An introduction to a monotone theory of co-design

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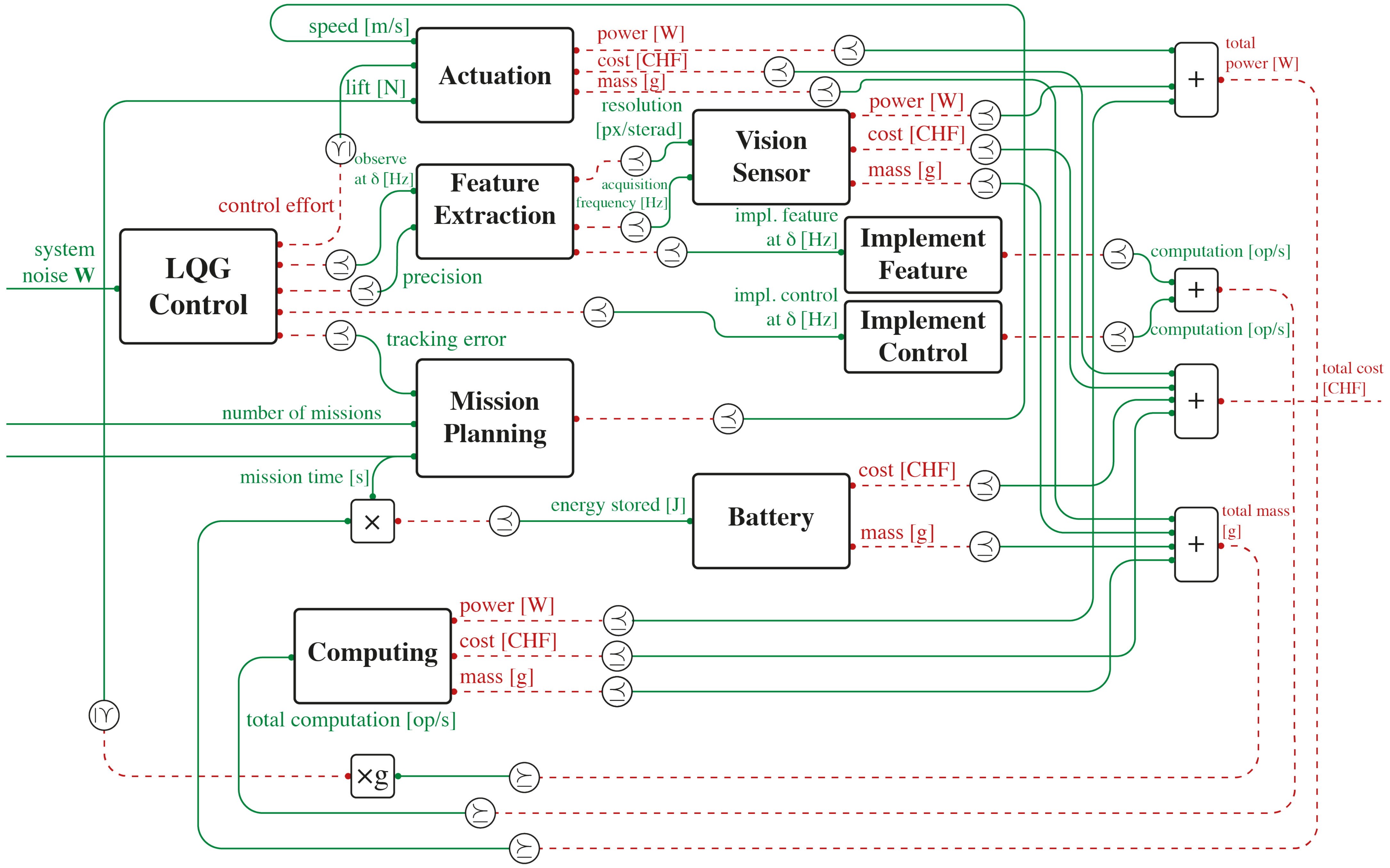
Desiderata for the automation of complex systems co-design

- **Formal**
- **Computationally tractable**
  - Need to compute solutions efficiently
- **Compositional, hierarchical**
  - My system is a component of somebody else's system
- **Collaborative**
  - Pooling knowledge from experts across fields.
- **Intellectually tractable**
  - Not exclusively accessible to system architects
- **Continuous**
  - Design is not static: it should be reactive to changes in goals and contexts

A new approach to multi-disciplinary engineering “co”-design

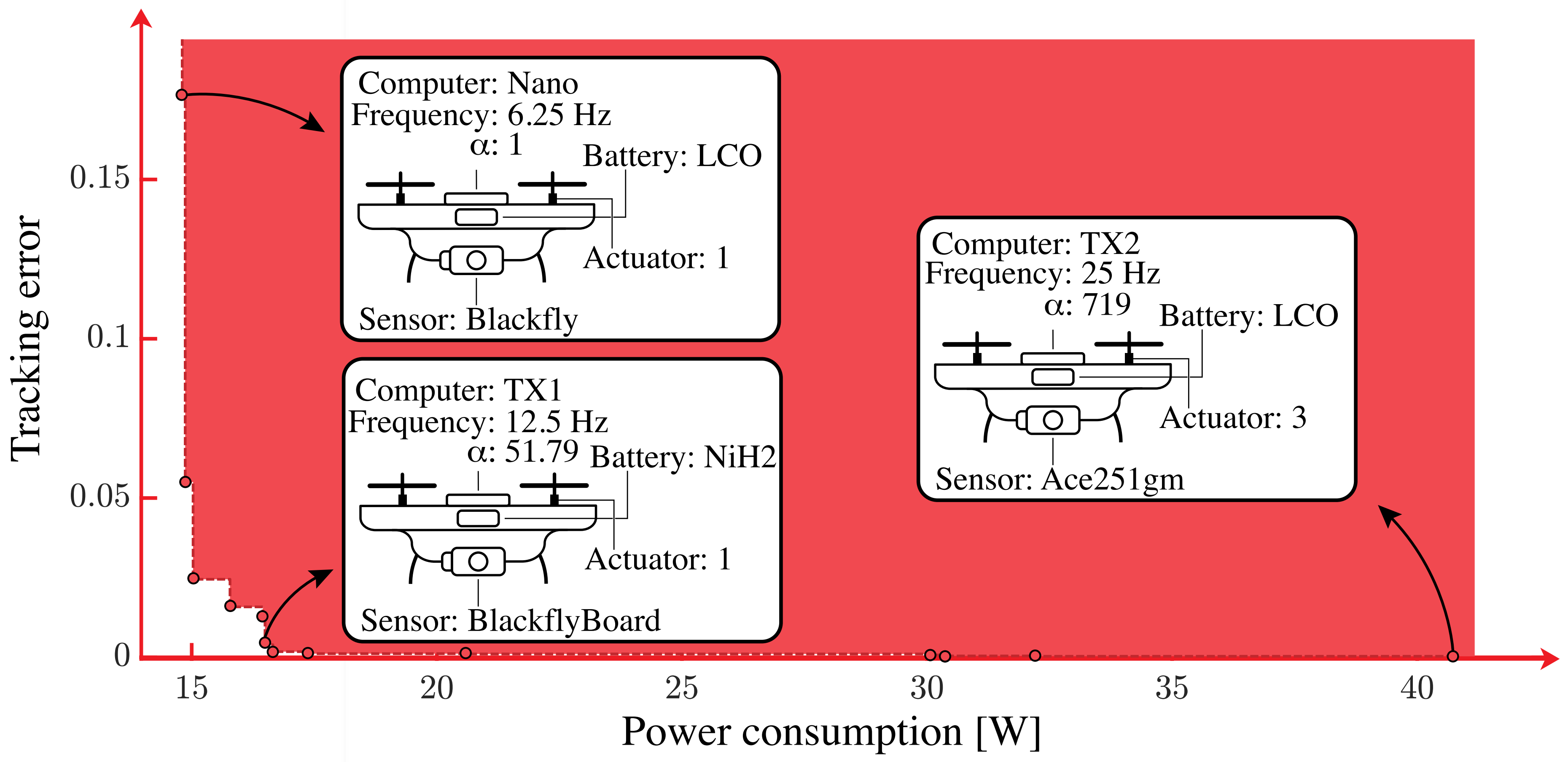
- A new approach to **collaborative**, **computational**, **compositional**, **continuous** design designed to work **across fields** and **across scales**.
- Leverages **domain theory**, **applied category theory**, and **optimization**
- Roadmap:
  - Defining **“design problems”** for **components.**
  - Modeling **co-design constraints** in a complex **system**.
  - **Efficient** solution to design queries.

“Co-design diagram”



optimization for a task

Pareto front of optimal designs



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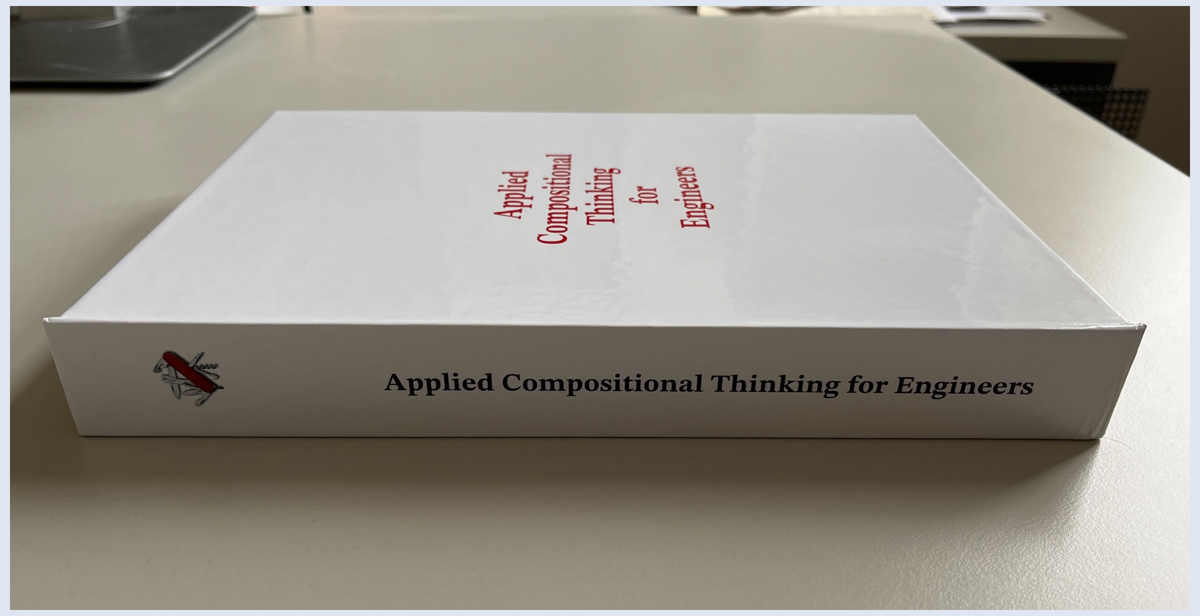
a “pro” box



Access the book at:
https://bit.ly/3qQNrdR

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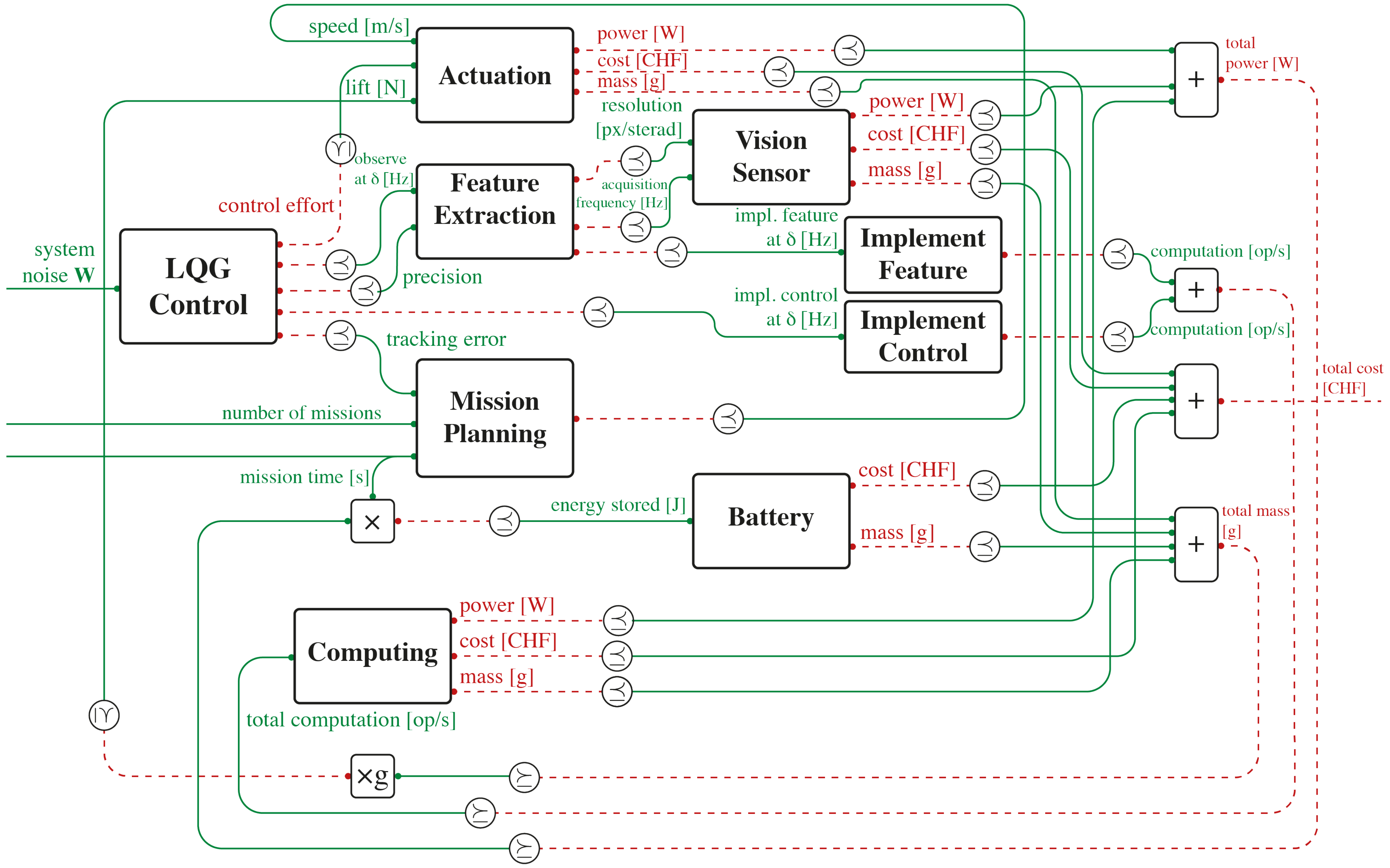

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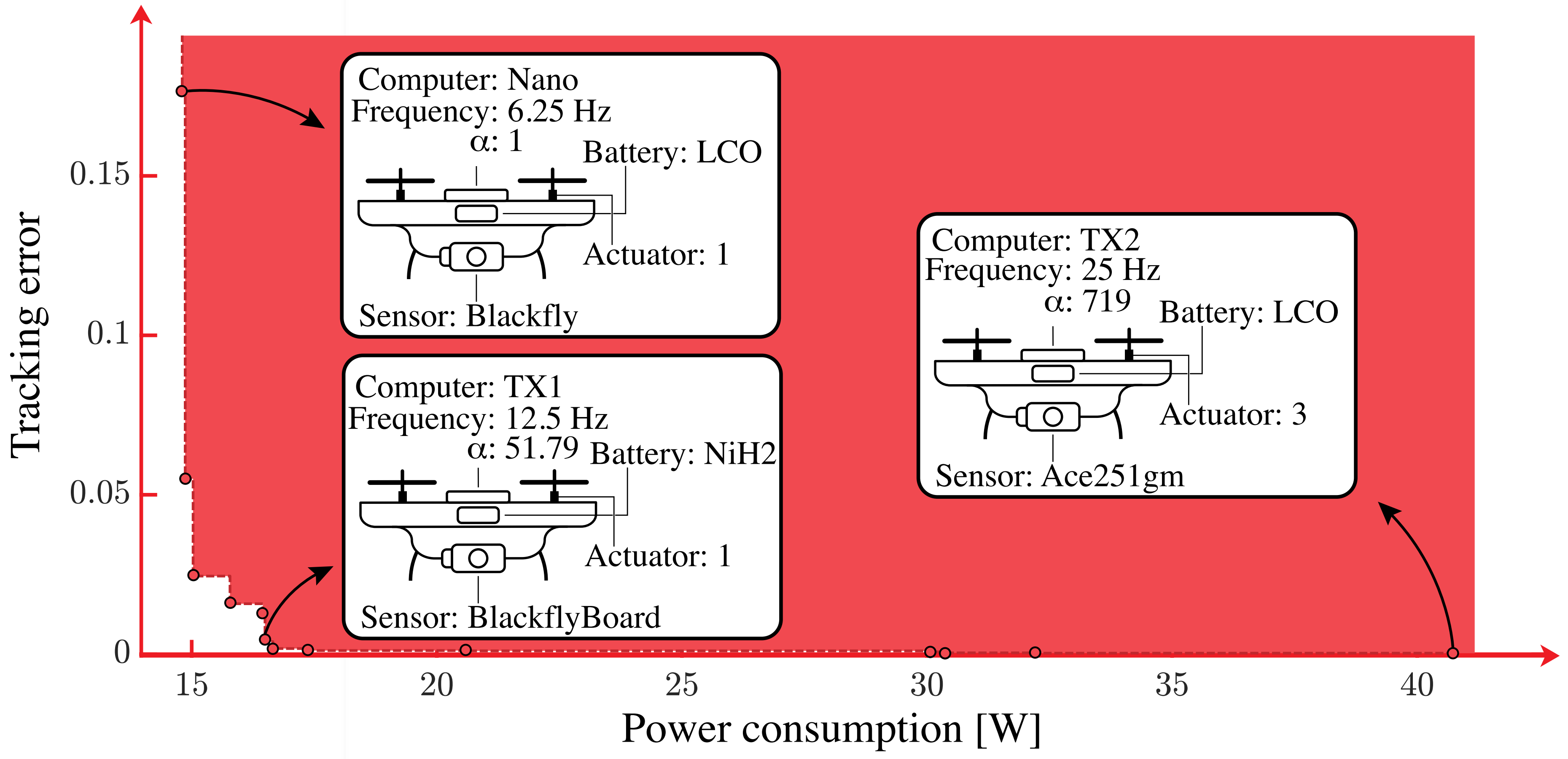
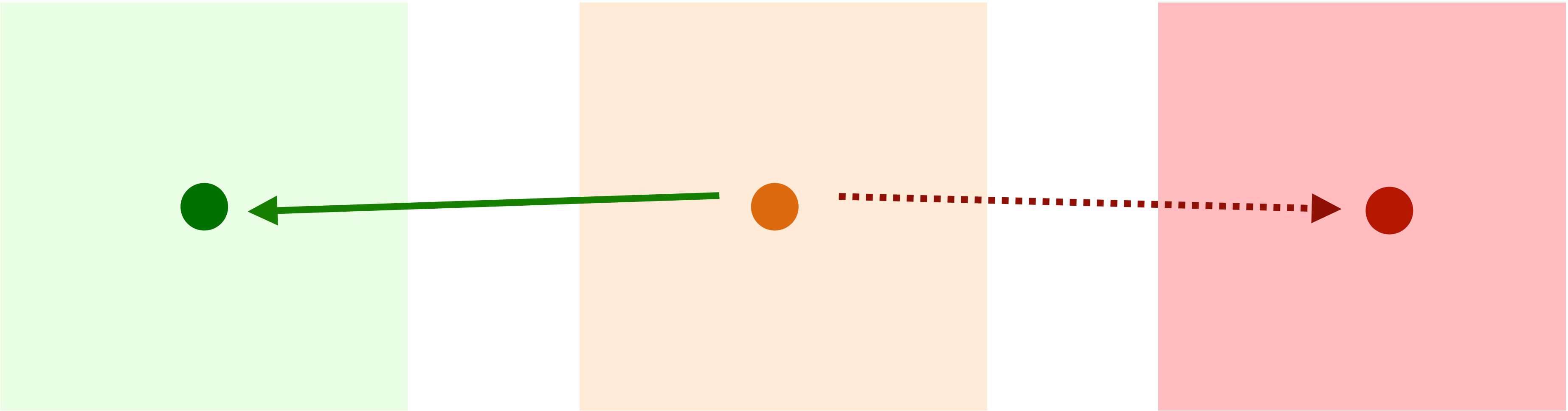


An abstract view of design problems

- Across fields, design or synthesis problems are defined with **three spaces**:
  - **implementation space**: the **options** we can choose from;
  - **functionality space**: what we need to **provide/achieve**;
  - **requirements/costs space**: the **resources** we need to have available;

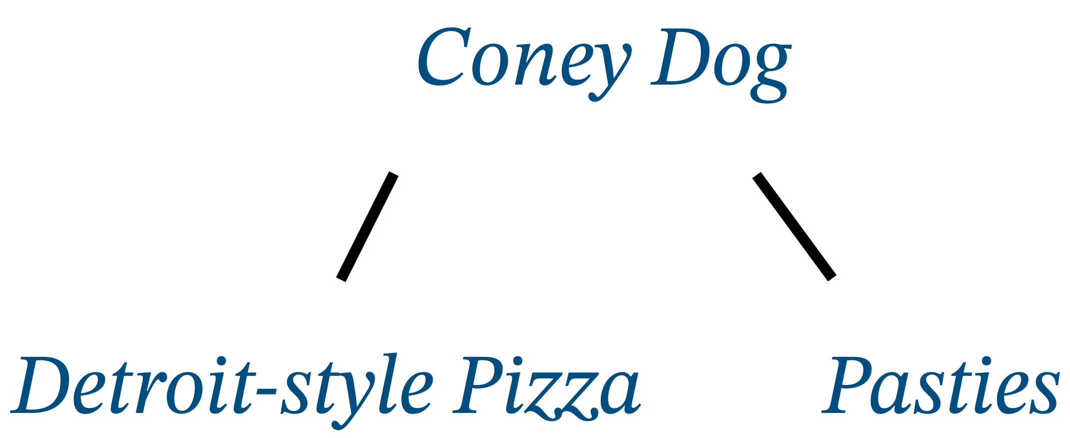


functionality *(provided)*	***implementations***	***costs, resources*** *(required)*
desired behavior	*choices*	*budget*
specifications	*plans*	*requirements*
objectives	*blueprints*	*dependencies*
guarantees	*decision variables*	*assumptions*
conclusions		
"function"	*"form"*	*"function"*

Partially ordered sets model trade-offs, across fields

- Posets model standard costs in engineering $\langle \mathbb{R}_{\geq 0}, \leq \rangle$, $\langle \mathbb{N}, \leq \rangle$
- ... but also enable **richer** cost structures:

