`rbprobit`: Recursive bivariate probit estimation and decomposition of marginal effects

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```
net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main/")
```

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Motivation

Effects of Interest

1. What we want
   - Estimate: Effect of binary or treatment variable on binary outcome variable
   - Treatment variable itself is endogenous
   - Unobservables may correlate with treatment and outcome equation
   - Compute average treatment effect
   - Compute average marginal effect for covariates
2. What doesn't work:
   - `margins` gives incorrect treatment effect
   - `margins` gives incorrect average marginal effect for covariates
   - `ivprobit` inappropriate; treatment variable is binary
3. What we need
   - Correct Estimation of a RBPM
   - Considering recursive nature of the model for postestimation commands

Contribution

A new Stata Command

- `rbprobit` estimates RBPMs like `biprobit` or `cmp`
- `rbprobit` accounts for recursive nature in postestimation commands
  - `predict` and `predictnl`
  - `margdec`
  - `tmeffect`
- `margdec` incorporates `margins` command, enabling
  - Decomposition of average marginal effects of covariates
  - Standard errors using the delta method
- `tmeffect` incorporates `margins` command, enabling
  - Different definitions of treatment effects
  - Standard errors using the delta method

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Recursive bivariate probit model

The Model

A structural model with endogenous explanatory treatment variable y_2 correlated with the unobservables

$$y_1^\star = \boldsymbol{x'}\boldsymbol{\beta} + \alpha y_2 + \epsilon_1 \quad , y_1 = 1\left[y_1^\star > 0\right] \tag{1}$$

$$y_2^\star = \boldsymbol{z'}\boldsymbol{\gamma} + \epsilon_2 \qquad\qquad , y_2 = 1\left[y_2^\star > 0\right] \tag{2}$$

$$\text{with} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right]$$

- correlation between ϵ_1 and ϵ_2 induces the endogeneity
- $\boldsymbol{x}$ and $\boldsymbol{z}$ can share some or all covariates
- Greene (2018) notes that endogenous nature of y_2 can be ignored
- Han and Lee (2019): estimates are at best weakly identified if $\boldsymbol{x} = \boldsymbol{z}$

Recursive bivariate probit model

Treatment Effects

1. Average Treatment Effect (ATE)

$$\text{ATE} = \frac{1}{n}\sum_{i=n}^{n} \Phi(x_i'\beta + \alpha) - \Phi(x_i'\beta)$$

2. Average Treatment Effect on the Treated (ATT)

$$\text{ATT} = \frac{1}{n_2}\sum_{i=1}^{n_2} \Phi\left(\frac{x_i'\beta + \alpha - \rho z_i'\gamma}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_i'\beta - \rho z_i'\gamma}{\sqrt{1-\rho^2}}\right) \quad \forall\ y_{2i} = 1$$

3. Average Treatment Effect on Conditional Probability (ATC)

$$\text{ATC} = \frac{1}{n}\sum_{i=1}^{n} \frac{\Phi_2(x_i'\beta + \alpha, z_i'\gamma, \rho)}{\Phi(z_i'\gamma)} - \frac{\Phi_2(x_i'\beta, -z_i'\gamma, -\rho)}{\Phi(-z_i'\gamma)}$$

Decomposition of Marginal Effects

Joint and Conditional Probabilities

- Covariate d appears in both $\boldsymbol{x}$ and $\boldsymbol{z}$
- Decomposition of total marginal effects on the probabilities (except marginal probabilities) are then
  1. Continuous Variables (see Greene, 2018)

$$\text{ME} = \frac{\partial \Pr}{\partial \begin{pmatrix} x_d \\ z_d \end{pmatrix}} = \underbrace{\frac{\partial \Pr}{\partial x_d}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pr}{\partial z_d}}_{\text{indirect effect}}$$

  2. Discrete Variables (see Hasebe, 2013; Edwards et al., 2019)

$$\text{ME} = \underbrace{[\Pr|_{x_d=1} - \Pr|_{x_d=0}]}_{\text{direct effect}} + \underbrace{[\Pr|_{z_d=1} - \Pr|_{z_d=0}]}_{\text{indirect effect}}$$

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Basic Syntax

rbprobit *depvar* [=] [*indepvars*] [*if*] [*in*]
    , endogenous(*depvar_en* [=] [*indepvars_en*])

- *depvar_en* automatically added to outcome equation as factor-variable
- `rbprobit` implemented as an `lf1 ml` evaluator
- `depvar` and `depvar_en` have to be binary (*current version*)
- factor variables and time-series operators allowed
- `rbprobit postestimation` available for features after estimation

rbprobit Output

