Type Checking

Cosmin E. Oancea
cosmin.oancea@diku.dk

Department of Computer Science (DIKU)
University of Copenhagen

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Structure of a Compiler

Program text
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Lexical analysis
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Syntax tree
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Interpretation Recap

Compiler	*vs.*	Interpreter
source program	*input*	*source program* *input*
$\downarrow$	$\downarrow$	$\downarrow$ $\downarrow$
$\boxed{\text{Compiler}}$	$\boxed{\text{Target Program}}$	$\boxed{\text{Interpreter}}$
$\downarrow$	$\downarrow$	$\downarrow$
target program	*output*	*output*

The interpreter directly *executes* one by one the operations specified in the *source program* on the *input* supplied by the user, by using the facilities of its implementation language.

Why interpret? Debugging, Prototype-Language Implementation, etc.

Synthesized vs Inherited Attributes

A compiler phase consists of one or several traversals of the AbsSyn. We formalize it via *attributes*:

Inherited: info passed downwards on the AbsSyn traversal, i.e., from root to leaves. Think: helper structs. Example?

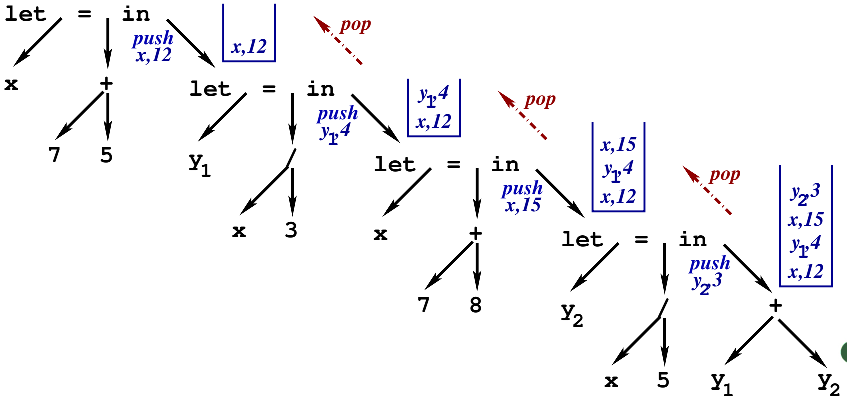
Synthesized: info passed upwards in the AbsSyn traversal, i.e., from leaves to the root. Think: the result. Example?

Both: Information may be synthesized from one subtree and may be inherited/used in another subtree (or at a latter parse of the same subtree). Example?

Example of Inherited Attributes

The variable and function symbol tables, i.e., *vtable* and *ftable*, in the interpretation of an expression:

$Eval_{Exp}(Exp, vtable, ftable) = \ldots$



Example of Synthesized Attributes

The interpreted value of an expression / program is synthesized.

Example of both *synthesized* and *inherited* attributes:

$$vtable = Bind_{TypeIds}(TypeIds, args)$$
$$ftable = Build_{ftable}(Funs)$$

and used in the interpretation of an expression.

Interpretation vs Compilation Pros and Cons

+ Simple (good for impatient people).

+ Allows easy modification / inspection of the program at run time.

– Typically, it does not discover all type errors. Example?

– Inefficient execution:

  - Inspects the AbSyn repeatedly, e.g., symbol table lookup.
  - Values must record their types.
  - The same types are checked over and over again.
  - No "global" optimizations are performed.

Idea: Type check and optimize as much as you can statically, i.e., before running the program, and generate optimized code.

Type System / Type Checking

Type System: a set of logical rules that a legal program must respect.

