

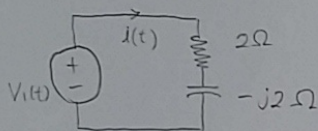
# <전기회로>

(Chapter 5.)

20112442

김용성

2. In the network in Figure P5.2,  $v_1(t) = 24 \cos(377t + 20^\circ) \text{ V}$ . Find the equations for the current and instantaneous power as a function of time.

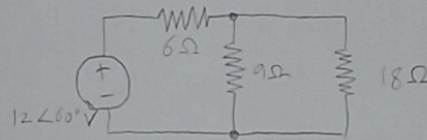


$$\begin{aligned} \hat{i}(t) &= \frac{V_1(t)}{Z} = \frac{24 \cos(377t + 20^\circ)}{2 - j2} \\ &= \frac{24 \angle 20^\circ}{2 - j2} = \frac{24 \angle 20^\circ}{2.83 \angle -45^\circ} \\ &= 8.485 \angle 65^\circ \end{aligned}$$

$$p(t) = v_1(t) \cdot i(t)$$

$$\begin{aligned} &= \frac{V_M I_M}{2} (\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)) \\ &= \frac{(24)(8.485)}{2} (\cos(-45^\circ) + \cos(754t + 85^\circ)) \\ &= 101.8 (0.707 + \cos(754t + 85^\circ)) \\ &= 71.98 + 101.8 \cos(754t + 85^\circ) \text{ W} \end{aligned}$$

4. Determine the total average power absorbed and supplied in the network in Figure P5.4

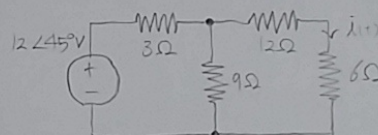


$$\text{총 저항 } R = 12 \Omega$$

$$I = \angle 60^\circ \text{ V}$$

$$P = \frac{1}{2} \cdot I^2 \cdot R = 6 \text{ W}$$

6. Determine the average power absorbed in the 6 ohm resistor in the network in Figure P5.6.



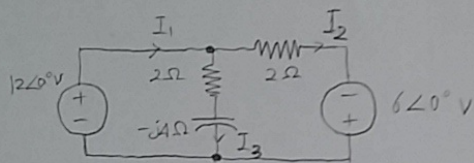
$$\text{총 저항 } R = 9 \Omega$$

$$\text{전체 전류 } I = \frac{12 \angle 45^\circ}{9} \text{ A}$$

$$i_1(t) = \frac{12 \angle 45^\circ}{9} \times \frac{9}{21} = \frac{4 \angle 45^\circ}{7} \text{ A}$$

$$P = \frac{1}{2} \cdot \left(\frac{4}{7}\right)^2 \cdot 6 \Omega = 0.59 \text{ W}$$

9. Find the total average power absorbed and supplied in the circuit in Figure p5.9.



$$I_2 = \frac{12 + 6}{2} = 9 \angle 0^\circ \text{ A}$$

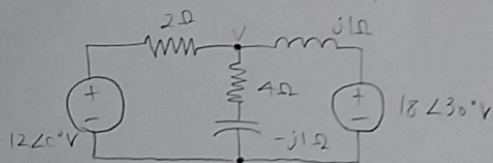
$$I_3 = \frac{12}{2 - j4} = \frac{6}{1 - j2} = \frac{6(1 + j2)}{5} \text{ A}$$

$$I_1 = I_2 + I_3$$

$$P_{\text{supplies}} = \frac{12 \times I_1}{2} \cos(\theta_V - \theta_I) \text{ W}$$

$$P_{\text{absorb}} = \left( \frac{2 \cdot (I_2)^2}{2} + \frac{2 \cdot (-j4 \cdot (I_3)^2)}{2} \right) \text{ W}$$

10. Show that the conservation of power holds for the network shown in Figure p5.10.



$$\frac{V - 12}{2} + \frac{V}{4 - j1} + \frac{V - 18 \angle 30^\circ}{j1} = 0$$

$$V = 18.11 \angle 5.9^\circ \text{ V}$$

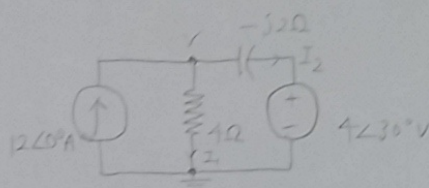
$$I_1 = 3.14 \angle 19.22^\circ \text{ A}$$

$$I_2 = 4.39 \angle 19.24^\circ \text{ A}$$

$$I_3 = 7.54 \angle -161.2^\circ \text{ A}$$

$$P = P_{12\angle 0^\circ} + P_{18\angle 30^\circ} + P_{2\Omega} + P_{4\Omega} \text{ W}$$

