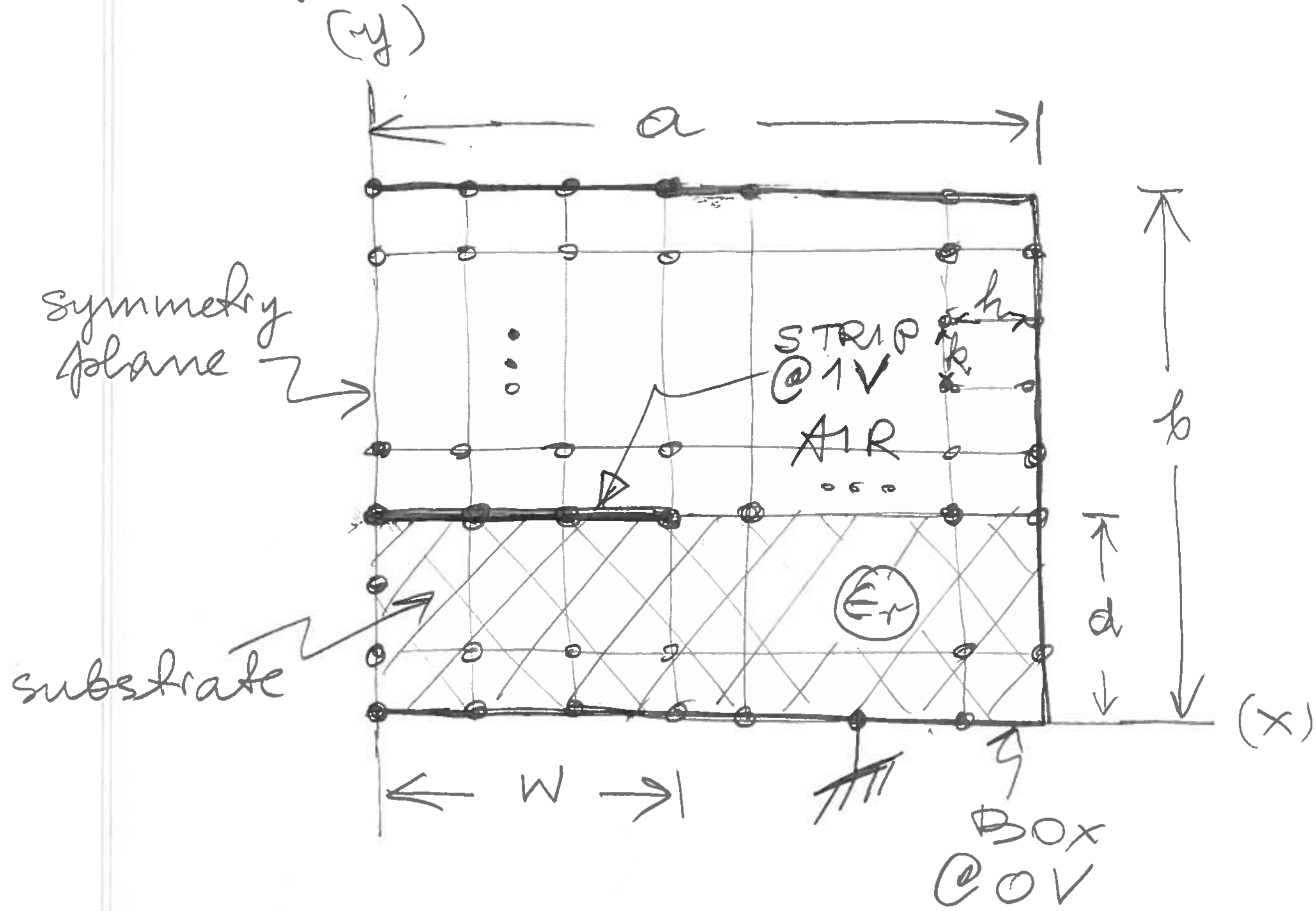
Computer Project

Stripline transmission line




The potential satisfies

$$\underline{\nabla}_T \cdot \epsilon \underline{\nabla}_T \phi = 0 \quad \text{inside box}$$

