# Mathematical Proof

*John Shea*

*February 15, 2019*

## Assignment #6

### Question 1

**Theorem 1.** *If $A$ is a set and $\{B_i | i \in I\}$ is an indexed family of sets. $A \times (\cup_{i \in I} B_i) = \cup_{i \in I})(A \times B)$.*

*Proof.* Suppose $(a, b) \in A \times (\cup_{i \in I} B_i)$. Then $a \in A$ and $b \in \cup_{i \in I}$. so $A \times (\cup_{i \in I} B_i) \subseteq \cup_{i \in I})(A \times B)$.

Suppose $(x, y) \in \cup_{i \in I}(A \times B)$. Then $x \in$ So, $\cup_{i \in I})(A \times B) \subseteq A \times (\cup_{i \in I} B_i)$. Therefore $A \times (\cup_{i \in I} = \cup_{i \in I})(A \times B)$. □

### Question 2

**Theorem 2.**

*Proof.* □

a $S^{-1} \circ R$

$S = \{(4, a), (4, d), (5, b), (5, c)\}$, and is a relation from $B$ to $C$, so $S^{-1} = \{(a, 4), (d, 4), (b, 5), (c, 5)\}$ and is a relation from $C$ to $B$. $R = \{(1, b), (2, a), (2, b), (2, c), (3, d)\}$ and is a relation from $A$ to $C$. Therefore $S^{-1} \circ R = \{(1, 5), (2, 4), (2, 5)\}$ and is a relation from $A$ to $B$. This results because we now have a relation that begins in $A$, connects through $C$ and arrives at $B$. The final relation $S^{-1} \circ R$ becomes a set of ordered pairs that have elements of $A$ as the first coordinate and elements of $B$ as the second.

b $R^{-1} \circ S$

$R = \{(1, b), (2, a), (2, b), (2, c), (3, d)\}$ and is a relation from $A$ to $C$, $Rso^{-1} = \{(b, 1), (a, 2), (b, 2), (c, 2), (d, 3)$ and is a relation from $C$ to $A$. $S = \{(4, a), (4, d), (5, b), (5, c)\}$ and is a relation from $B$ to $C$. Therefore $R^{-1} \circ S$ is a relation from $B$ to $A$. This results because we now have a relation that begins in $B$, connects through $C$ and arrives at $A$. The final relation $R^{-1} \circ S$ becomes a set of ordered pairs that have elements of $B$ as the first coordinate and elements of $A$ as the second.

## Question 3

**Theorem 3.**

*Proof.* □

a $R = Dom(R) \times Ran(R)$

Suppose $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ and $R = \{(1, a), (2, b), (3, c)\}$. Then $R$ is a relation from $A$ to $B$. In this scenario, $Dom(R) = \{1, 2, 3\}$ and Ran(R)=$\{a, b, c\}$. The Cartesian product of $Dom(R)$ and $Ran(R)$ would feature all possible combinations of the two sets. Hence, $Dom(R) \times Ran(R) = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$. Therefore, $R$ does not necessarily equal $Dom(R) \times Ran(R)$, and the statement $R = Dom(R) \times Ran(R)$ is untrue. It would be true to state that $R \subseteq Dom(R) \times Ran(R)$.

b $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

Suppose $(b, a) \in (R \cap S)^{-1}$. Then $(a, b) \in R \cap S$. For any $(a, b) \in R, (b, a) \in R^{-1}$ and for any $(a, b) \in S, (b, a) \in S^{-1}$. Hence, $(b, a) \in R^{-1}$ and $S^{-1}$, and consequently $(b, a) \in R^{-1} \cap S^{-1}$. Therefore, $(R \cap S)^{-1} \subseteq R^{-1} \cap S^{-1}$

Suppose $(b, a) \in R^{-1} \cap S^{-1}$. Then $(b, a) \in R^{-1}$ and $(b, a) \in S^{-1}$. Hence, $(a, b) \in R$ and $(a, b) \in S$, which means $(a, b) \in (R \cap S)$. Hence, $(b, a) \in (R \cap S)^{-1}$. Therefore, $R^{-1} \cap S^{-1} \subseteq (R \cap S)^{-1}$. Taken together this proves that $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$.

## Question 4

## Question 5

## Question 6