1 逐层展开

$$T(n) = 3T(n-1)$$
$$= 3*3T(n-2)$$
$$= 3^n T(0)$$
$$= 3^n * 5$$
$$= \Theta(3^n)$$

4

$$T(n) = \sqrt{n}T(\sqrt{n}) + n$$

令$m = \log_2 n$,则$n = 2^m$,得

$$T(n) = T(2^m)$$
$$= 2^{\frac{m}{2}}T(2^{\frac{m}{2}}) + 2^m$$

令$S(m) = T(2^m)$,则$S(\frac{m}{2}) = T(2^{\frac{m}{2}})$,得

$$S(m) = 2^{\frac{m}{2}}S(\frac{m}{2}) + 2^m$$
$$= 2^{\frac{m}{2}}(2^{\frac{m}{2}}S(\frac{m}{4}) + 2^m) + 2^m$$
$$= 2^{\frac{m}{2}*\log_2 m}S(1) + \cdots$$
$$= \Theta(2^{\frac{\log_2 n}{2}*\log_2(\log_2 n)})$$
$$= \Theta(n^{\frac{1}{2}*\log(\log n)})$$

5 Master(1)

根据Master定理，$a = 5, b = 3, f(n) = n$有

$$log_3 5 > 1$$

则

$$T(n) = \Theta(n^{\log_3 5})$$

6 Master(2)

根据Master定理，$a = 2, b = 2, f(n) = n^2$有

$$log_2 2 < 2$$

则

$$T(n) = \Theta(n^2)$$

7 Master(3)

根据Master定理，$a = 7, b = 7, f(n) = n$有

$$log_7 7 = 1$$

则

$$T(n) = \Theta(n \log n)$$

9 变量替换

令$m = \log_2 n$,则$n = 2^m$,得

$$\begin{aligned} T(n) &= T(2^m) \\ &= 2T(2^{\frac{m}{3}}) + 1 \end{aligned}$$

令$S(m) = T(2^m)$,则$S(\frac{m}{3}) = T(2^{\frac{m}{3}})$,得

$$S(m) = 2S(\frac{m}{3}) + 1$$
$$= 2 * (2S(\frac{m}{3}) + 1) + 1$$
$$= 2^{log_3 m} S(1) + \cdots$$
$$= \Theta(2^{log_3 m})$$
$$= \Theta(2^{log_3(log_2 n)})$$
$$= \Theta(2^{\frac{log_2(log_2 n)}{\log_2 3}})$$
$$= \Theta(\log_2 n^{\frac{1}{\log_2 3}})$$
$$= \Theta(\log_2 n^{\log_3 2})$$
$$= \Theta(\log n^{\log_3 2})$$