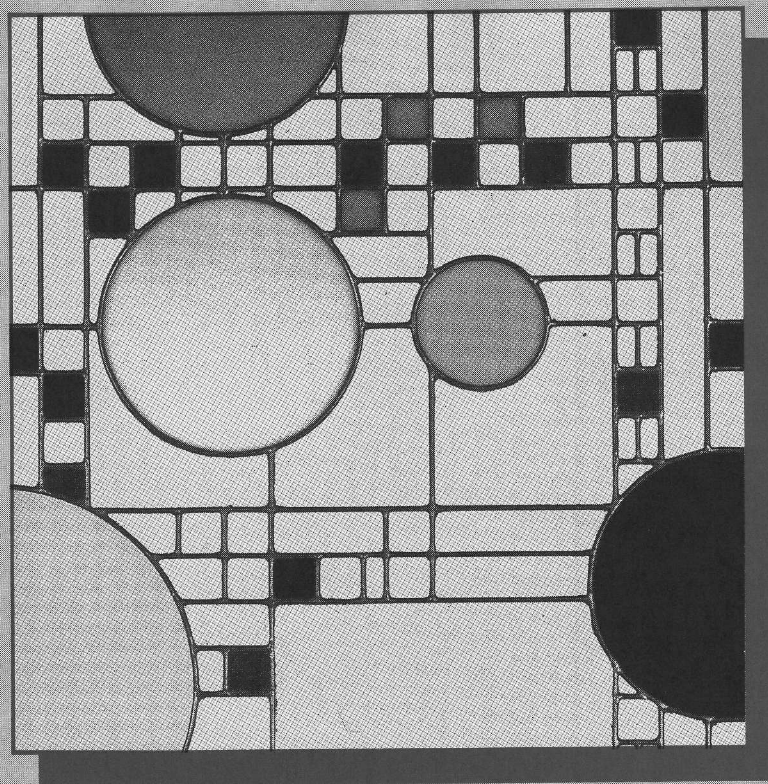
4

Introduction to Fractions and Multiplication and Division of Fractions



After completing this chapter, you should

Section 4.1 Fractions of Whole Numbers
- understand the concept of fractions of whole numbers
- be able to recognize the parts of a fraction

Section 4.2 Proper Fractions, Improper Fractions, and Mixed Numbers
- be able to distinguish between proper fractions, improper fractions, and mixed numbers
- be able to convert an improper fraction to a mixed number
- be able to convert a mixed number to an improper fraction

Section 4.3 Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Terms
- be able to recognize equivalent fractions
- be able to reduce a fraction to lowest terms
- be able to raise a fraction to higher terms

Section 4.4 Multiplication of Fractions
- understand the concept of multiplication of fractions
- be able to multiply one fraction by another
- be able to multiply mixed numbers
- be able to find powers and roots of various fractions

Section 4.5 Division of Fractions
- be able to determine the reciprocal of a number
- be able to divide one fraction by another

Section 4.6 Applications Involving Fractions
- be able to solve missing product statements
- be able to solve missing factor statements

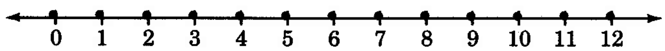
4.1 Fractions of Whole Numbers

Section Overview

- ❑ MORE NUMBERS ON THE NUMBER LINE
- ❑ FRACTIONS OF WHOLE NUMBERS
- ❑ THE PARTS OF A FRACTION
- ❑ READING AND WRITING FRACTIONS

❑ MORE NUMBERS ON THE NUMBER LINE

In Chapters 1, 2, and 3, we studied the whole numbers and methods of combining them. We noted that we could visually display the whole numbers by drawing a number line and placing closed circles at whole number locations.



By observing this number line, we can see that the whole numbers do not account for every point on the line. What numbers, if any, can be associated with these points? In this section we will see that many of the points on the number line, including the points already associated with whole numbers, can be associated with numbers called *fractions*.

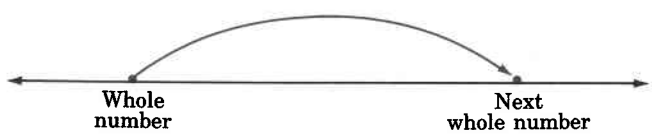
❑ FRACTIONS OF WHOLE NUMBERS

The Nature of the Positive Fractions

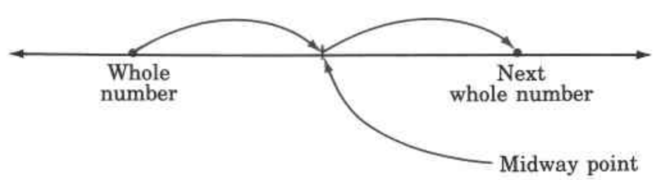
We can extend our collection of numbers, which now contains only the whole numbers, by including fractions of whole numbers. We can determine the nature of these fractions using the number line.

If we place a pencil at some whole number and proceed to travel to the right to the next whole number, we see that our journey can be *broken* into different types of equal parts as shown in the following examples.

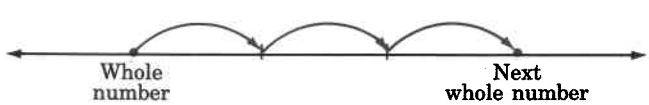
(a) 1 part.



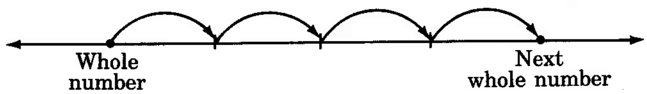
(b) 2 equal parts.



(c) 3 equal parts.



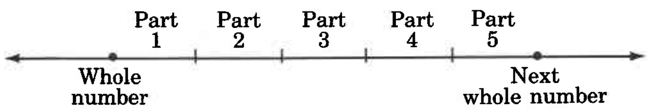
(d) 4 equal parts.



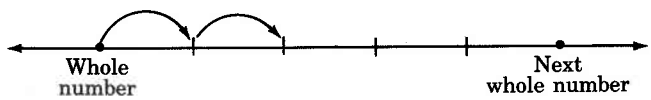
Notice that the number of parts, 2, 3, and 4, that we are breaking the original quantity into is always a *nonzero whole number.* The idea of breaking up a whole quantity gives us the word *fraction.* The word fraction comes from the Latin word "fractio" which means a breaking, or fracture.

The Latin Word Fractio

Suppose we break up the interval from some whole number to the next whole number into five equal parts.



After starting to move from one whole number to the next, we decide to stop after covering only two parts. We have covered 2 parts of 5 equal parts. This situation is described by writing $\frac{2}{5}$.



Positive Fraction

A number such as $\frac{2}{5}$ is called a **positive fraction,** or more simply, a **fraction.**

❑ THE PARTS OF A FRACTION

A fraction has *three parts.*

Fraction Bar

1. The fraction bar ——.

   The **fraction bar** serves as a grouping symbol. It separates a quantity into individual groups. These groups have names, as noted in 2 and 3 below.
2. The nonzero number below the fraction bar.

   Denominator

   This number is called the **denominator** of the fraction, and it indicates the number of parts the whole quantity has been divided into. Notice that the denominator must be a nonzero whole number since the least number of parts any quantity can have is one.
3. The number above the fraction bar.

   Numerator

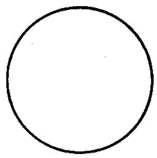
   This number is called the **numerator** of the fraction, and it indicates how many of the specified parts are being considered. Notice that the numerator can be any whole number (including zero) since any number of the specified parts can be considered.

$$\frac{\text{whole number}}{\text{nonzero whole number}} \longleftrightarrow \frac{\text{numerator}}{\text{denominator}}$$

☆ SAMPLE SET A

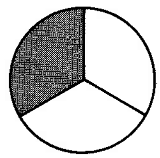
The diagrams in the following problems are illustrations of fractions.

1.



A whole circle

The whole circle divided into 3 equal parts



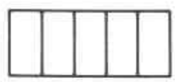
1 of the 3 equal parts

$\frac{1}{3}$ ← $\boxed{1}$ of $\boxed{3}$ equal parts

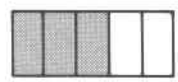
The fraction $\frac{1}{3}$ is read as "one third."

2.

A whole rectangle



The whole rectangle divided into 5 equal parts



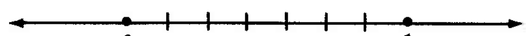
3 of the 5 equal parts

$\frac{3}{5}$ ← $\boxed{3}$ of $\boxed{5}$ equal parts

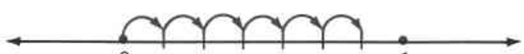
The fraction $\frac{3}{5}$ is read as "three fifths."

3.

The number line between 0 and 1



The number line between 0 and 1 divided into 7 equal parts

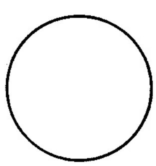


6 of the 7 equal parts

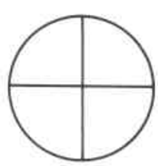
$\frac{6}{7}$ ← $\boxed{6}$ of the $\boxed{7}$ equal parts

The fraction $\frac{6}{7}$ is read as "six sevenths."

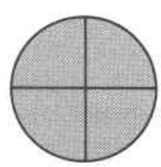
4.



A whole circle



The whole circle divided into 4 equal parts



4 of the 4 equal parts

$\frac{4}{4}$ ← $\boxed{4}$ of the $\boxed{4}$ equal parts

When the numerator and denominator are equal, the fraction represents the entire quantity, and its value is 1.

$$\frac{\text{nonzero whole number}}{\text{same nonzero whole number}} = 1$$

★ PRACTICE SET A

Specify the numerator and denominator of the following fractions.

1. $\frac{4}{7}$
2. $\frac{5}{8}$
3. $\frac{10}{15}$
4. $\frac{1}{9}$
5. $\frac{0}{2}$

☐ READING AND WRITING FRACTIONS

In order to properly translate fractions from word form to number form, or from number form to word form, it is necessary to understand the use of the *hyphen.*

Use of the Hyphen

One of the main uses of the **hyphen** is to tell the reader that two words not ordinarily joined are to be taken in combination as a unit. Hyphens are *always* used for numbers between and including 21 and 99 (except those ending in zero).

☆ SAMPLE SET B

Write each fraction using whole numbers.

1. Fifty three-hundredths. The hyphen joins the words three and hundredths and tells us to consider them as a single unit. Therefore,

   fifty three-hundredths translates as $\frac{50}{300}$

2. Fifty-three hundredths. The hyphen joins the numbers fifty and three and tells us to consider them as a single unit. Therefore,

   fifty-three hundredths translates as $\frac{53}{100}$

3. Four hundred seven-thousandths. The hyphen joins the words seven and thousandths and tells us to consider them as a single unit. Therefore,

   four hundred seven-thousandths translates as $\frac{400}{7{,}000}$

4. Four hundred seven thounsandths. The absence of hyphens indicates that the words *seven* and *thousandths* are to be considered individually.

   four hundred seven thousandths translates as $\frac{407}{1{,}000}$

Continued

Write each fraction using words.

5. $\frac{21}{85}$ translates as twenty-one eighty-fifths.

6. $\frac{200}{3{,}000}$ translates as two hundred three-thousandths. A hyphen is needed between the words three and thousandths to tell the reader that these words are to be considered as a single unit.

7. $\frac{203}{1{,}000}$ translates as two hundred three thousandths.

★ PRACTICE SET B

Write the following fractions using whole numbers.

1. one tenth

2. eleven fourteenths

3. sixteen thirty-fifths

4. eight hundred seven-thousandths

Write the following using words.

5. $\frac{3}{8}$

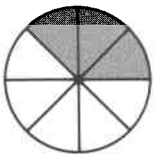
6. $\frac{1}{10}$

7. $\frac{3}{250}$

8. $\frac{114}{3{,}190}$

Name the fraction that describes each shaded portion.

9.



10.

In problems 11 and 12, state the numerator and denominator, and write each fraction in words.

11. The number $\frac{5}{9}$ is used in converting from Fahrenheit to Celsius.

12. A dime is $\frac{1}{10}$ of a dollar.

Answers to Practice Sets are on p. 144.

Section 4.1 EXERCISES

For problems 1–10, specify the numerator and denominator in each fraction.

1. $\frac{3}{4}$

2. $\frac{9}{10}$

3. $\frac{1}{5}$

4. $\frac{5}{6}$

5. $\frac{7}{7}$

6. $\frac{4}{6}$

7. $\frac{0}{12}$

8. $\frac{25}{25}$

9. $\frac{18}{1}$

10. $\frac{0}{16}$

For problems 11–20, write the fractions using whole numbers.

11. four fifths

12. two ninths

13. fifteen twentieths

14. forty-seven eighty-thirds

15. ninety-one one hundred sevenths

16. twenty-two four hundred elevenths

17. six hundred five eight hundred thirty-fourths

18. three thousand three forty-four ten-thousandths

19. ninety-two one-millionths

20. one three-billionths

For problems 21 – 30, write the fractions using words.

21. $\frac{5}{9}$

22. $\frac{6}{10}$

23. $\frac{8}{15}$

24. $\frac{10}{13}$

25. $\frac{75}{100}$

26. $\frac{86}{135}$

27. $\frac{916}{1,014}$

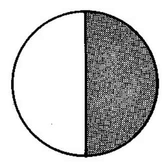
28. $\frac{501}{10,001}$

29. $\frac{18}{31,608}$

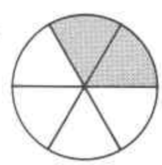
30. $\frac{1}{500,000}$

For problems 31 – 34, name the fraction corresponding to the shaded portion.

