Mathematical Proof

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Assignment #4

Question 1

Theorem 1. *If* $0 < \frac{1}{a} < \frac{1}{b}$*, then* $b < a$*.*

Proof. Suppose $0 < \frac{1}{a} < \frac{1}{b}$ Because both $\frac{1}{a}$ and $\frac{1}{b}$ are both greater than zero, both a and b must be positive. Therefore, all terms can be multiplied by a and b without needing to change the direction of the inequality. Thus, when all three terms are multiplied by a, the result is $0 < 1 < \frac{a}{b}$. Then when all three terms are multiplied by b, the result is $0 < b < a$. Therefore b is less than a. □

A more visual depiction:

$$
\begin{aligned}
&0 < \frac{1}{a} < \frac{1}{b} \\
&\qquad \equiv 0 \cdot a < \frac{1}{a} \cdot a < \frac{1}{b} \cdot a && \text{(multiply all terms by a)} \\
&\qquad \equiv 0 < 1 < \frac{a}{b} && \text{(simplify)} \\
&\qquad \equiv 0 \cdot b < 1 \cdot b < \frac{a}{b} \cdot b && \text{(multiply all terms by b)} \\
&\qquad \equiv 0 < b < a && \text{(simplify)} \\
&\qquad b < a && \text{(conclusion)}
\end{aligned}
$$

Question 2

Theorem 2. *Given* $A \subseteq B$ *and* $x \in A$*, if* $x \notin B \setminus C$ *then* $x \in C$*.*

Proof. Suppose $x \notin C$. Then □

Question 3

Theorem 3. *Given* $A \setminus B \subseteq C \cap D$ *and* $x \in A$*, if* $x \notin C$*, then* $x \in B$

Proof. Suppose □

Question 4

Theorem 4. *If* x *is a negative real number and* $x < \frac{1}{x}$*, then* $x < -1$*.*

Proof. Suppose a is the absolute value of x. Then the given statement can be written as $-a < -\frac{1}{a}$. Then multiply both sides by $-a$ and we get $a^2 > 1$. We then take the principle square root of both sides and arrive at $a > 1$. Then we multiply both sides by -1 and get $-a < -1$. Finally, we insert x for $-a$ and we get the desired $x < -1$. □

Question 5

Theorem 5. *If* x *is a real number and* $x \neq 2$*, then there is a real number* y *such that* $x = (2y+1)/(y-1)$*.*

Proof. Suppose □

Question 6

Theorem 6. *If* F *is a non-empty family of sets,* B *is a set, and* $\forall A \in F(A \subseteq B)$*. Is* $\cup F \subseteq B$*? Either provide a proof to show that this is true or provide a counterexample to show that this is false.*

Proof. Suppose □