BC: $\phi = 0$ on box, $\phi = 1$ on strip

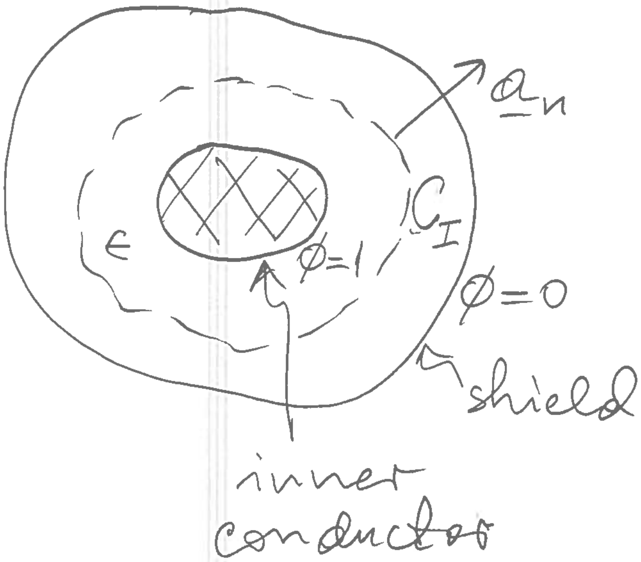
$$\underline{\nabla}_T = \underline{a}_x \frac{\partial}{\partial x} + \underline{a}_y \frac{\partial}{\partial y}$$

$$\underline{E} = -\underline{\nabla}_T \phi$$

BC: $\epsilon \dfrac{\partial \phi}{\partial y}$ continuous across the dielectric-air interface

We will use a finite-difference (FD) method to solve this problem.

We will then find the per-unit-length (p.u.l) capacitance $C$




$$C = \frac{Q}{V} = \frac{1}{V} \oint_{C_I} \epsilon \underline{E} \cdot \underline{a}_n \, dl$$

$$= -\frac{1}{V} \oint_{C_I} \epsilon \frac{\partial \phi}{\partial n} \, dl$$

In our case $V=1$, hence

$$\frac{C}{\epsilon_0} = - \oint_{C_I} \epsilon_r \frac{\partial \phi}{\partial n} \, dl ,$$

where $C_I$ is any closed contour enclosing the strip.

The capacitance of the same TL but with the substrate replaced by air will be denoted by $C_0$.

The p.u.l. inductance, $L$, is unaffected by the presence of the dielectric substrate.

From TL theory, we have

$$L C_0 = \mu_0 \epsilon_0 \quad \text{(air line)}$$

$$L C = \mu_0 \epsilon_0 \epsilon_{eff} \quad \text{(with substrate)}$$

$$\Longrightarrow \epsilon_{eff} = \frac{C}{C_0} \quad \text{(effective dielectric constant)}$$

$$\frac{\beta}{k_0} = \sqrt{\epsilon_{eff}} \quad \text{(phase constant)}$$

$$k_0 = \omega \sqrt{\mu_0 \epsilon_0} = \frac{\omega}{c}$$

Characteristic impedance

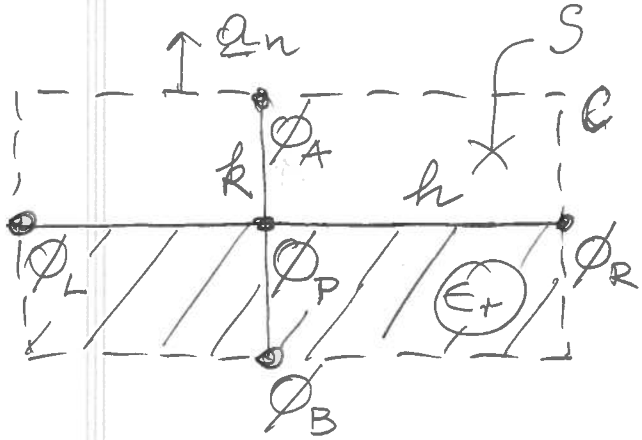
$$\tilde{Z}_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \sqrt{\frac{\mu_0 \epsilon_0}{C C_0}} = \frac{\eta_0}{\sqrt{\frac{C}{\epsilon_0} \cdot \frac{C_0}{\epsilon_0}}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \simeq 120\pi \ [\Omega]$$

Hence, all we need is $C$ (capacitance with substrate present) and $C_0$ (capacitance of an air line).

