OpenCL Day 2: Loop Reasoning, Optimizing Temporal and Spatial Locality

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Content of Day2 Lecture:

Dependence Analysis
- Problem Statement, Dependency Definition
- Dependency Matrix, Loop Parallelism and Interchange
- Loop Distribution, Array Expansion/Privatization
- Exercise: Putting it all together

Optimizing Spatial Locality (Coalesced Memory)
- Motivating Example on A Contrived Program
- Uncoalesced Matrix Transposition Kernel
- Coding Exercise: Entirely-Coalesced Transposition
- Program Optimized By Transposition + Coding Exercise
- Fully Fused Program + Coding Exercise

Optimizing Temporal Locality
- Naive Matrix-Matrix Multiplication
- Block-Tiled Matrix Multiplication + Coding Exercise
- Register-Tiled Matrix Multiplication + Coding Exercise

Problem Statement

Three Loop Examples

```
DO i = 1, N
 DO j = 1, N
   A[j,i] = A[j,i] ..
 ENDDO
ENDDO
```

```
DO i = 2, N
 DO j = 2, N
   A[j,i] = A[j-1,i-1]...
   B[j,i] = B[j-1,i]...
ENDDO ENDDO
```

```
DO i = 2, N
 DO j = 1, N
  A[i,j] = ...
    A[i-1,j+1]
ENDDO ENDDO
```

Iterations are ordered *lexicographically*, in the order they occur in the sequential execution: $\vec{k} = (\texttt{i=2}, \texttt{j=4}) < \vec{l} = (\texttt{i=3}, \texttt{j=3})$.

- Which of the three loop nests is amenable to parallelization?
- Loop interchange is one of the most simple and useful code transformations, e.g., used to enhance locality of reference, and even to "create" parallelism.
- In which loop nest is it safe to interchange the loops?

Definition of a Dependency

Load-Store Classification of Dependencies

```
True Dependency(RAW)  Anti Dependency(WAR)  Output dependency(WAW)
   S1    X  = ..          S1    .. = X          S1    X = ...
   S2    .. = X           S2    X  = ..         S2    X = ...
```

Def. Loop Dependence: There is a dependence from statement $S1$ to $S2$ in a loop nest *iff* there exist iterations $\vec{k} \leq \vec{l}$ and an execution path from statement $S1$ to statement $S2$ such that:

1. $S1$ accesses memory location M on iteration $\vec{k}$, and
2. $S2$ accesses memory location M on iteration $\vec{l}$, and
3. one of these accesses is a write.

We say that $S1$ is the source and $S2$ is the sink of the dependence, because $S1$ executes before $S2$ in the sequential program execution. Dependence depicted with an arrow pointing from source to sink.

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3. one of these accesses is a write.

We say that $S1$ is the source and $S2$ is the sink of the dependence, because $S1$ executes before $S2$ in the sequential program execution. Dependence depicted with an arrow pointing from source to sink.

We are most interested in cross iteration dependencies, i.e., $\vec{k} < \vec{l}$.

Loop-Nest Dependencies

Lexicographic ordering, e.g., $\vec{k}$ =(i=2,j=4) < $\vec{l}$ =(i=3,j=3).

Three Loop Examples

```
DO i = 1, N
 DO j = 1, N
   A[j,i] = A[j,i] ..
 ENDDO
ENDDO
```

```
DO i = 2, N
 DO j = 2, N
   A[j,i] = A[j-1,i-1]...
   B[j,i] = B[j-1,i]...
ENDDO ENDDO
```

```
DO i = 2, N
 DO j = 1, N
  A[i,j] = ...
    A[i-1,j+1]
ENDDO ENDDO
```

Loop-Nest Dependencies

Lexicographic ordering, e.g., $\vec{k}$ =(i=2,j=4) < $\vec{l}$ =(i=3,j=3).

Three Loop Examples

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DO i = 1, N
 DO j = 1, N
   A[j,i] = A[j,i] ..
 ENDDO
ENDDO
```

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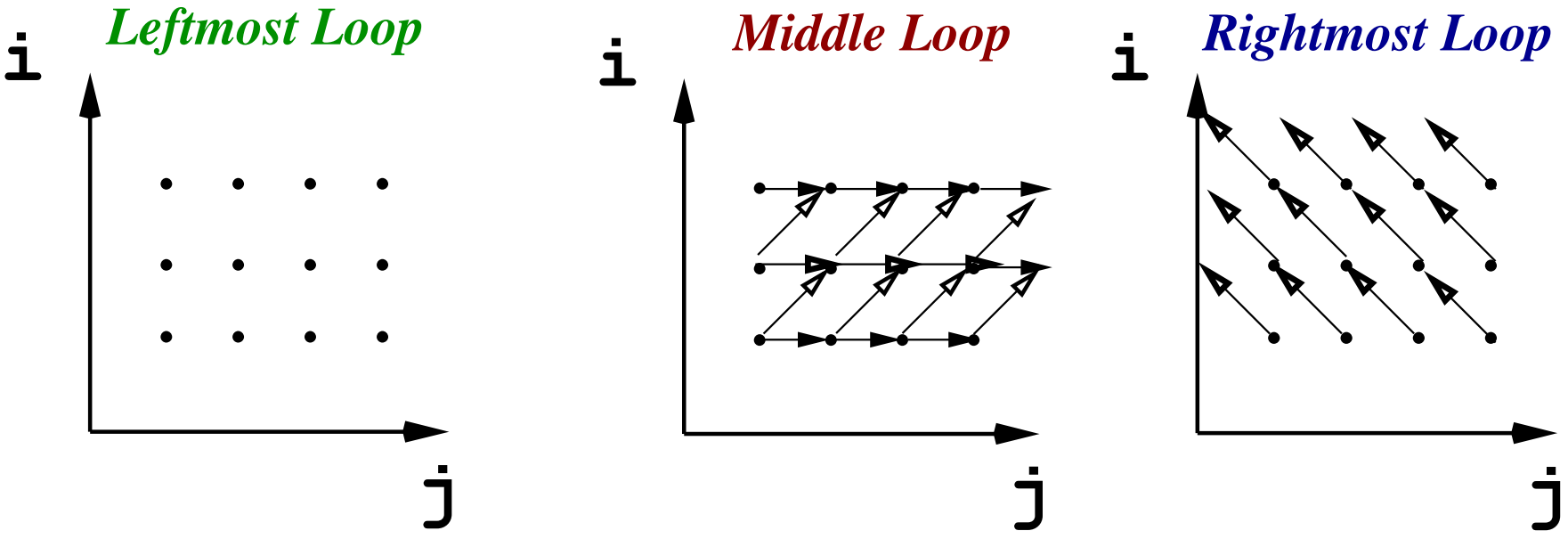
DO i = 2, N
 DO j = 2, N
   A[j,i] = A[j-1,i-1]...
   B[j,i] = B[j-1,i]...
ENDDO ENDDO
```

```
DO i = 2, N
 DO j = 1, N
  A[i,j] = ...
    A[i-1,j+1]
ENDDO ENDDO
```



How can I summarize this information?