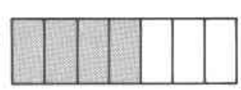
31.



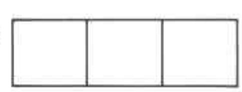
32.



33.

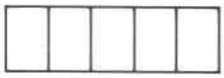


34.

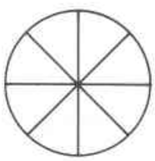


For problems 35 – 38, shade the portion corresponding to the given fraction on the given figure.

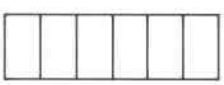
35. $\frac{3}{5}$



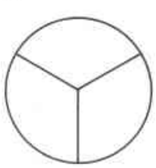
36. $\frac{1}{8}$



37. $\frac{6}{6}$



38. $\frac{0}{3}$



State the numerator and denominator and write in words each of the fractions appearing in the statements for problems 39 – 48.

39. A contractor is selling houses on $\frac{1}{4}$ acre lots.

40. The fraction $\frac{22}{7}$ is sometimes used as an approximation to the number π. (The symbol is read "pi.")

41. The fraction $\frac{4}{3}$ is used in finding the volume of a sphere.

42. One inch is $\frac{1}{12}$ of a foot.

43. About $\frac{2}{7}$ of the students in a college statistics class received a "B" in the course.

44. The probability of randomly selecting a club when drawing one card from a standard deck of 52 cards is $\frac{13}{52}$.

45. In a box that contains eight computer chips, five are known to be good and three are known to be defective. If three chips are selected at random, the probability that all three are defective is $\frac{1}{56}$.

46. In a room of 25 people, the probability that at least two people have the same birthdate (date and month, not year) is $\frac{569}{1,000}$.

47. The mean (average) of the numbers 21, 25, 43, and 36 is $\frac{125}{4}$.

48. If a rock falls from a height of 20 meters on Jupiter, the rock will be $\frac{32}{25}$ meters high after $\frac{6}{5}$ seconds.

EXERCISES FOR REVIEW

(1.6) 49. Use the numbers 3 and 11 to illustrate the commutative property of addition.

(2.3) 50. Find the quotient. $676 \div 26$.

(3.1) 51. Write $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ using exponents.

(3.2) 52. Find the value of $\frac{8 \cdot (6+20)}{8} + \frac{3 \cdot (6+16)}{22}$.

(3.5) 53. Find the least common multiple of 12, 16, and 18.

★ Answers to Practice Sets (4.1)

A. 1. 4, 7 2. 5, 8 3. 10, 15 4. 1, 9 5. 0, 2

B. 1. $\frac{1}{10}$ 2. $\frac{11}{14}$ 3. $\frac{16}{35}$ 4. $\frac{800}{7,000}$

5. three eighths 6. one tenth 7. three two hundred fiftieths

8. one hundred fourteen three thousand one hundred ninetieths 9. $\frac{3}{8}$ 10. $\frac{1}{16}$

11. 5, 9, five ninths 12. 1, 10, one tenth

4.2 Proper Fractions, Improper Fractions, and Mixed Numbers

Section Overview

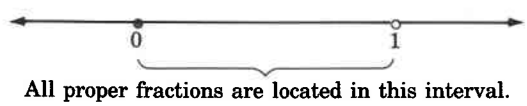
- ❑ POSITIVE PROPER FRACTIONS
- ❑ POSITIVE IMPROPER FRACTIONS
- ❑ POSITIVE MIXED NUMBERS
- ❑ RELATING POSITIVE IMPROPER FRACTIONS AND POSITIVE MIXED NUMBERS
- ❑ CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER
- ❑ CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

Now that we know what positive fractions are, we consider three types of positive fractions: proper fractions, improper fractions, and mixed numbers.

❑ POSITIVE PROPER FRACTIONS

Positive Proper Fractions

Fractions in which the whole number in the numerator is strictly less than the whole number in the denominator are called **positive proper fractions.** On the number line, proper fractions are located in the interval from 0 to 1. Positive proper fractions are always less than one.



The closed circle at 0 indicates that 0 is included, while the open circle at 1 indicates that 1 is not included.

Some examples of positive proper fractions are

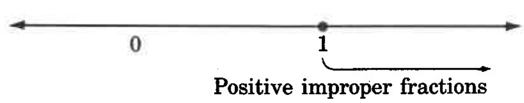
$$\frac{1}{2}, \quad \frac{3}{5}, \quad \frac{20}{27}, \quad \text{and} \quad \frac{106}{255}$$

Note that $1 < 2$, $3 < 5$, $20 < 27$, and $106 < 225$.

❑ POSITIVE IMPROPER FRACTIONS

Positive Improper Fractions

Fractions in which the whole number in the numerator is greater than or equal to the whole number in the denominator are called **positive improper fractions.** On the number line, improper fractions lie to the right of (and including) 1. Positive improper fractions are always greater than or equal to 1.



Some examples of positive improper fractions are

$$\frac{3}{2}, \quad \frac{8}{5}, \quad \frac{4}{4}, \quad \text{and} \quad \frac{105}{16}$$

Note that $3 \geq 2$, $8 \geq 5$, $4 \geq 4$, and $105 \geq 16$.

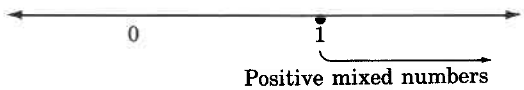
❑ POSITIVE MIXED NUMBERS

A number of the form

nonzero whole number + proper fraction

Positive Mixed Number

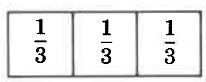
is called a **positive mixed number.** For example, $2\frac{3}{5}$ is a mixed number. On the number line, mixed numbers are located in the interval to the right of (and including) 1. Mixed numbers are always greater than or equal to 1.



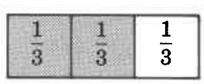
❑ RELATING POSITIVE IMPROPER FRACTIONS AND POSITIVE MIXED NUMBERS

A relationship between improper fractions and mixed numbers is suggested by two facts. The first is that improper fractions and mixed numbers are located in the same interval on the number line. The second fact, that mixed numbers are the sum of a natural number and a fraction, can be seen by making the following observations.

Divide a whole quantity into 3 equal parts.

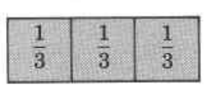


Now, consider the following examples by observing the respective shaded areas.



In the shaded region, there are 2 one thirds, or $\frac{2}{3}$.

$$2\left(\frac{1}{3}\right) = \frac{2}{3}$$



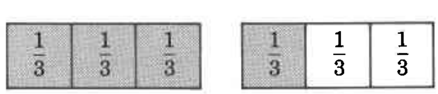
There are 3 one thirds, or $\frac{3}{3}$, or 1.

$$3\left(\frac{1}{3}\right) = \frac{3}{3} \quad \text{or} \quad 1$$

Thus,

$$\frac{3}{3} = 1$$

Improper fraction = whole number.



There are 4 one thirds, or $\frac{4}{3}$, or 1 and $\frac{1}{3}$.

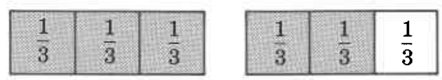
$$4\left(\frac{1}{3}\right) = \frac{4}{3} \quad \text{or} \quad 1 \text{ and } \frac{1}{3}$$

The terms 1 and $\frac{1}{3}$ can be represented as $1 + \frac{1}{3}$ or $1\frac{1}{3}$.

Thus,

$$\frac{4}{3} = 1\frac{1}{3}.$$

Improper fraction = mixed number.



There are 5 one thirds, or $\frac{5}{3}$, or 1 and $\frac{2}{3}$.

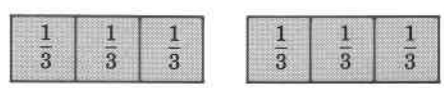
$$5\left(\frac{1}{3}\right) = \frac{5}{3} \quad \text{or} \quad 1 \text{ and } \frac{2}{3}$$

The terms 1 and $\frac{2}{3}$ can be represented as $1 + \frac{2}{3}$ or $1\frac{2}{3}$.

Thus,

$$\frac{5}{3} = 1\frac{2}{3}.$$

Improper fraction = mixed number.



There are 6 one thirds, or $\frac{6}{3}$, or 2.

$$6\left(\frac{1}{3}\right) = \frac{6}{3} = 2$$

Thus,

$$\frac{6}{3} = 2$$

Improper fraction = whole number.

The following important fact is illustrated in the preceding examples.

Mixed Number = Natural Number + Proper Fraction

> **Mixed numbers** are the *sum* of a natural number and a proper fraction.
>
> Mixed number = (natural number) + (proper fraction)

For example $1\frac{1}{3}$ can be expressed as $1 + \frac{1}{3}$. The fraction $5\frac{7}{8}$ can be expressed as $5 + \frac{7}{8}$.

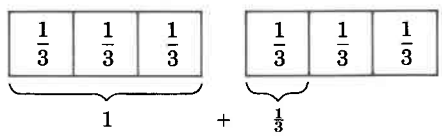
It is important to note that a number such as $5\frac{7}{8}$ does *not* indicate multiplication. To indicate multiplication, we would need to use a multiplication symbol (such as $\cdot$).

Note: $5\frac{7}{8}$ means $5+\frac{7}{8}$ and *not* $5 \cdot \frac{7}{8}$, which means 5 times $\frac{7}{8}$ or 5 multiplied by $\frac{7}{8}$.

Thus, mixed numbers may be represented by improper fractions, and improper fractions may be represented by mixed numbers.

❑ CONVERTING IMPROPER FRACTIONS TO MIXED NUMBERS

To understand how we might convert an improper fraction to a mixed number, let's consider the fraction, $\frac{4}{3}$.



$$\frac{4}{3} = \underbrace{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}_{1}+\frac{1}{3}$$

$$= 1+\frac{1}{3}$$

$$= 1\frac{1}{3}$$

Thus, $\frac{4}{3} = 1\frac{1}{3}$.

We can illustrate a procedure for converting an improper fraction to a mixed number using this example. However, the conversion is *more easily* accomplished by dividing the numerator by the denominator and using the result to write the mixed number.

Converting an Improper Fraction to a Mixed Number

> To convert an improper fraction to a mixed number, divide the numerator by the denominator.
>
> 1. The whole number part of the mixed numer is the quotient.
> 2. The fractional part of the mixed number is the remainder written over the divisor (the denominator of the improper fraction).

☆ SAMPLE SET A

Convert each improper fraction to its corresponding mixed number.

1. $\frac{5}{3}$. Divide 5 by 3.

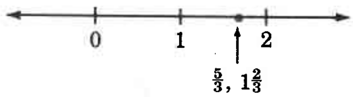
$$\begin{array}{r} 1 \\ 3\overline{)5} \\ \underline{3} \\ 2 \end{array}$$

1 ← whole number part

2 ← numerator of the fractional part

3 ← denominator of the fractional part

The improper fraction $\frac{5}{3} = 1\frac{2}{3}$.



2. $\frac{46}{9}$. Divide 46 by 9.

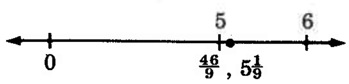
$$\begin{array}{r} 5 \\ 9\overline{)46} \\ \underline{45} \\ 1 \end{array}$$

5 ← whole number part

1 ← numerator of the fractional part

9 ← denominator of the fractional part

The improper fraction $\frac{46}{9} = 5\frac{1}{9}$.



3. $\frac{83}{11}$. Divide 83 by 11.

$$\begin{array}{r} 7 \\ 11\overline{)83} \\ \underline{77} \\ 6 \end{array}$$

7 ← whole number part

6 ← numerator of the fractional part

11 ← denominator of the fractional part

The improper fraction $\frac{83}{11} = 7\frac{6}{11}$.

4. $\frac{104}{4}$. Divide 104 by 4.

$$\begin{array}{r} 26 \\ 4\overline{)104} \\ \underline{8} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

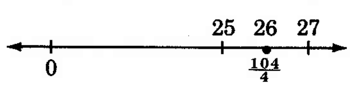
26 ← whole number part

0 ← numerator of the fractional part

4 ← denominator of the fractional part

$$\frac{104}{4} = 26\frac{0}{4} = 26$$

The improper fraction $\frac{104}{4} = 26$.



★ PRACTICE SET A

Convert each improper fraction to its corresponding mixed number.

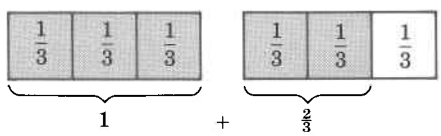
1. $\frac{9}{2}$
2. $\frac{11}{3}$
3. $\frac{14}{11}$
4. $\frac{31}{13}$
5. $\frac{79}{4}$
6. $\frac{496}{8}$

❑ CONVERTING MIXED NUMBERS TO IMPROPER FRACTIONS

To understand how to convert a mixed number to an improper fraction, we'll recall

mixed number = (natural number) + (proper fraction)

and consider the following diagram.



$$1\frac{2}{3} = 1 + \frac{2}{3}$$

$$= \underbrace{\overbrace{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}^{1}+\overbrace{\frac{1}{3}+\frac{1}{3}}^{\frac{2}{3}}}_{5\cdot\frac{1}{3}=\frac{5}{3}}$$

Recall that multiplication describes repeated addition.

