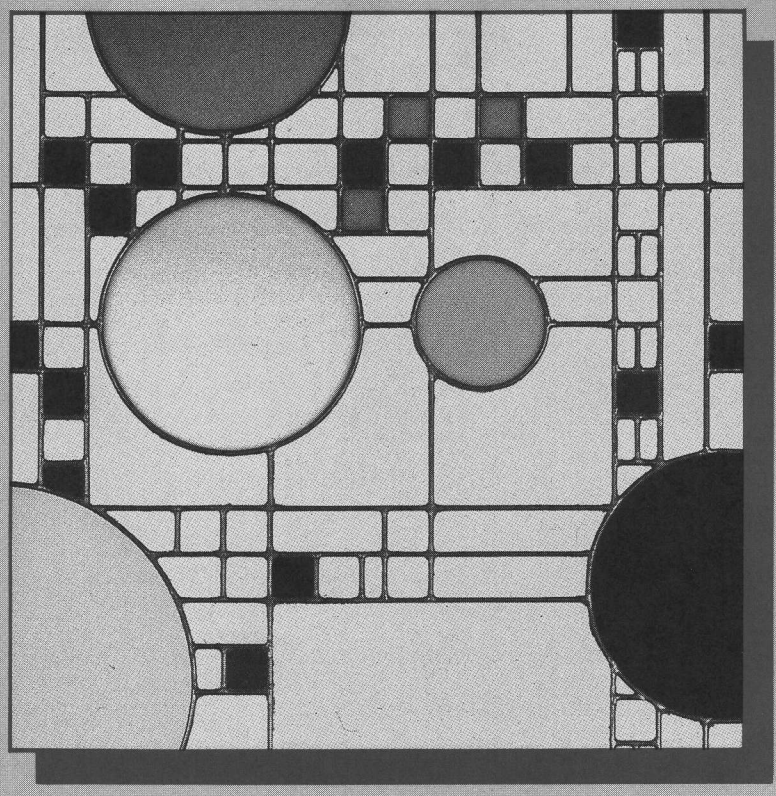
2 Multiplication and Division of Whole Numbers



After completing this chapter, you should

Section 2.1 Multiplication of Whole Numbers
- understand the process of multiplication
- be able to multiply whole numbers
- be able to simplify multiplications with numbers ending in zero
- be able to use a calculator to multiply one whole number by another

Section 2.2 Concepts of Division of Whole Numbers
- understand the process of division
- understand division of a nonzero number into zero
- understand why division by zero is undefined
- be able to use a calculator to divide one whole number by another

Section 2.3 Division of Whole Numbers
- be able to divide a whole number by a single or multiple digit divisor
- be able to interpret a calculator statement that a division results in a remainder

Section 2.4 Some Interesting Facts about Division
- be able to recognize a whole number that is divisible by 2, 3, 4, 5, 6, 8, 9, or 10

Section 2.5 Properties of Multiplication
- understand and appreciate the commutative and associative properties of multiplication
- understand why 1 is the multiplicative identity

2.1 Multiplication of Whole Numbers

Section Overview

- ❑ MULTIPLICATION
- ❑ THE MULTIPLICATION PROCESS WITH A SINGLE DIGIT MULTIPLIER
- ❑ THE MULTIPLICATION PROCESS WITH A MULTIPLE DIGIT MULTIPLIER
- ❑ MULTIPLICATIONS WITH NUMBERS ENDING IN ZERO
- ❑ CALCULATORS

❑ MULTIPLICATION

Multiplication is a description of repeated addition.

In the addition of

$5 + 5 + 5$

the number 5 is repeated **3 times.** Therefore, we say we have **three times five** and describe it by writing

3×5

Thus,

$3 \times 5 = 5 + 5 + 5$

Multiplicand

Multiplier

In a multiplication, the repeated addend (number being added) is called the **multiplicand.** In 3×5, the 5 is the multiplicand. Also, in a multiplication, the number that records the number of times the multiplicand is used is called the **multiplier.** In 3×5, the 3 is the multiplier.

☆ SAMPLE SET A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

1. $7 + 7 + 7 + 7 + 7 + 7$

6×7. Multiplier is 6. Multiplicand is 7.

2. $18 + 18 + 18$

3×18. Multiplier is 3. Multiplicand is 18.

★ PRACTICE SET A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

1. $12 + 12 + 12 + 12$

_____. Multiplier is _____. Multiplicand is _____.

2. $36 + 36 + 36 + 36 + 36 + 36 + 36 + 36$

_____. Multiplier is _____. Multiplicand is _____.

3. $0+0+0+0+0$

_____. Multiplier is _____. Multiplicand is _____.

4. $\underbrace{1847+1847+\cdots+1847}_{\text{12,000 times}}$

_____. Multiplier is _____. Multiplicand is _____.

Factors
Product

In a multiplication, the numbers being multiplied are also called **factors.** The result of a multiplication is called the **product.** In $3 \times 5 = 15$, the 3 and 5 are not only called the multiplier and multiplicand, but they are also called factors. The product is 15.

Indicators of Multiplication
$\times$, $\cdot$, ( )

The multiplication symbol ($\times$) is not the only symbol used to indicate multiplication. Other symbols include the dot ($\cdot$) and pairs of parentheses ( ). The expressions

$3 \times 5, \quad 3 \cdot 5, \quad 3(5), \quad (3)5, \quad (3)(5)$

all represent the same product.

❑ THE MULTIPLICATION PROCESS WITH A SINGLE DIGIT MULTIPLIER

Since multiplication is repeated addition, we should not be surprised to notice that **carrying** can occur. Carrying occurs when we find the product of 38 and 7.

$$\begin{array}{r} {\scriptstyle 5} \\ 38 \\ \times\ \ 7 \\ \hline 266 \end{array}$$

First, we compute $7 \times 8 = 56$. Write the 6 in the ones column. Carry the 5. Then take $7 \times 3 = 21$. Add to 21 the 5 that was carried: $21 + 5 = 26$. The product is 266.

☆ SAMPLE SET B

Find the following products.

1.
$$\begin{array}{r} {\scriptstyle 1} \\ 64 \\ \times\ \ 3 \\ \hline 192 \end{array}$$

$3 \times 4 = 12$. Write the 2, carry the 1.

$3 \times 6 = 18$. Add to 18 the 1 that was carried: $18 + 1 = 19$.

The product is 192.

2.
$$\begin{array}{r} {\scriptstyle 13} \\ 526 \\ \times\ \ 5 \\ \hline 2{,}630 \end{array}$$

$5 \times 6 = 30$. Write the 0, carry the 3.

$5 \times 2 = 10$. Add to 10 the 3 that was carried: $10 + 3 = 13$. Write the 3, carry the 1.

$5 \times 5 = 25$. Add to 25 the 1 that was carried: $25 + 1 = 6$.

The product is 2,630.

Continued

3.
$$\begin{array}{r} {\scriptstyle 7\ \ 3} \\ 1{,}804 \\ \times\quad 9 \\ \hline 16{,}236 \end{array}$$

$9 \times 4 = 36$. Write the 6, carry the 3.
$9 \times 0 = 0$. Add to the 0 the 3 that was carried. $0 + 3 = 3$. Write the 3.
$9 \times 8 = 72$. Write the 2, carry the 7.
$9 \times 1 = 9$. Add to the 9 the 7 that was carried. $9 + 7 = 16$. Since there are no more multiplications to perform, write both the 1 and 6.

The product is 16,236.

★ PRACTICE SET B

Find the following products.

1. $\begin{array}{r} 37 \\ \times\ 5 \\ \hline \end{array}$ **2.** $\begin{array}{r} 78 \\ \times\ 8 \\ \hline \end{array}$ **3.** $\begin{array}{r} 536 \\ \times\ 7 \\ \hline \end{array}$ **4.** $\begin{array}{r} 40{,}019 \\ \times\quad 8 \\ \hline \end{array}$ **5.** $\begin{array}{r} 301{,}599 \\ \times\quad 3 \\ \hline \end{array}$

❑ THE MULTIPLICATION PROCESS WITH A MULTIPLE DIGIT MULTIPLIER

In a multiplication in which the multiplier is composed of two or more digits, the *multiplication must take place in parts.* The process is as follows:

First Partial Product

Part 1: Multiply the multiplicand by the ones digit of the multiplier. This product is called the **first partial product.**

Second Partial Product

Part 2: Multiply the multiplicand by the tens digit of the multiplier. This product is called the **second partial product.** Since the tens digit is used as a factor, the second partial product is written below the first partial product so that its rightmost digit appears in the tens column.