Aggregate Dependencies via Direction Vectors

Three Loop Examples

```
DO i = 1, N
 DO j = 1, N
   A[j,i] = A[j,i] ..
 ENDDO
ENDDO
```

```
DO i = 2, N
 DO j = 2, N
   A[j,i] = A[j-1,i-1]...
   B[j,i] = B[j-1,i]...
ENDDO ENDDO
```

```
DO i = 2, N
 DO j = 1, N
  A[i,j] = ...
    A[i-1,j+1]
ENDDO ENDDO
```

Dependencies depicted via an edge *from* the stmt that executes first in the loop nest (*source*), *to* the one that executes later (*sink*).

Def. Dependence Direction: Assume there exists a dependence from $S1$ in iteration $\vec{k}$ to $S2$ in $\vec{l}$ ($\vec{k} \leq \vec{l}$). *The direction vector is*:

1. $\vec{D}(\vec{k},\vec{l})_m =$ "<" if $\vec{k}_m < \vec{l}_m$,
2. $\vec{D}(\vec{k},\vec{l})_m =$ "=" if $\vec{k}_m = \vec{l}_m$,
3. $\vec{D}(\vec{k},\vec{l})_m =$ ">" if $\vec{k}_m > \vec{l}_m$.
4. $\vec{D}(\vec{k},\vec{l})_m =$ "*" otherwise.

If the source is a write and the sink a read then RAW dependency, if the source is a read then WAR, if both are writes then WAW.

How to Compute the Direction Vectors?

- For any two statements $S1$ and $S2$ that may access the same array A (and one of the accesses is a write),
- in two symbolic iterations $I^1 \equiv (i_1^1, \ldots i_m^1)$ and $I^2 = (i_1^2, \ldots i_m^2)$ (assuming $I^1 \leq I^2$)
- on indices $A[e_1^1, \ldots, e_n^1]$ and $A[e_1^2, \ldots, e_n^2]$, respectivelly,
- then *the direction vectors may be derived* from the equations

$$\begin{cases} e_1^1 = e_1^2 \\ \ldots \\ e_n^1 = e_n^2 \end{cases}$$

(The system of equations models the definition of a dependency: both accesses need to refer to the same memory location!)

When is a Loop Parallel?

Direction Vectors/Matrix for Three Loops

```
  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO
```

When is a Loop Parallel?

Direction Vectors/Matrix for Three Loops

```
  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO
For S1→S1:
    (j1,i1)=(j2,i2)
    i1 = i2 & j1 = j2

Direction matrix:
S1→S1: [=,=]
```

```
    DO i = 2, N
      DO j = 2, N
S1      A[j,i]=A[j-1,i]...
S2      B[j,i]=B[j-1,i-1]...
      ENDDO
    ENDDO
```

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Direction Vectors/Matrix for Three Loops

```
  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO
For S1→S1:
    (j1,i1)=(j2,i2)
    i1 = i2 & j1 = j2

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```
  DO i = 2, N
    DO j = 2, N
S1    A[j,i]=A[j-1,i]...
S2    B[j,i]=B[j-1,i-1]...
    ENDDO
  ENDDO
S1→S1: (j1,i1)=(j2-1,i2)
       i1 = i2 & j1 < j2
S2→S2: (j1,i1)=(j2-1,i2-1)
       i1 < i2 & j1 < j2
S1→S1: [=,<]
S2→S2: [<,<]
```

```
DO i = 2, N
  DO j = 1, N
S1    A[i,j]=A[i-1,j+1]..
  ENDDO
ENDDO
For S1→S1:
```

When is a Loop Parallel?

Direction Vectors/Matrix for Three Loops

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  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO
For S1→S1:
    (j1,i1)=(j2,i2)
    i1 = i2 & j1 = j2

Direction matrix:
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    DO i = 2, N
      DO j = 2, N
S1      A[j,i]=A[j-1,i]...
S2      B[j,i]=B[j-1,i-1]...
      ENDDO
    ENDDO
S1→S1: (j1,i1)=(j2-1,i2)
       i1 = i2 & j1 < j2
S2→S2: (j1,i1)=(j2-1,i2-1)
       i1 < i2 & j1 < j2
S1→S1: [=,<]
S2→S2: [<,<]
```

```
  DO i = 2, N
    DO j = 1, N
S1    A[i,j]=A[i-1,j+1]..
    ENDDO
  ENDDO
For S1→S1:
    (i1,j1) = (i2-1,j2+1)
    i1 < i2 & j1 > j2

Direction matrix:
 S1→S1: [<,>]
```

Th. Parallelism: A loop in a loop nest is parallel *iff* for each entry in its direction column, the entry is either = or there exists an outer loop whose corresponding direction is <.

A direction vector cannot have > as the first non-= symbol, as that would mean that I depend on something in the future.

Parallelism and Loop Interchange

Direction Vectors/Matrix for Three Loops

```
  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO
For S1→S1:
    (j1,i1)=(j2,i2)
    i1 = i2 & j1 = j2

Direction matrix:
S1→S1: [=,=]
```

```
    DO i = 2, N
      DO j = 2, N
S1      A[j,i]=A[j-1,i]...
S2      B[j,i]=B[j-1,i-1]...
      ENDDO
    ENDDO
S1→S1: (j1,i1)=(j2-1,i2)
       i1 = i2 & j1 < j2
S2→S2: (j1,i1)=(j2-1,i2-1)
       i1 < i2 & j1 < j2
S1→S1: [=,<]
S2→S2: [<,<]
```

```
  DO i = 2, N
   DO j = 1, N
S1   A[i,j] = ...
       A[i-1,j+1]
  ENDDO ENDDO
For S1→S1:
  (i1,j1) = (i1-1,j2+1)
  i1 < i2 & j1 > j2