```
. webuse class10, clear
(Class of 2010 profile)

. rbprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

Univariate probits for starting values
Comparison:       log likelihood = -2673.8688

Recursive Binary Probit Estimation               Number of obs     =      2,500
                                                 Wald chi2(12)     =     964.07
Log likelihood = -2667.5268                      Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
graduate     |
   1.program |   .3522094   .1770159     1.99   0.047     .0052646    .6991542
      income |   .1434782   .0142911    10.04   0.000     .1154681    .1714882
             |
    roommate |
        yes  |    .267713   .0588568     4.55   0.000     .1523559    .3830701
             |
    hsgpagrp |
    2.5-2.9  |   .9451679   .1357869     6.96   0.000     .6790305    1.211305
    3.0-3.4  |   1.939513    .147325    13.16   0.000     1.650761    2.228264
    3.5-4.0  |   6.535829   127.5038     0.05   0.959     -243.367    256.4387
             |
       _cons |  -2.076232   .2181295    -9.52   0.000    -2.503758   -1.648706
-------------+----------------------------------------------------------------
program      |
      campus |
        yes  |   .7465297   .0747092     9.99   0.000     .6001024    .8929569
             |
     scholar |
        yes  |   .9007975   .0579886    15.53   0.000      .787142    1.014453
      income |  -.0785837   .0096477    -8.15   0.000    -.0974928   -.0596746
             |
    hsgpagrp |
    2.5-2.9  |   .0586754   .1099653     0.53   0.594    -.1568526    .2742035
    3.0-3.4  |   .0651845   .1152074     0.57   0.572    -.1606179    .2909869
    3.5-4.0  |  -.0970995   .1780755    -0.55   0.586    -.4461211    .2519222
             |
       _cons |  -.4441949   .1276995    -3.48   0.001    -.6944812   -.1939085
-------------+----------------------------------------------------------------
    /atanrho |   .4138925    .118934     3.48   0.001     .1807862    .6469988
-------------+----------------------------------------------------------------
         rho |   .3917727   .1006793                       .178842    .5696461
------------------------------------------------------------------------------
Wald test of rho=0: chi2(1) = 12.1105                 Prob > chi2 = 0.0005
```

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Postestimation Commands

Predictions

predict [*type*] *newvar* [*if*] [*in*] [, *statistic*]

statistic

p11	Pr(depvar = 1, depvar_en = 1); the default
p10	Pr(depvar = 1, depvar_en = 0)
p01	Pr(depvar = 0, depvar_en = 1)
p00	Pr(depvar = 0, depvar_en = 0)
pmarg1	Pr(depvar = 1); marginal success probability for outcome eq.
pmarg2	Pr(depvar_en = 1); marginal success probability for endogenous eq.
pcond1	Pr(depvar = 1 | depvar_en = 1)
pcond2	Pr(depvar_en = 1 | depvar = 1)
xb1	linear prediction for outcome eq.
xb2	linear prediction for endogenous eq.
stdp1	standard error of the linear prediction for outcome eq.
stdp2	standard error of the linear prediction for endogenous eq.

Postestimation Commands

Margins and Treatment Effects

margdec [*if*] [*in*] [, *response_options*]

tmeffect [*if*] [*in*] [, tmeffect(*effecttype*)]

margdec options

effect(*effecttype*)	specify type of effect; *effecttype* may be total, direct, or indirect; default is total
predict(*pred_opt*)	estimate margins for predict, *pred_opt* ; multiple predict not applicable
dydx(*varlist*)	estimate marginal effect of variables in *varlist*
continuous	treat factor-level indicators as continuous

tmeffect options

tmeffect(*effecttype*)	specify type of effect; *effecttype* may be ate, att, or atc; default is ate

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Post-Estimation: predict

Comparison: biprobit vs. rbprobit

