

Mathematical Proof

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Assignment #3

Question 1

- a. True
 - For all persons, it is not true that the given person is his or her own twin.
 - More succinctly: No one is their own twin.
- b. False
 - There exists at least one person who is his or her own twin.
- c. False
 - There exists at least one person who is twins with every other person.
- d. False
 - For all people, there is not a single person with whom they are twins.
- e. True
 - There exists at least one person who does not have a twin.
- f. True
 - for all people, it is not true that they are twins with everyone.
 - More succinctly: No one is twins with everyone.

Question 2

- a. $\forall x(5 < x < 10 \rightarrow \exists a \exists b \exists c(a^2 + b^2 + c^2 = x))$.
- b. $\exists! x((x - 4)^2 = 36)$.
- c. $\exists! x((x = 11)^2 = 49)$.
- d. $\exists x \exists y(((x \neq y)(x - 4)^2 = 36)(x - 4)^2 = 36)$.

Question #3

$$\neg \forall x \in A \neg P(x) \equiv \exists x \in P(x).$$

$$\equiv x, y, z \quad \text{(rule)}$$

$$\equiv a, b, c \quad \text{(rule)}$$

$$\equiv zzzqx \quad \text{(conclusion)}$$

Question 4

- a. The identity element for multiplication is 1.
- b. Every positive real number has a positive multiplicative inverse.
- c. No positive real number has a negative multiplicative inverse.

Question 5

- a.
 - $A_2 = \{2, 3, 4, 6\}$
 - $j = 2$
 - $j + 1 = 3$
 - $j + 2 = 4$
 - $2j = 4$ (only one 4 is included in the set because two is superfluous)
 - $3j = 6$
 - $A_3 = \{3, 4, 5, 6, 9\}$
 - $j = 3$
 - $j + 1 = 4$
 - $j + 2 = 5$
 - $2j = 6$
 - $3j = 9$
 - $A_4 = \{4, 5, 6, 8, 12\}$
 - $j = 4$
 - $j + 1 = 5$
 - $j + 2 = 6$
 - $2j = 8$
 - $3j = 12$
- b.
 - $\cap_{j \in J} A_j = \{4, 6\}$
 - $\cup_{j \in J} A_j = \{2, 3, 4, 5, 6, 8, 9, 12\}$

Question 6

- a. $\wp(A) \cup \wp(B) \subseteq \wp(A \cup B)$
- b. $\wp(A) \cup \wp(B) = \wp(A \cup B)$