Christian Zinck
Theory
9/18/18

13.

$<\text{LIT}(3), xi_0, phi, rho_0> \downarrow <3, xi_0, phi, rho_0>$ (LITERAL)
$X \in dom(rho)$

$<\text{SET}(\text{VAR}(X), \text{LIT}(3)), xi_0, phi, rho_0> \downarrow <3, xi_1, phi, rho_1\{X \rightarrow 3\}>$ (FORMALASSIGN)
$<\text{VAR}(X)), xi_1, phi, rho_1> \downarrow <3, xi_1, phi, rho_1>$ (FORMALVAR)

$<\text{BEGIN}(\text{SET}(\text{VAR}(X), \text{LIT}(3)), \text{VAR}(X)), xi_0, phi, rho_0> \downarrow <3, xi_1, phi, rho_1>$ (BEGIN)

14.

$X \in dom(rho)$
$rho(X) = v_1 \mathrel{!}= 0$

$<\text{VAR}(X)), xi_o, phi, rho_0> \downarrow <v_1, xi_o, phi, rho_0>$ (FORMALVAR)
$<\text{VAR}(X)), xi_o, phi, rho_0> \downarrow <v_1, xi_o, phi, rho_0>$ (FORMALVAR)

$<\text{IF}(\text{VAR}(X), \text{VAR}(X), \text{LIT}(0)), xi_o, phi, rho_0> \downarrow <v_1, xi_o, phi, rho_0>$ (IFTRUE)

$X \in dom(rho)$
$rho(X) = 0$

$<\text{VAR}(X)), xi_o, phi, rho_0> \downarrow <0, xi_o, phi, rho_0>$ (FORMALVAR)
$<\text{LIT}(X), xi_o, phi, rho_0> \downarrow <0, xi_o, phi, rho_0>$ (LITERAL)

$<\text{IF}(\text{VAR}(X), \text{VAR}(X), \text{LIT}(0)), xi_o, phi, rho_0> \downarrow <0, xi_o, phi, rho_0>$ (IFFALSE)

$X \in dom(rho)$
$rho(X) = v_2$

$<\text{VAR}(X)), xi_o, phi, rho_0> \downarrow <v_2, xi_o, phi, rho_0>$ (FORMALVAR)

When $rho(X) = 0$, $v_1 = v_2 = 0$, and when $rho(X) \mathrel{!}= 0$, $v_1 = v_2 = rho(X)$, so $v_1 = v_2$

21.

A.

$$\frac{X \notin dom(rho) \quad X \notin dom(xi)}{\langle VAR(X), xi_0, phi, rho_0\rangle \downarrow \langle 0, xi_1\{X \rightarrow 0\}, phi, rho_0\rangle} \quad (IMPLICITASSIGN)$$

B.

$$\frac{X \notin dom(rho) \quad X \notin dom(xi)}{\langle VAR(X), xi_0, phi, rho_0\rangle \downarrow \langle 0, xi_0, phi, rho_1\{X \rightarrow 0\}\rangle} \quad (IMPLICITASSIGN)$$

C.

I prefer the change to give Impcore Icon-like semantics because if an unbound variable is referenced, its scope should only be the scope of the current process because it is unfair to assume the variable will still be wanted after the procedure terminates.

20.

An expression is deterministic if it evaluates to the same value every time it is evaluated in a given set of environments. In notation:

Given $\langle e, xi_0, phi, rho_0\rangle \downarrow \langle v_1, xi_1, phi, rho_1\rangle$ and $\langle e, xi_0, phi, rho_0\rangle \downarrow \langle v_2, xi_2, phi, rho_2\rangle$
$v_1 = v_2$, $xi_1 = xi_2$, and $rho_1 = rho_2$

Induction hypothesis: An evaluation of an expression is deterministic if it evaluates to at most one value given the same starting machine state with premises that have deterministic derivations.