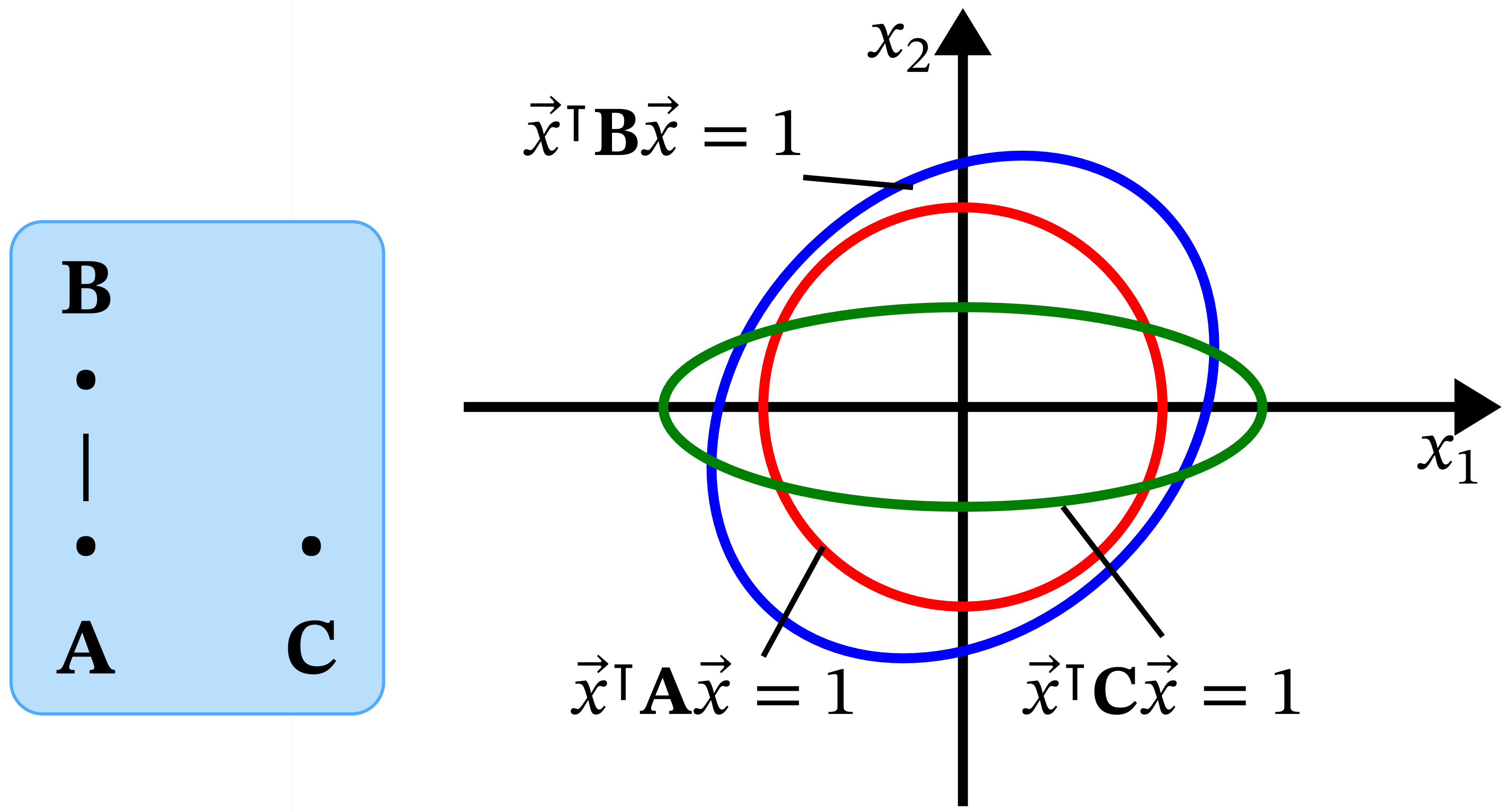
A poset of beach preferences in Michigan.



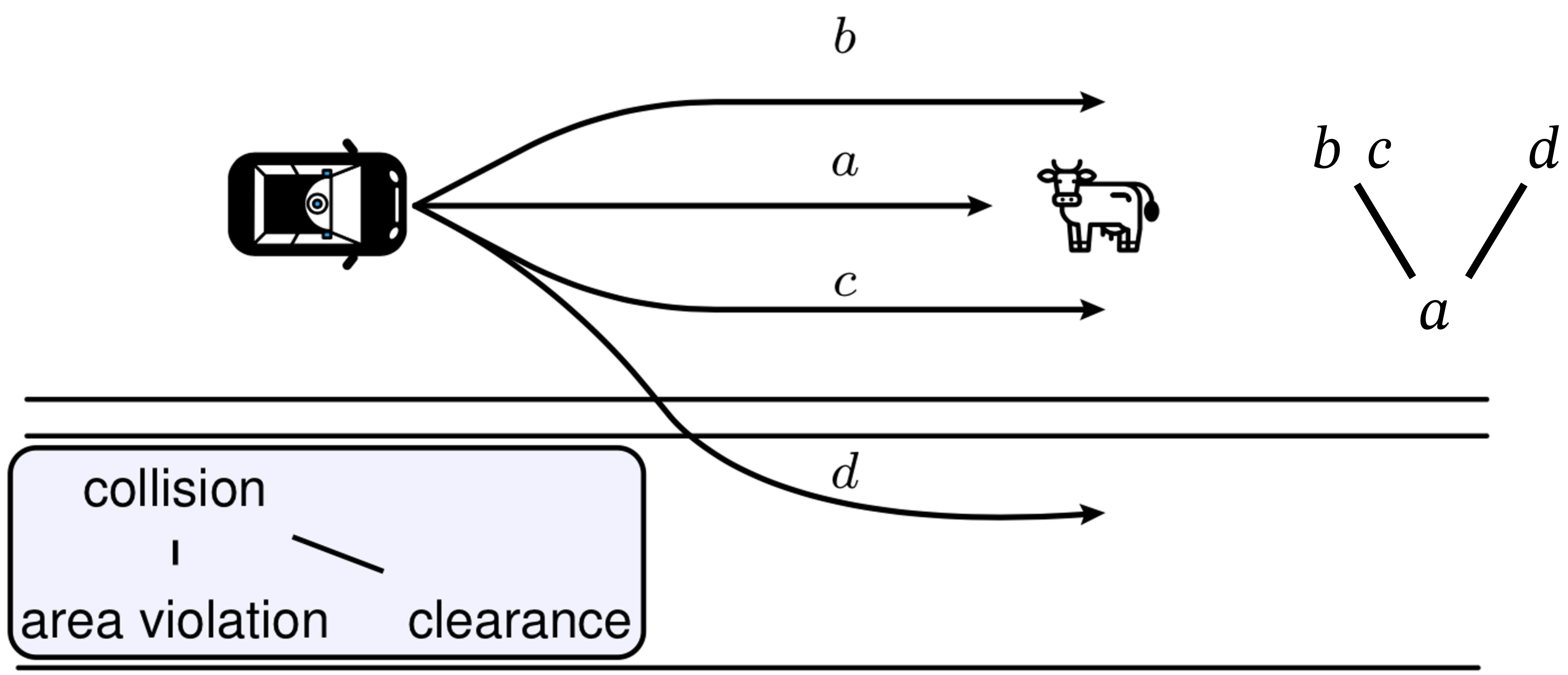
A poset of positive-definite matrices

$$\frac{\mathbf{A} \preceq_{\mathsf{PDM}(n)} \mathbf{B}}{\vec{x}^{\intercal}\mathbf{A}\vec{x} \leq \vec{x}^{\intercal}\mathbf{B}\vec{x} \quad \forall \vec{x} \in \mathbb{R}^n}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3/4 & -1/8 \\ -1/8 & 3/4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$



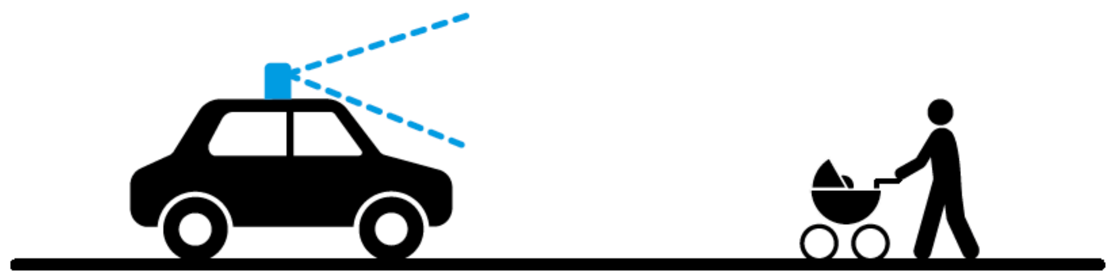
Posets of rules, which induce priorities over behaviors

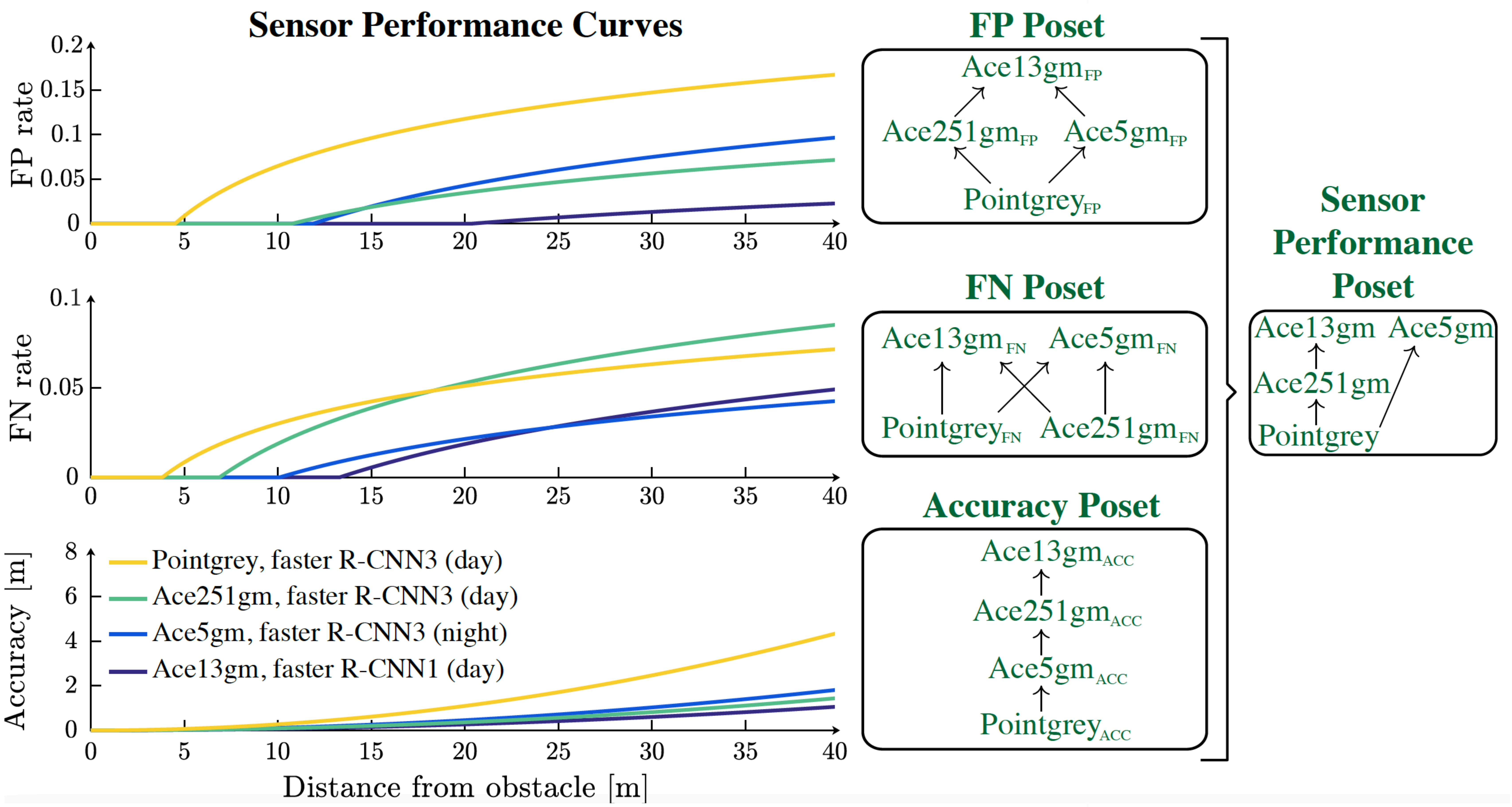


[RAL'22, CDC'22, IROS'21, ECC'21]

Partially ordered sets model trade-offs, across fields

A poset of sensor/algorithm pairs





[CDC'22, IROS'21]

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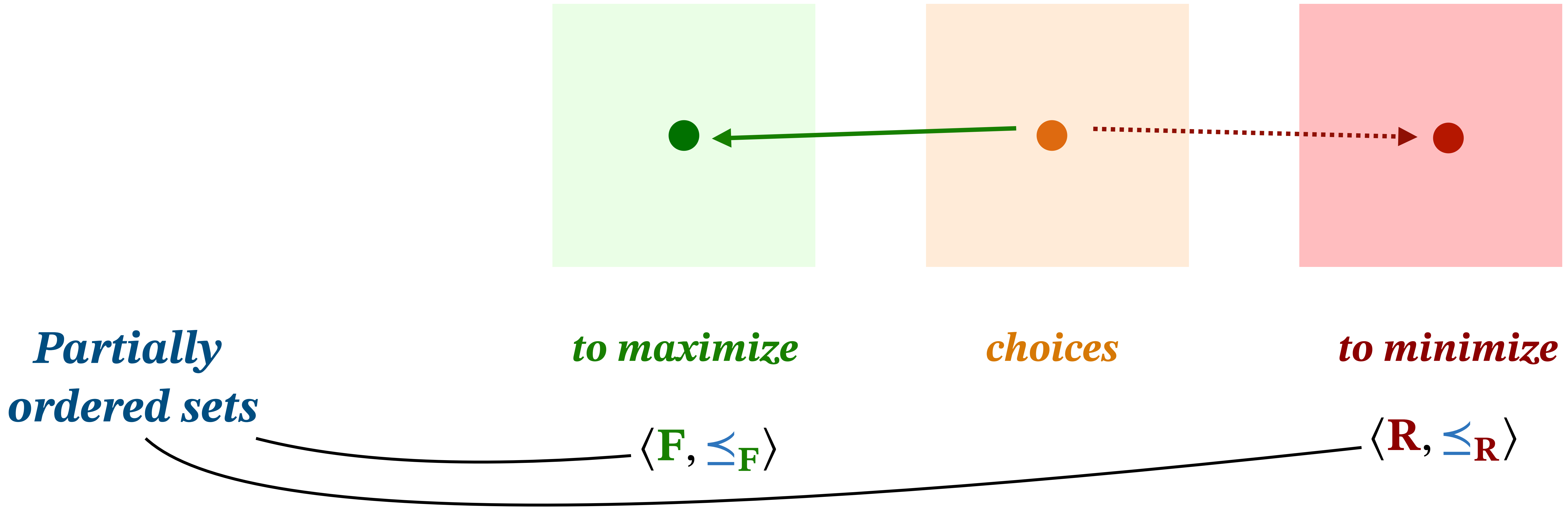


Function beyond the strictly technical meaning

- **Law of successful products**: at equilibrium, in an efficient and free market, no product completely dominates another by both functionality and costs.
(Otherwise, the dominated product would not sell.)

Casio F-91W Patek Philippe

"watchiness"

tells time,
multiple alarms

tells the time

signaling

?

I am somebody
who can afford
a *Patek Philippe*
and I chose to
buy one.

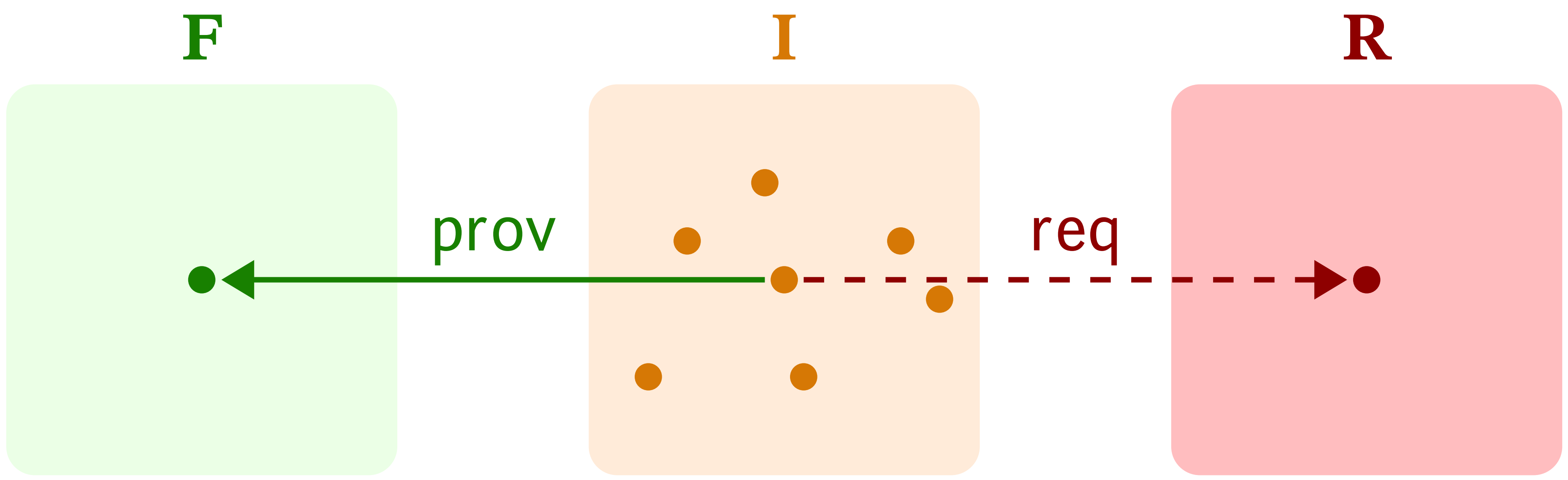
price

a lot

much less

Transparent vs black-box models

- The **"Design Problems with Implementations" model** is a **"transparent" model:**



- **DP** model: **direct feasibility relation** between functionality and resources ("black box") as a monotone map:

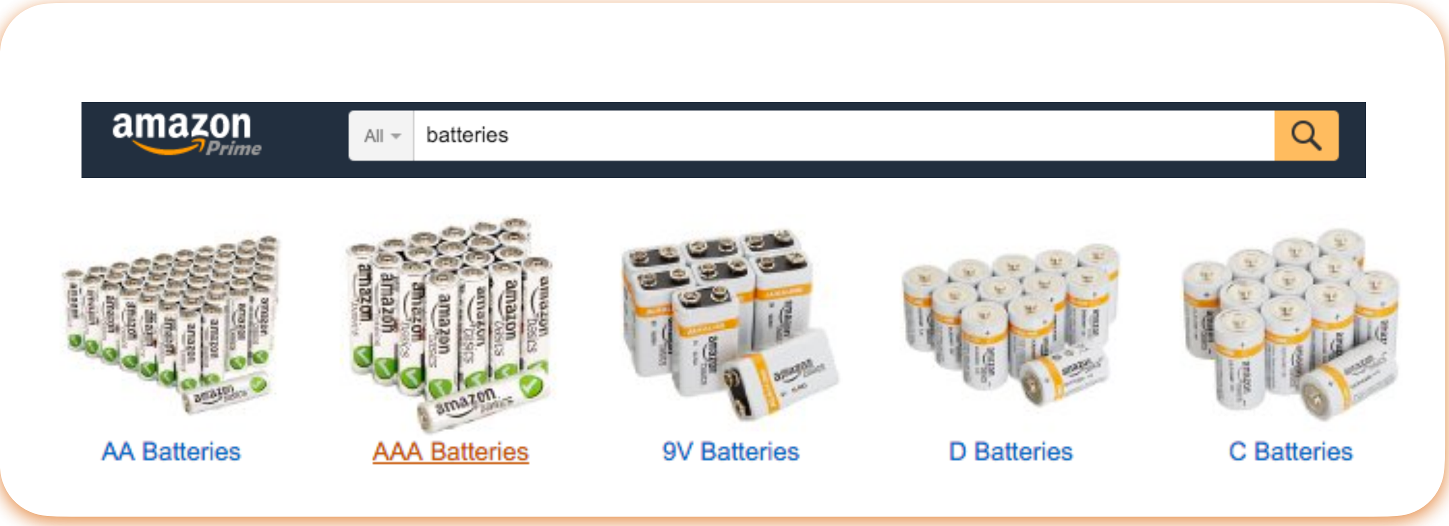
$$\mathbf{d} : \mathbf{F}^{\mathrm{op}} \times \mathbf{R} \to_{\mathbf{Pos}} \mathbf{Bool}$$
$$\langle f^*, r \rangle \mapsto \exists i \in \mathbf{I} : (f \preceq_{\mathbf{F}} \mathtt{prov}(i)) \wedge (\mathtt{req}(i) \preceq_{\mathbf{R}} r)$$

... a "boolean profunctor"

- **Monotonicity**:
  - **Lower functionality** does **not** require **more resources**;
  - More **resources** do not provide **less functionality.**

Co-design enables a rich class of model population techniques

- “**Catalogues**”: off-the-shelf designs.



