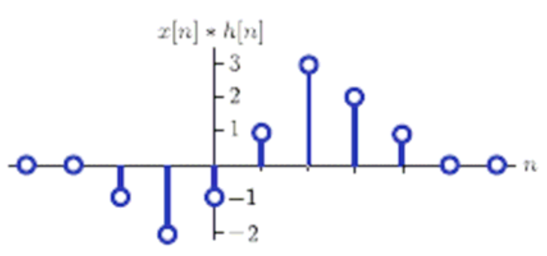
Signals and Systems

Solutions to Homework Assignment #2

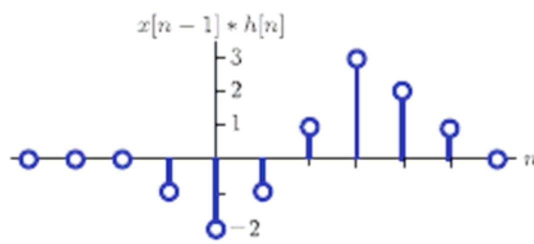
Problem 1.

a. This is easy to do by superposition:

n :	-4	-3	-2	-1	0	1	2	3	4
$x[n]$:	0	0	-1	-1	1	1	1	0	0
$x[n-1]$:	0	0	0	-1	-1	1	1	1	0
$x[n]$:	0	0	0	0	-1	-1	1	1	1
sum:	0	0	-1	-2	-1	1	3	2	1

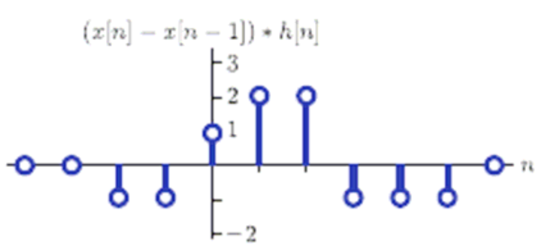


b. Delaying the input by one just delays the output by one.



c. If the input is the difference of two signals, the output is the difference of the corresponding outputs.

n :	-4	-3	-2	-1	0	1	2	3	4	5	6
$y[n]$:	0	0	-1	-2	-1	1	3	2	1	0	0
$y[n-1]$:	0	0	0	-1	-2	-1	1	3	2	1	0
diff:	0	0	-1	-1	1	2	2	-1	-1	-1	0

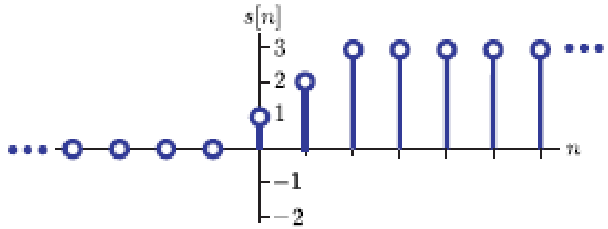


d. The convolution can be expressed as

$$s[n] = \sum_{k=-\infty}^{\infty} u[n-k]h[k].$$

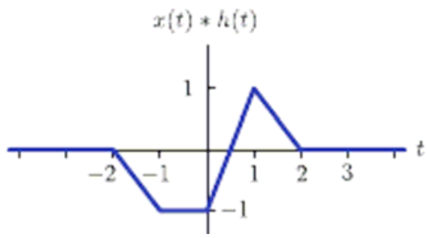
Since $h[k]$ is zero except for $k = 0$, 1, and 2,

$$s[n] = u[n-k]|_{k=0} + u[n-k]|_{k=1} + u[n-k]|_{k=2} = u[n] + u[n-1] + u[n-2].$$

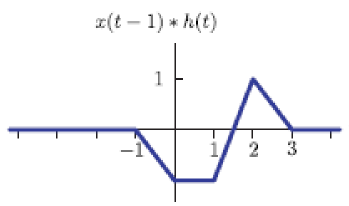


Problem 2.

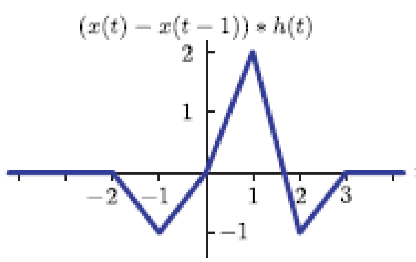
a. Easy.



b. Delaying the input by one just delays the output by one.



c. If the input is the difference of two signals, the output is the difference of the corresponding outputs.