13. Determine the average power absorbed and supplied by each element in the network in Figure p5.13.



$$-12 + \frac{1}{4} - \frac{V - 4 \angle 30^\circ}{-j2} = 0$$

$$V = 19.92 \angle -54.49^\circ \text{ V}$$

$$I_1 = 4.98 \angle -54.49^\circ \text{ A}$$

$$I_2 = 9.97 \angle 23.91^\circ \text{ A}$$

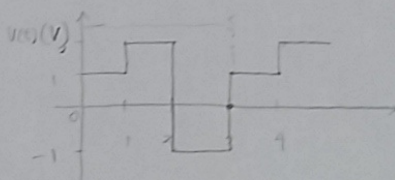
$$P_{12\angle 0^\circ} = -69.42 \text{ W}$$

$$P_{4\Omega} = 49.6 \text{ W}$$

$$P_{4\angle 30^\circ} = 19.83 \text{ W}$$

$$P = P_{12\angle 0^\circ} + P_{4\Omega} + P_{4\angle 30^\circ} \text{ W}$$

14. Determine the rms value of the waveform in Figure p5.14

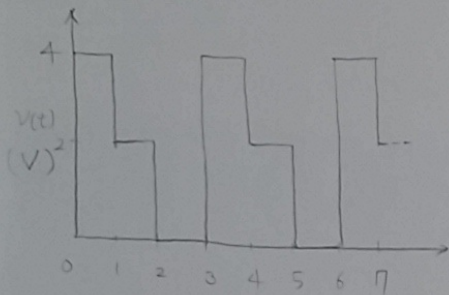


$$\begin{aligned} V_{\text{rms}} &= \left[ \frac{1}{3} \left( \int_0^1 1^2 dt + \int_1^2 (-1)^2 dt + \int_2^3 1^2 dt \right) \right]^{\frac{1}{2}} \\ &= \frac{1}{3} \left( [t]_0^1 + [t]_1^2 + [t]_2^3 \right)^{\frac{1}{2}} \\ &= \frac{1}{3} \left( 1 + 2 + 1 \right)^{\frac{1}{2}} = \left( \frac{4}{3} \right)^{\frac{1}{2}} \end{aligned}$$



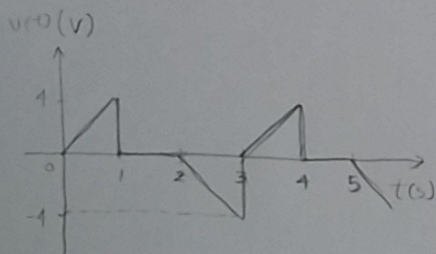
$$= (2)^{\frac{1}{2}} \approx 1.414 \text{ V}$$

15. Calculate the rms value of the wave form shown in Figure P5.15



$$\begin{aligned} V_{rms} &= \left[ \frac{1}{3} \left( \int_0^1 (4)^2 dt + \int_1^2 (2)^2 dt + \int_2^3 (0)^2 dt \right) \right]^{\frac{1}{2}} \\ &= \frac{1}{3} \left( [16t]_0^1 + [4t]_1^2 \right)^{\frac{1}{2}} \\ &= \frac{1}{3} \left( 16 + 8 - 4 \right)^{\frac{1}{2}} = \sqrt{\frac{20}{3}} \text{ V} \end{aligned}$$

17. Compute the rms value of the voltage waveform shown in Figure P5.17.

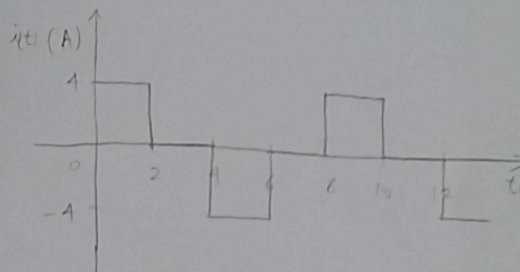


$$\begin{aligned} V_{rms} &= \left[ \frac{1}{3} \left( \int_0^1 (4t)^2 dt + \int_2^3 (-4t+8)^2 dt \right) \right]^{\frac{1}{2}} \\ &= \frac{1}{3} \left( \left[ \frac{16}{3} t^3 \right]_0^1 + \left[ \frac{16}{3} t^3 - 32t^2 + 64t \right]_2^3 \right)^{\frac{1}{2}} \end{aligned}$$

