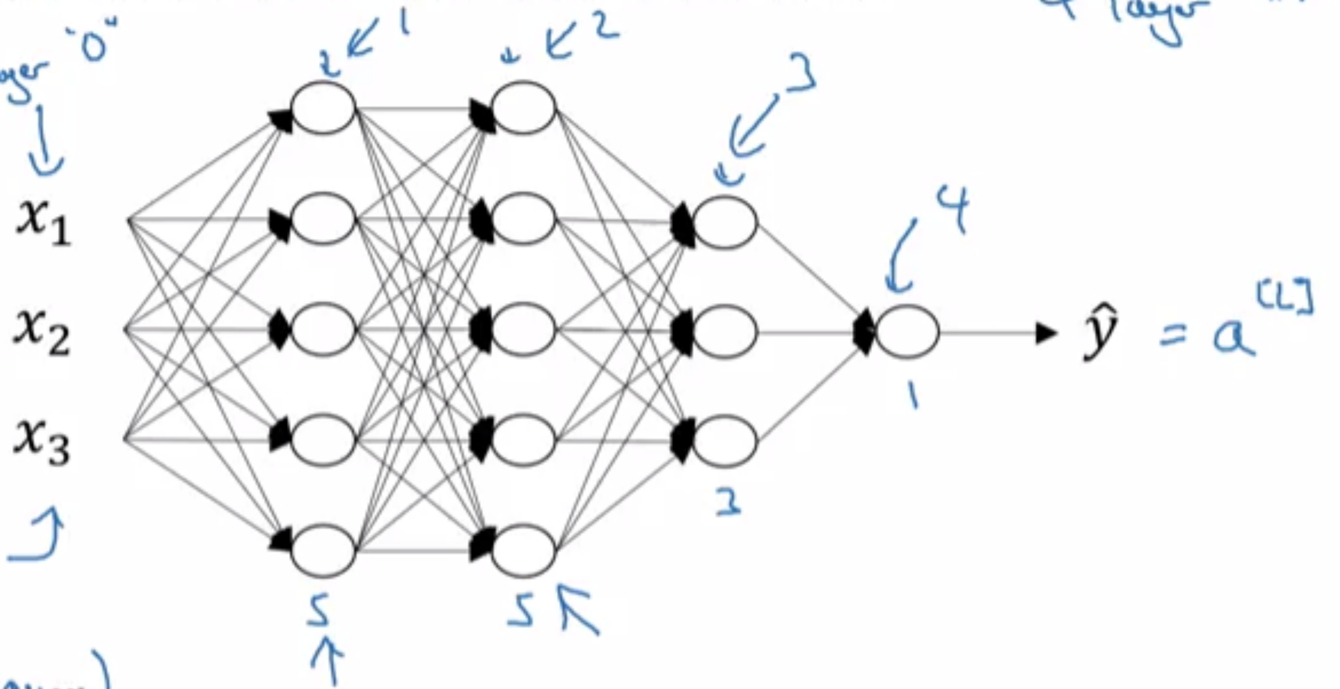
Deep neural network notation

4 layer NN

layer "0"



$X = a^{[0]}$

$L = 4$ (#layers)

$n^{[l]}$ = #units in layer l

$n^{[1]} = 5$, $n^{[2]} = 5$, $n^{[3]} = 3$, $n^{[4]} = n^{[L]} = 1$

$n^{[0]} = n_x = 3$

$a^{[l]}$ = activations in layer l

$a^{[l]} = g^{[l]}(z^{[l]})$, $W^{[l]}$ = weights for $z^{[l]}$

$b^{[l]}$

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Forward propagation for layer l

→ Input $a^{[l-1]}$ ← $W^{[l]}, b^{[l]}$

→ Output $a^{[l]}$, cache ($z^{[l]}$)

$z^{[l]} = W^{[l]} \cdot a^{[l-1]} + b^{[l]}$

$a^{[l]} = g^{[l]}(z^{[l]})$

Vectorized:

