Discrete Random Variables: Homework

Exercise 1

Complete the PDF and answer the questions.

X	P(X = x)	X∗P(X = x)
0	0.3	
1	0.2	
2		
3	0.4	

a. Find the probability that X = 2.
b. Find the expected value.

Exercise 2

Suppose that you are offered the following "deal." You roll a die. If you roll a 6, you win $10. If you roll a 4 or 5, you win $5. If you roll a 1, 2, or 3, you pay $6.

a. What are you ultimately interested in here (the value of the roll or the money you win)?
b. In words, define the Random Variable X.
c. List the values that X may take on.
d. Construct a PDF.
e. Over the long run of playing this game, what are your expected average winnings per game?
f. Based on numerical values, should you take the deal?
g. Explain your decision in (f) in complete sentences.

Exercise 3

A venture capitalist, willing to invest $1,000,000, has three investments to choose from. The first investment, a software company, has a 10% chance of returning $5,000,000 profit, a 30% chance of returning $1,000,000 profit, and a 60% chance of losing the million dollars. The second company, a hardware company, has a 20% chance of returning $3,000,000 profit, a 40% chance of returning $1,000,000 profit, and a 40% chance of losing the million dollars. The third company, a biotech firm, has a 10% chance of returning $6,000,000 profit, a 70% of no profit or loss, and a 20% chance of losing the million dollars.

a. Construct a PDF for each investment.
b. Find the expected value for each investment.
c. Which is the safest investment? Why do you think so?
d. Which is the riskiest investment? Why do you think so?
e. Which investment has the highest expected return, on average?

EXERCISE 4

A theater group holds a fund-raiser. It sells 100 raffle tickets for $5 apiece. Suppose you purchase 4 tickets. The prize is 2 passes to a Broadway show, worth a total of $150.

a. What are you interested in here?
b. In words, define the Random Variable X.
c. List the values that X may take on.
d. Construct a PDF.
e. If this fund-raiser is repeated often and you always purchase 4 tickets, what would be your expected average winnings per game?

EXERCISE 5

Suppose that 20,000 married adults in the United States were randomly surveyed as to the number of children they have. The results are compiled and are used as theoretical probabilities. Let X = the number of children

X	P(X = x)	X∗P(X = x)
0	0.10	
1	0.20	
2	0.30	
3		
4	0.10	
5	0.05	
6 (or more)	0.05	

a. Find the probability that a married adult has 3 children.
b. In words, what does the expected value in this example represent?
c. Find the expected value.
d. Is it more likely that a married adult will have 2 – 3 children or 4 – 6 children? How do you know?

EXERCISE 6

Suppose that the PDF for the number of years it takes to earn a Bachelor of Science (B.S.) degree is given below.

X	P(X = x)
3	0.05
4	0.40
5	0.30
6	0.15
7	0.10

a. In words, define the Random Variable X.
b. What does it mean that the values 0, 1, and 2 are not included for X on the PDF?
c. On average, how many years do you expect it to take for an individual to earn a B.S.?

For each problem, (7) - (29), below:

a. In words, define the Random Variable X.
b. List the values that X may take on.
c. Give the distribution of X. X~ _____

Then, answer questions specific to each individual problem.

EXERCISE 7

Six different colored dice are rolled. Of interest is the number of dice that show a "1."

d. On average, how many dice would you expect to show a "1"?
e. Find the probability that all six dice show a "1."
f. Is it more likely that 3 or that 4 dice will show a "1"? Use numbers to justify your answer numerically.

EXERCISE 8

According to a 2003 publication by Waits and Lewis (source: http://nces.ed.gov/pubs2003/2003017.pdf), by the end of 2002, 92% of U.S. public two-year colleges offered distance learning courses. Suppose you randomly pick 13 U.S.

public two-year colleges. We are interested in the number that offer distance learning courses.

d. On average, how many schools would you expect to offer such courses?
e. Find the probability that at most 6 offer such courses.
f. Is it more likely that 0 or that 13 will offer such courses? Use numbers to justify your answer numerically and answer in a complete sentence.