Direction matrix:
S1→S1: [<,>]
```

Th. Loop Interchange: Interchanging two loops in a loop nest is legal *iff* interchanging the columns of the direction matrix in the same way *does NOT result* in a > direction as the leftmost non-= direction in a row.

Parallelism and Loop Interchange

Direction Vectors/Matrix for Three Loops

```
  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO

For S1→S1: j1 = j2
             i1 = i2
(i2,j2)-(i1,j1)=
[=,=]
```

```
 DO i = 2, N
  DO j = 2, N
S1  A[j,i]=A[j-1,i]...
S2  B[j,i]=B[j-1,i-1]...
 ENDDO ENDDO

For S1→S1: j1 = j2-1
             i1 = i2
  (i2,j2)-(i1,j1)=[=,<]
For S2→S2: j1 = j2-1
             i1 = i2-1
  (i2,j2)-(i1,j1)=[<,<]
```

```
DO i = 2, N
 DO j = 1, N
S1 A[i,j]=A[i-1,j+1]..
 ENDDO
ENDDO

For S1→S1: i1 = i2-1
             j1 = j2+1
(i2,j2)-(i1,j1)=
    [<,>]
```

Interchange is safe for the 1st and 2nd nests, but not for the 3rd!

e.g., $[=,<] \rightarrow [<,=]$ (for the second loop nest)

$[<,<] \quad [<,<]$

Parallelism and Loop Interchange

Direction Vectors/Matrix for Three Loops

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  DO i = 1, N
    DO j = 1, N
S1    A[j,i]=A[j,i]..
    ENDDO
  ENDDO

For S1→S1: j1 = j2
             i1 = i2
(i2,j2)-(i1,j1)=
[=,=]
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 DO i = 2, N
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S1  A[j,i]=A[j-1,i]...
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 ENDDO ENDDO

For S1→S1: j1 = j2-1
             i1 = i2
  (i2,j2)-(i1,j1)=[=,<]
For S2→S2: j1 = j2-1
             i1 = i2-1
  (i2,j2)-(i1,j1)=[<,<]
```

```
DO i = 2, N
 DO j = 1, N
S1 A[i,j]=A[i-1,j+1]..
 ENDDO
ENDDO

For S1→S1: i1 = i2-1
             j1 = j2+1
(i2,j2)-(i1,j1)=
    [<,>]
```

Interchange is safe for the 1st and 2nd nests, but not for the 3rd!

e.g., $\begin{matrix}[=,<] \\ [<,<]\end{matrix} \rightarrow \begin{matrix}[<,=] \\ [<,<]\end{matrix}$ (for the second loop nest)

After interchange, loop *j* of the second loop nest is parallel.

Corollary: A parallel loop can be always interchanged inwards.

Dependency Graph and Loop Distribution

Def. Dependency Graph: edges from the source of the dependency, i.e., early iteration, to the sink, i.e., later iteration.

Th. Loop Distribution: Statements that are in a dependence cycle remain in one (sequential) loop. The others are distributed to separate loops in graph order; if no cycle then parallel loops.

Vectorization Example

```
  DO i = 3, N
S1  A[i] = B[i-2] ...
S2  B[i] = B[i-1] ...
  ENDDO

For S2→S1: i1 = i2-2, [<]
For S2→S2: i1 = i2-1, [<]
```

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S1  A[i] = B[i-2] ...
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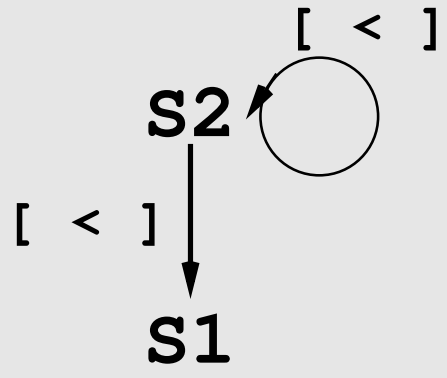
```




```
  DO i = 3, N
S2  B[i] = B[i-1] ...
  ENDDO

  DOALL i = 3, N
S1  A[i] = B[i-2] ...
  ENDDOALL
```

Corollary: It is always legal to distribute a parallel loop; but requires array expansion for local variables or if output dependencies are present.

Loop Distribution May Require Array Expansion

```
float tmp;
for(i=2; i<N; i++) {
  tmp = 2*B[i-2];
  A[i] = tmp;
  B[i] = tmp+B[i-1]
}
```

```
float tmp[N];
for(int i=2; i<N; i++) {
  tmp[i] = 2*B[i-2];
  B[i] = tmp[i]+B[i-1];
}

forall(int i=2; i<N; i++) {
  A[i] = tmp[i];
}
```

No matter where `tmp` is declared (inside or outside the loop) it needs to be expanded into an array in order to do loop distribution.

If `tmp` is declared outside the loop then requires **privatization**,

Loop Distribution May Require Array Expansion

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```
float tmp[N];
for(int i=2; i<N; i++) {
  tmp[i] = 2*B[i-2];
  B[i] = tmp[i]+B[i-1];
}

forall(int i=2; i<N; i++) {
  A[i] = tmp[i];
}
```

No matter where `tmp` is declared (inside or outside the loop) it needs to be expanded into an array in order to do loop distribution.

If `tmp` is declared outside the loop then requires **privatization**, because it actually causes frequent waw dependencies. However its value is written before being used within the same iteration. Hence it is semantically equivalent to a locally declared variable, which will remove the output (waw) dependency.

Distribution requires array expansion of the scalar `tmp`.

False Dependencies (WAR/WAW)

- Cross-Iteration Anti Dependencies (WAR) correspond to a read from the array as it was before the loop $\Rightarrow$ can be eliminated by reading from a copy of the array.

- Cross-Iteration WAW Dependencies (WAW):
  If they correspond to the case in which every **read** from a scalar or array location is covered by a **previous same-iteration write** $\Rightarrow$ can be eliminated **privatization (renaming)**, which semantically moves the declaration of the variable (scalar or array) inside the loop.

- Direction-vectors reasoning is limited to relatively simple loop nests, e.g., difficult to reason about privatization in such a way.