Before we can compute the capacitance, we need to find the potential distribution.

We will use a rectangular grid as illustrated in the figure above.




Integrate the Laplace's eqn.

$$\nabla_T \cdot \epsilon \nabla_T \phi = 0$$

over the surface S bounded by contour C and apply the <u>divergence theorem</u>

$$\int_S \nabla_T \cdot \epsilon \nabla_T \phi \, dS$$

$$= \oint_C \epsilon \nabla_T \phi \cdot a_n \, dl = \oint_C \epsilon \frac{\partial \phi}{\partial n} \, dl = 0$$

Using the FD method, we can <u>approximate</u> the contour integral as

$$\frac{\phi_A - \phi_P}{k} 2h + \frac{\phi_B - \phi_P}{k} 2h \epsilon_r$$

$$+ \frac{\phi_R - \phi_P}{h} (k + k\epsilon_r) + \frac{\phi_L - \phi_P}{h} (k + k\epsilon_r) = 0$$

Let $\alpha = \frac{h}{k}$

and solve for $\phi_P$:

$$\left[\phi_A - \phi_P + \epsilon_r(\phi_B - \phi_P)\right] 2\alpha^2$$
$$+ (\phi_R - 2\phi_P + \phi_L)(1+\epsilon_r) = 0$$

$$\frac{2\alpha^2}{1+\epsilon_r}\left[\phi_A - \phi_P + \epsilon_r(\phi_B - \phi_P)\right]$$
$$+ \phi_R - 2\phi_P + \phi_L = 0$$

$$\phi_L + \phi_R + \frac{2\alpha^2}{1+\epsilon_r}(\phi_A + \epsilon_r \phi_B)$$
$$- 2(1+\alpha^2)\phi_P = 0$$

$$\therefore \quad \phi_P = \frac{1}{2(1+\alpha^2)}\left[\phi_L + \phi_R + \frac{2\alpha^2}{1+\epsilon_r}(\phi_A + \epsilon_r \phi_B)\right]$$

This formula relates the potential at node P to the potentials at the nearby nodes L (left), R (right), A (above), and B (below).

If node P is away from the interface, either in the air or in the substrate region, we can still use the above formula, but with $\epsilon_r = 1$.

We can use the above formula in an iterative "successive relaxation" procedure, as follows.

First, erect a suitable grid and initialize the grid potentials on the box to zero and the grid potentials on the strip to 1V. The interior region grid potentials can be initially set to zero (or to 0.5V, say).

Next, in a systematic sweep through the grid, update each interior node potential as

$$\phi_P^{(new)} = \frac{1}{2(1+\alpha^2)} \underbrace{\left[\phi_L + \phi_R + \frac{2\alpha^2}{1+\epsilon_r}(\phi_A + \epsilon_r \phi_B)\right]}_{\text{use most up-to-date potentials available}}$$

or

$$\phi_P^{(new)} = \phi_P^{(old)} + R_P$$

$$\underbrace{R_P}_{\text{"residual"}} = \frac{1}{2(1+\alpha^2)} \underbrace{\left[\phi_L + \phi_R + \frac{2\alpha^2}{1+\epsilon_r}(\phi_A + \epsilon_r \phi_B)\right]}_{\text{use the most up-to-date potentials available at a given step}} - \phi_P^{(old)}$$

Continue the sweeps until the potentials converge (the maximum residual is smaller than some prescribed tolerance)

$$|R_P|_{max} < TOL \;(= 10^{-4}, \text{ say})$$

The convergence can be accelerated by "overrelaxing" the potentials:

$$\phi_P^{(new)} = \phi_P^{(old)} + \Omega R_P$$

relaxation factor

$$1 \leq \Omega < 2$$

The optimum value of $\Omega$ can be found by trial-and-error, or you can try the formula

$$\Omega_{opt} \simeq 2\left(1 - \frac{\pi}{\sqrt{2}} \sqrt{\frac{1}{n_a^2} + \frac{1}{n_b^2}}\right)$$

where $n_a$ and $n_b$ denote the number of subdivisions along x and y, respectively.

On the symmetry line ($x=0$), the appropriate boundary condition is

$$\left.\frac{\partial \phi}{\partial n}\right|_P \simeq \frac{\phi_L - \phi_R}{2h} = 0 \Rightarrow \phi_L = \phi_R$$

Hence, for the $x=0$ nodes, use $\phi_R$ in place of $\phi_L$.