	Spark	Phantom 3 Std	Phantom 4 Adv	Phantom 4 Pro	Mavic	Inspire
Flight time	16 mins	25 mins	30 mins	30 mins	27 mins	27 mins
Top Speed	31 mph (50 km/h)	36 mph (58 km/h)	45 mph (72 km/h)	45 mph (72 km/h)	40 mph (65 km/h)	58 mph (94 km/h)
Range	1.2 miles (2 km)	0.6 miles (1 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)	4.3 miles (7 km)
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Size	5.6 x 5.6 x 2.1 in (14.3 x 14.3 x 5.5 cm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.8 in diagonal (350 mm)	13.2 in diagonal (350 mm)	16.8 x 12.5 x 16.7 in (42.7 x 31.7 x 42.5 cm)
Takeoff weight	11.6 oz (330 g)	2.6 lb (1.2 kg)	3 lb (1.4 kg)	3 lb (1.4 kg)	1.6 lb (743 kg)	8.8 lb (4 kg)
Other features	Follow me, Return home, Obstacle avoidance, FPV	Follow me, Return home	Follow me, Return home, Obstacle avoidance	Follow me, Return home, 3 Direction Obstacle avoidance	Follow me, Return home, Obstacle avoid-ance, folding arms	Obstacle avoidance, Spotlight Pro/Broadcast/ Composition mode
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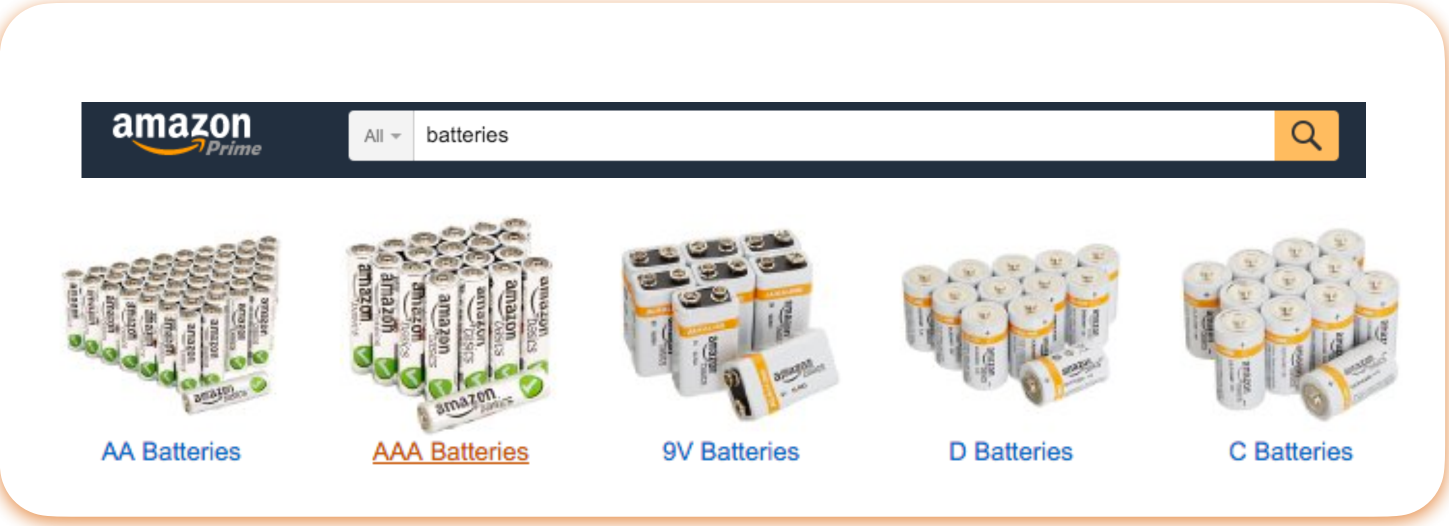

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- "**First-principles**": analytical relations.

mission energy ≥ *mission duration* x *power consumption*

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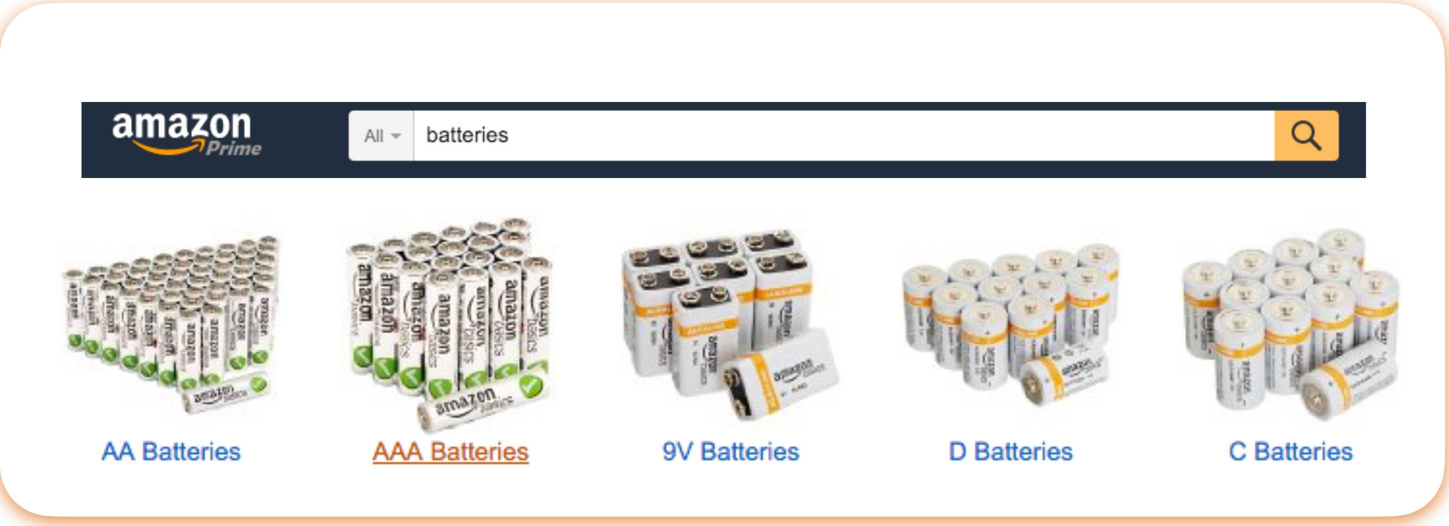

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- “**Data-driven**”, “**on-demand**”
  - The optimization will ask for a **sequence** of data points. The model is constructed **incrementally** (**experiments**, black-box **simulations**, solutions of **optimization problems**).

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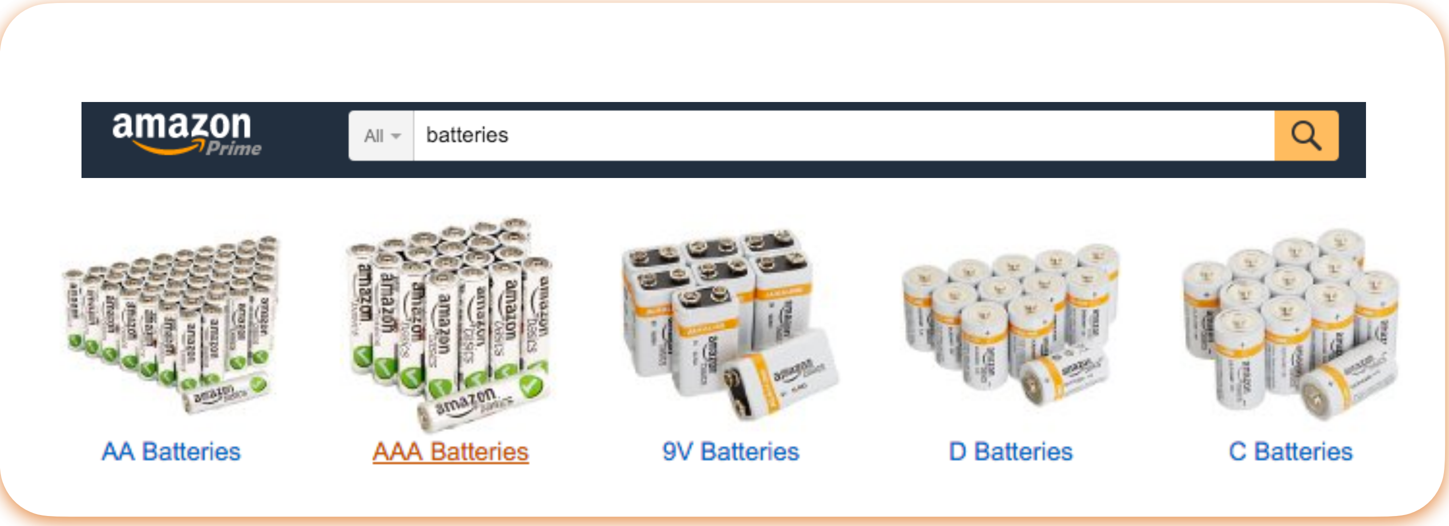

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- **"Data-driven", "on-demand"**
  - The optimization will ask for a **sequence** of data points. The model is constructed **incrementally** (**experiments**, black-box **simulations**, solutions of **optimization problems**).

- **Uncertain models**

Design problems arise naturally in many domains, across scales

Infrastructure level

demand

investments

externalities

Mobility system

Service level

area to cover [m2]

agents' speed [m/s]

agents

time [s]

total cost [$]

Swarm operations

Platform level

speed [m/s]

task

fix cost

operational cost

AV

Component level

speed [rad/s]

torque [Nm]

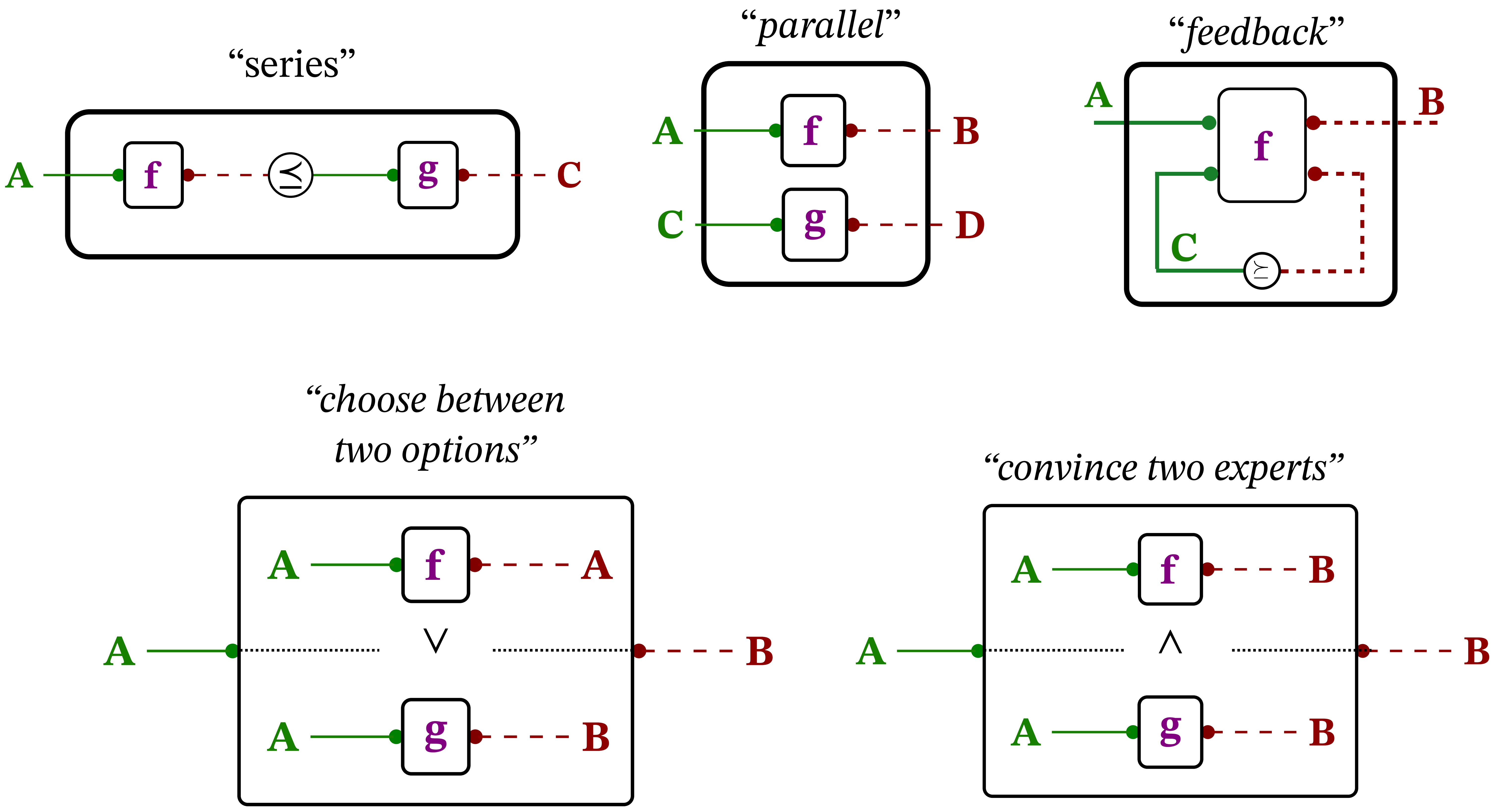
cost [$]

mass [kg]

current [A]

Motor

Design problems can be composed in various ways, preserving properties



- The **composition** of any two **DPs** returns a **DP** (closure)

- Very practical tool to **decompose** large **problems** into **subproblems**

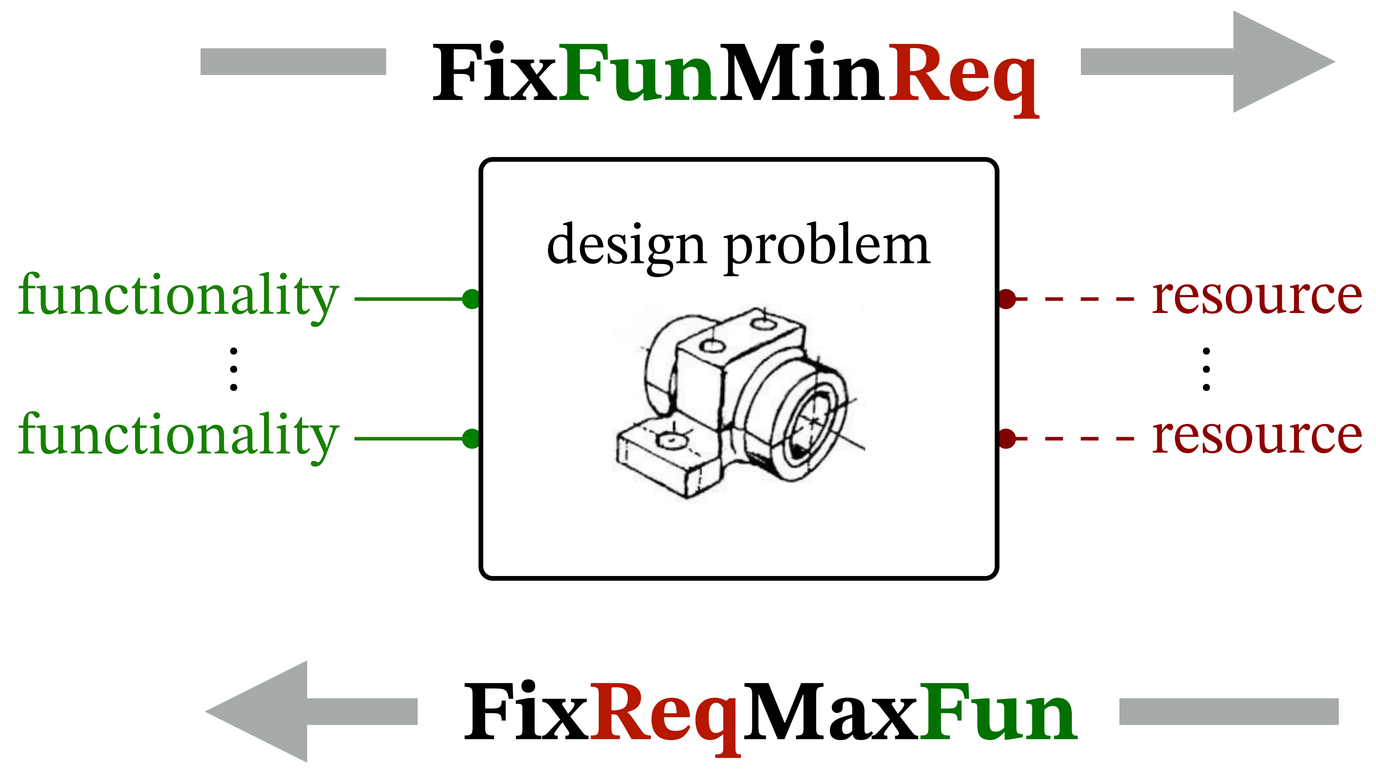
*There is a category **DP** which is traced monoidal, and locally posetal*

✔ ***Formal Compositional/hierarchical***

Multiple queries from the same design problem

- Two basic design queries are:

Given the functionality to be provided,
what are the **minimal resources** required?



Given the resources that are available, what is
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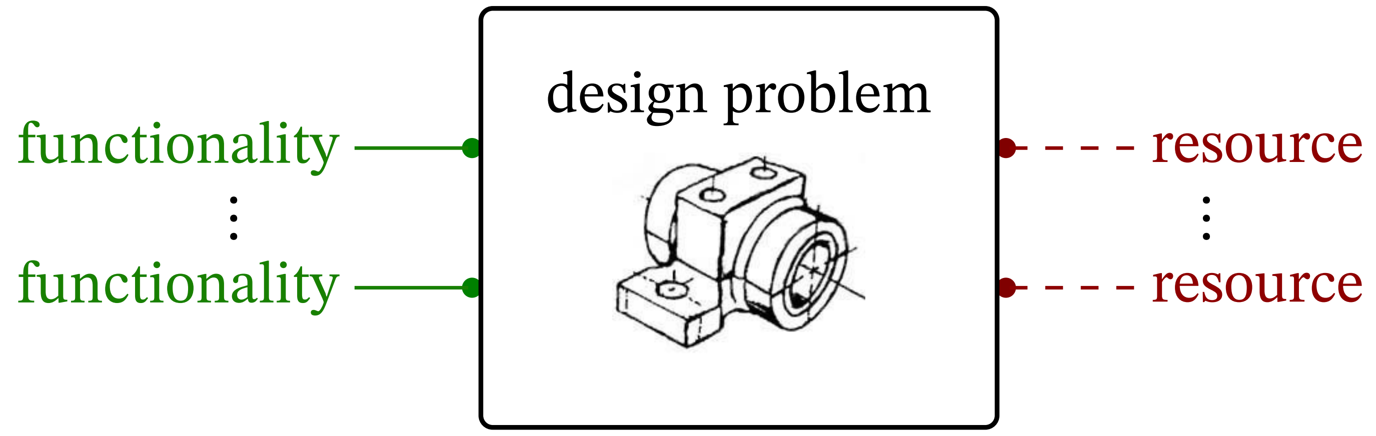
FixFunMinReq →



← **FixReqMaxFun**

Given the resources that are available, what is the **maximal functionality** that can be provided?

- The two problems are **dual**
- From the solutions, one can retrieve the **implementations** (design choices)

Multiple queries from the same design problem

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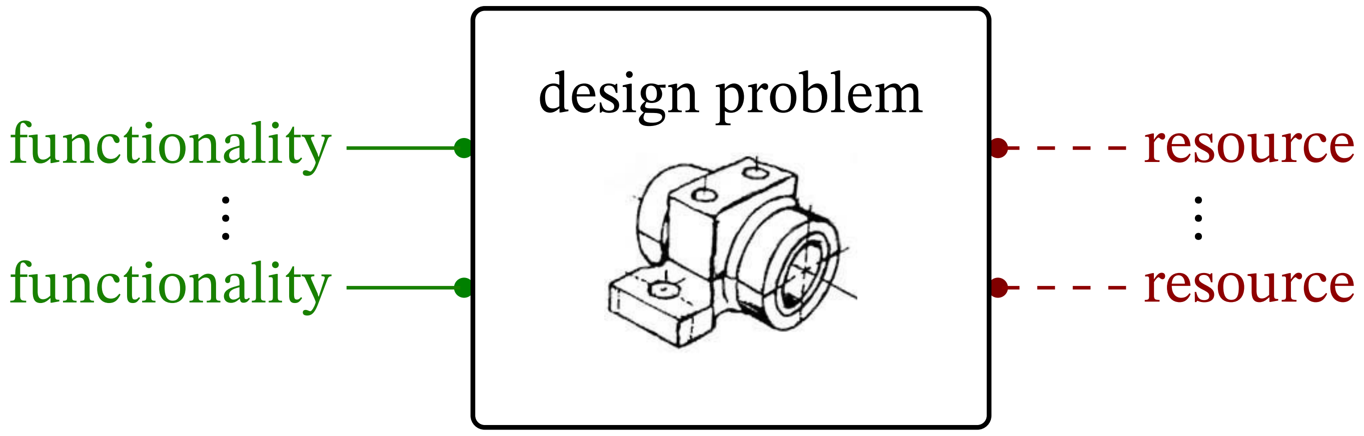
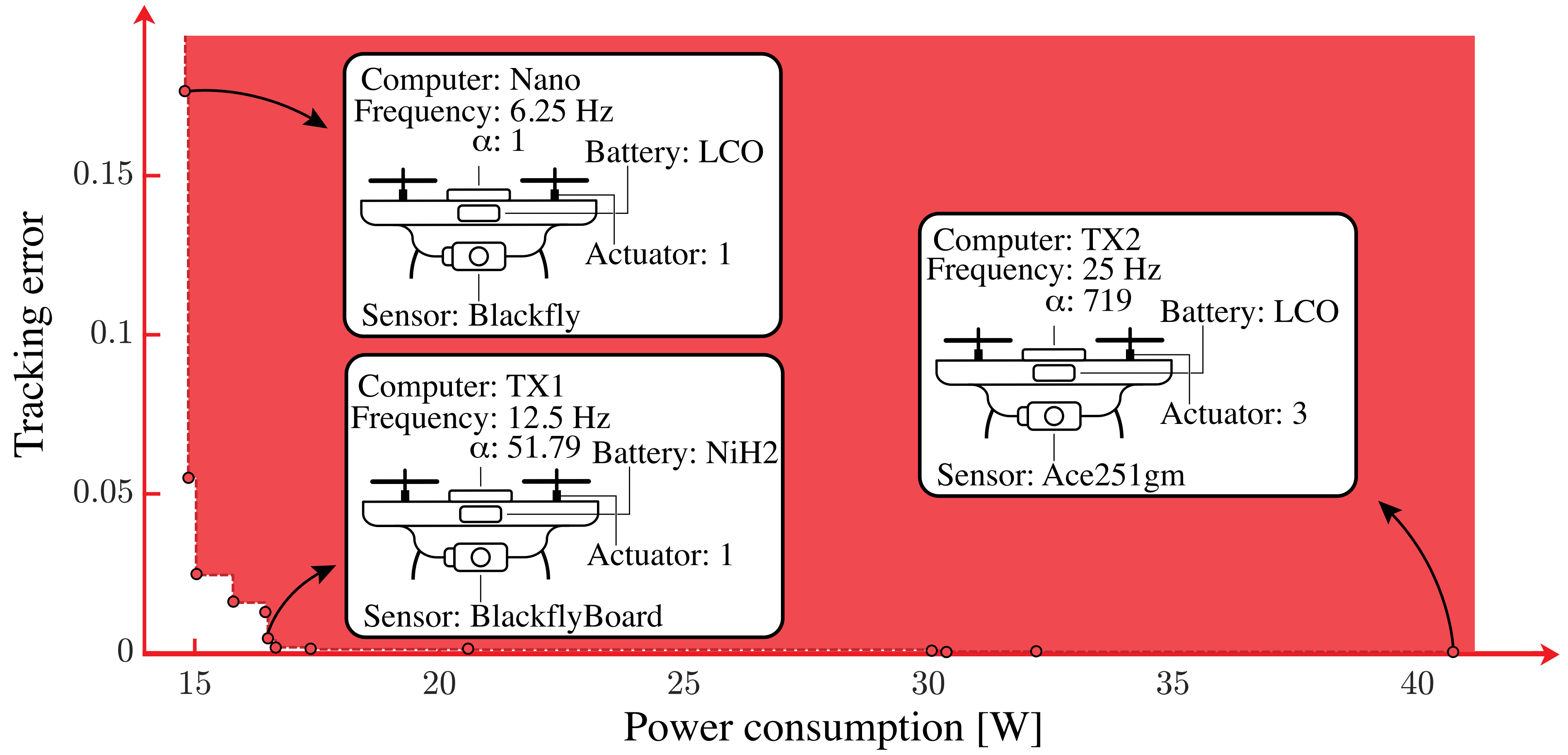
FixFunMinReq →



← **FixReqMaxFun**

Given the resources that are available, what is the **maximal functionality** that can be provided?