$$= \frac{1}{3} \left( \frac{16}{3} + \left( 144 - 288 + 192 \right) - \left( \frac{128}{3} - 128 + 128 \right) \right)^{\frac{1}{2}}$$

$$\begin{aligned} &= \frac{1}{3} \left( \frac{16}{3} + 48 - \frac{128}{3} \right)^{\frac{1}{2}} \\ &= \sqrt{\frac{32}{9}} = \frac{4\sqrt{2}}{3} \text{ V.} \end{aligned}$$

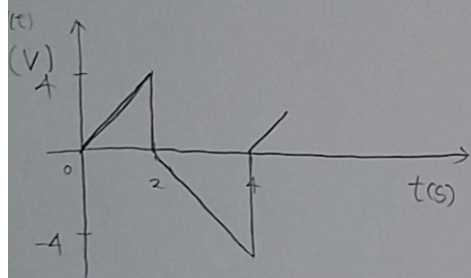
18. The current waveform in Figure P5.18 exists in a  $10\Omega$  resistor. Determine the average power delivered to the resistor.



$$\begin{aligned} i_{rms} &= \left[ \frac{1}{8} \left( \int_0^4 (1)^2 dt + \int_4^8 (-1)^2 dt \right) \right]^{\frac{1}{2}} \\ &= \frac{1}{8} \left( [16t]_0^4 + [16t]_4^8 \right)^{\frac{1}{2}} \\ &= \frac{1}{8} \left( 32 + 96 - 64 \right)^{\frac{1}{2}} = \sqrt{8} \text{ A} \end{aligned}$$

$$P_{avg} = (\sqrt{8})^2 \cdot 10 = 80 \text{ W}$$

21. The voltage across a  $4\Omega$  resistor is given by the waveform shown in Figure p5.21. Find the average power absorbed by the resistor.



$$V_{rms} = \left[ \frac{1}{4} \left( \int_0^2 (2t)^2 dt + \int_2^4 (-2t+4)^2 dt \right) \right]^{\frac{1}{2}}$$

$$= \frac{1}{4} \left( \left[ \frac{4}{3} t^3 \right]_0^2 + \left[ \frac{4}{3} t^3 - 8t^2 + 16t \right]_2^4 \right)^{\frac{1}{2}}$$

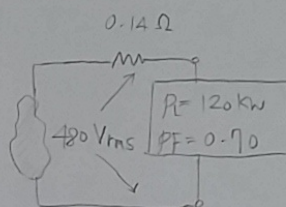
$$= \frac{1}{4} \left( \frac{32}{3} + \left( \frac{256}{3} - 128 + 64 \right) - \left( \frac{32}{3} - 32 + 32 \right) \right)^{\frac{1}{2}}$$

$$= \frac{1}{4} \left( \frac{128}{3} \right)^{\frac{1}{2}} = \sqrt{\frac{32}{3}} \text{ V}$$

$$P_{4\Omega} = \frac{\left( \sqrt{\frac{32}{3}} \right)^2}{4} = \frac{8}{3} \text{ W}$$

22. The power consumed by an industrial load is  $120\text{ kW}$  at  $0.9\text{ pf}$  lagging from a  $480\text{ V}$  rms line. If the transmission line resistance between the generator and the load is  $0.14\Omega$ ,

find the power that must be supplied by the power company.



$$I_{rms} = \frac{P_L}{(PF)(V_{rms})} = \frac{120000}{(0.9)(480)}$$

$$= 359.14 \text{ A rms}$$

$$P_{supplied} = P_L + (0.14)(359.14)^2$$

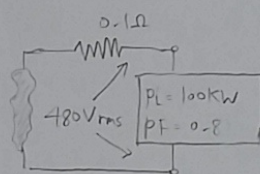
$$= 120000 + 17859$$

$$= 137859 \text{ kW.}$$

23.  $100\text{ kW}$ ,  $0.8\text{ pf}$  lagging.

The line voltage is  $480\text{ Vrms}$ .

$0.1\Omega$ , find the transmission line losses.