Note that the strip and boundary nodes potentials are not updated: they remain at $\phi = 1$ and $\phi = 0$, respectively.

Also note that $\epsilon_r$ is only used at the interface nodes (outside the strip).

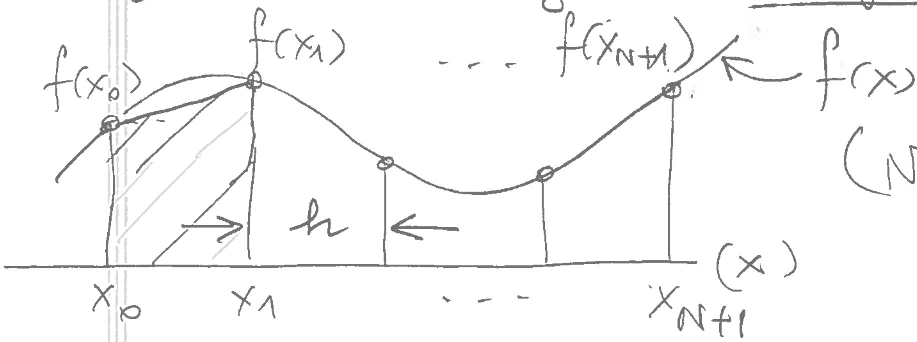
~~The~~ above procedure is referred to as "successive over-relaxation" (SOR).

Once the potentials have converged, the capacitance can be computed. Only for a symmetric air line an analytical formula is available:

$$\frac{C}{\epsilon_0} = 8\left\{\frac{W}{b} + \frac{1}{\pi}\ln\left(1 + \coth\left[\pi\left(\frac{a}{b} - \frac{W}{b}\right)\right]\right)\right\}$$

You can use it as a check of the numerical procedure, which must be used when the symmetry is not present.

The integral involved in the capacitance computation can be done using the <u>trapezoidal rule</u>:

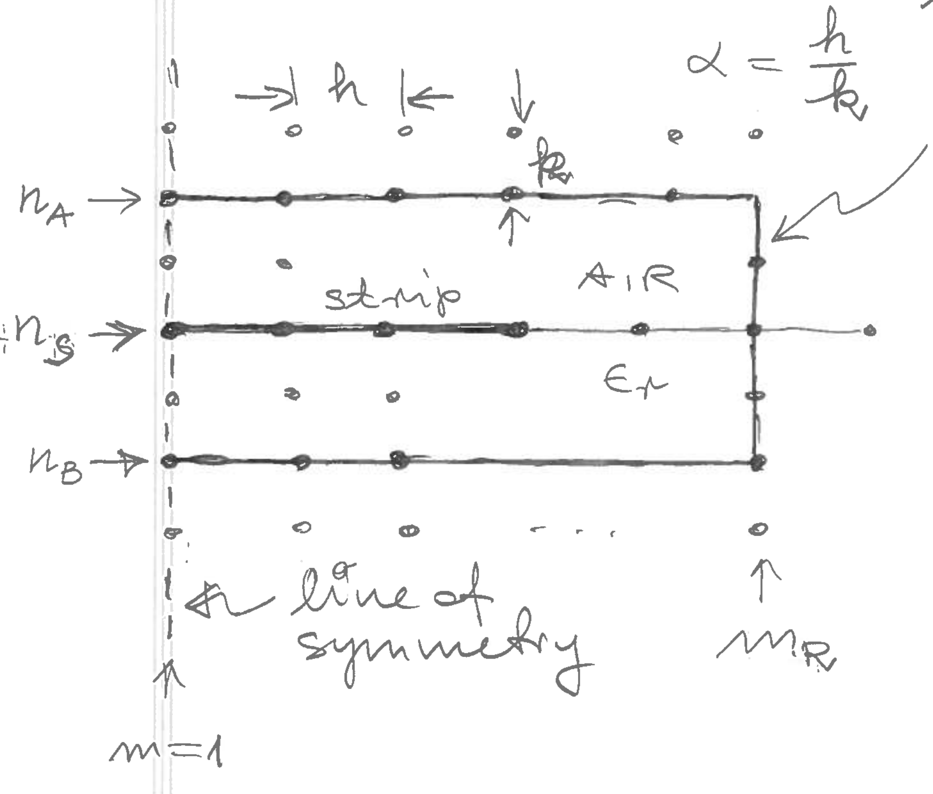



$$\int_{x_0}^{x_{N+1}} f(x)\,dx = \sum_{n=1}^{N} S_n \simeq \sum_{n=1}^{N+1} h\,\frac{f_{n-1}+f_n}{2}$$