Notice that $\frac{5}{3}$ can be obtained from $1\frac{2}{3}$ using multiplication in the following way.

Multiply: $3 \cdot 1 = 3$

$$1\frac{2}{3}$$

Add: $3 + 2 = 5$. Place the 5 over the 3: $\frac{5}{3}$.

The procedure for converting a mixed number to an improper fraction is illustrated in this example.

Converting a Mixed Number to an Improper Fraction

To convert a mixed number to an improper fraction,

1. Multiply the denominator of the fractional part of the mixed number by the whole number part.
2. To this product, add the numerator of the fractional part.
3. Place this result over the denominator of the fractional part.

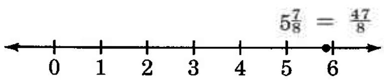
☆ SAMPLE SET B

Convert each mixed number to an improper fraction.

1. $5\frac{7}{8}$

1. Multiply: $8 \cdot 5 = 40$.
2. Add: $40 + 7 = 47$.
3. Place 47 over 8: $\frac{47}{8}$.

Thus, $5\frac{7}{8} = \frac{47}{8}$.



2. $16\frac{2}{3}$

1. Multiply: $3 \cdot 16 = 48$.
2. Add: $48 + 2 = 50$.
3. Place 50 over 3: $\frac{50}{3}$.

Thus, $16\frac{2}{3} = \frac{50}{3}$

★ PRACTICE SET B

Convert each mixed number to its corresponding improper fraction.

1. $8\frac{1}{4}$ **2.** $5\frac{3}{5}$ **3.** $1\frac{4}{15}$ **4.** $12\frac{2}{7}$

Answers to Practice Sets are on p. 154.

Section 4.2 EXERCISES

For problems 1–15, identify each expression as a proper fraction, an improper fraction, or a mixed number.

1. $\frac{3}{2}$

2. $\frac{4}{9}$

3. $\frac{5}{7}$

4. $\frac{1}{8}$

5. $6\frac{1}{4}$

6. $\frac{11}{8}$

7. $\frac{1,001}{12}$

8. $191\frac{4}{5}$

9. $1\frac{9}{13}$

10. $31\frac{6}{7}$

11. $3\frac{1}{40}$

12. $\frac{55}{12}$

13. $\frac{0}{9}$

14. $\frac{8}{9}$

15. $101\frac{1}{11}$

For problems 16–30, convert each of the improper fractions to its corresponding mixed number.

16. $\frac{11}{6}$

17. $\frac{14}{3}$

18. $\frac{25}{4}$

19. $\frac{35}{4}$

20. $\frac{71}{8}$

21. $\frac{63}{7}$

22. $\frac{121}{11}$

23. $\frac{165}{12}$

24. $\frac{346}{15}$

25. $\frac{5,000}{9}$

26. $\frac{23}{5}$

27. $\frac{73}{2}$

28. $\frac{19}{2}$

29. $\frac{316}{41}$

30. $\frac{800}{3}$

For problems 31–45, convert each of the mixed numbers to its corresponding improper fraction.

31. $4\frac{1}{8}$

32. $1\frac{5}{12}$

33. $6\frac{7}{9}$

34. $15\frac{1}{4}$

35. $10\frac{5}{11}$

36. $15\frac{3}{10}$

37. $8\frac{2}{3}$

38. $4\frac{3}{4}$

39. $21\frac{2}{5}$

40. $17\frac{9}{10}$

41. $9\frac{20}{21}$

42. $5\frac{1}{16}$

43. $90\frac{1}{100}$

44. $300\frac{43}{1,000}$

45. $19\frac{7}{8}$

46. Why does $0\frac{4}{7}$ not qualify as a mixed number? (*Hint:* See the definition of a mixed number.)

47. Why does 5 qualify as a mixed number? (*Hint:* See the definition of a mixed number.)

Calculator Problems

For problems 48–55, use a calculator to convert each mixed number to its corresponding improper fraction.

48. $35\frac{11}{12}$

49. $27\frac{5}{61}$

50. $83\frac{40}{41}$

51. $105\frac{21}{23}$

52. $72\frac{605}{606}$

53. $816\frac{19}{25}$

54. $708\frac{42}{51}$

55. $6,012\frac{4,216}{8,117}$

EXERCISES FOR REVIEW

(1.3) 56. Round 2,614,000 to the nearest thousand.

(2.1) 57. Find the product. $1{,}004 \cdot 1{,}005$.

(2.4) 58. Determine if 41,826 is divisible by 2 and 3.

(3.5) 59. Find the least common multiple of 28 and 36.

(4.1) 60. Specify the numerator and denominator of the fraction $\frac{12}{19}$.

★ Answers to Practice Sets (4.2)

A. 1. $4\frac{1}{2}$ 2. $3\frac{2}{3}$ 3. $1\frac{3}{11}$ 4. $2\frac{5}{13}$ 5. $19\frac{3}{4}$ 6. 62

B. 1. $\frac{33}{4}$ 2. $\frac{28}{5}$ 3. $\frac{19}{15}$ 4. $\frac{86}{7}$

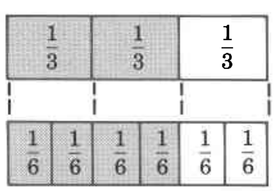
4.3 Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Terms

Section Overview

- ☐ EQUIVALENT FRACTIONS
- ☐ REDUCING FRACTIONS TO LOWEST TERMS
- ☐ RAISING FRACTIONS TO HIGHER TERMS

☐ EQUIVALENT FRACTIONS

Let's examine the following two diagrams.



$\frac{2}{3}$ of the whole is shaded.

$\frac{4}{6}$ of the whole is shaded.

Notice that both $\frac{2}{3}$ and $\frac{4}{6}$ represent the *same part* of the whole, that is, they represent the same number.

Equivalent Fractions

> Fractions that have the same value are called **equivalent fractions.** Equivalent fractions may look different, but they are still the same point on the number line.

There is an interesting property that equivalent fractions satisfy.

$$\frac{2}{3} \quad \frac{4}{6}$$

A Test for Equivalent Fractions Using the Cross Product

These pairs of products are called **cross products.**

$2 \cdot 6 \stackrel{?}{=} 3 \cdot 4$
$12 \stackrel{\checkmark}{=} 12$

If the cross products are equal, the fractions are equivalent. If the cross products are not equal, the fractions are not equivalent.

Thus, $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent, that is, $\frac{2}{3} = \frac{4}{6}$.

☆ SAMPLE SET A

Determine if the following pairs of fractions are equivalent.

1. $\frac{3}{4}$ and $\frac{6}{8}$. **Test for equality of the cross products.**

$\frac{3}{4} \times \frac{6}{8}$

$3 \cdot 8 \stackrel{?}{=} 6 \cdot 4$
$24 \stackrel{\checkmark}{=} 24$ **The cross products are equal.**

The fractions $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent, so $\frac{3}{4} = \frac{6}{8}$.

2. $\frac{3}{8}$ and $\frac{9}{16}$. **Test for equality of the cross products.**

$\frac{3}{8} \times \frac{9}{16}$

$3 \cdot 16 \stackrel{?}{=} 9 \cdot 8$
$48 \neq 72$ **The cross products are *not* equal.**

The fractions $\frac{3}{8}$ and $\frac{9}{16}$ are not equivalent.

★ PRACTICE SET A

Determine if the pairs of fractions are equivalent.

1. $\frac{1}{2}, \frac{3}{6}$
2. $\frac{4}{5}, \frac{12}{15}$
3. $\frac{2}{3}, \frac{8}{15}$
4. $\frac{1}{8}, \frac{5}{40}$
5. $\frac{3}{12}, \frac{1}{4}$

☐ REDUCING FRACTIONS TO LOWEST TERMS

It is often very useful to *convert* one fraction to an equivalent fraction that has reduced values in the numerator and denominator. We can suggest a method for doing so by considering the equivalent fractions $\frac{9}{15}$ and $\frac{3}{5}$. First, divide both the numerator and denominator of $\frac{9}{15}$ by 3. The fractions $\frac{9}{15}$ and $\frac{3}{5}$ are equivalent.

(Can you prove this?) So, $\frac{9}{15} = \frac{3}{5}$. We wish to convert $\frac{9}{15}$ to $\frac{3}{5}$. Now divide the numerator and denominator of $\frac{9}{15}$ by 3, and see what happens.

$$\frac{9 \div 3}{15 \div 2} = \frac{3}{5}$$

The fraction $\frac{9}{15}$ is converted to $\frac{3}{5}$.

A natural question is "Why did we choose to divide by 3?" Notice that

$$\frac{9}{15} = \frac{3 \cdot 3}{5 \cdot 3}$$

We can see that the *factor* 3 is common to both the numerator and denominator.

From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has reduced values in the numerator and denominator. The method is called **reducing a fraction.**

Reducing a Fraction

A fraction can be **reduced** by dividing *both* the numerator and denominator by the *same* nonzero whole number.

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4} \qquad \frac{16}{30} = \frac{16 \div 2}{30 \div 2} = \frac{8}{15}$$

Notice that $\frac{3}{3} = 1$ and $\frac{2}{2} = 1$

Consider the collection of equivalent fractions

$$\frac{5}{20}, \quad \frac{4}{16}, \quad \frac{3}{12}, \quad \frac{2}{8}, \quad \frac{1}{4}$$

Notice that each of the first four fractions can be *reduced* to the last fraction, $\frac{1}{4}$, by dividing both the numerator and denominator by, respectively, 5, 4, 3, and 2. When a fraction is converted to the fraction that has the smallest numerator and denominator in its collection of equivalent fractions, it is said to be **reduced to lowest terms.** The fractions $\frac{1}{4}, \frac{3}{8}, \frac{2}{5}$, and $\frac{7}{10}$ are all reduced to lowest terms.

Reduced to Lowest Terms

Observe a very important property of a fraction that has been reduced to lowest terms. The *only* whole number that divides *both* the numerator and denominator without a remainder is the number 1. When 1 is the only whole number that divides two whole numbers, the two whole numbers are said to be **relatively prime.**

Relatively Prime

A fraction is reduced to lowest terms if its numerator and denominator are relatively prime.

METHODS OF REDUCING FRACTIONS TO LOWEST TERMS

Method 1: DIVIDING OUT COMMON PRIMES

1. Write the numerator and denominator as a product of primes.
2. Divide the numerator and denominator by each of the common prime factors. We often indicate this division by drawing a slanted line through each divided out factor. This process is also called **cancelling common factors.**

Dividing Out (Cancelling) Common Factors

3. The product of the remaining factors in the numerator and the product of remaining factors of the denominator are relatively prime, and this fraction is reduced to lowest terms.

☆ SAMPLE SET B

Reduce each fraction to lowest terms.

1. $\frac{6}{18} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{1}{3}.$ **1 and 3 are relatively prime.**

2. $\frac{16}{20} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 5} = \frac{4}{5}.$ **4 and 5 are relatively prime.**

3. $\frac{56}{104} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 7}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 13} = \frac{7}{13}.$ **7 and 13 are relatively prime (and also truly prime).**

4. $\frac{315}{336} = \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot 5 \cdot \overset{1}{\cancel{7}}}{2 \cdot 2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{7}}} = \frac{15}{16}.$ **15 and 16 are relatively prime.**

5. $\frac{8}{15} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 5}.$ **No common prime factors, so 8 and 15 are relatively prime.**

   The fraction $\frac{8}{15}$ is reduced to lowest terms.

★ PRACTICE SET B

Reduce each fraction to lowest terms.

1. $\frac{4}{8}$
2. $\frac{6}{15}$
3. $\frac{6}{48}$
4. $\frac{21}{48}$
5. $\frac{72}{42}$
6. $\frac{135}{243}$

Method 2: DIVIDING OUT COMMON FACTORS

1. Mentally divide the numerator and the denominator by a factor that is common to each. Write the quotient above the original number.
2. Continue this process until the numerator and denominator are relatively prime.

☆ SAMPLE SET C

Reduce each fraction to lowest terms.

1. $\frac{25}{30}$. **5 divides into both 25 and 30.**

$\frac{\overset{5}{\cancel{25}}}{\underset{6}{\cancel{30}}} = \frac{5}{6}$ **5 and 6 are relatively prime.**

2. $\frac{18}{24}$. **Both numbers are even so we can divide by 2.**

$\frac{\overset{9}{\cancel{18}}}{\underset{12}{\cancel{24}}}$ **Now, both 9 and 12 are divisible by 3.**

$\frac{\overset{\overset{3}{\cancel{9}}}{\cancel{18}}}{\underset{\underset{4}{\cancel{12}}}{\cancel{24}}} = \frac{3}{4}$ **3 and 4 are relatively prime.**

3. $\frac{\overset{\overset{7}{\cancel{21}}}{\cancel{210}}}{\underset{\underset{5}{\cancel{15}}}{\cancel{150}}} = \frac{7}{5}$. **7 and 5 are relatively prime.**

4. $\frac{36}{96} = \frac{18}{48} = \frac{9}{24} = \frac{3}{8}$. **3 and 8 are relatively prime.**

★ PRACTICE SET C

Reduce each fraction to lowest terms.

1. $\frac{12}{16}$ 2. $\frac{9}{24}$ 3. $\frac{21}{84}$ 4. $\frac{48}{64}$ 5. $\frac{63}{81}$ 6. $\frac{150}{240}$

❑ RAISING FRACTIONS TO HIGHER TERMS

Equally as important as reducing fractions is raising fractions to higher terms. Raising a fraction to higher terms is the process of constructing an equivalent fraction that has higher values in the numerator and denominator than the original fraction.