Type Checking verifies that the type system's rules are respected. Example of type rules and type errors:

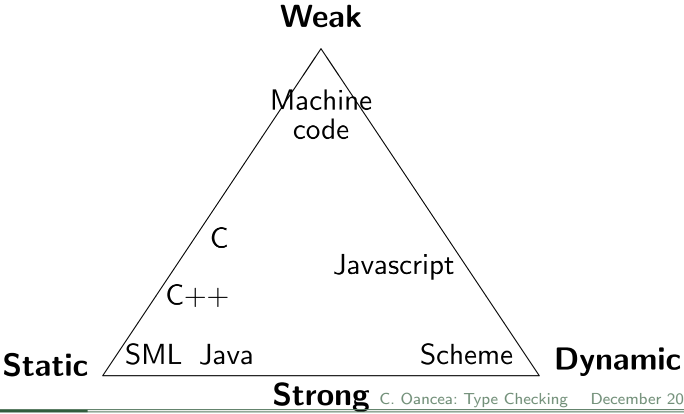
- $+, -$ expect integral arguments: `a + (b=c)`
- `if`-*branch expressions have the same type*:
  `let a = ( if (b = 3) then 'b' else 11 ) in ...`
- *the type and number of formal and actual arguments match:*
  ```
  fun int sum ([int] x) = reduce(op +, 0, x)
  fun [bool] main() = map(sum, iota(4))
  ```
- other rules?

Some language invariants cannot be checked statically: Examples?

Type System

Static: Type checking is performed before running the program.
Dynamic: Type checking is performed while running the program.

Strong: All type errors are caught.
Weak: Operations may be performed on values of wrong types.



1. Interpretation Recap: Synthesized/Inherited Attributes
2. Type-System Characterization
3. Type Checker for Fasto Without Arrays (Generic Notation)
4. Advanced Concepts: Type Inference
5. Type Checker for Fasto With Arrays (SML Code)

What Is The Plan

The type checker builds (statically) unique types for each expression, and reports whenever a type rule is violated.

As before, we logically split the ABSYN representation into different *syntactic categories*: expressions, function decl, etc.,

and implement each syntactic category via one or several functions that use case analysis on the ABSYN-type constructors.

In practice we work on ABSYN, but here we keep implementation generic by using a notation that resembles the language grammar.

For symbols representing variable names, we use *name*(**id**) to get the name as a string. A type error is signaled via function **error()**.

Symbol Tables Used by the Type Checker

vtable binds variable names to their *types*, e.g., `int`, `char`, `bool` or arrays, e.g., `[[[int]]]`.

ftable binds function names to their *types*. The type of a function is written $(t_1, ..., t_n) \rightarrow t_0$, where t_1, ..., t_n are the argument types and t_0 is the result type.

Type Checking an Expression (Part 1)

Inherited attributes: *vtable* and *ftable*.

Synthesized attribute: the expression's type.

$Check_{Exp}(Exp, vtable, ftable) =$ `case` Exp `of`	
num	`int`
id	$t = lookup(vtable, name(\textbf{id}))$ $if\ (\ t = unbound\ )\ then$ **error**(); `int` $else\ \ t$
Exp_1 + Exp_2	$t_1 = Check_{Exp}(Exp_1, vtable, ftable)$ $t_2 = Check_{Exp}(Exp_2, vtable, ftable)$ $if\ (\ t_1 =$ `int` $and\ t_2 =$ `int` $)\ then$ `int` $else$ **error**(); `int`
Exp_1 = Exp_2	$t_1 = Check_{Exp}(Exp_1, vtable, ftable)$ $t_2 = Check_{Exp}(Exp_2, vtable, ftable)$ $if\ (\ t_1 = t_2\ )\ then$ `bool` $else$ **error**(); `bool`
...	

