

Mathematical Proof

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Assignment #4

Question 1

Theorem 1. *If $0 < \frac{1}{a} < \frac{1}{b}$, then $b < a$.*

Proof. Suppose $0 < \frac{1}{a} < \frac{1}{b}$. Because both $\frac{1}{a}$ and $\frac{1}{b}$ are both greater than zero, both a and b must be positive. Therefore, all terms can be multiplied by a and b without needing to change the direction of the inequality. Thus, when all three terms are multiplied by a , the result is $0 < 1 < \frac{a}{b}$. Then when all three terms are multiplied by b , the result is $0 < b < a$. Therefore b is less than a . \square

A more visual depiction:

$$\begin{aligned} 0 < \frac{1}{a} < \frac{1}{b} \\ \equiv 0 \cdot a < \frac{1}{a} \cdot a < \frac{1}{b} \cdot a & \quad \text{(multiply all terms by } a) \\ \equiv 0 < 1 < \frac{a}{b} & \quad \text{(simplify)} \\ \equiv 0 \cdot b < 1 \cdot b < \frac{a}{b} \cdot b & \quad \text{(multiply all terms by } b) \\ \equiv 0 \cdot b < 1 \cdot b < \frac{a}{b} \cdot b & \quad \text{(simplify)} \\ \equiv 0 < b < a & \quad \text{(result)} \\ b < a & \quad \text{(conclusion)} \end{aligned}$$

Question 2

Theorem 2.

Proof.

\square

A more visual depiction:

Question 3

Question 4

Question 5

Question 6