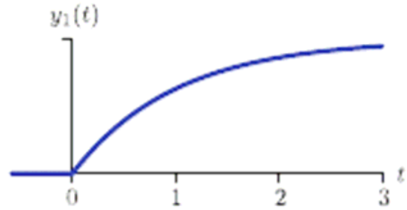
Problem 3.

a. From the definition of convolution,

$$y_1(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) u(t-\tau) d\tau.$$

If $\tau > 0$, then $u(\tau)$ is 1. If $\tau < 0$, then $u(\tau)$ is 0. Also, if $\tau < t$, $u(t-\tau) = 1$. If $\tau > t$, $u(t-\tau) = 0$. Therefore,

$$y_1(t) = \int_0^t e^{-\tau} d\tau = -e^{-\tau}\Big|_0^t = (1 - e^{-t})u(t).$$

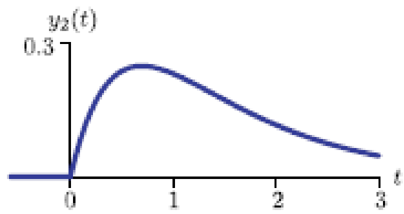


b. From the definition of convolution,

$$y_2(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau) e^{-2\tau} u(\tau) d\tau.$$

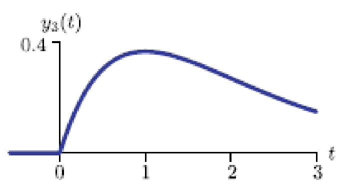
As in part a, $u(\tau)$ and $u(t - \tau)$ are both equal to one if $0 < \tau < t$ and their product is zero outside this range,

$$y_2(t) = \int_0^t e^{-(t-\tau)} e^{-2\tau} d\tau = e^{-t} \int_0^t e^{-\tau} d\tau = e^{-t}\left[-e^{-\tau}\right]_0^t = e^{-t}(1 - e^{-t})u(t) = (e^{-t} - e^{-2t})u(t).$$



c. From the definition of convolution,

$$y_3(t) = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau) e^{-\tau} u(\tau) d\tau = \int_0^t e^{-(t-\tau)} e^{-\tau} d\tau = e^{-t} \int_0^t 1 d\tau = e^{-t}[t]_0^t = te^{-t}u(t).$$



Notice that the shapes of $y_2(t)$ and $y_3(t)$ are similar even though their mathematical expressions are quite different.

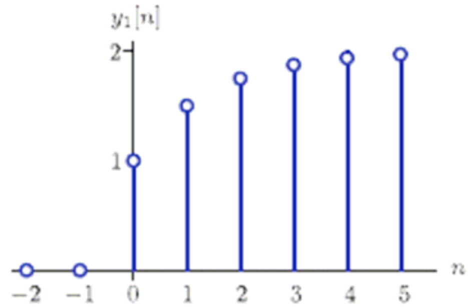
Problem 4.

a. From the definition of convolution,

$$y_1[n] = \sum_{k=-\infty}^{\infty} 0.5^k u[k] u[n-k].$$

If $0 \le k \le n$ then $u[k]u[n - k] = 1$. Otherwise it is 0. Therefore,

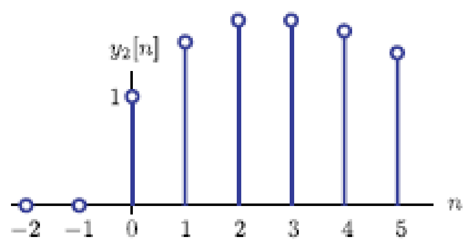
$$y_1[n] = \sum_{k=0}^{n} 0.5^k = \left(\frac{1 - 0.5^{n+1}}{1 - 0.5}\right) = (2 - 0.5^n)u[n].$$



b.

$$y_2[n] = \sum_{k=-\infty}^{\infty} 0.7^k u[k] 0.8^{n-k} u[n-k] = \sum_{k=0}^{n} 0.7^k 0.8^{n-k} = 0.8^n \sum_{k=0}^{n} \left(\frac{7}{8}\right)^k$$

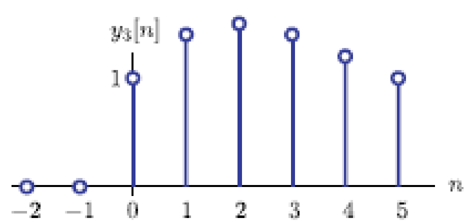
$$= 0.8^n \left(\frac{1 - (7/8)^{n+1}}{1 - 7/8}\right) = 8 \times 0.8^n (1 - (7/8)^{n+1}) u[n].$$



c.

$$y_3[n] = \sum_{k=-\infty}^{\infty} 0.7^k u[k] 0.7^{n-k} u[n-k] = \sum_{k=0}^{n} 0.7^k 0.7^{n-k} = 0.7^n \sum_{k=0}^{n} 1^k u[n]$$

$$= (n+1) 0.7^n u[n].$$



Notice that the shapes of $y_2[n]$ and $y_3[n]$ are similar even though the expressions look quite different.