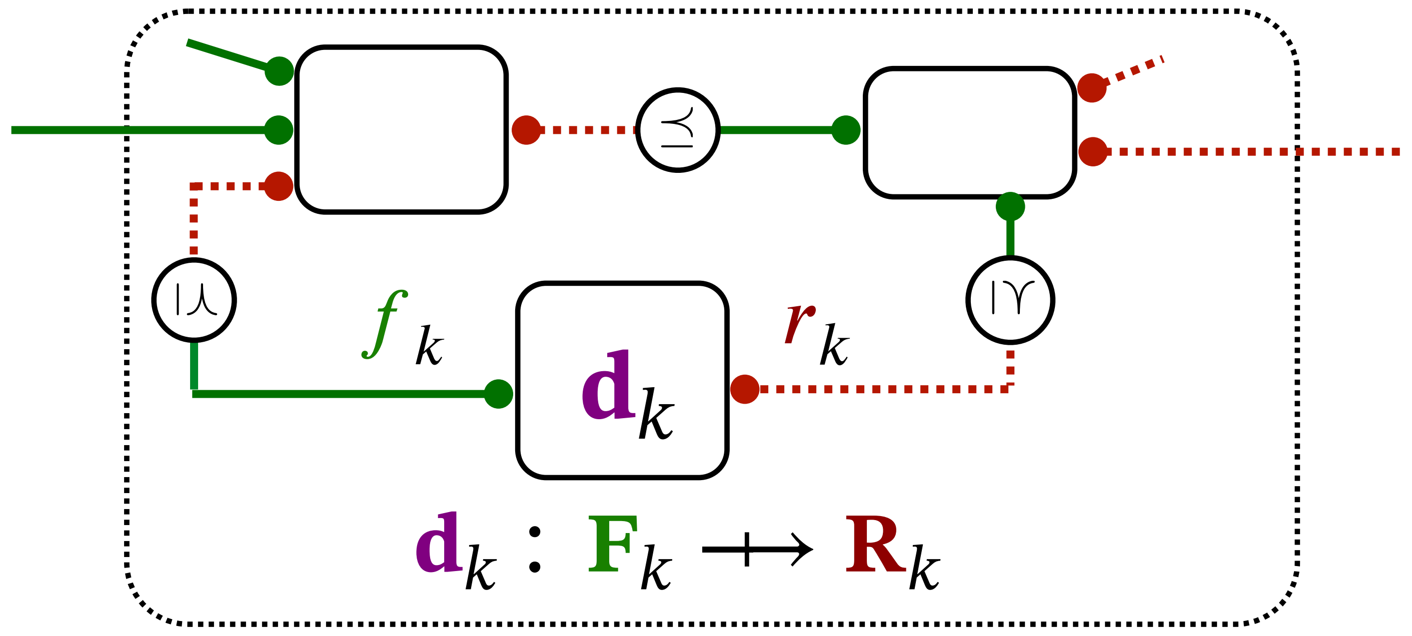
- We are looking for:
  - A map from functionality to **upper sets** of feasible resources: $h : \mathbf{F} \to \mathcal{U}\mathbf{R}$
  - A map from functionality to **antichains** of minimal resources: $h : \mathbf{F} \to \mathcal{A}\mathbf{R}$



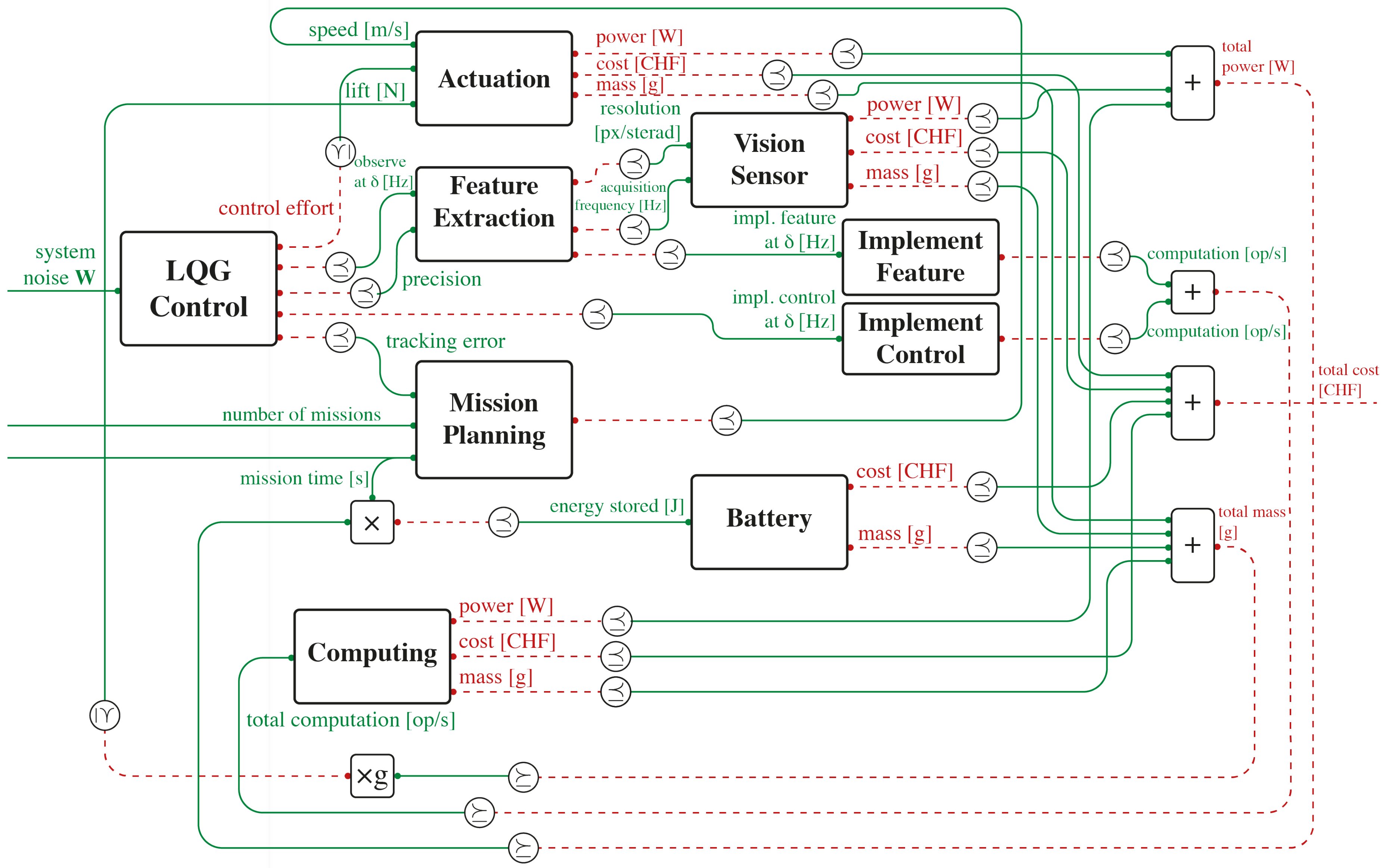
Optimization semantics

- This is the semantics of **FixFunMinReq** as a **family of optimization problems.**

chosen by user $\bar{f}$

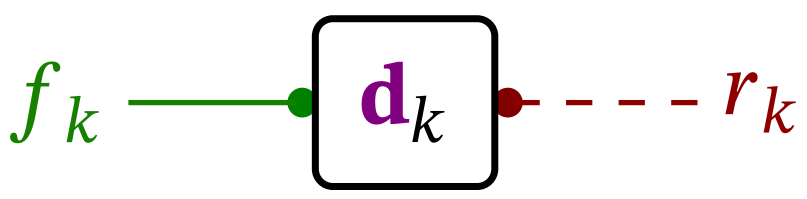


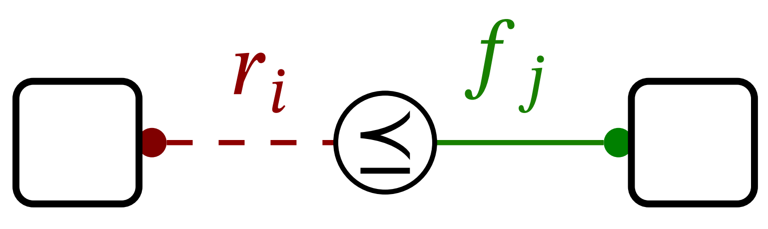
$\bar{r}$ **to minimize**



variables $f_k \in \langle \mathbf{F}_k, \preceq_{\mathbf{F}_k} \rangle$ $r_k \in \langle \mathbf{R}_k, \preceq_{\mathbf{R}_k} \rangle$

constraints *for each node:* *for each edge:*





$\mathbf{d}_k(f_k^*, r_k) = \top$ $r_i \preceq f_j$

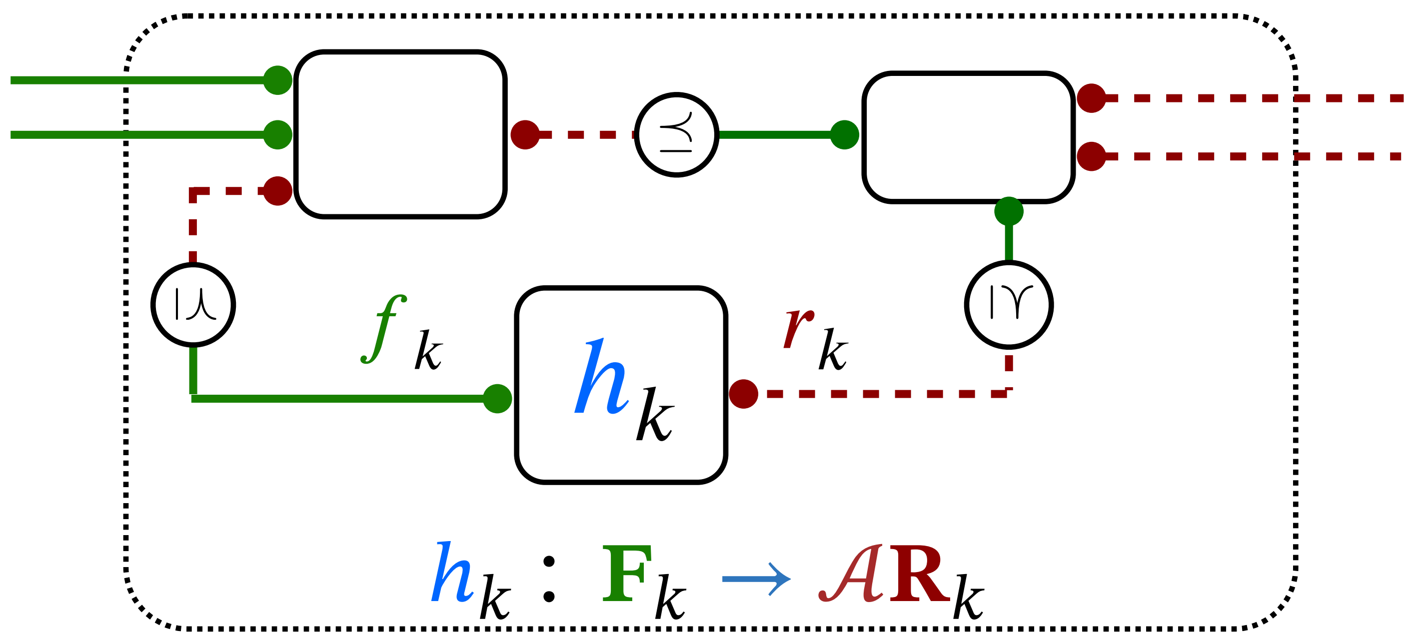
component feasibility *co-design constraint*

objective $\operatorname{Min}_{\preceq} \bar{r}$

! **not convex**
! **not differentiable**
! **not continuous**
! **not even defined on continuous spaces**

Compositional solution of design problem queries

- Suppose that we are given the map $h_k : \mathbf{F}_k \to \mathcal{A}\mathbf{R}_k$ for all nodes in the co-design graph



- Can we find the map $h : \mathbf{F} \to \mathcal{A}\mathbf{R}$ for the entire graph?

✔ ***Computationally tractable***

- **Compositional approach**: just need to work out the composition formulas for all operations

... a functor

solution(**composition**(a, b)) = **composition**(**solution**(a), **solution**(b))

- The set of **minimal** feasible resources can be obtained as the **least fixed point** of a monotone function in the space of anti-chain

- We have a **complete solution**: guaranteed to find the set of **all** optimal solutions (if empty, **certificate of infeasibility**)
- The complexity is **not combinatorial in the number of options** for each component
- The complexity depends on the **complexity of the interactions**: the co-design **constraints**

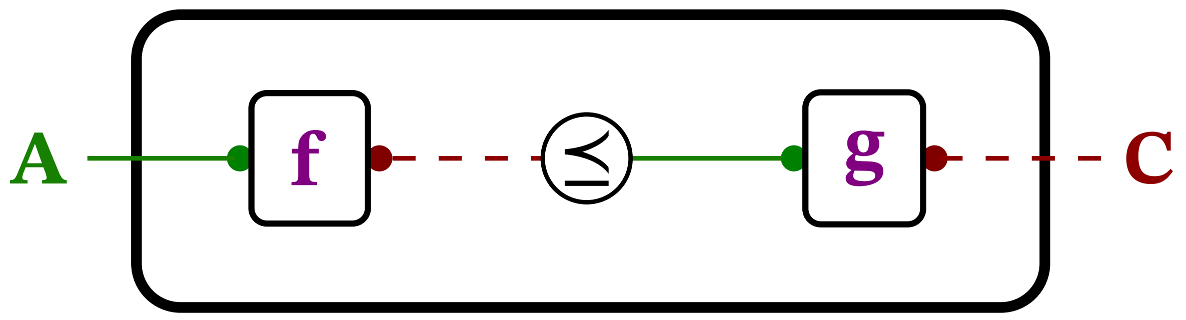
a options | *c* options

b options

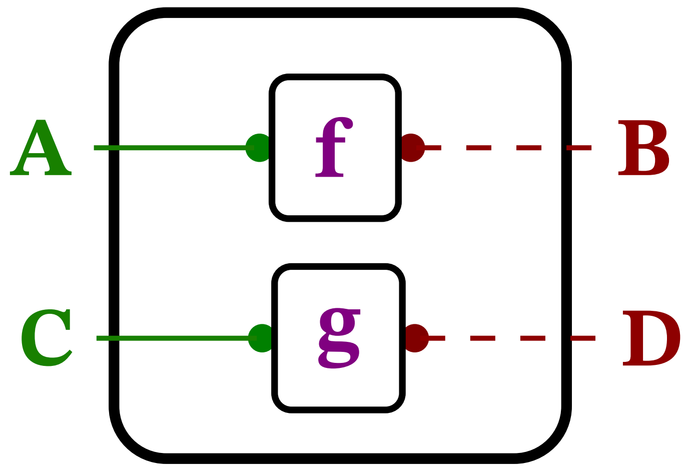
$O(a + b + c)$

Compositional solution of design problem queries

- We can *easily* write the solution for all composition operations *except feedback*.

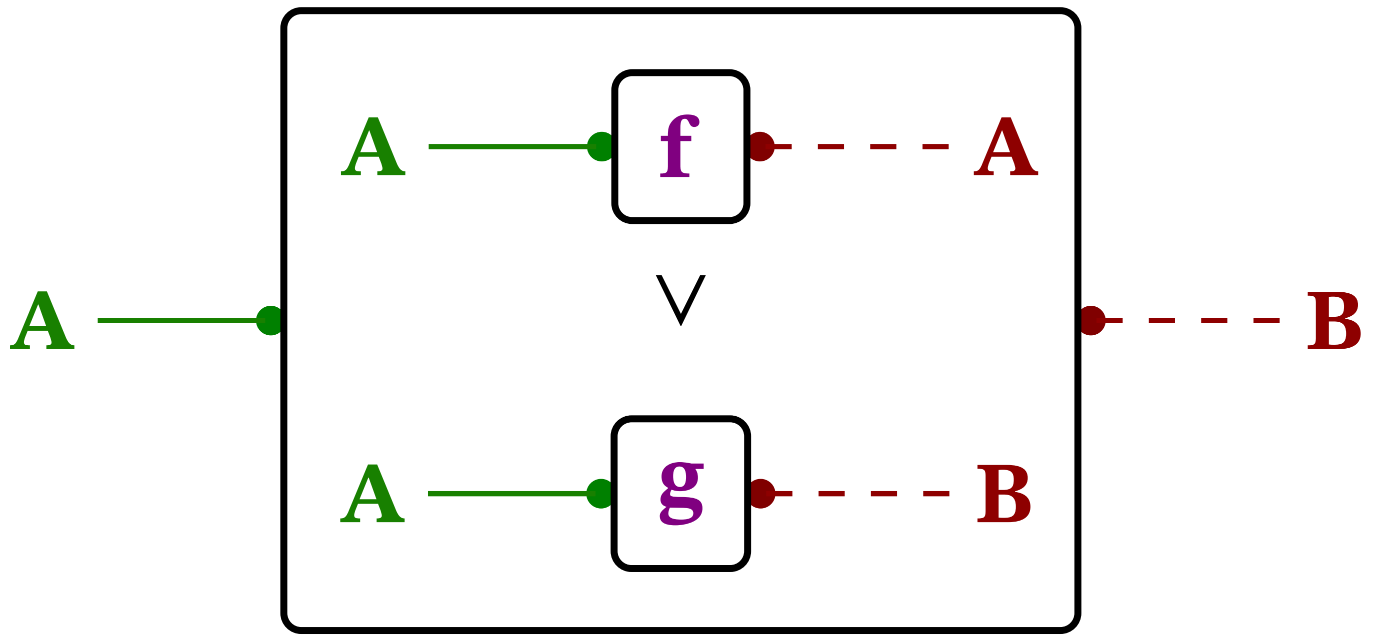


$$h_{\mathbf{f}\,;\,\mathbf{g}} : \mathbf{A} \to \mathsf{A}\,\mathbf{C}$$
$$a \mapsto \operatorname{Min}_{\preceq_{\mathbf{C}}} \bigcup_{s \in h_{\mathbf{f}}(a)} h_{\mathbf{g}}(s).$$

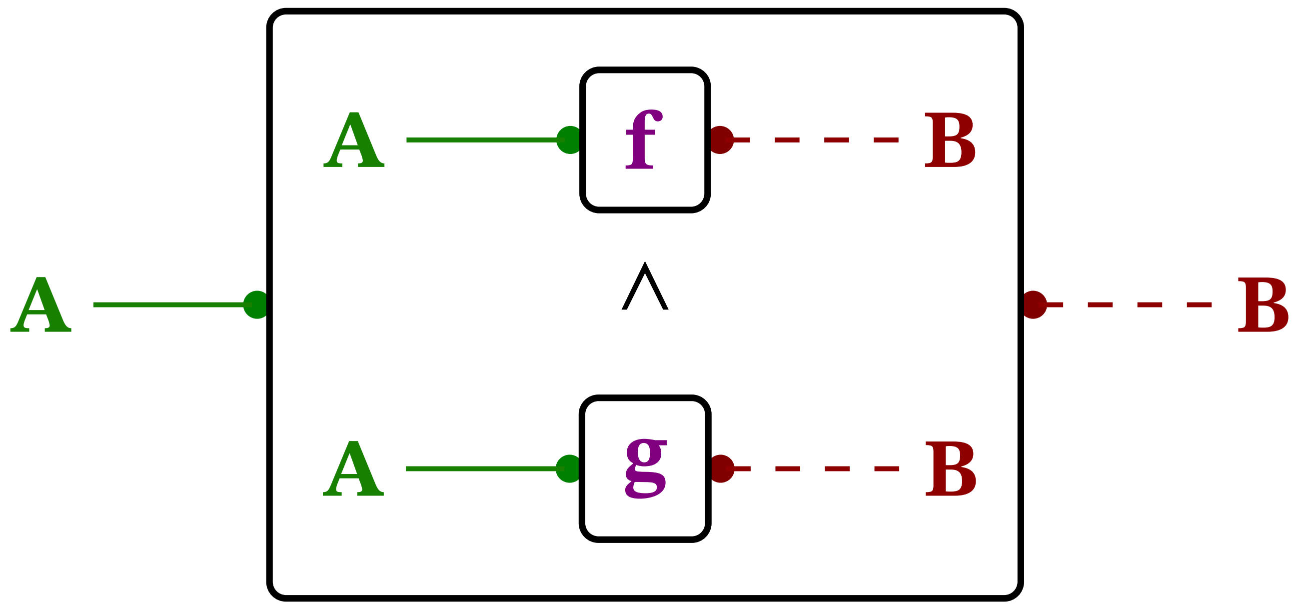


$$h_{\mathbf{f}} \otimes h_{\mathbf{g}} : (\mathbf{A} \times \mathbf{C}) \to \mathsf{A}\,(\mathbf{B} \times \mathbf{D}),$$
$$\langle a, c\rangle \mapsto h_{\mathbf{f}}(a) \times h_{\mathbf{g}}(c),$$

feedback is always the problem...