Example: Eliminating WAW Dependencies

```
float A[M];
for(i=0; i<N; i++){
  for(int j=0, j<M; j++)
      A[j] = (4*i+4*j) % M;
  for(int k=0; k<N; k++)
      X[i][k] = X[i][k-1] *
         A[A[(2*i+k)%M]%M];
}
```

Example: Eliminating WAW Dependencies

```
                                        // The write to A[j] causes many WAWs,

  float A[M];
  for(i=0; i<N; i++){
    for(int j=0, j<M; j++)
        A[j] = (4*i+4*j) % M;
    for(int k=0; k<N; k++)
        X[i][k] = X[i][k-1] *
           A[A[(2*i+k)%M]%M];
  }
```

Example: Eliminating WAW Dependencies

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float A[M];
for(i=0; i<N; i++){
  for(int j=0, j<M; j++)
      A[j] = (4*i+4*j) % M;
  for(int k=0; k<N; k++)
      X[i][k] = X[i][k-1] *
         A[A[(2*i+k)%M]%M];
}
```

```
// The write to A[j] causes many WAWs,
// but A is fully written in loop j
// float A[N][M];
forall(int i=0; i<N; i++){
  float A[M];
  for(int j=0, j<M; j++)
      A[j] = (4*i+4*j) % M; // A[i][j]
  for(int k=0; k<N; k++)
      X[i][k]=X[i][k-1] *
         A[A[(2*i+k)%M]%M];
         //A[i][A[i,...]]
}
```

For OpenCL programming you would likely use array expansion (`float A[N,M]`) because the size M is likely unknown at compile time.

Putting Everything Together

```
float X[M][N][N]; /* ... compute X ... */
float A[N][N];
for(int i = 0; i < M; i++) {
  //initialize A
  for(int k = 0; k < N; k++) {
    for(int j = 0; j < N; j++) {
        float x = X[i][j][k];
        A[k][j] = x * x;
  } }
  // convergence loop
  for(int t = 0; t < T; t++) {
    for(int j = 0; j < N; j++) {
      for(int k = 0; k < N; k++) {
        A[k][j] = fsqrt(A[k][j] * A[k-1][j]);
        X[i][k][j] += fsqrt(B[i][k-1][j-1]);
      }
    }
} }
```

Exercise: Assume the code above runs on a dataset having `M = 128` and `N = 512` and, less important, say `T = 256`.

(1) Is this code suitable for GPU execution?

(2) Transform the code to one suitable for GPU execution.

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Motivation: Coalesced Accesses to Global Memory

Which are the parallel/sequential loops?

```
real A[height][width];
real B[height][width];
// Non-Coalesced Access
for(i=0; i<height; i++) {
  real accum  = 0.0;

  for(j=0; j<width; j++) {
    real tmpA = A[i][j];
    accum = tmpA*tmpA - accum;
    B[i][j] = accum;
  }
}
```

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    real tmpA = A[i][j];
    accum = tmpA*tmpA - accum;
    B[i][j] = accum;
  }
}
```

```
A' = transpose(A);
// Coalesced Accesses
for(i=0; i<height; i++) {
  real accum = 0.0;

  for(int j=0; j<width; j++) {
    real tmpA = A'[j][i];
    accum = tmpA*tmpA - accum;
    B'[j][i] = accum;
  }
}
B = transpose(B');
```

- Loop `i` is parallel;
- loop `j` is sequential because of the RAW cross-iteration dependencies on `accum`.
- The transformed program performs about $3\times$ the number of global-memory accesses than the original.

Motivation: Coalesced Accesses to Global Memory

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real A[height][width];
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    real tmpA = A[i][j];
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    B[i][j] = accum;
  }
}
```

```
A' = transpose(A);
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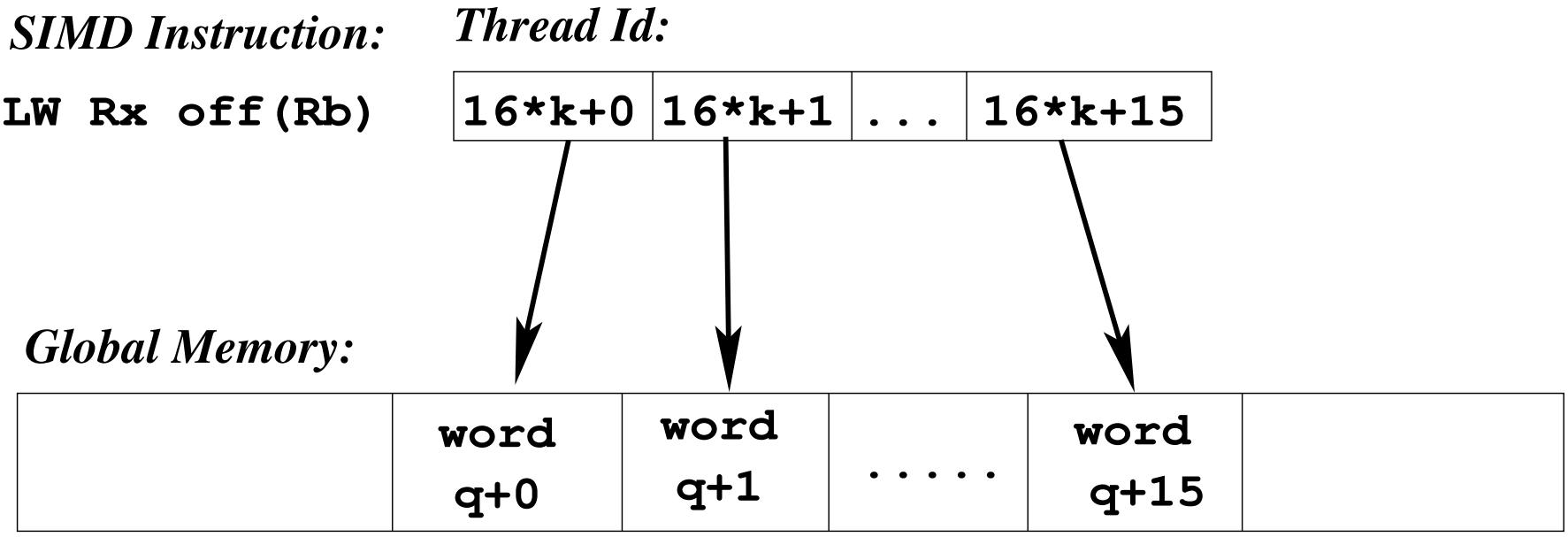
  for(int j=0; j<width; j++) {
    real tmpA = A'[j][i];
    accum = tmpA*tmpA - accum;
    B'[j][i] = accum;
  }
}
B = transpose(B');
```

- Loop `i` is parallel;
- loop `j` is sequential because of the RAW cross-iteration dependencies on `accum`.

- The transformed program performs about $3\times$ the number of global-memory accesses than the original.
- But it is significantly faster than the original (on GPUs).

Motivation: What Are Coalesced Accesses?

Coalesced access: a (quarter) wave accesses in a SIMD instruction consecutive words in global-memory.



Uncoalesced Transposition Kernel

- Semantically we chunk the matrix into T×T blocks;
- Each block is processed by one workgroup, e.g., T=16.

