Using Sparse FFT to Decrease Computation Time in Discrete Multitone Communication

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Problem

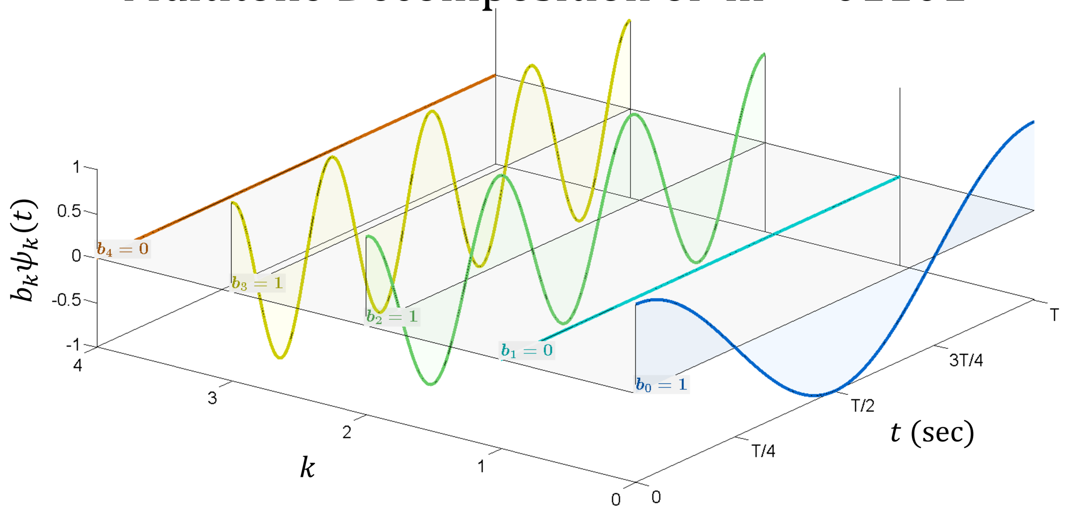
Modern communication schemes rely on the use of optimized FFT algorithms to compute length-N transforms in $O(N \log_2 N)$, long thought to be the best achievable run time. Recently, a class of Sparse FFT algorithms has been developed that feature run time complexities that are sub-linear in N. We propose FFAST, a novel Sparse FFT algorithm, as a method to significantly decrease computation time in digital multitone modulation.

Digital Multitone (DMT)

- Send B bits in bit interval $1/f_0$ in message $m(t)$
- k^{th} bit indicated by presence of $\psi_k(t)$ in message

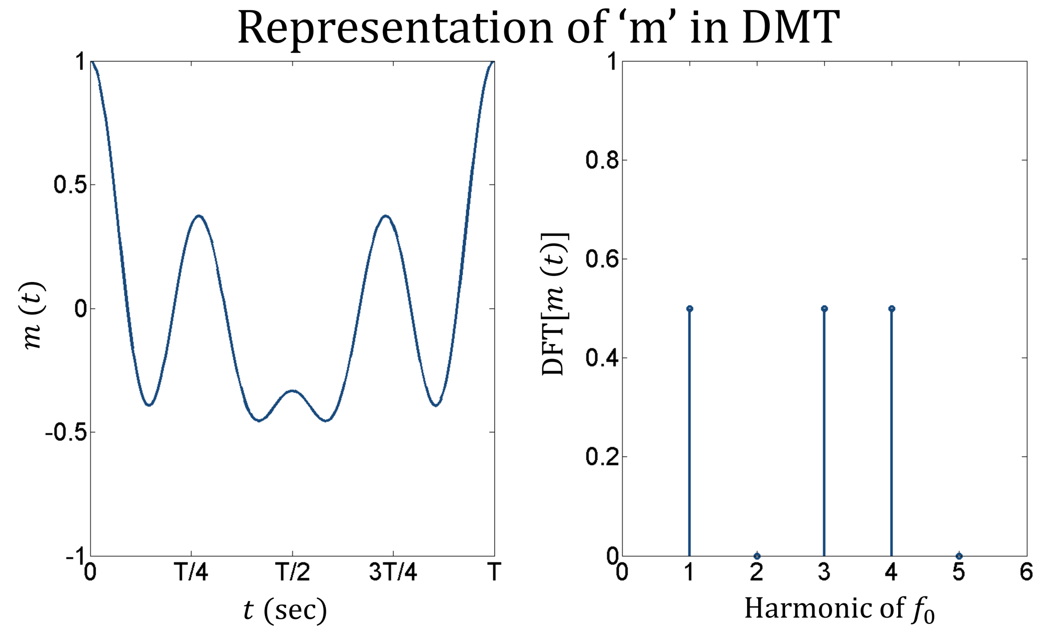
$$\psi_k(t) = b_k \cos(2\pi(k+1)f_0 t)$$