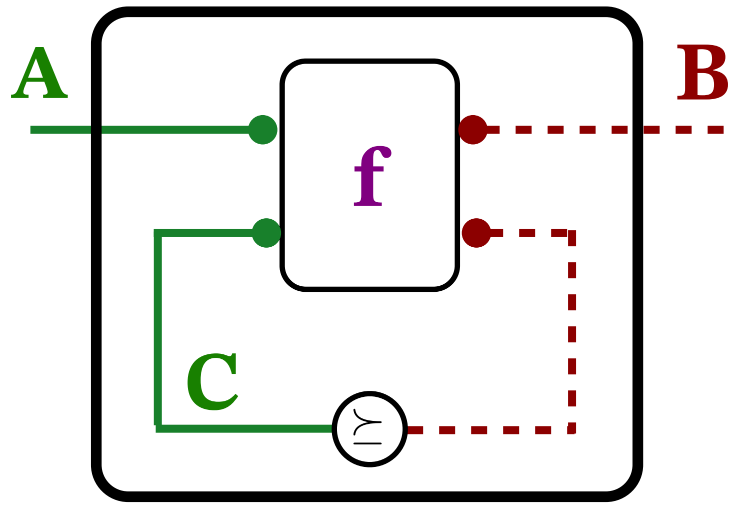


$$h_{\mathbf{f}} \vee h_{\mathbf{g}} : \mathbf{A} \to \mathsf{A}\,\mathbf{B},$$
$$a \mapsto \operatorname{Min}_{\preceq_{\mathbf{B}}} \left(h_{\mathbf{f}}(a) \cup h_{\mathbf{g}}(a)\right).$$

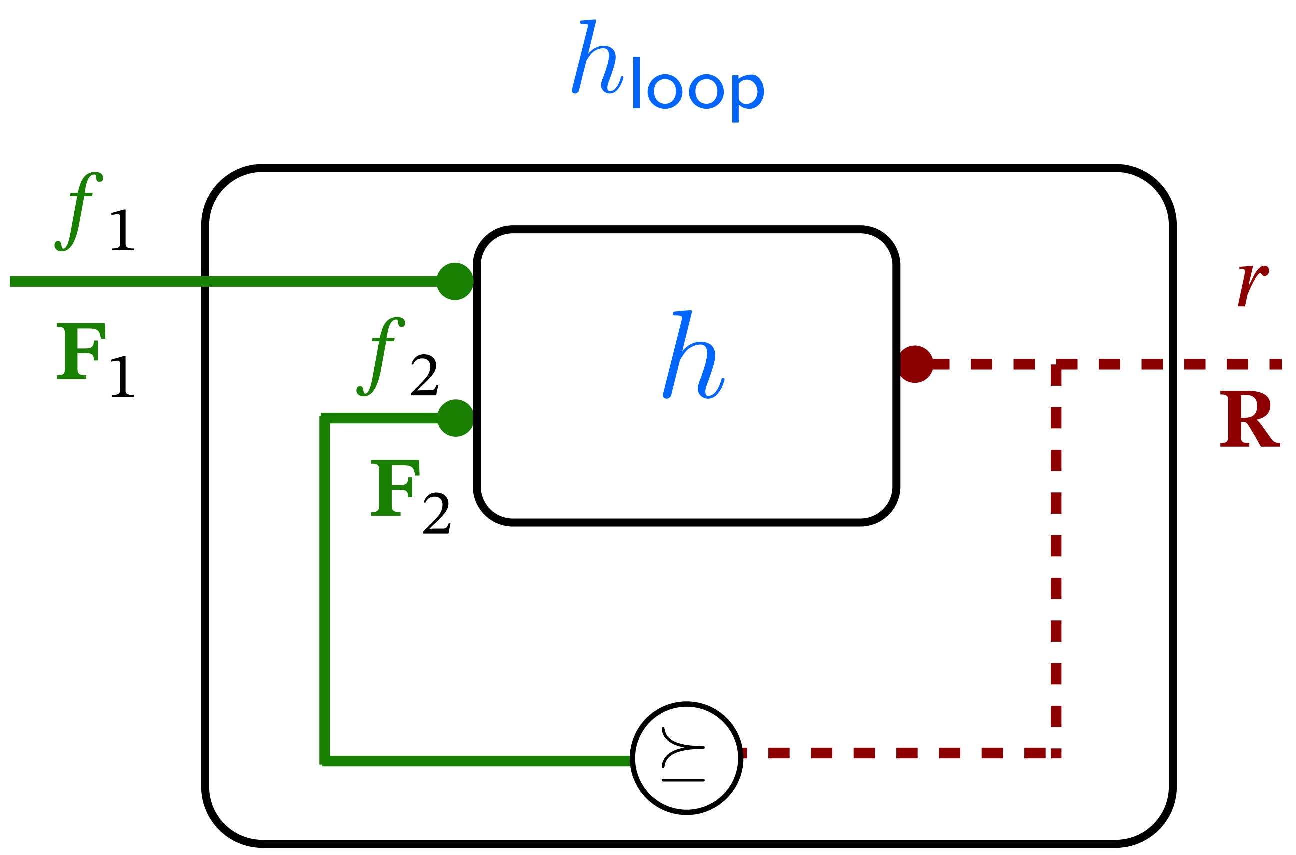


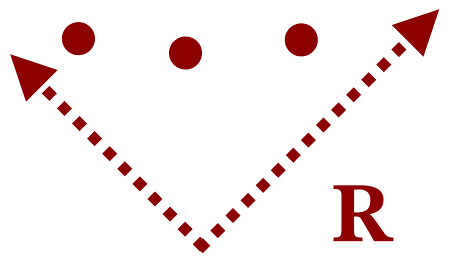
$$h_{\mathbf{f}} \wedge h_{\mathbf{g}} : \mathbf{A} \to \mathsf{A}\,\mathbf{B},$$
$$a \mapsto \operatorname{Min}_{\preceq_{\mathbf{B}}} \left(h_{\mathbf{f}}(a) \cap h_{\mathbf{g}}(a)\right).$$

"*feedback*"



Solution for feedback



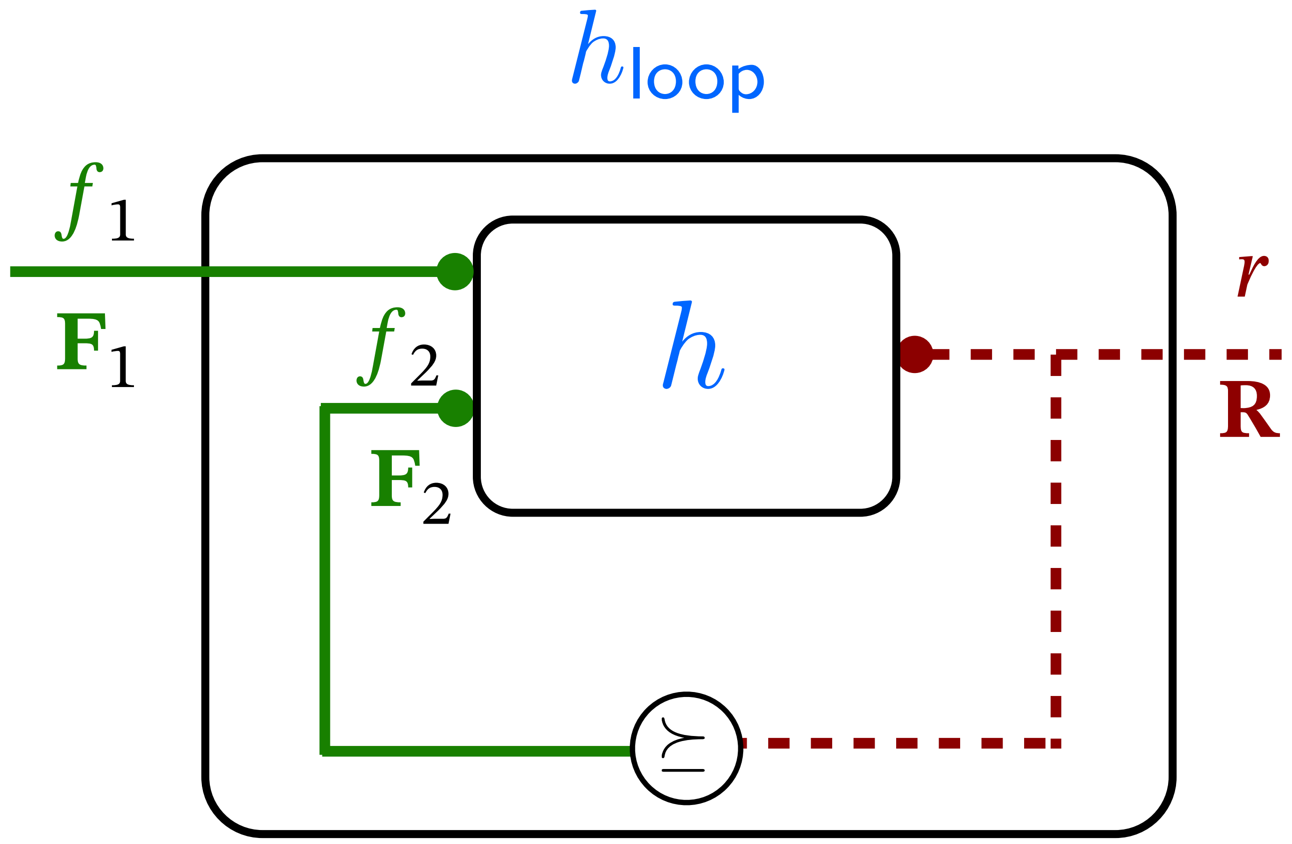


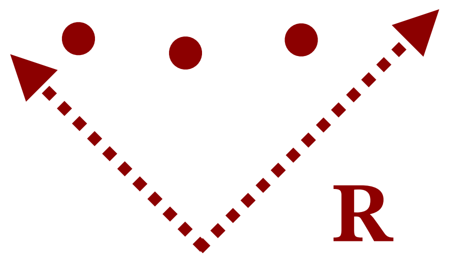
Theorem. The **set of minimal feasible resources** can be obtained as the least fixed point of a monotone function in the space of anti-chains.

$$h_{\text{loop}} : \mathbf{F}_1 \to \mathcal{A}(\mathbf{R})$$
$$f_1 \mapsto \underbrace{\text{least-fixed-point}(\Phi_{f_1})}$$

$$\Phi_{f_1} : \mathcal{A}(\mathbf{R}) \to \mathcal{A}(\mathbf{R})$$
$$S \mapsto \operatorname*{Min}_{\preceq_{\mathbf{R}}} \bigcup_{r \in S} h(f_1, r) \cap \uparrow r$$

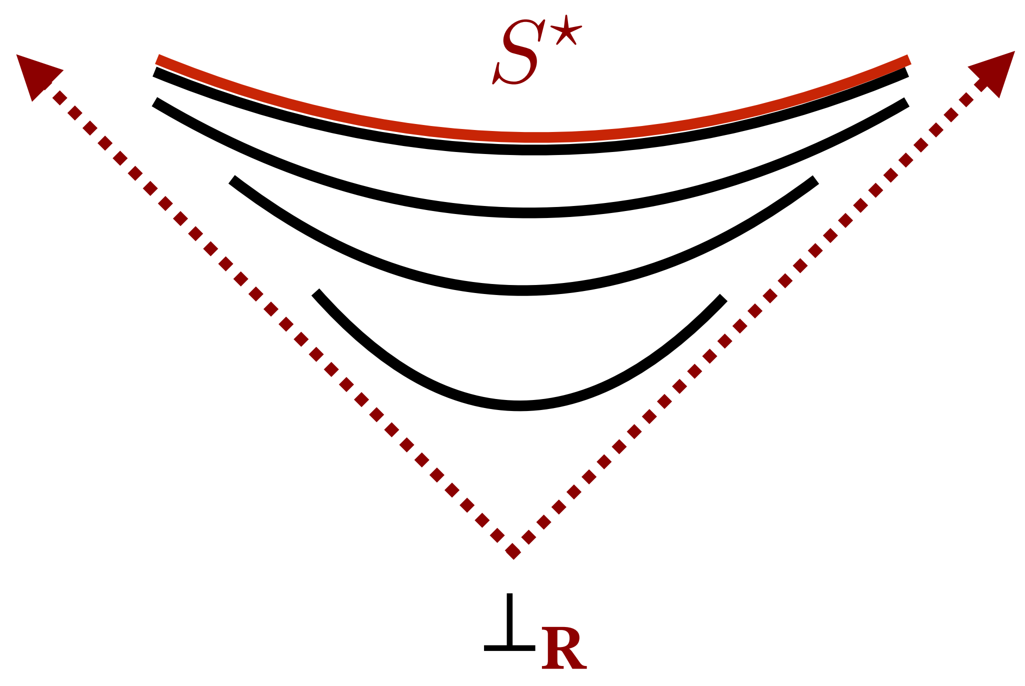
Solution for feedback





Corollary. The set of minimal solutions can be found using Kleene's algorithm.*

$$
\begin{aligned}
S &\subset \mathcal{A}(\mathbf{R}) \\
S_0 &= \{\perp_{\mathbf{R}}\} \\
S_{k+1} &= \Phi_{f_1}(S_k)
\end{aligned}
$$



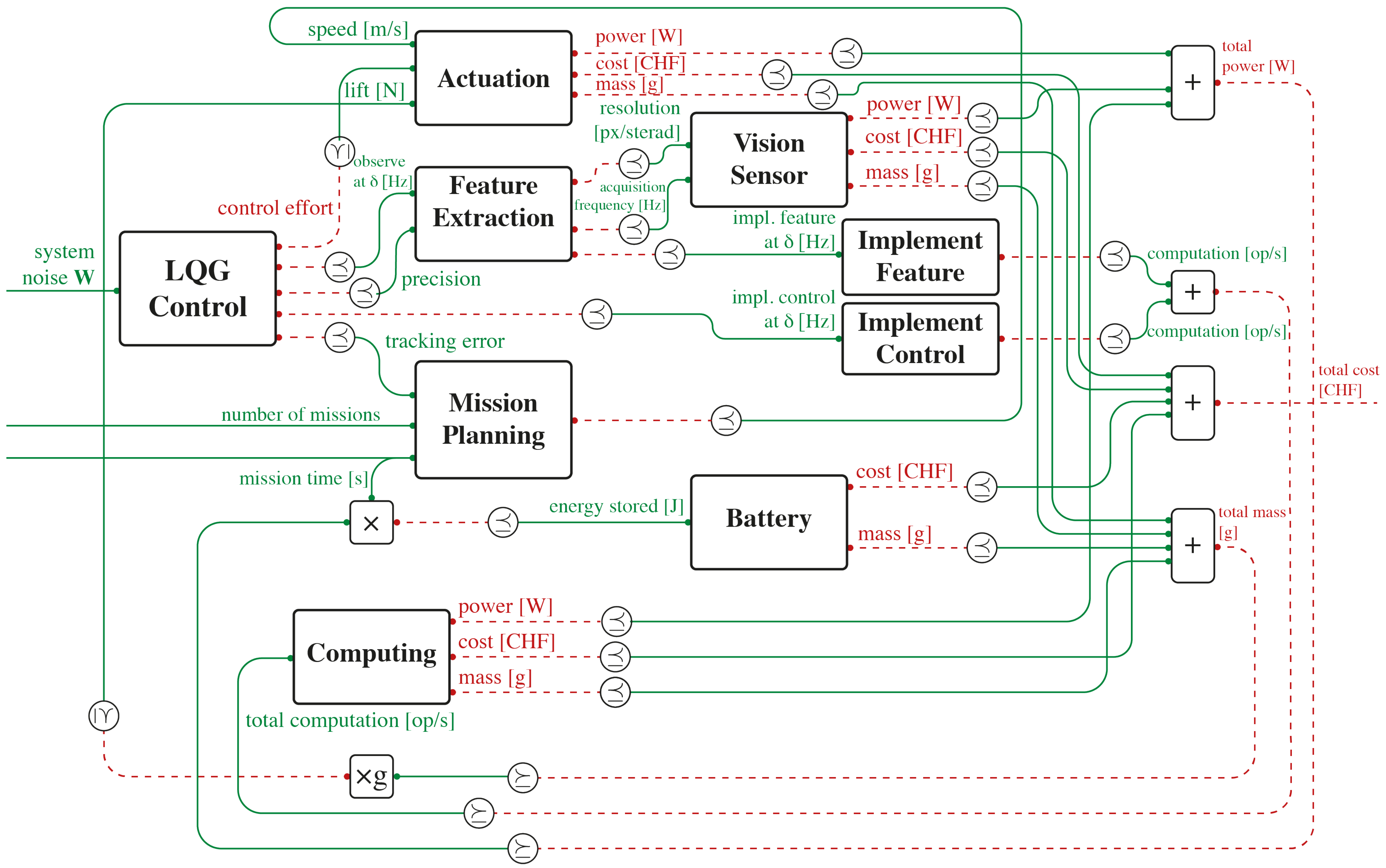
This always converges to the set of optimal solutions.
The sequence produced is a certificate of optimality/infeasibility.

A systematic process for task-driven co-design of complex systems

- A new approach to **co-design** designed to work **across fields** and **across scales**.
- What we have seen:
  - Defining **“design problems”** for **components.**
  - Modeling **co-design constraints** in a complex **system**.
  - **Efficient** solution to design queries.

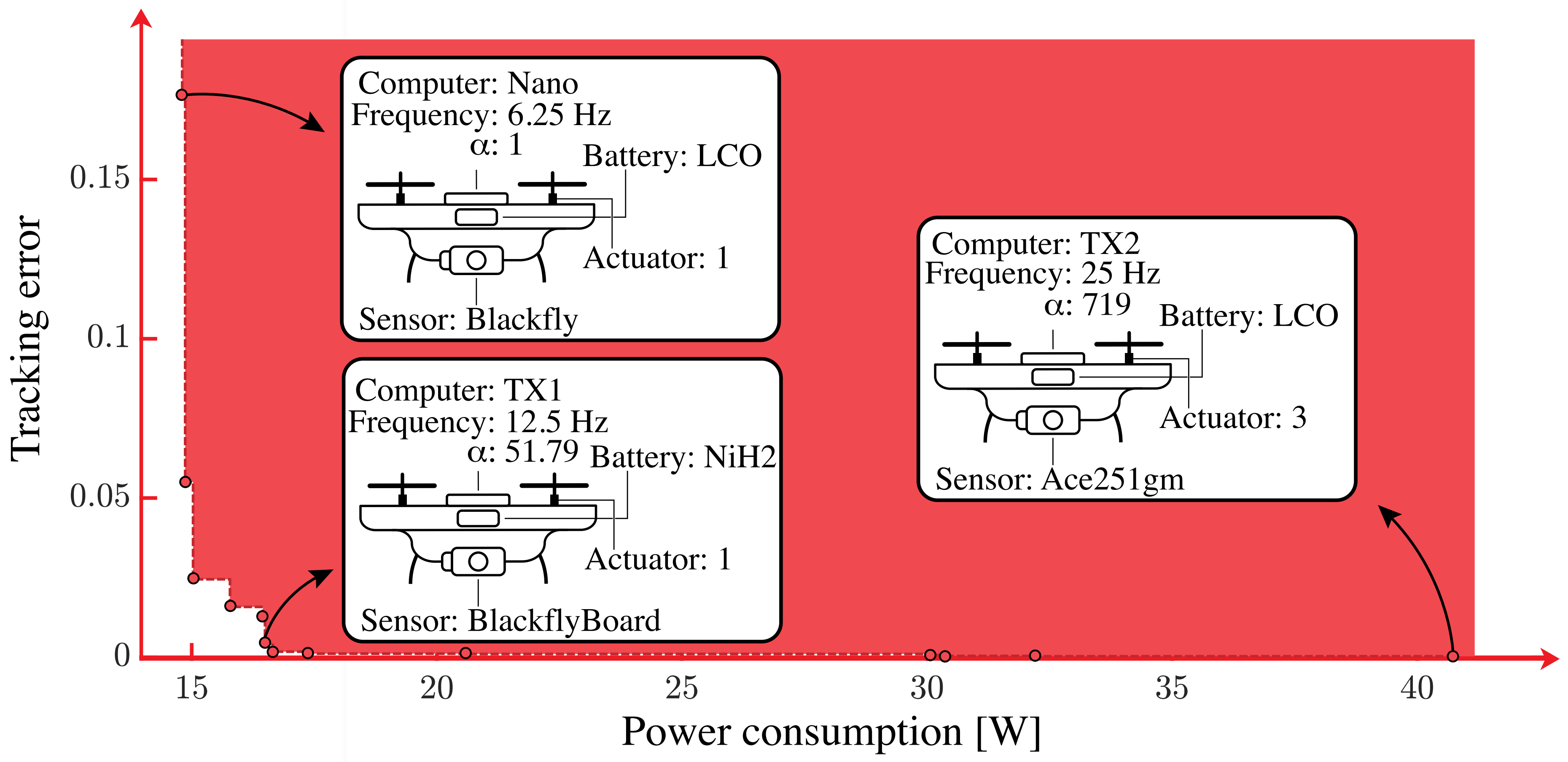
- **Modeling approach**
- **Implementation**
  - Coming up with the skeleton/diagram
  - Populating the models

“Co-design diagram”



optimization for a task

Pareto front of optimal designs



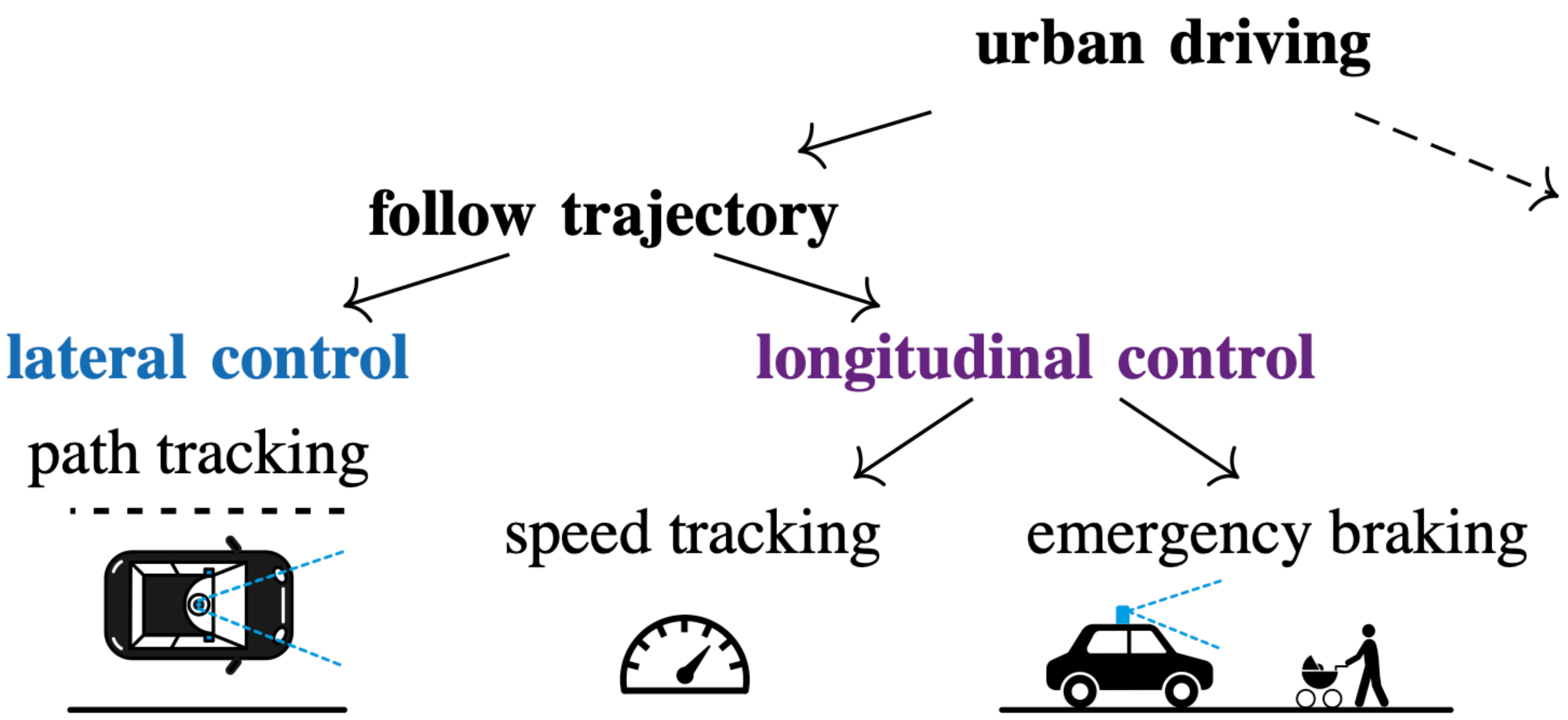
A systematic process for task-driven co-design of complex systems