Part 3: If necessary, continue this way finding partial products. Write each one below the previous one so that the rightmost digit appears in the column directly below the digit that was used as a factor.

Total Product

Part 4: Add the partial products to obtain the **total product.**

Note: It may be necessary to carry when finding each partial product.

☆ SAMPLE SET C

1. Multiply 326 by 48.

Part 1:
$$\begin{array}{r} {\scriptstyle 24} \\ 326 \\ \times\ 48 \\ \hline 2608 \end{array}$$
← First partial product.

Part 2:
$$\begin{array}{r} {\scriptstyle 12} \\ {\scriptstyle 24} \\ 326 \\ \times\ 48 \\ \hline 2608 \\ 1304 \end{array}$$
← Second partial product.

Part 3: This step is unnecessary since all of the digits in the multiplier have been used.

Part 4: Add the partial products to obtain the total product.

```
  12
  24
 326
×  48
2608
+1304
15648   ← Total product.
```

The product is 15,648.

2. Multiply 5,369 by 842.

Part 1:

```
  11
5369
×842
10738   ← First partial product.
```

Part 2:

```
 123
  11
5369
× 842
10738
21476   ← Second partial product.
```

Part 3:

```
 257
 123
  11
5369
× 842
10738
21476
42952   ← Third partial product.
4520698 ← Total product (Part 4).
```

The product is 4,520,698.

3. Multiply 1,508 by 206.

Part 1:

```
 3 4
1508
× 206
9048    ← First partial product (in first column from the right).
```

Part 2:

```
 3 4
1508
× 206
9048
```

Since 0 times 1508 is 0, the partial product will not change the identity of the total product (which is obtained by addition).
Go to the next partial product.

Part 3:

```
 1 1
 3 4
1508
× 206
9048
3016    ← Third partial product (in third column from the right).
310648  ← Total product (Part 4).
```

The product is 310,648

★ PRACTICE SET C

1. Multiply 73 by 14.

2. Multiply 86 by 52.

3. Multiply 419 by 85.

4. Multiply 2,376 by 613.

5. Multiply 8,107 by 304.

6. Multiply 66,260 by 1,008.

7. Multiply 209 by 501.

8. Multiply 24 by 10.

9. Multiply 3,809 by 1,000.

10. Multiply 813 by 10,000.

❑ MULTIPLICATIONS WITH NUMBERS ENDING IN ZERO

Often, when performing a multiplication, one or both of the factors will end in zeros. Such multiplications can be done quickly by aligning the numbers so that the rightmost nonzero digits are in the same column.

☆ SAMPLE SET D

Perform the multiplication (49,000)(1,200).

$$(49{,}000)(1{,}200) = \begin{array}{r} 49000 \\ \times\ 1200 \\ \hline \end{array}$$

Since 9 and 2 are the rightmost nonzero digits, put them in the same column.

$$\begin{array}{r} 49000 \\ \times 1200 \\ \hline \end{array}$$

Draw (perhaps mentally) a vertical line to separate the zeros from the nonzeros.

$$\begin{array}{r} 49\,|\,000 \\ \times 12\,|\,00 \\ \hline \end{array}$$

Multiply the numbers to the left of the vertical line as usual, then attach to the right end of this product the total number of zeros.

$$\begin{array}{r} 49\,|\,000 \\ \times\ 12\,|\,00 \\ \hline 98 \\ 49 \\ \hline 588\,|\,00000 \end{array}$$

Attach these 5 zeros to 588.

The product is 58,800,000.

★ PRACTICE SET D

1. Multiply 1,800 by 90.
2. Multiply 420,000 by 300.
3. Multiply 20,500,000 by 140,000.

❑ CALCULATORS

Most multiplications are performed using a calculator.

☆ SAMPLE SET E

1. Multiply 75,891 by 263.

		Display Reads
Type	75891	75891
Press	×	75891
Type	263	263
Press	=	19959333

The product is 19,959,333.

2. Multiply 4,510,000,000,000 by 1,700.

		Display Reads
Type only	451	451
Press	×	451
Type only	17	17
Press	=	7667

The display now reads 7667. We'll have to add the zeros ourselves. There are a total of 12 zeros. Attaching 12 zeros to 7667, we get 7,667,000,000,000,000.

The product is 7,667,000,000,000,000.

3. Multiply 57,847,298 by 38,976.

		Display Reads
Type	57847298	57847298
Press	×	57847298
Type	38976	38976
Press	=	2.2546563 12

The display now reads 2.2546563 12. What kind of number is this? This is an example of a whole number written in **scientific notation.** We'll study this concept when we get to decimal numbers.

★ PRACTICE SET E

Use a calculator to perform each multiplication.

1. 52×27 **2.** $1{,}448 \times 6{,}155$ **3.** $8{,}940{,}000 \times 205{,}000$

Answers to Practice Sets are on p. 65.

Section 2.1 EXERCISES

For problems 1–55, perform the multiplications. You may check each product with a calculator.