Multitone Decomposition of 'm' = 01101



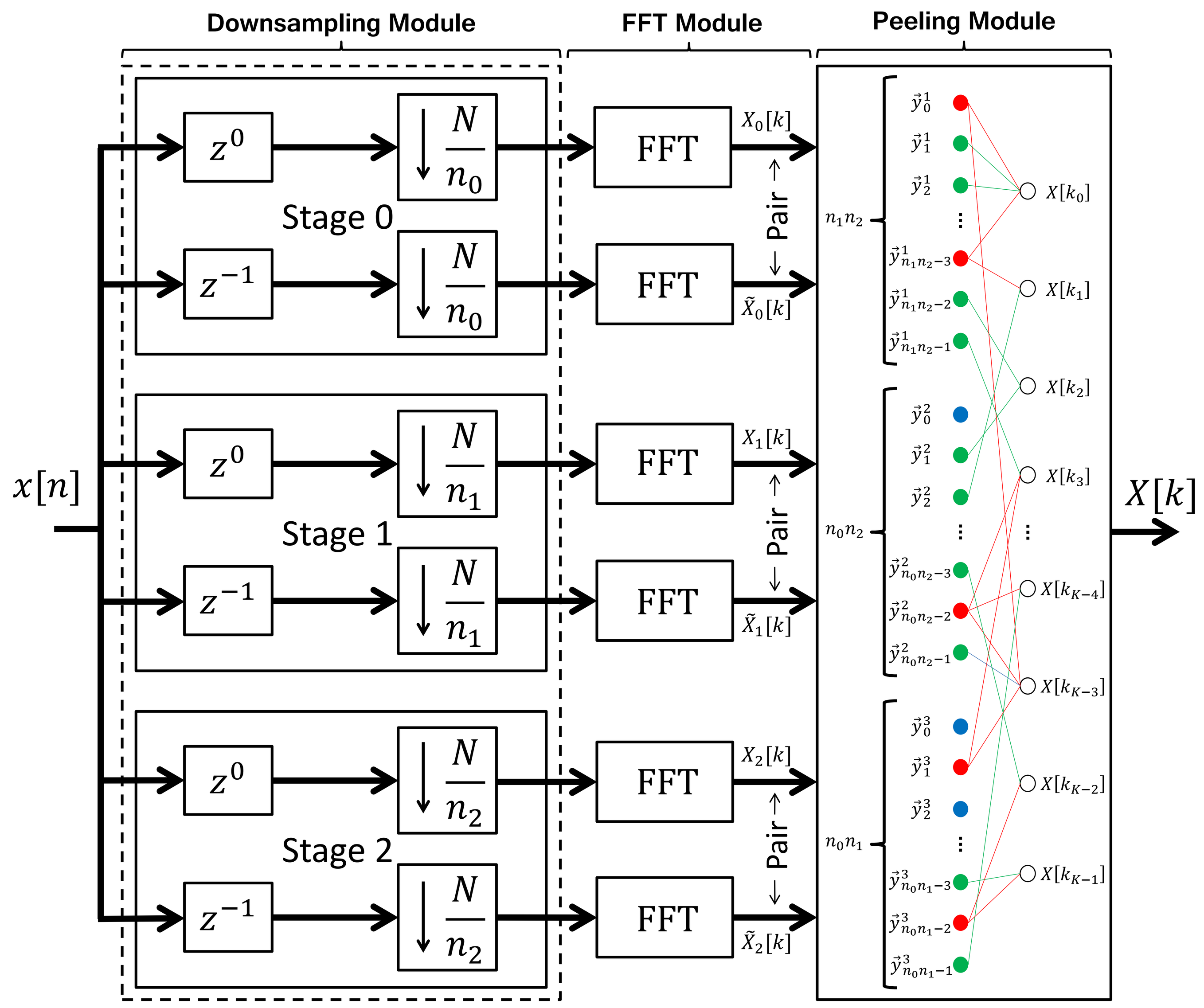
$$m(t) = \frac{1}{\sum_{k=0}^{B-1} b_k} \sum_{k=0}^{B-1} \psi_k(t)$$