- **Systematic modeling approach:**
  - **Define the task -** *what do we need to do?*

urban driving

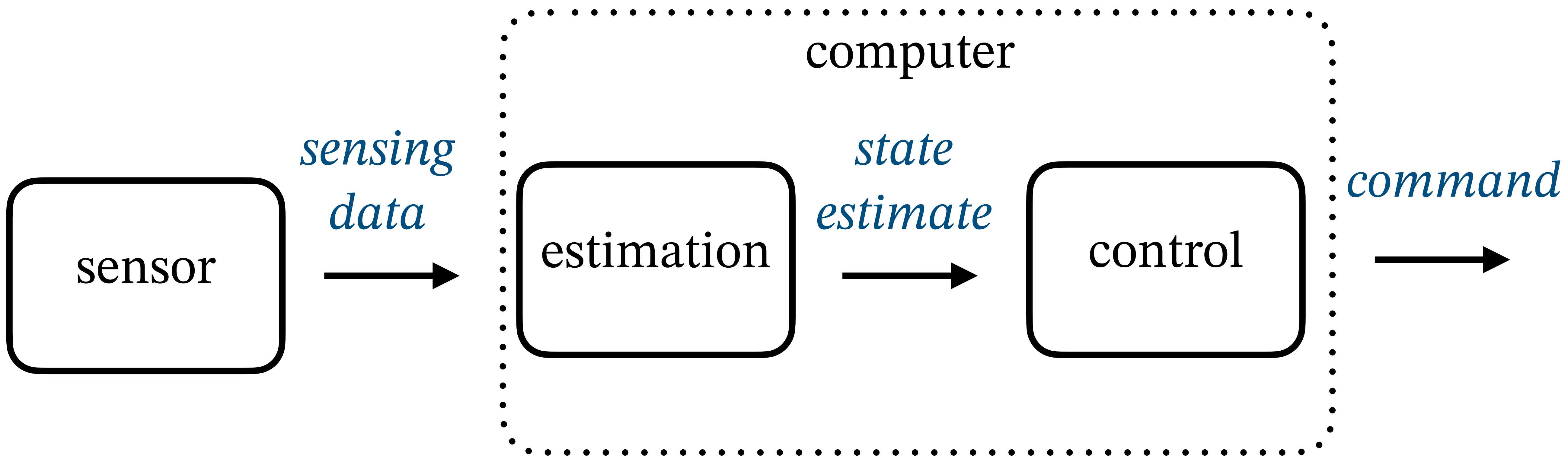
A systematic process for task-driven co-design of complex systems

- **Systematic modeling approach:**
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*

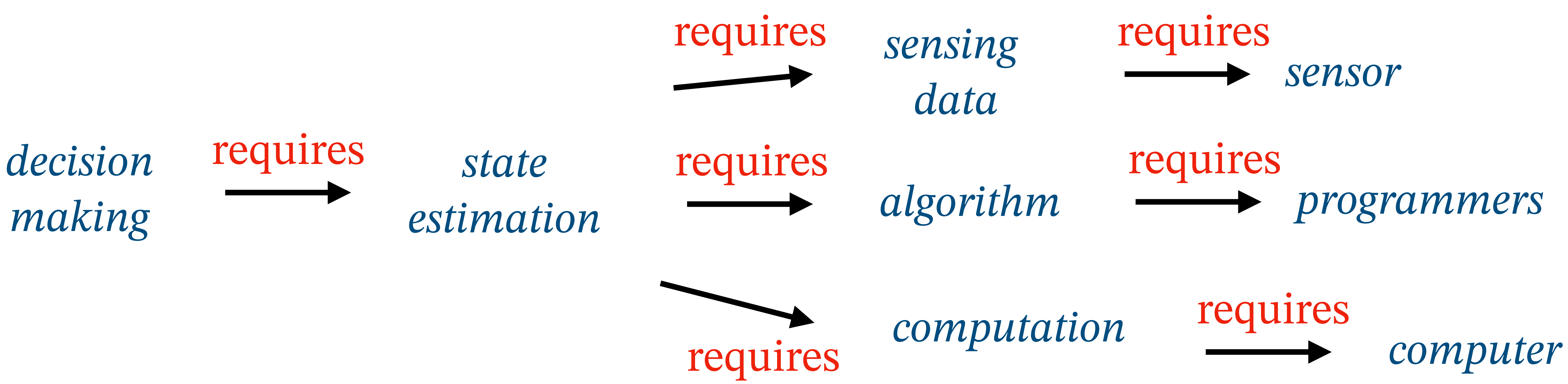


A systematic process for task-driven co-design of complex systems

- **Systematic modeling approach:**
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*
  - **Find components** - *decompose until you find components* (hardware and software)



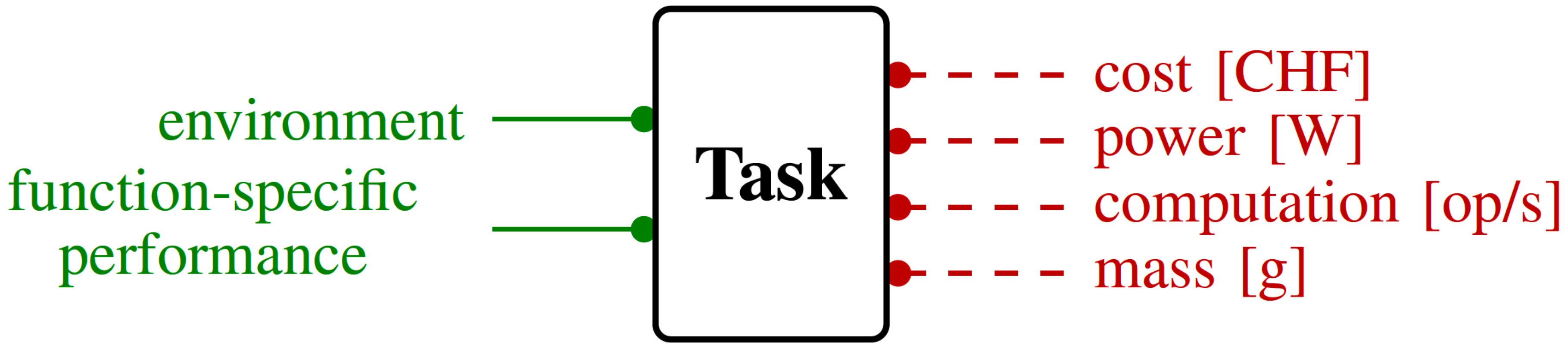
Data/Information flow

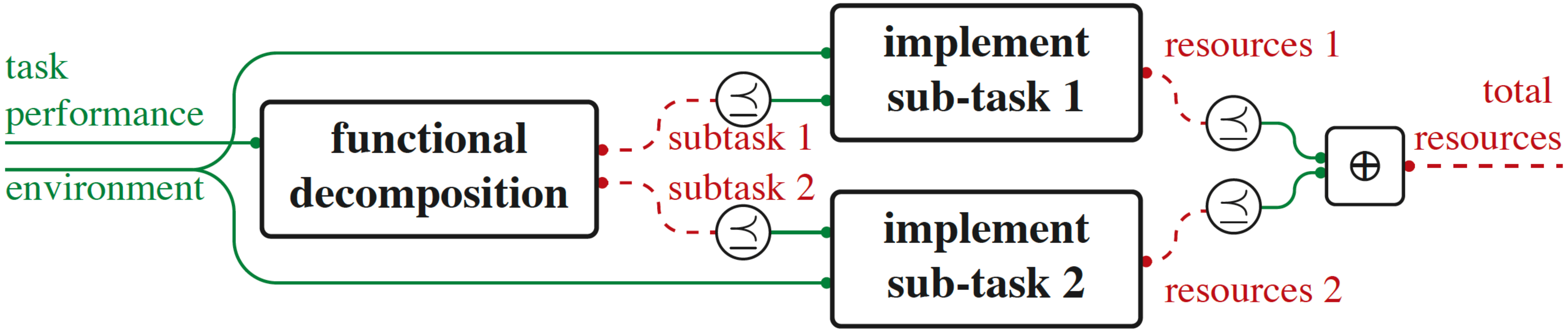


Logical dependencies

A systematic process for task-driven co-design of complex systems

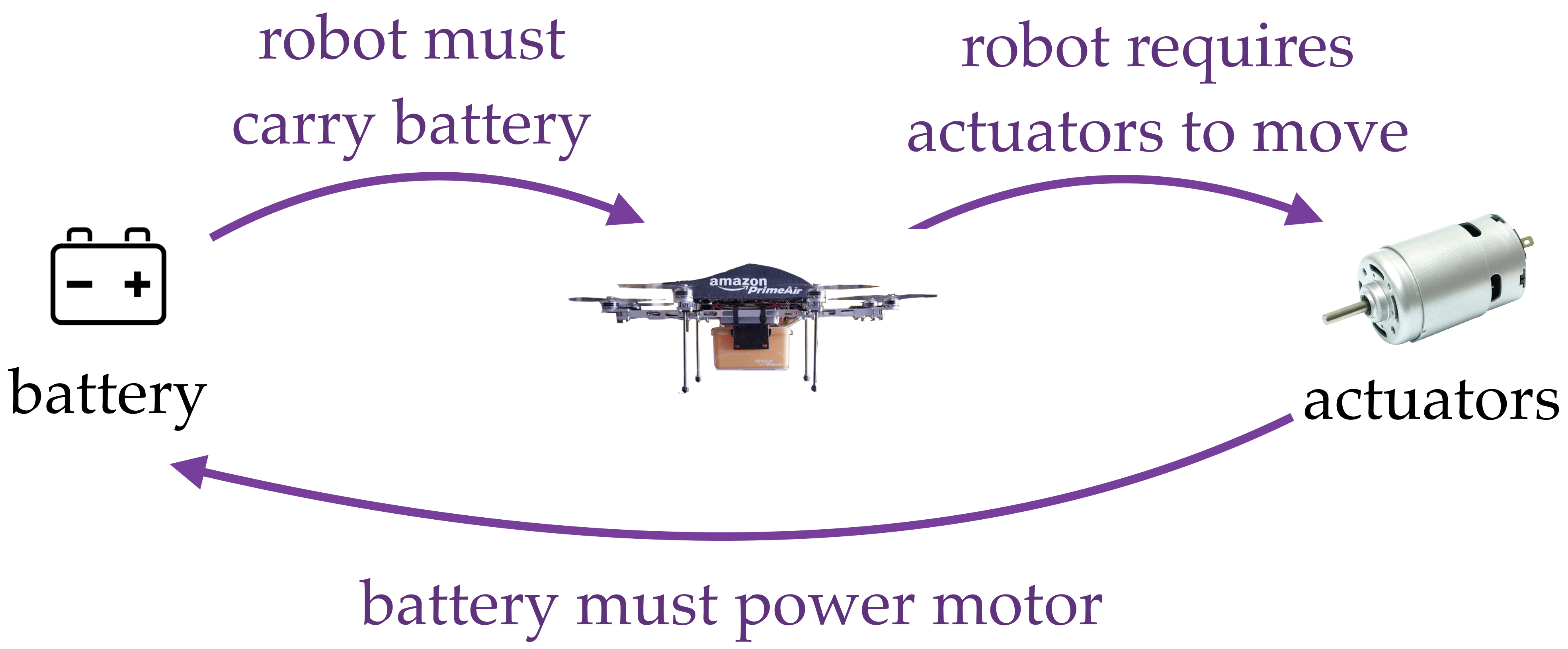
- **Systematic modeling approach:**
  - **Define the task** - *what do we need to do?*
  - **Functional decomposition** - *how to decompose the functionality?*
  - **Find components** - *decompose until you find components* (hardware and software)
  - **Find common resources** and **add** them
    In autonomy, **size**, **cost**, **weight**, **power**, **computation**

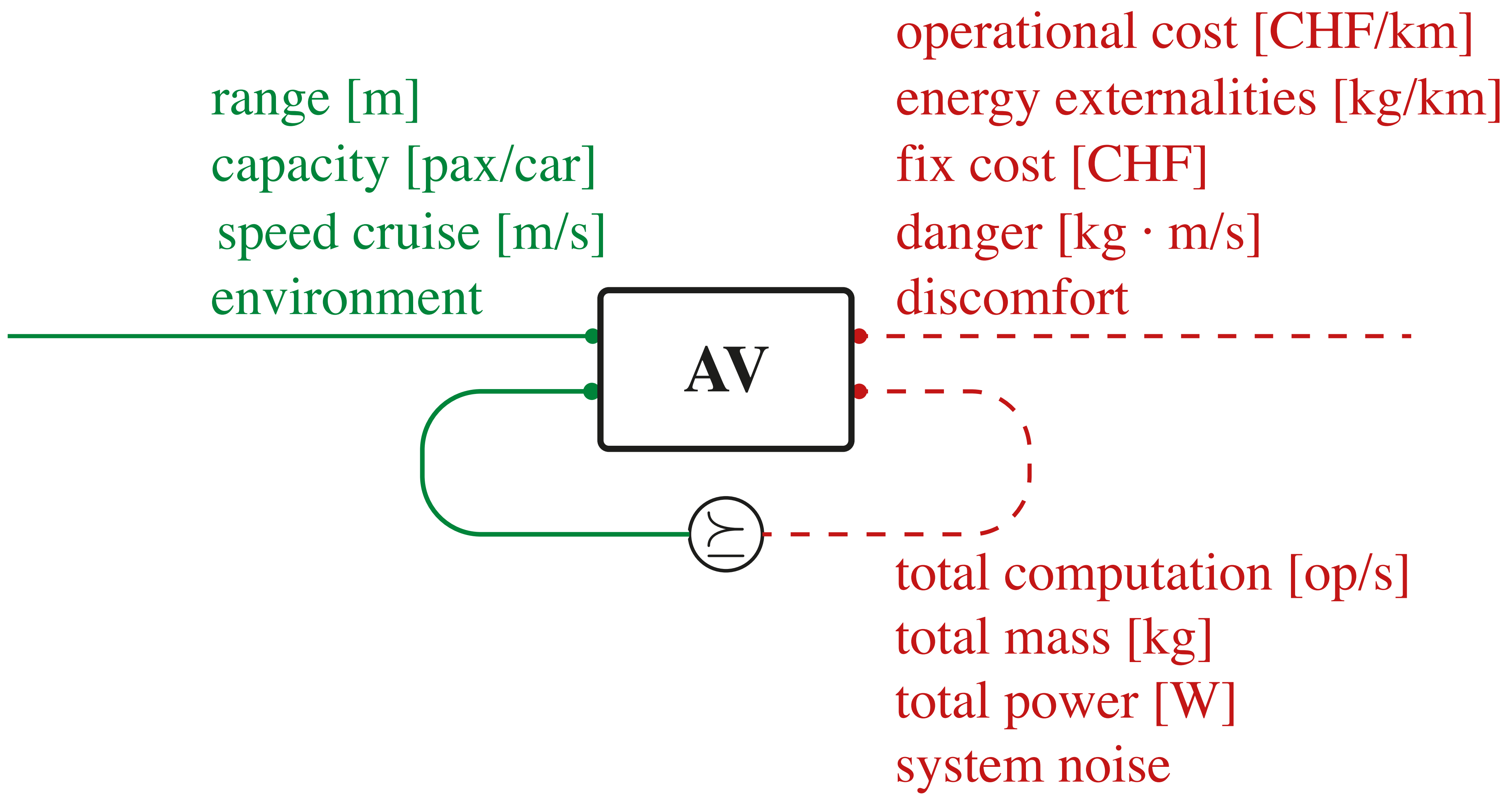


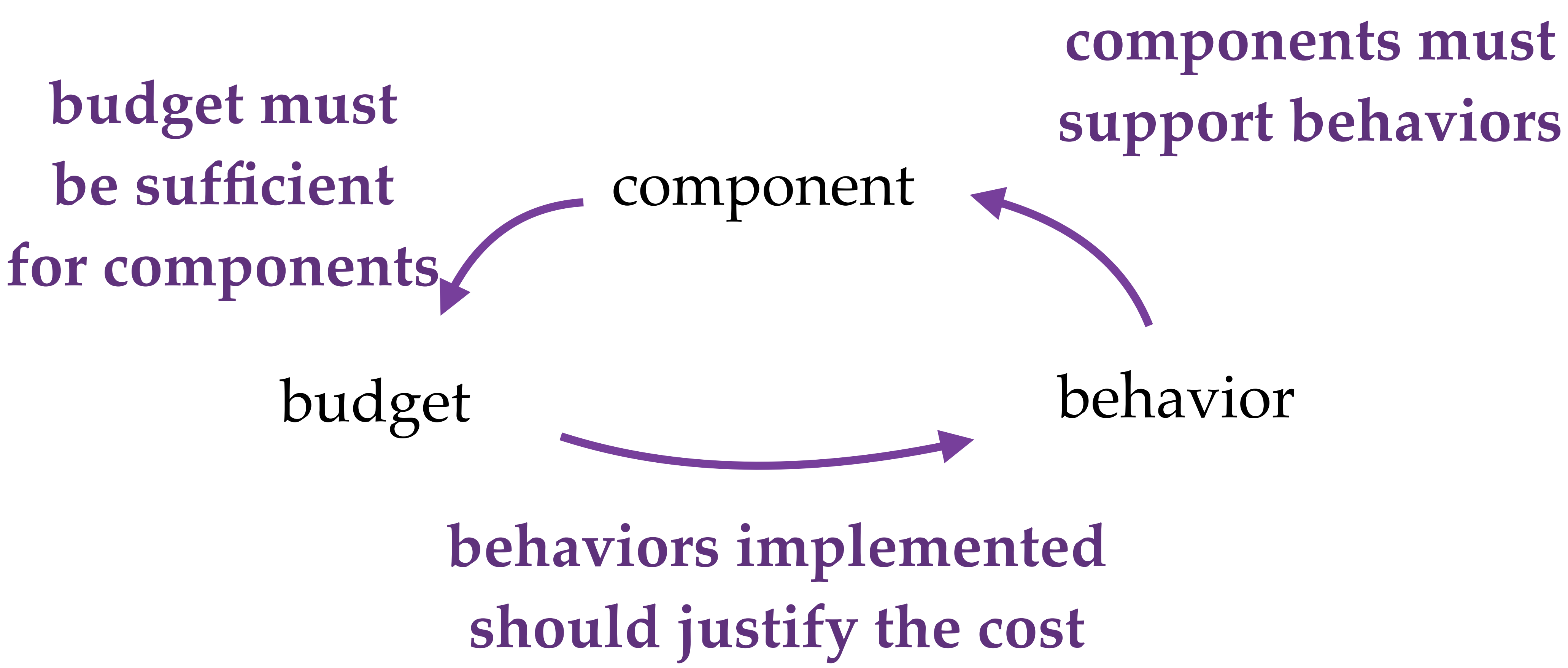


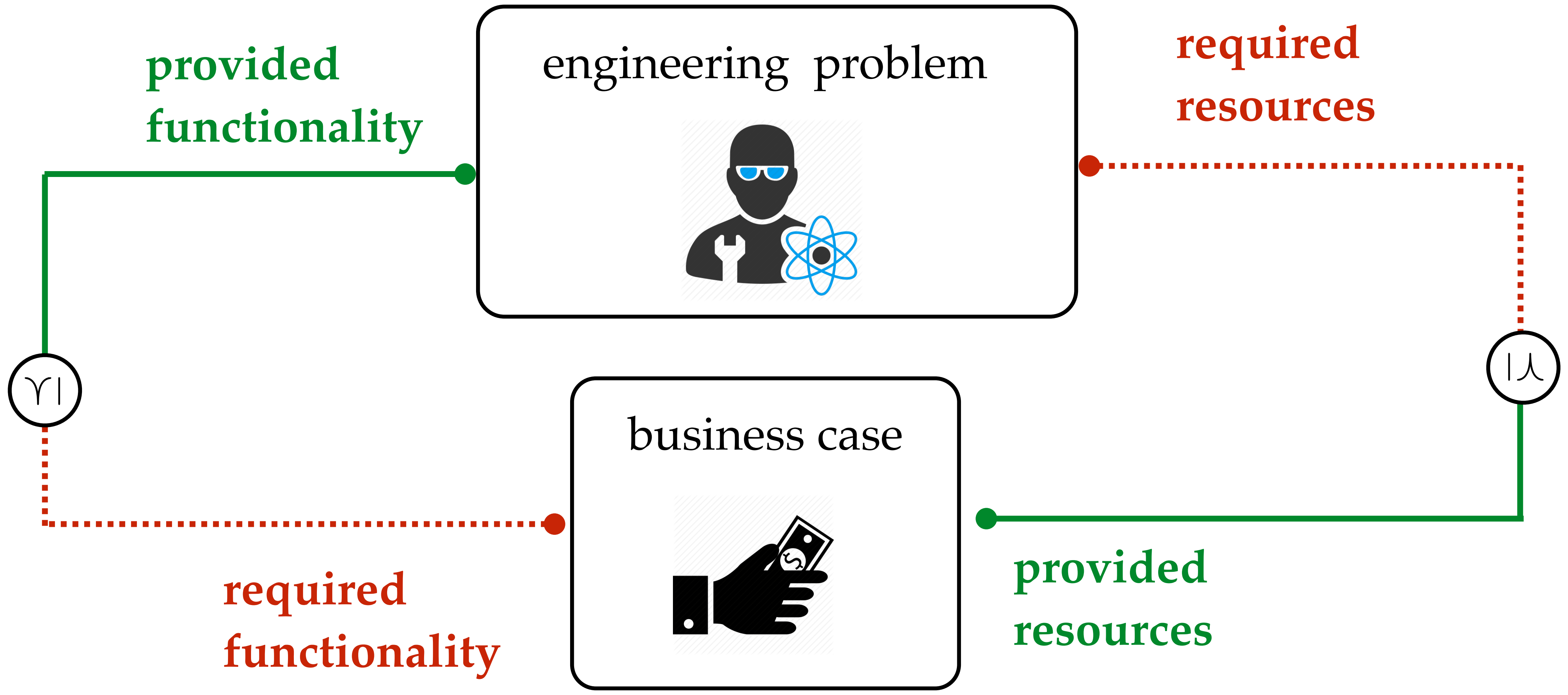
Feedback as the irreducible complexity of system design

