

# Mathematical Proof

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## Assignment #5

### Question 1

**Theorem 1.** *If  $\mathcal{F}$  and  $\mathcal{G}$  are families of sets, and  $A$  in  $\mathcal{F}$  and  $B$  in  $\mathcal{G}$ , then  $\cup\mathcal{F}$  and  $\cup\mathcal{G}$  are not disjoint if  $A$  and  $B$  are not disjoint.*

*Proof.* Suppose  $A$  and  $B$  are not disjoint. Then there exists at least one  $x$  which exists in  $A$  and  $B$ . Since  $A$  is an element of  $\mathcal{F}$  and  $\cup\mathcal{F}$  consists of every member of the elements of  $\mathcal{F}$ , then  $x$  exists in  $\cup\mathcal{F}$ . And since  $B$  is an element of  $\mathcal{G}$  and  $\cup\mathcal{G}$  consists of every member of every element of  $\mathcal{G}$ , then  $x$  exists in  $\cup\mathcal{G}$ . Therefore,  $\cup\mathcal{F}$  and  $\cup\mathcal{G}$  have  $x$  in common and are subsequently not disjoint.  $\square$

### Question 2

**Theorem 2.** *For every integer  $n$ ,  $30|n$  if, and only if,  $5|n$  and  $6|n$ .*

*Proof.* Suppose  $30|n$ . Then there exists an integer  $k$  such that  $30k = n$ . Therefore  $n = 30k = 5(6k)$ , so  $5|n$ . Similarly  $n = 30k = 6(5k)$ , so  $6|n$ .

Suppose  $5|n$  and  $6|n$ . Then there exists integers  $j$  and  $k$  such that  $n = 5j$  and  $n = 6k$ . Therefore  $30(j - k) = 30j - 30k = 5(6j) - 6(5k = 6n - 5n = n)$ , so  $30|n$ .  $\square$

### Question 3

**Theorem 3.** *There is a unique real number  $x$  such that for every real number  $y$ ,  $xy + x - 17 = 17y$*

*Proof.* First, take  $xy + x - 17 = 17y$  and add 17 to both sides, the result is  $xy + x = 17y + 17$ . Then factor  $x + 1$  out of both sides and get  $x(y + 1) = 17(y + 1)$ . Then divided both sides by  $y + 1$  and get  $x = 17$ . This proves that  $x = 17$  for all real values of  $y$  except  $-1$ . Because, if  $y = -1$  then dividing by  $y + 1$  constitutes dividing by zero which is undefined. To prove that  $x = 17$  holds as true for  $y = -1$ , take the point where the division by zero would occur and insert  $x = 17$  and  $y = -1$  to test for truth. That results in the statement  $17(-1 + 1) = 17(-1 + 1)$ , which is clearly identical and leads to the true statement that  $0 = 0$ .  $\square$

Visual proof that  $x = 17$  for all real numbers except  $-1$ .

$$\begin{aligned}
 zy + x - 17 &= 17y \\
 &\equiv xy + x - 17 + 17 = 17y + 17 && \text{(add 17 to both sides)} \\
 &\equiv xy + x = 17y + 17 && \text{(simplify)} \\
 &\equiv x(y + 1) = 17(y + 1) && \text{(factor both sides)} \\
 &\equiv \frac{x(y + 1)}{y + 1} y + 1 = \frac{17(y + 1)}{y + 1} && \text{(divide both sides by } y-1) \\
 &\equiv x = 17 && \text{(conclusion)}
 \end{aligned}$$

Visual proof that  $x = 17$  for  $-10$ .

$$\begin{aligned}
 zy + x - 17 &= 17y \\
 &\equiv xy + x - 17 + 17 = 17y + 17 && \text{(add 17 to both sides)} \\
 &\equiv xy + x = 17y + 17 && \text{(simplify)} \\
 &\equiv x(y + 1) = 17(y + 1) && \text{(factor both sides)} \\
 &\equiv 17(-1 + 1) = 17(-1 + 1) && \text{(insert } x = 17 \text{ and } y = -1) \\
 &\equiv 17(0) = 17(0) && \text{(simplify)} \\
 &\equiv 0 = 0 && \text{(true statement)}
 \end{aligned}$$

## Question 4

**Theorem 4.** *For any set  $U$ , for every  $B \in \wp(U)$  there is a unique  $D$  such that for every  $C \in \wp(U)$ ,  $C \setminus B = C \cap D$ .*

*Proof.* Because  $B$ ,  $D$ , and  $C$  are within  $\wp(U)$ ,  $\wp(U)$  is the universal set within the context of this problem. Suppose  $D$  is the complement of  $B$  (i.e.  $B^c$ ) within set  $U$ . Then  $D$  is the set of all  $x \notin B$  within  $\wp(U)$ . Therefore, for any possible  $C \in \wp(U)$  if the members of  $C$  which also exists in  $B$  are removed (i.e.  $C \setminus B$ ), the remainder of the set will exist in both  $C$  and  $D$ . Therefore, there exists a unique set  $D$  such that for all  $C$ ,  $C \setminus B = C \cap D$ . Furthermore, that unique set  $D$  is the complement of  $B$ ,  $B^c$ .  $\square$

## Question 5

**Theorem 5.** *For every positive integer  $n$ , there is a sequence of  $2n$  consecutive positive integers containing no primes.*

*Proof.*

$\square$

## Question 6

**Theorem 6.** For  $f(x) = x^2$  with domain  $0 \leq x \leq 10$  the  $\lim_{x \rightarrow 5} f(x) = 25$

*Proof.*

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