1. $\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$

2. $\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$

3. $\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$

4. $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$

5. 6×1

6. 4×5

7. 75×3

8. 35×5

9. $\begin{array}{r} 45 \\ \times\ 6 \\ \hline \end{array}$

10. $\begin{array}{r} 31 \\ \times\ 7 \\ \hline \end{array}$

11. $\begin{array}{r} 97 \\ \times\ 6 \\ \hline \end{array}$

12. $\begin{array}{r} 75 \\ \times 57 \\ \hline \end{array}$

13. $\begin{array}{r} 64 \\ \times 15 \\ \hline \end{array}$

14. $\begin{array}{r} 73 \\ \times 15 \\ \hline \end{array}$

15. $\begin{array}{r} 81 \\ \times 95 \\ \hline \end{array}$

16. $\begin{array}{r} 31 \\ \times 33 \\ \hline \end{array}$

17. 57×64

18. 76×42

19. 894×52

20. 684×38

21. $\begin{array}{r} 115 \\ \times\ 22 \\ \hline \end{array}$

22. $\begin{array}{r} 706 \\ \times\ 81 \\ \hline \end{array}$

23. $\begin{array}{r} 328 \\ \times\ 21 \\ \hline \end{array}$

24. $\begin{array}{r} 550 \\ \times\ 94 \\ \hline \end{array}$

25. 930×26

26. 318×63

27. $\begin{array}{r} 582 \\ \times 127 \\ \hline \end{array}$

28. $\begin{array}{r} 247 \\ \times 116 \\ \hline \end{array}$

29. $\begin{array}{r} 305 \\ \times 225 \\ \hline \end{array}$

30. $\begin{array}{r} 782 \\ \times 547 \\ \hline \end{array}$

31. $\begin{array}{r} 771 \\ \times 663 \\ \hline \end{array}$

32. $\begin{array}{r} 638 \\ \times 516 \\ \hline \end{array}$

33. $1{,}905 \times 710$

34. $5{,}757 \times 5{,}010$

35. $\begin{array}{r} 3{,}106 \\ \times 1{,}752 \\ \hline \end{array}$

36. $\begin{array}{r} 9{,}300 \\ \times 1{,}130 \\ \hline \end{array}$

37. $\begin{array}{r} 7{,}057 \\ \times 5{,}229 \\ \hline \end{array}$

38. $\begin{array}{r} 8{,}051 \\ \times 5{,}580 \\ \hline \end{array}$

39. $\begin{array}{r} 5{,}804 \\ \times 4{,}300 \\ \hline \end{array}$

40. $\begin{array}{r} 357 \\ \times 16 \\ \hline \end{array}$

41. $\begin{array}{r} 724 \\ \times 0 \\ \hline \end{array}$

42. $\begin{array}{r} 2{,}649 \\ \times 41 \\ \hline \end{array}$

43. $\begin{array}{r} 5{,}173 \\ \times 8 \\ \hline \end{array}$

44. $\begin{array}{r} 1{,}999 \\ \times 0 \\ \hline \end{array}$

45. $\begin{array}{r} 1{,}666 \\ \times 0 \\ \hline \end{array}$

46. $\begin{array}{r} 51{,}730 \\ \times 142 \\ \hline \end{array}$

47. $\begin{array}{r} 387 \\ \times 190 \\ \hline \end{array}$

48. $\begin{array}{r} 3{,}400 \\ \times 70 \\ \hline \end{array}$

49. $\begin{array}{r} 460{,}000 \\ \times 14{,}000 \\ \hline \end{array}$

50. $\begin{array}{r} 558{,}000{,}000 \\ \times 81{,}000 \\ \hline \end{array}$

51. $\begin{array}{r} 37{,}000 \\ \times 120 \\ \hline \end{array}$

52. $\begin{array}{r} 498{,}000 \\ \times 0 \\ \hline \end{array}$

53. $\begin{array}{r} 4{,}585{,}000 \\ \times 140 \\ \hline \end{array}$

54. $\begin{array}{r} 30{,}700{,}000 \\ \times 180 \\ \hline \end{array}$

55. $\begin{array}{r} 8{,}000 \\ \times 10 \\ \hline \end{array}$

56. Suppose a theater holds 426 people. If the theater charges \$4 per ticket and sells every seat, how much money would they take in?

57. In an English class, a student is expected to read 12 novels during the semester and prepare a report on each one of them. If there are 32 students in the class, how many reports will be prepared?

58. In a mathematics class, a final exam consists of 65 problems. If this exam is given to 28 people, how many problems must the instructor grade?

59. A business law instructor gives a 45 problem exam to two of her classes. If each class has 37 people in it, how many problems will the instructor have to grade?

60. An algebra instructor gives an exam that consists of 43 problems to four of his classes. If the classes have 25, 28, 31, and 35 students in them, how many problems will the instructor have to grade?

61. In statistics, the term "standard deviation" refers to a number that is calculated from certain data. If the data indicate that one standard deviation is 38 units, how many units is three standard deviations?

62. Soft drinks come in cases of 24 cans. If a supermarket sells 857 cases during one week, how many individual cans were sold?

63. There are 60 seconds in 1 minute and 60 minutes in 1 hour. How many seconds are there in 1 hour?

64. There are 60 seconds in 1 minute, 60 minutes in one hour, 24 hours in one day, and 365 days in one year. How many seconds are there in 1 year?

65. Light travels 186,000 miles in one second. How many miles does light travel in one year? (*Hint:* Can you use the result of the previous problem?)

66. An elementary school cafeteria sells 328 lunches every day. Each lunch costs \$1. How much money does the cafeteria bring in in 2 weeks?

67. A computer company is selling stock for \$23 a share. If 87 people each buy 55 shares, how much money would be brought in?

EXERCISES FOR REVIEW

(1.1) **68.** In the number 421,998, how may ten thousands are there?

(1.3) **69.** Round 448,062,187 to the nearest hundred thousand.

(1.4) **70.** Find the sum. 22,451 + 18,976.

(1.5) **71.** Subtract 2,289 from 3,001.

(1.6) **72.** Specify which property of addition justifies the fact that (a first whole number + a second whole number) = (the second whole number + the first whole number)

★ Answers to Practice Sets (2.1)

A. **1.** 4×12. Multiplier is 4. Multiplicand is 12. **2.** 8×36. Multiplier is 8. Multiplicand is 36. **3.** 5×0. Multiplier is 5. Multiplicand is 0. **4.** $12{,}000 \times 1{,}847$. Multiplier is 12,000. Multiplicand is 1,847.

B. **1.** 185 **2.** 624 **3.** 3,752 **4.** 320,152 **5.** 904,797

C. **1.** 1,022 **2.** 4,472 **3.** 35,615 **4.** 1,456,488 **5.** 2,464,528 **6.** 66,790,080 **7.** 104,709 **8.** 240 **9.** 3,809,000 **10.** 8,130,000

D. **1.** 162,000 **2.** 126,000,000 **3.** 2,870,000,000,000

E. **1.** 1,404 **2.** 8,912,440 **3.** 1,832,700,000,000

2.2 Concepts of Division of Whole Numbers

Section Overview

- ❑ DIVISION
- ❑ DIVISION INTO ZERO (ZERO AS A DIVIDEND: $\frac{0}{a}$, $a \neq 0$)
- ❑ DIVISION BY ZERO (ZERO AS A DIVISOR: $\frac{a}{0}$, $a \neq 0$)
- ❑ DIVISION BY AND INTO ZERO (ZERO AS A DIVIDEND AND DIVISOR: $\frac{0}{0}$)
- ❑ CALCULATORS

❑ DIVISION

Division is a description of repeated subtraction.

In the process of division, the concern is how many times one number is contained in another number. For example, we might be interested in how many 5's are contained in 15. The word *times* is significant because it implies a relationship between division and multiplication.

There are several notations used to indicate division. Suppose Q records the number of times 5 is contained in 15. We can indicate this by writing

$\underbrace{5\overline{)15}^{\,Q}}_{\text{5 into 15}}$ $\qquad$ $\underbrace{\frac{15}{5} = Q}_{\text{15 divided by 5}}$

$\underbrace{15/5 = Q}_{\text{15 divided by 5}}$ $\qquad$ $\underbrace{15 \div 5 = Q}_{\text{15 divided by 5}}$

Each of these division notations describes the *same* number, represented here by the symbol Q. Each notation also converts to the same multiplication form. It is

$15 = 5 \times Q$

In division,

Dividend
Divisor
Quotient

1. the number being divided into is called the **dividend.**
2. the number dividing into the dividend is the **divisor.**
3. the result of the division is called the **quotient.**

$$\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

$$\text{dividend/divisor} = \text{quotient} \qquad \text{dividend} \div \text{divisor} = \text{quotient}$$

☆ SAMPLE SET A

Find the following quotients using multiplication facts.

1. $18 \div 6$

Since $6 \times 3 = 18$,

$18 \div 6 = 3$

Notice also that

$$\begin{array}{r} 18 \\ -\ 6 \\ \hline 12 \\ -\ 6 \\ \hline 6 \\ -\ 6 \\ \hline 0 \end{array} \Bigg\} \text{Repeated subtraction}$$

Thus, 6 is contained in 18 three times.

2. $\frac{24}{3}$

Since $3 \times 8 = 24$,

$\frac{24}{3} = 8$

Notice also that 3 could be subtracted exactly 8 times from 24. This implies that 3 is contained in 24 eight times.

3. $\frac{36}{6}$

Since $6 \times 6 = 36$,

$\frac{36}{6} = 6$

Thus, there are 6 sixes in 36.

4. $9\overline{)72}$

Since $9 \times 8 = 72$,

$$9\overline{)72}^{\,8}$$

Thus, there are 8 nines in 72.

★ PRACTICE SET A

Use multiplication facts to determine the following quotients.

1. $32 \div 8$ **2.** $18 \div 9$ **3.** $\frac{25}{5}$ **4.** $\frac{48}{8}$ **5.** $\frac{28}{7}$ **6.** $4\overline{)36}$

❑ DIVISION INTO ZERO (ZERO AS A DIVIDEND: $\frac{0}{a}$, $a \neq 0$)

Let's look at what happens when the dividend (the number being divided into) is zero, and the divisor (the number doing the dividing) is any whole number except zero. The question is

What number, if any, is $\frac{0}{\text{any nonzero whole number}}$?

Let's represent this unknown quotient by Q. Then,

$$\frac{0}{\text{any nonzero whole number}} = Q$$

Converting this division problem to its corresponding multiplication problem, we get

$0 = Q \times$ (any nonzero whole number)

From our knowledge of multiplication, we can understand that if the product of two whole numbers is zero, then one or both of the whole numbers must be zero. Since any nonzero whole number is certainly not zero, Q must represent zero. Then,

$$\frac{0}{\text{any nonzero whole number}} = 0$$

Zero Divided By Any Nonzero Whole Number Is Zero

Zero divided by any nonzero whole number is zero.