- Where is **feedback**? In the **co-design constraints**





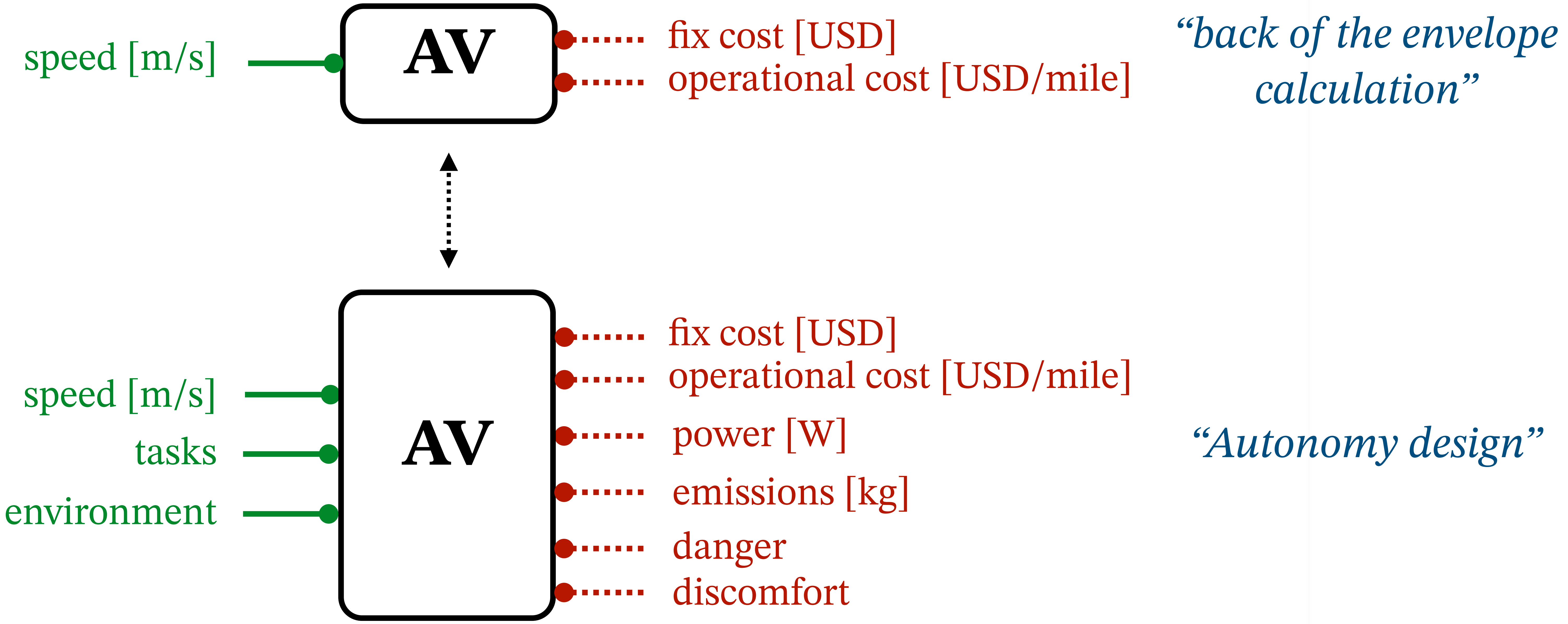




A systematic process for task-driven co-design of complex systems

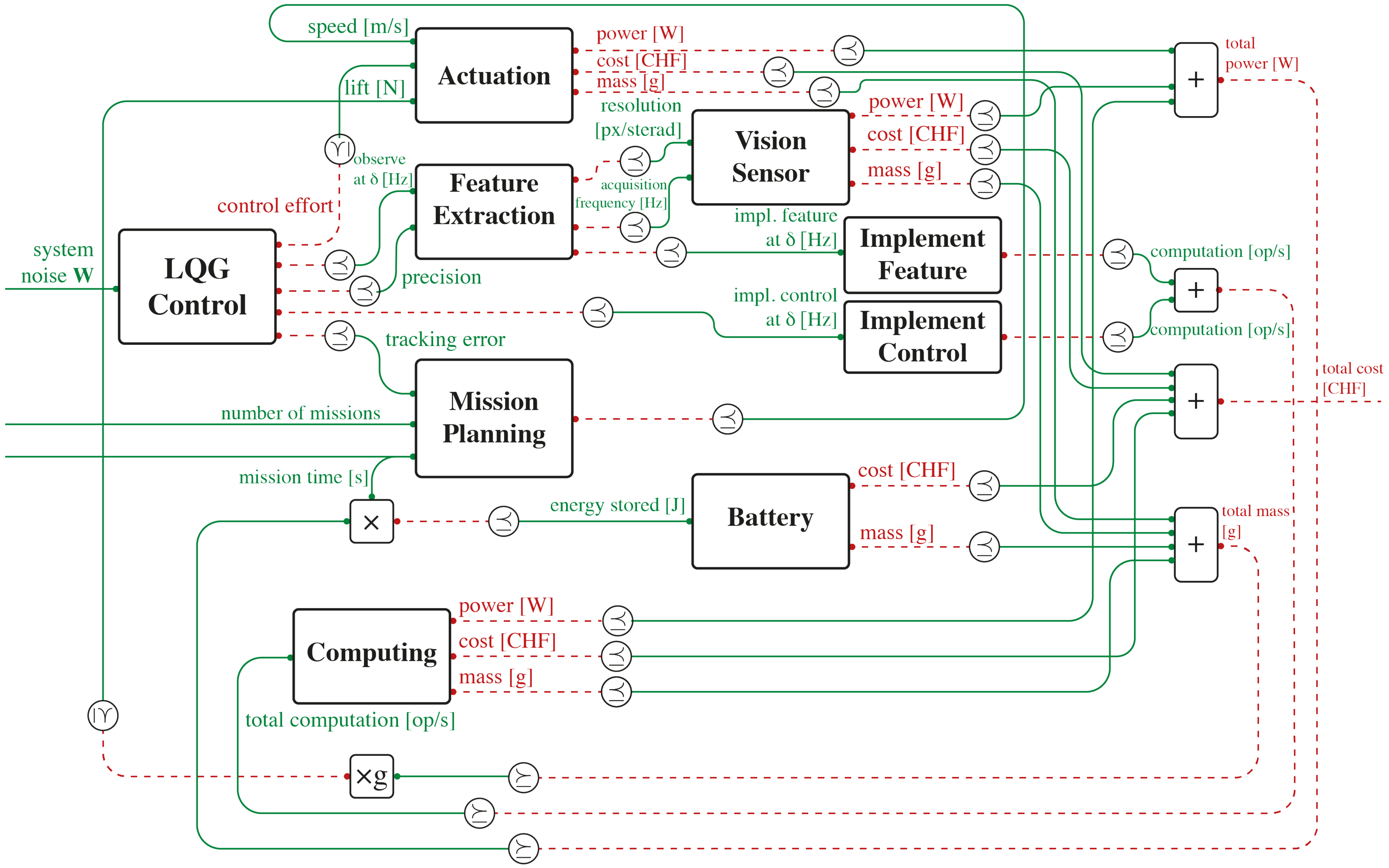
- **Implementation:**
  - **Write a skeleton** - *write the structure using the formal language and the* ***logical*** *dependencies.*

Context informs **level of detail**:



A systematic process for task-driven co-design of complex systems

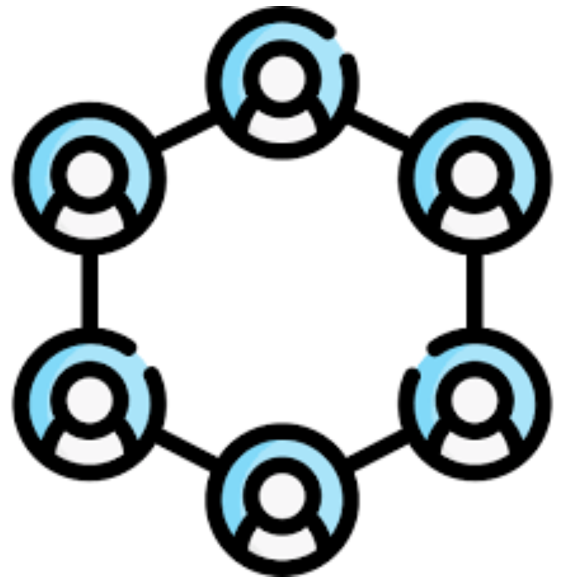
- **Implementation:**
  - **Populate the models:**
    catalogues, analytic models, data-driven



Continuous
Collaborative
Intellectually tractable

If unsure about a component, easy to embed ***assumptions***

Technologies don't need to exist already - parametric with ***time***



Decentralized - ***humans*** *in the loop*

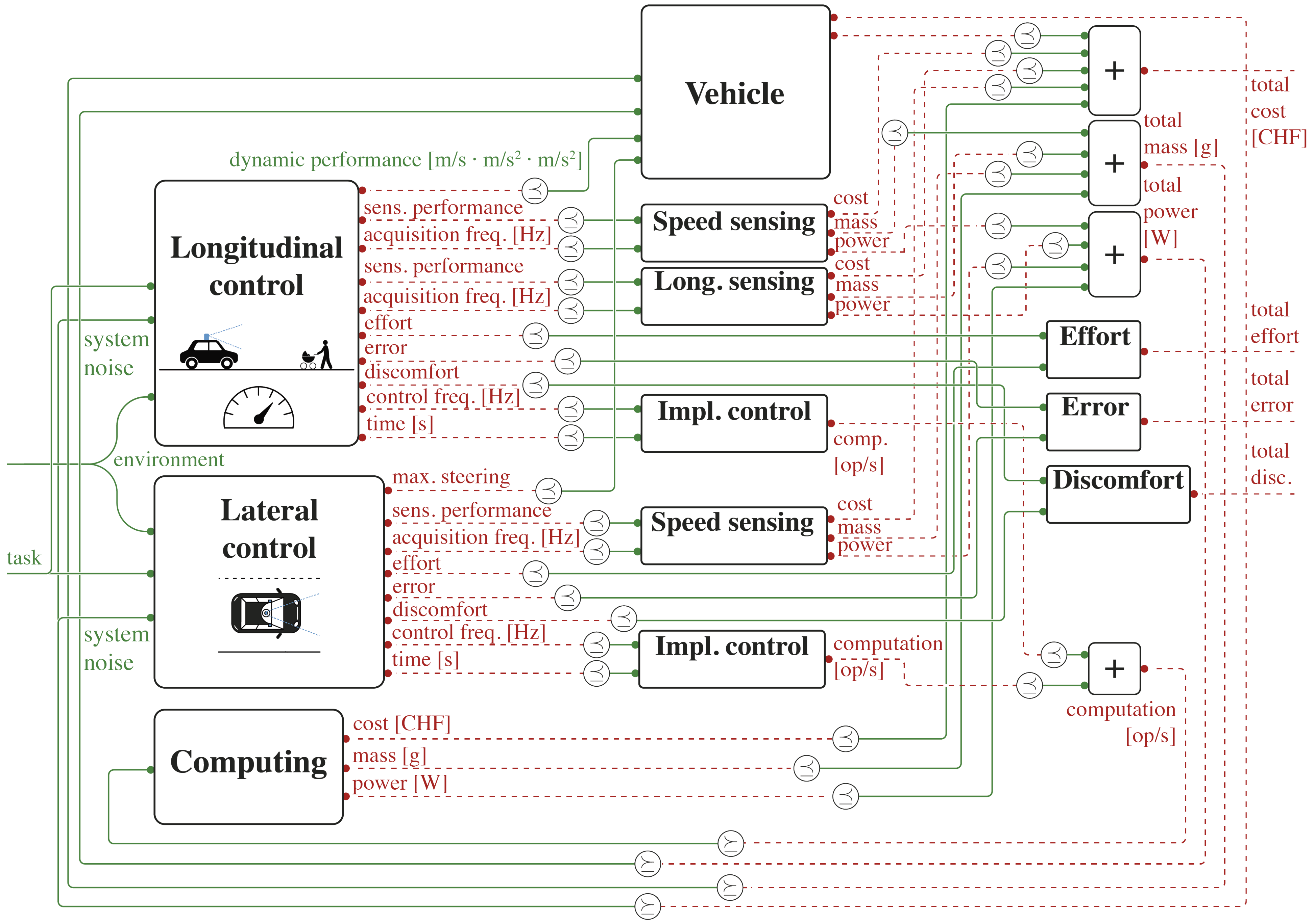
Task-driven co-design of an autonomous vehicle

[CDC'22, IROS'21, ECC'21]

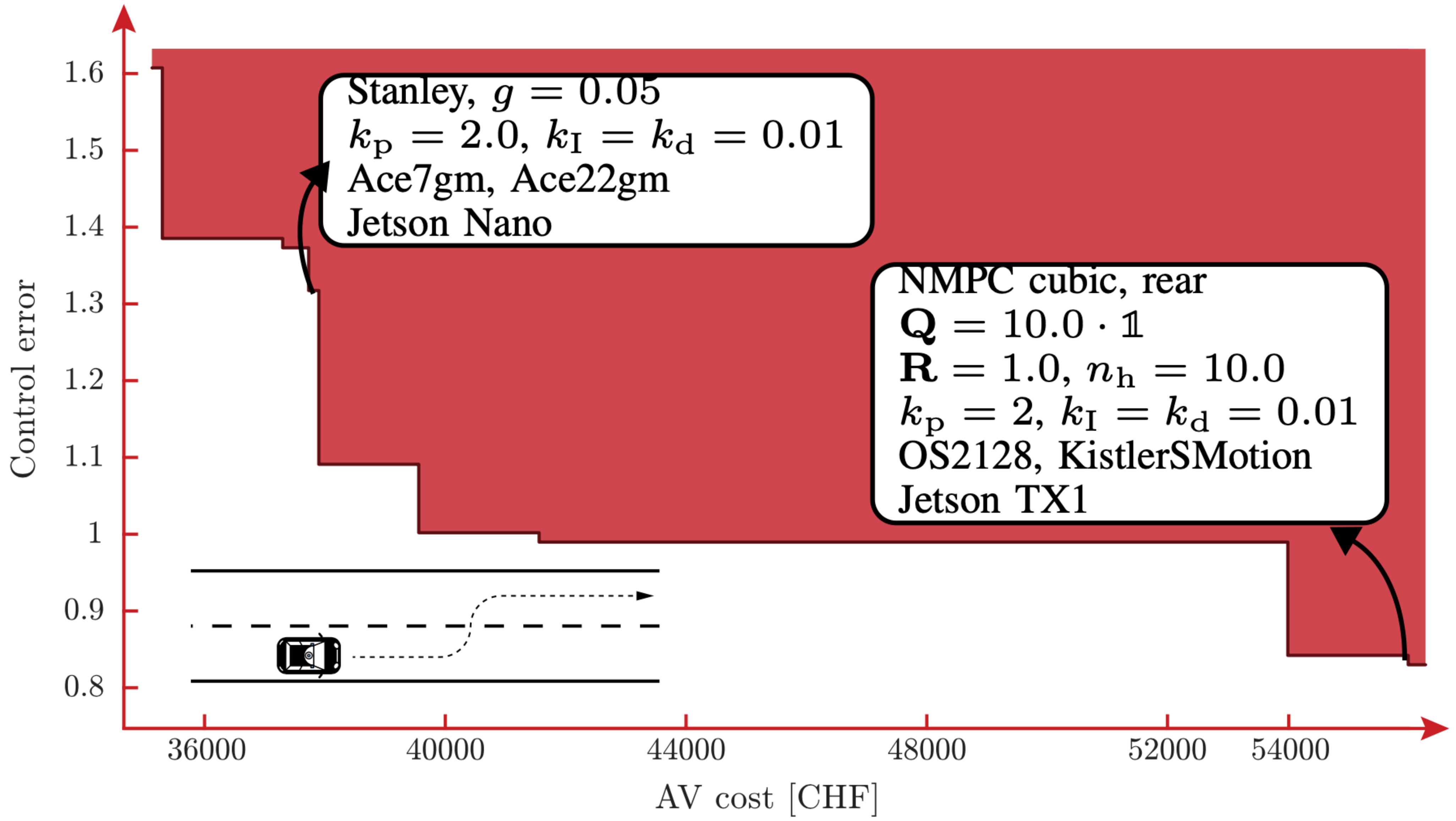
Task-driven co-design of an autonomous vehicle

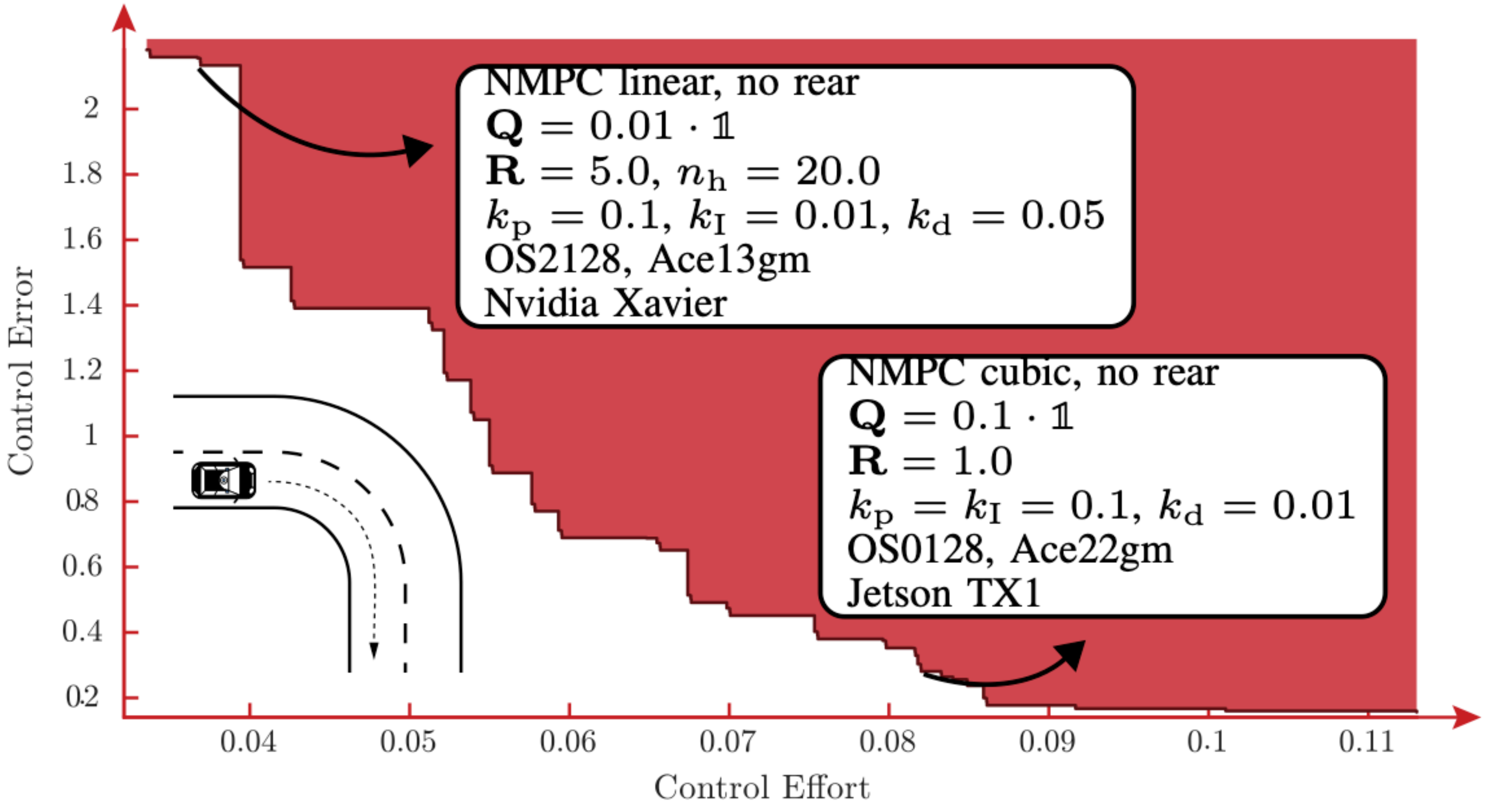
[CDC’22, IROS’21, ECC’21]