```
. webuse class10, clear
(Class of 2010 profile)

. qui: rbprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. predict p11r, p11

. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///
> (program = i.campus i.scholar income i.hsgpagrp)

. predict p11b, p11

. compare p11r p11b

                                        ---------- difference ----------
                            count       minimum      average     maximum
------------------------------------------------------------------------
p11r<p11b                     678      -.0000178    -.0000104   -1.49e-08
p11r=p11b                       1
p11r>p11b                    1821       2.98e-08      .026773    .1206536
                       ----------
jointly defined              2500      -.0000178     .0194987    .1206536
                       ----------
total                        2500
```

Post-Estimation: margdec

Continuous Covariate: Total Average Marginal Effects

```
. margdec, dydx(income) predict(p11) effect(total)

Average marginal effects                          Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0032146    .002856     1.13   0.260    -.0023831    .0088123
------------------------------------------------------------------------------
```

Post-Estimation: margdec

Continuous Covariate: Direct Average Marginal Effects

```
. margdec, dydx(income) predict(p11) effect(direct)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0207027   .0017927    11.55   0.000     .0171891    .0242163
------------------------------------------------------------------------------
```

Post-Estimation: margdec

Continuous Covariate: Indirect Average Marginal Effects

```
. margdec, dydx(income) predict(p11) effect(indirect)

Average marginal effects                          Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |  -.0174881     .00214    -8.17   0.000    -.0216825   -.0132937
------------------------------------------------------------------------------
```

Post-Estimation: tmeffect

Average Treatment Effect

```
. tmeffect, tmeffect(ate)

Treatment effect                                 Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1), predict(pmarg1)
Effect       : Average treatment effect
dydx w.r.t.  : 1.program

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ate |   .0981233   .0476266     2.06   0.039     .0047769    .1914697
------------------------------------------------------------------------------
```

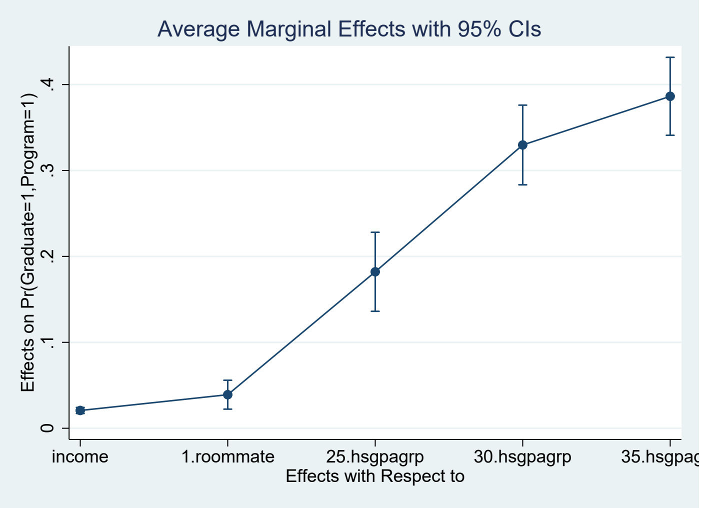
Post-Estimation: marginsplot

Graph results from margdec and tmeffect

```
. qui: margdec, dydx(income roommate hsgpagrp) predict(p11) effect(direct)