Type Checking an Expression (Part 2)

$Check_{Exp}(Exp, vtable, ftable) =$ `case` Exp `of`	
...	
`if` Exp_1 `then` Exp_2 `else` Exp_3	$t_1 = Check_{Exp}(Exp_1, vtable, ftable)$ $t_2 = Check_{Exp}(Exp_2, vtable, ftable)$ $t_3 = Check_{Exp}(Exp_3, vtable, ftable)$ $if\ (\ t_1 = \texttt{bool}\ and\ t_2 = t_3\ )\ \ then\ \ t_2$ $else\ \ \textbf{error}();\ t_2$
`let` **id** `=` Exp_1 `in` Exp_2	$t_1 = Check_{Exp}(Exp_1, vtable, ftable)$ $vtable' = bind(vtable, name(\textbf{id}), t_1)$ $Check_{Exp}(Exp_2, vtable', ftable)$
id ($Exps$)	$t = lookup(ftable, name(\textbf{id}))$ $if\ (\ t = unbound\ )\ then\ \ \textbf{error}();\ \texttt{int}$ $else\ \ ((t_1, \ldots, t_n) \rightarrow t_0) = t$ $[t'_1, \ldots, t'_m] = Check_{Exps}(Exps, vtable, ftable)$ $if\ (\ m = n\ and\ \ t_1 = t'_1, \ldots, t_n = t'_n\ )$ $then\ \ t_0$ $else\ \ \textbf{error}();\ t_0$

Type Checking a Function (Declaration)

- creates a *vtable* that binds the formal args to their types,
- computes the type of the function-body expression, named t_1,
- and checks that the function's return type equals t_1.

$Check_{Fun}(Fun, ftable) =$ `case` Fun `of`	
$Type$ **id** $($ $TypeIds$ $)$ $=$ Exp	$vtable = Check_{TypeIds}(TypeIds)$ $t_1 = Check_{Exp}(Exp, vtable, ftable)$ $if\ (\ t_1 \neq Type\ )$ $then$ **error**(); `int`

$Check_{TypeIds}(TypeIds) =$ `case` $TypeIds$ `of`	
$Type$ **id**	$bind(SymTab.empty(),$ **id**$, Type)$
$Type$ **id** , $TypeIds$	$vtable = Check_{TypeIds}(TypeIds)$ $if\ (\ lookup(vtable,$ **id**$) = unbound\ )$ $then\ \ bind(vtable,$ **id**$,\ Type)$ $else$ **error**(); $vtable$

Type Checking the Whole Program

- builds the functions' symbol table,
- type-checks all functions,
- checks that a `main` function of no args exists.

$Check_{Program}(Program) = \texttt{case}\ Program\ \texttt{of}$	
$Funs$	$ftable = Get_{Funs}(Funs)$ $Check_{Funs}(Funs, ftable)$ $if\ (\ lookup(ftable, \texttt{main}) \neq (\ ) \rightarrow \alpha\ )$ $then\ \ \textbf{error}()$

$Check_{Funs}(Funs, ftable) = \texttt{case}\ Funs\ \texttt{of}$	
Fun	$Check_{Fun}(Fun, ftable)$
$Fun\ Funs$	$Check_{Fun}(Fun, ftable)$ $Check_{Funs}(Funs, ftable)$

Building the Functions' Symbol Table

$Get_{Funs}(Funs) = $ case $Funs$ of	
Fun	$(f, t) = Get_{Fun}(Fun)$ $bind(SymTab.empty(),\ f,\ t)$
$Fun\ Funs$	$ftable = Get_{Funs}(Funs)$ $(f, t) = Get_{Fun}(Fun)$ $if\ (\ lookup(ftable, f) = unbound\ )$ $then\ \ bind(ftable, f, t)$ $else\ \ $ **error**$()$; $ftable$

$Get_{Fun}(Fun) = $ case Fun of	
$Type$ **id** $(\ TypeIds\ )\ =\ Exp$	$[t_1, \ldots, t_n] = Get_{Types}(TypeIds)$ $(\ \mathbf{id},\ (t_1, \ldots, t_n)\ \rightarrow\ Type)$

$Get_{Types}(TypeIds) = $ case $TypeIds$ of	
$Type$ **id**	$[Type]$
$Type$ **id** $,\ TypeIds$	$Type :: Get_{Types}(TypeIds)$

Advanced Type Checking

Data-Structures: Represent the data-structure type in the symbol table and check operations on the values of this type.

Overloading: Check all possible types. If multiple matches, select a default typing or report errors.