The fractions $\frac{3}{5}$ and $\frac{9}{15}$ are equivalent, that is, $\frac{3}{5} = \frac{9}{15}$. Notice also,

$$\frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15}$$

Notice that $\frac{3}{3} = 1$ and that $\frac{3}{5} \cdot 1 = \frac{3}{5}$. We are not changing the value of $\frac{3}{5}$.

From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has higher values in the numerator and denominator. This method is called **raising a fraction to higher terms.**

Raising a Fraction to Higher Terms

A fraction can be raised to an equivalent fraction that has higher terms in the numerator and denominator by multiplying both the numerator and denominator by the same nonzero whole number.

The fraction $\frac{3}{4}$ can be raised to $\frac{24}{32}$ by multiplying both the numerator and denominator by 8.

$$\frac{3}{4} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32}$$

Notice that $\frac{8}{8} = 1$.

Most often, we will want to convert a given fraction to an equivalent fraction with a higher specified denominator. For example, we may wish to convert $\frac{5}{8}$ to an equivalent fraction that has denominator 32, that is,

$$\frac{5}{8} = \frac{?}{32}$$

This is possible to do because we know the process. We must multiply *both* the numerator and denominator of $\frac{5}{8}$ by the *same* nonzero whole number in order to obtain an equivalent fraction.

We have some information. The denominator 8 was raised to 32 by multiplying it by some nonzero whole number. Division will give us the proper factor. Divide the original denominator into the new denominator.

$32 \div 8 = 4$

Now, multiply the numerator 5 by 4.

$5 \cdot 4 = 20$

Thus,

$$\frac{5}{8} = \frac{5 \cdot 4}{8 \cdot 4} = \frac{20}{32}$$

So,

$$\frac{5}{8} = \frac{20}{32}$$

☆ SAMPLE SET D

Determine the missing numerator or denominator.

1. $\frac{3}{7} = \frac{?}{35}$. **Divide the original denominator into the new denominator.**

   $35 \div 7 = 5$ **The quotient is 5. Multiply the original numerator by 5.**

   $\frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$ **The missing numerator is 15.**

Continued

2. $\frac{5}{6}=\frac{45}{?}$. Divide the original numerator into the new numerator.

$45 \div 5 = 9$ The quotient is 9. Multiply the original denominator by 9.

$\frac{5}{6}=\frac{5 \cdot 9}{6 \cdot 9}=\frac{45}{54}$ The missing denominator is 45.

★ PRACTICE SET D

Determine the missing numerator or denominator.

1. $\frac{4}{5}=\frac{?}{40}$

2. $\frac{3}{7}=\frac{?}{28}$

3. $\frac{1}{6}=\frac{?}{24}$

4. $\frac{3}{10}=\frac{45}{?}$

5. $\frac{8}{15}=\frac{?}{165}$

Answers to Practice Sets are on p. 164.

Section 4.3 EXERCISES

For problems 1–15, determine if the pairs of fractions are equivalent.

1. $\frac{1}{2}, \frac{5}{10}$

2. $\frac{2}{3}, \frac{8}{12}$

3. $\frac{5}{12}, \frac{10}{24}$

4. $\frac{1}{2}, \frac{3}{6}$

5. $\frac{3}{5}, \frac{12}{15}$

6. $\frac{1}{6}, \frac{7}{42}$

7. $\frac{16}{25}, \frac{49}{75}$

8. $\frac{5}{28}, \frac{20}{112}$

9. $\frac{3}{10}, \frac{36}{110}$

10. $\frac{6}{10}, \frac{18}{32}$

11. $\frac{5}{8}, \frac{15}{24}$

12. $\frac{10}{16}, \frac{15}{24}$

13. $\frac{4}{5}, \frac{3}{4}$

14. $\frac{5}{7}, \frac{15}{21}$

15. $\frac{9}{11}, \frac{11}{9}$

For problems 16–35, determine the missing numerator or denominator.

16. $\frac{1}{3} = \frac{?}{12}$

17. $\frac{1}{5} = \frac{?}{30}$

18. $\frac{2}{3} = \frac{?}{9}$

19. $\frac{3}{4} = \frac{?}{16}$

20. $\frac{5}{6} = \frac{?}{18}$

21. $\frac{4}{5} = \frac{?}{25}$

22. $\frac{1}{2} = \frac{4}{?}$

23. $\frac{9}{25} = \frac{27}{?}$

24. $\frac{3}{2} = \frac{18}{?}$

25. $\frac{5}{3} = \frac{80}{?}$

26. $\frac{1}{8} = \frac{3}{?}$

27. $\frac{4}{5} = \frac{?}{100}$

28. $\frac{1}{2} = \frac{25}{?}$

29. $\frac{3}{16} = \frac{?}{96}$

30. $\frac{15}{16} = \frac{225}{?}$

31. $\frac{11}{12} = \frac{?}{168}$

32. $\frac{9}{13} = \frac{?}{286}$

33. $\frac{32}{33} = \frac{?}{1518}$

34. $\frac{19}{20} = \frac{1045}{?}$

35. $\frac{37}{50} = \frac{1369}{?}$

For problems 36–85, reduce, if possible, each of the fractions to lowest terms.

36. $\frac{6}{8}$

37. $\frac{8}{10}$

38. $\frac{5}{10}$

39. $\frac{6}{14}$

40. $\frac{3}{12}$

41. $\frac{4}{14}$

42. $\frac{1}{6}$

43. $\frac{4}{6}$

44. $\frac{18}{14}$

45. $\frac{20}{8}$

46. $\frac{4}{6}$

47. $\frac{10}{6}$

48. $\frac{6}{14}$

49. $\frac{14}{6}$

50. $\frac{10}{12}$

51. $\frac{16}{70}$

52. $\frac{40}{60}$

53. $\frac{20}{12}$

54. $\frac{32}{28}$

55. $\frac{36}{10}$

56. $\frac{36}{60}$

57. $\frac{12}{18}$

58. $\frac{18}{27}$

59. $\frac{18}{24}$

60. $\frac{32}{40}$

61. $\frac{11}{22}$

62. $\frac{27}{81}$

63. $\frac{17}{51}$

64. $\frac{16}{42}$

65. $\frac{39}{13}$

66. $\frac{44}{11}$

67. $\frac{66}{33}$

68. $\frac{15}{1}$

69. $\frac{15}{16}$

70. $\frac{15}{40}$

71. $\frac{36}{100}$

72. $\frac{45}{32}$

73. $\frac{30}{75}$

74. $\frac{121}{132}$

75. $\frac{72}{64}$

76. $\frac{30}{105}$

77. $\frac{46}{60}$

78. $\frac{75}{45}$

79. $\frac{40}{18}$

80. $\frac{108}{76}$

81. $\frac{7}{21}$

82. $\frac{6}{51}$

83. $\frac{51}{12}$

84. $\frac{8}{100}$

85. $\frac{51}{54}$

86. A ream of paper contains 500 sheets. What fraction of a ream of paper is 200 sheets? Be sure to reduce.

87. There are 24 hours in a day. What fraction of a day is 14 hours?

88. A full box contains 80 calculators. How many calculators are in $\frac{1}{4}$ of a box?

89. There are 48 plants per flat. How many plants are there in $\frac{1}{3}$ of a flat?

90. A person making $18,000 per year must pay $3,960 in income tax. What fraction of this person's yearly salary goes to the IRS?

For problems 91–95, find the mistake.

91. $\frac{3}{24} = \frac{\cancel{3}}{\cancel{3} \cdot 8} = \frac{0}{8} = 0$

92. $\frac{8}{10} = \frac{\cancel{2}+6}{\cancel{2}+8} = \frac{6}{8} = \frac{3}{4}$

93. $\frac{7}{15} = \frac{\cancel{7}}{\cancel{7}+8} = \frac{1}{8}$

94. $\frac{6}{7} = \frac{\cancel{5}+1}{\cancel{5}+2} = \frac{1}{2}$

95. $\frac{\cancel{9}}{\cancel{9}} = \frac{0}{0} = 0$

EXERCISES FOR REVIEW

(1.3) **96.** Round 816 to the nearest thousand.

(2.2) **97.** Perform the division: $0 \div 6$.

(3.3) **98.** Find all the factors of 24.

(3.4) **99.** Find the greatest common factor of 12 and 18.

(4.2) **100.** Convert $\frac{15}{8}$ to a mixed number.

★ Answers to Practice Sets (4.3)

A. **1.** $6 \stackrel{?}{=} 6$, yes **2.** $60 \stackrel{?}{=} 60$, yes **3.** $30 \neq 24$, no **4.** $40 \stackrel{?}{=} 40$, yes **5.** $12 \stackrel{?}{=} 12$, yes

B. **1.** $\frac{1}{2}$ **2.** $\frac{2}{5}$ **3.** $\frac{1}{8}$ **4.** $\frac{7}{16}$ **5.** $\frac{12}{7}$ **6.** $\frac{5}{9}$

C. **1.** $\frac{3}{4}$ **2.** $\frac{3}{8}$ **3.** $\frac{1}{4}$ **4.** $\frac{3}{4}$ **5.** $\frac{7}{9}$ **6.** $\frac{5}{8}$

D. **1.** 32 **2.** 12 **3.** 4 **4.** 150 **5.** 88

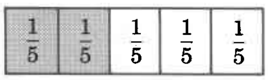
4.4 Multiplication of Fractions

Section Overview

- ❑ FRACTIONS OF FRACTIONS
- ❑ MULTIPLICATION OF FRACTIONS
- ❑ MULTIPLICATION OF FRACTIONS BY DIVIDING OUT COMMON FACTORS
- ❑ MULTIPLICATION OF MIXED NUMBERS
- ❑ POWERS AND ROOTS OF FRACTIONS

❑ FRACTIONS OF FRACTIONS

We know that a fraction represents a part of a whole quantity. For example, two fifths of one unit can be represented by

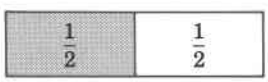


$\frac{2}{5}$ of the whole is shaded.

A natural question is, what is a fractional part of a fractional quantity, or, what is a fraction of a fraction? For example, what is $\frac{2}{3}$ of $\frac{1}{2}$?

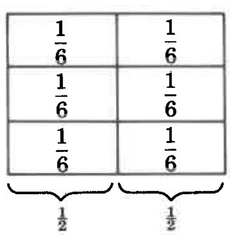
We can suggest an answer to this question by using a picture to examine $\frac{2}{3}$ of $\frac{1}{2}$.

First, let's represent $\frac{1}{2}$.



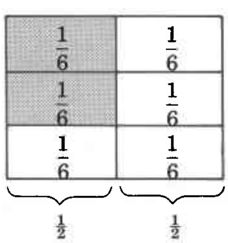
$\frac{1}{2}$ of the whole is shaded

Then divide each of the $\frac{1}{2}$ parts into 3 equal parts.



Each part is $\frac{1}{6}$ of the whole.

Now we'll take $\frac{2}{3}$ of the $\frac{1}{2}$ unit.



$\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$, which reduces to $\frac{1}{3}$.

MULTIPLICATION OF FRACTIONS

Now we ask, what arithmetic operation $(+, -, \times, \div)$ will produce $\frac{2}{6}$ from $\frac{2}{3}$ of $\frac{1}{2}$?

Notice that, if in the fractions $\frac{2}{3}$ and $\frac{1}{2}$, we multiply the numerators together and the denominators together, we get precisely $\frac{2}{6}$.

$$\frac{2 \cdot 1}{3 \cdot 2} = \frac{2}{6}$$

This reduces to $\frac{1}{3}$ as before.

Using this observation, we can suggest the following:

The Word "OF" Indicates Multiplication

The Method of Multiplying Fractions

1. The word "of" translates to the arithmetic operation "times."
2. To multiply two or more fractions, multiply the numerators together and then multiply the denominators together. Reduce if necessary.

$$\frac{\text{numerator 1}}{\text{denominator 1}} \cdot \frac{\text{numerator 2}}{\text{denominator 2}} = \frac{\text{numerator 1} \cdot \text{numerator 2}}{\text{denominator 1} \cdot \text{denominator 2}}$$

☆ SAMPLE SET A

Perform the following multiplications.

1. $\frac{3}{4} \cdot \frac{1}{6} = \frac{3 \cdot 1}{4 \cdot 6} = \frac{3}{24}$. **Now, reduce.**

$$= \frac{\overset{1}{\cancel{3}}}{\underset{8}{\cancel{24}}} = \frac{1}{8}$$

Thus,

$$\frac{3}{4} \cdot \frac{1}{6} = \frac{1}{8}$$

This means that $\frac{3}{4}$ of $\frac{1}{6}$ is $\frac{1}{8}$, that is, $\frac{3}{4}$ of $\frac{1}{6}$ of a unit is $\frac{1}{8}$ of the original unit.

2. $\frac{3}{8} \cdot 4$. **Write 4 as a fraction by writing $\frac{4}{1}$.**

$$\frac{3}{8} \cdot \frac{4}{1} = \frac{3 \cdot 4}{8 \cdot 1} = \frac{12}{8} = \frac{\overset{3}{\cancel{12}}}{\underset{2}{\cancel{8}}} = \frac{3}{2}$$

$$\frac{3}{8} \cdot 4 = \frac{3}{2}$$

This means that $\frac{3}{8}$ of 4 whole units is $\frac{3}{2}$ of one whole unit.