```
// Initial
for(i=0; i<height; i++) {
  for(j=0; j<width; j++) {
    B[j*height+i] =
           A[i*width+j];
  }
}
```

```
// Blocked
for(ii=0; ii<height; ii+=T) {
  for(jj=0; jj<width; jj+=T) {
    for(i=ii; i<min(ii+T,height); i++) {
      for(j=jj; j<min(jj+T,width); j++) {
        B[j*height+i] = A[i*width+j];
} } } }
```

- Local workgroup: $T \times T$;
- Global(1): $\lceil \texttt{width}/T \rceil * T$; Global 1: $\lceil \texttt{height}/T \rceil * T$.

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  }
}
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```
// Blocked
for(ii=0; ii<height; ii+=T) {
  for(jj=0; jj<width; jj+=T) {
    for(i=ii; i<min(ii+T,height); i++) {
      for(j=jj; j<min(jj+T,width); j++) {
        B[j*height+i] = A[i*width+j];
} } } }
```

- Local workgroup: $T \times T$;
- Global(1): $\lceil \texttt{width}/T \rceil * T$; Global 1: $\lceil \texttt{height}/T \rceil * T$.

```
__kernel void naiveTransp( __global real* A, __global real* B
                         , int height, int width ) {
    unsigned int i = get_global_id(1);
    unsigned int j = get_global_id(0);
    if( (j >= width) || (i >= height) ) return;
    B[j*height + i] = A[i*width + j];  // uncoalesced access!
}
```

Exercise 1: Fill In Coalesced Transposition Kernel

```
// verifies that workgroup size is: TILE*TILE
__kernel __attribute__((reqd_work_group_size(TILE, TILE, 1)))
__kernel void coalsTransp( __global real* A,
        __global real* B, uint32_t height, uint32_t width ) {
  __local float tile[TILE][TILE];                    // local memory
  int gidx = get_global_id(0); int gidy = get_global_id(1);
  int lidx = get_local_id(0);  int lidy = get_local_id(1);

  // 1. read current element from global to local memory.
  //    make sure to NOT access arrays out of bounds!
  // 2. current thread loads from local memory the transposed element
  //    at workgroup level, and writes it in the proper position in B.
  //    Useful functions: get_group_id(0), get_local_size(0); or 1.
  // 3. Insert necessary workgroup-level synchronization
  //    barrier(CLK_LOCAL_MEM_FENCE);
}
```

Transposing a (blocked) matrix $\equiv$ moving each block to the transposed position $+$ transposing each block!

Exercise 1: Fill In Coalesced Transposition Kernel

```
// verifies that workgroup size is: TILE*TILE
__kernel __attribute__((reqd_work_group_size(TILE, TILE, 1)))
__kernel void coalsTransp( __global real* A,
        __global real* B, uint32_t height, uint32_t width ) {
  __local float tile[TILE][TILE];                    // local memory
  int gidx = get_global_id(0); int gidy = get_global_id(1);
  int lidx = get_local_id(0);  int lidy = get_local_id(1);

  // 1. read current element from global to local memory.
  //    make sure to NOT access arrays out of bounds!
  // 2. current thread loads from local memory the transposed element
  //    at workgroup level, and writes it in the proper position in B.
  //    Useful functions: get_group_id(0), get_local_size(0); or 1.
  // 3. Insert necessary workgroup-level synchronization
  //    barrier(CLK_LOCAL_MEM_FENCE);
}
```

Transposing a (blocked) matrix $\equiv$ moving each block to the transposed position $+$ transposing each block!

- Implement in `Day2-Exercises/Transp/kernels.cl`
- Are the read and write accesses to global memory coalesced?
- Why/Where should you place the barrier?

Exercise 2: Coalescing By Transposition

```
real A[height][width];
real B[height][width];
// Non-Coalesced Access
for(i=0; i<height; i++) {
  real accum  = 0.0;

  for(j=0; j<width; j++) {
    real tmpA = A[i][j];
    accum = tmpA*tmpA - accum;
    B[i][j] = accum;
} }
```

Exercise 2: Coalescing By Transposition

```
real A[height][width];
real B[height][width];
// Non-Coalesced Access
for(i=0; i<height; i++) {
  real accum  = 0.0;

  for(j=0; j<width; j++) {
    real tmpA = A[i][j];
    accum = tmpA*tmpA - accum;
    B[i][j] = accum;
} }
```

```
A' = transpose(A);
// Coalesced Accesses
for(i=0; i<height; i++) {
  real accum = 0.0;

  for(int j=0; j<width; j++) {
    real tmpA = A'[j][i];
    accum = tmpA*tmpA - accum;
    B'[j][i] = accum;
} }
B = transpose(B');
```

- Open File "Day2-Exercises/Transp/coalescing.c", function named "runGPUcoalsProgram".
- Fill the correct sizes for the "coalsProgrm" kernel call.
- Open File "Day2-Exercises/Transp/kernels.cl" and implement kernel named "coalsProgrm".
- Fix bugs until it validates! :-)
- What is the speedup with respect to the naive program?
- What fraction of the memcpy bandwidth is achieved?

Exercise 3: Fused Program With Coalesced Accesses

- Open File ”Day2-Exercises/Transp/kernels.cl” and implement the missing pieces from kernel named ”optimProgrm”;
- The idea is to have coalesced accesses ”fused” into the kernel by each thread processing CHUNK elements at a time by:
  - collectively copying `CHUNK * WORKGROUP_SIZE` input elements from global to local memory in coalesced fashion;
  - each thread now processes CHUNK elements from its row;
  - collectively copying `CHUNK * WORKGROUP_SIZE` result elements from local to global memory in coalesced fashion;
  - repeat in a loop (inside the kernel) until all row’s elements have been processed!
- Fix bugs until it validates! :-)
- What is the speedup with respect to the transposition-optimized program?
- What fraction of the memcpy bandwidth is achieved?