EXERCISE 9

A school newspaper reporter decides to randomly survey 12 students to see if they will attend Tet festivities this year. Based on past years, she knows that 18% of students attend Tet festivities. We are interested in the number of students who will attend the festivities.

d. How many of the 12 students do we expect to attend the festivities?
e. Find the probability that at most 4 students will attend.
f. Find the probability that more than 2 students will attend.

EXERCISE 10

Suppose that about 85% of graduating students attend their graduation. A group of 22 graduating students is randomly chosen.

d. How many are expected to attend their graduation?
e. Find the probability that 17 or 18 attend.
f. Based on numerical values, would you be surprised if all 22 attended graduation? Justify your answer numerically.

EXERCISE 11

At The Fencing Center, 60% of the fencers use the foil as their main weapon. We randomly survey 25 fencers at The Fencing Center. We are interested in the number that do NOT use the foil as their main weapon.

d. How many are expected to NOT use the foil as their main weapon?
e. Find the probability that six do NOT use the foil as their main weapon.
f. Based on numerical values, would you be surprised if all 25 did NOT use foil as their main weapon? Justify your answer numerically.

EXERCISE 12

Approximately 8% of students at a local high school participate in after-school sports all four years of high school. A group of 60 seniors is randomly chosen. Of interest is the number that participated in after-school sports all four years of high school.

d. How many seniors are expected to have participated in after-school sports all four years of high school?
e. Based on numerical values, would you be surprised if none of the seniors participated in after-school sports all four years of high school? Justify your answer numerically.
f. Based upon numerical values, is it more likely that 4 or that 5 of the seniors participated in after-school sports all four years of high school? Justify your answer numerically.

EXERCISE 13

The chance of having an extra fortune in a fortune cookie is about 3%. Given a bag of 144 fortune cookies, we are interested in the number of cookies with an extra fortune. Two distributions may be used to solve this problem. Use one distribution to solve the problem.

d. How many cookies do we expect to have an extra fortune?
e. Find the probability that none of the cookies have an extra fortune.
f. Find the probability that more than 3 have an extra fortune.
g. As n increases, what happens involving the probabilities using the two distributions? Explain in complete sentences.

EXERCISE 14

There are two games played for Chinese New Year and Vietnamese New Year. They are almost identical. In the Chinese version, fair dice with numbers 1, 2, 3, 4, 5, and 6 are used, along with a board with those numbers. In the Vietnamese version, fair dice with pictures of a gourd, fish, rooster, crab, crayfish, and deer are used. The board has those six objects on it, also. We will play with bets being $1. The player places a bet on a number or object. The "house" rolls three dice. If none of the dice show the number or object that was bet, the house keeps the $1 bet. If one of the dice shows the number or object bet (and the other two do not show it), the player gets back his $1 bet, plus $1 profit. If two of the dice show the number or object bet (and the third die does not show it), the player gets back his $1 bet, plus $2 profit. If all three dice show the number or object bet, the player gets back his $1 bet, plus $3 profit.

Let X = number of matches. Let Y = profit per game.

d. List the values that Y may take on. Then, construct one PDF table that includes both X & Y and their probabilities.

e. Calculate the average expected matches over the long run of playing this game for the player.
f. Calculate the average expected earnings over the long run of playing this game for the player.
g. Determine who has the advantage, the player or the house.

EXERCISE 15

According to the South Carolina Department of Mental Health web site, for every 200 U.S. women, the average number who suffer from anorexia is one (http://www.state.sc.us/dmh/anorexia/statistics.htm). Out of a randomly chosen group of 600 U.S. women:

d. How many are expected to suffer from anorexia?
e. Find the probability that no one suffers from anorexia.
f. Find the probability that more than four suffer from anorexia.

EXERCISE 16

The average number of children of middle-aged Japanese couples is 2.09 (Source: *The Yomiuri Shimbun*, June 28, 2006). Suppose that one middle-aged Japanese couple is randomly chosen.

d. Find the probability that they have no children.
e. Find the probability that they have fewer children than the Japanese average.
f. Find the probability that they have more children than the Japanese average.