$$\simeq h\left\{\frac{f_0+f_1}{2}+\frac{f_1+f_2}{2}+\cdots+\frac{f_{N-1}+f_N}{2}+\frac{f_N+f_{N+1}}{2}\right\}$$

$$= h\left\{\frac{f_0}{2}+\sum_{n=1}^{N} f_n+\frac{f_{N+1}}{2}\right\} = h\sum_{n=0}^{N+1}{}' f_n$$

(The prime over $\Sigma$ indicates that the first and the last term should be halved.)




$C_I$ contour for capacitance calculation can be anywhere between the strip and the box (preferably not next to the strip)

Let $\quad \phi_{mn} = \phi(x_m, y_n)$

$$\frac{C}{\epsilon_0} \simeq -2 \left\{ \epsilon_r h \sum_{m=1}^{m_R}{}' \frac{\phi_{m,n_B-1} - \phi_{m,n_B+1}}{2k} \right.$$

$$+ h \sum_{m=1}^{m_R}{}' \frac{\phi_{m,n_A+1} - \phi_{m,n_A-1}}{2k}$$

$$+ \epsilon_r k \sum_{n=n_B}^{n_S}{}' \frac{\phi_{m_R+1,n} - \phi_{m_R-1,n}}{2h}$$

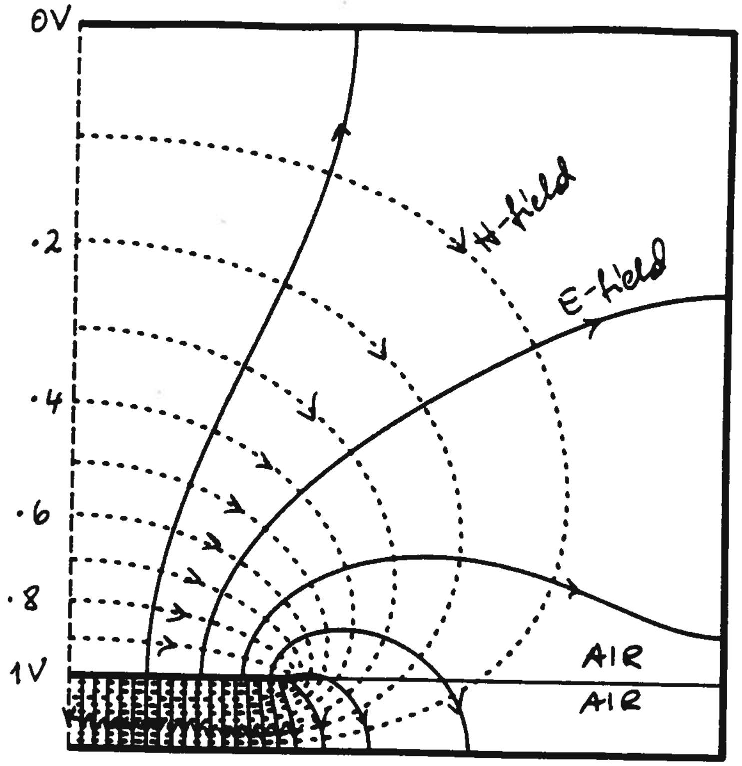
$$\left. + k \sum_{n=n_S}^{n_A}{}' \frac{\phi_{m_R+1,n} - \phi_{m_R-1,n}}{2h} \right\}$$

(because we only integrate half of the closed contour)

$$-\frac{C}{\epsilon_0} \simeq \alpha \sum_{m=1}^{m_R}{}' \Big[ \epsilon_r \left( \phi_{m,n_B-1} - \phi_{m,n_B+1} \right) + \left( \phi_{m,n_A+1} - \phi_{m,n_A-1} \right) \Big]$$