❑ DIVISON BY ZERO (ZERO AS A DIVISOR: $\frac{a}{0}$, $a \neq 0$)

Now we ask,

What number, if any, is $\frac{\text{any nonzero whole number}}{0}$?

Letting Q represent a possible quotient, we get

$$\frac{\text{any nonzero whole number}}{0} = Q$$

Converting to the corresponding multiplication form, we have

(any nonzero whole number) $= Q \times 0$

Since $Q \times 0 = 0$, (any nonzero whole number) $= 0$. But this is absurd. This would mean that $6 = 0$, or $37 = 0$. A nonzero whole number *cannot* equal 0! Thus,

$\frac{\text{any nonzero whole number}}{0}$ *does not* name a number

Division by Zero is Undefined

Division by zero does not name a number. It is, therefore, undefined.

❑ DIVISION BY AND INTO ZERO $\left(\text{ZERO AS A DIVIDEND AND DIVISOR: } \frac{0}{0}\right)$

We are now curious about zero divided by zero $\left(\frac{0}{0}\right)$. If we let Q represent a potential quotient, we get

$$\frac{0}{0} = Q$$

Converting to the multiplication form,

$$0 = Q \times 0$$

This results in

$$0 = 0$$

This is a statement that is true regardless of the number used in place of Q. For example,

$\frac{0}{0} = 5$, since $0 = \quad 5 \times 0$.

$\frac{0}{0} = 31$, since $0 = \quad 31 \times 0$.

$\frac{0}{0} = 286$, since $0 = 286 \times 0$.

A *unique* quotient cannot be determined.

Indeterminant

Since the result of the division is inconclusive, we say that $\frac{0}{0}$ is **indeterminant.**

$\frac{0}{0}$ is Indeterminant

The division $\frac{0}{0}$ is indeterminant.

☆ SAMPLE SET B

Perform, if possible, each division.

1. $\frac{19}{0}$. Since division by 0 does not name a whole number, no quotient exists, and we state

$\frac{19}{0}$ is undefined

2. $0\overline{)14}$. Since division by 0 does not name a defined number, no quotient exists, and we state

$0\overline{)14}$ is undefined

3. $9\overline{)0}$. Since division into 0 by any nonzero whole number results in 0, we have

$9\overline{)0}^{\,0}$

4. $\frac{0}{7}$. Since division into 0 by any nonzero whole number results in 0, we have

$\frac{0}{7} = 0$

★ PRACTICE SET B

Perform, if possible, the following divisions.

1. $\frac{5}{0}$ 2. $\frac{0}{4}$ 3. $0\overline{)0}$ 4. $0\overline{)8}$ 5. $\frac{9}{0}$ 6. $\frac{0}{1}$

❑ CALCULATORS

Divisions can also be performed using a calculator.

☆ SAMPLE SET C

1. Divide 24 by 3.

		Display Reads
Type	24	24
Press	÷	24
Type	3	3
Press	=	8

The display now reads 8, and we conclude that $24 \div 3 = 8$.

2. Divide 0 by 7.

		Display Reads
Type	0	0
Press	÷	0
Type	7	7
Press	=	0

The display now reads 0, and we conclude that $0 \div 7 = 0$.

3. Divide 7 by 0.

Since division by zero is undefined, the calculator should register some kind of error message.

		Display Reads
Type	7	7
Press	÷	7
Type	0	0
Press	=	Error

The error message indicates an undefined operation was attempted, in this case, division by zero.

★ PRACTICE SET C

Use a calculator to perform each division.

1. $35 \div 7$ **2.** $56 \div 8$ **3.** $0 \div 6$ **4.** $3 \div 0$ **5.** $0 \div 0$

Answers to Practice Sets are on p. 71.

Section 2.2 EXERCISES

For problems 1–25, determine the quotients (if possible). You may use a calculator to check the result.

1. $4\overline{)32}$

2. $7\overline{)42}$

3. $6\overline{)18}$

4. $2\overline{)14}$

5. $3\overline{)27}$

6. $1\overline{)6}$

7. $4\overline{)28}$

8. $\frac{30}{5}$

9. $\frac{16}{4}$

10. $24 \div 8$

11. $10 \div 2$

12. $21 \div 7$

13. $21 \div 3$

14. $0 \div 6$

15. $8 \div 0$

16. $12 \div 4$

17. $3\overline{)9}$

18. $0\overline{)0}$

19. $7\overline{)0}$

20. $6\overline{)48}$

21. $\frac{15}{3}$

22. $\frac{35}{0}$

23. $56 \div 7$

24. $\frac{0}{9}$

25. $72 \div 8$

26. Write $\frac{16}{2} = 8$ using three different notations.

27. Write $\frac{27}{9} = 3$ using three different notations.

28. In the statement $6\overline{)\overset{4}{24}}$

6 is called the _____,
24 is called the _____,
4 is called the _____.

29. In the statement $56 \div 8 = 7$,

7 is called the _____,
8 is called the _____,
56 is called the _____.

EXERCISES FOR REVIEW

(1.1) **30.** What is the largest digit?

(1.4) **31.** Find the sum.
$$\begin{array}{r} 8{,}006 \\ +4{,}118 \\ \hline \end{array}$$

(1.5) **32.** Find the difference.
$$\begin{array}{r} 631 \\ -589 \\ \hline \end{array}$$

(1.6) **33.** Use the numbers 2, 3, and 7 to illustrate the associative property of addition.

(2.1) **34.** Find the product.
$$\begin{array}{r} 86 \\ \times 12 \\ \hline \end{array}$$

★ ANSWERS TO PRACTICE SETS (2.2)

A. **1.** 4 **2.** 2 **3.** 5 **4.** 6 **5.** 4 **6.** 9

B. **1.** undefined **2.** 0 **3.** indeterminant **4.** undefined **5.** undefined **6.** 0

C. **1.** 5 **2.** 7 **3.** 0

4. An error message tells us that this operation is undefined. The particular message depends on the calculator.

5. An error message tells us that this operation cannot be performed. Some calculators actually set $0 \div 0$ equal to 1. We know better! $0 \div 0$ is indeterminant.

2.3 Division of Whole Numbers

Section Overview

- ❑ DIVISION WITH A SINGLE DIGIT DIVISOR
- ❑ DIVISION WITH A MULTIPLE DIGIT DIVISOR
- ❑ DIVISION WITH A REMAINDER
- ❑ CALCULATORS

❑ DIVISION WITH A SINGLE DIGIT DIVISOR

Our experience with multiplication of whole numbers allows us to perform such divisions as $75 \div 5$. We perform the division by performing the corresponding multiplication, $5 \times Q = 75$. Each division we considered in Section 2.2 had a one-digit quotient. Now we will consider divisions in which the quotient may consist of two or more digits. For example, $75 \div 5$.

Let's examine the division $75 \div 5$. We are asked to determine how many 5's are contained in 75. We'll approach the problem in the following way.

1. Make an educated guess based on experience with multiplication.
2. Find how close the estimate is by multiplying the estimate by 5.
3. If the product obtained in step 2 is less than 75, find out how much less by subtracting it from 75.
4. If the product obtained in step 2 is greater than 75, decrease the estimate until the product is less than 75. Decreasing the estimate makes sense because we do not wish to exceed 75.

We can suggest from this discussion that the process of division consists of

The Four Steps in Division

1. an educated guess
2. a multiplication
3. a subtraction
4. bringing down the next digit (if necessary)

The educated guess can be made by determining how many times the divisor is contained in the dividend by using only one or two digits of the dividend.

☆ SAMPLE SET A

1. Find $75 \div 5$.

$5\overline{)75}$ **Rewrite the problem using a division bracket.**

$\begin{array}{r} 10 \\ 5\overline{)75} \end{array}$ **Make an educated guess by noting that one 5 is contained in 75 at most 10 times.**
Since 7 is the tens digit, we estimate that 5 goes into 75 at most 10 times.

$\begin{array}{r} 10 \\ 5\overline{)\ 75} \\ -50 \\ \hline 25 \end{array}$ **Now determine how close the estimate is.**
10 fives is $10 \times 5 = 50$. Subtract 50 from 75.
Estimate the number of 5's in 25.
There are exactly 5 fives in 25.

