1 Expansion

1. Θ is the set of old parameters
2. Θ^* is the set of new parameters
3. We sample variances
4. u_Θ have size such as $\dim(u_\Theta) = \dim(\Theta)$
5. u_n have size such as $\dim(u_n) = \dim(\Theta^*) - \dim(\Theta)$
6. We sample u_Θ and u_n from the prior $p(\Theta)$
7. $\Theta^* = (\text{mean}(\Theta, u_\Theta),\ u_n)$ (set addition)
8. $h(U) = \prod^{\forall(u_\Theta \cup u_n)} p(u)$
9. We keep $U^* = \frac{\Theta - u_\Theta}{2}$

$j(\Theta, U)$ is the function from the old parameters Θ and the auxiliary variables $U = (u_\Theta, u_n)$ to the new set of parameters Θ^* ans the auxiliary U^*.

1.1 Example

$$(\theta_1,\ \theta_2) \to (\theta_1^* \dots \theta_4^*)$$

$$j(\Theta, u_\Theta, u_n) = (\Theta^*, U^*) = (\frac{\theta_1 + u_{\Theta 1}}{2},\ \frac{\theta_2 + u_{\Theta 2}}{2},\ u_{n1},\ u_{n2},\ \frac{\theta_1 - u_{\Theta 1}}{2},\ \frac{\theta_2 - u_{\Theta 2},}{2}\ )$$

$$J = \left|\frac{\partial j(\Theta,U)}{\partial(\Theta,U)}\right| = \begin{vmatrix} \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial\theta_1} & \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial\theta_2} & \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial u_{\Theta 1}} & \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial u_{\Theta 2}} & \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial u_{n1}} & \frac{\partial(\theta_1+u_{\Theta 1})/2}{\partial u_{n2}} \\ \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial\theta_1} & \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial\theta_2} & \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial u_{\Theta 1}} & \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial u_{\Theta 2}} & \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial u_{n1}} & \frac{\partial(\theta_2+u_{\Theta 2})/2}{\partial u_{n2}} \\ \frac{\partial u_{n1}}{\partial\theta_1} & \frac{\partial u_{n1}}{\partial\theta_2} & \frac{\partial u_{n1}}{\partial u_{\Theta 1}} & \frac{\partial u_{n1}}{\partial u_{\Theta 2}} & \frac{\partial u_{n1}}{\partial u_{n1}} & \frac{\partial u_{n1}}{\partial u_{n2}} \\ \frac{\partial u_{n2}}{\partial\theta_1} & \frac{\partial u_{n2}}{\partial\theta_2} & \frac{\partial u_{n2}}{\partial u_{\Theta 1}} & \frac{\partial u_{n2}}{\partial u_{\Theta 2}} & \frac{\partial u_{n2}}{\partial u_{n1}} & \frac{\partial u_{n2}}{\partial u_{n2}} \\ \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial\theta_1} & \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial\theta_2} & \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial u_{\Theta 1}} & \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial u_{\Theta 2}} & \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial u_{n1}} & \frac{\partial(\theta_1-u_{\Theta 1})/2}{\partial u_{n2}} \\ \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial\theta_1} & \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial\theta_2} & \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial u_{\Theta 1}} & \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial u_{\Theta 2}} & \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial u_{n1}} & \frac{\partial(\theta_2-u_{\Theta 2})/2}{\partial u_{n2}} \end{vmatrix} =$$

$$= \begin{vmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 \end{vmatrix} = \frac{1}{4}$$

We expected $2^{-\dim(\Theta)}$ from Jin's code. Matches.

2 Shrinkage

1. Θ is the set of old parameters
2. We sample variances
3. Θ_0 are the parameters we keep
4. Θ_d are the parameters we discard
5. We sample U of size $\dim(\Theta_0)$ from $N(0, \sigma^2_{a|b})$
6. $\Theta^* = \Theta_0 + U$ (vector addition)
   - $\theta_i^* = \theta_i + U_i$
7. $h(U) = \prod^{\forall U} p(U_i)$
8. We keep $U^* = (\Theta_0 - U, \Theta_d)$ (set addition)

$j(\Theta, U)$ is the function from the old parameters Θ and the auxiliary variables U to the new set of parameters Θ^* and the auxiliary U^*.

2.1 Example

$$(\theta_1 \ldots \theta_4) \rightarrow (\theta_1^*,\ \theta_2^*)$$

$$j(\Theta, U) = (\Theta^*, U^*) = (\theta_1 + U_1,\ \theta_2 + U_2,\ \theta_1 - U_1,\ \theta_2 - U_2,\ \theta_3,\ \theta_4)$$

$$J=\left|\frac{\partial j(\Theta,U)}{\partial(\Theta,U)}\right|=\begin{vmatrix}\frac{\partial(\theta_1+U_1)}{\partial\theta_1} & \frac{\partial(\theta_1+U_1)}{\partial\theta_2} & \frac{\partial(\theta_1+U_1)}{\partial\theta_3} & \frac{\partial(\theta_1+U_1)}{\partial\theta_4} & \frac{\partial(\theta_1+U_1)}{\partial U_1} & \frac{\partial(\theta_1+U_1)}{\partial U_2}\\ \frac{\partial(\theta_2+U_2)}{\partial\theta_1} & \frac{\partial(\theta_2+U_2)}{\partial\theta_2} & \frac{\partial(\theta_2+U_2)}{\partial\theta_3} & \frac{\partial(\theta_2+U_2)}{\partial\theta_4} & \frac{\partial(\theta_2+U_2)}{\partial U_1} & \frac{\partial(\theta_2+U_2)}{\partial U_2}\\ \frac{\partial(\theta_1-U_1)}{\partial\theta_1} & \frac{\partial(\theta_1-U_1)}{\partial\theta_2} & \frac{\partial(\theta_1-U_1)}{\partial\theta_3} & \frac{\partial(\theta_1-U_1)}{\partial\theta_4} & \frac{\partial(\theta_1-U_1)}{\partial U_1} & \frac{\partial(\theta_1-U_1)}{\partial U_2}\\ \frac{\partial(\theta_2-U_2)}{\partial\theta_1} & \frac{\partial(\theta_2-U_2)}{\partial\theta_2} & \frac{\partial(\theta_2-U_2)}{\partial\theta_3} & \frac{\partial(\theta_2-U_2)}{\partial\theta_4} & \frac{\partial(\theta_2-U_2)}{\partial U_1} & \frac{\partial(\theta_2-U_2)}{\partial U_2}\\ \frac{\partial\theta_3}{\partial\theta_1} & \frac{\partial\theta_3}{\partial\theta_2} & \frac{\partial\theta_3}{\partial\theta_3} & \frac{\partial\theta_3}{\partial\theta_4} & \frac{\partial\theta_3}{\partial U_1} & \frac{\partial\theta_3}{\partial U_2}\\ \frac{\partial\theta_4}{\partial\theta_1} & \frac{\partial\theta_4}{\partial\theta_2} & \frac{\partial\theta_4}{\partial\theta_3} & \frac{\partial\theta_4}{\partial\theta_4} & \frac{\partial\theta_4}{\partial U_1} & \frac{\partial\theta_4}{\partial U_2}\end{vmatrix}=$$

$$=\begin{vmatrix}1&0&0&0&1&0\\0&1&0&0&0&1\\1&0&0&0&-1&0\\0&1&0&0&0&-1\\0&0&1&0&0&0\\0&0&0&1&0&0\end{vmatrix}=4$$

We expected $2^{\dim(\Theta^*)}$ from Jin's code. Matches.