Type Conversion: if an operator takes arguments of wrong types then, if possible, convert to values of the right type.

Polymorphic/Generic Types: Check whether a polymorphic function is correct for all instances of type parameters. Instantiate the type parameters of a polymorphic function, which gives a monomorphic type.

Type Inference: Refine the type of a variable/function according to how it is used. If not used consistently then report error.

Type Inference for Polymorphic Functions

Key difference: type rules check whether types can be "unified", rather than type equality.

```
if ...  then ([],      [1,2,3], [])
        else (['a','b'], [],    [])
```

When we do not know a type we use a (fresh) type variable:

then: $\forall\alpha.\forall\beta.list(\alpha) * list(int) * list(\beta)$

else: $\forall\gamma.\forall\delta.list(char) * list(\gamma) * list(\delta)$

notation: use Greeks for type vars, omit $\forall$ but use fresh names.

Types t_1 and t_2 can be unified $\Leftrightarrow \exists$ substitution $S \mid S(t_1) = S(t_2)$.

Most-General Unifier: $list(char) * list(int) * list(\beta)$

$$S = \{\alpha \leftarrow char, \gamma \leftarrow int, \alpha \leftarrow \beta\}$$

Example: Inferring the Type of SML's `length`

```
fun length(x) = if null(x) then 0
                else length( tl(x) ) + 1
```

EXPRESSION	:	*TYPE*	*UNIFY*
length	:	$\beta \rightarrow \gamma$	
x	:	β	
if	:	$bool * \alpha_i * \alpha_i \rightarrow \alpha_i$	
null	:	$list(\alpha_n) \rightarrow bool$	
null(x)	:	$bool$	$list(\alpha_n) \equiv \beta$
0	:	int	$\alpha_i \equiv int$
$+$	:	$int * int \rightarrow int$	
tl	:	$list(\alpha_t) \rightarrow list(\alpha_t)$	
$tl(x)$	:	$list(\alpha_t)$	$list(\alpha_t) \equiv list(\alpha_n)$
$length(tl(x))$	:	γ	$\gamma \equiv int$
$length(tl(x)) + 1$	:	int	
if(..) then .. else ..	:	int	

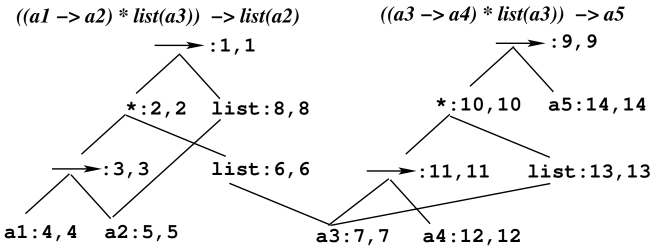
Most-General Unifier Algorithm

- a type expression is represented by a graph (typically acyclic),
- a set of unified nodes has one representative, REP, (initially each node is its own representative),
- `find(n)` returns the representative of node `n`.
- `union(m,n)` merges the equivalence classes of `n` and `m`:
  - if `n` is a type constructor then REP = `n`, (and similar for `m`),
  - otherwise REP = either `n` or `m`.

boolean unify(Node m, Node n)

$s = \texttt{find}(m);\ \ t = \texttt{find}(n);$
if ($s = t$) **then return** *true*;
else if (*s and t are the same basic type*) **then return** *true*;
else if (*s or t represent a type variable*) **then** $\texttt{union}(s,t)$; **return** *true*;
else if (*s and t are the same type – constructor*
with children s_1, .., s_k *and* t_1, .., t_k, $\forall\, k$) **then**
$\texttt{union}(s,t)$; **return** $\texttt{unify}(s_1,\ t_1)$ *and* .. *and* $\texttt{unify}(s_k, t_k)$;
else **return** *false*;