$Z^{[l]} = W^{[l]} \cdot A^{[l-1]} + b^{[l]}$

$A^{[l]} = g^{[l]}(Z^{[l]})$

$a^{[0]}$

$A^{[0]}$

$X = A^{[0]} \rightarrow \square \rightarrow \square \rightarrow \square \rightarrow$

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Backward propagation for layer l

→ Input $da^{[l]}$

→ Output $\boxed{da^{[l-1]}}$, $dW^{[l]}$, $db^{[l]}$

$$dz^{[l]} = da^{[l]} * g^{[l]\prime}(z^{[l]})$$

$$dW^{[l]} = dz^{[l]} \cdot a^{[l-1]}$$

$$db^{[l]} = dz^{[l]}$$

$$\boxed{da^{[l-1]}} = W^{[l]T} \cdot dz^{[l]}$$

$$dz^{[l]} = W^{[l+1]T} dz^{[l+1]} * g^{[l]\prime}(z^{[l]})$$

$$dZ^{[l]} = dA^{[l]} * g^{[l]\prime}(Z^{[l]})$$

$$dW^{[l]} = \frac{1}{m} dZ^{[l]} \cdot A^{[l-1]T}$$

$$db^{[l]} = \frac{1}{m} np.sum(dZ^{[l]}, axis=1, keepdims=True)$$

$$\boxed{dA^{[l-1]}} = W^{[l]T} \cdot dZ^{[l]}$$

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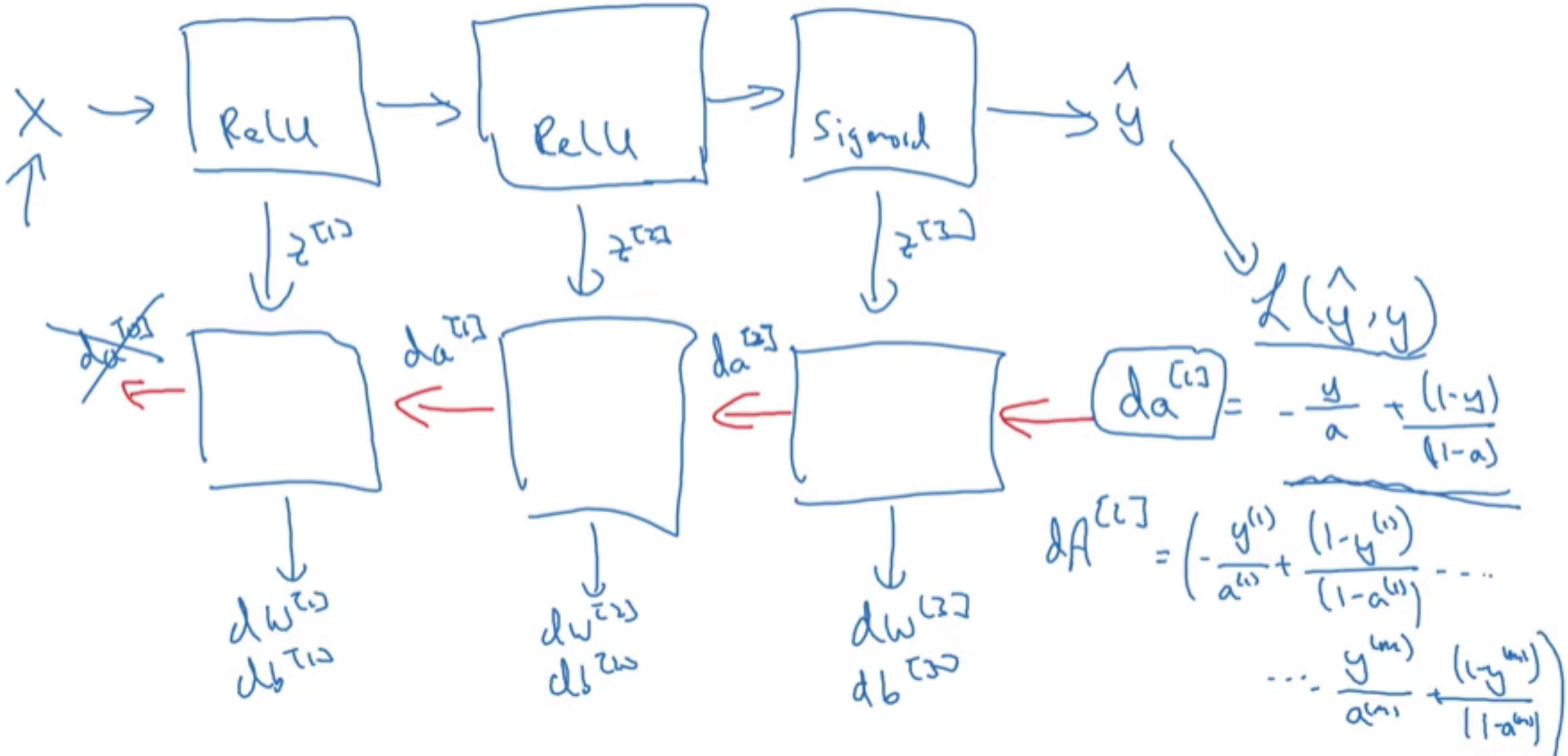
Summary



