



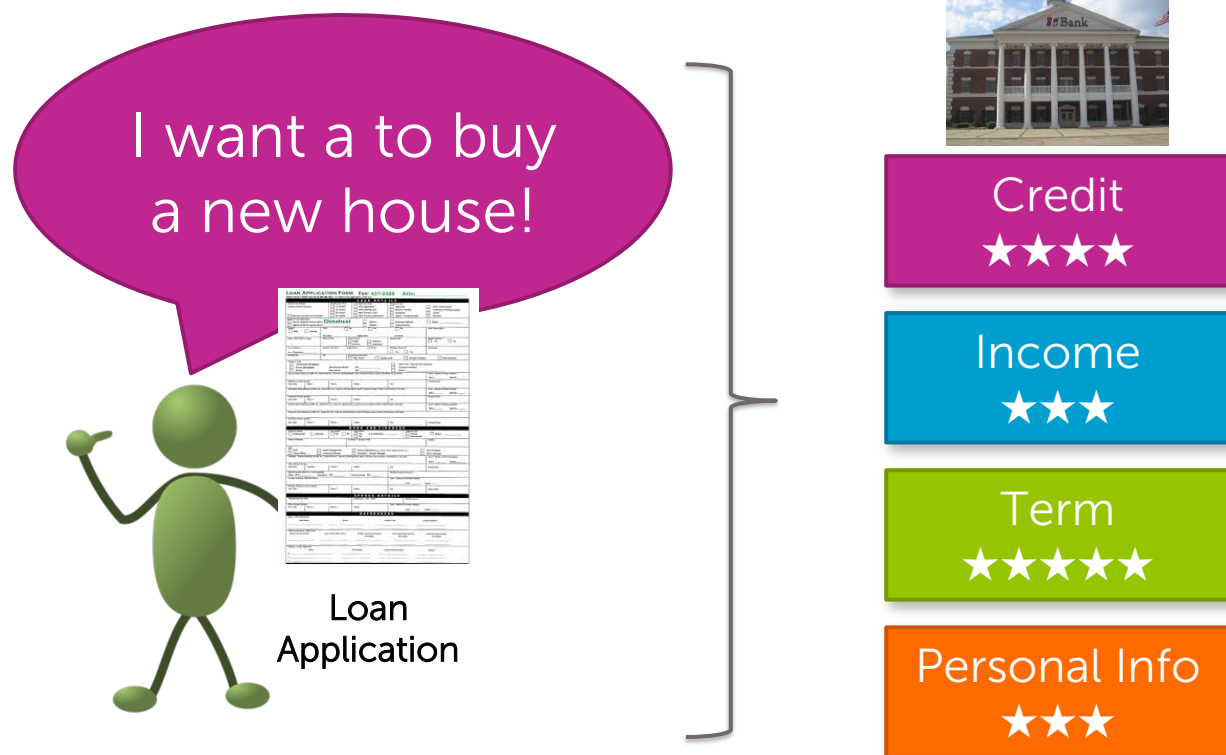
Decision Trees

Emily Fox & Carlos Guestrin
Machine Learning Specialization
University of Washington



Predicting potential loan defaults

What makes a loan risky?

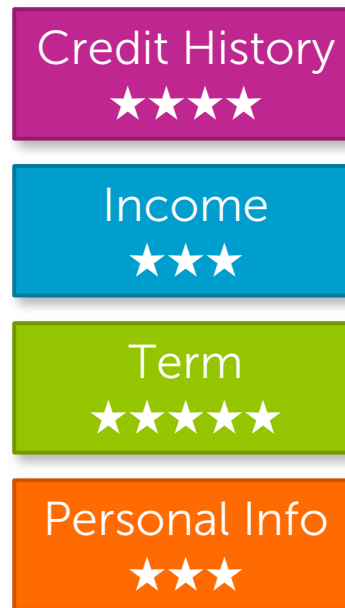


Credit history explained

Did I pay previous
loans on time?



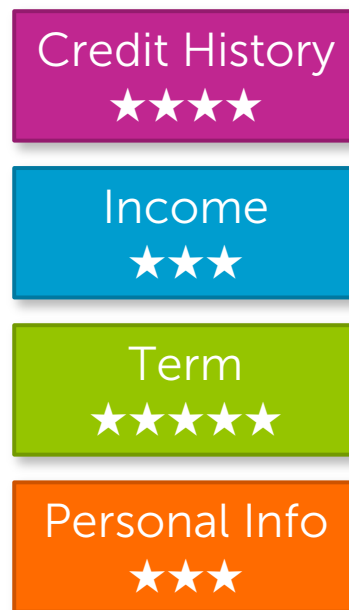
Example: excellent,
good, or fair



Income

What's my income?

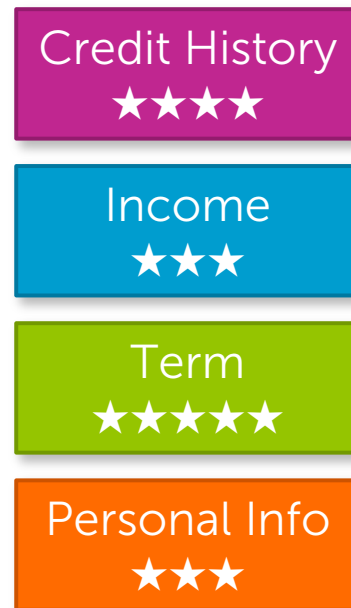
Example:
\$80K per year



Loan terms

How soon do I need to pay the loan?

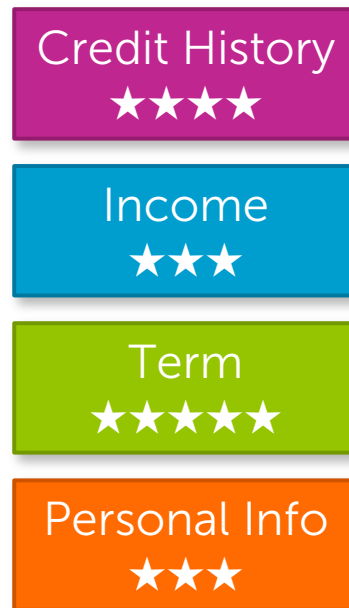
Example: 3 years,
5 years,...



Personal information

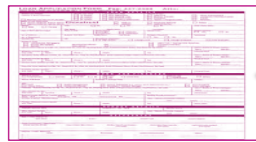
Age, reason for the
loan, marital status,...

Example: Home loan
for a married couple



Intelligent application

Loan
Applications

A pink loan application form with various fields and checkboxes.A blue loan application form with various fields and checkboxes.A green loan application form with various fields and checkboxes.

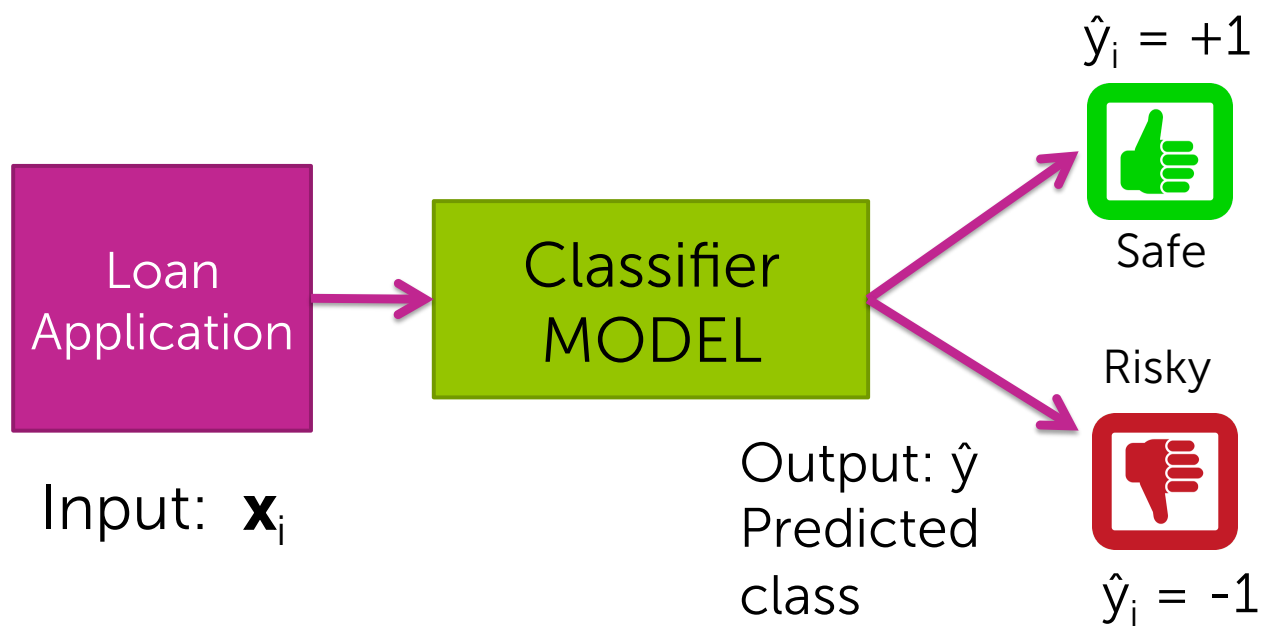
Intelligent loan application
review system

Safe
✓

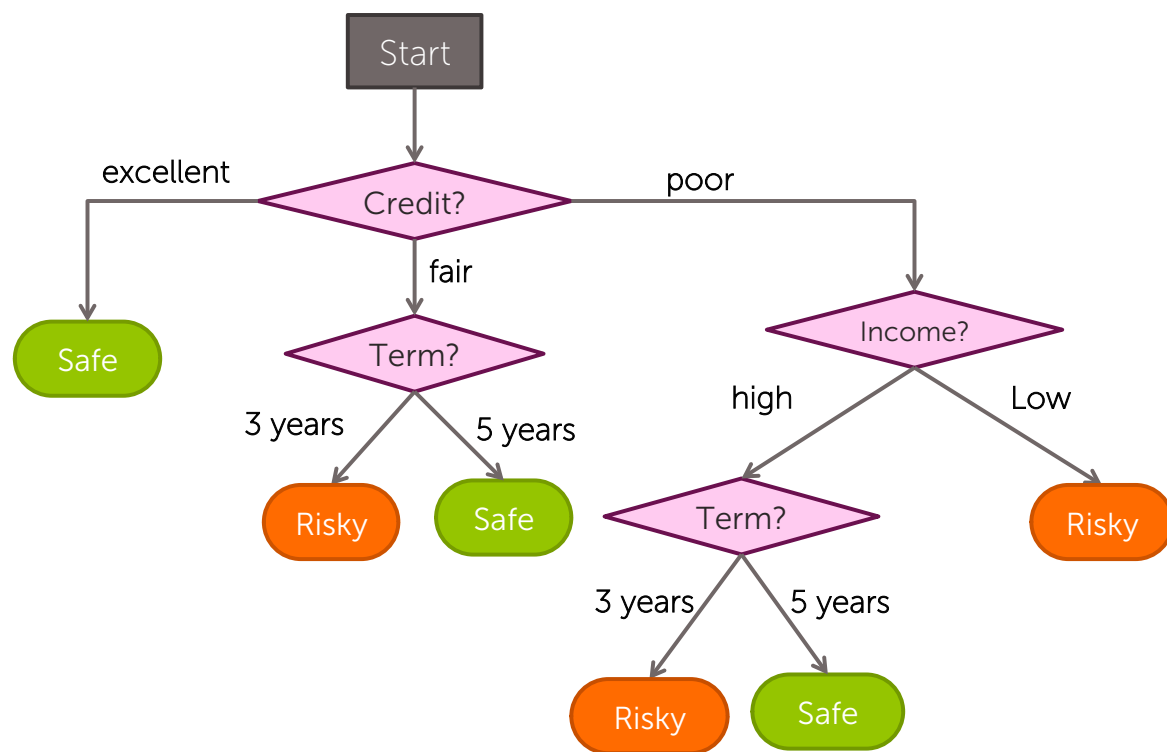
Risky
✗

Risky
✗

Classifier review

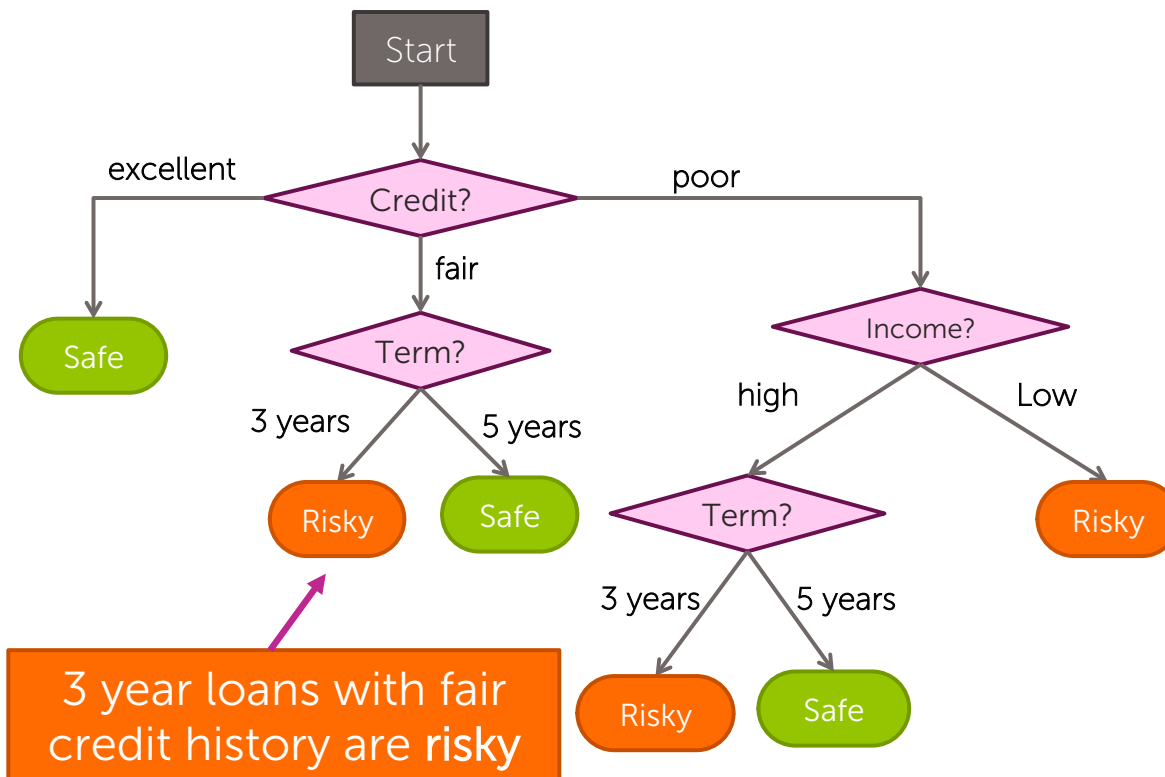


This module ... decision trees

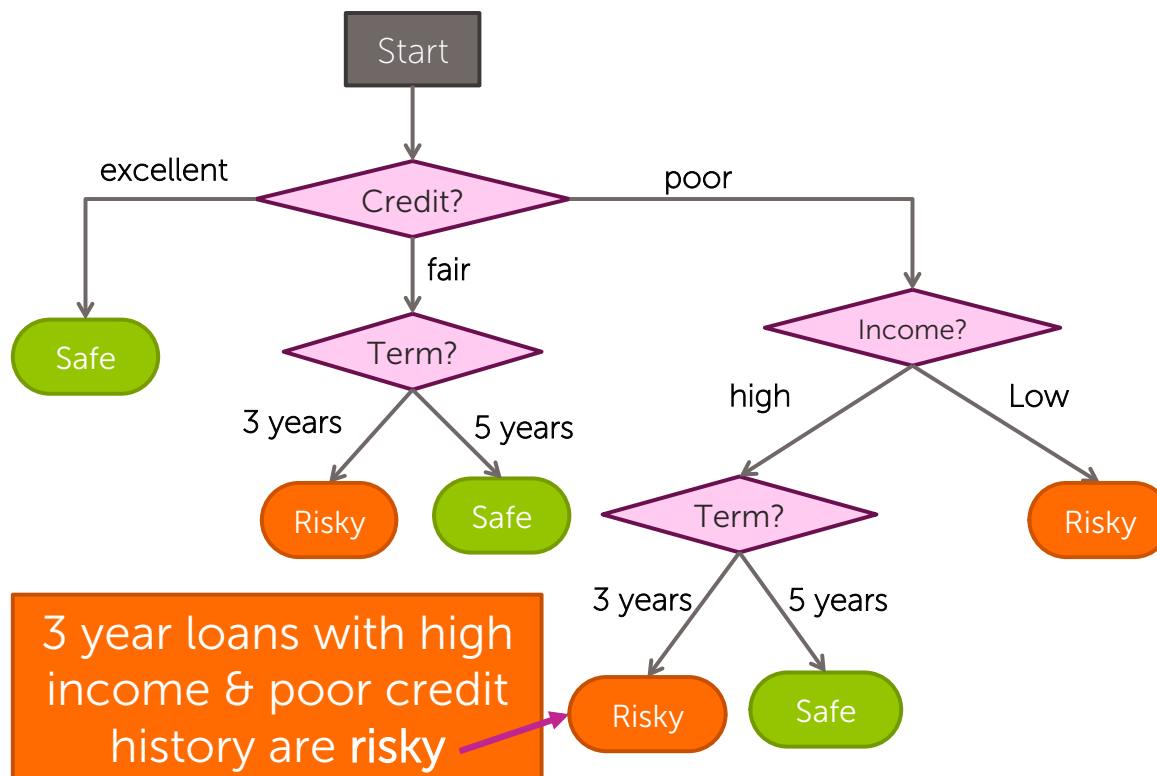


Decision trees: *Intuition*

What does a decision tree represent?

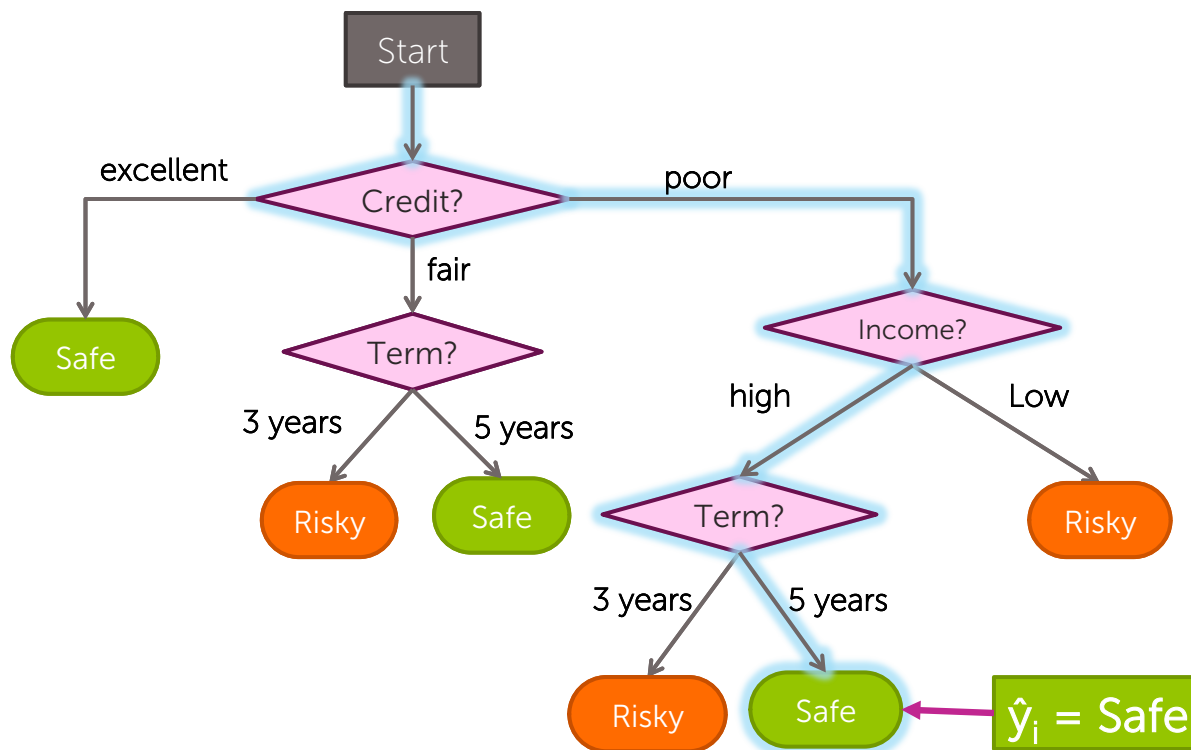


What does a decision tree represent?