. marginsplot

  Variables that uniquely identify margins: _deriv

. qui: tmeffect, tmeffect(ate)

. marginsplot

  Variables that uniquely identify margins:
```

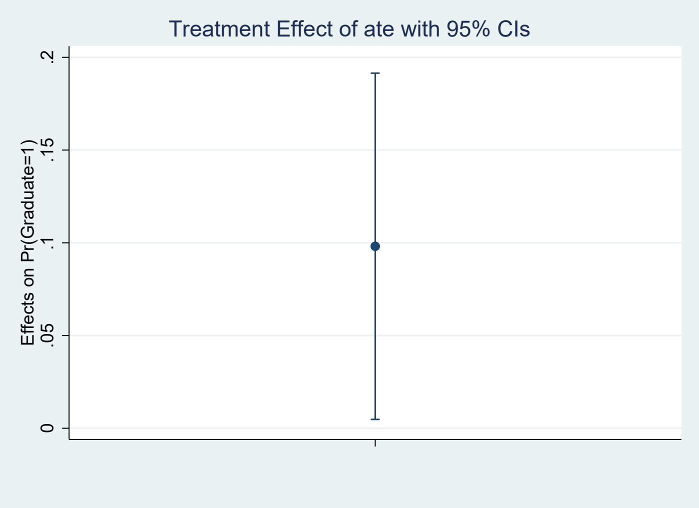
Post-Estimation: marginsplot

Graph results from margdec



Post-Estimation: marginsplot

Graph results from tmeffect



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Future Additions

Estimation and Post-Estimation Options

1. Estimation Options
   - Weights
   - Model and SE options
   - Reporting options
   - Maximization options
2. Post-Estimation Options
   - Approriate `margins` options (`at()`, `contrast`, etc.)
   - Weights
   - SE options
   - Reporting options
   - Maximization options
3. Post-Estimation Commands
   - `bphltest`
   - `scoregof`

Thank you

Version 1.0.0 available

net install rbiprobit, from("https://raw.githubusercontent.com/cobanomics/rbiprobit/main/")

GitHub: github.com/cobanomics/rbprobit
Email: mustafa.coban@iab.de
Web: mustafacoban.de

References

Alrasheed, D. S. (2019). The relationship between neighborhood design and social capital as measured by carpooling. *Journal of Regional Science*, 59(5):962–987.

Blasch, J., Filippini, M., and Kumar, N. (2019). Boundedly rational consumers, energy and investment literacy, and the display of information on household appliances. *Resource and Energy Economics*, 56(C):39–58.

Edwards, L. N., Hasebe, T., and Sakai, T. (2019). Education and Marriage Decisions of Japanese Women and the Role of the Equal Employment Opportunity Act. *Journal of Human Capital*, 13(2):260–292.

Greene, W. H. (2018). *Econometric Analysis*. Pearson, New York.

Han, S. and Lee, S. (2019). Estimation in a generalization of bivariate probit models with dummy endogenous regressors. *Journal of Applied Econometrics*, 34(6):994–1015.

Hasebe, T. (2013). Marginal effects of a bivariate binary choice model. *Economics Letters*, 121(2):298–301.

Appendix

Predictions of Interest

1. Joint Probabilities

$$\Pr(y_1 = 1, y_2 = 1 | x, z) = \Phi_2(x'\beta + \alpha, z'\gamma, \rho)$$
$$\Pr(y_1 = 1, y_2 = 0 | x, z) = \Phi_2(x'\beta, -z'\gamma, -\rho)$$
$$\Pr(y_1 = 0, y_2 = 1 | x, z) = \Phi_2(-x'\beta + \alpha, z'\gamma, -\rho)$$
$$\Pr(y_1 = 0, y_2 = 0 | x, z) = \Phi_2(-x'\beta, -z'\gamma, \rho)$$

2. Conditional Probabilities

$$\Pr(y_1 = 1 | y_2 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(z'\gamma)}$$
$$\Pr(y_2 = 1 | y_1 = 1, x, z) = \frac{\Phi_2(x'\beta + \alpha, z'\gamma, \rho)}{\Phi(x'\beta + \alpha)}$$

Appendix

Predictions of Interest

3. Marginal Probabilities

$$\Pr(y_1 = 1|x) = \Phi(x'\beta + \alpha y_2)$$
$$\Pr(y_2 = 1|z) = \Phi(z'\gamma)$$

4. Unconditional Mean Function (see Blasch et al., 2019; Alrasheed, 2019)

$$\begin{aligned} E[y_1|x,z] &= \Pr(y_2 = 1|z) \cdot E[y_1|y_2 = 1, x, z] \\ &+ \Pr(y_2 = 0|z) \cdot E[y_1|y_2 = 0, x, z] \\ &= \Pr(y_1 = 1, y_2 = 1|x, z) + \Pr(y_1 = 1, y_2 = 0|x, z) \\ &= \Phi_2(x'\beta + \alpha, z'\gamma, \rho) + \Phi_2(x'\beta, -z'\gamma, -\rho) \end{aligned}$$

Appendix

Manual Changes of Dependent Variables for Predictions