Representation of 'm' in DMT



- Bits detected by observing $m(t)$ in fourier domain

Fast Fourier Aliasing-based Sparse Transform (FFAST)



FFAST Architecture

- Downsampling Module
  - In stage j, signal is delayed and downsampled by n_j
- FFT Module
  - Compute DFT each delayed and downsampled signal
- Peeling Module
  - Construct bipartite graph and backsolve via peeling methods

Convergence Conditions

- Sufficient Sparsity
  - K nonzero DFT coefficients
  - $K < N^{1/3}$
- Downsampling Coefficients
  - $n_0\, n_1\, n_2 = N$
  - n_0, n_1, n_2 are coprime
  - $O(n_0) = O(n_1) = O(n_2)$

Experiment Design and Results

Experiment Design:

- Optimized FFT Algorithm
  - Mixed-Radix FFT
  - Winograd (Short-Length Transforms)
  - Rader's FFT (Long-Length Transforms)
- Operation Count Methodology
  - Complex Multiplications and Additions
  - Lookup Table for Twiddle Factors and $\tan^{-1}$
- Partial DFT
  - DFT calculated only at nonzero frequencies
- DMT Demodulation Demo

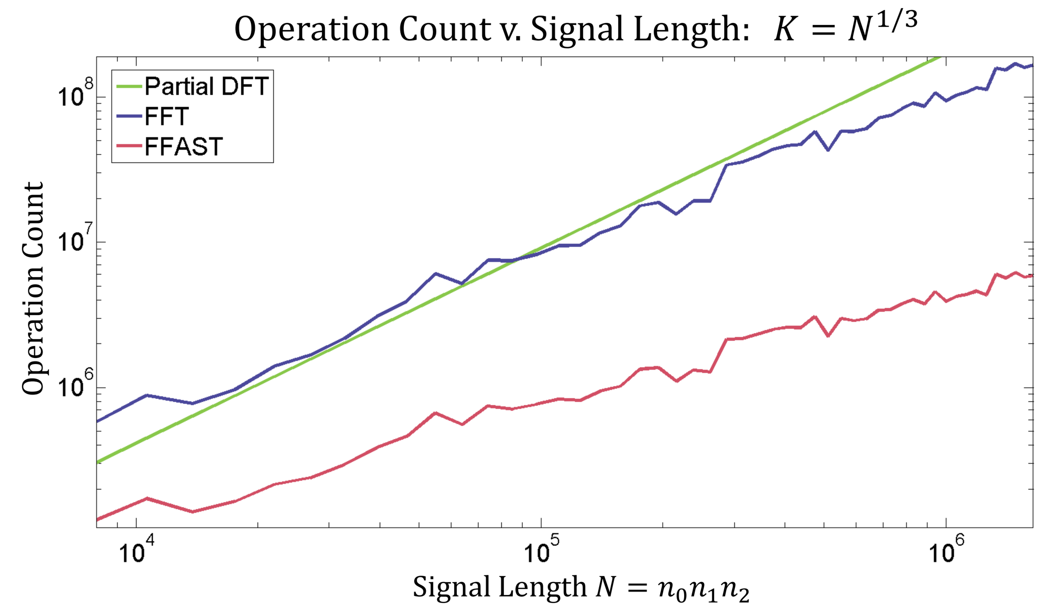
"The quick brown fox jumps over the lazy dog!"