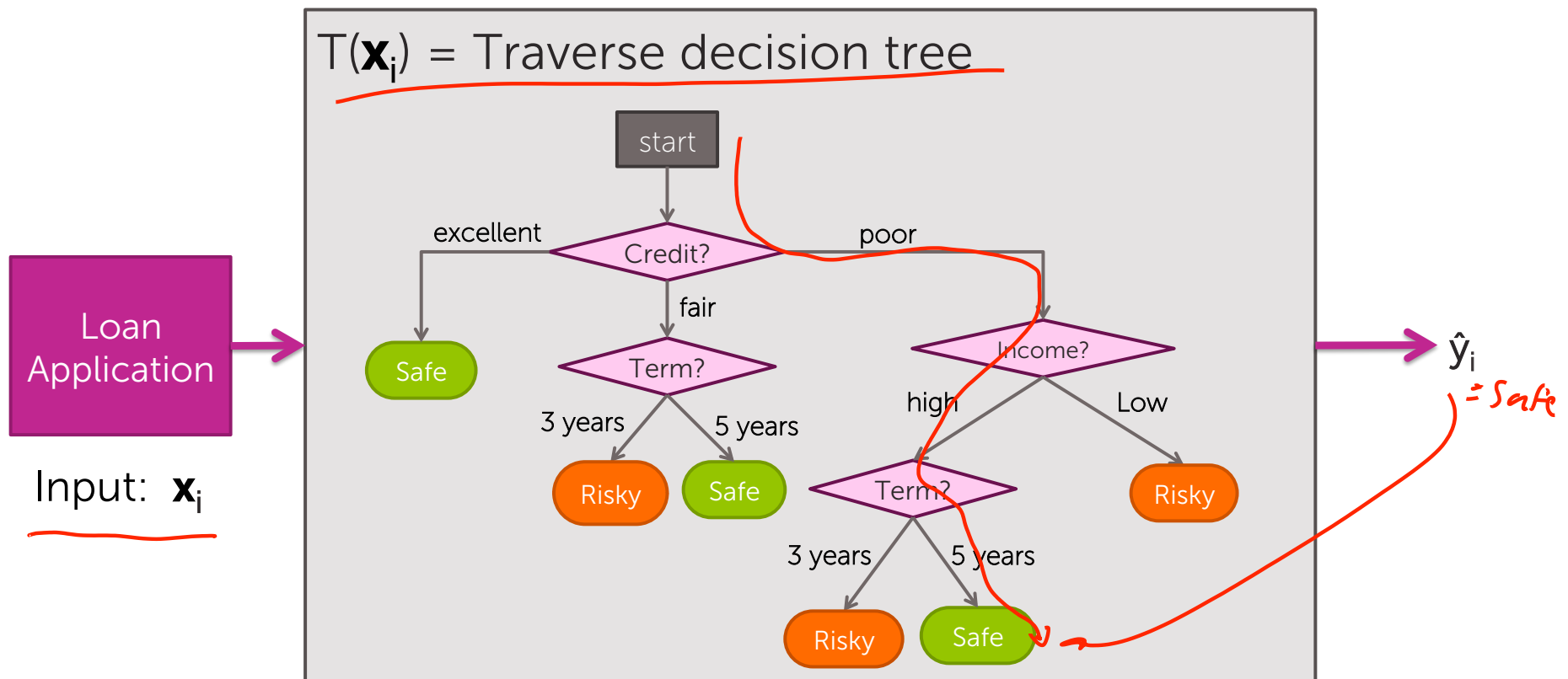


Scoring a loan application

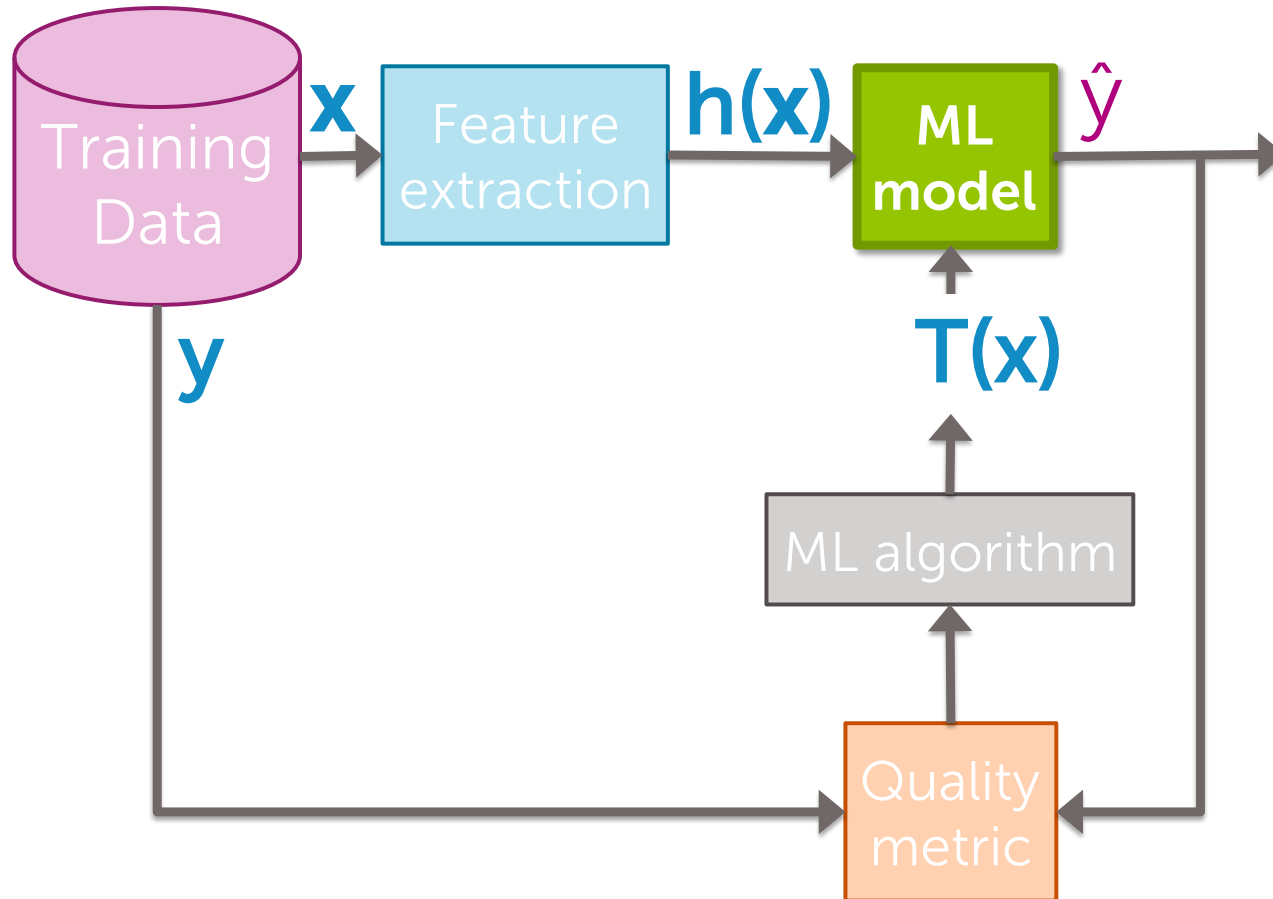
$\mathbf{x}_i = (\text{Credit} = \text{poor}, \text{Income} = \text{high}, \text{Term} = 5 \text{ years})$



Decision tree model

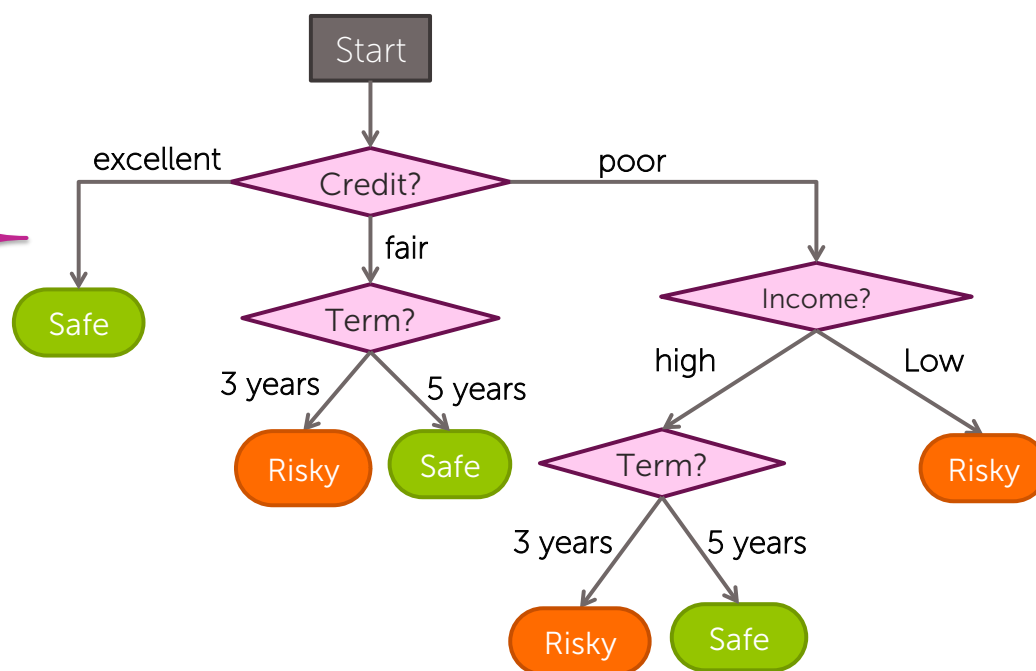


Decision tree learning task



Learn decision tree from data?

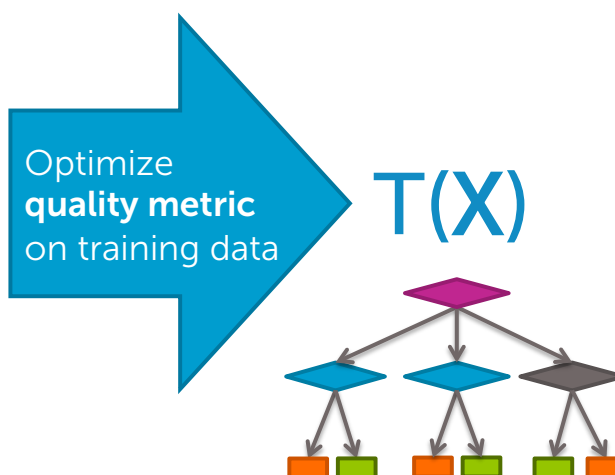
$h_1(x)$	$h_2(x)$	$h_3(x)$	Loan Status
Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe



Decision tree learning problem

Training data: N observations (\mathbf{x}_i, y_i)

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe





Quality metric: Classification error

- Error measures fraction of mistakes

$$\text{Error} = \frac{\text{\# incorrect predictions}}{\text{\# examples}}$$

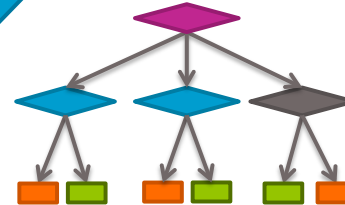
- Best possible value : 0.0
- Worst possible value: 1.0

Find the tree with lowest classification error

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

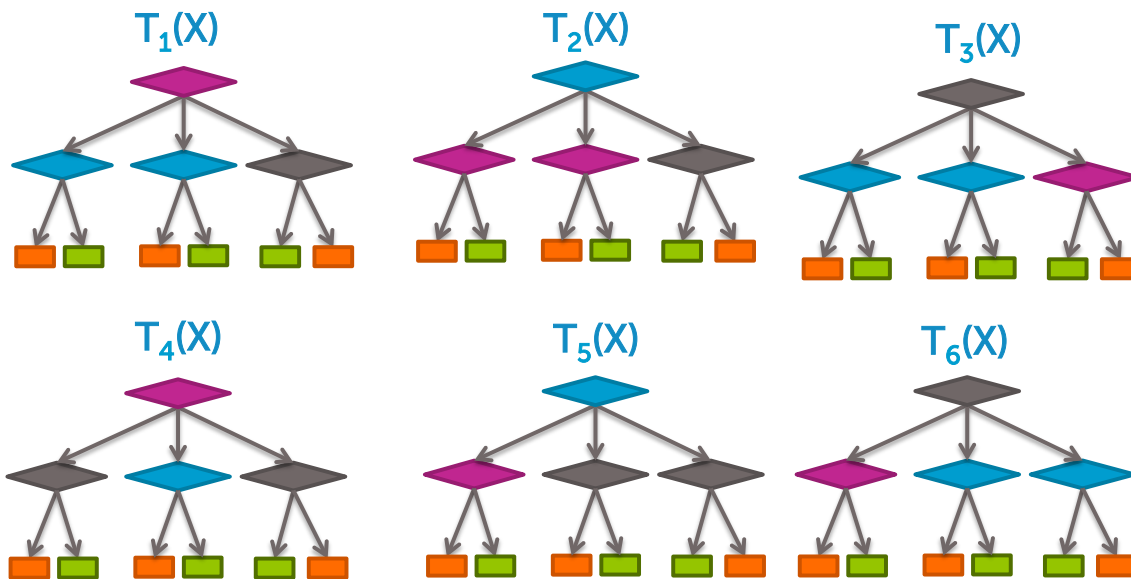
Minimize
classification error
on training data

$T(X)$



How do we find the best tree?

Exponentially large number of possible trees makes decision tree learning **hard**!
(NP-hard problem)

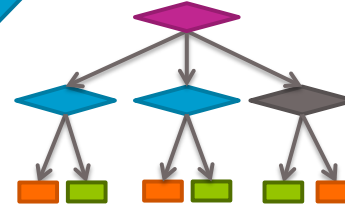


Simple (greedy) algorithm finds “good” tree

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

Approximately
minimize
classification error
on training data

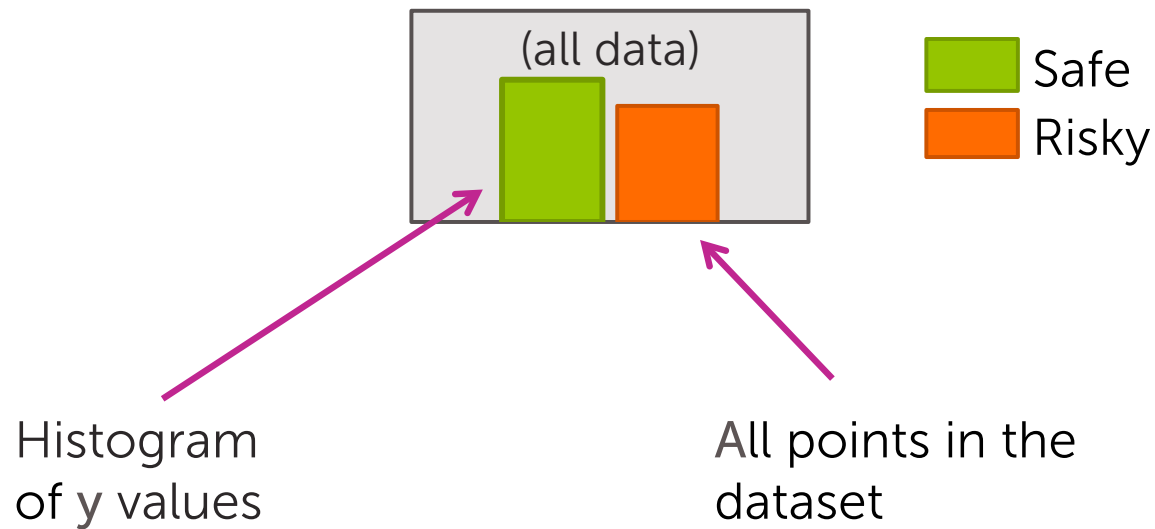
$T(X)$



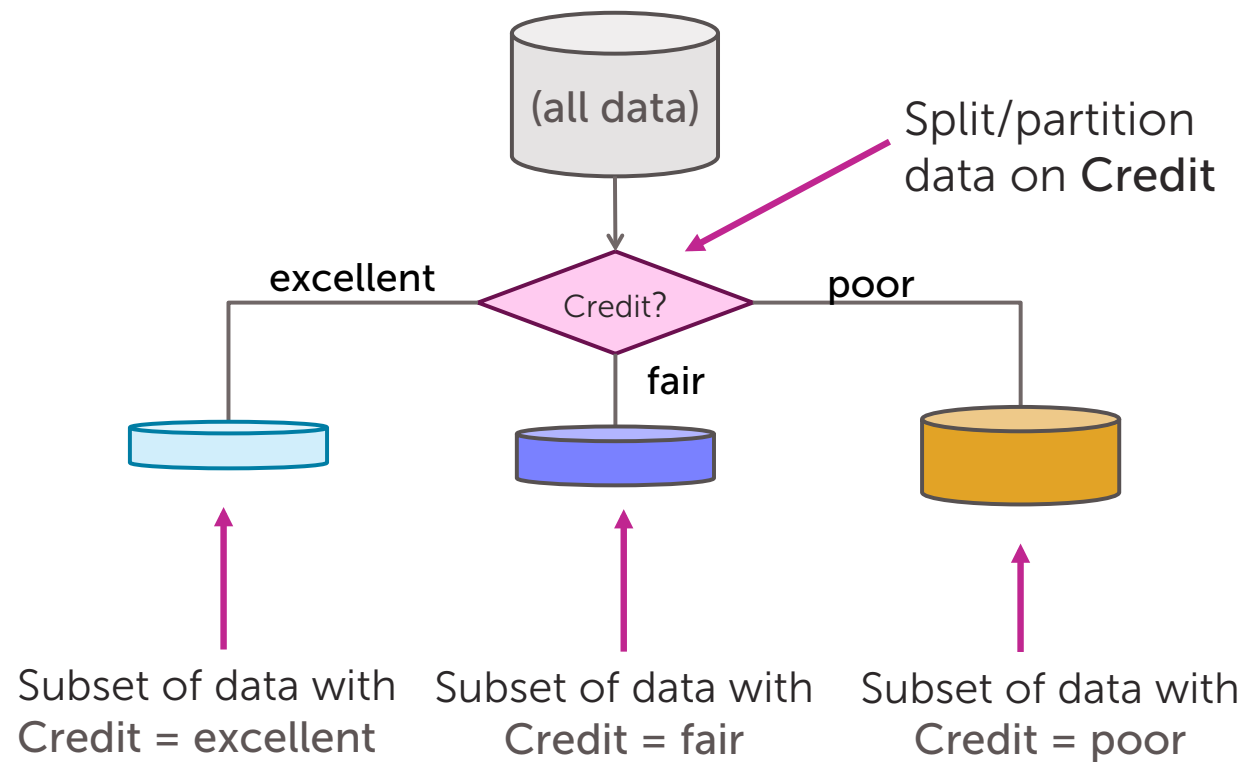
Greedy decision tree learning:

Algorithm outline

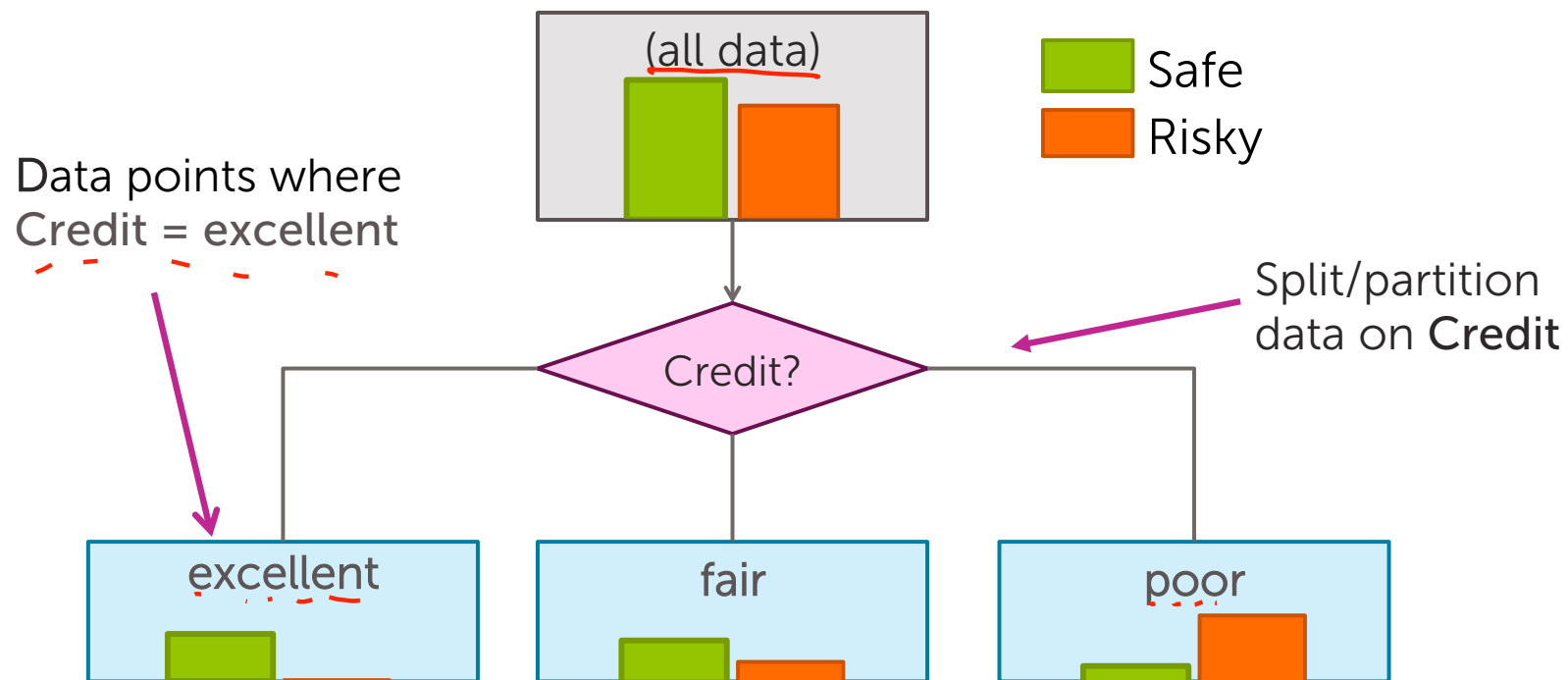
Step 1: Start with an empty tree



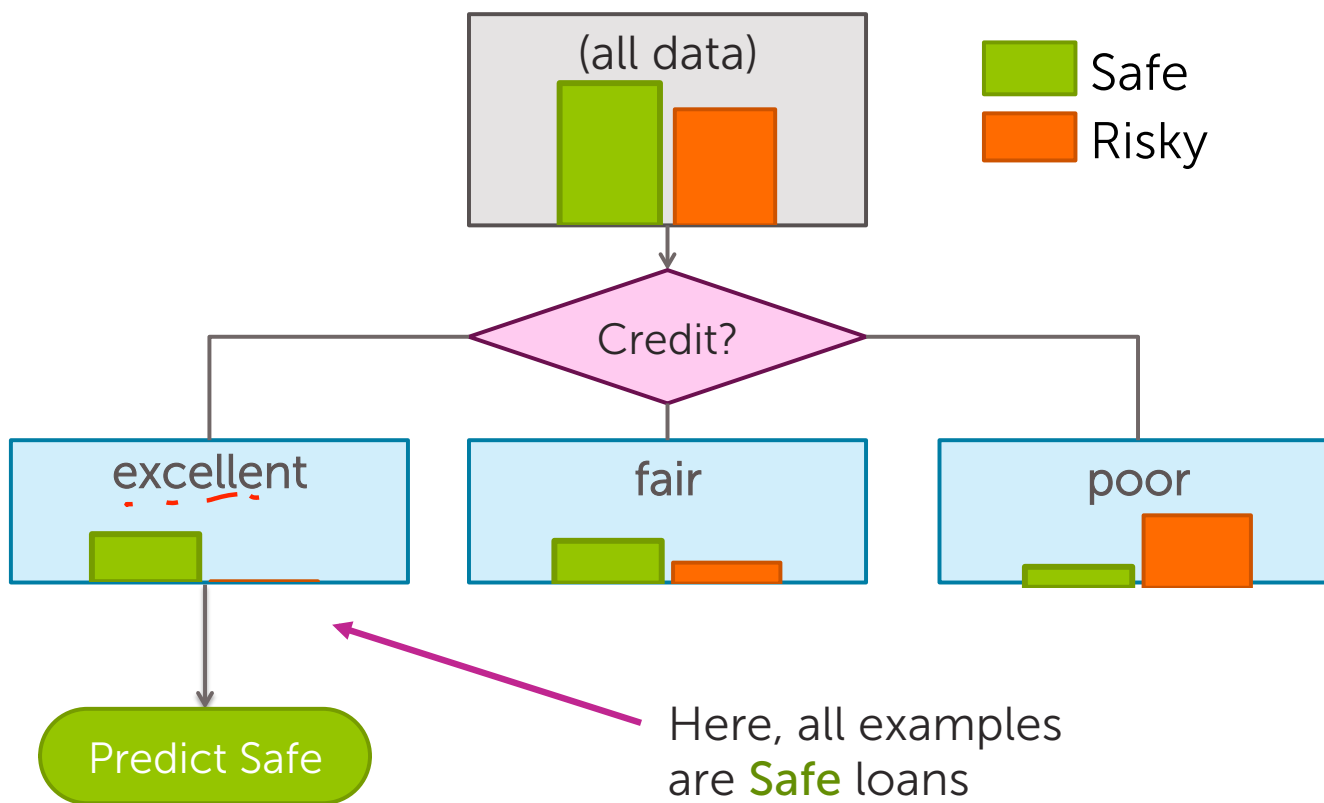
Step 2: Split on a feature



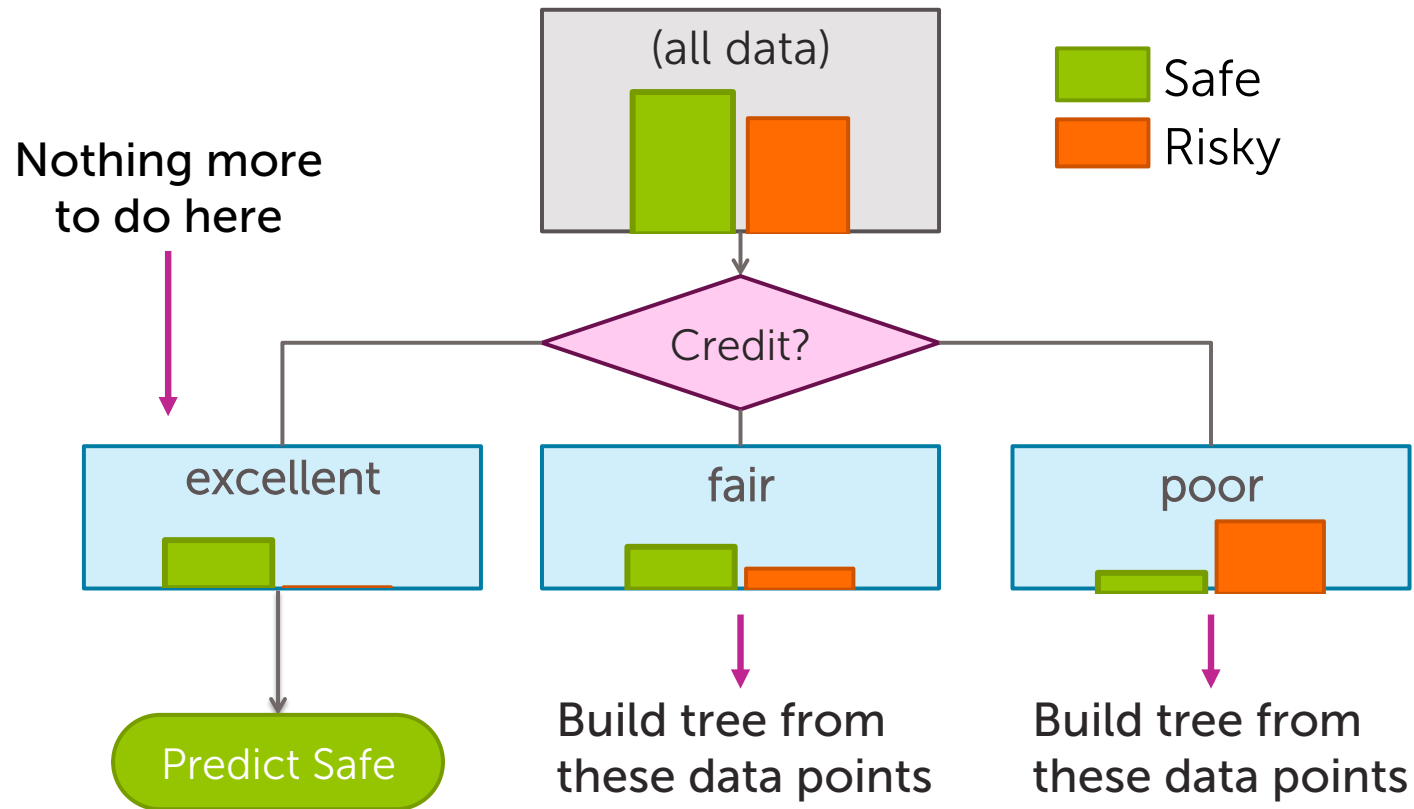
Feature split explained



Step 3: Making predictions



Step 4: Recursion



Greedy decision tree learning

- **Step 1:** Start with an empty tree

- **Step 2:** Select a feature to split data

- For each split of the tree:

