

Theory

9/24/18

1.

$LIST(SEXP_{FG}) = \{ '() \} \cup \{ (cons\ ATOM\ LIST(SEXP_{FG})) \} \cup \{ (cons\ SEXP_{FG}\ LIST(SEXP_{FG})) \}$
 $\{ '() \} \in ATOM \in SEXP_{FG}$
 $\{ (cons\ ATOM\ LIST(SEXP_{FG})) \} \in$ with $v_1 = ATOM \in SEXP_{FG}$ and $v_2 = LIST(SEXP_{FG}) \in SEXP_{FG}$
 $\{ (cons\ SEXP_{FG}\ LIST(SEXP_{FG})) \}$ with $v_1 = SEXP_{FG}$ and $v_2 = LIST(SEXP_{FG}) \in SEXP_{FG}$
 $\therefore LIST(SEXP_{FG}) \in SEXP_{FG}$

35.

Base:

0

$= (length\ (reverse\ '()))$

substituting

$= (reverse\ '()) = '()$

reverse null rule

$= (length\ '()) = 0$

Inductive

n

$= (length\ (reverse\ (cons\ x\ xs)))$

substituting

$= (length\ (cons\ (reverse\ xs)\ x))$

reverse cons rule

$= (length\ reverse\ xs) + (length\ x)$

length cons rule

$= (n - 1) + 1$

simplifying

$= n$

By induction, if after one recurse of reverse both the base and inductive cases have the same length, then
 $(length\ (reverse\ xs)) = (length\ xs)$.

A.

1.

$X \in \text{rho}$

$XS \in \text{rho}$

$\langle \text{CONS}, \text{rho}, \text{sigma}_1 \rangle \downarrow \langle \text{PRIMITIVE}(\text{CONS}), \text{sigma}_2 \rangle$

$\langle \text{VAR}(X), \text{rho}, \text{sigma}_2 \rangle \downarrow \langle x, \text{sigma}_3 \rangle$ (FORMAL)

$\langle \text{VAR}(XS), \text{rho}, \text{sigma}_3 \rangle \downarrow \langle xs, \text{sigma}_4 \rangle$ (FORMAL)

$l_1 \notin \text{sigma}_4 \quad l_2 \notin \text{sigma}_4 \quad l_1 \neq l_2$

$\langle \text{CDR}, \text{rho}, \text{sigma}_0 \rangle \downarrow \langle \text{PRIMITIVE}(\text{CDR}), \text{sigma}_1 \rangle$

$\langle \text{CONS}(\text{VAR}(X) \text{ VAR}(XS)), \text{rho}, \text{sigma}_1 \rangle \downarrow \langle (\text{PAIR}(l_1, l_2)), \text{sigma}_4 \{l_1 \rightarrow x, l_2 \rightarrow xs\} \rangle (\text{CONS})$

$\langle \text{CDR}(\text{CONS}(\text{VAR}(X) \text{ VAR}(XS))), \text{rho}, \text{sigma}_0 \rangle \downarrow \langle xs, \text{sigma}_4 \rangle$ (CDR)

2.

Induction hypothesis:

If e_1 and e_2 are terminating expressions and cdr and cons can both terminate given terminating expressions, then $(\text{cdr} (\text{cons } e_1 e_2))$ terminates and equals e_2

$\langle \text{CONS}, \text{rho}, \text{sigma}_1 \rangle \downarrow \langle \text{PRIMITIVE}(\text{CONS}), \text{sigma}_2 \rangle$

$\langle e_1, \text{rho}, \text{sigma}_2 \rangle \downarrow \langle e_1, \text{sigma}_3 \rangle$ (IH)

$\langle e_2, \text{rho}, \text{sigma}_3 \rangle \downarrow \langle e_2, \text{sigma}_4 \rangle$ (IH)

$l_1 \notin \text{sigma}_4 \quad l_2 \notin \text{sigma}_4 \quad l_1 \neq l_2$

$\langle \text{CDR}, \text{rho}, \text{sigma}_0 \rangle \downarrow \langle \text{PRIMITIVE}(\text{CDR}), \text{sigma}_1 \rangle$

$\langle \text{CONS}(e_1 e_2), \text{rho}, \text{sigma}_1 \rangle \downarrow \langle (\text{PAIR}(l_1, l_2)), \text{sigma}_4 \{l_1 \rightarrow e_1, l_2 \rightarrow e_2\} \rangle$ (CONS)

$\langle \text{CDR}(\text{CONS}(e_1 e_2)), \text{rho}, \text{sigma}_0 \rangle \downarrow \langle \text{sigma}_4(e_2), \text{sigma}_4 \rangle$ (CDR)

Terminates and evaluates to e_2 given terminating e_1 and e_2 .