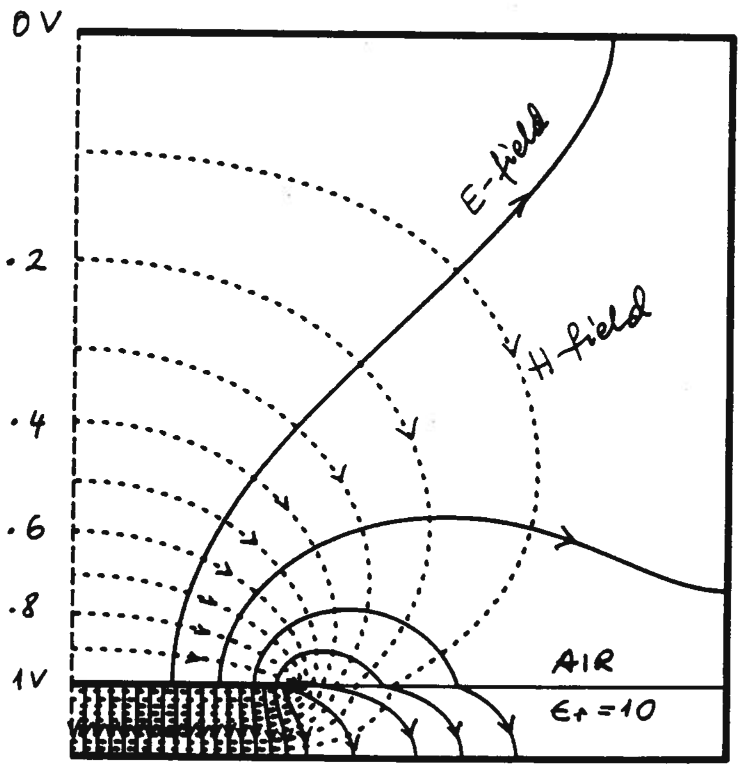
$$+ \frac{1}{\alpha} \left[ \sum_{n=n_B}^{n_S}{}' \epsilon_r \left( \phi_{m_R+1,n} - \phi_{m_R-1,n} \right) + \sum_{n=n_S}^{n_A}{}' \left( \phi_{m_R+1,n} - \phi_{m_R-1,n} \right) \right]$$