- **Step 3:** If nothing more to, make predictions

- **Step 4:** Otherwise, go to **Step 2** & continue (recurse) on this split

Problem 1: Feature split selection

Problem 2: Stopping condition

Recursion



Feature split learning

=

Decision stump learning



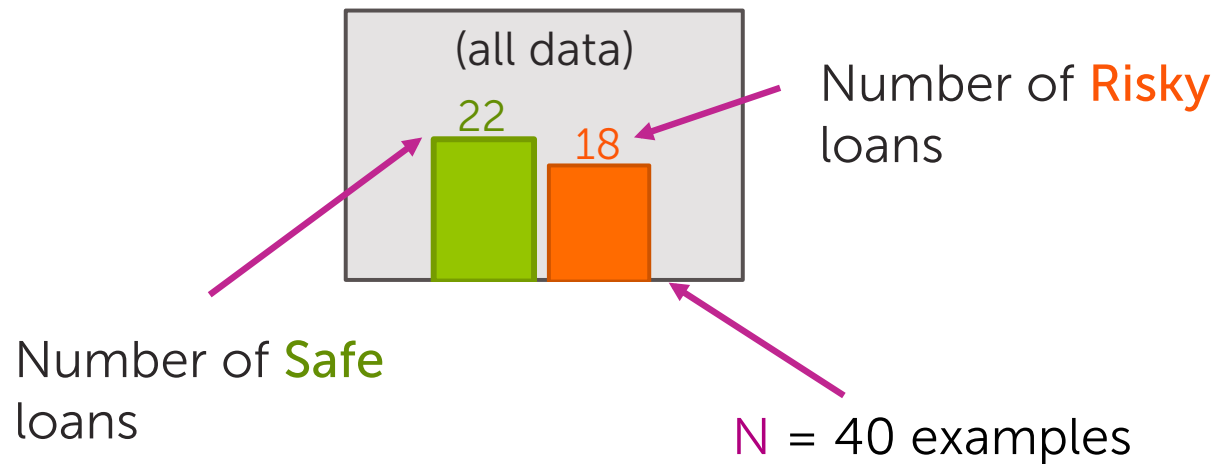
Start with the data

Assume $N = 40$, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

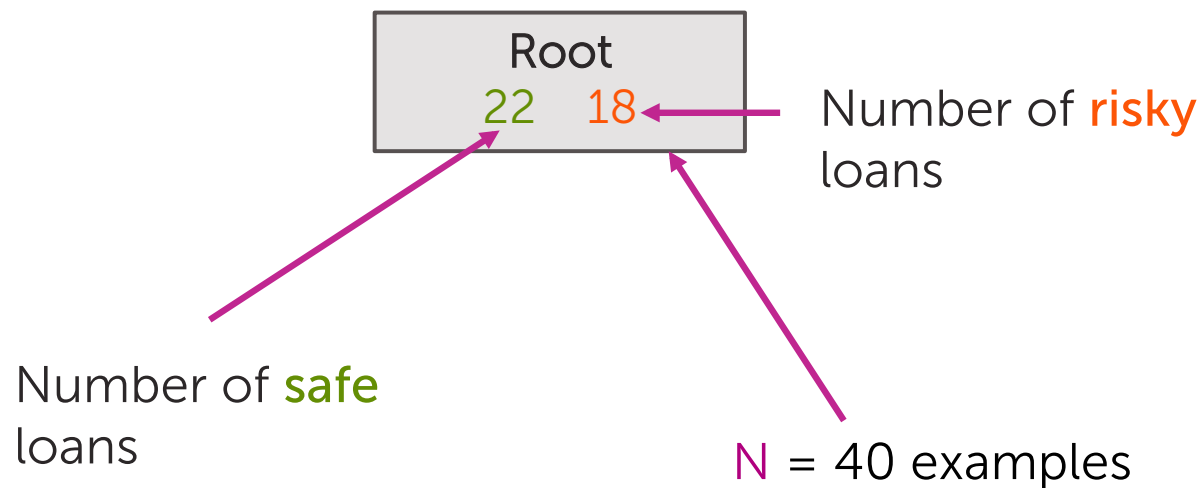
Start with all the data

Loan status: **Safe** **Risky**

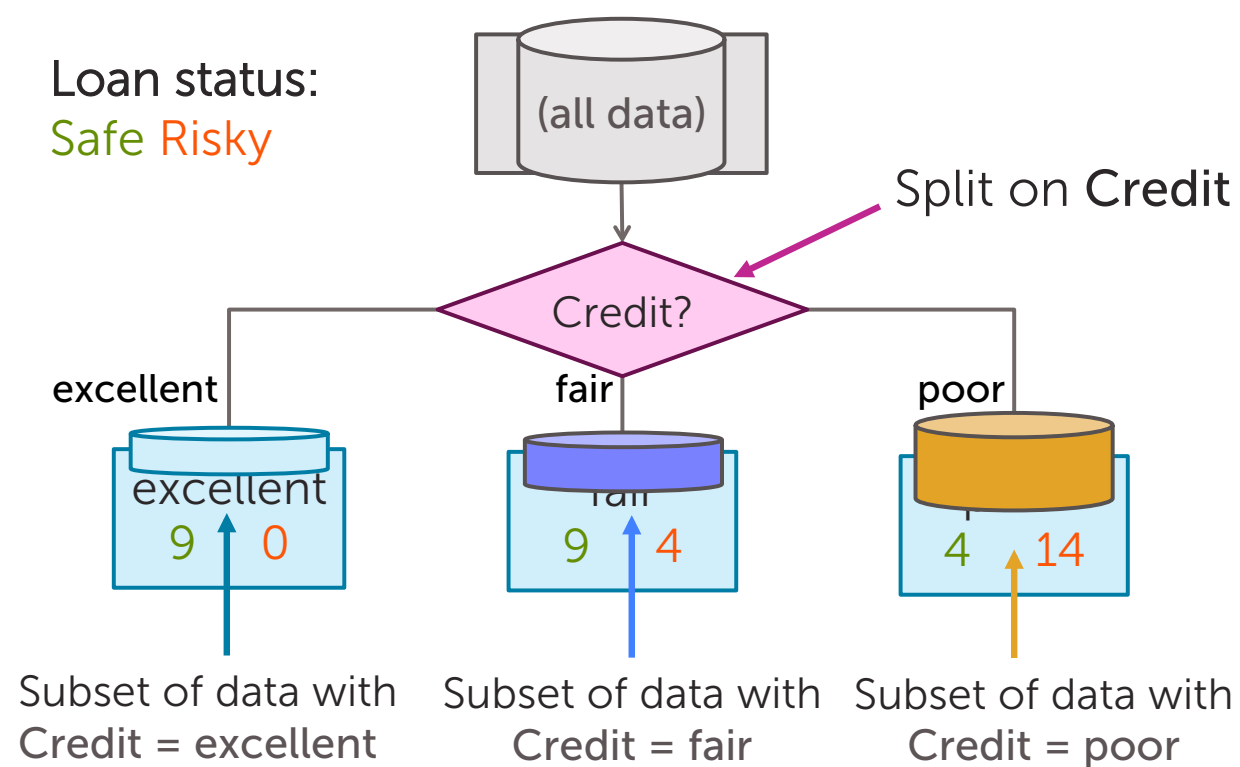


Compact visual notation: Root node

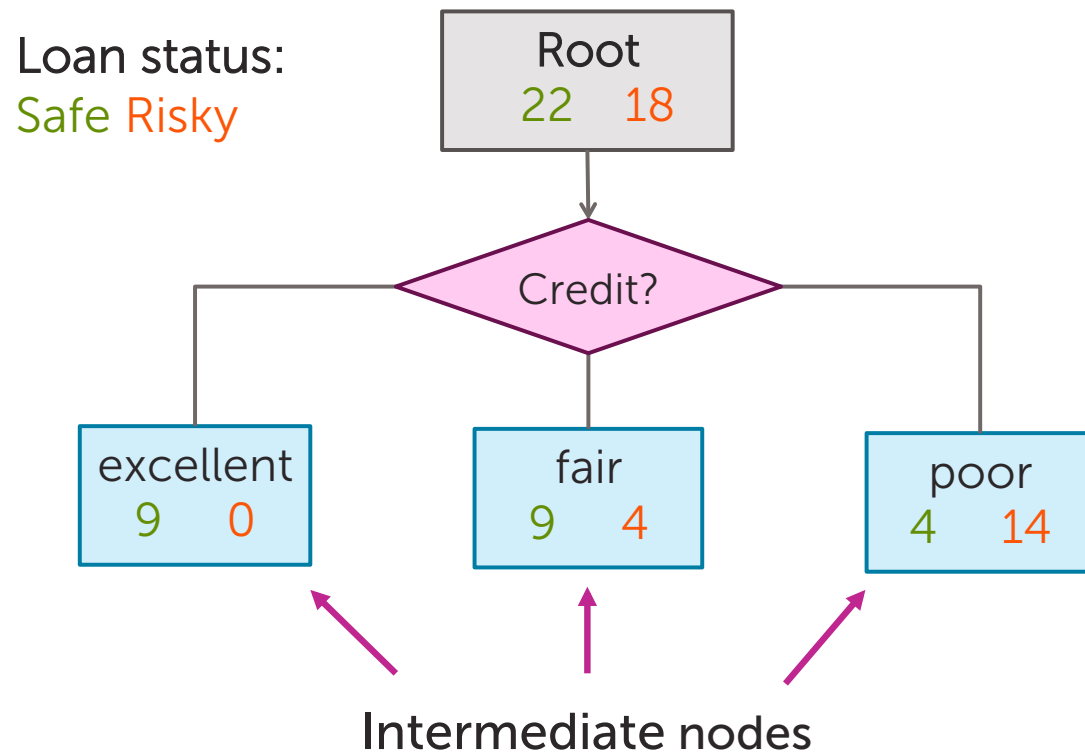
Loan status: **Safe** **Risky**



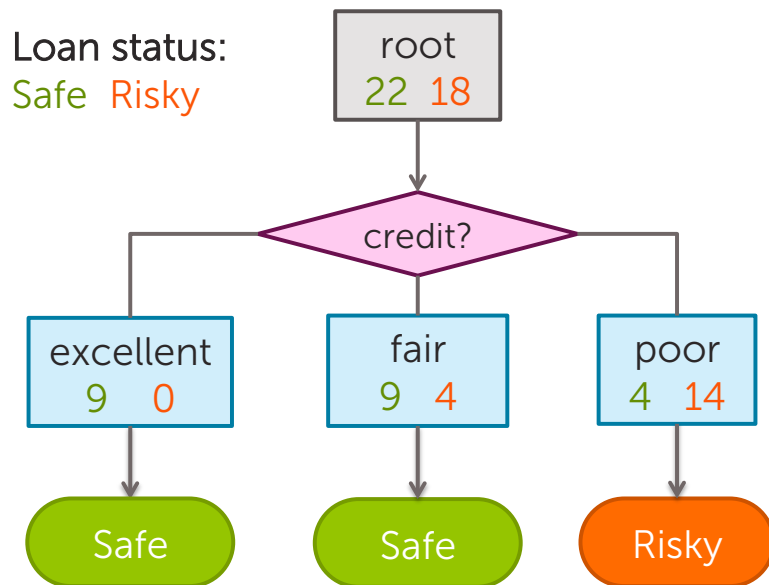
Decision stump: Single level tree



Visual Notation: Intermediate nodes



Making predictions with a decision stump

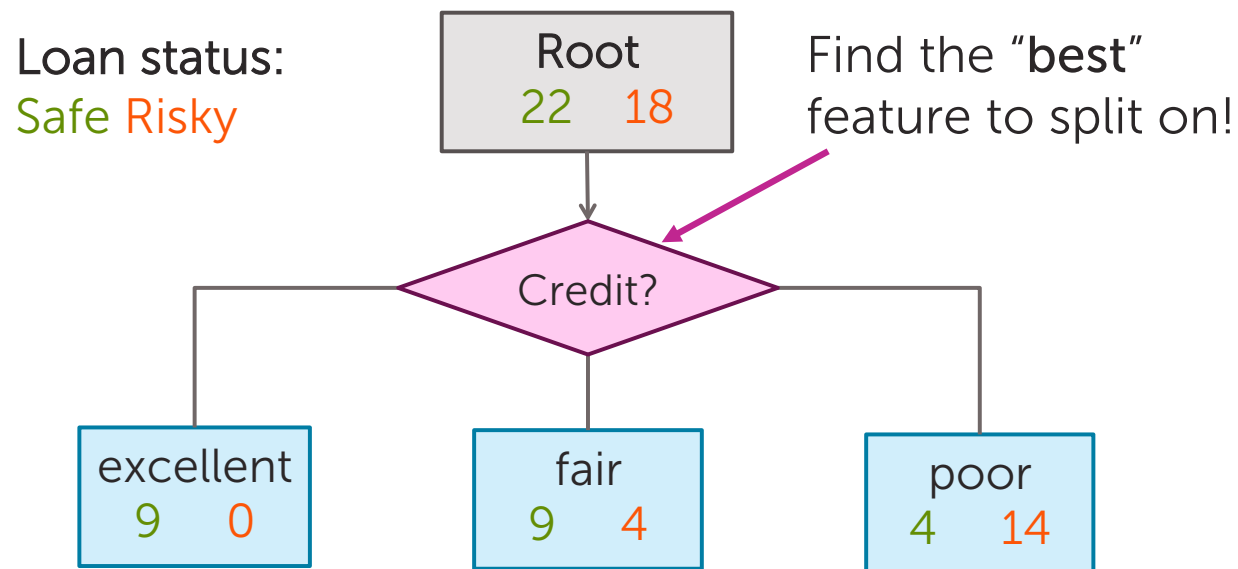


For each intermediate node,
set \hat{y} = majority value

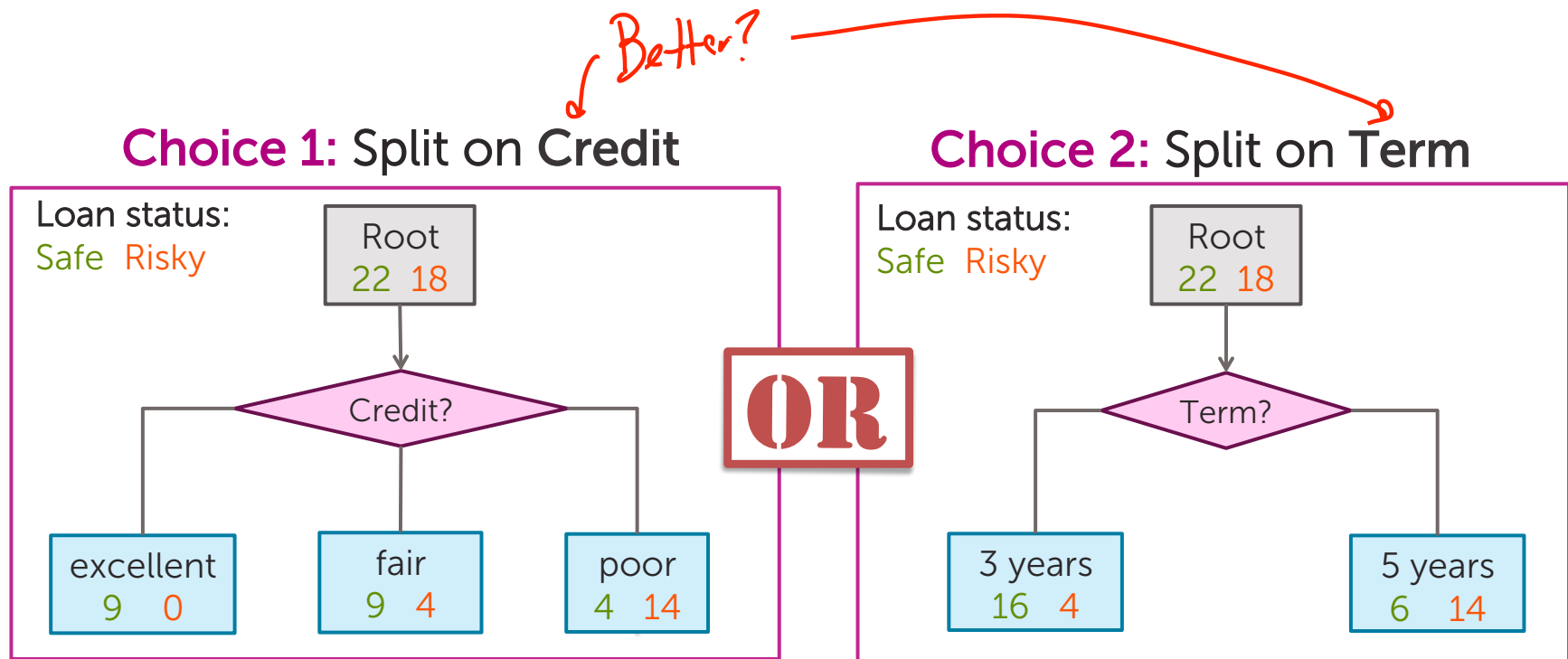


Selecting best feature to split on

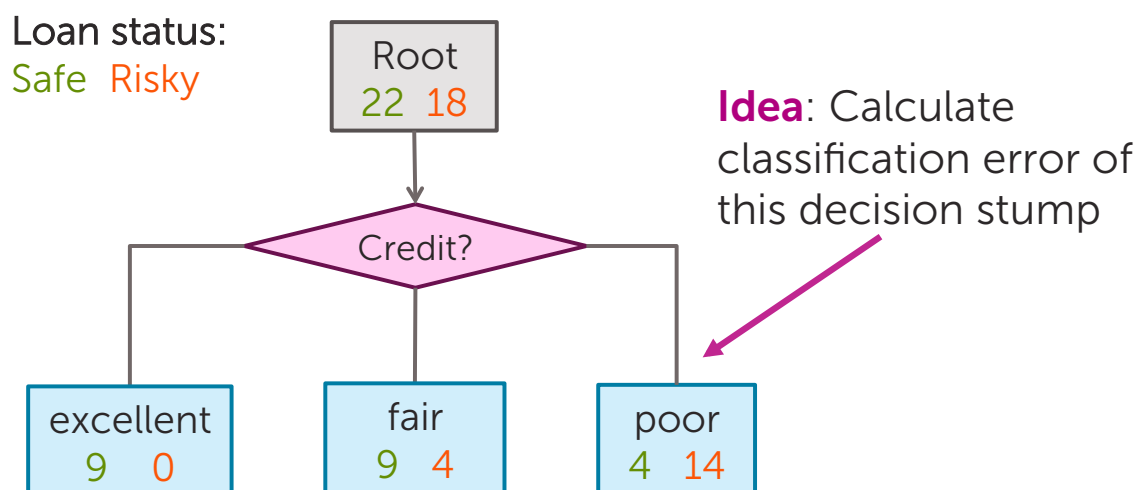
How do we learn a decision stump?



How do we select the best feature?



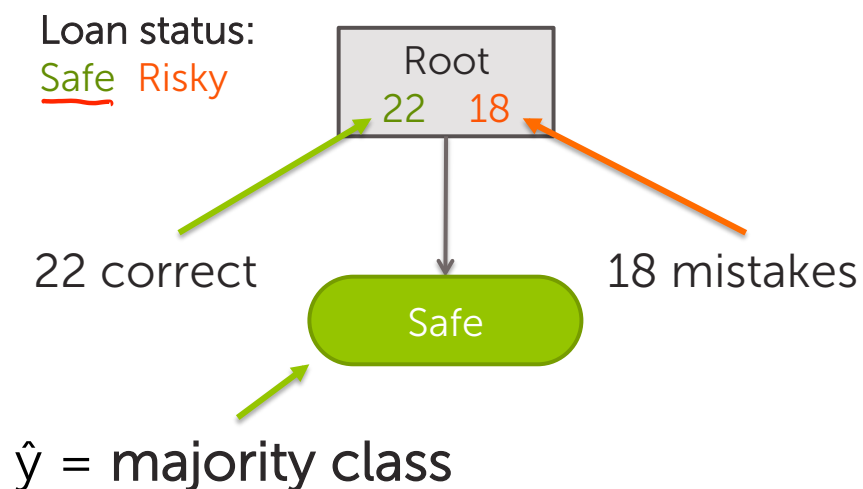
How do we measure effectiveness of a split?



$$\text{Error} = \frac{\text{\# mistakes}}{\text{\# data points}}$$

Calculating classification error

- **Step 1:** \hat{y} = class of majority of data in node
- **Step 2:** Calculate classification error of predicting \hat{y} for this data

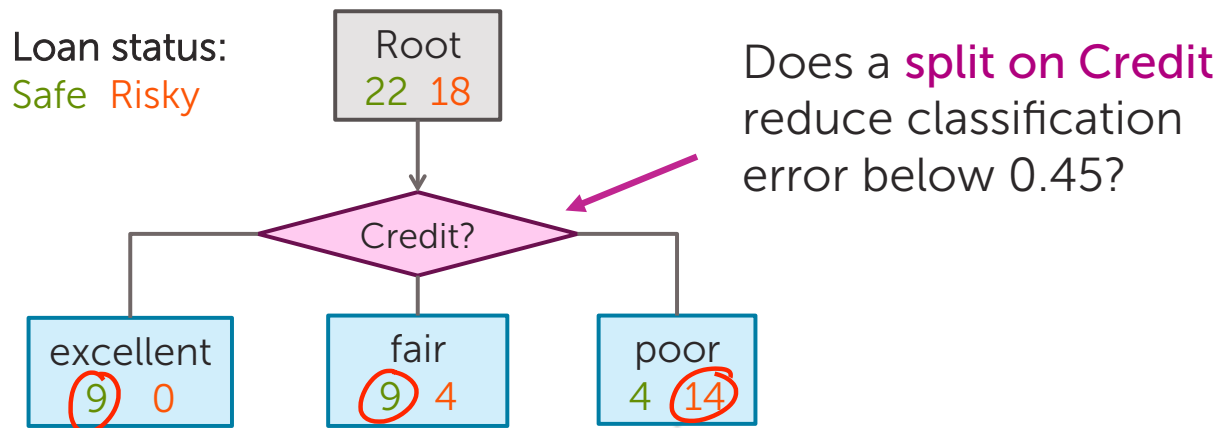


$$\text{Error} = \frac{18}{22+18}$$
$$= 0.45$$

Tree	Classification error
(root)	0.45

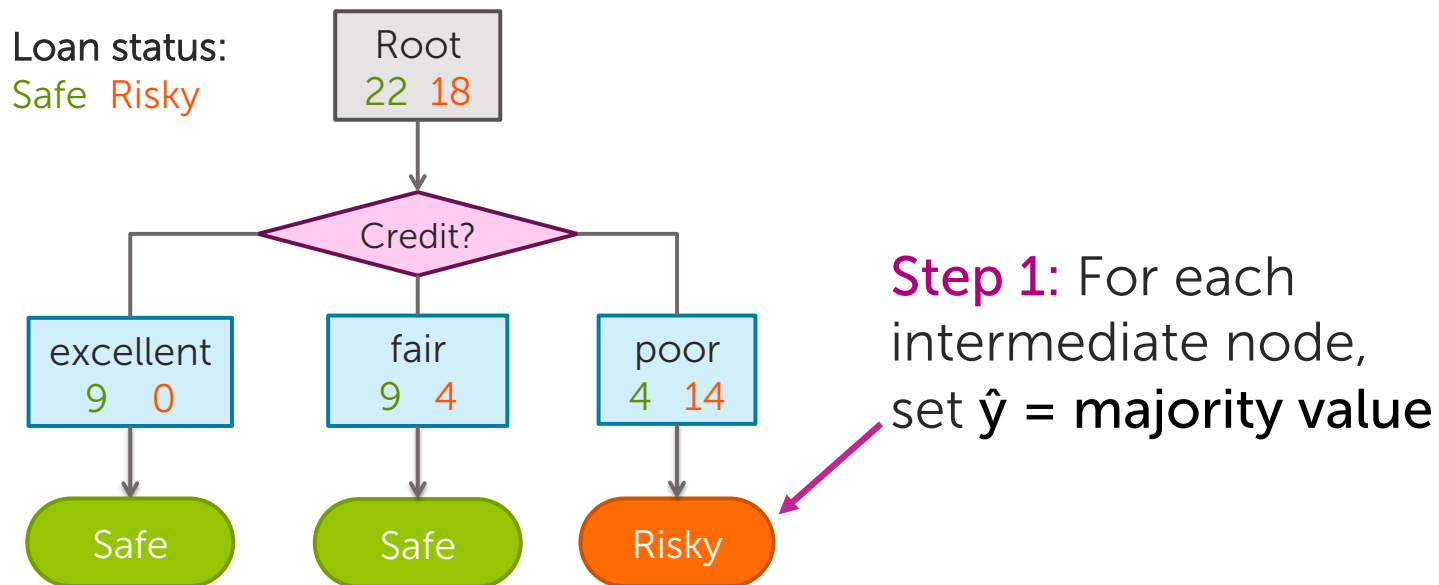
Choice 1: Split on credit history?

Choice 1: Split on Credit



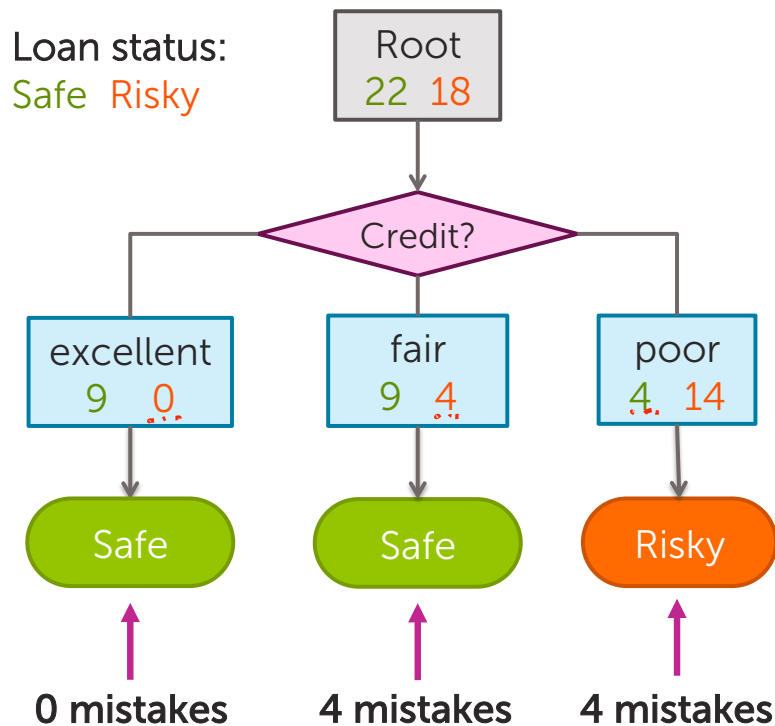
How good is the split on Credit?

Choice 1: Split on Credit



Split on Credit: Classification error

Choice 1: Split on Credit

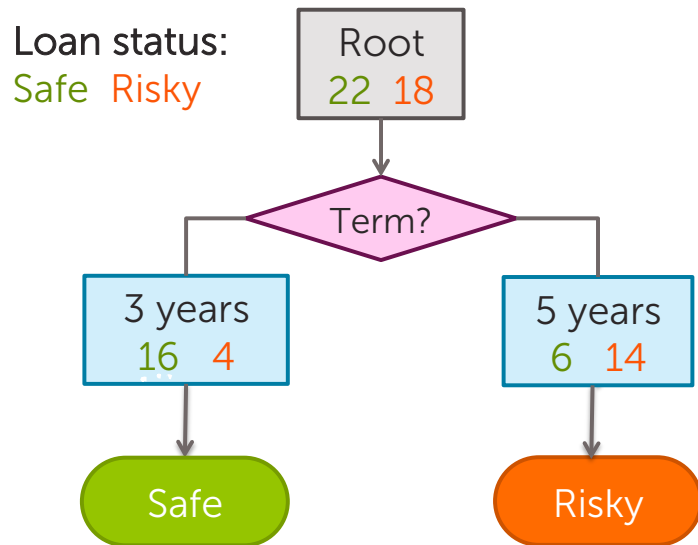


$$\text{Error} = \frac{4 + 4}{40} = 0.20$$

Tree	Classification error
(root)	0.45
Split on credit	0.2

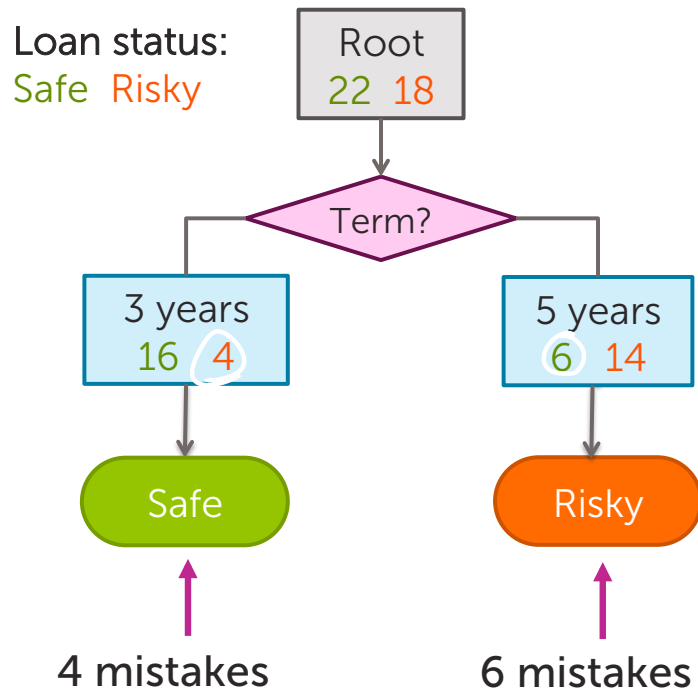
Choice 2: Split on Term?

Choice 2: Split on Term



Evaluating the split on Term

Choice 2: Split on Term



$$\text{Error} = \frac{4 + 6}{40} = 0.25$$

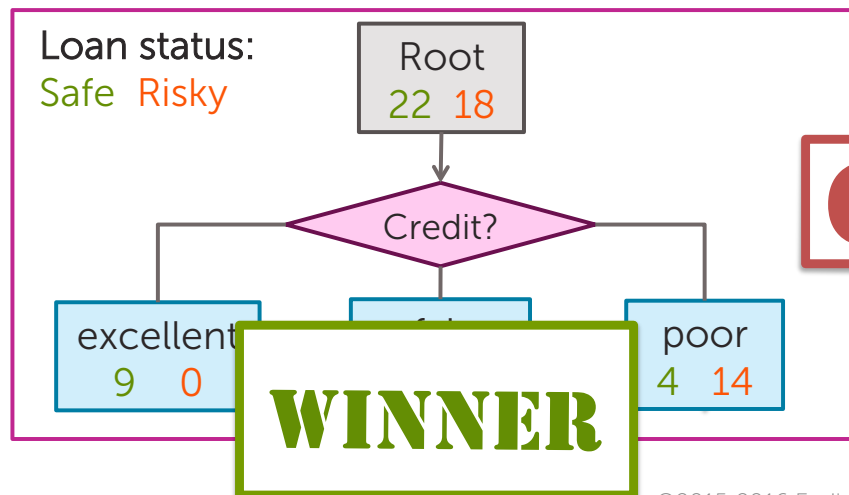
Tree	Classification error
(root)	0.45
Split on credit	0.2
Split on term	0.25

Choice 1 vs Choice 2

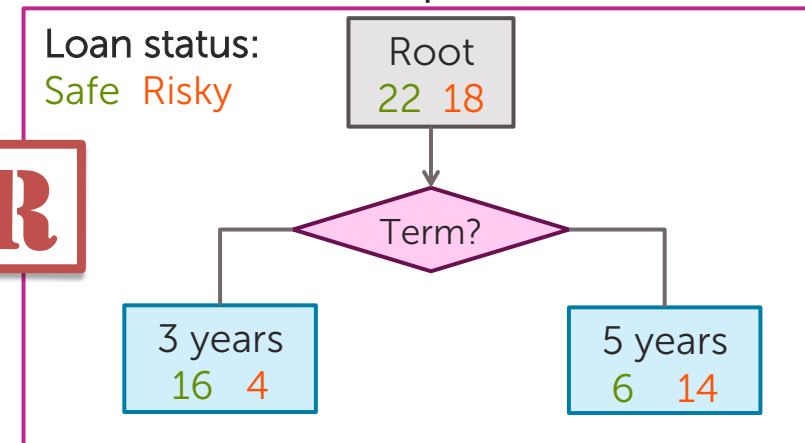
Tree	Classification error
(root)	0.45
split on <u>credit</u>	0.2
split on loan term	0.25

← First split!

Choice 1: Split on Credit



Choice 2: Split on Term




OR

Feature split selection algorithm

- Given a subset of data M (a node in a tree)
- For each feature $h_j(x)$: *credit, term, income*
 1. Split data of M according to feature $h_j(x)$
 2. Compute classification error split
- Chose feature $h^*(x)$ with lowest classification error *credit*

Greedy decision tree learning

- Step 1: Start with an empty tree
- Step 2: Select a feature to split data
- For each split of the tree:
 - Step 3: If nothing more to, make predictions
 - Step 4: Otherwise, go to Step 2 & continue (recurse) on this split



Pick feature split
leading to lowest
classification error

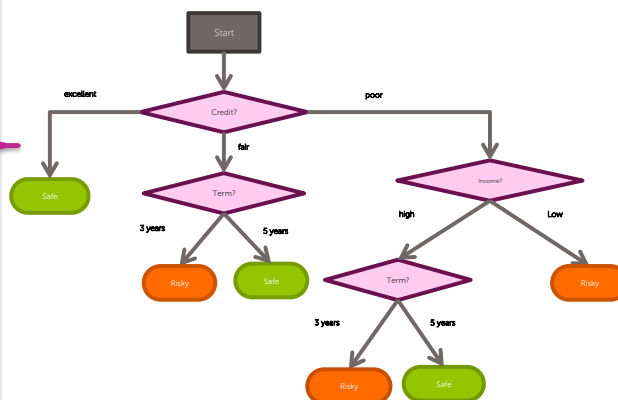


Decision Tree Learning:

Recursion & Stopping conditions

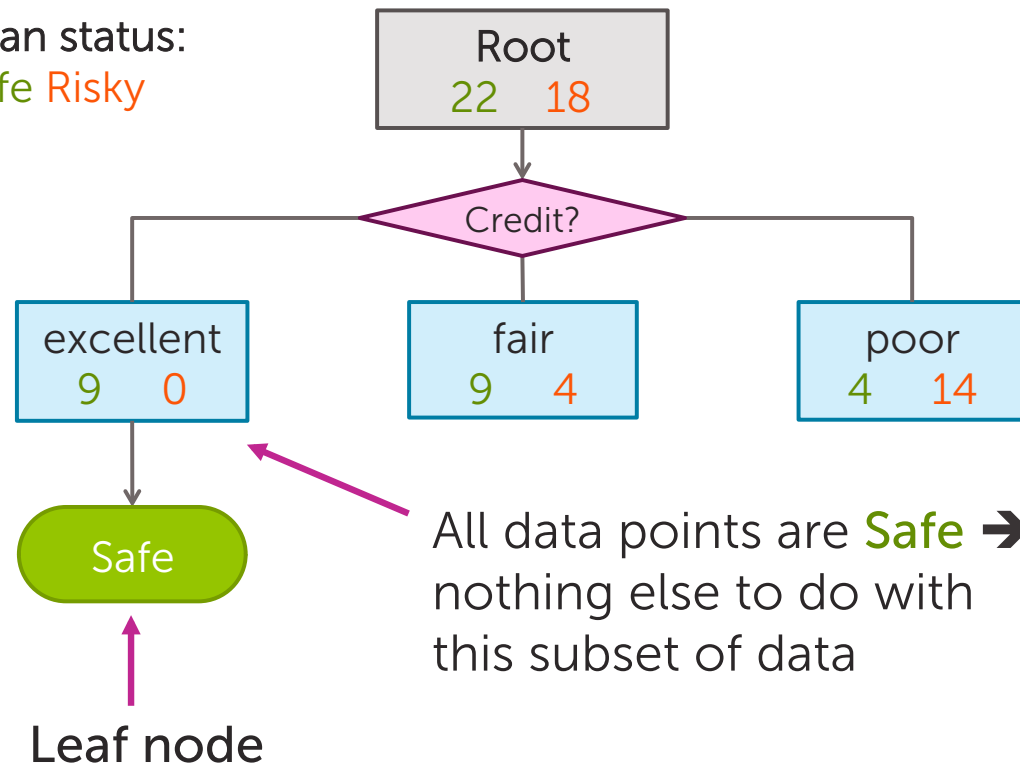
Learn decision tree from data?