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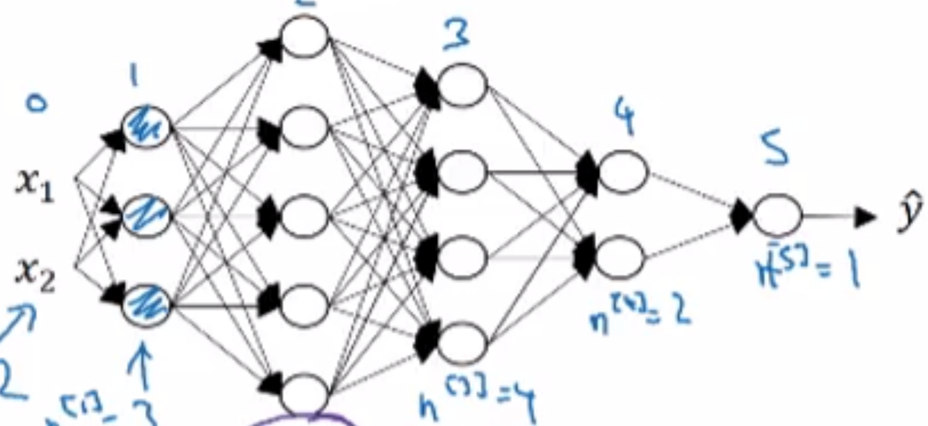
Parameters $W^{[l]}$ and $b^{[l]}$

$L = 5$



$n^{[0]} = n_x = 2$ $\quad n^{[1]} = 3$ $\quad n^{[2]} = 5$ $\quad n^{[3]} = 4$ $\quad n^{[4]} = 2$ $\quad n^{[5]} = 1$

$$z^{[l]} = g^{[l]}(a^{[l]})$$

$$a^{[l]}$$

$$W^{[l]} : (n^{[l]}, n^{[l-1]})$$
$$b^{[l]} : (n^{[l]}, 1)$$
$$dW^{[l]} : (n^{[l]}, n^{[l-1]})$$
$$db^{[l]} : (n^{[l]}, 1)$$