We can find optimal designs, with insights at heterogeneous abstraction levels

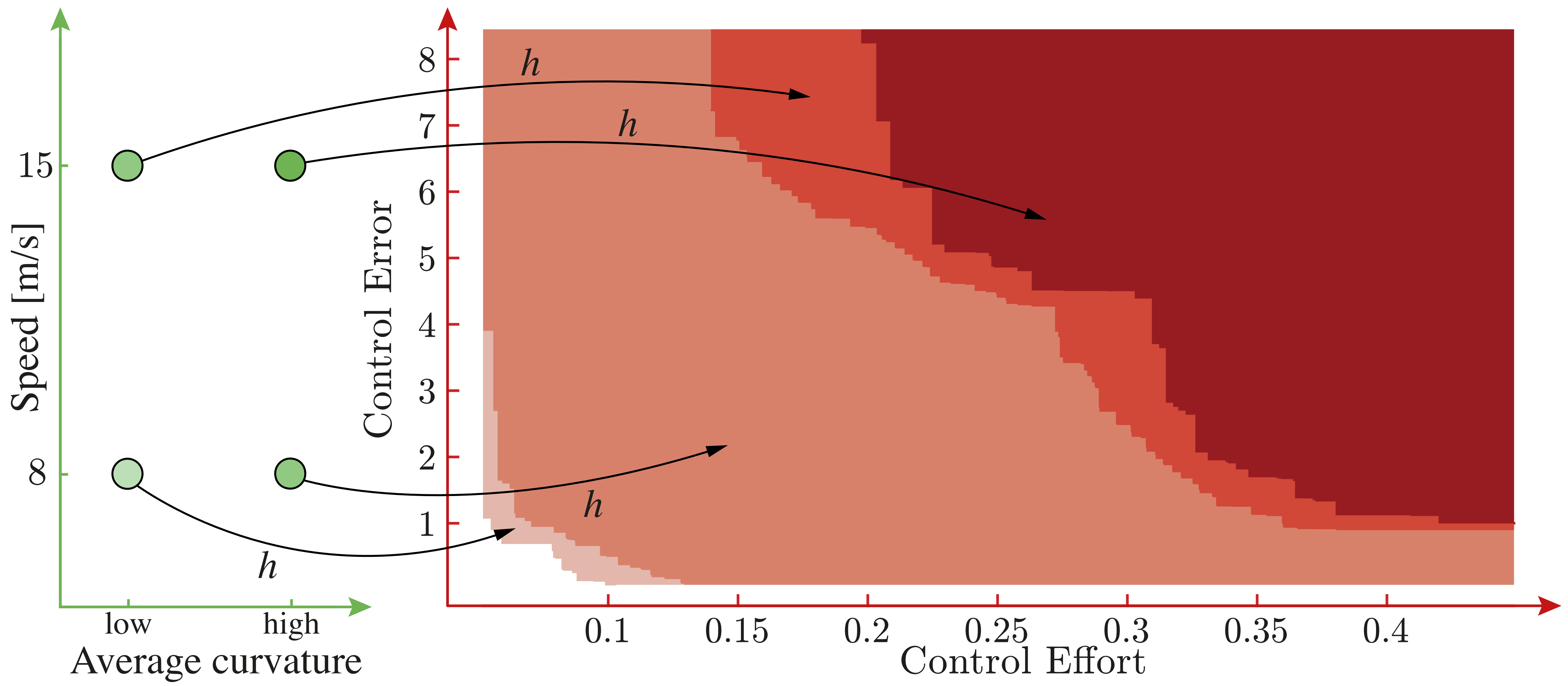


Fix an environment
Fix a task





Monotonicity in task complexity

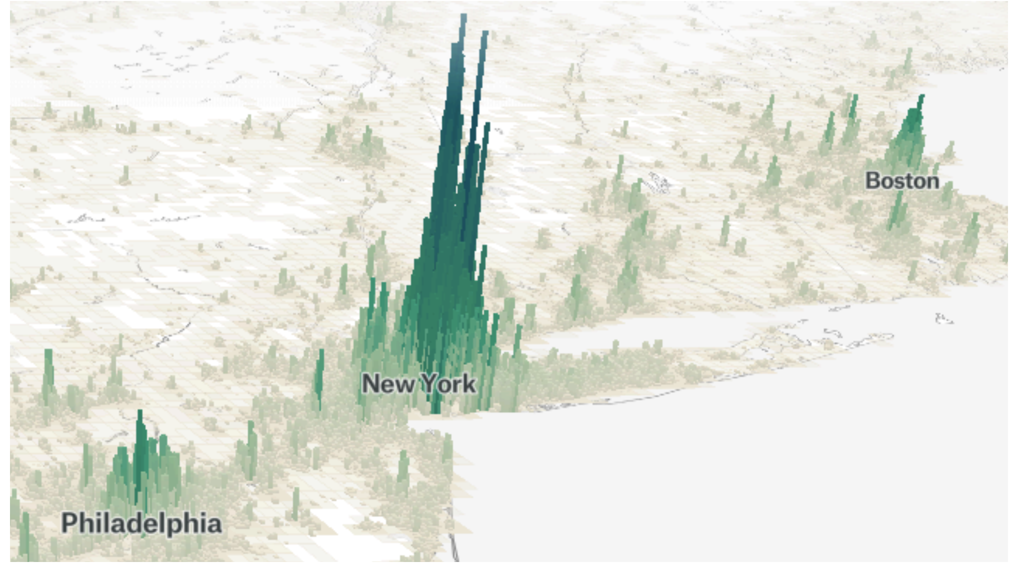


[CDC'22, IROS'21, ECC'21]

Co-design across scales: from autonomy to mobility systems

- Mobility systems are **under pressure**

Travel demand is changing
By 2050, 68% of population in cities



Need for **service design** and **regulations**
Over 1,000% ride-hailing increase in 2012-22

Need to meet **sustainability goals**
Cities cause 60% of GHGs, 30% from mobility

- We look at the problem from the perspective of **municipalities** and **policy makers**

How many vehicles should we allow?

Which infrastructure investments?

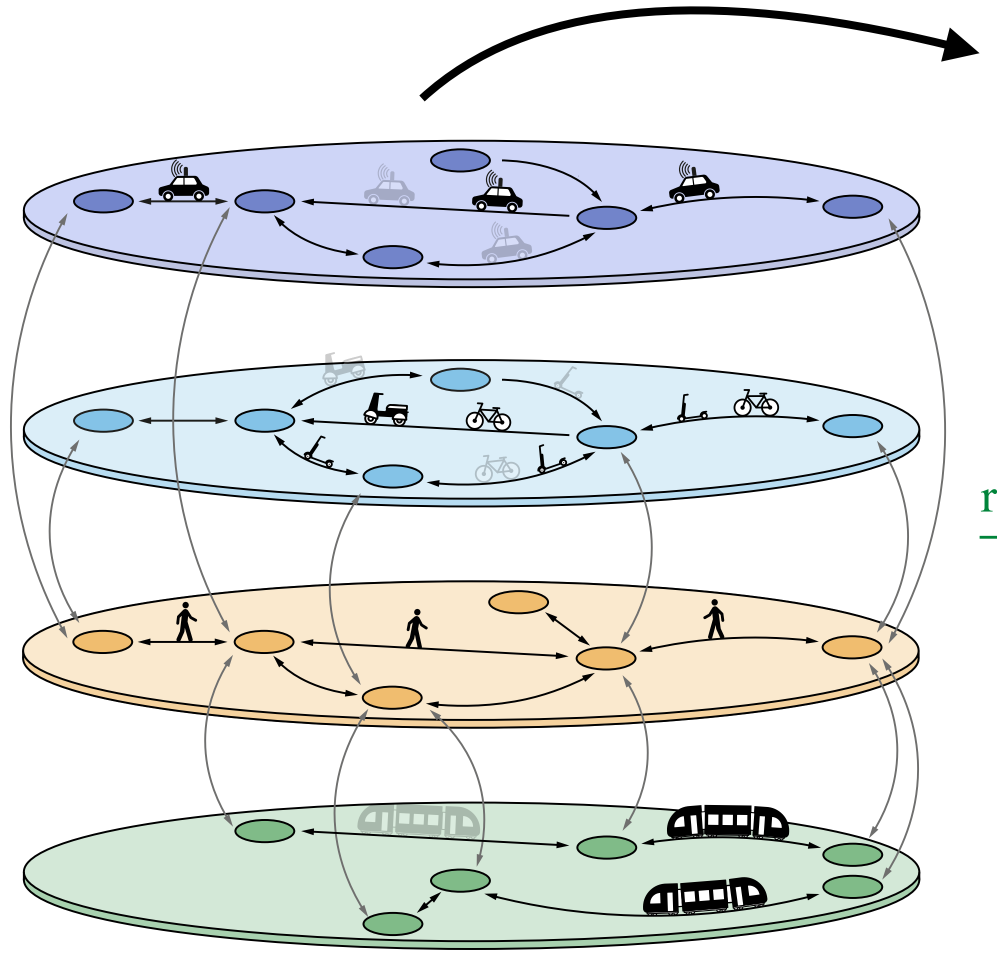
How performant?

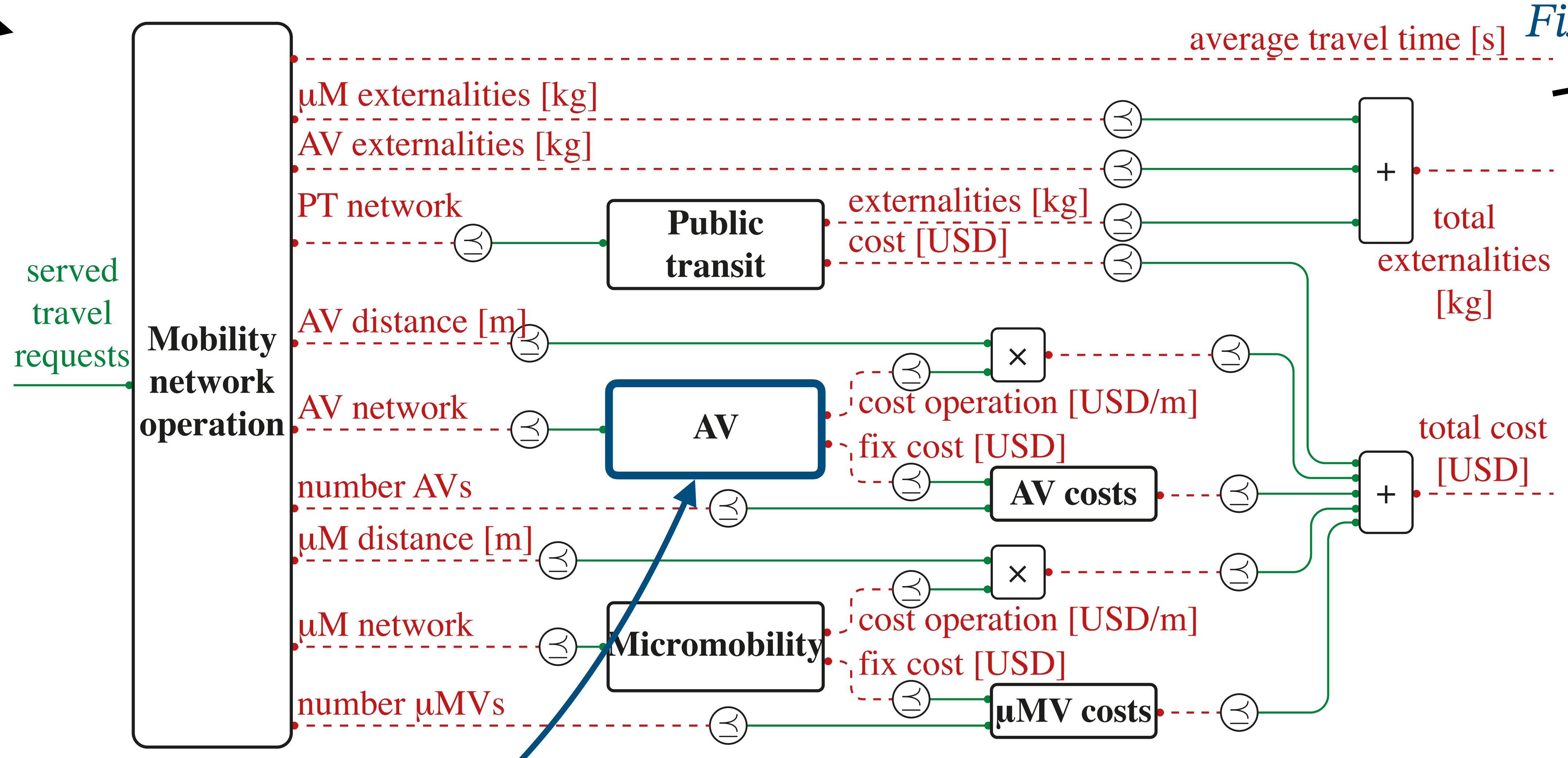
Which services to encourage?

- Need for **demand-driven** co-design of **mobility solutions** and the **intermodal network** they enable

- Several **disciplines** involved (transportation science, autonomy, economics, policy-making)

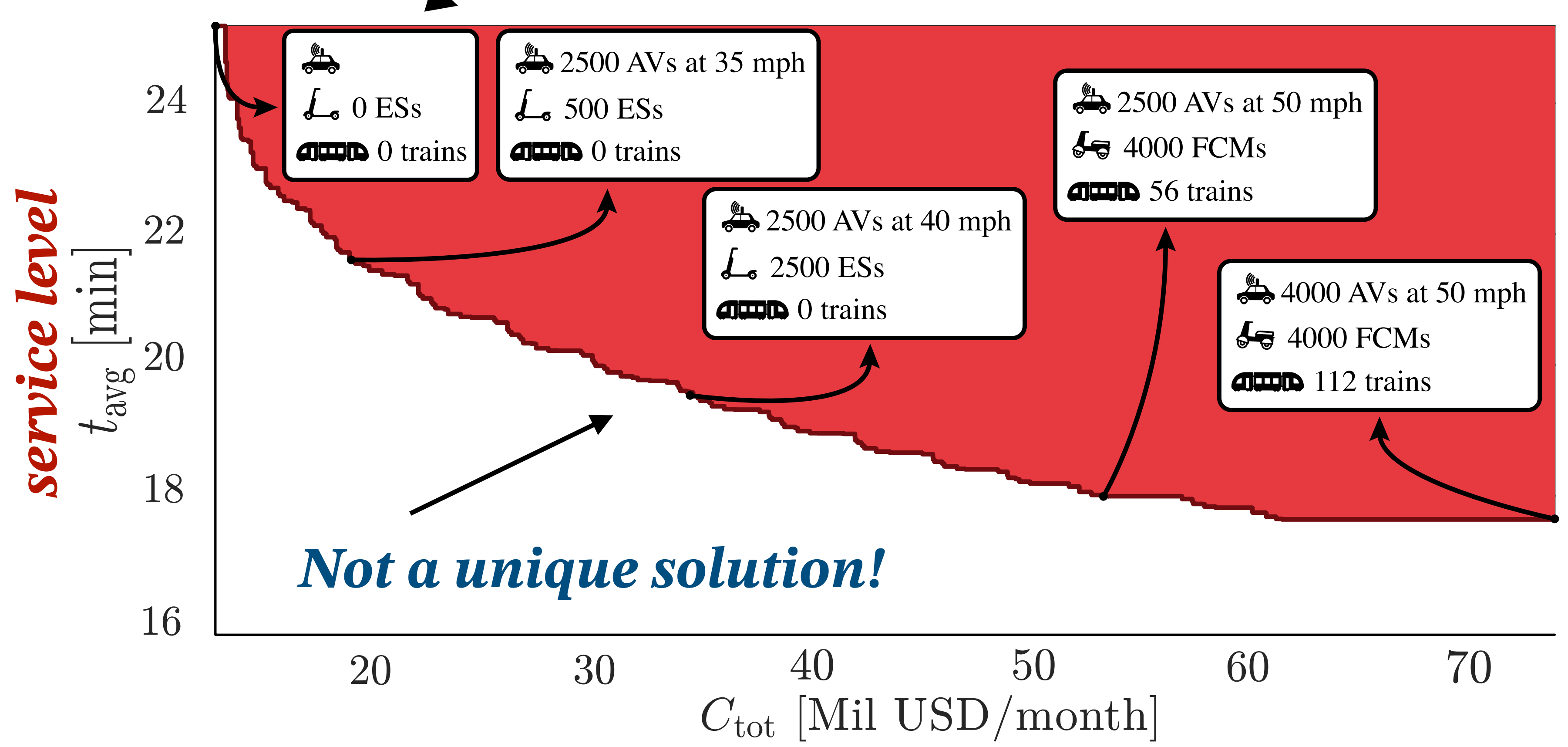
Co-design to enable user-friendly tools to assess the impact of future mobility solutions





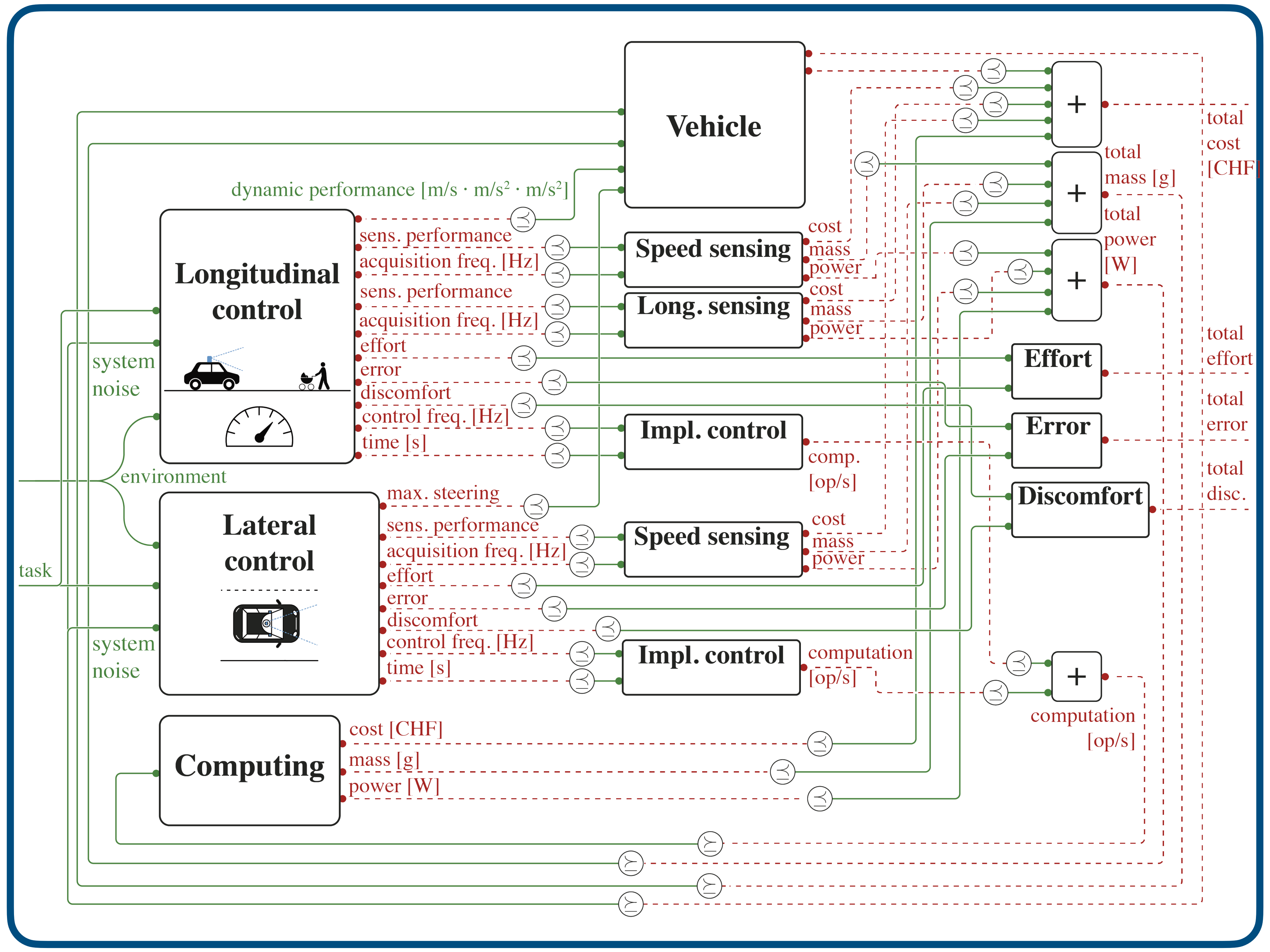
Fix demand, minimize costs

Washington D.C. in 2030

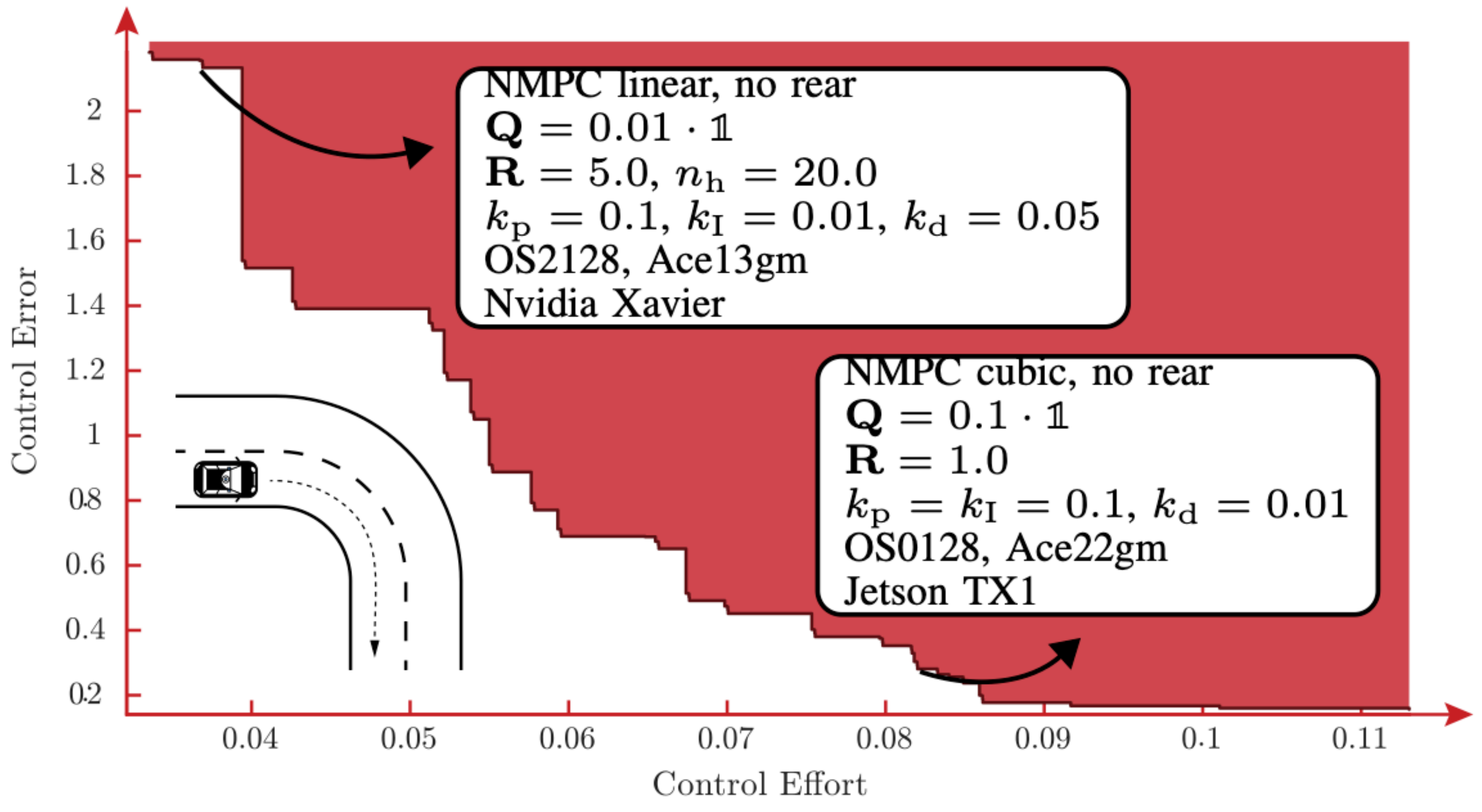


investments

compositionality (vertical composition)



Fix environment, task



Details about software and hardware implementations, in a way that was not possible before

[TNSE’23, CDC’22, IROS’21, ECC’21, ITSC’20]

Related references

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Co-Design basics

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Co-Design of autonomy, mobility

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Strategic Interactions

Take-aways

- A new approach to **co-design** designed to work **across fields** and **scales**.
- It is:
  - **Compositional** horizontally and hierarchically.
  - Supports both **data-driven** and **model-based** components.
  - **Computationally tractable**.
  - **Intellectually tractable.**

- We need to account for **strategic interactions** of **designers**
  - **Posetal games**: A new class of games, where **utilities** are **posets**
  - We can deal with **hierarchical interactions** via Stackelberg games

Questions?