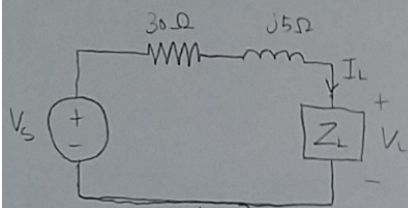
$$I_{rms} = \frac{P_L}{(PF)(V_{rms})} = \frac{100000}{(0.8)(480)} = 260.42 \text{ A}$$

$$P_{losses} = (0.1)(260.42)^2$$

$$= 6.782 \text{ kW.}$$



28. Calculate the voltage  $V_s$  that must be supplied to obtain 2kW,  $240 \angle 0^\circ$  V rms, and a power factor of 0.8 leading at the load  $Z_L$  in the network in Figure p5.28.



$$I_{rms} = \frac{P_L}{(PF) \cdot V_{rms}} = \frac{2000}{(0.8) \cdot 240} \cdot \cos^{-1}(0.8)$$

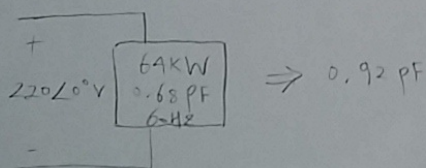
$$= 10.42 \angle 36.87^\circ \text{ A rms}$$

$$V_s = (30 + j5) 10.42 \angle 36.87^\circ$$

$$+ 240 \angle 0^\circ$$

$$= 512.8 \angle 26.54^\circ \text{ V}$$

32. A small plant has a bank of induction motors that consume 64kW at a PF of 0.68 lagging. The 60 Hz line voltage across the motors is  $220 \angle 0^\circ$  V rms. The local power company has told the plant to raise the PF to 0.92 lagging. What value of capacitance is required?



$$Q_{old} = P_{old} \tan \theta_{old} = (64)(10^3)(\tan(\cos^{-1} 0.68))$$

$$= 69503.2 \text{ var}$$

$$S_{old} = 64000 + j 69503.2$$

$$S_{new} = 64000 + j 64000 \times \tan(\cos^{-1} 0.92)$$

$$= 64000 + j 27260$$

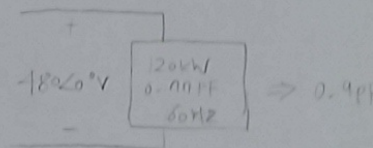
$$S_{cap} = S_{new} - S_{old}$$

$$= -j 41743$$

$$C = \frac{S_{cap}}{(-j\omega)(V^2)_{rms}} = \frac{-j 41743}{(j 377)(220)^2}$$

$$= 2289.9 \mu\text{F}$$

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$$\theta_{old} = \cos^{-1} 0.77$$

$$Q_{old} = P_{old} \tan \theta_{old} = (120)(10^3)(\tan(\cos^{-1} 0.77))$$

$$= 99440 \text{ var}$$

$$S_{old} = 120000 + j 99440$$

$$S_{new} = 120000 + j 120000 \cdot \tan(\cos^{-1} 0.9)$$

$$= 120000 + j 58120$$

$$S_{cap} = -j 41320$$

$$C = \frac{41320}{(377)(180)^2} = 510.2 \mu\text{F}$$

42. An abc-sequence balanced three-phase wye-connected source supplies power to a balanced wye-connected load. The line impedance per phase is  $1 + j0 \Omega$  and the load impedance per phase is  $20 + j20 \Omega$ . If the source line voltage  $V_{ab}$  is  $100 \angle 0^\circ \text{ V}_{rms}$ , find the line currents.

$$V_{an} = \frac{100 \angle -30^\circ}{\sqrt{3}} \text{ V}_{rms}$$

$$V_{bn} = \frac{100 \angle -150^\circ}{\sqrt{3}} \text{ V}_{rms}$$

$$V_{cn} = 100 \angle 90^\circ \text{ V}_{rms}$$

$$I_{aA} = \frac{100 \angle -30^\circ}{\sqrt{3}(20 + j20)} \text{ A}_{rms}$$

$$I_{bB} = \frac{100 \angle -150^\circ}{\sqrt{3}(20 + j20)} \text{ A}_{rms}$$

$$I_{cC} = \frac{100 \angle 90^\circ}{\sqrt{3}(20 + j20)} \text{ A}_{rms}$$

46. three phase wye-delta system,  
 $V_{an} = 120 \angle 10^\circ \text{ V}_{rms}$ , load impedance per phase in the  $\Delta$  is  $24 + j18 \Omega$ .