Content of Day2 Lecture:

Dependence Analysis
- Problem Statement, Dependency Definition
- Dependency Matrix, Loop Parallelism and Interchange
- Loop Distribution, Array Expansion/Privatization
- Exercise: Putting it all together

Optimizing Spatial Locality (Coalesced Memory)
- Motivating Example on A Contrived Program
- Uncoalesced Matrix Transposition Kernel
- Coding Exercise: Entirely-Coalesced Transposition
- Program Optimized By Transposition + Coding Exercise
- Fully Fused Program + Coding Exercise

Optimizing Temporal Locality
- Naive Matrix-Matrix Multiplication
- Block-Tiled Matrix Multiplication + Coding Exercise
- Register-Tiled Matrix Multiplication + Coding Exercise

Matrix Multiplication: Loop Strip Mining

```
for (int i = 0; i < M; i++) {  // Parallel
  for (int j = 0; j < N; j++) {  // Parallel
    float tmp = 0.0
    for(int k = 0; k < U; k++) { // Reduction
      tmp += A[i,k]*B[k,j]
    }
    C[i,j] = tmp;
  }
}
```

Matrices:

- A $\in \mathcal{M}^{M \times U}$
- B $\in \mathcal{M}^{U \times N}$
- C$\in \mathcal{M}^{M \times N}$

Loops of indices `i` and `j` are parallel.

Accesses to A and B invariant to loops `i` and `j` $\Rightarrow$

The idea is to apply Block Tiling to optimize temporal locality!

Loop Strip Mining

```
for(int i = 0; i < N; i++) {
    loop_body(i)
}
```

$\Rightarrow$

```
for(int ii = 0; ii < N; ii += T) {
  for(int i = ii; i<MIN(ii+T,N); i++) {
    loop_body(i)
} }
```

Strip Mining is always safe: the transformed loop executes the same instructions in the same order as the original loop!

Strip mining all loops by some tile T result in:

Loop Strip Mining

```
for(int i = 0; i < N; i++) {
    loop_body(i)
}
```

⇒

```
for(int ii = 0; ii < N; ii += T) {
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Strip Mining is always safe: the transformed loop executes the same instructions in the same order as the original loop!

Strip mining all loops by some tile T result in:

```
for (int ii = 0; ii < M; ii += T) {              // Parallel
  for (int i = ii; i < MIN(ii+T,M); i++) {       // Parallel
    for (int jj = 0; jj < N; jj += T) {          // Parallel
      for (int j = jj; j < MIN(jj+T,N); j++) {   // Parallel
          float tmp = 0.0;
          for(int kk = 0; kk < U; k += T) {      // Reduction
            for(int k=kk; k<MIN(kk+T,U); k++) {// Reduction
                tmp += A[i,k]*B[k,j];
            }
          }
          C[i,j] = tmp;
      }
    }
  }
}
```

Block Tiling: Strip Mining + Interchange

Now interchange the loops if index `i` and `jj`. Why is this safe?

```
for (int ii = 0; ii < M; ii += T) {                 // Global 1
  for (int jj = 0; jj < N; jj += T) {               // Global 0
    for (int i = ii; i < MIN(ii+T,M); i++) {        // Local 1
      for (int j = jj; j < MIN(jj+T,N); j++) {      // Local 0
          float tmp = 0.0;
          for(int kk = 0; kk < U; k += T) {         // Sequential

            //What slices of A and B are used in the loop below?

            for(int k=kk; k<MIN(kk+T,U); k++) { // Sequential
                tmp += A[i,k]*B[k,j];
            }
          }
          C[i,j] = tmp;
} } } }
```

Block Tiling: Strip Mining + Interchange

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    for (int i = ii; i < MIN(ii+T,M); i++) {        // Local 1
      for (int j = jj; j < MIN(jj+T,N); j++) {      // Local 0
          float tmp = 0.0;
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            //What slices of A and B are used in the loop below?

            for(int k=kk; k<MIN(kk+T,U); k++) { // Sequential
                tmp += A[i,k]*B[k,j];
            }
          }
          C[i,j] = tmp;
} } } }
```

`A[ii : MIN(ii+T,M)][kk : MIN(kk+T,U)]`
`B[kk : MIN(kk+T,U)][jj : MIN(jj+T,N)]`

Each array slice has `T`×`T` elements and there are `T`×`T` threads in a workgroup; so each thread can copy one element to local memory. Thus, we cut the number of global-memory accesses by a `T` factor.

Exercise 4: Implement Block-Tiled Matrix Multiplication

- Open File ”Day2-Exercises/MMMult/mmm.c”, function named ”runGPUverMMM”.
- Place there the correct sizes for the matrix multiplication kernels. That would make the naive version work correctly.
- Open File ”Day2-Exercises/MMMult/mmm.cl” and implement kernel named ”blockMMM”.
- Fix bugs until it validates! :-)

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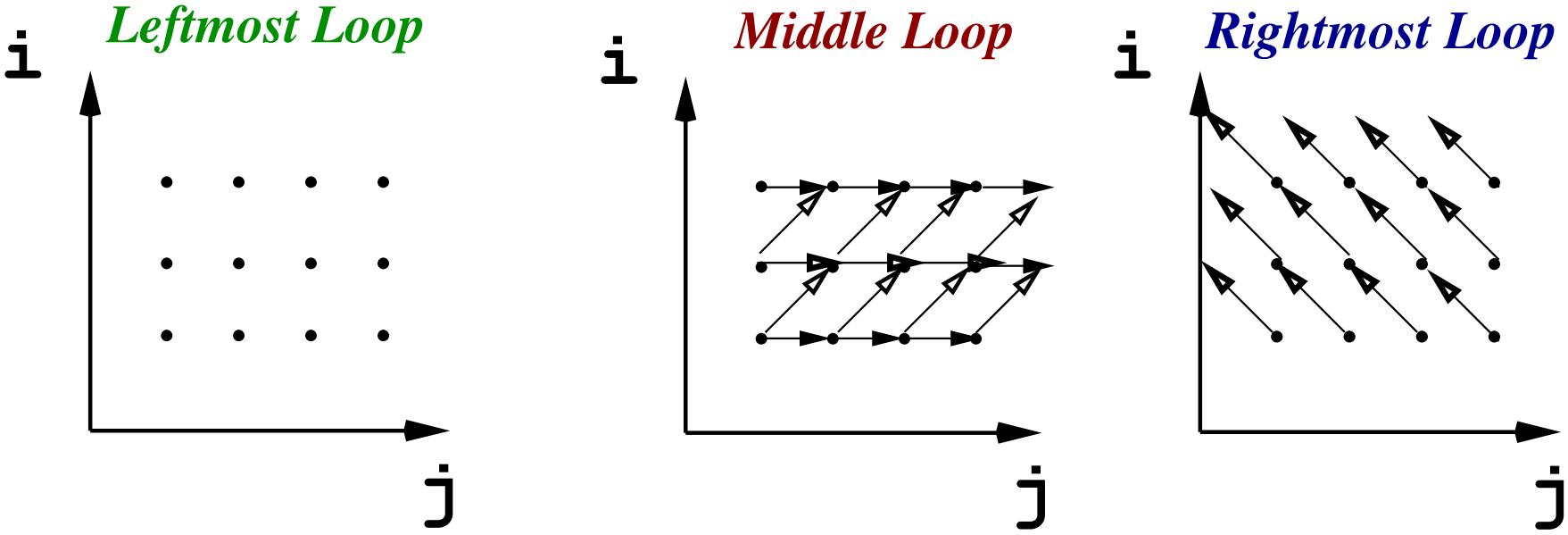
- Place there the correct sizes for the matrix multiplication kernels. That would make the naive version work correctly.
- Open File ”Day2-Exercises/MMMult/mmm.cl” and implement kernel named ”blockMMM”.
- Fix bugs until it validates! :-)