W/D = 3.000  B/D = 10.000  A/W = 3.000  ER = 1.000

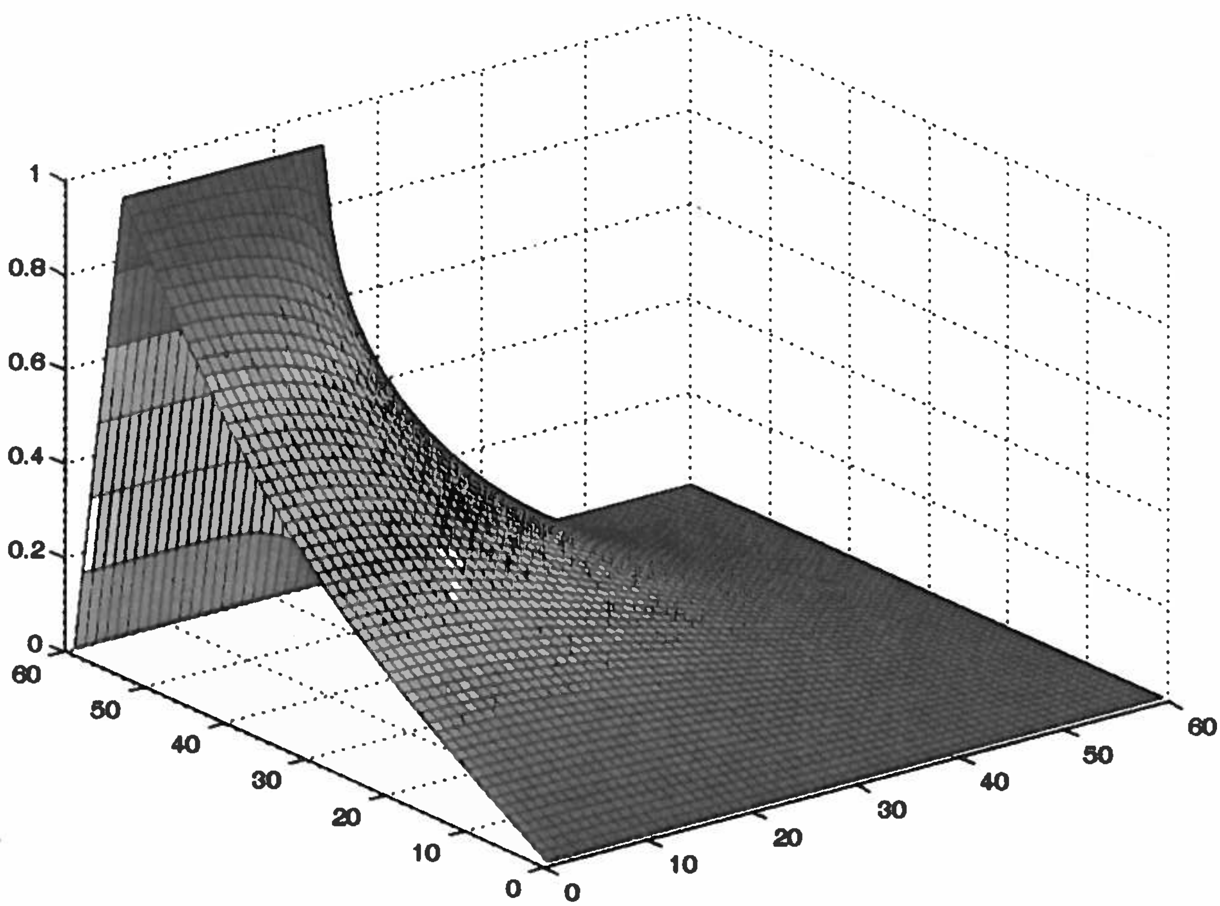


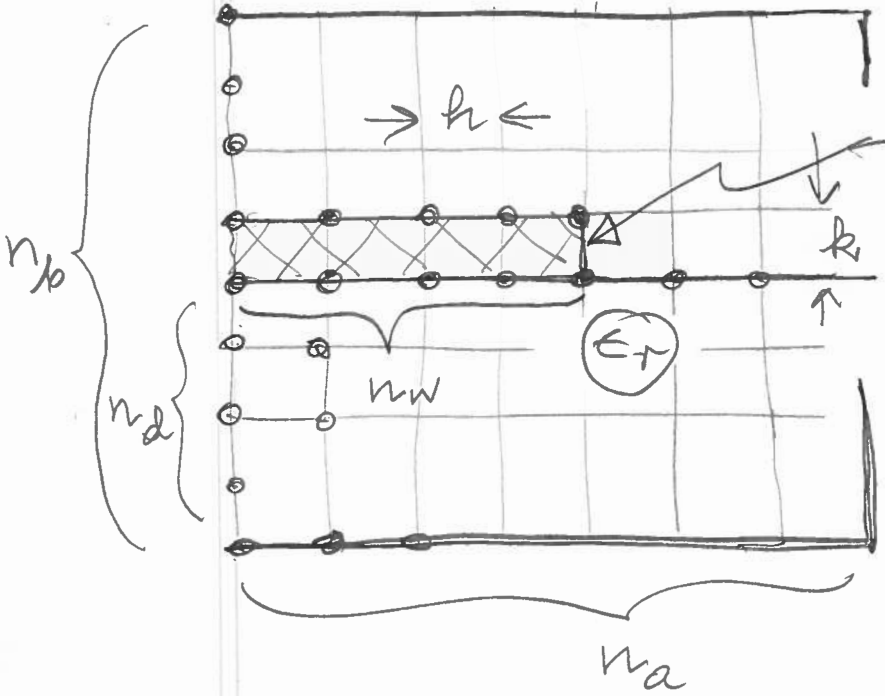

C/EO = 9.060
FLAG = 0
KOUNT = 143
OMEGA = 1.895

W/D = 3.000  B/D = 10.000  A/W = 3.000  ER = 10.000




C/EO = 71.978
FLAG = 0
KOUNT = 133
OMEGA = 1.895


1
0.8
0.6
0.4
0.2
0
60
50
40
30
20
10
0
0
10
20
30
40
50
60




assuming uniform grid, the conductor thickness must equal the vertical grid step, k.

keep the potential of all conductor nodes @ 1V