20398315

Cole Garnor

I pledge to support the mission of Foothill College and to demonstrate the core values by upholding academic Integrity.

Signed Cole Garnor

1.A

$$A=\begin{bmatrix}1&1&-1\\1&2&K\\1&-1&1\end{bmatrix}\quad b=\begin{bmatrix}1\\h\\3\end{bmatrix}\rightarrow[A\mid b]$$

$$\left[\begin{array}{ccc|c}1&1&-1&1\\1&2&K&h\\1&-1&1&3\end{array}\right]\underset{R_3-R_1}{\overset{R_2-R_1}{\sim}}\left[\begin{array}{cccc}1&1&-1&1\\0&1&K+1&h-1\\0&-2&0&3\end{array}\right]$$

$$\overset{R_3+2R_2}{\sim}\left[\begin{array}{ccc|c}1&1&-1&1\\0&1&K+1&h\\0&0&2K+2&3+2h\end{array}\right]\text{ REF}$$

1.B

		$K\in\mathbb{R}$	$h\in\mathbb{R}$
No solution $000\neq$ non zero	so	-1	any except $-\frac{3}{2}$
A unique solution pivot in all row	so	not -1	any
Many solutions free variable	so	-1	$-\frac{3}{2}$

1.C

$$\left[\begin{array}{ccc|c}1&1&-1&1\\0&1&K+1&h\\0&0&0&0\end{array}\right]\overset{R_1-R_2}{\sim}\left[\begin{array}{cccc}1&0&-2-K&1\\0&1&K+1&h\\0&0&0&0\end{array}\right]\text{ RREF}$$

$x=1-x_3(-2-K)$

$y=h-x_3(K+1)$

$z=$ free

~~subs K~~

cont. retry remember to subs $K=-1$ $h=-\frac{3}{2}$

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~~P.2~~ 1.C cont.

z free

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 \end{array}\right] \overset{R_1 - R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 2.5 + \cancel{R_3}\, z$
$y = -1.5$
z :: free

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2.5 \\ -1.5 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \therefore \vec{x} = \underbrace{\begin{bmatrix} 2.5 \\ -1.5 \\ 0 \end{bmatrix}}_{=\vec{v}} + k \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{=\vec{u}}$$

~~P.2~~ 1.D

~~$\vec{v}$ and $\vec{u}$ are linearly independent and so~~

It's a line that does not touch the origin. span$\{\vec{u}\}$ except translated by $\vec{v}$.

2.A Do cols of A span $\mathbb{R}^3$? Yes.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 3 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

let us have $b = \emptyset$.
we will find pivot points
I will omit [A | b] since b always $\emptyset$

$$\overset{R_1 \text{ swap } R_2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & \emptyset \\ 1 & 1 & 0 & 1 & \emptyset \\ 2 & 1 & 2 & 4 & \emptyset \end{array}\right] \underset{R_3 - 2R_1}{\overset{R_2 - R_1}{\sim}} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -2 & -2 \\ 0 & 1 & 0 & -1 \end{bmatrix} \text{ swap } R_2, R_3$$

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -2 & -2 \end{bmatrix} \overset{R_3 - R_2}{\sim} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -1 \end{bmatrix}$$

3 pivot points in REF

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

can control x, y, z with c_1, c_2, c_3

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2.B skipped

2.C linear independance is when trivial solution is the only solution to a homogeneous system. I think? See below

$A\vec{x} = \vec{0}$ ONLY when $x = \vec{0}$

Let us take the REF matrix from last page

$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}$$
$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -1 \end{bmatrix}$$

I will now demonstrate that I can generate v_4 with a linear combination of only v_1, v_2, and v_3.

$$\begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

v_4 is in Span$\{v_1, v_2, v_3\}$ so cols of A are <u>Linearly Dependant</u>

2.D skipped

Cont...

