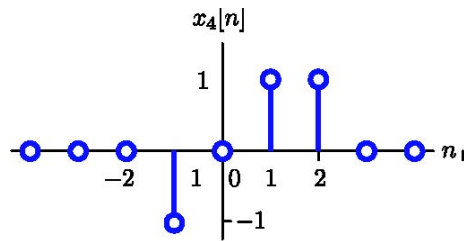


# Signals and Systems

## Homework Assignment #9

**Problem 1.** Determine the Z Transform (including region of convergence) for each of the following signals:

- $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$
- $x_2[n] = (1+n) \left(\frac{1}{3}\right)^n u[n]$
- $x_3[n] = n \left(\frac{1}{2}\right)^{|n|}$
- 



**Problem 2.** Determine and sketch all possible signals with Z Transforms of the following forms:

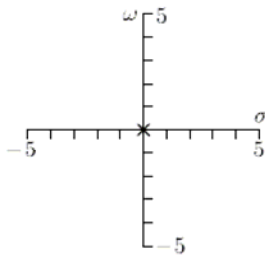
- $X_1(z) = \frac{1}{z-1}$
- $X_2(z) = \frac{1}{z(z-1)^2}$
- $X_3(z) = \frac{1}{z^2+z+1}$
- $X_4(z) = \left(\frac{1-e^{-z}}{z}\right)^2$

For each signal, indicate the associated region of convergence.

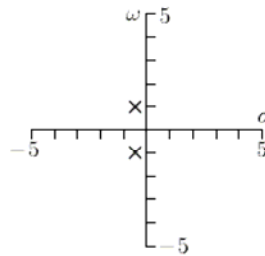
**Problem 3.** The following plots show pole-zero diagrams, impulse responses, Bode magnitude plots, and Bode angle plots for six causal CT LTI systems. Determine which corresponds to which and fill in the following table.

	$h(t)$	Magnitude	Angle
PZ diagram 1 =			
PZ diagram 2 =			
PZ diagram 3 =			
PZ diagram 4 =			
PZ diagram 5 =			
PZ diagram 6 =			

