Chapter 1 Notes

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1 Linear Equations (in General)

Each variable within the equation such as x, y, or z, for example, must have either exponent 1 or 0.

- ✓ $2x + 1 = 7$
- ✗ $x^2 = 2y$
- ✗ $\sin(x) = 2x$
- ✗ $\sqrt{x} = 3x + 1$

Def: Linear Equation

$$a_1v_1 + a_2v_2 + a_3v_3 + \cdots + a_nv_n = b$$

where:

$$\forall a \in \mathbb{R}$$

$$b \in \mathbb{R}$$

Def: System of Linear Equations / Linear System / L.S.
A collection of one or more linear equations.
(One linear equation only would be kinda silly, though it technically qualifies.)

1.1 Solutions

1.1.1 A Solution

Def: *A* Solution of a Linear System
Given a L.S. in $x_1, x_2, x_3 \cdots x_n$ or $\mathbb{R}^n$, *A* solution is an n-tuple corresponding to each respective variable.
$(S_1, S_2, S_3, \cdots, S_n)$,

Let's work an example:

$$\left\{\begin{matrix} x + 2y & = & 5 \\ 2x + y & = & 10 \end{matrix}\right\} \quad \begin{array}{l} x = 5 - 2y \to 2[x = 5 - 2y] + y = 10 \to \cancel{10} - 2y + y = \cancel{10} \to \mathbf{y = 0} \\ x = 5 - 2[y = 0] \to \mathbf{x = 5} \\ \text{The solution is } (x, y) = (5, 0) \end{array} \tag{ex. A}$$

1.1.2 Solution Sets

Def: The Solution Set of a Linear System
The set of all possible solutions for a given L.S.

In the example in 1.1.1, there was only one possible solution: (5,0). Therefore, the solution set of ex. A is {(5, 0)}.

$$\begin{array}{l} \{x + 2y = 1\} \\ \Rightarrow \ \{(x, y) \mid x + 2y = 1\} \\ or \ \left\{(x, \dfrac{1-x}{2}) \mid \forall x \in \mathbb{R}\right\} \\ or \ \{(1 - 2y, y) \mid \forall y \in \mathbb{R}\} \end{array} \tag{ex. B}$$

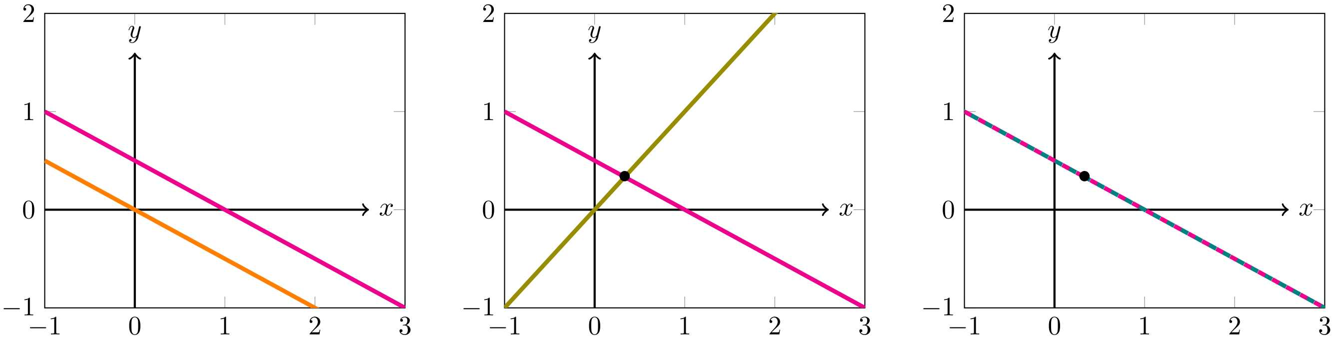
The notation you see above is the Set builder notation.
$\{x \mid \forall\ x \in \mathbb{Z}, \text{modulo}(x, 2) = 0\}$ should be read as "the set of all integers x where x mod two is zero.". This is the set of all even numbers.

1.1.3 The Existence and Uniqueness Questions

So when we have a linear system, something we like to ask is "Does a solution exist?". If a solution exists, "Does a unique solution exist?". These are *The Existence and Uniqueness Questions*. There are three solution states for any given linear system.

- No Solution $\nexists$
- Unique Solution $\exists!$
- Infinitely Many Solutions $\exists\infty$-many

$$\left\{\begin{matrix} x+2y &=& 1 \\ 2x-4y &=& 0 \end{matrix}\right\} \nexists \text{ sln.} \qquad \left\{\begin{matrix} x+2y &=& 1 \\ x-y &=& 0 \end{matrix}\right\} \exists! \text{ sln.} \qquad \left\{\begin{matrix} x+2y &=& 1 \\ 2x+4y &=& 2 \end{matrix}\right\} \exists\infty \text{ sln.}$$



1.2 Determination

$$\left\{\begin{matrix} 2x+y &=& 3-x \\ x-y &=& 5 \\ x+2y &=& 1 \end{matrix}\right\}$$

Linear System of x and y.
Overdetermined; three equations when there are only two variables.

$$\left\{\begin{matrix} x-y+z &=& 1 \\ 2x+y-z &=& 3 \end{matrix}\right\}$$

Linear System of x, y, and z.
Underdetermined; two equations when there are three variables.

Keep in mind that determination alone is not a sufficient indicator of the existence of a solution. We'll touch on this more in