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe



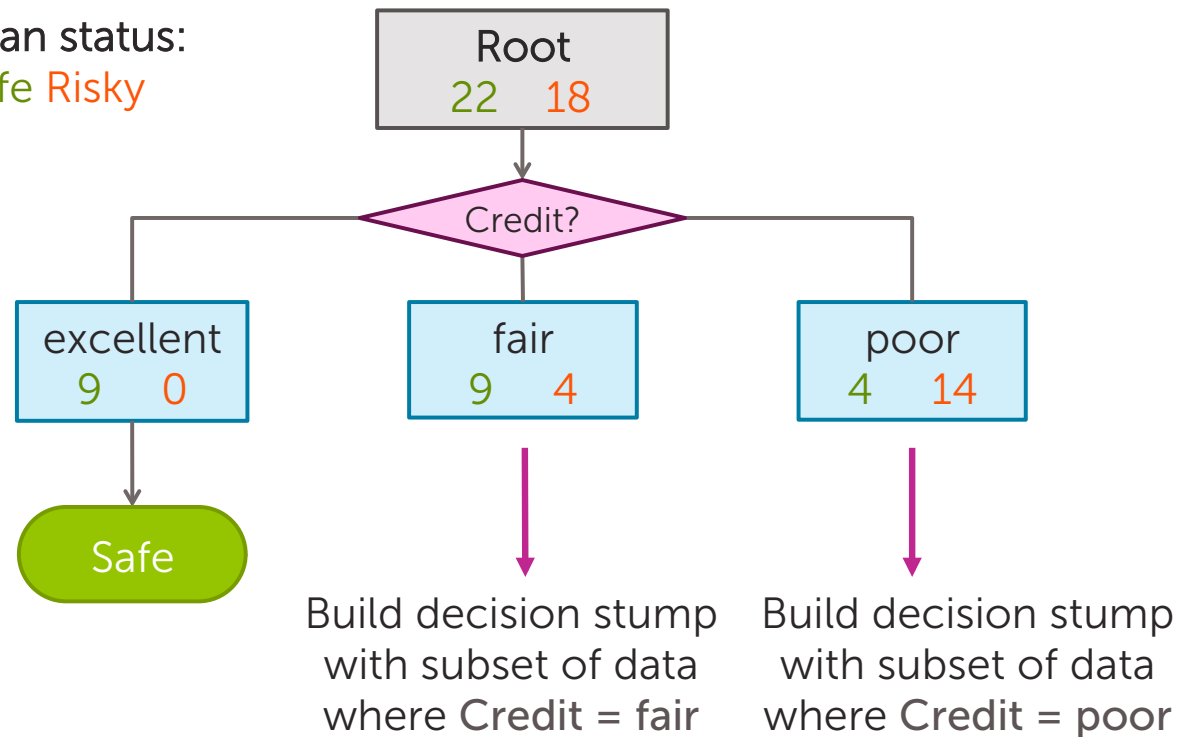
We've learned a decision stump, what next?

Loan status:
Safe Risky

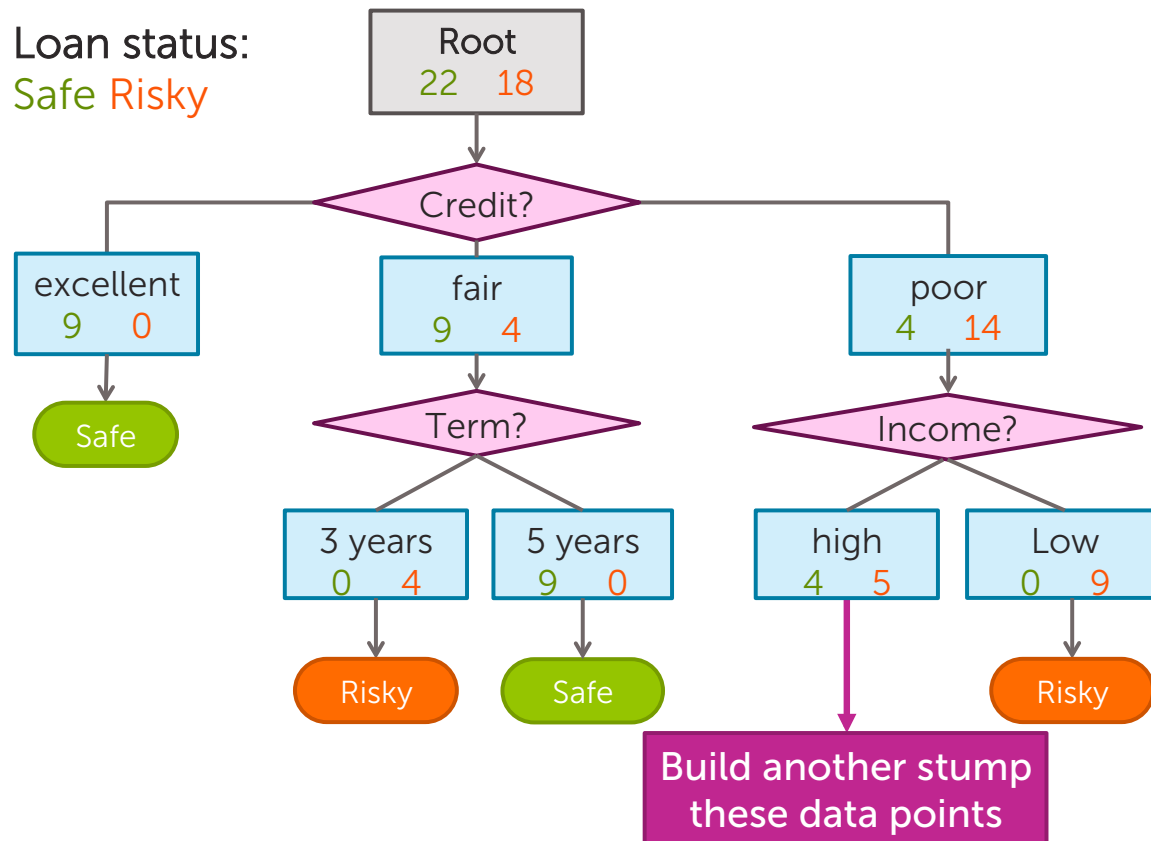


Tree learning = Recursive stump learning

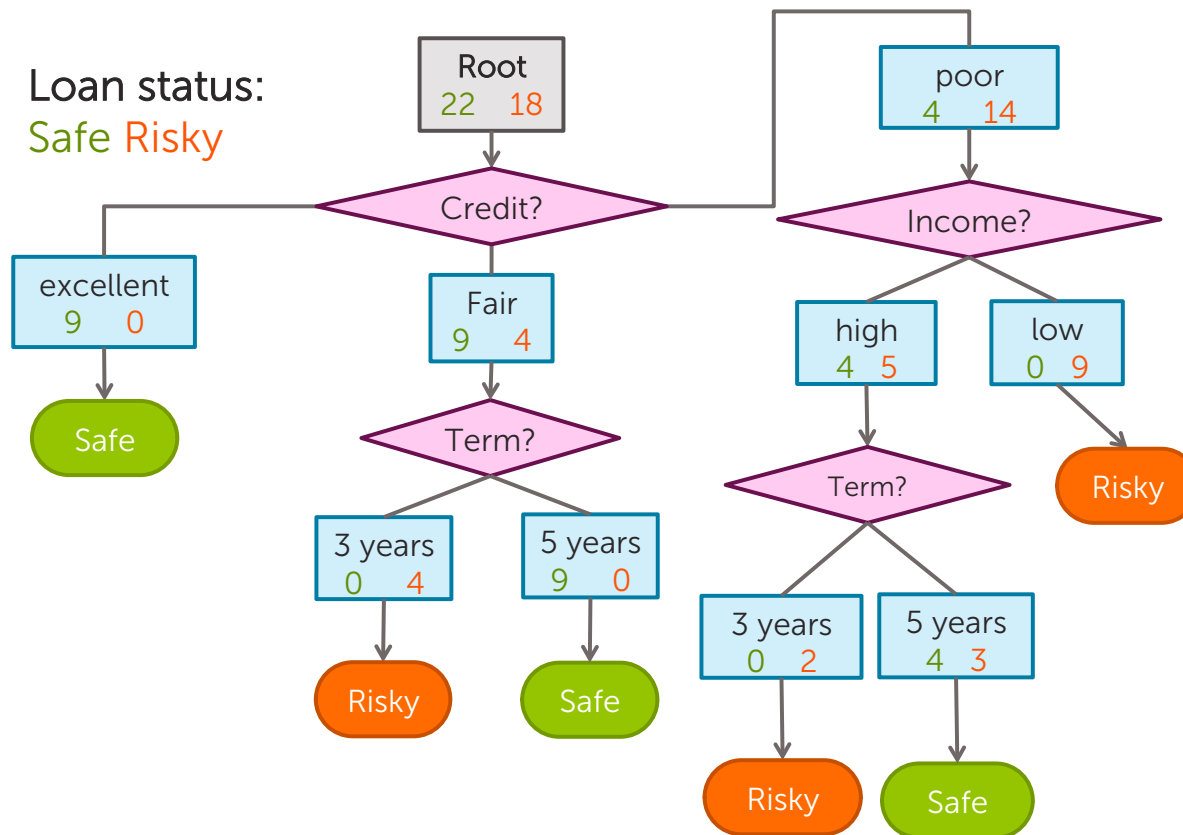
Loan status:
Safe Risky



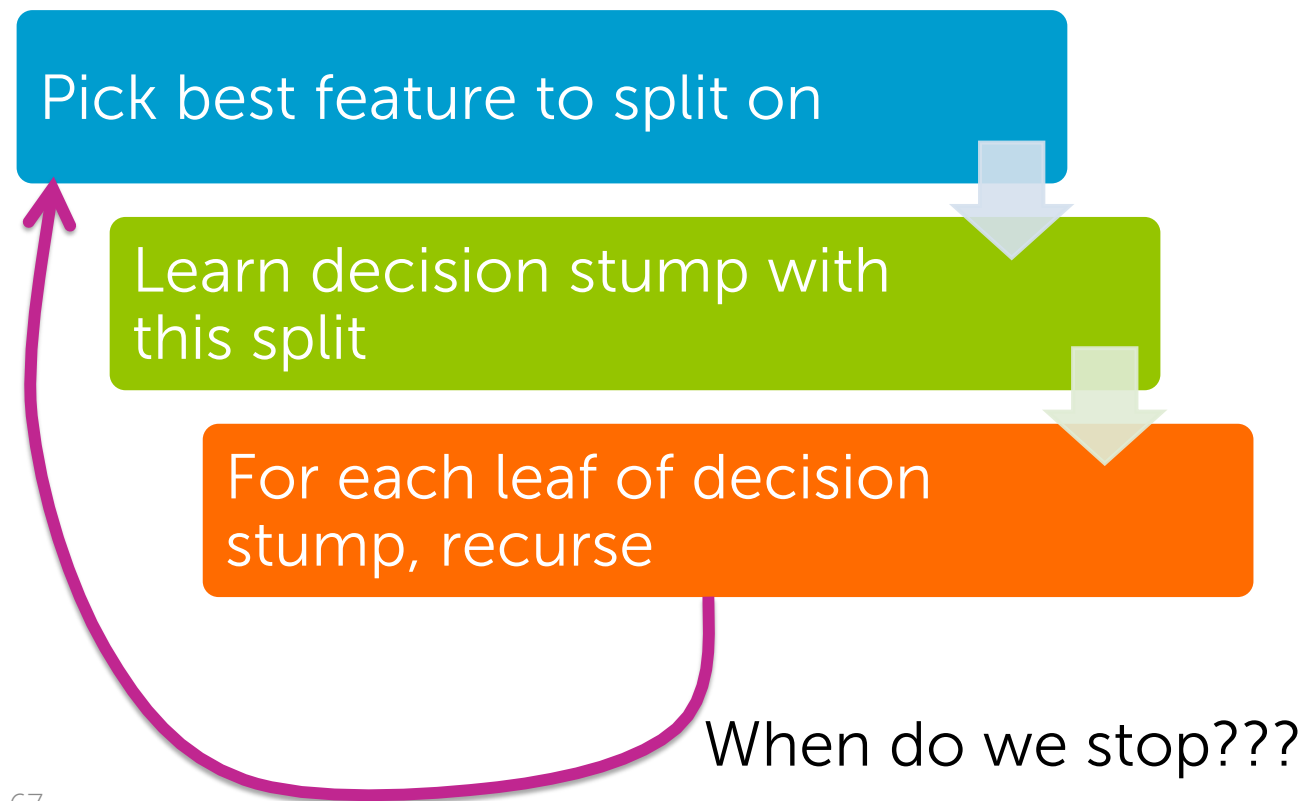
Second level



Final decision tree

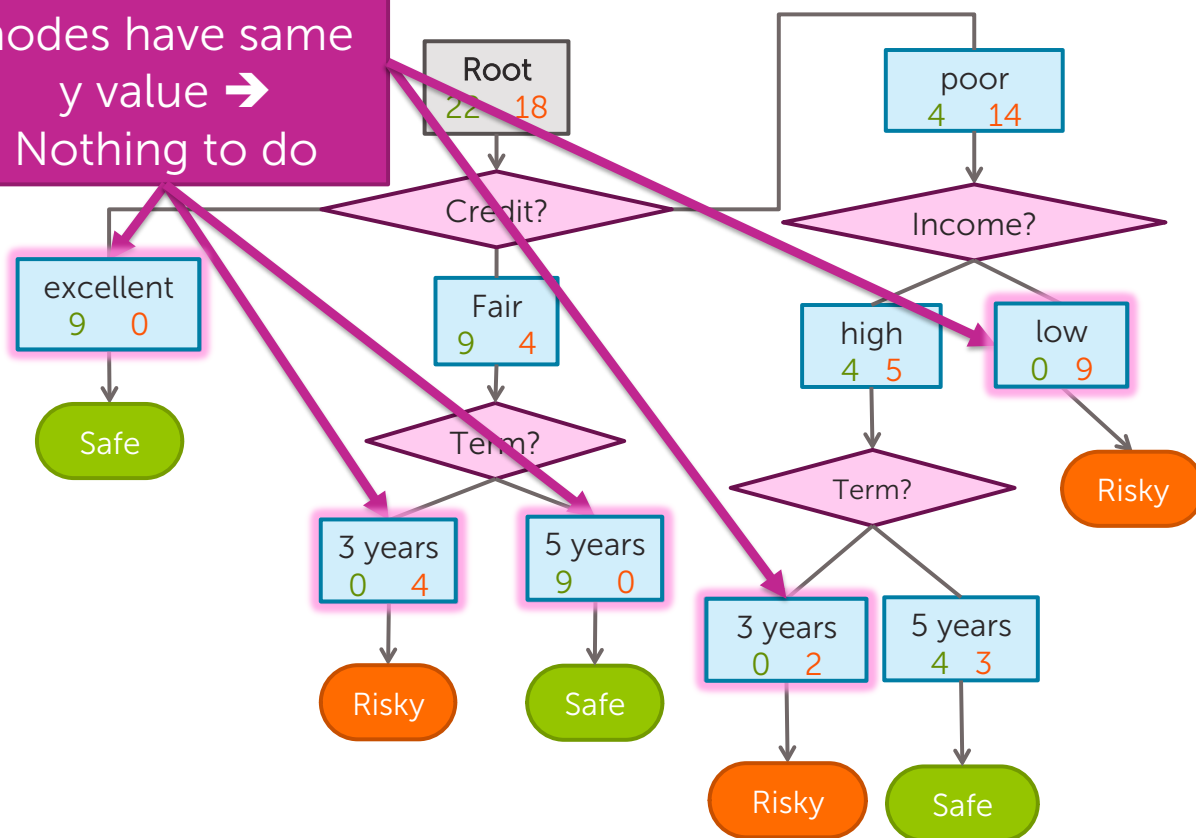


Simple greedy decision tree learning



Stopping condition 1: All data agrees on y

All data in these nodes have same y value → Nothing to do

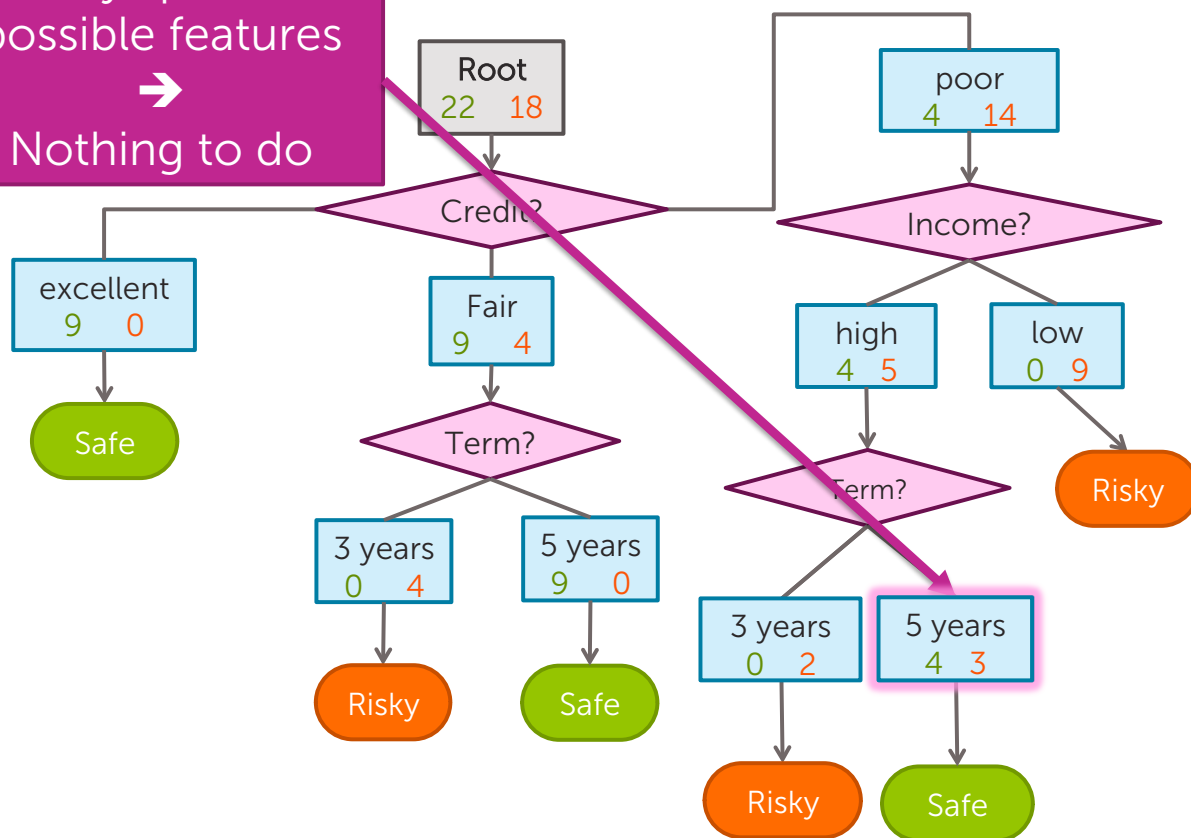


Stopping condition 2: Already split on all features

Already split on all
possible features



Nothing to do



Greedy decision tree learning

- **Step 1:** Start with an empty tree

- **Step 2:** Select a feature to split data

- For each split of the tree:

- **Step 3:** If nothing more to, make predictions

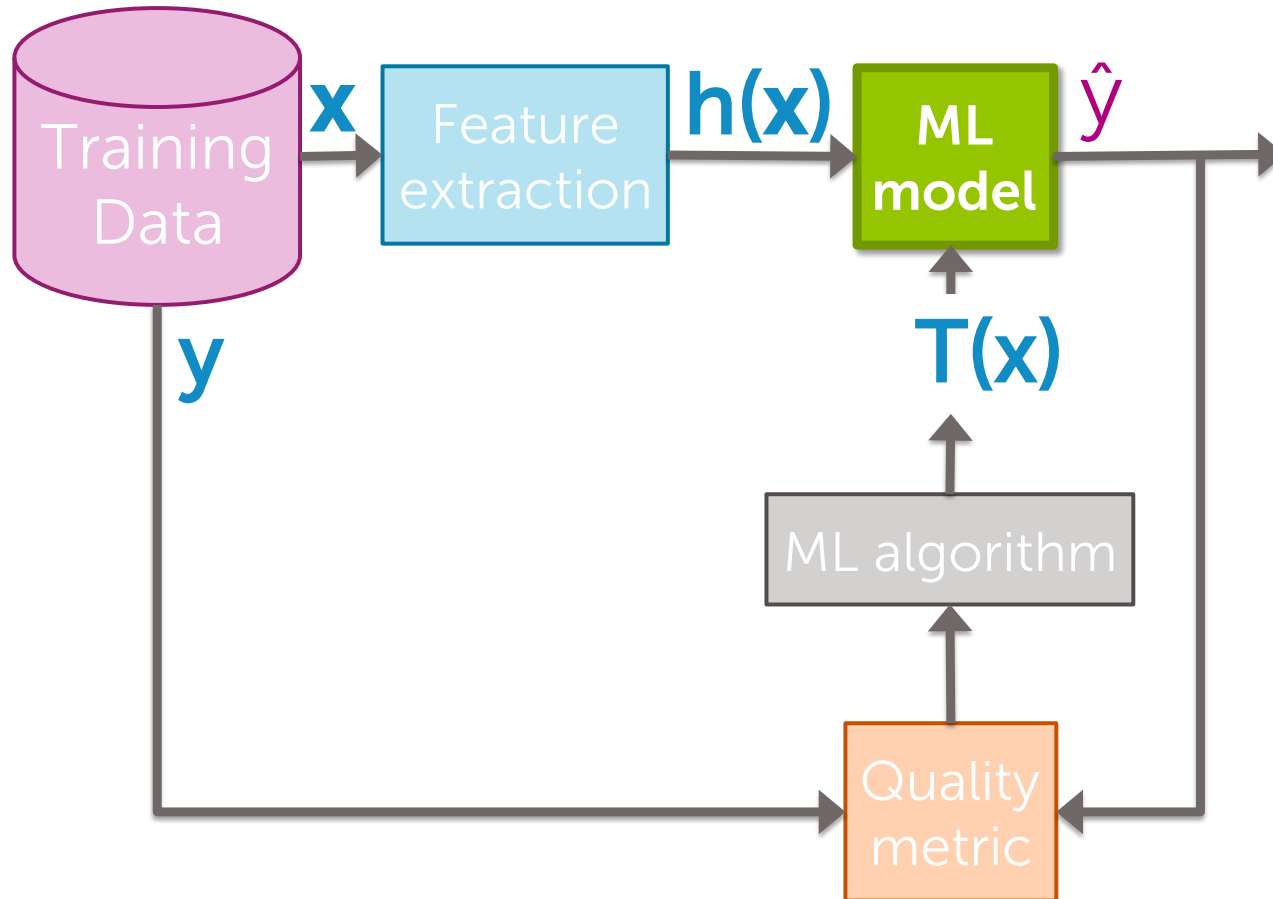
- **Step 4:** Otherwise, go to **Step 2** & continue (recurse) on this split