Base Cases:
LIT:

$$D = \frac{}{\langle LIT(v), xi_0, phi, rho_0\rangle \downarrow \langle v, xi_0, phi, rho_0\rangle} \quad (LITERAL)$$

A literal expression evaluates to its equivalent value, which is the same value regardless of the context the literal is used in, so for any v, so $v = v_1 = v_2$, $xi_0 = xi_1 = xi_2$, and $rho_0 = rho_1 = rho_2$.

VAR:

$$D = \frac{\begin{array}{l} X \in \mathrm{dom}(rho) \\ rho(X) = v \end{array}}{\langle VAR(X), xi_0, phi, rho_0\rangle \downarrow \langle v, xi_0, phi, rho_0\rangle} \quad \text{(FORMALVAR)}$$

Therefore, $v = v_1 = v_2$, $xi_0 = xi_1 = xi_2$, and $rho_0 = rho_1 = rho_2$, so evaluating a formal variable expression is deterministic.

$$D = \frac{\begin{array}{l} X \notin \mathrm{dom}(rho) \\ X \in \mathrm{dom}(xi) \\ xi(X) = v \end{array}}{\langle VAR(X), xi_0, phi, rho_0\rangle \downarrow \langle v, xi_0, phi, rho_0\rangle} \quad \text{(GLOBALVAR)}$$

Therefore, $v = v_1 = v_2$, $xi_0 = xi_1 = xi_2$, and $rho_0 = rho_1 = rho_2$, so evaluating a global variable expression is deterministic.

Induction Cases:

SET:

$$D = \frac{\begin{array}{l} X \in \mathrm{dom}(rho) \\ D_1 \end{array}}{\langle SET(X, e), xi_0, phi, rho_0\rangle \downarrow \langle v, xi_{set}, phi, rho_{set}\{X \rightarrow v\}\rangle} \quad \text{(FORMALASSIGN)}$$

By induction hypothesis, if D_1 is a deterministic derivation, then $v = v_1 = v_2$, $xi_{set} = xi_1 = xi_2$, and $rho_{set}\{X \rightarrow v\} = rho_1 = rho_2$.

$$D = \frac{\begin{array}{l} X \notin \mathrm{dom}(rho) \\ X \in \mathrm{dom}(xi) \\ D_1 \end{array}}{\langle SET(X, e), xi_0, phi, rho_0\rangle \downarrow \langle v, xi_{set}, phi, rho_{set}\{X \rightarrow v\}\rangle} \quad \text{(GLOBALASSIGN)}$$

By induction hypothesis, if D_1 is a deterministic derivation, then $v = v_1 = v_2$, $xi_{set}\{X \rightarrow v\} = xi_1 = xi_2$, and $rho_{set} = rho_1 = rho_2$.

IF:

$$D = \frac{D_1 \quad v_1 \mathrel{!}= 0 \quad D_2}{\langle IF(e, e_t, e_f), xi_0, phi, rho_0\rangle \downarrow \langle v_t, xi_t, phi, rho_t\rangle} \quad \text{(IFTRUE)}$$

By induction hypothesis, if D_1 and D_2 are deterministic derivations, then $v_t = v_1 = v_2$, $xi_t = xi_1 = xi_2$, and $rho_t = rho_1 = rho_2$.

$$D = \frac{D_1 \quad v_1 = 0 \quad D_3}{\langle IF(e, e_t, e_f), xi_0, phi, rho_0\rangle \downarrow \langle v_f, xi_f, phi, rho_f\rangle} \quad \text{(IFFALSE)}$$

By induction hypothesis, if D_1 and D_3 are deterministic derivations, then $v_f = v_1 = v_2$, $xi_f = xi_1 = xi_2$, and $rho_f = rho_1 = rho_2$.

APPLYADD:

$$D = \frac{D_1 \quad D_2 \quad -2^{31} <= v_a + v_b < 2^{31}}{\langle APPLY(+, e_a, e_b), xi_0, phi, rho_0\rangle \downarrow \langle v_a + v_b, xi_{add}, phi, rho_{add}\rangle}$$

By induction hypothesis, if D_1 and D_2 are deterministic derivations, then $v_a + v_b = v_1 = v_2$, $xi_{add} = xi_1 = xi_2$, and $rho_{add} = rho_1 = rho_2$.