The parallel iteration space is block tiled, not the matrices:



Exercise 4: Implement Block-Tiled Matrix Multiplication

Copy A[ii:MIN(ii+T,M)][kk:MIN(kk+T,U)] in A_loc of size [T][T]
Copy B[kk:MIN(kk+T,U)][jj:MIN(jj+T,N)] in B_loc of size [T][T]

```
for (int ii = 0; ii < M; ii += T) {                 // Global 1
  for (int jj = 0; jj < N; jj += T) {               // Global 0
    for (int i = ii; i < MIN(ii+T,M); i++) {        // Local 1
      for (int j = jj; j < MIN(jj+T,N); j++) {      // Local 0
          float tmp = 0.0;
          for(int kk = 0; kk < U; k += T) {         // Sequential

            // Collectively copy to local memory.
            // Loop of index  k should only use local memory.
            // Add barrier here. Why?

            for(int k=kk; k<MIN(kk+T,U); k++) { // Sequential
                tmp += A_loc[i-ii,k-kk] * B_loc[k-kk,j-jj];
          } } // Add barrier here. Why?
          C[i,j] = tmp;
} } } }
```

Modify loops of index `i`, `j` and `k` to go from `0` to `T` with stride 1. Unroll loop `k`.

OpenCL equivalences:
Global workgroup size:

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Modify loops of index i, j and k to go from 0 to T with stride 1. Unroll loop k.

OpenCL equivalences:
Global workgroup size: $(\lceil M/T \rceil * T) \times (\lceil N/T \rceil * T)$. Local Workgroup size: $T \times T$.
Local ids:

Exercise 4: Implement Block-Tiled Matrix Multiplication

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          C[i,j] = tmp;
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Modify loops of index i, j and k to go from 0 to T with stride 1. Unroll loop k.

OpenCL equivalences:
Global workgroup size: $(\lceil M/T \rceil * T) \times (\lceil N/T \rceil * T)$. Local Workgroup size: $T \times T$.
Local ids: `i - ii = get_local_id(1), j - jj = get_local_id(0)`
Global ids: `i = get_global_id(1), j = get_global_id(0)`
Similar: `ii = get_group_id(1)*T, jj = get_group_id(0)*T`

Local memory declared with: `__local float A_loc[T][T];`

Exercise 4: Implement Block-Tiled Matrix Multiplication

Loops of index `i` and `j` form the local workgroup. Interchange them innermost and distribute them:

```
for (int ii = 0; ii < M; ii += T) {              // Global 1
  for (int jj = 0; jj < N; jj += T) {            // Global 0
    float A_loc[T][T], B_loc[T][T]; // local memory
    float tmp_reg[T][T];  // register memory: still represented as a scalar for a thread

    for (int i = ii; i < ii+T; i++) {    // Local 1
      for (int j = jj; j < jj+T; j++) {  // Local 0
          tmp[i-ii][j-jj] = 0.0;
    } }
    for(int kk = 0; kk < U; k += T) {      // Sequential
      for (int i = ii; i < ii+T; i++) {    // Local
        for (int j = jj; j < jj+T; j++) {  // Local
          A_loc[i-ii][k-kk] = (i < M && k < U)? A[i][kk+j-jj] : 0.0; // copy to local mem
          B_loc[k-kk][j-jj] = (j < N && k < U)? B[kk+i-ii][j] : 0.0; // copy to local mem
      } } // needs barrier here, why?
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
            for(int k=kk; k < kk+T; k++) { // Sequential
                tmp[i][j] += A_loc[i-ii,k-kk] * B[k-kk,j-jj]; // use local memory
            }
      } }
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
          C[i,j] = tmp;
      } } // needs barrier here, why?
} } }
```

Modify loops of index `i`, `j` and `k` to go from `0` to `T` with stride 1. Unroll loop `k`.

OpenCL equivalences:
Global workgroup size:

Exercise 4: Implement Block-Tiled Matrix Multiplication

Loops of index `i` and `j` form the local workgroup. Interchange them innermost and distribute them:

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for (int ii = 0; ii < M; ii += T) {              // Global 1
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    float A_loc[T][T], B_loc[T][T]; // local memory
    float tmp_reg[T][T];  // register memory: still represented as a scalar for a thread

    for (int i = ii; i < ii+T; i++) {    // Local 1
      for (int j = jj; j < jj+T; j++) {  // Local 0
          tmp[i-ii][j-jj] = 0.0;
    } }
    for(int kk = 0; kk < U; k += T) {      // Sequential
      for (int i = ii; i < ii+T; i++) {    // Local
        for (int j = jj; j < jj+T; j++) {  // Local
          A_loc[i-ii][k-kk] = (i < M && k < U)? A[i][kk+j-jj] : 0.0; // copy to local mem
          B_loc[k-kk][j-jj] = (j < N && k < U)? B[kk+i-ii][j] : 0.0; // copy to local mem
      } } // needs barrier here, why?
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
            for(int k=kk; k < kk+T; k++) { // Sequential
                tmp[i][j] += A_loc[i-ii,k-kk] * B[k-kk,j-jj]; // use local memory
            }
      } }
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
          C[i,j] = tmp;
      } } // needs barrier here, why?
} } }
```

Modify loops of index `i`, `j` and `k` to go from `0` to `T` with stride 1. Unroll loop `k`.

OpenCL equivalences:
Global workgroup size: $(\lceil M/T \rceil * T) \times (\lceil N/T \rceil * T)$. Local Workgroup size: $T \times T$.
Local ids:

Exercise 4: Implement Block-Tiled Matrix Multiplication

Loops of index `i` and `j` form the local workgroup. Interchange them innermost and distribute them:

```
for (int ii = 0; ii < M; ii += T) {              // Global 1
  for (int jj = 0; jj < N; jj += T) {            // Global 0
    float A_loc[T][T], B_loc[T][T]; // local memory
    float tmp_reg[T][T];  // register memory: still represented as a scalar for a thread

    for (int i = ii; i < ii+T; i++) {    // Local 1
      for (int j = jj; j < jj+T; j++) {  // Local 0
          tmp[i-ii][j-jj] = 0.0;
    } }
    for(int kk = 0; kk < U; k += T) {      // Sequential
      for (int i = ii; i < ii+T; i++) {    // Local
        for (int j = jj; j < jj+T; j++) {  // Local
          A_loc[i-ii][k-kk] = (i < M && k < U)? A[i][kk+j-jj] : 0.0; // copy to local mem
          B_loc[k-kk][j-jj] = (j < N && k < U)? B[kk+i-ii][j] : 0.0; // copy to local mem
      } } // needs barrier here, why?
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
            for(int k=kk; k < kk+T; k++) { // Sequential
                tmp[i][j] += A_loc[i-ii,k-kk] * B[k-kk,j-jj]; // use local memory
            }
      } }
      for (int i = ii; i < ii+T; i++) {    // Local 1
        for (int j = jj; j < jj+T; j++) {  // Local 0
          C[i,j] = tmp;
      } } // needs barrier here, why?
} } }
```

Modify loops of index `i`, `j` and `k` to go from `0` to `T` with stride 1. Unroll loop `k`.

OpenCL equivalences:
Global workgroup size: $(\lceil M/T \rceil * T) \times (\lceil N/T \rceil * T)$. Local Workgroup size: $T \times T$.
Local ids: `i - ii = get_local_id(1), j - jj = get_local_id(0)`
Global ids: `i = get_global_id(1), j = get_global_id(0)`

Local memory declared with: `__local float A_loc[T][T];`

Exercise 5: Register+Block Tiled Matrix Multiplication

```
for (int i = 0; i < M; i++) {  // Parallel
  for (int j = 0; j < N; j++) {  // Parallel
    float c = 0.0
    for(int k = 0; k < U; k++) { // Reduction
      c += A[i,k]*B[k,j]
    }
    C[i,j] = c;
  }
}
```

Matrices:

- A $\in \mathcal{M}^{M \times U}$
- B $\in \mathcal{M}^{U \times N}$
- C$\in \mathcal{M}^{M \times N}$

Loops of indices `i` and `j` are parallel.

We can do even better by sacrificing some parallelism in excess:

- stripmine parallel dimension `i` by a tile `T` and stride 1; then move it innermost and sequentialize it;
- stripmine parallel dimension `j` by a tile `T`$\times$`T` and stride `T` and
- stripmine the result by a tile `T` and stride 1;
- the two tiles of parallel dimension `j` form the local workgroup.

Do not forget to perform array expansion on `c`, which we will rename `cs`.

Exercise 5: Register+Block Tiled Matrix Multiplication

```
unsigned int ii, i, jjj, jj, j, kk, k;
forall (ii = 0; ii < M; ii += T ) {            // Parallel Global 1
  forall (jjj = 0; jjj < N; jjj += T*T ) {     // Parallel Global 0
    float cs[T][T][T];

    forall(jj=jjj; jj<min(jjj+T*T,N); jj+=T){ // Parallel Local 1
      forall(j=jj; j<min(jj+T,N); j++) {       // Parallel Local 0
        for (i=ii; i<min(ii+T,M); i++) {       // sequential
          cs[(jj-jjj)/T][j-jj][i-ii] = 0.0;
    } } }

    for (kk = 0; kk < U; kk += T) {                  // sequential
      // here we will insert a collective copy to local
      // memory of the slice: A[ii : ii+T, kk : kk+T]
      for (k = kk; k < min(kk+T,U); k++) {           // sequential
        forall (jj=jjj; jj<min(jjj+T*T,N); jj+=T) { // Parallel Local 1
          forall (j=jj; j<min(jj+T,N); j++) {       // Parallel Local 0
            for (i = ii; i < min(ii+T,M); i++) {    // sequential
              // please modify here to read A from local memory
              // hoist out B[k][j] outside loop of index i (invariant)
              cs[(jj-jjj)/T][j-jj][i-ii] += A[i][k] * B[k][j];
    } } } } }

    forall (jj=jjj; jj<min(jjj+T*T,N); jj+=T) {     // Parallel Local 1
      forall (j=jj; j<min(jj+T,N); j++) {           // Parallel Local 0
        for (i = ii; i < min(ii+T, M); i++) {       // sequential
          C[i][j] = cs[(jj-jjj)/T][j-jj][i-ii];
    } } }
} }
```

Exercise 5: Register+Block Tiled Matrix Multiplication

- Open File ”Day2-Exercises/MMMult/mmm.c”, function named ”runGPUverMMM”.
- Fill the correct sizes for the ”rgblkMMM” kernel.
- Open File ”Day2-Exercises/MMMult/mmm.cl” and implement kernel named ”rgblkMMM”.
- Fix bugs until it validates! :-) Tile size is called RT and not T!

OpenCL equivalences:

Exercise 5: Register+Block Tiled Matrix Multiplication

- Open File "Day2-Exercises/MMMult/mmm.c", function named "runGPUverMMM".
- Fill the correct sizes for the "rgblkMMM" kernel.
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- Fix bugs until it validates! :-) Tile size is called RT and not T!

OpenCL equivalences:

- Global worksize 1: $\lceil M/RT \rceil$; Global worksize 0: $\lceil N/RT^2 \rceil * RT^2$;
- Local worksize 0: RT^2; Local worksize 1: 1;
- ```
  ii = get_group_id(1) * RT;
  j = jj + get_global_id(0);
  locx = get_local_id(0) % RT;
  locy = get_local_id(0) / RT;
  ```
- Loops of index `i` should be normalized `[0..RT-1]` and unrolled (`#pragma unroll`);
- Array `cs` should be one dimensional: `float cs[RT];`
- `__local real A_loc[RT][RT+1];` for bank conflicts.
  ```