$\left.\begin{array}{r} 5 \\ 10 \end{array}\right\}$ **10 fives + 5 fives = 15 fives.**
There are 15 fives contained in 75.

$\begin{array}{r} \\ 5\overline{)\ 75} \\ -50 \\ \hline 25 \\ -25 \\ \hline 0 \end{array}$

Check: $75 \stackrel{?}{=} 15 \times 5$
$75 \checkmark 75$

Thus, $75 \div 5 = 15$.

The notation in this division can be shortened by writing.

$\begin{array}{r} 15 \\ 5\overline{)75} \\ 5\downarrow \\ \hline 25 \\ 25 \\ \hline 0 \end{array}$

Divide: 5 goes into 7 at most 1 time.
Multiply: $1 \times 5 = 5$. Write 5 below 7.
Subtract: $7 - 5 = 2$. Bring down 5.

Divide: 5 goes into 25 exactly 5 times.
Multiply: $5 \times 5 = 25$. Write 25 below 25.
Subtract: $25 - 25 = 0$.

2. Find $4{,}944 \div 8$.

$8\overline{)4944}$ **Rewrite the problem using a division bracket.**

$\begin{array}{r} 600 \\ 8\overline{)\ 4944} \\ -4800 \\ \hline 144 \end{array}$ **8 goes into 49 at most 6 times, and 9 is in the hundreds column. We'll guess 600.**
Then, $8 \times 600 = 4800$.

```
      10
     600
8)  4944
   -4800
     144
   -  80
      64
```

8 goes into 14 at most 1 time, and 4 is in the tens column. We'll guess 10.

```
       8
      10
     600
8)  4944
   -4800
     144
   -  80
      64
   -  64
       0
```

8 goes into 64 exactly 8 times.
600 eights + 10 eights + 8 eights = 618 eights.

Check: $4944 \stackrel{?}{=} 8 \times 618$
$4944 = 4944$

Thus, $4{,}944 \div 8 = 618$.

As in the first problem, the notation in this division can be shortened by eliminating the subtraction signs and the zeros in each educated guess.

```
     618
8)4944
 48↓
  14
   8↓
   64
   64
    0
```

Divide: 8 goes into 49 at most 6 times.
Multiply: $6 \times 8 = 48$. Write 48 below 49.
Subtract: $49 - 48 = 1$. Bring down the 4.

Divide: 8 goes into 14 at most 1 time.
Multiply: $1 \times 8 = 8$. Write 8 below 14.
Subtract: $14 - 8 = 6$. Bring down the 4.

Divide: 8 goes into 64 exactly 8 times.
Multiply: $8 \times 8 = 64$. Write 64 below 64.
Subtract: $64 - 64 = 0$.

NOTE: Not all divisions end in zero. We will examine such divisions in a subsequent subsection.

★ PRACTICE SET A

Perform the following divisions.

1. $126 \div 7$ **2.** $324 \div 4$ **3.** $2{,}559 \div 3$ 4. $5{,}645 \div 5$ **5.** $757{,}125 \div 9$

❑ DIVISION WITH A MULTIPLE DIGIT DIVISOR

The process of division also works when the divisor consists of two or more digits. We now make educated guesses using the first digit of the divisor and one or two digits of the dividend.

☆ SAMPLE SET B

1. Find 2,232 ÷ 36.

$36\overline{)2232}$

Use the first digit of the divisor and the first two digits of the dividend to make the educated guess.

3 goes into 22 at most 7 times.
Try 7: $7 \times 36 = 252$ which is greater than 223. Reduce the estimate.
Try 6: $6 \times 36 = 216$ which is less than 223.

$$\begin{array}{r} 6 \\ 36\overline{)\ 2232} \\ -216\downarrow \\ \hline 72 \end{array}$$

Multiply: $6 \times 36 = 216$. Write 216 below 223.
Subtract: $223 - 216 = 7$. Bring down the 2.

Divide 3 into 7 to estimate the number of times 36 goes into 72. The 3 goes into 7 at most 2 times.
Try 2: $2 \times 36 = 72$.

$$\begin{array}{r} 62 \\ 36\overline{)2232} \\ 216\downarrow \\ \hline 72 \\ -72 \\ \hline 0 \end{array}$$

Check: $2232 \stackrel{?}{=} 36 \times 62$
$2232 \stackrel{\checkmark}{=} 2232$

Thus, 2,232 ÷ 36 = 62.

2. Find 2,417,228 ÷ 802.

$802\overline{)2417228}$

First, the educated guess: $24 \div 8 = 3$. Then $3 \times 802 = 2406$, which is less than 2417. Use 3 as the guess. Since $3 \times 802 = 2406$, and 2406 has four digits, place the 3 above the fourth digit of the dividend.

$$\begin{array}{r} 3 \\ 802\overline{)\ 2417228} \\ -2406\downarrow \\ \hline 112 \end{array}$$

Subtract: $2417 - 2406 = 11$.
Bring down the 2.

The divisor 802 goes into 112 at most 0 times. Use 0.

$$\begin{array}{r} 30 \\ 802\overline{)\ 2417228} \\ -2406\downarrow\ \ \\ \hline 112\ \ \\ -0\downarrow \\ \hline 1122 \end{array}$$

Multiply: $0 \times 802 = 0$.
Subtract: $112 - 0 = 112$.
Bring down the 2.

The 8 goes into 11 at most 1 time, and $1 \times 802 = 802$, which is less than 1122. Try 1.

$$\begin{array}{r} 301 \\ 802\overline{)\ 2417228} \\ -2406\downarrow\ \ \\ \hline 112\ \ \\ -0\ \downarrow \\ \hline 1122\ \ \\ -802\downarrow \\ \hline 3208 \end{array}$$

Subtract $1122 - 802 = 320$.
Bring down the 8.

8 goes into 32 at most 4 times.
$4 \times 802 = 3208$.
Use 4.

$$
\begin{array}{r}
3014 \\
802\overline{)\,2417228} \\
-2406\downarrow \\
\hline
112 \\
-0\downarrow \\
\hline
1122 \\
-802\downarrow \\
\hline
3208 \\
-3208 \\
\hline
0
\end{array}
$$

Check: $2417228 \stackrel{?}{=} 3014 \times 802$
$2417228 \checkmark 2417228$

Thus, 2,417,228 ÷ 802 = 3,014.

★ PRACTICE SET B

Perform the following divisions.

1. 1,376 ÷ 32 **2.** 6,160 ÷ 55 **3.** 18,605 ÷ 61 **4.** 144,768 ÷ 48

❑ DIVISION WITH A REMAINDER

We might wonder how many times 4 is contained in 10.
Repeated subtraction yields

$$
\begin{array}{r}
10 \\
-\ 4 \\
\hline
6 \\
-\ 4 \\
\hline
2
\end{array}
$$

Since the remainder is less than 4, we stop the subtraction. Thus, 4 goes into 10 two times with 2 remaining. We can write this as a division as follows.

$$
\begin{array}{r}
2 \\
4\overline{)\,10} \\
-\ 8 \\
\hline
2
\end{array}
$$

Divide: **4 goes into 10 at most 2 times.**
Multiply: **$2 \times 4 = 8$. Write 8 below 0.**
Subtract: **$10 - 8 = 2$.**

Since 4 does not divide into 2 (the remainder is less than the divisor) and there are no digits to bring down to continue the process, we are done. We write

$$
\begin{array}{r}
2\ \mathrm{R}2 \\
4\overline{)\,10} \\
-\ 8 \\
\hline
2
\end{array}
\qquad \text{or} \qquad 10 \div 4 = \underbrace{2\ \mathrm{R}2}_{\text{2 with remainder 2}}
$$

☆ SAMPLE SET C

1. Find $85 \div 3$.

$$\begin{array}{r} 28 \\ 3\overline{)85} \\ \underline{6}\downarrow \\ 25 \\ \underline{24} \\ 1 \end{array}$$

Divide: 3 goes into 8 at most 2 times.
Multiply: $2 \times 3 = 6$. Write 6 below 8.
Subtract: $8 - 6 = 2$. Bring down the 5.