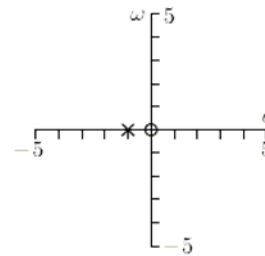
Pole-zero diagram 1



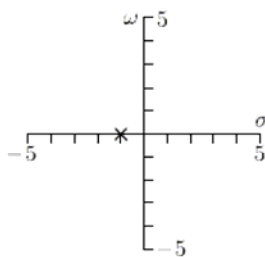
Pole-zero diagram 2



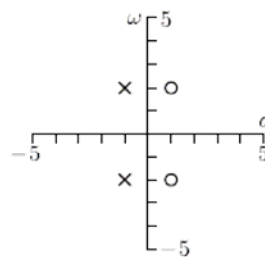
Pole-zero diagram 3



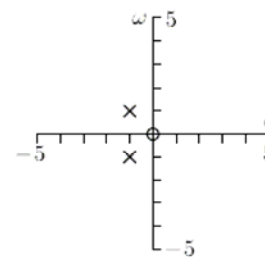
Pole-zero diagram 4



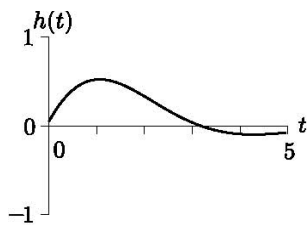
Pole-zero diagram 5



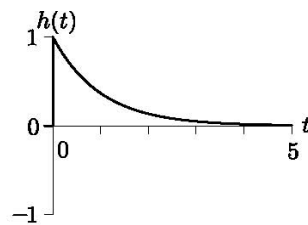
Pole-zero diagram 6



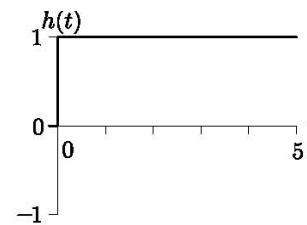
Impulse response 1



Impulse response 2



Impulse response 3



Impulse response 4

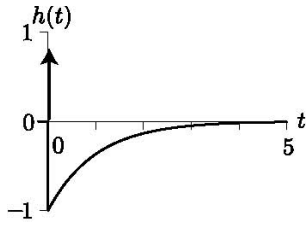


Impulse response 5

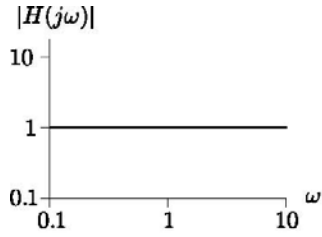


Impulse response 6

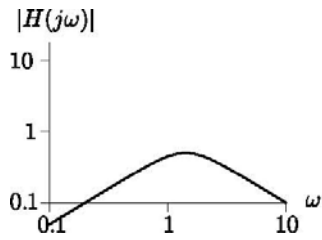




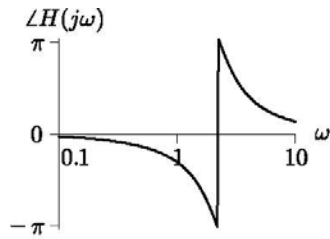
Bode Magnitude 1



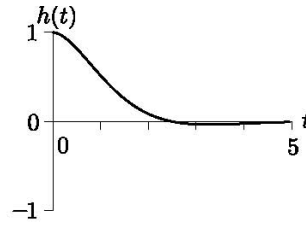
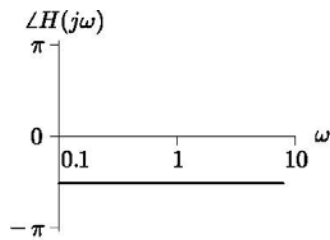
Bode Magnitude 4



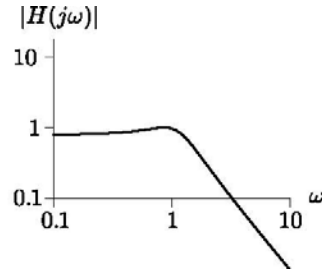
Bode Angle 1



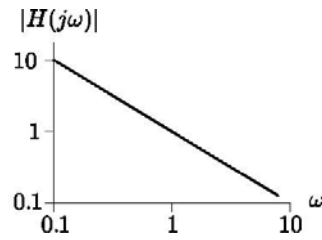
Bode Angle 4



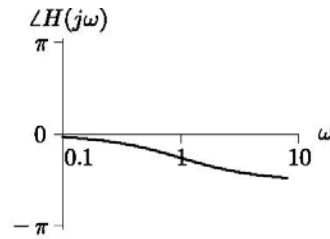
Bode Magnitude 2



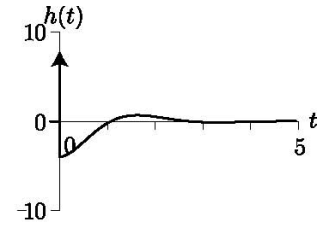
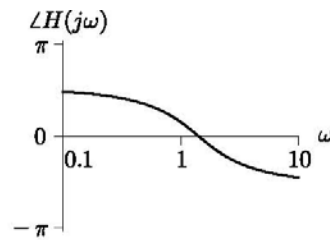
Bode Magnitude 5