EXERCISE 17

The average number of children per Spanish couples was 1.34 in 2005. Suppose that one Spanish couple is randomly chosen. (Source: http://www.typicallyspanish.com/news/publish/article_4897.shtml, June 16, 2006).

d. Find the probability that they have no children.
e. Find the probability that they have fewer children than the Spanish average.
f. Find the probability that they have more children than the Spanish average .

EXERCISE 18

Fertile (female) cats produce an average of 3 litters per year. (Source: The Humane Society of the United States). Suppose that one fertile, female cat is randomly chosen. In one year, find the probability she produces:

d. No litters.
e. At least 2 litters.
f. Exactly 3 litters.

EXERCISE 19

A consumer looking to buy a used red Miata car will call dealerships until she finds a dealership that carries the car. She estimates the probability that any independent dealership will have the car will be 28%. We are interested in the number of dealerships she must call.

d. On average, how many dealerships would we expect her to have to call until she finds one that has the car?
e. Find the probability that she must call at most 4 dealerships.:
f. Find the probability that she must call 3 or 4 dealerships.

EXERCISE 20

Suppose that the probability that an adult in America will watch the Super Bowl is 40%. Each person is considered independent. We are interested in the number of adults in America we must survey until we find one who will watch the Super Bowl.

d. How many adults in America do you expect to survey until you find one who will watch the Super Bowl?
e. Find the probability that you must ask 7 people.
f. Find the probability that you must ask 3 or 4 people.

EXERCISE 21

A group of Martial Arts students is planning on participating in an upcoming demonstration. 6 are students of Tae Kwon Do; 7 are students of Shotokan Karate. Suppose that 8 students are randomly picked to be in the first demonstration. We are interested in the number of Shotokan Karate students in that first demonstration.

d. How many Shotokan Karate students do we expect to be in that first demonstration?
e. Find the probability that 4 students of Shotokan Karate are picked.
f. Find the probability that no more than 6 students of Shotokan Karate are picked.

EXERCISE 22

The chance of a IRS audit for a tax return with over $25,000 in income is about 2% per year. We are interested in the expected number of audits a person with that income has in a 20 year period. Assume each year is independent.

d. How many audits are expected in a 20 year period?
e. Find the probability that a person is not audited at all.
f. Find the probability that a person is audited more than twice.

EXERCISE 23

Refer to (22). Suppose that 100 people with tax returns over $25,000 are randomly picked. We are interested in the number of people audited in 1 year. One way to solve this problem is by using the Binomial Distribution. Since n is large and p is small, another discrete distribution could be used to solve the following problems. Solve **d** - **f** using that distribution.

d. How many are expected to be audited?
e. Find the probability that no one was audited.
f. Find the probability that more than 2 were audited.

EXERCISE 24

Suppose that a technology task force is being formed to study technology awareness among instructors. Assume that 10 people will be randomly chosen to be on the committee from a group of 28 volunteers, 20 who are technically proficient and 8 who are not. We are interested in the number on the committee who are **not** technically proficient.

d. How many instructors do you expect on the committee who are **not** technically proficient?
e. Find the probability that at least 5 on the committee are not technically proficient.
f. Find the probability that at most 3 on the committee are not technically proficient.

EXERCISE 25

Refer back to problem (12). Solve this problem again, using a different acceptable distribution.

EXERCISE 26

Suppose that 9 Massachusetts athletes are scheduled to appear at a charity benefit. The 9 are randomly chosen from 8 volunteers from the Boston Celtics and 4 volunteers from the New England Patriots. We are interested in the number of Patriots picked.

d. Is it more likely that there will be 2 Patriots or 3 Patriots picked?
e. What is the probability that all of the volunteers will be from the Celtics?

f. Is it more likely that more of the volunteers will be from the Patriots or from the Celtics? How do you know?

