

Chapter 1 Notes

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1 Linear Equations (in General)

Each variable within the equation such as x , y , or z , for example, must have either exponent 1 or 0.

- ✓ $2x + 1 = 7$
- ✗ $x^2 = 2y$
- ✗ $\sin(x) = 2x$
- ✗ $\sqrt{x} = 3x + 1$

Def: Linear Equation

$$a_1v_1 + a_2v_2 + a_3v_3 + \cdots + a_nv_n = b$$

where:

$$\forall a \in \mathbb{R}$$

$$b \in \mathbb{R}$$

Def: System of Linear Equations / Linear System / L.S.

A collection of one or more linear equations.

(One linear equation only would be kinda silly, though it technically qualifies.)

1.1 Solutions

1.1.1 A Solution

Def: A Solution of a Linear System

Given a L.S. in $x_1, x_2, x_3 \cdots x_n$ or \mathbb{R}^n , A solution is an n-tuple corresponding to each respective variable.

$$(S_1, S_2, S_3, \cdots, S_n),$$

Let's work an example:

$$\left\{ \begin{array}{l} x + 2y = 5 \\ 2x + y = 10 \end{array} \right\} \quad \begin{array}{l} x = 5 - 2y \rightarrow 2[x = 5 - 2y] + y = 10 \rightarrow \cancel{4x} - 2y + y = \cancel{4x} \rightarrow y = 0 \\ x = 5 - 2[y = 0] \rightarrow x = 5 \end{array} \quad \text{The solution is } (x, y) = (5, 0) \quad (0e,f)$$

1.1.2 Solution Sets

Def: The Solution Set of a Linear System

The set of all possible solutions for a given L.S.

In the example in 1.1.1, there was only one possible solution: $()$

1.1.3 The Existence Question

1.2 Determination

$$\left\{ \begin{array}{l} 2x + y = 3 - x \\ x - y = 5 \\ x + 2y = 1 \end{array} \right\} \quad \begin{array}{l} \text{Linear System of } x \text{ and } y. \\ \text{Overdetermined; three equations when there are only two variables.} \end{array}$$

$$\left\{ \begin{array}{l} x - y + z = 1 \\ 2x + y - z = 3 \end{array} \right\} \quad \begin{array}{l} \text{Linear System of } x, y, \text{ and } z. \\ \text{Underdetermined; two equations when there are three variables.} \end{array}$$

Keep in mind that determination alone is not a sufficient indicator of the existence of a solution. We'll touch on this more in