Mathematical Proof

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Assignment #5

Question 1

Theorem 1. *If $\mathscr{F}$ and $\mathscr{G}$ are families of sets, and A in $\mathscr{F}$ and B in $\mathscr{G}$, then $\cup\mathscr{F}$ and $\cup\mathscr{G}$ are not disjoin if A and B are not disjoint.*

Proof. Suppose A and B are not disjoint. Then there exists at least one x which exists in A and B. Since A is an element of $\mathscr{F}$ and $\cup\mathscr{F}$ consists of every member of the elements of $\mathscr{F}$, then x exists in $\cup\mathscr{F}$. And since B is an element of $\mathscr{G}$ and $\cup\mathscr{G}$ consists of every member of every element of $\mathscr{G}$, then x exists in $\cup\mathscr{G}$. Therefore, $\cup\mathscr{F}$ and $\cup\mathscr{G}$ have x in common and are subsequently not disjoint. □

Question 2

Theorem 2. *For every integer n, $30|n$ if, and only inf, $5|n$ and $6|n$.*

Proof. Suppose □

Question 3

Theorem 3. *There is a unique real number x such that for every real number y, $xy + x\breve{}17 = 17y$*

Proof. First, take $xy + x - 17 = 17y$ and add 17 to both sides, the result is $xy + x = 17y + 17$. Then factor $x+1$ out of both sides and get $x(y+1) = 17(y+1)$. Then dived both sides by $y+1$ and get $x = 17$. This proves that $x = 17$ for all real values of y except -1. Because, if $y = -1$ then dividing by $y + 1$ constitutes dividing by zero which is undefined. To prove that $x = 17$ holds as true for $y = -1$, take the point where the division by zero would occur and insert $x = 17$ and $y = -1$ to test for truth. That results in the statement $17(-1 + 1) = 17(-1 + 1)$, which is clearly identical and leads to the true statement that $0 = 0$. □

Visual proof that $x = 17$ for all real numbers except -1.

$$
\begin{aligned}
& zy + x - 17 = 17y \\
& \qquad \equiv xy + x - 17 + 17 = 17y + 17 && \text{(add 17 to both sides)} \\
& \qquad \equiv xy + x = 17y + 17 && \text{(simplify)} \\
& \qquad \equiv x(y+1) = 17(y+1) && \text{(factor both sides)} \\
& \qquad \equiv \frac{x(y+1)}{y+1} y + 1 = \frac{17(y+1)}{y+1} && \text{(divide both sides by y-1)} \\
& \qquad \equiv x = 17 && \text{(conclusion)}
\end{aligned}
$$

Visual proof that $x = 17$ for -10.

$$
\begin{aligned}
& zy + x - 17 = 17y \\
& \qquad \equiv xy + x - 17 + 17 = 17y + 17 && \text{(add 17 to both sides)} \\
& \qquad \equiv xy + x = 17y + 17 && \text{(simplify)} \\
& \qquad \equiv x(y+1) = 17(y+1) && \text{(factor both sides)} \\
& \qquad \equiv 17(-1+1) = 17(-1+1) && \text{(insert } x = 17 \text{ and } y = -1\text{)} \\
& \qquad \equiv 17(0) = 17(0) && \text{(simplify)} \\
& \qquad \equiv 0 = 0 && \text{(true statement)}
\end{aligned}
$$

Question 4

Question 5

Question 6