EXERCISE 27

On average, Pierre, an amateur chef, drops 3 pieces of egg shell into every 2 batters of cake he makes. Suppose that you buy one of his cakes.

d. On average, how many pieces of egg shell do you expect to be in the cake?
e. What is the probability that there will not be any pieces of egg shell in the cake?
f. Let's say that you buy one of Pierre's cakes each week for 6 weeks. What is the probability that there will not be any egg shell in any of the cakes?
g. Based upon the average given for Pierre, is it possible for there to be 7 pieces of shell in the cake? Why?

EXERCISE 28

It has been estimated that only about 30% of California residents have adequate earthquake supplies. Suppose we are interested in the number of California residents we must survey until we find a resident who does NOT have adequate earthquake supplies.

d. What is the probability that we must survey just 1 or 2 residents until we find a California resident who does not have adequate earthquake supplies?
e. What is the probability that we must survey at least 3 California residents until we find a California resident who does not have adequate earthquake supplies?
f. How many California residents do you expect to need to survey until you find a California resident who **does not** have adequate earthquake supplies?
g. How many California residents do you expect to need to survey until you find a California resident who **does** have adequate earthquake supplies?

EXERCISE 29

Refer to the above problem. Suppose you randomly survey 11 California residents. We are interested in the number who have adequate earthquake supplies.

d. What is the probability that at least 8 have adequate earthquake supplies?
e. Is it more likely that none or that all of the residents surveyed will have adequate earthquake supplies? Why?
f. How many residents do you expect will have adequate earthquake supplies?

Questions 30 - 32 refer to the following: In one of its Spring catalogs, L.L. Bean® advertised footwear on 29 of its 192 catalog pages.

EXERCISE 30

Suppose we randomly survey 20 pages. We are interested in the number of pages that advertise footwear. Each page may be picked at most once.

d. How many pages do you expect to advertise footwear on them?
e. Is it probable that all 20 will advertise footwear on them? Why or why not?
f. What is the probability that less than 10 will advertise footwear on them?

EXERCISE 31

Suppose we randomly survey 20 pages. We are interested in the number of pages that advertise footwear. This time, each page may be picked more than once.

d. How many pages do you expect to advertise footwear on them?
e. Is it probable that all 20 will advertise footwear on them? Why or why not?
f. What is the probability that less than 10 will advertise footwear on them?
g. Suppose that a page may be picked more than once. We are interested in the number of pages that we must randomly survey until we find one that has footwear advertised on it. Define the random variable X and give its distribution.
h. Do you expect to survey more than 10 pages in order to find one that advertises footwear on it? Why?
i. What is the probability that you only need to survey at most 3 pages in order to find one that advertises footwear on it?
j. How many pages do you expect to need to survey in order to find one that advertises footwear?

EXERCISE 32

Suppose that you roll a fair die until each face has appeared at least once. It does not matter in what order the numbers appear. Find the expected number of rolls you must make until each face has appeared at least once.

Try these multiple choice problems.

Questions 33 – 35: The probability that the San Jose Sharks will win any given game is 0.3694 based on their 13 year win history of 382 wins out of 1034 games played (as of a certain date). Their 2005 schedule for November contains 12 games. Let X = number of games won in November 2005

EXERCISE 33

The expected number of wins for the month of November 2005 is

A. 1.67
B. 12
C. 382/1034
D. 4.43

EXERCISE 34

What is the probability that the San Jose Sharks win 6 games in November?

A. 0.1476
B. 0.2336
C. 0.7664
D. 0.8903

EXERCISE 35

Find the probability that the San Jose Sharks win at least 5 games in November.

A. 0.3694
B. 0.5266
C. 0.4734
D. 0.2305

Questions 36 – 37: The average number of times per week that Mrs. Plum's cats wake her up at night because they want to play is 10. We are interested in the number of times her cats wake her up each week.

EXERCISE 36

In words, the random variable X =

A. number of times Mrs. Plum's cats wake her up each week
B. number of times Mrs. Plum's cats wake her up each hour
C. number of times Mrs. Plum's cats wake her up each night
D. number of times Mrs. Plum's cats wake her up

EXERCISE 37

Find the probability that her cats will wake me up no more than 5 times next week.

A. 0.5000
B. 0.9329
C. 0.0378
D. 0.0671