Pick feature split leading to lowest classification error

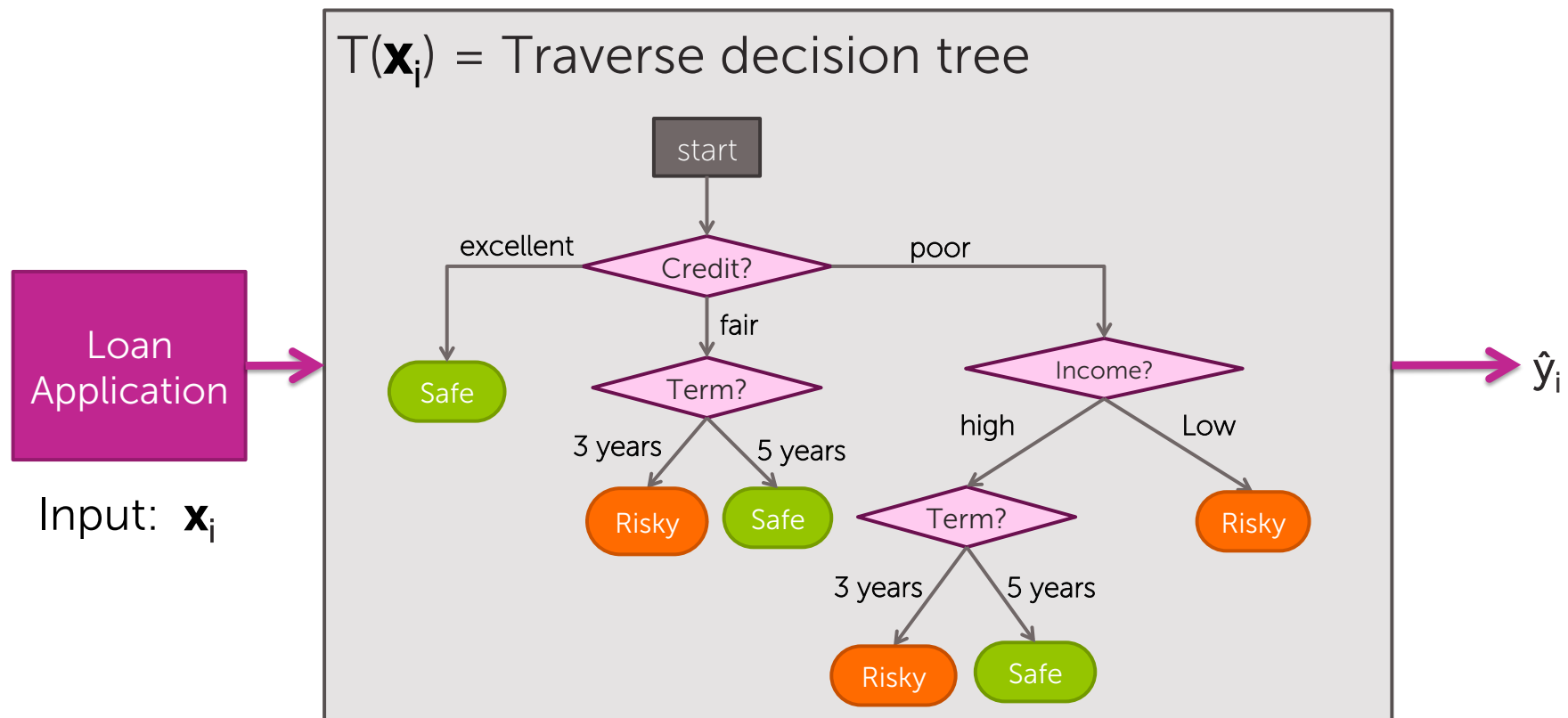
Stopping conditions 1 & 2

Recursion

Predictions with decision trees

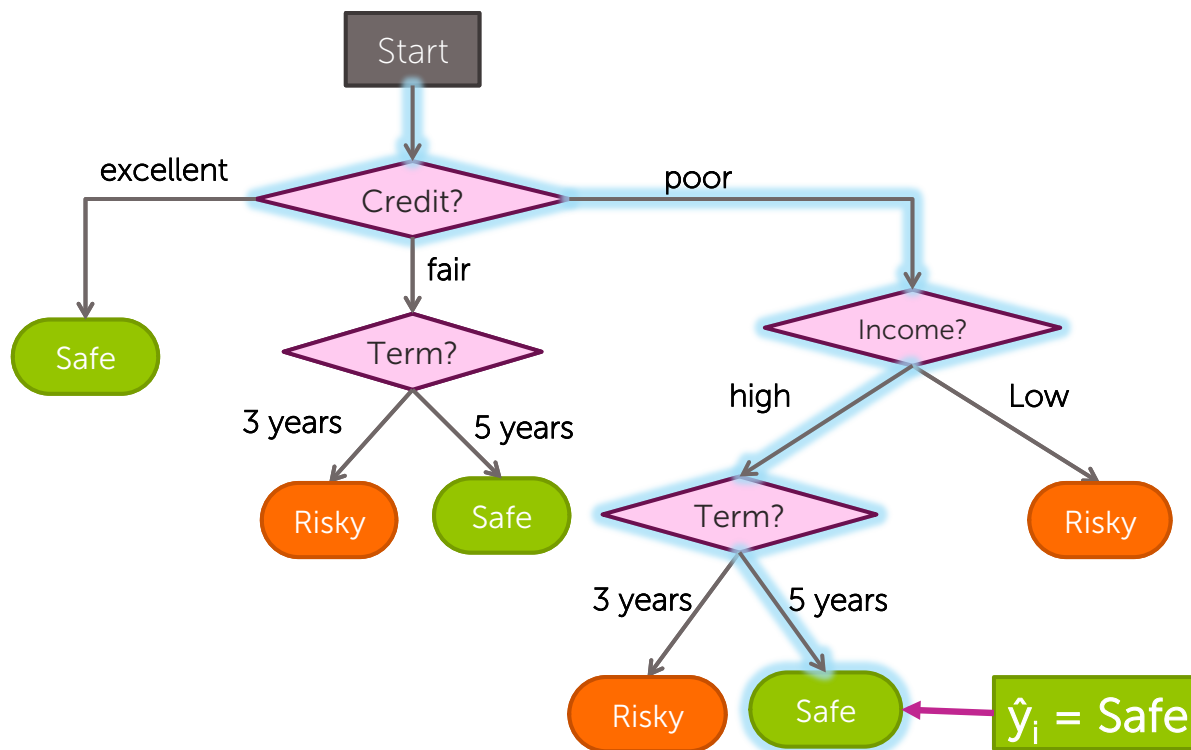


Decision tree model



Traversing a decision tree

$\mathbf{x}_i = (\text{Credit} = \text{poor}, \text{Income} = \text{high}, \text{Term} = 5 \text{ years})$





Decision tree prediction algorithm

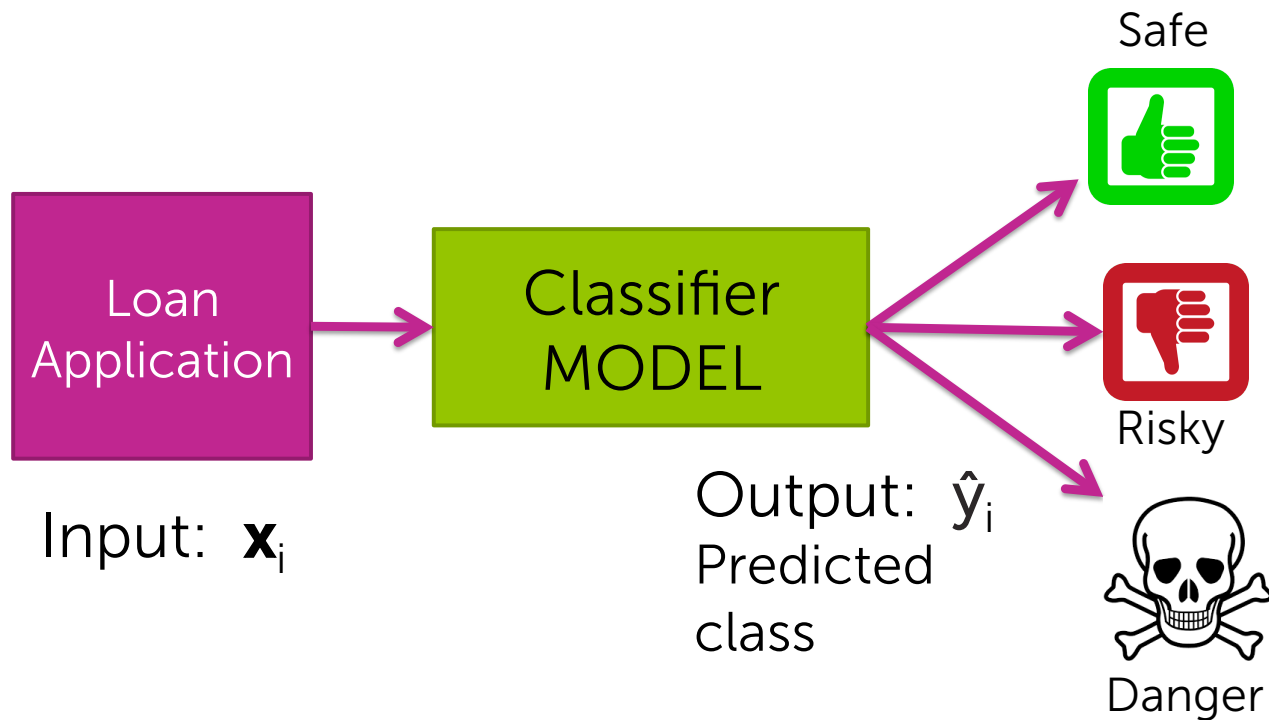
predict(tree_node, input)

- If current tree_node is a leaf:
 - **return** majority class of data points in leaf
- else:
 - next_note = child node of tree_node whose feature value agrees with input
 - **return predict**(next_note, input)



Multiclass classification & predicting probabilities

Multiclass prediction

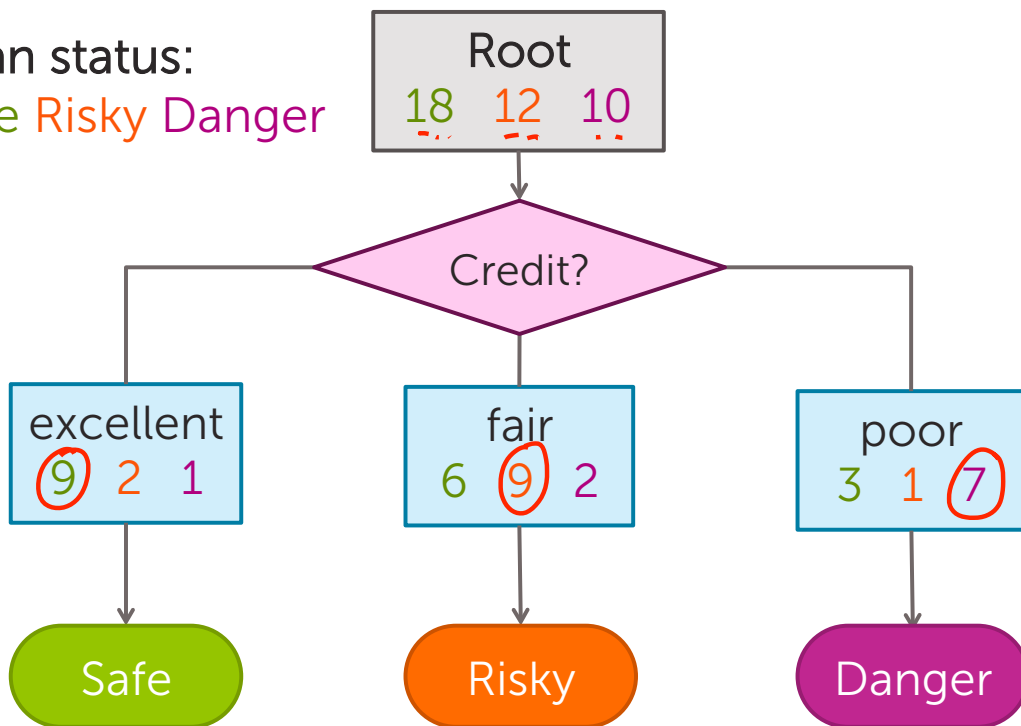


Multiclass decision stump

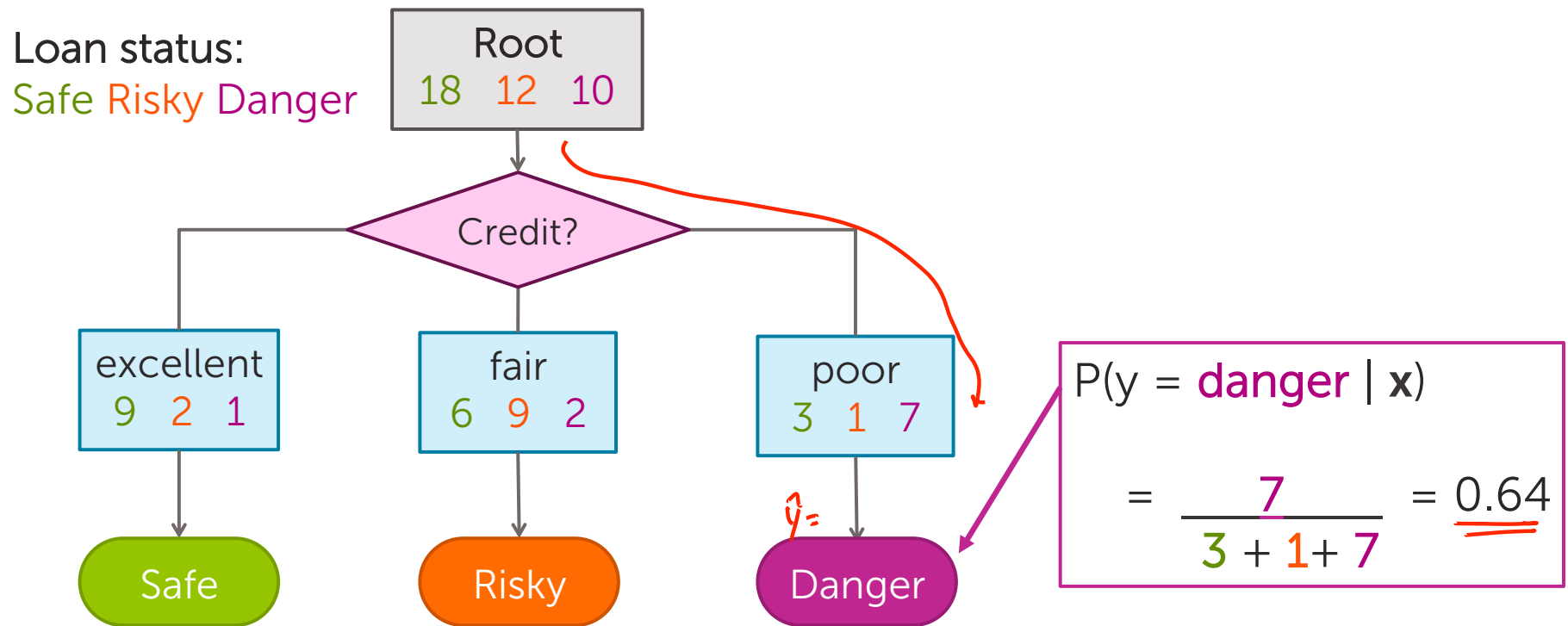
N = 40,
1 feature,
3 classes

Credit	y
excellent	<u>safe</u>
fair	<u>risky</u>
fair	safe
poor	<u>danger</u>
excellent	risky
fair	safe
poor	danger
poor	safe
fair	safe
...	...

Loan status:
Safe Risky Danger



Predicting probabilities with decision trees





Decision tree learning:

Real valued features

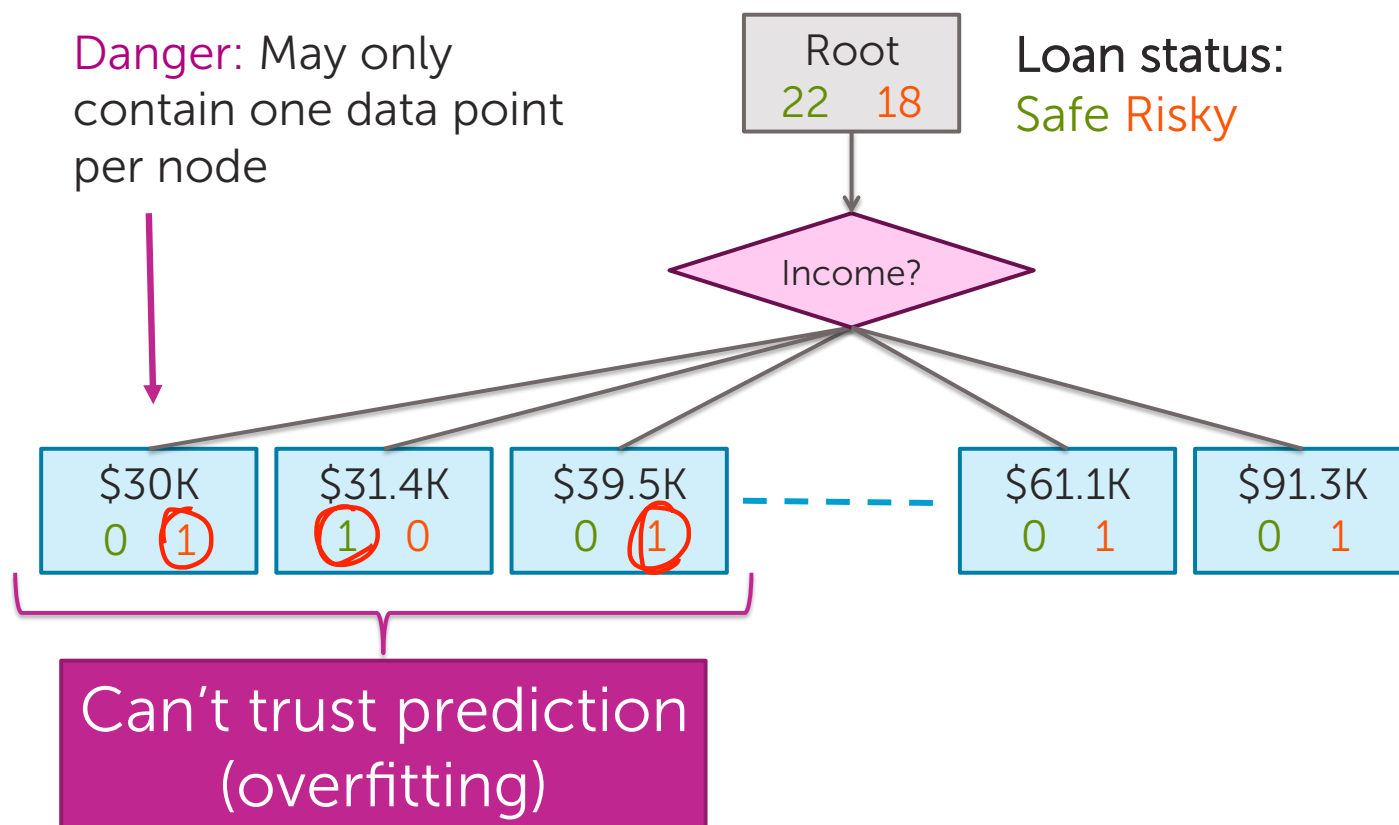
How do we use real values inputs?