Divide: 3 goes into 25 at most 8 times.
Multiply: $3 \times 8 = 24$. Write 24 below 25.
Subtract: $25 - 24 = 1$

There are no more digits to bring down to continue the process. We are done. One is the remainder.

Check: Multiply 28 and 3, then add 1.

$$\begin{array}{r} 28 \\ \times\ 3 \\ \hline 84 \\ +\ 1 \\ \hline 85 \end{array}$$

Thus, $85 \div 3 = 28$ R1.

2. Find $726 \div 23$.

$$\begin{array}{r} 31 \\ 23\overline{)726} \\ \underline{69}\downarrow \\ 36 \\ \underline{23} \\ 13 \end{array}$$

Check: Multiply 31 by 23, then add 13.

$$\begin{array}{r} 31 \\ \times\ 23 \\ \hline 93 \\ \underline{62} \\ 713 \\ +\ 13 \\ \hline 726 \end{array}$$

Thus, $726 \div 23 = 31$ R13.

★ PRACTICE SET C

Perform the following divisions.

1. $75 \div 4$ **2.** $346 \div 8$ **3.** $489 \div 21$ **4.** $5{,}016 \div 82$ **5.** $41{,}196 \div 67$

❑ CALCULATORS

The calculator can be useful for finding quotients with single and multiple digit divisors. If, however, the division should result in a remainder, the calculator is unable to provide us with the particular value of the remainder. Also, some calcula-

tors (most nonscientific) are unable to perform divisions in which one of the numbers has more than eight digits.

☆ SAMPLE SET D

Use a calculator to perform each division.

1. $328 \div 8$

Type 328

Press [÷]

Type 8

Press [=]

The display now reads 41.

2. $53{,}136 \div 82$

Type 53136

Press [÷]

Type 82

Press [=]

The display now reads 648.

3. $730{,}019{,}001 \div 326$

We first try to enter 730,019,001 but find that we can only enter 73001900. If our calculator has only an eight-digit display (as most nonscientific calculators do), we will be unable to use the calculator to perform this division.

4. $3727 \div 49$

Type 3727

Press [÷]

Type 49

Press [=]

The display now reads 76.061224.

This number is an example of a decimal number (see Chapter 6). When a decimal number results in a calculator division, we can conclude that the division produces a remainder.

★ PRACTICE SET D

Use a calculator to perform each division.

1. $3{,}330 \div 74$

2. $63{,}365 \div 115$

3. $21{,}996{,}385{,}287 \div 53$

4. $4{,}558 \div 67$

Answers to Practice Sets are on p. 81.

Section 2.3 EXERCISES

For problems 1–55, perform the divisions.

Problems 1–38 can be checked with a calculator by multiplying the divisor and quotient then adding the remainder.

1. $52 \div 4$

2. $776 \div 8$

3. $603 \div 9$

4. $240 \div 8$

5. $208 \div 4$

6. $576 \div 6$

7. $21 \div 7$

8. $0 \div 0$

9. $140 \div 2$

10. $528 \div 8$

11. $244 \div 4$

12. $0 \div 7$

13. $177 \div 3$

14. $96 \div 8$

15. $67 \div 1$

16. $896 \div 56$

17. $1{,}044 \div 12$

18. $988 \div 19$

19. $5{,}238 \div 97$

20. $2{,}530 \div 55$

21. $4{,}264 \div 82$

22. $637 \div 13$

23. $3{,}420 \div 90$

24. $5{,}655 \div 87$

25. $2{,}115 \div 47$

26. $9{,}328 \div 22$

27. $55{,}167 \div 71$

28. $68{,}356 \div 92$

29. $27{,}702 \div 81$

30. $6{,}510 \div 31$

31. 60,536 ÷ 94

32. 31,844 ÷ 38

33. 23,985 ÷ 45

34. 60,606 ÷ 74

35. 2,975,400 ÷ 285

36. 1,389,660 ÷ 795

37. 7,162,060 ÷ 879

38. 7,561,060 ÷ 909

39. 38 ÷ 9

40. 97 ÷ 4

41. 199 ÷ 3

42. 573 ÷ 6

43. 10,701 ÷ 13

44. 13,521 ÷ 53

45. 3,628 ÷ 90

46. 10,592 ÷ 43

47. 19,965 ÷ 30

48. 8,320 ÷ 21

49. 61,282 ÷ 64

50. 1,030 ÷ 28

51. 7,319 ÷ 11

52. 3,628 ÷ 90

53. 35,279 ÷ 77

54. 52,196 ÷ 55

55. 67,751 ÷ 68

For problems 56–60, use a calculator to find the quotients.

56. 4,346 ÷ 53

57. 3,234 ÷ 77

58. 6,771 ÷ 37

59. 4,272,320 ÷ 520

60. 7,558,110 ÷ 651

61. A mathematics instructor at a high school is paid $17,775 for 9 months. How much money does this instructor make each month?

62. A couple pays $4,380 a year for a one-bedroom apartment. How much does this couple pay each month for this apartment?

63. Thirty-six people invest a total of $17,460 in a particular stock. If they each invested the same amount, how much did each person invest?

64. Each of the 28 students in a mathematics class buys a textbook. If the bookstore sells $644 worth of books, what is the price of each book?

65. A certain brand of refrigerator has an automatic ice cube maker that makes 336 ice cubes in one day. If the ice machine makes ice cubes at a constant rate, how many ice cubes does it make each hour?

66. A beer manufacturer bottles 52,380 ounces of beer each hour. If each bottle contains the same number of ounces of beer, and the manufacturer fills 4,365 bottles per hour, how many ounces of beer does each bottle contain?

67. A computer program consists of 68,112 bits. 68,112 bits equals 8,514 bytes. How many bits in one byte?

68. A 26-story building in San Francisco has a total of 416 offices. If each floor has the same number of offices, how many floors does this building have?

69. A college has 67 classrooms and a total of 2,546 desks. How many desks are in each classroom if each classroom has the same number of desks?

EXERCISES FOR REVIEW

(1.1) **70.** What is the value of 4 in the number 124,621?

(1.3) **71.** Round 604,092 to the nearest hundred thousand.

(1.6) **72.** What whole number is the additive identity?

(2.1) **73.** Find the product. $6{,}256 \times 100$.

(2.2) **74.** Find the quotient. $0 \div 11$.

★ Answers to Practice Sets (2.3)

A. **1.** 18 **2.** 81 **3.** 853 **4.** 1,129 **5.** 84,125

B. **1.** 43 **2.** 112 **3.** 305 **4.** 3,016

C. **1.** 18 R3 **2.** 43 R2 **3.** 23 R6 **4.** 61 R14 **5.** 614 R58

D. **1.** 45 **2.** 551
3. Since the dividend has more than eight digits, this division cannot be performed on most nonscientific calculators. On others, the answer is 415,026,137.4.
4. This division results in 68.02985075, a decimal number, and therefore, we cannot, at this time, find the value of the remainder. Later, we will discuss decimal numbers.

2.4 Some Interesting Facts about Division

Section Overview

- ❑ **DIVISION BY 2, 3, 4, AND 5**
- ❑ **DIVISION BY 6, 8, 9, AND 10**

Quite often, we are able to determine if a whole number is divisible by another whole number just by observing some simple facts about the number. Some of these facts are listed in this section.

❑ DIVISION BY 2, 3, 4, AND 5

Division by 2

A whole number is **divisible by 2** if its *last digit* is 0, 2, 4, 6, or 8.

The numbers 80, 112, 64, 326, and 1,008 are all divisible by 2 since the last digit of each is 0, 2, 4, 6, or 8, respectively.

The numbers 85 and 731 are *not* divisible by 2.

Division by 3

A whole number is **divisible by 3** if the *sum of its digits* is divisible by 3.

The number 432 is divisible by 3 since $4 + 3 + 2 = 9$ and 9 is divisible by 3.

$432 \div 3 = 144$

The number 25 is *not* divisible by 3 since $2 + 5 = 7$, and 7 is not divisible by 3.

Division by 4

A whole number is **divisible by 4** if its *last two digits* form a number that is divisible by 4.

The number 31,048 is divisible by 4 since the last two digits, 4 and 8, form a number, 48, that is divisible by 4.

$31048 \div 4 = 7262$

The number 137 is not divisible by 4 since 37 is not divisible by 4.