Total Operations Saved:	2.43e+07
Average Operations Saved per Letter:	5.53e+05
Computation Decrease:	79.36%

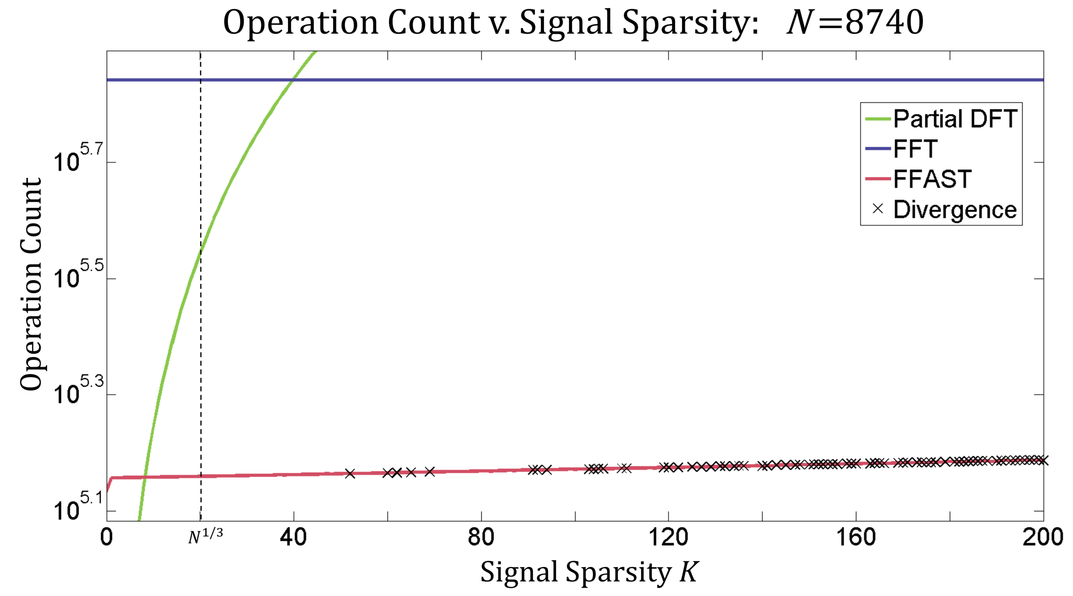
Results:

- FFAST is best demodulation method for $K < N^{1/3}$, but breaks down for $K > N^{1/3}$
- Main bottleneck in FFAST is FFT module
- Partial DFT demodulation is naïve so operation counts are optimistic

Operation Count v. Signal Length: $K = N^{1/3}$



Operation Count v. Signal Sparsity: N=8740



Discussion

From the results, the FFAST algorithm outperforms the mixed-radix FFT for all signals with lengths $N > 2^{14}$, and the additional condition that $2B < N^3$. For instance, if a digital multitone scheme with N = 2^{14} samples per $B = 12$ simultaneously transmitted bits, a sparse FFT will require fewer computations than existing frequency domain schemes, thus reducing demodulation time. This suggests that the FFAST and other sparse FFT algorithms can be practical and desirable in digital multitone communication schemes with appropriate design parameters, N and B.

Future Work

- Providing analysis and computational results for noisy case
- Researching compatible sampling schemes
- Physical implementation and experimental results

References

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[3] Clive Temperton, *Self-sorting mixed-radix fast fourier transforms*, Journal of computational physics 52 (1983), no. 1, 1-23.

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