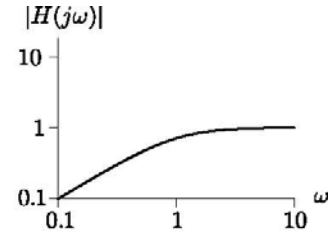
Bode Angle 2



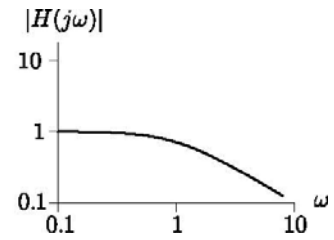
Bode Angle 5



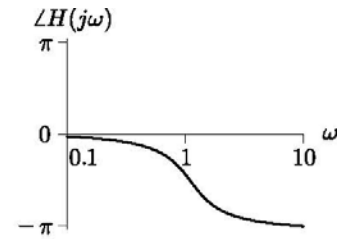
Bode Magnitude 3



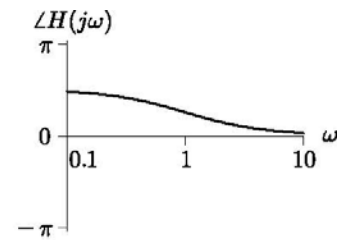
Bode Magnitude 6



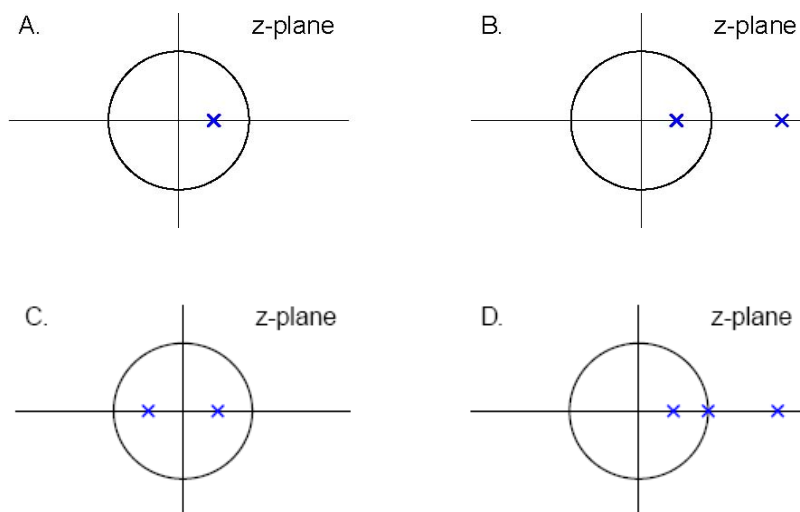
Bode Angle 3



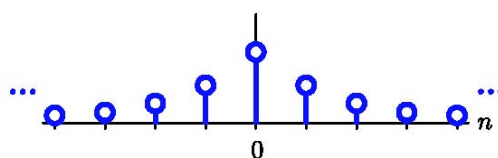
Bode Angle 6



**Problem 4.** Consider the following DT pole-zero diagrams, where each circle has a radius of one.



- a. Which if any of these pole-zero plots could represent the Z-transform of the following DT signal?



- b. Assuming that each of the pole-zero diagrams represents a system, which of the following statements is true?
- $x[n] = 1$ , for all  $n$ , is an eigenfunction of all 4 systems
  - $x[n] = 1$ , for all  $n$ , could be an eigenfunction of all 4 systems, if the regions of convergence are appropriately selected.
  - $x[n] = 1$ , for all  $n$ , cannot be an eigenfunction of all 4 systems.
  - None of the above.
- c. Which if any of the pole-zero diagrams could represent systems that are both causal and stable?

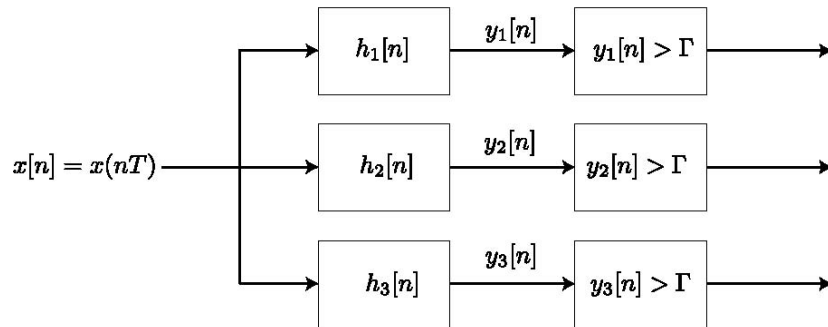
**Problem 5.** Pressing the buttons on a touch-tone phone generates tones that are used for dialing. Each button produces a pair of tones of the form

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$

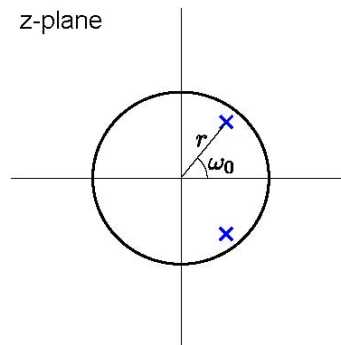
where  $f_1$  and  $f_2$  code the row and column of the button as shown in the following table.

$f_1(\text{Hz})$	$f_2(\text{Hz})$		
	1209	1336	1477
697	1	2	3
770	4	5	6
852	7	8	9
941	*	0	#

This problem concerns the design of a system to detect the row and column numbers that were pressed by analyzing the signal  $x(t)$ . The following block diagram illustrates the basic structure of such a system.



The input  $x(t)$  is first sampled with  $T = 0.001$  seconds. The samples are then passed through LTI systems that generate intermediate signals so that  $y_1[n]$  is large when a button in column 1 is pressed,  $y_2[n]$  is large when a button in column 2 is pressed, and  $y_3[n]$  is large when a button in column 3 is pressed. These intermediate signals are then passed through detectors that determine when the signals are bigger than a threshold value  $\Gamma$ . Your task is to design the LTI systems. Each should consist of a system with 2 poles of the form shown in the following pole-zero diagram.



Such systems can be simulated in Matlab by finding the difference equation that corresponds to the system and then iteratively solving that difference equation.

The following Matlab code generates a vector that contains bursts of each of the column frequencies.

```
x = zeros(1,3000);
x(1:500) = cos(2*pi*0.1209*(1:500));
x(1001:1500) = cos(2*pi*0.1336*(1001:1500));
```

$x(2001:2500) = \cos(2\pi \cdot 0.1477 \cdot (2001:2500));$

- a. Determine values of  $r$  and  $\omega_0$  for the  $h_1[n]$  system so that  $y_1[n]$  is large at the end of the first pulse and small at the end of the second and third pulses. Submit hard copies of your code to generate  $y_1[n]$  along with a plot of  $y_1[n]$ .
- b. Describe how the choice of  $\omega_0$  affects the output signal  $y_1[n]$ .
- c. Describe how the choice of  $r$  affects the output signal  $y_1[n]$ . In particular, what limits the maximum acceptable value of  $r$ ? Also, what limits the minimum acceptable value of  $r$ ?