$$z^{[1]} = \boxed{W^{[1]} \cdot x} + \boxed{b^{[1]}}$$

$(3,1) \leftarrow (3,2) \quad (2,1) \quad (3,1)$

$(n^{[1]}, 1) \quad (n^{[1]}, n^{[0]}) \quad (n^{[0]}, 1) \quad (n^{[1]}, 1)$

$$W^{[1]} : (n^{[1]}, n^{[0]})$$

$$W^{[2]} : (5, 3) \quad (n^{[2]}, n^{[1]})$$

$$z^{[2]} = \boxed{W^{[2]} \cdot a^{[1]}} + \boxed{b^{[2]}}$$

$\rightarrow (5,1) \quad (5,3) \quad (3,1) \quad (5,1)$

$(n^{[2]}, 1)$

$$W^{[3]} : (4, 5)$$

$$W^{[4]} : (2, 4), \quad W^{[5]} : (1, 2)$$

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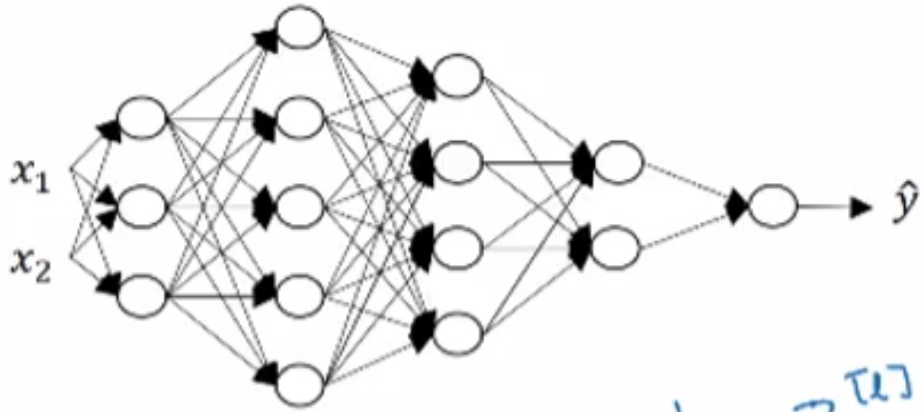
Vectorized implementation



$z^{[1]} = W^{[1]} \cdot x + b^{[1]}$

$(n^{[1]},1)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},1)$ $(n^{[1]},1)$