3. $\frac{2}{5} \cdot \frac{5}{8} \cdot \frac{1}{4} = \frac{2 \cdot 5 \cdot 1}{5 \cdot 8 \cdot 4} = \frac{\overset{1}{\cancel{10}}}{\underset{16}{\cancel{160}}} = \frac{1}{16}$

This means that $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{1}{4}$ of a whole unit is $\frac{1}{16}$ of the original unit.

★ PRACTICE SET A

Perform the following multiplications.

1. $\frac{2}{5} \cdot \frac{1}{6}$
2. $\frac{1}{4} \cdot \frac{8}{9}$
3. $\frac{4}{9} \cdot \frac{15}{16}$
4. $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$
5. $\left(\frac{7}{4}\right)\left(\frac{8}{5}\right)$
6. $\frac{5}{6} \cdot \frac{7}{8}$
7. $\frac{2}{3} \cdot 5$
8. $\left(\frac{3}{4}\right)(10)$
9. $\frac{3}{4} \cdot \frac{8}{9} \cdot \frac{5}{12}$

☐ MULTIPLYING FRACTIONS BY DIVIDING OUT COMMON FACTORS

We have seen that to multiply two fractions together, we multiply numerators together, then denominators together, then reduce to lowest terms, if necessary. The reduction can be tedious if the numbers in the fractions are large. For example,

$$\frac{9}{16} \cdot \frac{10}{21} = \frac{9 \cdot 10}{16 \cdot 21} = \frac{90}{336} = \frac{45}{168} = \frac{15}{28}$$

We avoid the process of reducing if we divide out common factors *before* we multiply.

$$\frac{9}{16} \cdot \frac{10}{21} = \frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{7}{\cancel{21}}} = \frac{3 \cdot 5}{8 \cdot 7} = \frac{15}{56}$$

Divide 3 into 9 and 21, and divide 2 into 10 and 16. The product is a fraction that is reduced to lowest terms.

The Process of Multiplication by Dividing Out Common Factors

To multiply fractions by dividing out common factors, divide out factors that are common to both a numerator and a denominator. The factor being divided out can appear in any numerator and any denominator.

☆ SAMPLE SET B

Perform the following multiplications.

1. $\frac{4}{5} \cdot \frac{5}{6}$

$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} = \frac{2 \cdot 1}{1 \cdot 3} = \frac{2}{3}$$

Divide 4 and 6 by 2.
Divide 5 and 5 by 5.

2. $\frac{8}{12} \cdot \frac{8}{10}$

$$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{12}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{10}}} = \frac{4 \cdot 2}{3 \cdot 5} = \frac{8}{15}$$

Divide 8 and 10 by 2.
Divide 8 and 12 by 4.

3. $8 \cdot \frac{5}{12} = \frac{\overset{2}{\cancel{8}}}{1} \cdot \frac{5}{\underset{3}{\cancel{12}}} = \frac{2 \cdot 5}{1 \cdot 3} = \frac{10}{3}$

4. $\frac{35}{18} \cdot \frac{63}{105}$

$$\frac{\overset{\overset{1}{\cancel{7}}}{\cancel{35}}}{\underset{2}{\cancel{18}}} \cdot \frac{\overset{7}{\cancel{63}}}{\underset{\underset{3}{\cancel{21}}}{\cancel{105}}} = \frac{1 \cdot 7}{2 \cdot 3} = \frac{7}{6}$$

5. $\frac{13}{9} \cdot \frac{6}{39} \cdot \frac{1}{12}$

$$\frac{\overset{1}{\cancel{13}}}{9} \cdot \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{39}}} \cdot \frac{1}{\underset{6}{\cancel{12}}} = \frac{1 \cdot 1 \cdot 1}{9 \cdot 1 \cdot 6} = \frac{1}{54}$$

★ PRACTICE SET B

Perform the following multiplications.

1. $\frac{2}{3} \cdot \frac{7}{8}$

2. $\frac{25}{12} \cdot \frac{10}{45}$

3. $\frac{40}{48} \cdot \frac{72}{90}$

4. $7 \cdot \frac{2}{49}$

5. $12 \cdot \frac{3}{8}$

6. $\left(\frac{13}{7}\right)\left(\frac{14}{26}\right)$

7. $\frac{16}{10} \cdot \frac{22}{6} \cdot \frac{21}{44}$

☐ MULTIPLICATION OF MIXED NUMBERS

Multiplying Mixed Numbers

To perform a multiplication in which there are mixed numbers, it is convenient to first convert each mixed number to an improper fraction, then multiply.

☆ SAMPLE SET C

Perform the following multiplications. Convert improper fractions to mixed numbers.

1. $1\frac{1}{8} \cdot 4\frac{2}{3}$.

Convert each mixed number to an improper fraction.

$$1\frac{1}{8} = \frac{8 \cdot 1 + 1}{8} = \frac{9}{8}.$$

$$4\frac{2}{3} = \frac{4 \cdot 3 + 2}{3} = \frac{14}{3}.$$

$$\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{7}{\cancel{14}}}{\underset{1}{\cancel{3}}} = \frac{3 \cdot 7}{4 \cdot 1} = \frac{21}{4} = 5\frac{1}{4}$$

2. $16 \cdot 8\frac{1}{5}$.

Convert $8\frac{1}{5}$ to an improper fraction.

$$8\frac{1}{5} = \frac{5 \cdot 8 + 1}{5} = \frac{41}{5}.$$

$$\frac{16}{1} \cdot \frac{41}{5}.$$

There are no common factors to divide out.

$$\frac{16}{1} \cdot \frac{41}{5} = \frac{16 \cdot 41}{1 \cdot 5} = \frac{656}{5} = 131\frac{1}{5}$$

3. $9\frac{1}{6} \cdot 12\frac{3}{5}$.

Convert to improper fractions.

$$9\frac{1}{6} = \frac{6 \cdot 9 + 1}{6} = \frac{55}{6}.$$

$$12\frac{3}{5} = \frac{5 \cdot 12 + 3}{5} = \frac{63}{5}.$$

$$\frac{\overset{11}{\cancel{55}}}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{21}{\cancel{63}}}{\underset{1}{\cancel{5}}} = \frac{11 \cdot 21}{2 \cdot 1} = \frac{231}{2} = 115\frac{1}{2}$$

4. $\frac{11}{8} \cdot 4\frac{1}{2} \cdot 3\frac{1}{3} = \frac{11}{8} \cdot \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}}$

$$= \frac{11 \cdot 3 \cdot 5}{8 \cdot 1 \cdot 1} = \frac{165}{8} = 20\frac{5}{8}$$

★ PRACTICE SET C

Perform the following multiplications. Convert improper fractions to mixed numbers.

1. $2\frac{2}{3} \cdot 2\frac{1}{4}$
2. $6\frac{2}{3} \cdot 3\frac{3}{10}$
3. $7\frac{1}{8} \cdot 12$
4. $2\frac{2}{5} \cdot 3\frac{3}{4} \cdot 3\frac{1}{3}$

❑ POWERS AND ROOTS OF FRACTIONS

☆ SAMPLE SET D

Find the value of each of the following.

1. $\left(\frac{1}{6}\right)^2 = \frac{1}{6} \cdot \frac{1}{6} = \frac{1 \cdot 1}{6 \cdot 6} = \frac{1}{36}$.

2. $\sqrt{\frac{9}{100}}$. We're looking for a number, call it ?, such that when it is squared, $\frac{9}{100}$ is produced.

$(?)^2 = \frac{9}{100}$

We know that

$3^2 = 9$ and $10^2 = 100$

We'll try $\frac{3}{10}$. Since

$\left(\frac{3}{10}\right)^2 = \frac{3}{10} \cdot \frac{3}{10} = \frac{3 \cdot 3}{10 \cdot 10} = \frac{9}{100}$

$\sqrt{\frac{9}{100}} = \frac{3}{10}$

3. $4\frac{2}{5} \cdot \sqrt{\frac{100}{121}}$

$\frac{\overset{2}{\cancel{22}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{11}}} = \frac{2 \cdot 2}{1 \cdot 1} = \frac{4}{1} = 4$

$4\frac{2}{5} \cdot \sqrt{\frac{100}{121}} = 4$

★ PRACTICE SET D

Find the value of each of the following.

1. $\left(\frac{1}{8}\right)^2$ **2.** $\left(\frac{3}{10}\right)^2$ **3.** $\sqrt{\frac{4}{9}}$ **4.** $\sqrt{\frac{1}{4}}$ **5.** $\frac{3}{8} \cdot \sqrt{\frac{1}{9}}$ **6.** $9\frac{1}{3} \cdot \sqrt{\frac{81}{100}}$

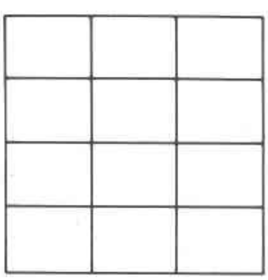
7. $2\frac{8}{13} \cdot \sqrt{\frac{169}{16}}$

Answers to Practice Sets are on p. 175.

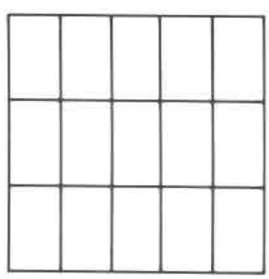
Section 4.4 EXERCISES

For problems 1–6, use the diagrams to find each of the following parts. Use multiplication to verify your result.

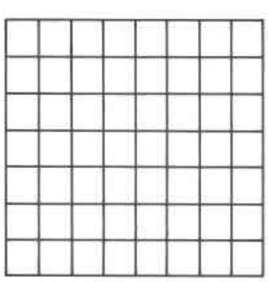
1. $\frac{3}{4}$ of $\frac{1}{3}$



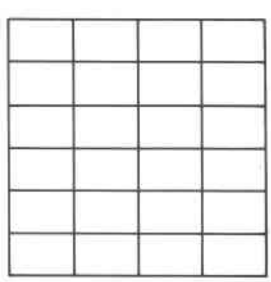
2. $\frac{2}{3}$ of $\frac{3}{5}$



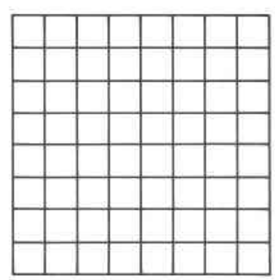
3. $\frac{2}{7}$ of $\frac{7}{8}$



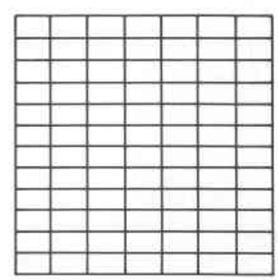
4. $\frac{5}{6}$ of $\frac{3}{4}$



5. $\frac{1}{8}$ of $\frac{1}{8}$



6. $\frac{7}{12}$ of $\frac{6}{7}$



For problems 7–25, find each part without using a diagram.

7. $\frac{1}{2}$ of $\frac{4}{5}$.

8. $\frac{3}{5}$ of $\frac{5}{12}$

9. $\frac{1}{4}$ of $\frac{8}{9}$

10. $\frac{3}{16}$ of $\frac{12}{15}$

11. $\frac{2}{9}$ of $\frac{6}{5}$

12. $\frac{1}{8}$ of $\frac{3}{8}$

13. $\frac{2}{3}$ of $\frac{9}{10}$

14. $\frac{18}{19}$ of $\frac{38}{54}$

15. $\frac{5}{6}$ of $2\frac{2}{5}$

16. $\frac{3}{4}$ of $3\frac{3}{5}$

17. $\frac{3}{2}$ of $2\frac{2}{9}$

18. $\frac{15}{4}$ of $4\frac{4}{5}$

19. $5\frac{1}{3}$ of $9\frac{3}{4}$

20. $1\frac{13}{15}$ of $8\frac{3}{4}$

21. $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{2}{3}$

22. $\frac{1}{6}$ of $\frac{12}{13}$ of $\frac{26}{36}$

23. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$

24. $1\frac{3}{7}$ of $5\frac{1}{5}$ of $8\frac{1}{3}$

25. $2\frac{4}{5}$ of $5\frac{5}{6}$ of $7\frac{5}{7}$

For problems 26–70, find the products. Be sure to reduce.

