

# Mathematical Proof

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## Assignment #3

### Question 1

- a. True
  - For all persons, it is not true that the given person is his or her own twin.
  - More succinctly: No one is their own twin.
- b. False
  - There exists at least one person who is his or her own twin.
- c. False
  - There exists at least one person who is twins with every other person.
- d. False
  - For all people, there is not a single person with whom they are twins.
- e. True
  - There exists at least one person who does not have a twin.
- f. False
  - for all perople
  - More succinctly: No one is twins with everyone.

### Question 2

- a.  $\forall x(5 < x < 10 \rightarrow \exists a \exists b \exists c(a^2 + b^2 + c^2 = x))$ .
- b.  $\exists! x((x - 4)^2 = 36)$ .
- c.  $\exists! x((x = 11)^2 = 49)$ .
- d.  $\exists x \exists y(((x \neq y)^{(x - 4)^2 = 36})^{(x - 4)^2 = 36})$ .

### Question #3

$$\neg \forall x \in A \neg P(x) \equiv \exists x \in P(x).$$

$$\equiv x, y, z \quad (\text{rule})$$

$$\equiv a, b, c \quad (\text{rule})$$

$$\equiv zzzqx \quad (\text{conclusion})$$

### Question 4

- a. The identity element for multiplication is 1.
- b. Every positive real number has a positive multiplicative inverse.
- c. No positive real number has a negative multiplicative inverse.

### Question 5

- a.
  - $A_2 = \{2, 3, 4, 6\}$
  - $A_3 = \{3, 4, 5, 6, 9\}$
  - $A_4 = \{4, 5, 6, 8, 12\}$
- b.
  - $\cap_{j \in J} A_j = \{4, 6\}$
  - $\cup_{j \in J} A_j = \{2, 3, 4, 5, 6, 8, 9, 12\}$

### Question 6

- a.  $\wp(A) \cup \wp(B) \subseteq \wp(A \cup B)$
- b.  $\wp(A) \cup \wp(B) = \wp(A \cup B)$