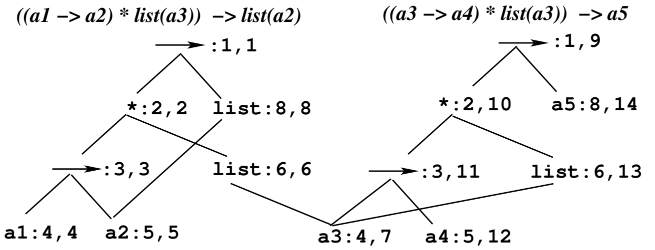
Most-General Unifier Example



Each node is annotated with two integer values:

– REP

– node's identifier



The unifier is constructed by combining nodes' REPs:

*((a1 -> a2) * list(a2)) -> list(a2)*

What Changes When Adding Arrays? (part 1)

Polymorphic Array Constructors and Combinators:

replicate: $\forall\ \alpha.\ \mathit{int}\ * \alpha\ \rightarrow\ [\alpha]$,
`replicate(3, a)` $\equiv$ `{a, a, a}`.

map: $\forall\ \alpha.\ \forall\ \beta.\ (\alpha\ \rightarrow\ \beta\ )\ *\ [\alpha]\ \rightarrow\ [\beta]$,
`map`$(f, \{x_1, .., x_n\})\ \equiv\ \{f(x_1), .., f(x_n)\}$

reduce: $\forall\ \alpha.\ (\alpha\ *\ \alpha\ \rightarrow\ \alpha)\ *\ \alpha\ *\ [\alpha]\ \rightarrow\ \alpha$
`reduce`$(g,\ e,\ \{x_1\ , ..,\ x_n\})\ \equiv\ g(..(g(e,\ x_1)..,\ x_n)$

Question 1: Do we need to implement type inference?

Answer 1: No! FASTO supports a fixed set of polymorphic function whose types are know (or if you like, very simple type inference).

What Changes When Adding Arrays? (part 1)

Polymorphic Array Constructors and Combinators:

replicate: $\forall\, \alpha.\ int\ * \alpha\ \rightarrow\ [\alpha]$,
`replicate(3, a)` $\equiv$ {`a, a, a`}.

map: $\forall\, \alpha. \forall\, \beta. (\alpha\ \rightarrow\ \beta\ )\ *\ [\alpha]\ \rightarrow\ [\beta]$,
`map`$(f, \{x_1, .., x_n\})\ \equiv\ \{f(x_1), .., f(x_n)\}$

reduce: $\forall\, \alpha.\ (\alpha\ *\ \alpha\ \rightarrow\ \alpha)\ *\ \alpha\ *\ [\alpha]\ \rightarrow\ \alpha$
`reduce`$(g,\ e,\ \{x_1\ , ..,\ x_n\})\ \equiv\ g(..(g(e,\ x_1)..,\ x_n)$

Question 2: Assuming type-checking is successful, can we forget the type of `replicate(3, a)`?

Answer 2: No, the type of a needs to be remembered for machine-code generation, e.g., `a : int` *vs* `a : char`.
Same for array literals, array indexing, map, reduce, etc.

What Changes When Adding Arrays? (part 2)

`map` : $\forall\,\alpha.\,\forall\,\beta.\,(\alpha \rightarrow \beta) * [\alpha] \rightarrow [\beta]$. Type rule for `map(f, x)`:

- compute t, the type of `x`, and check that $t \equiv [t_{in}]$ for some t_{in}.
- check that $f : t_{in} \rightarrow t_{out}$
- if so then `map(f, x)` : $[t_{out}]$.

ABSYN representation for `map`:

- `Exp =...| Map of string * Exp * Type * Type * pos`,
- Before type checking, both types are `UNKNOWN`. After:
- the first `Type` is the input-array element type, e.g., t_{in},
- the second `Type` is the output-array element type, e.g., t_{out}.

Type checking an expression/program now results in a new exp/prg, where all the `Type` fields of an expression are filled with known types.