$[z^{[1](1)} \; z^{[1](2)} \; \dots \; z^{[1](m)}]$

$Z^{[1]} = W^{[1]} \cdot X + b^{[1]}$

$(n^{[1]},m)$ $(n^{[1]},n^{[0]})$ $(n^{[0]},m)$ $(n^{[1]},1)$

$(n^{[1]},m)$

$z^{[l]}, a^{[l]} : (n^{[l]},1)$

$Z^{[l]}, A^{[l]} : (n^{[l]},m)$

$l=0$ $A^{[0]} = X = (n^{[0]},m)$

$dZ^{[l]}, dA^{[l]} : (n^{[l]},m)$

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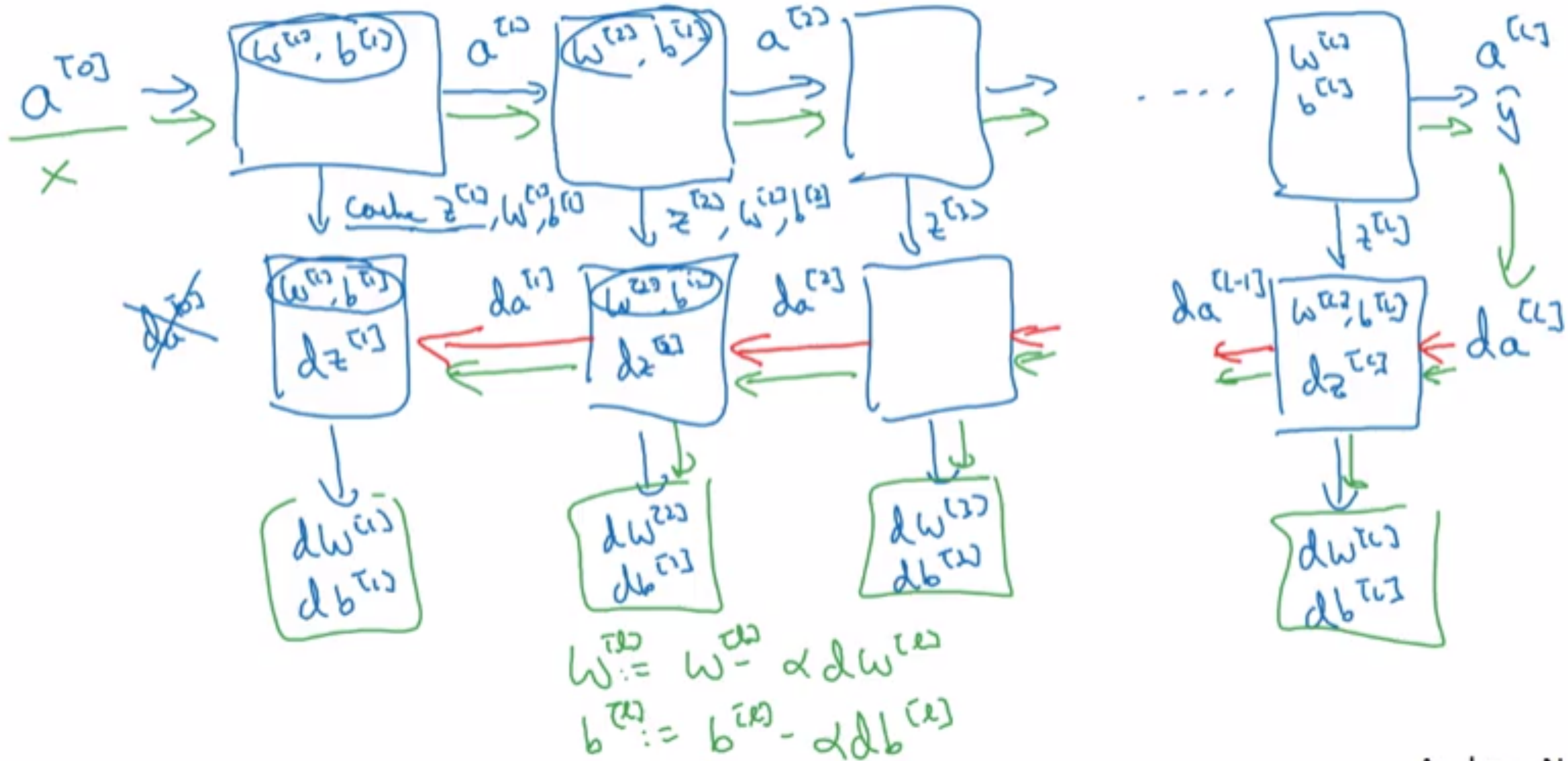
Forward and backward functions



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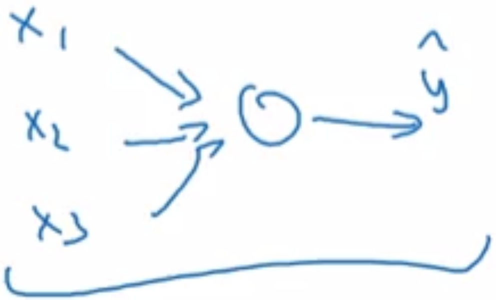
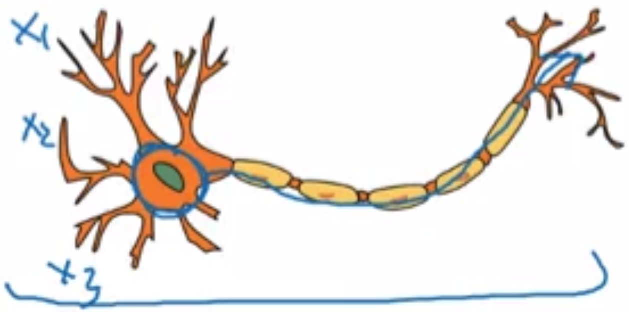
Forward and backward propagation

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = g^{[2]}(Z^{[2]})$$
$$\vdots$$
$$A^{[L]} = g^{[L]}(Z^{[L]}) = \hat{Y}$$

$$dZ^{[L]} = A^{[L]} - Y$$
$$dW^{[L]} = \frac{1}{m} dZ^{[L]} A^{[L]^T}$$
$$db^{[L]} = \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True)$$
$$dZ^{[L-1]} = dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]})$$
$$\vdots$$
$$dZ^{[1]} = dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]})$$
$$dW^{[1]} = \frac{1}{m} dZ^{[1]} A^{[1]^T}$$
$$db^{[1]} = \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)$$

"It's like the brain"





$X \longrightarrow y$

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