26. $\frac{1}{3} \cdot \frac{2}{3}$

27. $\frac{1}{2} \cdot \frac{1}{2}$

28. $\frac{3}{4} \cdot \frac{3}{8}$

29. $\frac{2}{5} \cdot \frac{5}{6}$

30. $\frac{3}{8} \cdot \frac{8}{9}$

31. $\frac{5}{6} \cdot \frac{14}{15}$

32. $\frac{4}{7} \cdot \frac{7}{4}$

33. $\frac{3}{11} \cdot \frac{11}{3}$

34. $\frac{9}{16} \cdot \frac{20}{27}$

35. $\frac{35}{36} \cdot \frac{48}{55}$

36. $\frac{21}{25} \cdot \frac{15}{14}$

37. $\frac{76}{99} \cdot \frac{66}{38}$

38. $\frac{3}{7} \cdot \frac{14}{18} \cdot \frac{6}{2}$

39. $\frac{4}{15} \cdot \frac{10}{3} \cdot \frac{27}{2}$

40. $\frac{14}{15} \cdot \frac{21}{28} \cdot \frac{45}{7}$

41. $\frac{8}{3} \cdot \frac{15}{4} \cdot \frac{16}{21}$

42. $\frac{18}{14} \cdot \frac{21}{35} \cdot \frac{36}{7}$

43. $\frac{3}{5} \cdot 20$

44. $\frac{8}{9} \cdot 18$

45. $\frac{6}{11} \cdot 33$

46. $\frac{18}{19} \cdot 38$

47. $\frac{5}{6} \cdot 10$

48. $\frac{1}{9} \cdot 3$

49. $5 \cdot \frac{3}{8}$

50. $16 \cdot \frac{1}{4}$

51. $\frac{2}{3} \cdot 12 \cdot \frac{3}{4}$

52. $\frac{3}{8} \cdot 24 \cdot \frac{2}{3}$

53. $\frac{5}{18} \cdot 10 \cdot \frac{2}{5}$

54. $\frac{16}{15} \cdot 50 \cdot \frac{3}{10}$

55. $5\frac{1}{3} \cdot \frac{27}{32}$

56. $2\frac{6}{7} \cdot 5\frac{3}{5}$

57. $6\frac{1}{4} \cdot 2\frac{4}{15}$

58. $9\frac{1}{3} \cdot \frac{9}{16} \cdot 1\frac{1}{3}$

59. $3\frac{5}{9} \cdot 1\frac{13}{14} \cdot 10\frac{1}{2}$

60. $20\frac{1}{4} \cdot 8\frac{2}{3} \cdot 16\frac{4}{5}$

61. $\left(\frac{2}{3}\right)^2$

62. $\left(\frac{3}{8}\right)^2$

63. $\left(\frac{2}{11}\right)^2$

64. $\left(\frac{8}{9}\right)^2$

65. $\left(\frac{1}{2}\right)^2$

66. $\left(\frac{3}{5}\right)^2 \cdot \frac{20}{3}$

67. $\left(\frac{1}{4}\right)^2 \cdot \frac{16}{15}$

68. $\left(\frac{1}{2}\right)^2 \cdot \frac{8}{9}$

69. $\left(\frac{1}{2}\right)^2 \cdot \left(\frac{2}{5}\right)^2$

70. $\left(\frac{3}{7}\right)^2 \cdot \left(\frac{1}{9}\right)^2$

For problems 71–80, find each value. Reduce answers to lowest terms or convert to mixed numbers.

71. $\sqrt{\frac{4}{9}}$

72. $\sqrt{\frac{16}{25}}$

73. $\sqrt{\frac{81}{121}}$

74. $\sqrt{\frac{36}{49}}$

75. $\sqrt{\frac{144}{25}}$

76. $\frac{2}{3} \cdot \sqrt{\frac{9}{16}}$

77. $\frac{3}{5} \cdot \sqrt{\frac{25}{81}}$

78. $\left(\frac{8}{5}\right)^2 \cdot \sqrt{\frac{25}{64}}$

79. $\left(1\frac{3}{4}\right)^2 \cdot \sqrt{\frac{4}{49}}$

80. $\left(2\frac{2}{3}\right)^2 \cdot \sqrt{\frac{36}{49}} \cdot \sqrt{\frac{64}{81}}$

EXERCISES FOR REVIEW

(1.1) **81.** How many thousands in 342,810?

(1.4) **82.** Find the sum of 22, 42, and 101.

(2.4) **83.** Is 634,281 divisible by 3?

(3.3) **84.** Is the whole number 51 prime or composite?

(4.3) **85.** Reduce $\frac{36}{150}$ to lowest terms.

★ Answers to Practice Sets (4.4)

A. 1. $\frac{1}{15}$ 2. $\frac{2}{9}$ 3. $\frac{5}{12}$ 4. $\frac{4}{9}$ 5. $\frac{14}{5}$ 6. $\frac{35}{48}$ 7. $\frac{10}{3}$ 8. $\frac{15}{2}$ 9. $\frac{5}{18}$

B. 1. $\frac{7}{12}$ 2. $\frac{25}{54}$ 3. $\frac{2}{3}$ 4. $\frac{2}{7}$ 5. $\frac{9}{2}$ 6. 1 7. $\frac{14}{5}$

C. 1. 6 2. 22 3. $85\frac{1}{2}$ 4. 30

D. 1. $\frac{1}{64}$ 2. $\frac{9}{100}$ 3. $\frac{2}{3}$ 4. $\frac{1}{2}$ 5. $\frac{1}{8}$ 6. $8\frac{2}{5}$ 7. $8\frac{1}{2}$

4.5 Division of Fractions

Section Overview

- ❑ RECIPROCALS
- ❑ DIVIDING FRACTIONS

❑ RECIPROCALS

Reciprocals

Two numbers whose product is 1 are called **reciprocals** of each other.

☆ SAMPLE SET A

The following pairs of numbers are reciprocals.

$$\underbrace{\frac{3}{4} \text{ and } \frac{4}{3}}_{\frac{3}{4}\cdot\frac{4}{3}=1} \qquad \underbrace{\frac{7}{16} \text{ and } \frac{16}{7}}_{\frac{7}{16}\cdot\frac{16}{7}=1} \qquad \underbrace{\frac{1}{6} \text{ and } \frac{6}{1}}_{\frac{1}{6}\cdot\frac{6}{1}=1}$$

Notice that we can find the reciprocal of a nonzero number in fractional form by inverting it (exchanging positions of the numerator and denominator).

★ PRACTICE SET A

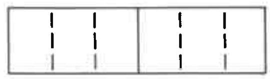
Find the reciprocal of each number.

1. $\frac{3}{10}$ **2.** $\frac{2}{3}$ **3.** $\frac{7}{8}$ **4.** $\frac{1}{5}$

5. $2\frac{2}{7}$ (*Hint:* Write this number as an improper fraction first.) **6.** $5\frac{1}{4}$ **7.** $10\frac{3}{16}$

❑ DIVIDING FRACTIONS

Our concept of division is that it indicates *how many times* one quantity is contained in another quantity. For example, using the diagram we can see that there are 6 one-thirds in 2.



There are 6 one-thirds in 2.

Since 2 contains six $\frac{1}{3}$'s, we express this as

$$2 \div \boxed{\frac{1}{3}} = 6$$

Note also that $2 \cdot \boxed{3} = 6$

$\frac{1}{3}$ and 3 are reciprocals

Using these observations, we can suggest the following method for dividing a number by a fraction.

Dividing One Fraction by Another Fraction

> To divide a first fraction by a second, nonzero fraction, multiply the first fraction by the reciprocal of the second fraction.

Invert and Multiply

This method is commonly referred to as **"invert the divisor and multiply."**

☆ SAMPLE SET B

Perform the following divisions.

1. $\frac{1}{3} \div \frac{3}{4}$.

The divisor is $\frac{3}{4}$. Its reciprocal is $\frac{4}{3}$. Multiply $\frac{1}{3}$ by $\frac{4}{3}$.

$$\frac{1}{3} \cdot \frac{4}{3} = \frac{1 \cdot 4}{3 \cdot 3} = \frac{4}{9}$$

$$\frac{1}{3} \div \frac{3}{4} = \frac{4}{9}$$

2. $\frac{3}{8} \div \frac{5}{4}$.

The divisor is $\frac{5}{4}$. Its reciprocal is $\frac{4}{5}$. Multiply $\frac{3}{8}$ by $\frac{4}{5}$.

$$\frac{3}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{4}}}{5} = \frac{3 \cdot 1}{2 \cdot 5} = \frac{3}{10}$$

$$\frac{3}{8} \div \frac{5}{4} = \frac{3}{10}$$

3. $\frac{5}{6} \div \frac{5}{12}$.

The divisor is $\frac{5}{12}$. Its reciprocal is $\frac{12}{5}$. Multiply $\frac{5}{6}$ by $\frac{12}{5}$.

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \cdot \frac{\overset{2}{\cancel{12}}}{\underset{1}{\cancel{5}}} = \frac{1 \cdot 2}{1 \cdot 1} = \frac{2}{1} = 2$$

$$\frac{5}{6} \div \frac{5}{12} = 2$$

4. $2\frac{2}{9} \div 3\frac{1}{3}$.

Convert each mixed number to an improper fraction.

$$2\frac{2}{9} = \frac{9 \cdot 2 + 2}{9} = \frac{20}{9}.$$

$$3\frac{1}{3} = \frac{3 \cdot 3 + 1}{3} = \frac{10}{3}.$$

$$\frac{20}{9} \div \frac{10}{3}$$

The divisor is $\frac{10}{3}$. Its reciprocal is $\frac{3}{10}$. Multiply $\frac{20}{9}$ by $\frac{3}{10}$.

$$\frac{\overset{2}{\cancel{20}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{10}}} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$$

$$2\frac{2}{9} \div 3\frac{1}{3} = \frac{2}{3}$$

5. $\frac{12}{11} \div 8$.

First conveniently write 8 as $\frac{8}{1}$.

$$\frac{12}{11} \div \frac{8}{1}$$

The divisor is $\frac{8}{1}$. Its reciprocal is $\frac{1}{8}$. Multiply $\frac{12}{11}$ by $\frac{1}{8}$.

$$\frac{\overset{3}{\cancel{12}}}{11} \cdot \frac{1}{\underset{2}{\cancel{8}}} = \frac{3 \cdot 1}{11 \cdot 2} = \frac{3}{22}$$

$$\frac{12}{11} \div 8 = \frac{3}{22}$$

6. $\frac{7}{8} \div \frac{21}{20} \cdot \frac{3}{35}$.

The divisor is $\frac{21}{20}$. Its reciprocal is $\frac{20}{21}$.

$$\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{20}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{21}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{35}}} = \frac{1 \cdot 1 \cdot 1}{2 \cdot 1 \cdot 7} = \frac{1}{14}$$

$$\frac{7}{8} \div \frac{21}{20} \cdot \frac{3}{25} = \frac{1}{14}$$

Continued

7. How many $2\frac{3}{8}$-inch-wide packages can be placed in a box 19 inches wide?

The problem is to determine how many two and three eighths are contained in 19, that is, what is $19 \div 2\frac{3}{8}$?

$2\frac{3}{8} = \frac{19}{8}$ **Convert the divisor $2\frac{3}{8}$ to an improper fraction.**

$19 = \frac{19}{1}$ **Write the dividend 19 as $\frac{19}{1}$.**

$\frac{19}{1} \div \frac{19}{8}$ **The divisor is $\frac{19}{8}$. Its reciprocal is $\frac{8}{19}$.**

$$\frac{\overset{1}{\cancel{19}}}{1} \cdot \frac{8}{\underset{1}{\cancel{19}}} = \frac{1 \cdot 8}{1 \cdot 1} = \frac{8}{1} = 8$$

Thus, 8 packages will fit into the box.

★ PRACTICE SET B

Perform the following divisions.

1. $\frac{1}{2} \div \frac{9}{8}$

2. $\frac{3}{8} \div \frac{9}{24}$

3. $\frac{7}{15} \div \frac{14}{15}$

4. $8 \div \frac{8}{15}$

5. $6\frac{1}{4} \div \frac{5}{12}$

6. $3\frac{1}{3} \div 1\frac{2}{3}$

7. $\frac{5}{6} \div \frac{2}{3} \cdot \frac{8}{25}$

8. A container will hold 106 ounces of grape juice. How many $6\frac{5}{8}$-ounce glasses of grape juice can be served from this container?

Determine each of the following quotients and then write a rule for this type of division.

9. $1 \div \frac{2}{3}$

10. $1 \div \frac{3}{8}$

11. $1 \div \frac{3}{4}$

12. $1 \div \frac{5}{2}$

13. When dividing 1 by a fraction, the quotient is the ______________________________

______________________________.

Answers to Practice Sets are on p. 181.

Section 4.5 EXERCISES

For problems 1–10, find the reciprocal of each number.

1. $\frac{4}{5}$

2. $\frac{8}{11}$

3. $\frac{2}{9}$

4. $\frac{1}{5}$

5. $3\frac{1}{4}$

6. $8\frac{1}{4}$

7. $3\frac{2}{7}$

8. $5\frac{3}{4}$

9. 1

10. 4

For problems 11–45, find each value.

11. $\frac{3}{8} \div \frac{3}{5}$

12. $\frac{5}{9} \div \frac{5}{6}$

13. $\frac{9}{16} \div \frac{15}{8}$

14. $\frac{4}{9} \div \frac{6}{15}$

15. $\frac{25}{49} \div \frac{4}{9}$

16. $\frac{15}{4} \div \frac{27}{8}$

17. $\frac{24}{75} \div \frac{8}{15}$

18. $\frac{5}{7} \div 0$

19. $\frac{7}{8} \div \frac{7}{8}$

20. $0 \div \frac{3}{5}$

21. $\frac{4}{11} \div \frac{4}{11}$

22. $\frac{2}{3} \div \frac{2}{3}$

23. $\frac{7}{10} \div \frac{10}{7}$

24. $\frac{3}{4} \div 6$

25. $\frac{9}{5} \div 3$

26. $4\frac{1}{6} \div 3\frac{1}{3}$

27. $7\frac{1}{7} \div 8\frac{1}{3}$

28. $1\frac{1}{2} \div 1\frac{1}{5}$

29. $3\frac{2}{5} \div \frac{6}{25}$

30. $5\frac{1}{6} \div \frac{31}{6}$

31. $\frac{35}{6} \div 3\frac{3}{4}$

32. $5\frac{1}{9} \div \frac{1}{18}$

33. $8\frac{3}{4} \div \frac{7}{8}$

34. $\frac{12}{8} \div 1\frac{1}{2}$

35. $3\frac{1}{8} \div \frac{15}{16}$

36. $11\frac{11}{12} \div 9\frac{5}{8}$

37. $2\frac{2}{9} \div 11\frac{2}{3}$

38. $\frac{16}{3} \div 6\frac{2}{5}$

39. $4\frac{3}{25} \div 2\frac{56}{75}$

40. $\frac{1}{1000} \div \frac{1}{100}$

41. $\frac{3}{8} \div \frac{9}{16} \cdot \frac{6}{5}$

42. $\frac{3}{16} \cdot \frac{9}{8} \cdot \frac{6}{5}$

43. $\frac{4}{15} \div \frac{2}{25} \cdot \frac{9}{10}$

44. $\frac{21}{30} \cdot 1\frac{1}{4} \div \frac{9}{10}$

45. $8\frac{1}{3} \cdot \frac{36}{75} \div 4$

EXERCISES FOR REVIEW

(1.1) **46.** What is the value of 5 in the number 504,216?

