Mathematical Proof

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Assignment #2

Question 1

a. Any of:
   - {x | x is a positive integer that is divisible by 5}
   - $\{x \in \mathbb{Z}^+ \mid 5 \equiv 0 \pmod 5)\}$
   - $\{5x \mid x \in \mathbb{Z}^+\}$

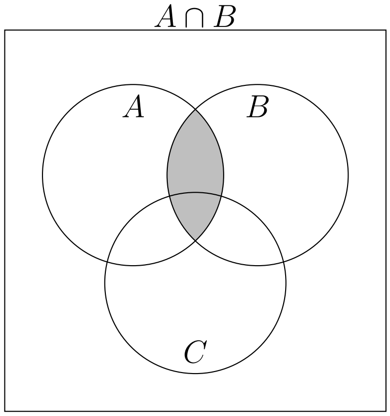
b. Any of:
   - {x | x is an integer between -4 and 3 inclusive}
   - $\{x \in \mathbb{Z} \mid -4 \leq x \leq 3\}$

c. Any of:
   - {x | x is a prime number}
   - $\{x \mid \pi(x)\}$

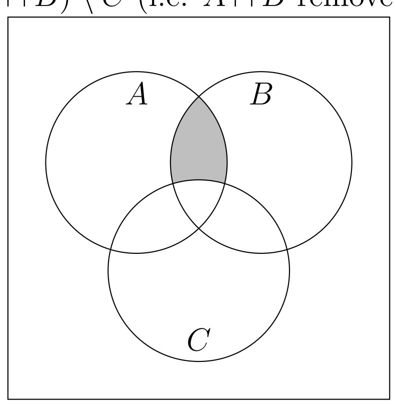
Question 2

a.

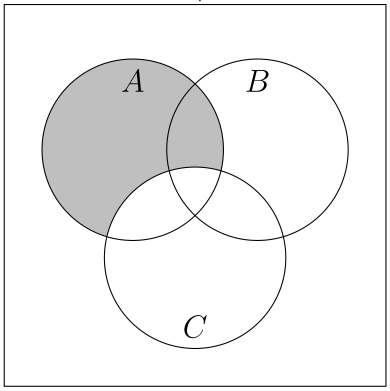
$A \cap B$



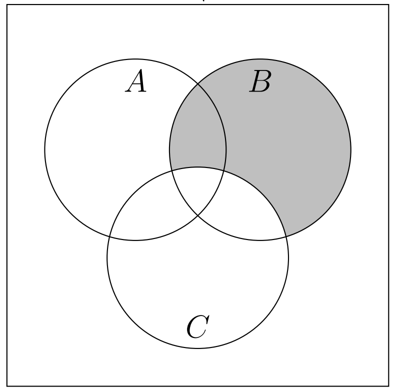
$(A \cap B) \setminus C$ (i.e. $A \cap B$ remove C)



$A \setminus C$

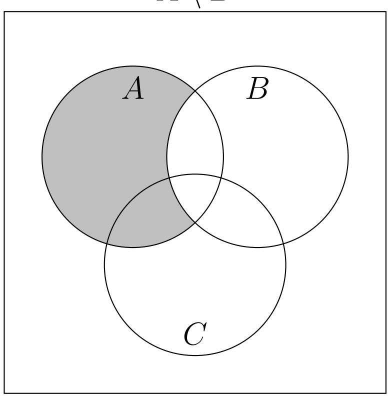


$B \setminus C$

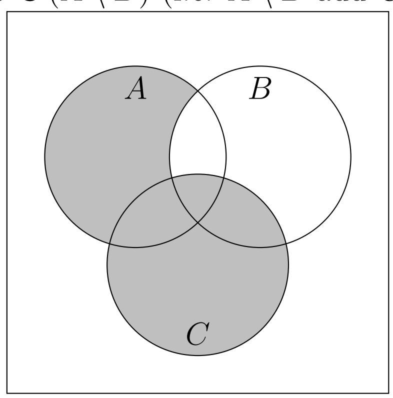


b.

$A \setminus B$

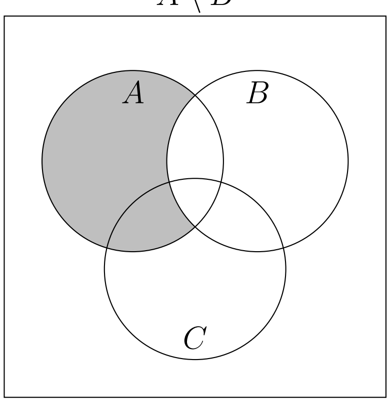


$C \cup (A \setminus B)$ (i.e. $A \setminus B$ add C)



Question 3

$A \setminus B$



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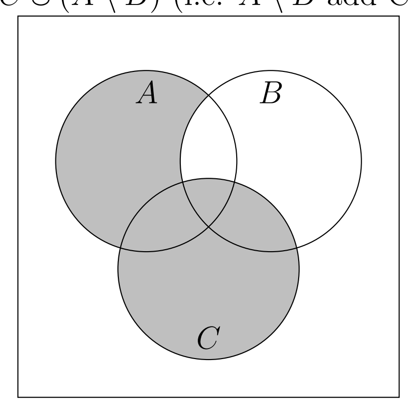


Question 4

a. $W \to (S \wedge \neg M)$
b. $(W \wedge \neg M) \to S$

c. $W \rightarrow (S \wedge \neg M)$

Explanation

- b and e are equivalent
  - using the distribution law, e become $(K \vee J) \wedge (K \vee \neg K)$
  - $K \vee \neg K$ is a tautology, therefore the above statement is equivalent to $J \vee K$, which is statement b
- c and d are equivalent
  - using DeMorgan's law, d becomes $\neg\neg J \vee \neg K$
  - using the Double Negation law, the above statement becomes $J \vee \neg K$, which is c

Question 5

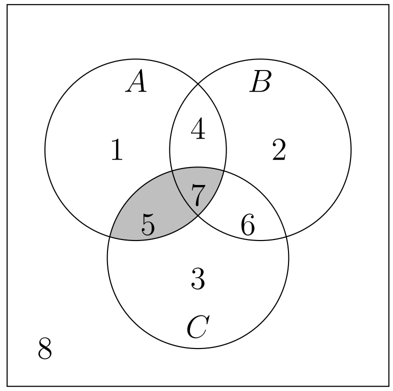
Preliminaries

M	N	$M \wedge \neg N$	$\neg M \wedge N$	$\neg M \vee N$	$\neg M \vee N$
TRUE	TRUE	FALSE	FALSE	FALSE	TRUE
TRUE	FALSE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	FALSE	TRUE	FALSE	TRUE
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE

Conclusion

a $(M \wedge \neg N) \vee (\neg M \wedge N)$	b $(M \wedge \neg N) \wedge (\neg M \wedge N)$	c $(\neg M \wedge \neg N) \vee (\neg M \vee N) \vee (M \wedge \neg N)$
FALSE	FALSE	TRUE
TRUE	FALSE	TRUE
TRUE	FALSE	TRUE
FALSE	FALSE	TRUE

Explanation



Question 6

a. $Q \wedge P$
b. $P \wedge \neg R$