Income	Credit	Term	y
\$105 K	excellent	3 yrs	Safe
\$112 K	good	5 yrs	Risky
\$73 K	fair	3 yrs	Safe
\$69 K	excellent	5 yrs	Safe
\$217 K	excellent	3 yrs	Risky
\$120 K	good	5 yrs	Safe
\$64 K	fair	3 yrs	Risky
\$340 K	excellent	5 yrs	Safe
\$60 K	good	3 yrs	Risky

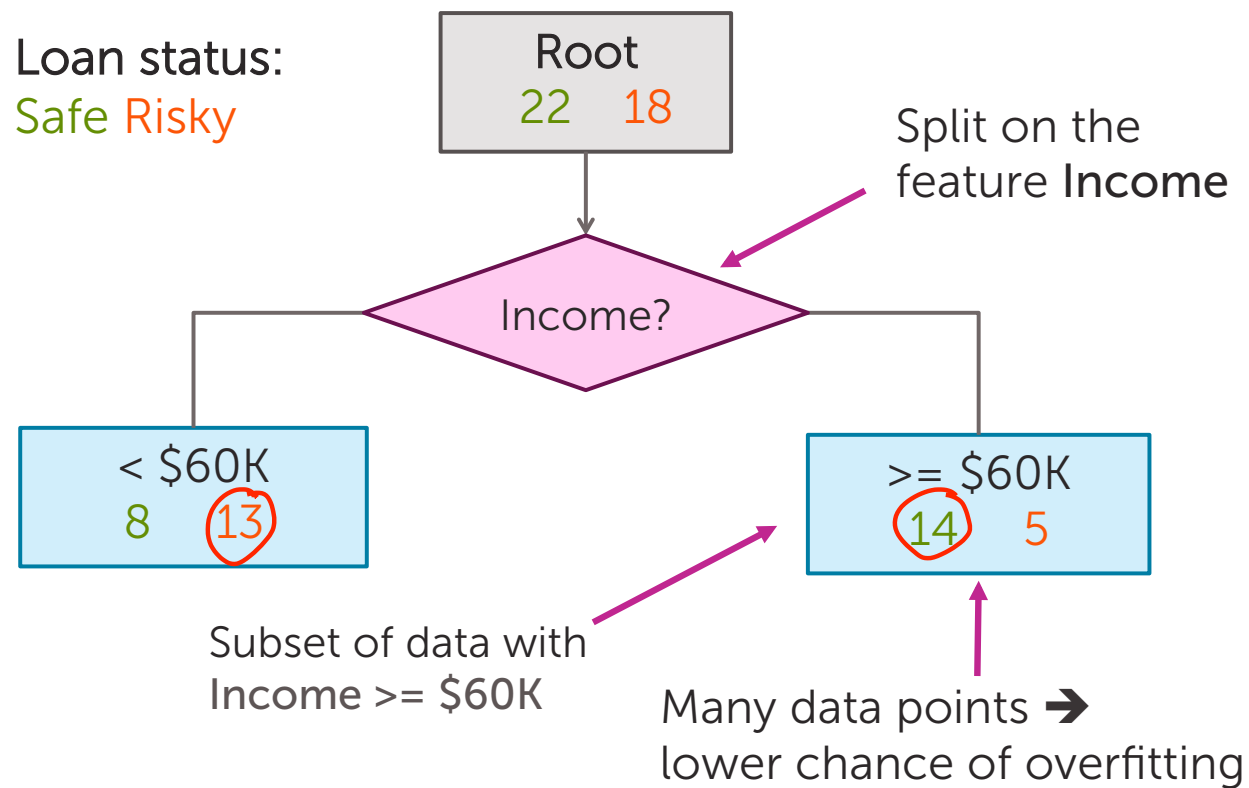
Split on each numeric value?

Danger: May only contain one data point per node

Loan status:
Safe Risky

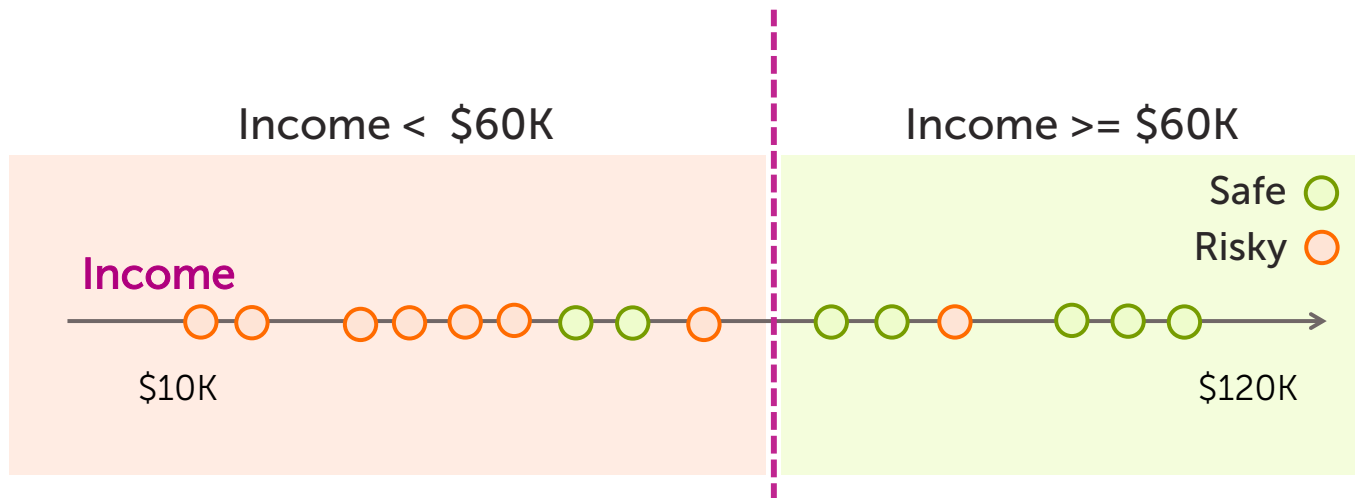


Alternative: Threshold split

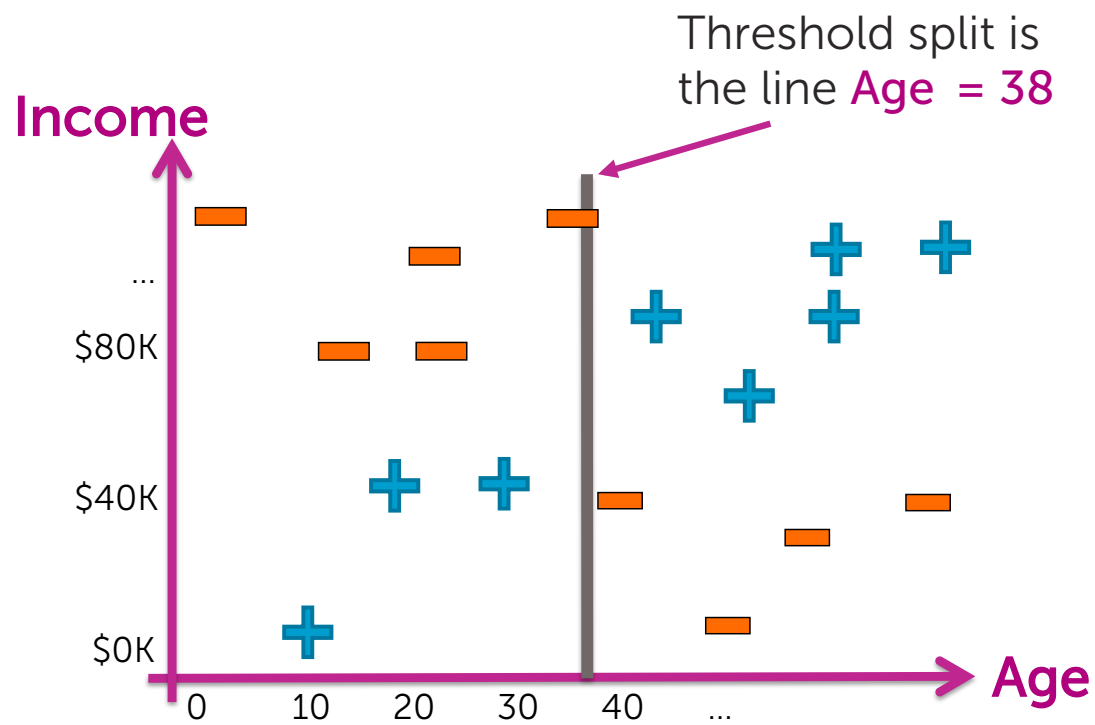


Threshold splits in 1-D

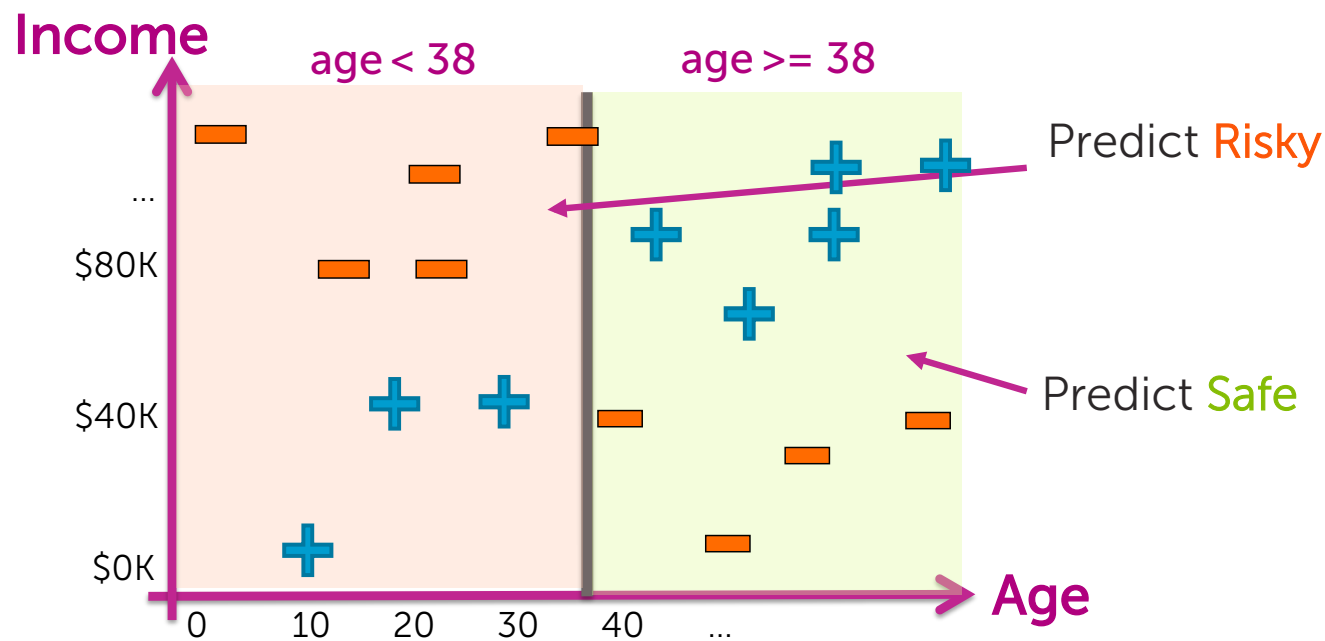
Threshold split is the line
 $\text{Income} = \$60\text{K}$



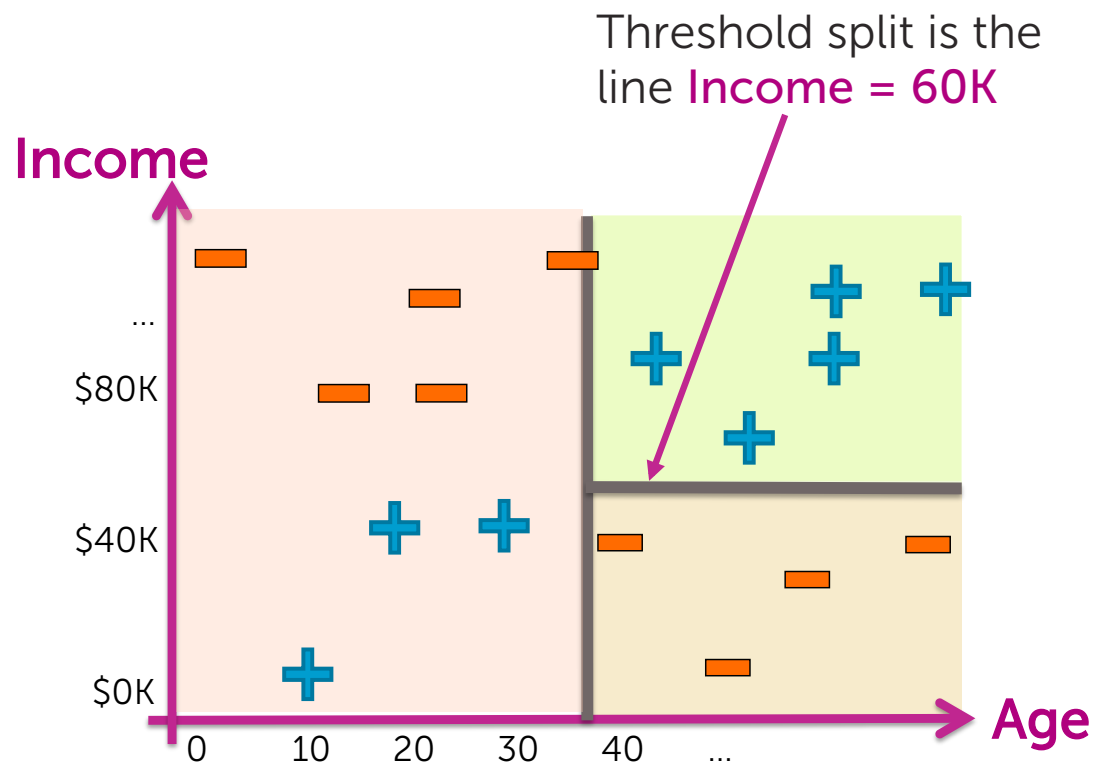
Visualizing the threshold split



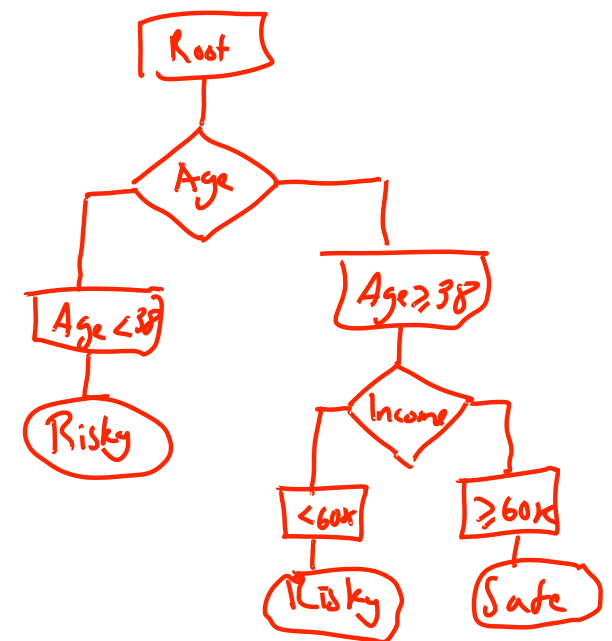
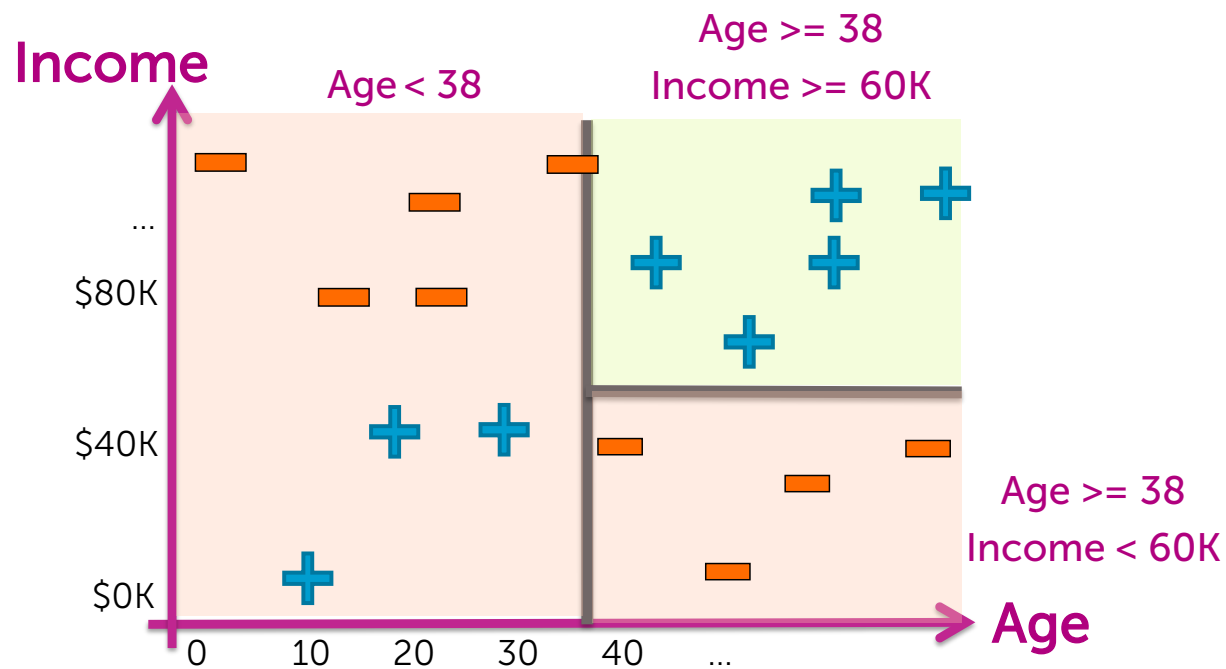
Split on Age ≥ 38



Depth 2: Split on Income \geq \$60K



Each split partitions the 2-D space





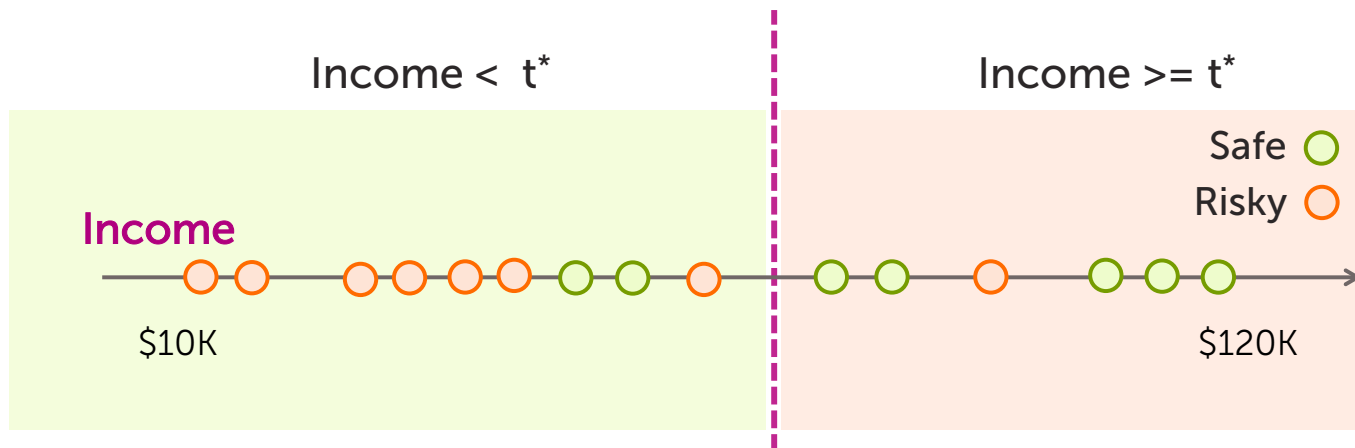
Finding the best threshold split

OPTIONAL

Finding the best threshold split

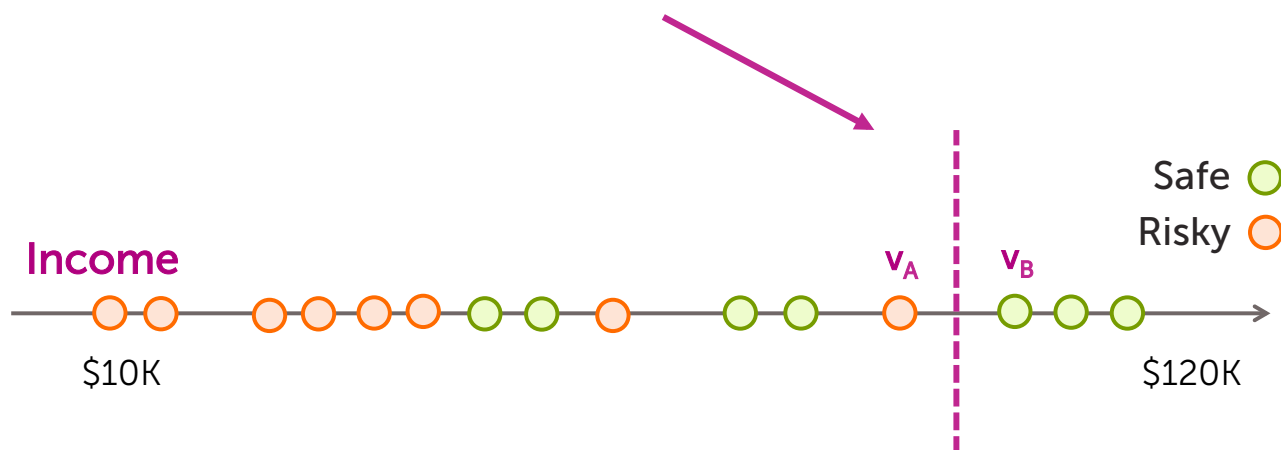
Infinite possible
values of t

Income = t^*



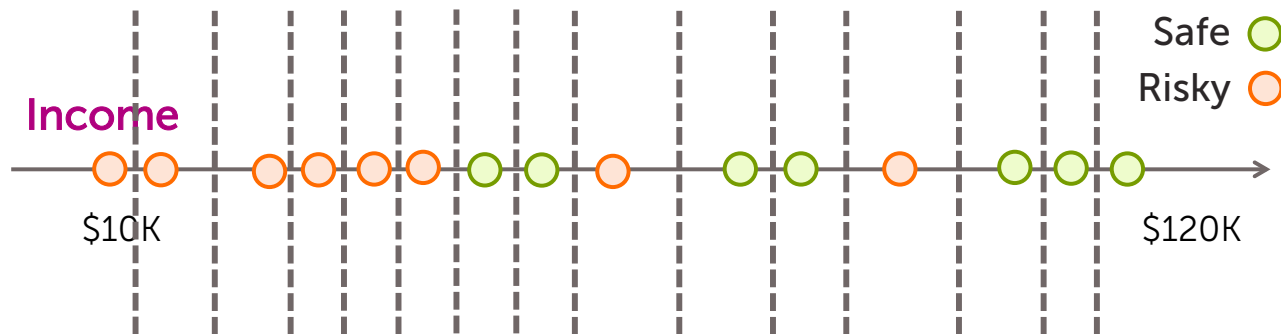
Consider a threshold between points

Same **classification error** for any threshold split between v_A and v_B



Only need to consider mid-points

Finite number of splits to consider



Threshold split selection algorithm

- **Step 1:** Sort the values of a feature $h_j(\mathbf{x})$:
Let $\{v_1, v_2, v_3, \dots, v_N\}$ denote sorted values
- **Step 2:**
 - For $i = 1 \dots N-1$
 - Consider split $t_i = (v_i + v_{i+1}) / 2$
 - Compute classification error for threshold
split $h_j(\mathbf{x}) \geq t_i$
 - Chose the t^* with the lowest classification error

