This Knowledge Check must be done with YOUR OWN WORK! Winter 2021

CH1 Knowledge Check

MATH 2B

Registered Name (print): BEN STEED CULT

I pledge to support the mission of Foothill College and to demonstrate its core values by upholding academic integrity in this assignment.

Signature: your mom

Materials permitted for this assessment: Course Textbook, Lecture Notes, Your Own Course Notes **ONLY**

No Calculator, No Online Resources, No Other Human Being, No STEM CENTER Help

Directions: Please read the following directions carefully.

1. Honor yourself with academic integrity.

2. Please write your **full name clearly** which is **registered** for the class.

3. Read all questions **carefully** before responding.

4. Please show **all your work clearly** to get partial credit, unless instructed otherwise. You are to defend all your answers for full credit in this assessment. That means you need to show work. You must defend your answers **analytically** (with algebra or calculus), unless stated otherwise. For open-ended questions, you can earn significant points for showing clear, well-organized work even if your answer is not correct. Similarly, you will lose significant points if you don't show your work or display an answer (even if correct) with little or no work. ***If your work is hard to read, it will be considered wrong.*** :) :)

5. Except for typographical errors and omissions, questions will neither be interpreted, nor explained. Comprehension of all questions is considered a part of the assessment..

6. You are graded for correct mathematical notation as well as for the correct answer(s) for each question.

7. Simplify your final answers and write appropriate units when applicable.

8. You must show your work and use concepts you have learned in Math1D.

ENJOY ☹

1. (5 points) Consider the following system: $\begin{cases} x+y+z=1 \\ 2x+y+z=h \\ x-y+kz=1 \end{cases}$

(A) Apply the elementary row operations to the corresponding augmented matrix to the system above in order to write it in an echelon form.
(solution)

$A=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & k \end{bmatrix}$ $b=\begin{bmatrix} 1 \\ h \\ 1 \end{bmatrix}$ $[A \mid b]$ $\begin{matrix} R_2-2R_1 \\ R_3-R_1 \end{matrix}$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & h-2 \\ 0 & -2 & k & 0 \end{array}\right]$ $\begin{matrix} R_2(-1) \\ R_3+2R_2 \end{matrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2-h \\ 0 & 0 & k & 4-2h \end{bmatrix}$ REF

(B) Determine the values of k and h such that the linear system has a specific number of solutions stated in the table given below.
(solution)

# of solutions	k	h
000 = non zero No solution	∅	not 2
pivot point in every row A unique solution	not ∅	ℝ
free variable Many solutions	∅	2

k=0 h=2

(C) When the system has many solutions in (B) above, find the general solution of the system in parametric vector form.
(solution)

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ REF; RREF would be nice
R_1-R_2

$\left[\begin{array}{ccc:c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$A \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{x}=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

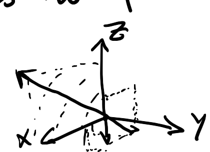
$x = 1 - z$
$y = 0$
z free

z varies

$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - z\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(D) Describe the solution set in (C) above geometrically.
(solution)

line does not pass thru origin, z goes up one, x goes down one
y is fixed at zero



2. (5 points) Let $A = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3] = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 2 & 0 & -1 \end{bmatrix}$.

span is all linear combinations $= x_1 a_1 + x_2 a_2 + x_3 a_3$ where $x_1 x_2 x_3 \in \mathbb{R}$

(A) Do the columns in A span R^3? Explain why or why not. can this linear combination generate all points in $\mathbb{R}^3$?