The Gist of Type.sml: Whole Program

Type-Checker Entry Point is Function CheckProgram

```
type TabEntry = string * (Type list * Type)
val funTab : TabEntry list ref = ref []

fun checkProgram(funDecs : Fasto.FunDec list) : Fasto.FunDec list =
  let val tab = ("ord",([Fasto.Char (0,0)], Fasto.Int (0,0)))::
                ("chr",([Fasto.Int (0,0)], Fasto.Char (0,0)))::
                (List.map getType funDecs) (*ftable for declared funs*)
      (* Oversimplified: what did I omit to check? *)

      val () = funTab := tab (*global,to avoid passing it as param*)

      (* type checking each FunDec results in a new FunDec *)
      val decorated = List.map checkAndDecorate funDecs

      (* check main function exists and has type () -> int *) ...
  in decorated
  end
```

The Gist of Type.sml: Type Checking a Function

Compute the type of fun's body, check that it matches the result type

```
fun checkAndDecorate (fid, ret type, args, body, pos) =

  (* args : (string * Type) list  can be used as vtable *)
  let val (body_type, newbody) = expType body args

  (* type rule: type of body equals the type of function's result *)
  in (fid, typesMatch(body type,ret type), args, newbody, pos)
  end


fun typesMatch( Fasto.Int  p1, Fasto.Int  p2 ) = Fasto.Int  p1
  | typesMatch( Fasto.Bool p1, Fasto.Bool p2 ) = Fasto.Bool p1
  | typesMatch( Fasto.Char p1, Fasto.Char p2 ) = Fasto.Char p1
  | typesMatch( Fasto.Array(t1,p1), Fasto.Array(t2,p2) ) =
                             Fasto.Array(typesMatch(t1,t2), p1)
  | typesMatch( t1 , t2 ) = raise Error("Type error!")
```

The Gist of Type.sml: Type Checking an Expression

Map Type Rule: the type of array's elems equals the type of fun's arg.

```
fun expType exp vtab = case exp of
    Fasto.Num (n, pos) => (Fasto.Int pos, exp) | ...
  | Fasto.Map (fid, arr, argtype, restype, pos) =>
      let val (arr_type, arr_new) = expType arr vtab

          val el_type = case arr_type of
                          Fasto.Array (t,p) => t
                        | other => raise Error ("Map argument not an array")

          val (f_arg_type, f_res_type) =
            case SymTab.lookup fid (!funTab) of
              NONE => raise Error ("Unknown identifier!")
            | SOME ([a1],res) => (a1,res)
            | SOME (args,res) => raise Error("Map: not unary fun!")

      in ( Fasto.Array(f_res_type, pos),
           Fasto.Map( fid, arr_new, typesMatch(el_type, f_arg_type),
                      f_res_type, pos ) )
      end
```

Dead-Function Elimination

Partial Pseudocode for live_funs

```
fun live_funs (
          exp    : Fasto.Exp,
          livefs : string list,
          ftab   : (string * Fasto.FunDec) list
    ) : string list =
  case exp of
```

Dead-Function Elimination: Recursive-Scan of Expressions

Partial Pseudocode for live_funs

```
fun live_funs (
          exp    : Fasto.Exp,
          livefs : string list,
          ftab   : (string * Fasto.FunDec) list
    ) : string list =
  case exp of
    Plus (e1, e2, p) =>
        live_funs(e2, live_funs(e1, livefs, ftab), ftab)

  | ...
```

Dead-Function Elimination: Scan any Reachable Call

Partial Pseudocode for live_funs

```
fun live_funs (
          exp    : Fasto.Exp,
          livefs : string list,
          ftab   : (string * Fasto.FunDec) list
    ) : string list =
  case exp of
    Plus (e1, e2, p) =>
        live_funs(e2, live_funs(e1, livefs, ftab), ftab)

  | ...

  | Map(fid, e, t1, t2, p) =>
        let val elives = live_funs(e, livefs, ftab)
        in  if( fid is already in elives ) then elives
            else live_funs( fid's body, fid::elives, ftab )
        end
```