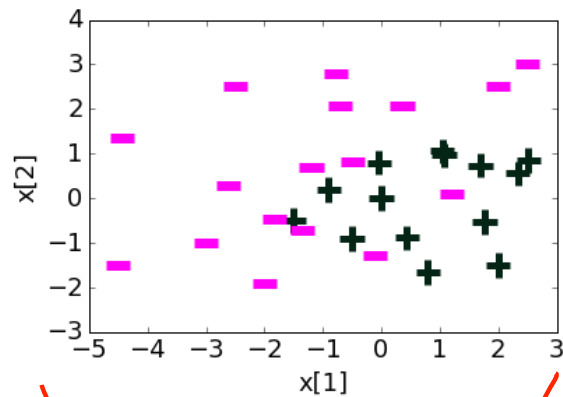
✓ income

Decision trees vs logistic regression:

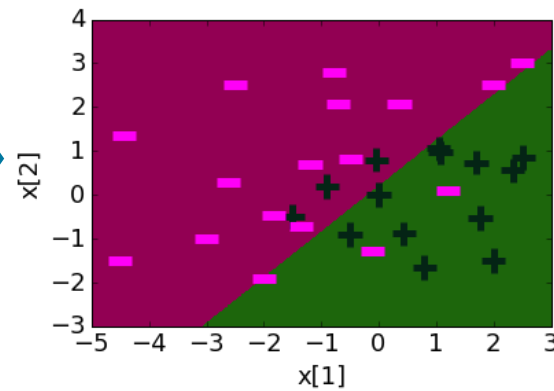
Example

Logistic regression

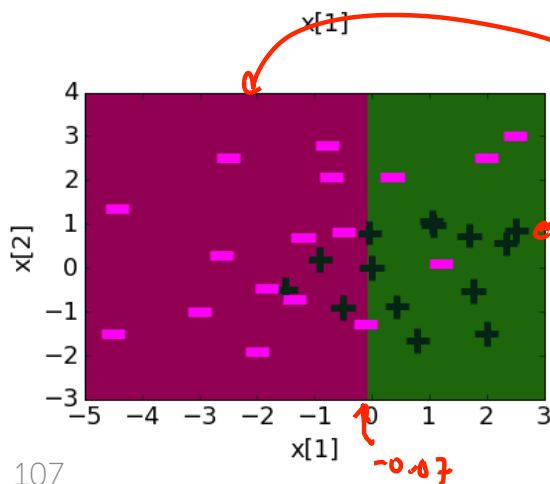
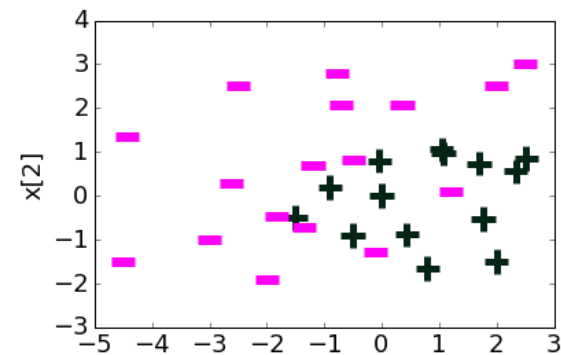
Feature	Value	Weight Learned
$h_0(\mathbf{x})$	1	0.22
$h_1(\mathbf{x})$	$x[1]$	1.12
$h_2(\mathbf{x})$	$x[2]$	-1.07



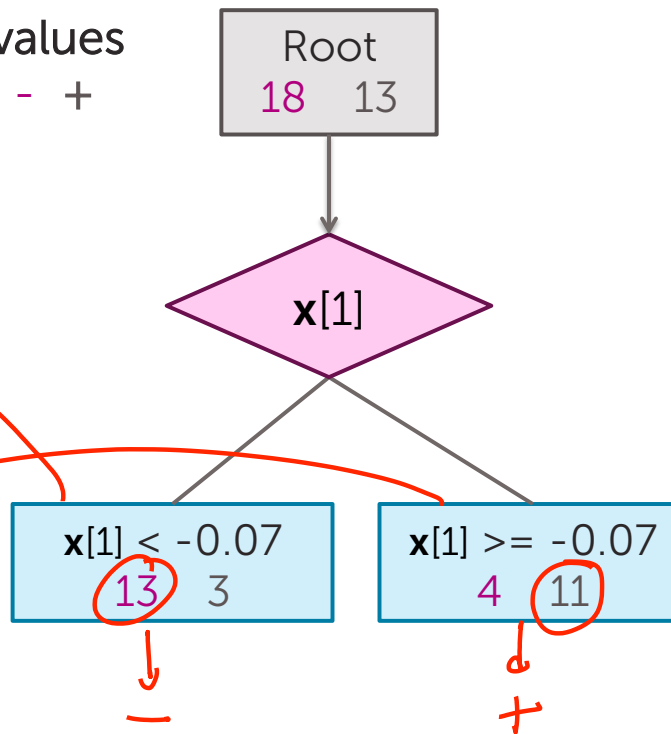
data



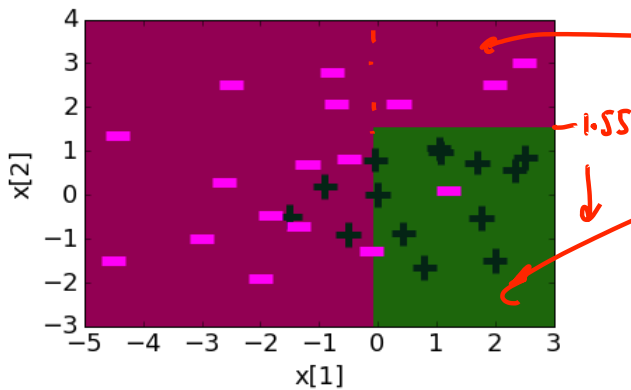
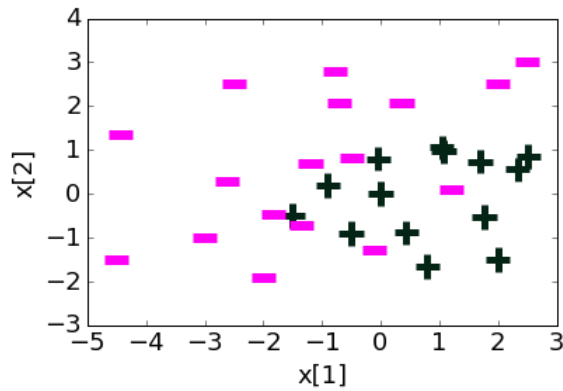
Depth 1: Split on $x[1]$



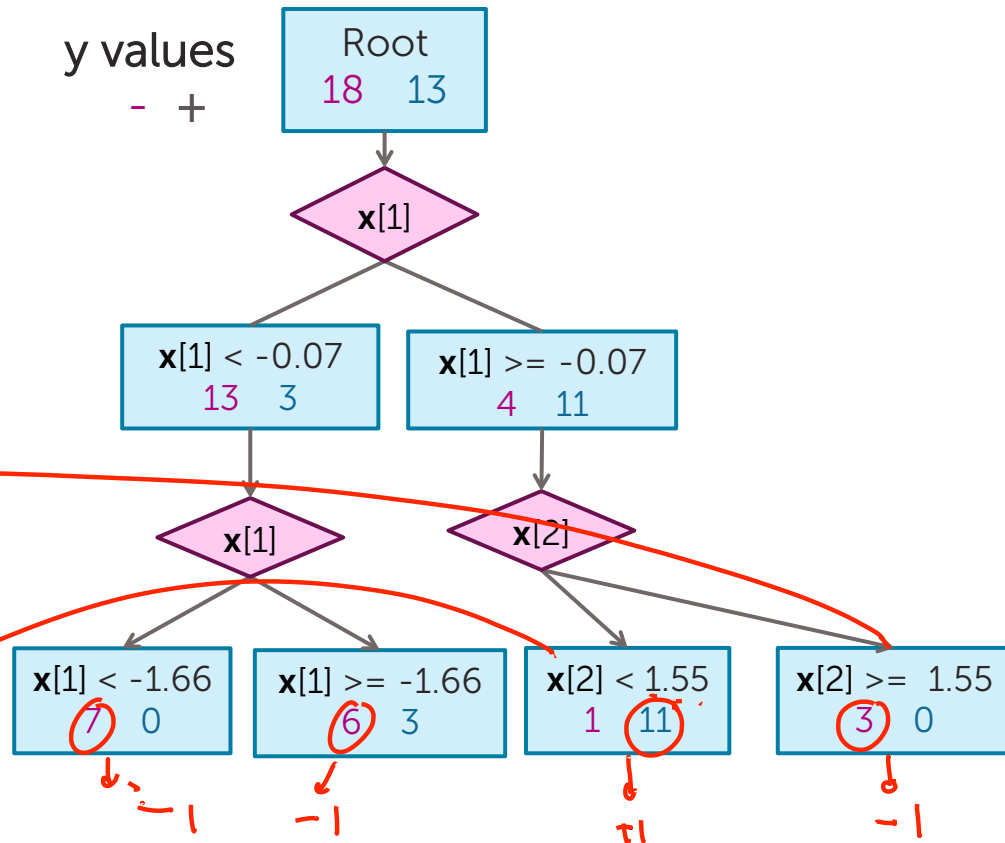
y values
- +



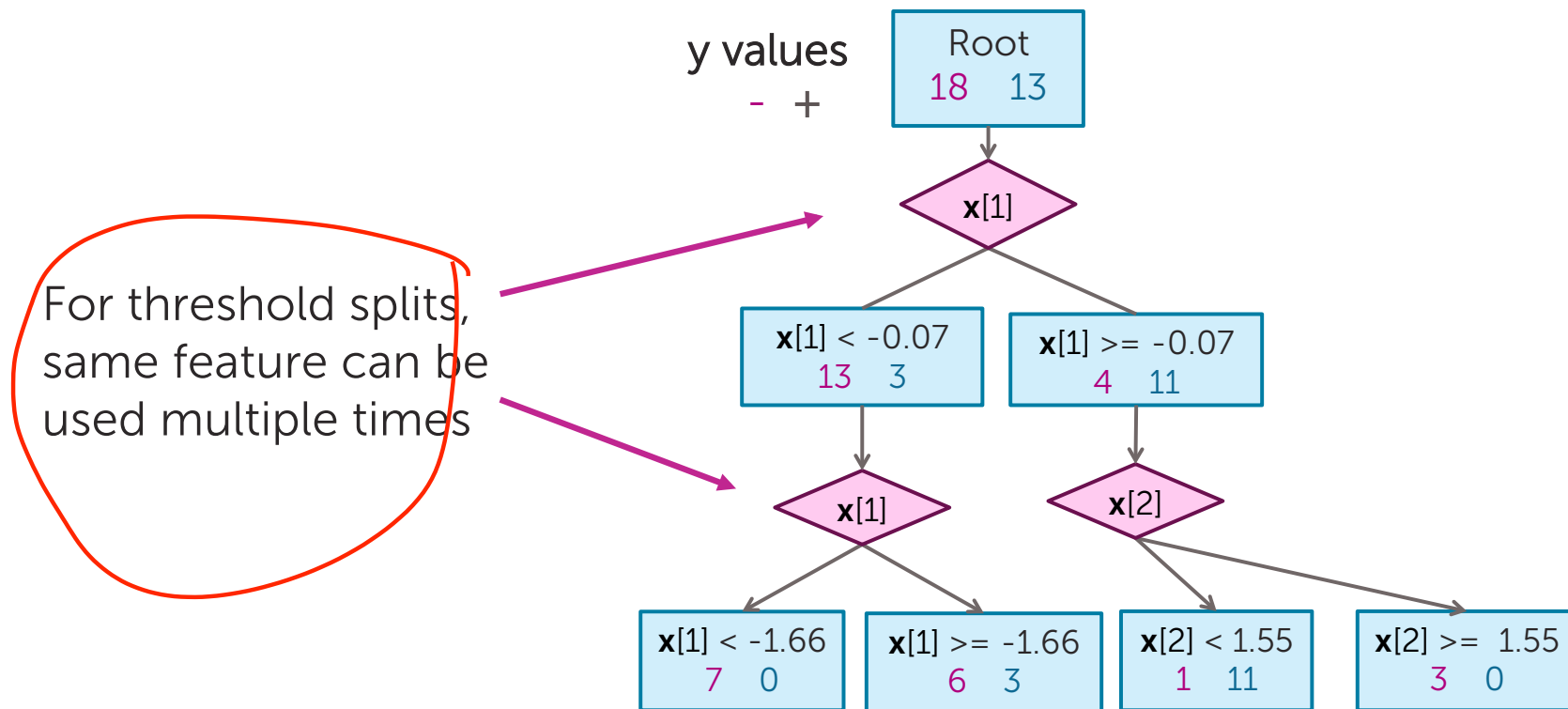
Depth 2



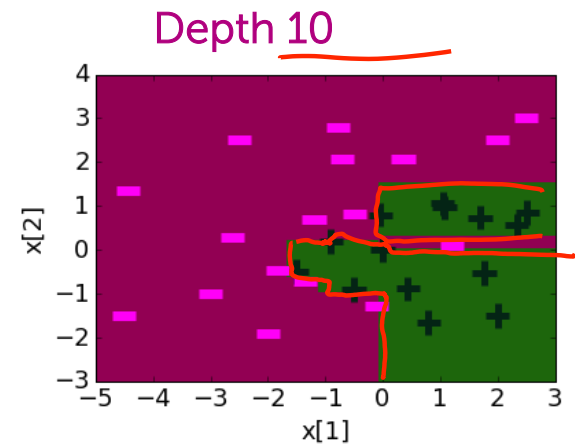
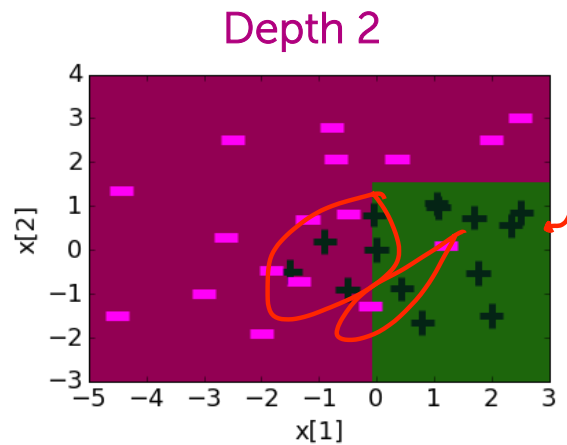
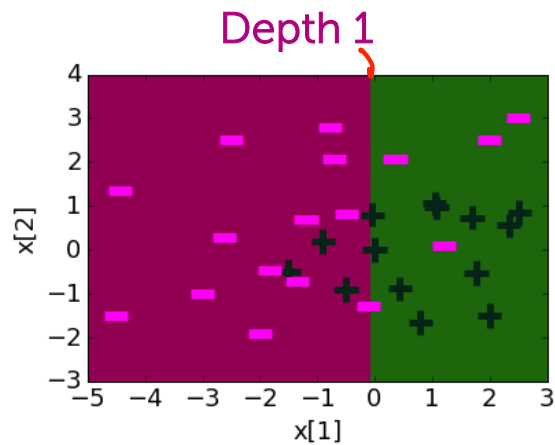
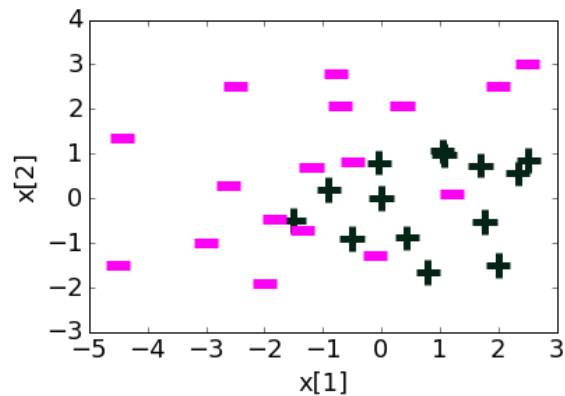
y values
- +



Threshold split caveat

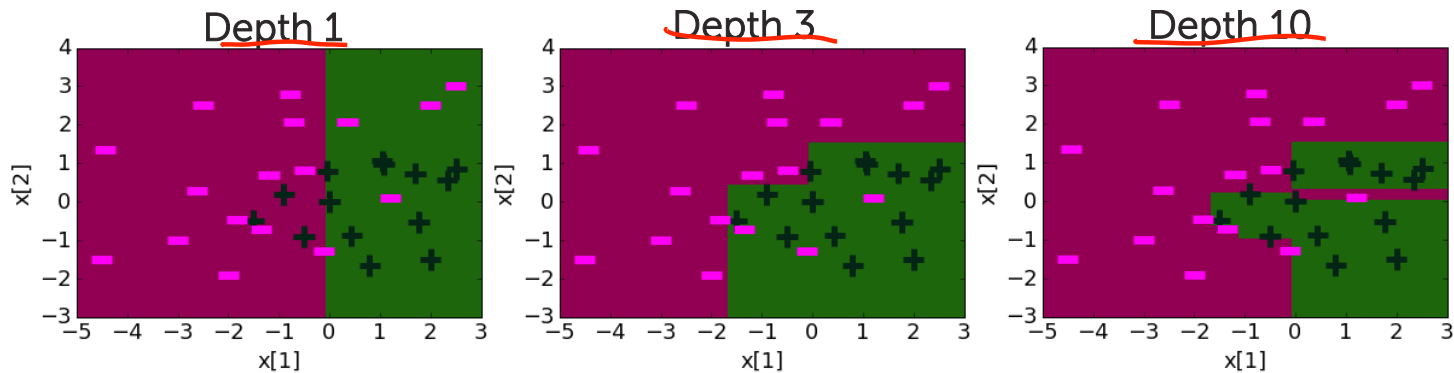


Decision boundaries

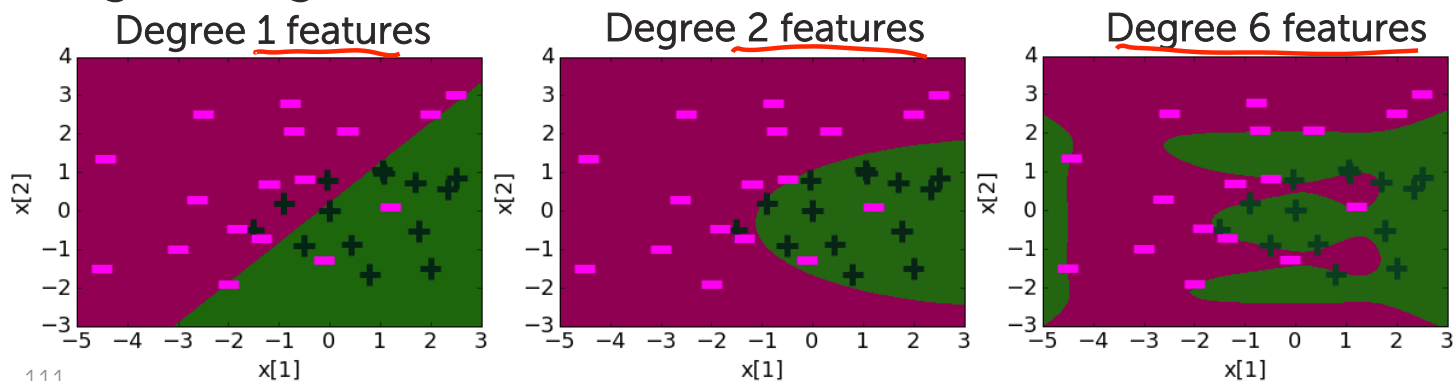


Comparing decision boundaries

Decision Tree



Logistic Regression





Summary of decision trees



What you can do now

- Define a decision tree classifier
- Interpret the output of a decision trees
- Learn a decision tree classifier using greedy algorithm
- Traverse a decision tree to make predictions
 - Majority class predictions
 - Probability predictions
 - Multiclass classification

Thank you to Dr. Krishna Sridhar



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