(solution)

$$\xrightarrow{R_3-2R_1} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \overset{R_3-R_2}{\sim} \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

two pivots no way to control bottom row

$$\longrightarrow c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

No!! Only R^2

Stuck! Bottom row is always zero no matter the coefficients

(B) If the answer to (A) above was 'No', then find all the vectors that are linear combinations of the columns in A. If the answer to (A) above was 'Yes', then skip this question.

w

(solution)

RREF

$$\overset{2R_1}{\sim} \begin{bmatrix} 2 & -2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_1+R_2}{\sim} \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underset{R_2/2}{\overset{R_1/2}{\sim}} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

Set

$\therefore \left\{ A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \mid \text{where } \forall c_1, c_2, c_3 \in \mathbb{R} \right\}$

(C) Determine whether the columns in A are linearly independent or not. Explain why or why not.

(solution)

$A\vec{c} = 0$ only if $\vec{c} = 0$

(D) If your answer to (C) was 'No', then write a dependence relation among the column vectors. If your answer to (C) was 'Yes', then skip this question.

(solution)

(E) Describe the set Span$\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ geometrically, where $\mathbf{a_1}, \mathbf{a_2}$ and $\mathbf{a_3}$ are the columns of A.

(solution)

3. (5 points) Let $T: R^3 \to R^2$ be a transformation defined by $T(x,y,z) = (x+2y,\ x+y+3z)$.

all power 1

(A) Determine whether T is a matrix transformation or not. Explain why or why notl
(solution) From class a linear transformation from $R^n \to R^m$ is always a matrix transformation.

Yes

(B) If the answer to (A) is 'YES', then find the standard matrix for T. If not, skip this question.
(solution)

$$\begin{bmatrix} x+2y \\ x+y+3z \end{bmatrix} = x\begin{bmatrix}1\\1\end{bmatrix} + y\begin{bmatrix}2\\1\end{bmatrix} + z\begin{bmatrix}0\\3\end{bmatrix} \quad \text{factor } \begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

(C) Use the definition of Linear Transformation to prove that T is linear.
(solution)

$\forall x_1, x_2 : T(x_1 + x_2) = T(x_1) + T(x_2)$

$\forall c, x : cT(x) = T(cx)$

combined not derivative, variable d.

$\forall c, d, x, y : T(cx + dy) = cT(x) + dT(y)$

$$(cx_1 + dy_1)\begin{bmatrix}1\\1\end{bmatrix} + (cx_2 + dy_2)\begin{bmatrix}2\\1\end{bmatrix} + (cx_3 + dy_3)\begin{bmatrix}0\\3\end{bmatrix}$$

$$=$$

$$c\left(x_1\begin{bmatrix}1\\1\end{bmatrix} + x_2\begin{bmatrix}2\\1\end{bmatrix} + x_3\begin{bmatrix}0\\3\end{bmatrix}\right) + d\left(y_1\begin{bmatrix}1\\1\end{bmatrix} + y_2\begin{bmatrix}2\\1\end{bmatrix} + y_3\begin{bmatrix}0\\3\end{bmatrix}\right)$$

Distribute c and d and factor by A_1, A_2, A_3

QED.

(D) Find all the preimage(s) of $(-1,1)$ under the transformation if there is any. If not, explain why not.
(solution)

input output $\vec{b}$

$$T(\vec{x}) = \begin{bmatrix}-1\\1\end{bmatrix}$$

$$A\vec{x} = \begin{bmatrix}-1\\1\end{bmatrix}$$

$$[A \mid \vec{b}] \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 1 & 1 & 3 & 1 \end{array}\right] \overset{R_2 - R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & -1 & 3 & 0 \end{array}\right] \text{ REF}$$

$$\overset{R_1 + 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 6 & -1 \\ 0 & -1 & 3 & 0 \end{array}\right] \quad R_2(-1) \left[\begin{array}{ccc|c} 1 & 0 & 6 & -1 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

x_3 free

$$\vec{x} = \begin{bmatrix}-1\\0\end{bmatrix} - x_3\begin{bmatrix}6\\-3\end{bmatrix}$$

Pre images of $(-1,1)$ $\therefore \left\{ \begin{bmatrix}-1\\0\end{bmatrix} - k\begin{bmatrix}6\\-3\end{bmatrix} \mid \forall k \in \mathbb{R} \right\}$