(2.1) **47.** Find the product of 2,010 and 160.

(2.5) **48.** Use the numbers 8 and 5 to illustrate the commutative property of multiplication.

(3.5) **49.** Find the least common multiple of 6, 16, and 72.

(4.4) **50.** Find $\frac{8}{9}$ of $6\frac{3}{4}$.

★ **Answers to Practice Sets (4.5)**

A. 1. $\frac{10}{3}$ 2. $\frac{3}{2}$ 3. $\frac{8}{7}$ 4. 5 5. $\frac{7}{16}$ 6. $\frac{4}{21}$ 7. $\frac{16}{163}$

B. 1. $\frac{4}{9}$ 2. 1 3. $\frac{1}{2}$ 4. 15 5. 15 6. 2 7. $\frac{2}{5}$ 8. 16 glasses 9. $\frac{3}{2}$ 10. $\frac{8}{3}$
11. $\frac{4}{3}$ 12. $\frac{2}{5}$ 13. is the reciprocal of the fraction.

4.6 Applications Involving Fractions

Section Overview

- ❑ MULTIPLICATION STATEMENTS
- ❑ MISSING PRODUCT STATEMENTS
- ❑ MISSING FACTOR STATEMENTS

❑ MULTIPLICATION STATEMENTS

Statement

A *statement* is a sentence that is either true or false. A mathematical statement of the form

product = (factor 1) · (factor 2)

Multiplication Statement

is a **multiplication statement.** Depending on the numbers that are used, it to can be either true or false.

Omitting exactly one of the three numbers in the statement will produce exactly one of the following three problems. For convenience, we'll represent the omitted (or missing) number with the letter M (M for Missing).

1. M = (factor 1) · (factor 2) Missing *product* statement.
2. M · (factor 2) = product Missing *factor* statement.
3. (factor 1) · M = product Missing *factor* statement.

We are interested in developing and working with methods to determine the missing number that makes the statement true. Fundamental to these methods is the ability to translate two words to mathematical symbols. The word

of translates to *times*
is translates to *equals*

❑ MISSING PRODUCT STATEMENTS

The equation $M = 8 \cdot 4$ is a *missing product* statement. We can find the value of M that makes this statement true by *multiplying* the known factors.

Missing product statements can be used to determine the answer to a question such as, "What number is fraction 1 of fraction 2?

☆ **SAMPLE SET A**

1. Find $\frac{3}{4}$ of $\frac{8}{9}$. We are being asked the question, "What number is $\frac{3}{4}$ of $\frac{8}{9}$?" We must translate from words to mathematical symbols.

Continued

$$\underbrace{\text{What number}}\ \text{is}\ \frac{3}{4}\ \text{of}\ \frac{8}{9} \qquad \textbf{becomes}$$

$$\underset{\text{missing product}}{M} = \underset{\text{known factor}}{\frac{3}{4}} \cdot \underset{\text{known factor}}{\frac{8}{9}} \qquad \textbf{Multiply.}$$

$$M = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{1 \cdot 2}{1 \cdot 3} = \frac{2}{3}$$

Thus, $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$.

2. $\underbrace{\text{What number}}\ \text{is}\ \frac{3}{4}\ \text{of}\ 24$

$$\underset{\text{missing product}}{M} = \underset{\text{known factor}}{\frac{3}{4}} \cdot \underset{\text{known factor}}{24}$$

$$M = \frac{3}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{6}{\cancel{24}}}{1} = \frac{3 \cdot 6}{1 \cdot 1} = \frac{18}{1} = 18$$

Thus, 18 is $\frac{3}{4}$ of 24.

★ PRACTICE SET A

1. Find $\frac{3}{8}$ of $\frac{16}{15}$.

2. What number is $\frac{9}{10}$ of $\frac{5}{6}$?

3. $\frac{11}{16}$ of $\frac{8}{33}$ is what number?

☐ MISSING FACTOR STATEMENTS

The equation $8 \cdot M = 32$ is a *missing factor* statement. We can find the value of M that makes this statement true by *dividing* (since we know that $32 \div 8 = 4$).

$8 \cdot M = 32$ means that $\underset{\text{missing factor}}{M} = \underset{\text{product}}{32} \div \underset{\text{known factor}}{8}$

Finding the Missing Factor

To find the missing factor in a missing factor statement, divide the product by the known factor.

missing factor = (product) ÷ (known factor)

Missing factor statements can be used to answer such questions as

1. $\frac{3}{8}$ of what number is $\frac{9}{4}$?
2. What part of $1\frac{2}{7}$ is $1\frac{13}{14}$?

☆ SAMPLE SET B

1. $\frac{3}{8}$ of $\underbrace{\text{what number}}$ is $\frac{9}{4}$?

$$\underset{\substack{\text{known}\\ \text{factor}}}{\frac{3}{8}} \cdot \underset{\substack{\text{missing}\\ \text{factor}}}{M} = \underset{\text{product}}{\frac{9}{4}}$$

Now, using

missing factor = (product) ÷ (known factor)

We get

$$M = \frac{9}{4} \div \frac{3}{8} = \frac{9}{4} \cdot \frac{8}{3} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}}$$

$$= \frac{3 \cdot 2}{1 \cdot 1}$$

$$= 6$$

Check: $\frac{3}{8} \cdot 6 \stackrel{?}{=} \frac{9}{4}$

$$\frac{3}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{3}{\cancel{6}}}{1} \stackrel{?}{=} \frac{9}{4}$$

$$\frac{3 \cdot 3}{4 \cdot 1} \stackrel{?}{=} \frac{9}{4}$$

$$\frac{9}{4} \stackrel{\checkmark}{=} \frac{9}{4}$$

Thus, $\frac{3}{8}$ of 6 is $\frac{9}{4}$.

2. $\underbrace{\text{What part}}$ of $1\frac{2}{7}$ is $1\frac{13}{14}$?

$$\underset{\substack{\text{missing}\\ \text{factor}}}{M} \cdot \underset{\substack{\text{known}\\ \text{factor}}}{1\frac{2}{7}} = \underset{\text{product}}{1\frac{13}{14}}$$

For convenience, let's convert the mixed numbers to improper fractions.

$$M \cdot \frac{9}{7} = \frac{27}{14}$$

Continued

Now, using

missing factor = (product) ÷ (known factor)

we get

$$M = \frac{27}{14} \div \frac{9}{7} = \frac{27}{14} \cdot \frac{7}{9} = \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{14}}} \cdot \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{9}}}$$

$$= \frac{3 \cdot 1}{2 \cdot 1}$$

$$= \frac{3}{2}$$

Check: $\frac{3}{2} \cdot \frac{9}{7} \stackrel{?}{=} \frac{27}{14}$

$\frac{3 \cdot 9}{2 \cdot 7} \stackrel{?}{=} \frac{27}{14}$

$\frac{27}{14} \stackrel{\checkmark}{=} \frac{27}{14}$

Thus, $\frac{3}{2}$ of $1\frac{2}{7}$ is $1\frac{13}{14}$.

★ **PRACTICE SET B**

1. $\frac{3}{5}$ of what number is $\frac{9}{20}$?

2. $3\frac{3}{4}$ of what number is $2\frac{2}{9}$?

3. What part of $\frac{3}{5}$ is $\frac{9}{10}$?

4. What part of $1\frac{1}{4}$ is $1\frac{7}{8}$?

Answers to Practice Sets are on p. 188.

Section 4.6 EXERCISES

1. Find $\frac{2}{3}$ of $\frac{3}{4}$.

2. Find $\frac{5}{8}$ of $\frac{1}{10}$.

3. Find $\frac{12}{13}$ of $\frac{13}{36}$.

4. Find $\frac{1}{4}$ of $\frac{4}{7}$.

5. $\frac{3}{10}$ of $\frac{15}{4}$ is what number?

6. $\frac{14}{15}$ of $\frac{20}{21}$ is what number?

7. $\frac{3}{44}$ of $\frac{11}{12}$ is what number?

8. $\frac{1}{3}$ of 2 is what number?

9. $\frac{1}{4}$ of 3 is what number?

10. $\frac{1}{10}$ of $\frac{1}{100}$ is what number?

11. $\frac{1}{100}$ of $\frac{1}{10}$ is what number?

12. $1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

13. $1\frac{7}{18}$ of $\frac{4}{15}$ is what number?

14. $1\frac{1}{8}$ of $1\frac{11}{16}$ is what number?

15. Find $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{9}{2}$.

16. Find $\frac{5}{8}$ of $\frac{9}{20}$ of $\frac{4}{9}$.

17. $\frac{5}{12}$ of what number is $\frac{5}{6}$?

18. $\frac{3}{14}$ of what number is $\frac{6}{7}$?

19. $\frac{10}{3}$ of what number is $\frac{5}{9}$?

20. $\frac{15}{7}$ of what number is $\frac{20}{21}$?

21. $\frac{8}{3}$ of what number is $1\frac{7}{9}$?

22. $\frac{1}{3}$ of what number is $\frac{1}{3}$?

23. $\frac{1}{6}$ of what number is $\frac{1}{6}$?

24. $\frac{3}{4}$ of what number is $\frac{3}{4}$?

25. $\frac{8}{11}$ of what number is $\frac{8}{11}$?

26. $\frac{3}{8}$ of what number is 0?

27. $\frac{2}{3}$ of what number is 1?

28. $3\frac{1}{5}$ of what number is 1?

29. $1\frac{9}{12}$ of what number is $5\frac{1}{4}$?

30. $3\frac{1}{25}$ of what number is $2\frac{8}{15}$?

31. What part of $\frac{2}{3}$ is $1\frac{1}{9}$?

32. What part of $\frac{9}{10}$ is $3\frac{3}{5}$?

33. What part of $\frac{8}{9}$ is $\frac{3}{5}$?

34. What part of $\frac{14}{15}$ is $\frac{7}{30}$?

35. What part of 3 is $\frac{1}{5}$?

36. What part of 8 is $\frac{2}{3}$?

37. What part of 24 is 9?

38. What part of 42 is 26?

39. Find $\frac{12}{13}$ of $\frac{39}{40}$.

40. $\frac{14}{15}$ of $\frac{12}{21}$ is what number?

41. $\frac{8}{15}$ of what number is $2\frac{2}{5}$?

42. $\frac{11}{15}$ of what number is $\frac{22}{35}$?

43. $\frac{11}{16}$ of what number is 1?

44. What part of $\frac{23}{40}$ is $3\frac{9}{20}$?

45. $\frac{4}{35}$ of $3\frac{9}{22}$ is what number?

EXERCISES FOR REVIEW

(1.6) 46. Use the numbers 2 and 7 to illustrate the commutative property of addition.

(2.2) 47. Is 4 divisible by 0?

(3.4) 48. Expand 3^7. Do not find the actual value.

(4.2) 49. Convert $3\frac{5}{12}$ to an improper fraction.

(4.5) 50. Find the value of $\frac{3}{8} \div \frac{9}{16} \cdot \frac{6}{5}$.

★ Answers to Practice Sets (4.6)

A. 1. $\frac{2}{5}$ 2. $\frac{3}{4}$ 3. $\frac{1}{6}$

B. 1. $\frac{3}{4}$ 2. $\frac{16}{27}$ 3. $1\frac{1}{2}$ 4. $1\frac{1}{2}$

Chapter 4 SUMMARY OF KEY CONCEPTS

Fraction (4.1)

The idea of breaking up a whole quantity into equal parts gives us the word *fraction.*

Fraction Bar
Denominator
Numerator (4.1)

A fraction has three parts:

1. The fraction bar ——.
2. The nonzero whole number below the fraction bar is the *denominator.*
3. The whole number above the fraction bar is the *numerator.*

$$\frac{4}{5}\begin{array}{l}\leftarrow \text{numerator}\\ \leftarrow \text{fraction bar}\\ \leftarrow \text{denominator}\end{array}$$

Proper Fraction (4.2)

Proper fractions are fractions in which the numerator is strictly less than the denominator.

$\frac{4}{5}$ is a proper fraction

Improper Fraction (4.2)

Improper fractions are fractions in which the numerator is greater than or equal to the denominator. Also, any nonzero number placed over 1 is an improper fraction.

$\frac{5}{4}, \frac{5}{5}$, and $\frac{5}{1}$ are improper fractions

Mixed Number (4.2)

A *mixed number* is a number that is the sum of a whole number and a proper fraction.

