Signals and Systems

Homework Assignment #6

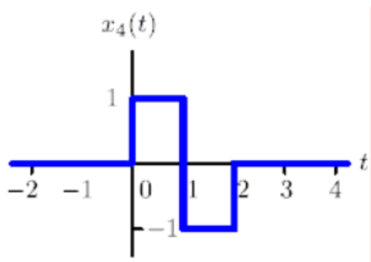
Problem 1. Determine the Laplace Transform (including region of convergence) for each of the following signals:

a. $x_1(t) = e^{-2t}u(t-3)$

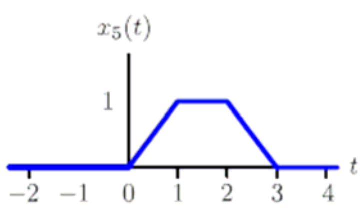
b. $x_2(t) = (1-(1-t)e^{-3t})u(t)$

c. $x_3(t) = te^{-|t|}$

d.



e.



Problem 2. Determine and sketch all possible signals with Laplace Transforms of the following forms:

a. $X_1(s) = \dfrac{s+2}{(s+1)^2}$

b. $X_2(s) = \dfrac{1}{s^2(s-1)}$

c. $X_3(s) = \dfrac{s+1}{s^2+2s+2}$

d. $X_4(s) = \left(\dfrac{1-e^{-s}}{s}\right)^2$

For each signal, indicate the associated region of convergence.

Problem 3. A causal, LTI system has the system function

$$H(s) = \frac{s+2}{(s+1)(s+3)}$$

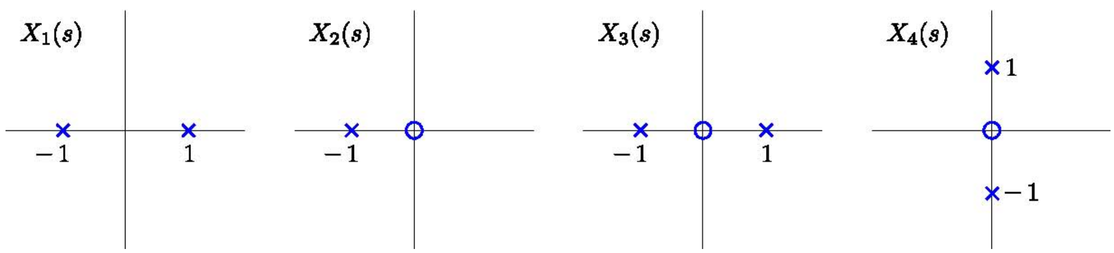
Determine the output signal that results when the input to this system is each of the following signals:

a. $x_1(t) = \cos(t)$

b. $x_2(t) = 1$

c. $x_3(t) = e^{-5t}$

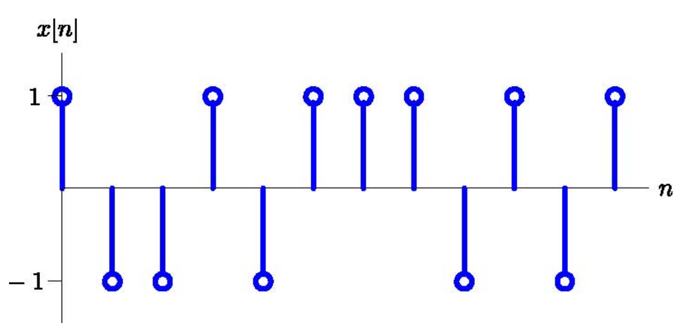
Problem 4. Determine which of the following pole-zero diagrams could represent Laplace Transforms of even functions of time. For those that can represent even functions of time, determine expressions for those functions. For those that cannot, explain why they cannot.



How can you determine if a signal is even or not by looking at its Laplace Transform and region of convergence?

Problem 5. Consider a linear, time-invariant system with an impulse response $h(t) = \delta(t) - 2e^{-t}u(t)$. How many different inputs to this system could give rise to an output $y(t) = e^{-2t}u(t)$? If one or more such inputs are possible, determine expressions for each. If no such inputs are possible, explain why not.

Problem 6. Consider the sequence of 1's and −1's shown below.



In this $x[n]$, there is a single occurrence of the pattern −1, −1, 1. It occurs starting at $n = 1$ and ending at $n = 3$. One method to automatically locate particular patterns of this type is called "matched filtering." Let $p[n]$ represent the pattern of interest flipped about $n = 0$. Then instances of the pattern can be found by finding the times when $y[n] = p[n]*x[n]$ is maximized.

a. Determine a matched filter $p[n]$ that will find occurrences of the sequence: −1, −1, 1. Design p[n] so that p[n]*x[n] has maxima at points that are centered on the desired pattern, i.e., at n = 2 for the sequence above.

b. Test that your answer to part a is robust using Matlab. The following code will generate a random sequence of 1's and −1's and then plot that sequence.

```
z = sign(randn(1,50));
stem(1:50,z);
```

Matlab's conv() function will carry out a convolution, and you can automatically find the times when a sequence y achieves it's maximum value using Matlab's find (y == max(y)) function. Show (1) a typical random sequence, (2) your code to find the pattern −1, −1, 1, and (3) the answer produced by your code.

The same approach can be used to find patterns in pictures. Matlab's conv2() function performs two-dimensional convolutions defined by

$$y[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} p[k,l]x[n-k,m-l].$$

Try the following exercise to figure out how conv2() works.

```
test = [0 0 0 0 0;
        0 0 0 0 0;
        0 0 1 0 0;
        0 0 0 0 0;
        0 0 0 0 0]
figure(1); imagesc(conv2([1 1 1],test)); colormap('gray');
figure(2); imagesc(conv2([1;1;1],test)); colormap('gray');
figure(3); imagesc(conv2([1 0 0;0 1 0;0 1 1],test)); colormap('gray');
```