2.E describe span$\{a_1, a_2, a_3, a_4\}$
Remembering 2.A we know that the $a_1 a_2 a_3$ can span R^3. Since this is a 3x4 matrix (3 rows 4 cols), we see that the output space is R^3 and the span is all of that space

Span is R^3

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A is a 3x2 matrix since $R^2 \to R^3$

$T\left(\begin{bmatrix}2\\0\end{bmatrix}\right) = \begin{bmatrix}2\\4\\-2\end{bmatrix}$ $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\6\\-3\end{bmatrix}$.

I feel like there are a lot of transformations that can do this. Maybe infinitely many?
Oh wait $\begin{bmatrix}2\\0\end{bmatrix}$ is $2\hat{i}$ and $\begin{bmatrix}1\\1\end{bmatrix}$ is $\hat{i}+\hat{j}$.

~~New $\hat{i}$ is~~ if $2T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = T\left(2\begin{bmatrix}1\\0\end{bmatrix}\right)$

new $\hat{i}$ basis vector is $\begin{bmatrix}1\\2\\-1\end{bmatrix}$ $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) + T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$

$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) =$ so $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) - T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$

cont.

new $\hat{j} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$

therefore $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{bmatrix}$ solution

$A \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$ ✓

$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$ ✓

3.B image of $\begin{bmatrix} x \\ y \end{bmatrix}$

$A \begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} x + 2y \\ 2x + 4y \\ -x - 2y \end{bmatrix} \rightarrow$

$(x + 2y,\ 2x + 4y,\ -x - 2y)$

3.C well, we already know that it is linear otherwise I could not get A. But I will prove it anyways. Cont...

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3.C combined theorem:

$$\forall a,b\in\mathbb{R}\ \forall \vec{x},\vec{y}\in\mathbb{R}^2: T(a\vec{x}+b\vec{y}) = aT(\vec{x})+bT(\vec{y})$$

$\cancel{a\vec{x}+b\vec{y} = a(x_1 + x_2 + x_3) + b(y_1+y_2+y_3)}$

x is a vector y is a vector

$\cancel{aT(x)+bT(y) = a\left(x_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+x_2\begin{bmatrix}2\\4\\-2\end{bmatrix}\right)+b\left(y_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+y_2\begin{bmatrix}2\\4\\-2\end{bmatrix}\right)}$

$\cancel{T(ax+by) = \begin{bmatrix}1&2\\2&4\\-1&-2\end{bmatrix}(a\vec{x}_1+a\vec{x}_2+by_1+by_2)}$

$$A\left(a\vec{x}+b\vec{y}\right) = A\left(\begin{bmatrix}ax_1+by_1\\ax_2+by_2\end{bmatrix}\right)$$

$$=(ax_1+by_1)\begin{bmatrix}1\\2\\-1\end{bmatrix}+(ax_2+by_2)\begin{bmatrix}2\\4\\-2\end{bmatrix}$$

$\Updownarrow$

$$aT(\vec{x})+bT(\vec{y}) = a\left(x_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+x_2\begin{bmatrix}2\\4\\-2\end{bmatrix}\right)+b\left(y_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+y_2\begin{bmatrix}2\\4\\-2\end{bmatrix}\right)$$

those are equivalent to

$$ax_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+ax_2\begin{bmatrix}2\\4\\-2\end{bmatrix}+by_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+by_2\begin{bmatrix}2\\4\\-2\end{bmatrix}$$

QED It is linear

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~~2.F~~ ~~[illegible]~~

~~[illegible]~~

3.D preimage of (1,2,-1)

$$A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 2 \\ -1 & -2 & -1 \end{array}\right] \underset{R_3+R_1}{\overset{R_2-2R_1}{\sim}} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$x = 1 - 2y$
$y =$ free

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\therefore \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -2 \\ 1 \end{bmatrix} \middle| \forall k \in \mathbb{R} \right\}$ is a preimage of (1,2-1)

3.E one to one is ~~one input~~ one output cannot have different inputs. Above, we see (1,2,-1) has different inputs

3.F ~~[illegible] is onto~~ T cannot be onto $\mathbb{R}^3$ because there are not enough $\mathbb{R}^2$ to cover $\mathbb{R}^3$.