Division by 5

A whole number is **divisible by 5** if its *last digit* is 0 or 5.

☆ SAMPLE SET A

The numbers 65, 110, 8,030, and 16,955 are each divisible by 5 since the last digit of each is 0 or 5.

★ PRACTICE SET A

State which of the following whole numbers are divisible by 2, 3, 4, or 5. A number may be divisible by more than one number.

1. 26 **2.** 81 **3.** 51 **4.** 385

5. 6,112 **6.** 470 **7.** 113,154

❑ DIVISION BY 6, 8, 9, AND 10

Division by 6

A number is **divisible by 6** if it is divisible by *both* 2 and 3.

The number 234 is divisible by 2 since its last digit is 4. It is also divisible by 3 since $2 + 3 + 4 = 9$ and 9 is divisible by 3. Therefore, 234 is divisible by 6.
The number 6,532 is *not* divisible by 6. Although its last digit is 2, making it divisible by 2, the sum of its digits, $6 + 5 + 3 + 2 = 16$, and 16 is not divisible by 3.

Division by 8

A whole number is **divisible by 8** if its *last three digits* form a number that is divisible by 8.

The number 4,000 is divisible by 8 since 000 is divisible by 8.
The number 13,128 is divisible by 8 since 128 is divisible by 8.
The number 1,170 is *not* divisible by 8 since 170 is not divisible by 8.

Division by 9

A whole number is **divisible by 9** is the *sum of its digits* is divisible by 9.

The number 702 is divisible by 9 since $7 + 0 + 2$ is divisible by 9.
The number 6588 is divisible by 9 since $6 + 5 + 8 + 8 = 27$ is divisible by 9.
The number 14,123 is *not* divisible by 9 since $1 + 4 + 1 + 2 + 3 = 11$ is not divisible by 9.

Division by 10

A whole number is **divisible by 10** if its *last digit* is 0.

☆ SAMPLE SET B

The numbers 30, 170, 16,240, and 865,000 are all divisible by 10.

★ PRACTICE SET B

State which of the following whole numbers are divisible 6, 8, 9, or 10. Some numbers may be divisible by more than one number.

1. 900 **2.** 6,402 **3.** 6,660 **4.** 55,116

Answers to Practice Sets are on p. 85.

Section 2.4 EXERCISES

For problems 1–30, specify if the whole number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10. Write "none" if the number is not divisible by any digit other than 1. Some numbers may be divisible by more than one number.

1. 48 _____

2. 85 _____

3. 30 _____

4. 83 _____

5. 98 _____

6. 972 _____

7. 892 _____

8. 676 _____

9. 903 _____

10. 800 _____

11. 223 _____

12. 836 _____

13. 665 _____

14. 4,381 _____

15. 2,195 _____

16. 2,544 _____

17. 5,172 _____

18. 1,307 _____

19. 1,050 _____

20. 3,898 _____

21. 1,621 _____

22. 27,808 _____

23. 45,764 _____

24. 49,198 _____

25. 296,122 _____

26. 178,656 _____

27. 5,102,417 _____

28. 16,990,792 _____

29. 620,157,659 _____

30. 457,687,705 _____

EXERCISES FOR REVIEW

(1.1) **31.** In the number 412, how many tens are there?

(1.5) **32.** Subtract 613 from 810.

(1.6) **33.** Add 35, 16, and 7 in two different ways.

(2.2) **34.** Find the quotient $35 \div 0$, if it exists.

(2.3) **35.** Find the quotient. $3654 \div 42$.

★ Answers to Practice Sets (2.4)

A. **1.** 2 **2.** 3 **3.** 3 **4.** 5 **5.** 2, 4 **6.** 2, 5 **7.** 2, 3

B. **1.** 6, 9, 10 **2.** 6 **3.** 6, 9, 10 **4.** 6, 9

2.5 Properties of Multiplication

Section Overview

- ❑ THE COMMUTATIVE PROPERTY OF MULTIPLICATION
- ❑ THE ASSOCIATIVE PROPERTY OF MULTIPLICATION
- ❑ THE MULTIPLICATIVE IDENTITY

We will now examine three simple but very important properties of multiplication.

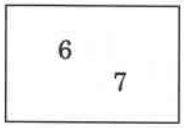
❑ THE COMMUTATIVE PROPERTY OF MULTIPLICATION

Commutative Property of Multiplication

The product of two whole numbers is the same regardless of the order of the factors.

☆ SAMPLE SET A

Multiply the two whole numbers.



$6 \cdot 7 = 42$

$7 \cdot 6 = 42$

The numbers 6 and 7 can be multiplied in any order. Regardless of the order they are multiplied, the product is 42.

★ PRACTICE SET A

Use the commutative property of multiplication to find the products in two ways.

1. 15 · 6

2. 432 · 428

❑ THE ASSOCIATIVE PROPERTY OF MULTIPLICATION

Associative Property of Multiplication

If three whole numbers are multiplied, the product will be the same if the first two are multiplied first and then that product is multiplied by the third, or if the second two are multiplied first and that product is multiplied by the first. Note that the order of the factors is maintained.

It is a common mathematical practice to *use parentheses* to show which pair of numbers is to be combined first.

☆ SAMPLE SET B

Multiply the whole numbers.

$8 \cdot 3 \cdot 14$

$(8 \cdot 3) \cdot 14 = 24 \cdot 14 = 336$

$8 \cdot (3 \cdot 14) = 8 \cdot 42 = 336$

★ PRACTICE SET B

Use the associative property of multiplication to find the products in two ways.

1. $7 \cdot 3 \cdot 8$

2. $73 \cdot 18 \cdot 126$

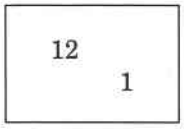
❑ THE MULTIPLICATIVE IDENTITY

The Multiplicative Identity is 1

The whole number 1 is called the **multiplicative identity,** since any whole number multiplied by 1 is not changed.

☆ SAMPLE SET C

Multiply the whole numbers.

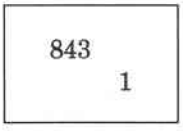


$12 \cdot 1 = 12$

$1 \cdot 12 = 12$

★ PRACTICE SET C

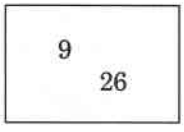
Multiply the whole numbers.

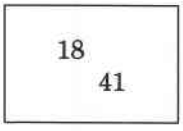


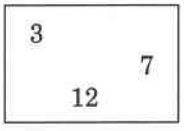
Answers to Practice Sets are on p. 88.

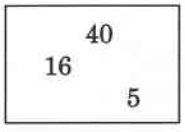
Section 2.5 EXERCISES

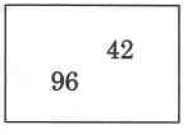
For problems 1–12, multiply the numbers.

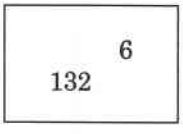
1. 

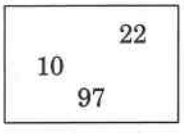
2. 

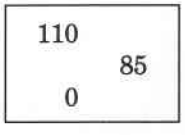
7. 

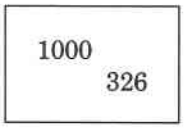
8. 

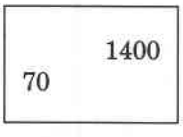
3. 

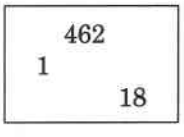
4. 

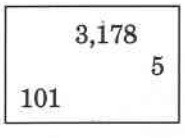
9. 

10. 

5. 

6. 

11. 

12. 

For problems 13–16, show that the quantities yield the same products by performing the multiplications.

13. $(4 \cdot 8) \cdot 2$ and $4 \cdot (8 \cdot 2)$

14. $(100 \cdot 62) \cdot 4$ and $100 \cdot (62 \cdot 4)$

15. $23 \cdot (11 \cdot 106)$ and $(23 \cdot 11) \cdot 106$

16. $1 \cdot (5 \cdot 2)$ and $(1 \cdot 5) \cdot 2$

17. The fact that

(a first number · a second number) · a third number = a first number · (a second number · a third number)

is an example of the _____ property of multiplication.