$1\frac{1}{5}$ is a mixed number $\left(1\frac{1}{5} = 1 + \frac{1}{5}\right)$

Correspondence Between Improper Fractions and Mixed Numbers (4.2)

Each improper fraction corresponds to a particular mixed number, and each mixed number corresponds to a particular improper fraction.

Converting an Improper Fraction to a Mixed Number (4.2)

A method, based on division, converts an improper fraction to an equivalent mixed number.

$\frac{5}{4}$ can be converted to $1\frac{1}{4}$

Converting a Mixed Number to an Improper Fraction (4.2)

A method, based on multiplication, converts a mixed number to an equivalent improper fraction.

$5\frac{7}{8}$ can be converted to $\frac{47}{8}$

Equivalent Fractions (4.3)

Fractions that represent the same quantity are *equivalent fractions.*

$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions

Test for Equivalent Fractions (4.3)

If the *cross products* of two fractions are equal, then the two fractions are equivalent.

$$\frac{3}{4} \times \frac{6}{8}$$

$3 \cdot 8 \stackrel{?}{=} 4 \cdot 6$
$24 = 24$

Thus, $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent.

Relatively Prime (4.3)

Two whole numbers are *relatively prime* when 1 is the only number that divides both of them.

3 and 4 are relatively prime

Reduced to Lowest Terms (4.3)

A fraction is *reduced to lowest terms* if its numerator and denominator are relatively prime.

The number $\frac{3}{4}$ is reduced to lowest terms, since 3 and 4 are relatively prime.

The number $\frac{6}{8}$ is *not* reduced to lowest terms since 6 and 8 are not relatively prime.

Reducing Fractions to Lowest Terms (4.3)

Two methods, one based on dividing out common primes and one based on dividing out any common factors, are available for reducing a fraction to lowest terms.

Raising Fractions to Higher Terms (4.3)

A fraction can be raised to higher terms by multiplying both the numerator and denominator by the same nonzero number.

$$\frac{3}{4} = \frac{3 \cdot 2}{4 \cdot 2} = \frac{6}{8}$$

The Word "OF" Means Multiplication (4.4)

In many mathematical applications, the word "of" means multiplication.

Multiplication of Fractions (4.4)

To multiply two or more fractions, multiply the numerators together and multiply the denominators together. Reduce if possible.

$$\frac{5}{8} \cdot \frac{4}{15} = \frac{5 \cdot 4}{8 \cdot 15} = \frac{20}{120} = \frac{1}{6}$$

Multiplying Fractions by Dividing Out Common Factors (4.4)

Two or more fractions can be multiplied by first dividing out common factors and then using the rule for multiplying fractions.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{1 \cdot 1}{2 \cdot 3} = \frac{1}{6}$$

Multiplication of Mixed Numbers (4.4)

To perform a multiplication in which there are mixed numbers, first convert each mixed number to an improper fraction, then multiply. This idea also applies to division of mixed numbers.

Reciprocals (4.5)

Two numbers whose product is 1 are reciprocals.

7 and $\frac{1}{7}$ are reciprocals

Division of Fractions (4.5)

To divide one fraction by another fraction, multiply the dividend by the reciprocal of the divisor.

$$\frac{4}{5} \div \frac{2}{15} = \frac{4}{5} \cdot \frac{15}{2}$$

Dividing 1 by a Fraction (4.5)

When dividing 1 by a fraction, the quotient is the reciprocal of the fraction.

$$\frac{1}{\frac{3}{7}} = \frac{7}{3}$$

Multiplication Statements (4.6)

A mathematical statement of the form

product = (factor 1) · (factor 2)

is a multiplication statement.

By omitting one of the three numbers, one of three following problems result:

1. $M =$ (factor 1) $\cdot$ (factor 2)	Missing product statement.
2. product $=$ (factor 1) $\cdot$ M	Missing factor statement.
3. product $= M \cdot$ (factor 2)	Missing factor statement.

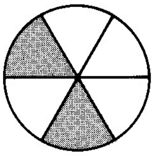
Missing products are determined by simply multiplying the known factors. *Missing factors* are determined by

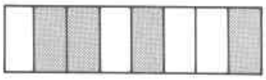
missing factor $=$ (product) $\div$ (known factor)

EXERCISE SUPPLEMENT

Section 4.1

For problems 1 and 2, name the suggested fraction.

1. 

2. 

For problems 3–5, specify the numerator and denominator.

3. $\frac{4}{5}$

4. $\frac{5}{12}$

5. $\frac{1}{3}$

For problems 6–10, write each fraction using digits.

6. Three fifths

7. Eight elevenths

8. Sixty-one forty-firsts

9. Two hundred six-thousandths

10. Zero tenths

For problems 11–15, write each fraction using words.

11. $\frac{10}{17}$

12. $\frac{21}{38}$

13. $\frac{606}{1431}$

14. $\frac{0}{8}$

15. $\frac{1}{16}$

For problems 16–18, state each numerator and denominator and write each fraction using digits.

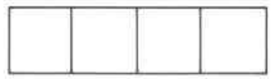
16. One minute is one sixtieth of an hour.

17. In a box that contains forty-five electronic components, eight are known to be defective. If three components are chosen at random from the box, the probability that all three are defective is fifty-six fourteen thousand one hundred ninetieths.

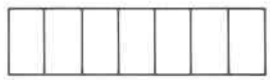
18. About three fifths of the students in a college algebra class received a "B" in the course.

For problems 19 and 20, shade the region corresponding to the given fraction.

19. $\frac{1}{4}$



20. $\frac{3}{7}$



Section 4.2

For problems 21–29, convert each improper fraction to a mixed number.

21. $\frac{11}{4}$

22. $\frac{15}{2}$

23. $\frac{51}{8}$

24. $\frac{121}{15}$

25. $\frac{356}{3}$

26. $\frac{3}{2}$

27. $\frac{5}{4}$

28. $\frac{20}{5}$

29. $\frac{9}{3}$

For problems 30–40, convert each mixed number to an improper fraction.

30. $5\frac{2}{3}$

31. $16\frac{1}{8}$

32. $18\frac{1}{3}$

33. $3\frac{1}{5}$

34. $2\frac{9}{16}$

35. $17\frac{20}{21}$

36. $1\frac{7}{8}$

37. $1\frac{1}{2}$

38. $2\frac{1}{2}$

39. $8\frac{6}{7}$

40. $2\frac{9}{2}$

41. Why does $0\frac{1}{12}$ not qualify as a mixed number?

42. Why does 8 qualify as a mixed number?

Section 4.3

For problems 43–47, determine if the pairs of fractions are equivalent.

43. $\frac{1}{2}, \frac{15}{30}$

44. $\frac{8}{9}, \frac{32}{36}$

45. $\frac{3}{14}, \frac{24}{110}$

46. $2\frac{3}{8}, \frac{38}{16}$

47. $\frac{108}{77}, 1\frac{5}{13}$

For problems 48–60, reduce, if possible, each fraction.

48. $\frac{10}{25}$

49. $\frac{32}{44}$

50. $\frac{102}{266}$

51. $\frac{15}{33}$

52. $\frac{18}{25}$

53. $\frac{21}{35}$

54. $\frac{9}{16}$

55. $\frac{45}{85}$

56. $\frac{24}{42}$

57. $\frac{70}{136}$

58. $\frac{182}{580}$

59. $\frac{325}{810}$

60. $\frac{250}{1000}$

For problems 61–72, determine the missing numerator or denominator.

61. $\frac{3}{7} = \frac{?}{35}$

62. $\frac{4}{11} = \frac{?}{99}$

63. $\frac{1}{12} = \frac{?}{72}$

64. $\frac{5}{8} = \frac{25}{?}$

65. $\frac{11}{9} = \frac{33}{?}$

66. $\frac{4}{15} = \frac{24}{?}$

67. $\frac{14}{15} = \frac{?}{45}$

68. $\frac{0}{5} = \frac{?}{20}$

69. $\frac{12}{21} = \frac{96}{?}$

70. $\frac{14}{23} = \frac{?}{253}$

71. $\frac{15}{16} = \frac{180}{?}$

72. $\frac{21}{22} = \frac{336}{?}$

Sections 4.4 and 4.5

For problems 73 – 95, perform each multiplication and division.

73. $\frac{4}{5} \cdot \frac{15}{16}$

74. $\frac{8}{9} \cdot \frac{3}{24}$

75. $\frac{1}{10} \cdot \frac{5}{12}$

76. $\frac{14}{15} \div \frac{7}{5}$

77. $\frac{5}{6} \cdot \frac{13}{22} \cdot \frac{11}{39}$

78. $\frac{2}{3} \div \frac{15}{7} \cdot \frac{5}{6}$

79. $3\frac{1}{2} \div \frac{7}{2}$

80. $2\frac{4}{9} \div \frac{11}{45}$

81. $\frac{8}{15} \cdot \frac{3}{16} \cdot \frac{5}{24}$

82. $\frac{8}{15} \div 3\frac{3}{5} \cdot \frac{9}{16}$

83. $\frac{14}{15} \div 3\frac{8}{9} \cdot \frac{10}{21}$

84. $18 \cdot 5\frac{3}{4}$

85. $3\frac{3}{7} \cdot 2\frac{1}{12}$

86. $4\frac{1}{2} \div 2\frac{4}{7}$

87. $6\frac{1}{2} \div 3\frac{1}{4}$

88. $3\frac{5}{16} \div 2\frac{7}{18}$

89. $7 \div 2\frac{1}{3}$

90. $17 \div 4\frac{1}{4}$

91. $\frac{5}{8} \div 1\frac{1}{4}$

92. $2\frac{2}{3} \cdot 3\frac{3}{4}$

93. $20 \cdot \frac{18}{4}$

94. $0 \div 4\frac{1}{8}$

95. $1 \div 6\frac{1}{4} \cdot \frac{25}{4}$

Section 4.6

96. Find $\frac{8}{9}$ of $\frac{27}{2}$.

97. What part of $\frac{3}{8}$ is $\frac{21}{16}$?

98. What part of $3\frac{1}{5}$ is $1\frac{7}{25}$?

99. Find $6\frac{2}{3}$ of $\frac{9}{15}$.

100. $\frac{7}{20}$ of what number is $\frac{14}{35}$?

101. What part of $4\frac{1}{16}$ is $3\frac{3}{4}$?

102. Find $8\frac{3}{10}$ of $16\frac{2}{3}$.

103. $\frac{3}{20}$ of what number is $\frac{18}{30}$?

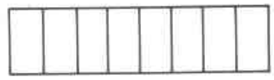
104. Find $\frac{1}{3}$ of 0.

105. Find $\frac{11}{12}$ of 1.

Chapter 4 PROFICIENCY EXAM

1. ______

1. **(4.1)** Shade a portion that corresponds to the fraction $\frac{5}{8}$.



2. ______

2. **(4.1)** Specify the numerator and denominator of the fraction $\frac{5}{9}$.

3. ______

3. **(4.1)** Write the fraction five elevenths.

4. ______

4. **(4.1)** Write, in words, $\frac{4}{5}$.

5. ______

5. **(4.2)** Which of the fractions is a proper fraction?

$4\frac{1}{12}, \frac{5}{12}, \frac{12}{5}$

6. ______

6. **(4.2)** Convert $3\frac{4}{7}$ to an improper fraction.

7. ______

7. **(4.2)** Convert $\frac{16}{5}$ to a mixed number.

8. ______

8. **(4.3)** Determine if $\frac{5}{12}$ and $\frac{20}{48}$ are equivalent fractions.

For problems 9–11, reduce, if possible, each fraction to lowest terms.

9. ______

9. **(4.3)** $\frac{21}{35}$

10. ____________________

10. **(4.3)** $\frac{15}{51}$

11. **(4.3)** $\frac{104}{480}$

11. ____________________

For problems 12 and 13, determine the missing numerator or denominator.

12. ____________________

12. **(4.3)** $\frac{5}{9} = \frac{?}{36}$

13. **(4.3)** $\frac{4}{3} = \frac{32}{?}$

13. ____________________

For problems 14–25, find each value.

14. ____________________

14. **(4.4)** $\frac{15}{16} \cdot \frac{4}{25}$

15. ____________________

15. **(4.4)** $3\frac{3}{4} \cdot 2\frac{2}{9} \cdot 6\frac{3}{5}$

16. ____________________

16. **(4.4)** $\sqrt{\frac{25}{36}}$

17. ____________________

17. **(4.4)** $\sqrt{\frac{4}{9}} \cdot \sqrt{\frac{81}{64}}$

18. ______________

18. **(4.4)** $\frac{11}{30} \cdot \sqrt{\frac{225}{121}}$

19. ______________

19. **(4.5)** $\frac{4}{15} \div 8$

20. ______________

20. **(4.5)** $\frac{8}{15} \cdot \frac{5}{12} \div 2\frac{4}{9}$

21. ______________

21. **(4.5)** $\left(\frac{6}{5}\right)^3 \div \sqrt{1\frac{11}{25}}$

22. ______________

22. **(4.6)** Find $\frac{5}{12}$ of $\frac{24}{25}$.

23. ______________

23. **(4.6)** $\frac{2}{9}$ of what number is $\frac{1}{18}$?

24. ______________

24. **(4.6)** $1\frac{5}{7}$ of $\frac{21}{20}$ is what number?

25. ______________

25. **(4.6)** What part of $\frac{9}{14}$ is $\frac{6}{7}$?