$$I_{aA} = \frac{120 \angle 10^\circ}{8 + j6} \text{ A}_{rms}$$

$$I_{bB} = \frac{120 \angle -110^\circ}{8 + j6} \text{ A}_{rms}$$

$$I_{cC} = \frac{120 \angle 130^\circ}{8 + j6} \text{ A}_{rms}$$

48. three-phase wye-delta system, the source has an abc phase sequence. The load impedance is  $12 + j8 \Omega$ .  
 $V_{AB} = 260 \angle 45^\circ \text{ V}_{rms}$ .

$$V_{an} = \frac{260 \angle 15^\circ}{\sqrt{3}} \text{ V}_{rms}$$

$$V_{bn} = \frac{260 \angle -105^\circ}{\sqrt{3}} \text{ V}_{rms}$$

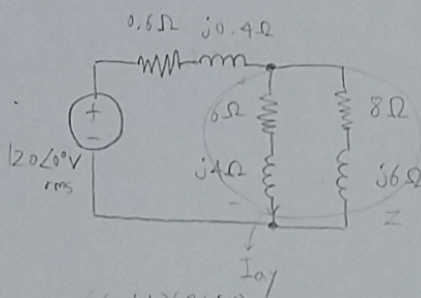
$$V_{cn} = \frac{260 \angle 135^\circ}{\sqrt{3}} \text{ V}_{rms}$$

$$I_{aA} = \frac{260 \angle 15^\circ}{\sqrt{3}(4 + j2.67)} \text{ A}_{rms}$$

$$I_{bB} = \frac{260 \angle -105^\circ}{\sqrt{3}(4 + j2.67)} \text{ A}_{rms}$$

$$I_{cC} = \frac{260 \angle 135^\circ}{\sqrt{3}(4 + j2.67)} \text{ A}_{rms}$$

54. Find  $I_{ay}$  in the circuit shown in Figure P5.54.



$$Z = \frac{(6 + j4)(8 + j6)}{(6 + j4) + (8 + j6)} = \frac{24 + j68}{14 + j10}$$

$$I_{ay} = \frac{V_a}{6 + j4} = \frac{120 \angle 0^\circ \cdot Z}{(6 + j4)(Z + 0.6 + j0.4)}$$

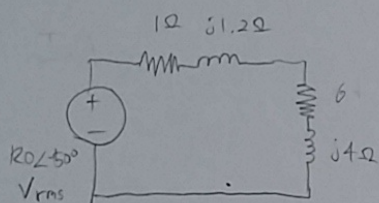


56. A balanced three-phase delta-delta system has the following parameters:  $V_{ab} = 207.84 \angle -20^\circ \text{ V rms}$ ,  $Z_{\text{Line}} = 1 + j1.2 \Omega$ , and  $Z_{\text{Load}} = 18 + j12 \Omega$ . Find the line currents.

$$V_{an} = \frac{207.84 \angle -50^\circ}{\sqrt{3}} = 120 \angle -50^\circ \text{ V rms}$$

$$V_{bn} = 120 \angle -170^\circ \text{ V rms}$$

$$V_{cn} = 120 \angle -290^\circ \text{ V rms}$$



$$I_{aA} = \frac{120 \angle -50^\circ}{(1 + j5.2) \Omega} \text{ A rms}$$

$$I_{bB} = \frac{120 \angle -170^\circ}{1 + j5.2} \text{ A rms}$$

$$I_{cC} = \frac{120 \angle -290^\circ}{1 + j5.2} \text{ A rms}$$

\*58. A 480V rms line feeds two balanced three-phase loads. If the two loads are rated as follows:

Load 1: 5kVA at 0.8 PF lagging

Load 2: 10kVA at 0.9 PF lagging

determine the magnitude of the line current from the 480V rms source.

$$S_1 = \frac{5k}{0.8} \angle \cos^{-1} 0.8 \text{ VA}$$

$$= 4k + j3k \text{ VA}$$

$$S_2 = \frac{10k}{0.9} \angle \cos^{-1} 0.9 \text{ VA}$$

$$= 9k + j4.358k \text{ VA}$$

$$S_T = S_1 + S_2 = 13k + j7.358k$$

$$I_L = \frac{S_T}{V_L \cdot \sqrt{3}} = \frac{13k + j7.358k}{480 \cdot \sqrt{3}}$$