18. The fact that

1 · any number = that particular number

is an example of the _____ property of multiplication.

19. Use the numbers 7 and 9 to illustrate the commutative property of multiplication.

20. Use the numbers 6, 4, and 7 to illustrate the associative property of multiplication.

EXERCISES FOR REVIEW

(1.1) **21.** In the number 84,526,098,441, how many millions are there?

(1.4) **22.** Replace the letter m with the whole number that makes the addition true.

$$\begin{array}{r} 85 \\ +\ m \\ \hline 97 \end{array}$$

(1.6) **23.** Use the numbers 4 and 15 to illustrate the commutative property of addition.

(2.2) **24.** Find the product. $8{,}000{,}000 \times 1{,}000$.

(2.4) **25.** Specify which of the digits 2, 3, 4, 5, 6, 8, 10 are divisors of the number 2,244.

★ Answers to Practice Sets (2.5)

A. **1.** $15.6 = 90$ and $6 \cdot 15 = 90$ **2.** $432 \cdot 428 = 184{,}896$ and $428 \cdot 432 = 184{,}896$

B. **1.** 168 **2.** 165,564

C. 843

Chapter 2 SUMMARY OF KEY CONCEPTS

Multiplication (2.1)

Multiplication is a description of repeated addition.

$$\underbrace{7+7+7+7}$$

7 appears 4 *times*

This expression is described by writing 4×7.

Multiplicand/Multiplier/Product (2.1)

In a multiplication of whole numbers, the repeated addend is called the *multiplicand,* and the number that records the number of times the multiplicand is used is the *multiplier.* The result of the multiplication is the *product.*

Factors (2.1)

In a multiplication, the numbers being multiplied are also called *factors.* Thus, the multiplicand and the multiplier can be called factors.

Division (2.2)

Division is a description of repeated subtraction.

Dividend/Divisor/Quotient (2.2)

In a division, the number divided into is called the *dividend,* and the number dividing into the dividend is called the *divisor.* The result of the division is called the *quotient.*

$$\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$$

Division into Zero (2.2)

Zero divided by any nonzero whole number is zero.

Division by Zero (2.2)

Division by zero does not name a whole number. It is, therefore, undefined. The quotient $\frac{0}{0}$ is indeterminant.

Division by 2, 3, 4, 5, 6, 8, 9, 10 (2.4)

Division by the whole numbers 2, 3, 4, 5, 6, 8, 9, and 10 can be determined by noting some certain properties of the particular whole number.

Commutative Property of Multiplication (2.5)

The product of two whole numbers is the same regardless of the order of the factors.

$3 \times 5 = 5 \times 3$

Associative Property of Multiplication (2.5)

If three whole numbers are to be multiplied, the product will be the same if the first two are multiplied first and then that product is multiplied by the third, or if the second two are multiplied first and then that product is multiplied by the first.

$(3 \times 5) \times 2 = 3 \times (5 \times 2)$

Note that the order of the factors is maintained.

Multiplicative Identity (2.5)

The whole number 1 is called the *multiplicative identity* since any whole number multiplied by 1 is not changed.

$4 \times 1 = 4$
$1 \times 4 = 4$

EXERCISE SUPPLEMENT

Section 2.1

1. In the multiplication $5 \times 9 = 45$, 5 and 9 are called _____ and 45 is called the _____.

2. In the multiplication $4 \times 8 = 32$, 4 and 8 are called _____ and 32 is called the _____.

Section 2.2

3. In the division $24 \div 6 = 4$, 6 is called the _____, and 4 is called the _____.

4. In the division $36 \div 2 = 18$, 2 is called the _____, and 18 is called the _____.

Section 2.4

5. A number is divisible by 2 only if its last digit is _____.

6. A number is divisible by 3 only if _____ of its digits is divisible by 3.

7. A number is divisible by 4 only if the rightmost two digits form a number that is _____.

Sections 2.1, 2.3

Find each product or quotient.

8. $\begin{array}{r} 24 \\ \times\ 3 \\ \hline \end{array}$

9. $\begin{array}{r} 14 \\ \times\ 8 \\ \hline \end{array}$

10. $21 \div 7$

11. $35 \div 5$

12. $\begin{array}{r} 36 \\ \times 22 \\ \hline \end{array}$

13. $\begin{array}{r} 87 \\ \times 35 \\ \hline \end{array}$

14. $\begin{array}{r} 117 \\ \times\ 42 \\ \hline \end{array}$

15. $208 \div 52$

16. $\begin{array}{r} 521 \\ \times\ 87 \\ \hline \end{array}$

17. $\begin{array}{r} 1005 \\ \times\ 15 \\ \hline \end{array}$

18. $1338 \div 446$

19. $2814 \div 201$

20. $\begin{array}{r} 5521 \\ \times\ 8 \\ \hline \end{array}$

21. $\begin{array}{r} 6016 \\ \times\ 7 \\ \hline \end{array}$

22. $576 \div 24$

23. $3969 \div 63$

24. $\begin{array}{r} 5482 \\ \times\ 322 \\ \hline \end{array}$

25. $\begin{array}{r} 9104 \\ \times\ 115 \\ \hline \end{array}$

26. $\begin{array}{r} 6102 \\ \times 1000 \\ \hline \end{array}$

27. $\begin{array}{r} 10101 \\ \times 10000 \\ \hline \end{array}$

28. $162{,}006 \div 31$

29. $0 \div 25$

30. $25 \div 0$

31. $4280 \div 10$

32. $2126000 \div 100$

33. $84 \div 15$

34. $126 \div 4$

35. $424 \div 0$

36. $1198 \div 46$

37. $995 \div 31$

38. $0 \div 18$

39. $\begin{array}{r} 2162 \\ \times 1421 \\ \hline \end{array}$

40. 0×0

41. 5×0

42. 64×1

43. 1×0

44. $0 \div 3$

45. $14 \div 0$

46. $35 \div 1$

47. $1 \div 1$

Section 2.5

48. Use the commutative property of multiplication to rewrite 36×128.

49. Use the commutative property of multiplication to rewrite 114×226.

50. Use the associative property of multiplication to rewrite $(5 \cdot 4) \cdot 8$.

51. Use the associative property of multiplication to rewrite $16 \cdot (14 \cdot 0)$.

Sections 2.1, 2.3

52. A computer store is selling diskettes for \$4 each. At this price, how much would 15 diskettes cost?

53. Light travels 186,000 miles in one second. How far does light travel in 23 seconds?

54. A dinner bill for eight people comes to exactly \$112. How much should each person pay if they all agree to split the bill equally?

55. Each of the 33 students in a math class buys a textbook. If the bookstore sells \$1089 worth of books, what is the price of each book?

Chapter 2 PROFICIENCY EXAM

1. _____

1. **(2.1)** In the multiplication of $8 \times 7 = 56$, what are the names given to the 8 and 7 and the 56?

2. _____

2. **(2.1)** Multiplication is a description of what repeated process?

3. **(2.2)** In the division $12 \div 3 = 4$, what are the names given to the 3 and the 4?

3. _____

4. **(2.4)** Name the digits that a number must end in to be divisible by 2.

4. _____

5. **(2.5)** Name the property of multiplication that states that the order of the factors in a multiplication can be changed without changing the product.

6. **(2.5)** Which number is called the multiplicative identity?

5. _____

For problems 7–17, find the product or quotient.

6. _____

7. **(2.1)** 14×6

7. _____

8. **(2.1)** 37×0

8. _____

9. **(2.1)** 352×1000

9. _____

10. **(2.1)** 5986×70

10. _____

11. **(2.1)** 21×12

11. _____

12. **(2.2)** $856 \div 0$

12. _____

13. **(2.2)** $0 \div 8$

13. _____

14. _____ **14. (2.3)** $136 \div 8$

15. _____ **15. (2.3)** $432 \div 24$

16. _____ **16. (2.3)** $5286 \div 37$

17. _____ **17. (2.5)** 211×1

For problems 18–20, use the numbers 216, 1,005, and 640.

18. _____ **18. (2.4)** Which numbers are divisible by 3?

19. _____ **19. (2.4)** Which number is divisible by 4?

20. _____ **20. (2.4)** Which number(s) is divisible by 5?