```
. qui: rbprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. predict p11r, p11

. qui: biprobit (graduate = income i.roommate i.hsgpagrp i.program) ///
> (program = i.campus i.scholar income i.hsgpagrp)

. replace graduate = 1
(972 real changes made)

. replace program = 1
(1,148 real changes made)

. predict p11b, p11

. compare p11r p11b

                                        ---------- difference ----------
                            count       minimum      average     maximum
------------------------------------------------------------------------
p11r<p11b                    1033     -.0000178    -8.81e-06   -1.49e-08
p11r=p11b                       7
p11r>p11b                    1460      2.98e-08     .0000105     .000084
                       ----------
jointly defined              2500     -.0000178     2.47e-06     .000084
                       ----------
total                        2500
```

Appendix

Incorrect Standard Errors after margins

```
. qui: rbprobit graduate = income i.roommate i.hsgpagrp ///
> , endog(program = i.campus i.scholar income i.hsgpagrp)

. margdec, dydx(income) predict(p11) effect(total)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0032146    .002856     1.13   0.260    -.0023831    .0088123
------------------------------------------------------------------------------

. margins, dydx(income) predict(p11)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : income

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0032146   .0031248     1.03   0.304    -.0029099    .0093391
------------------------------------------------------------------------------
```

Appendix

Discrete Covariate: Direct Average Marginal Effects

```
. margdec, dydx(hsgpagrp) predict(p11) effect(direct)

Average marginal effects                        Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : 25.hsgpagrp 30.hsgpagrp 35.hsgpagrp

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    hsgpagrp |
    2.5-2.9  |   .1821001   .0234585     7.76   0.000     .1361223    .2280779
    3.0-3.4  |   .3297082   .0236584    13.94   0.000     .2833386    .3760777
    3.5-4.0  |    .386345   .0231423    16.69   0.000     .3409869    .4317032
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Appendix

Discrete Covariate: Indirect Average Marginal Effects

```
. margdec, dydx(hsgpagrp) predict(p11) effect(indirect)

Average marginal effects                          Number of obs     =      2,500
Model VCE    : OIM

Expression   : Pr(graduate=1,program=1), predict(p11)
dy/dx w.r.t. : 25.hsgpagrp 30.hsgpagrp 35.hsgpagrp

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    hsgpagrp |
    2.5-2.9  |   .0131266   .0246783     0.53   0.595     -.035242    .0614953
    3.0-3.4  |   .0145757   .0258243     0.56   0.572     -.036039    .0651905
    3.5-4.0  |  -.0219075   .0402565    -0.54   0.586    -.1008087    .0569938
------------------------------------------------------------------------------
Note: dy/dx for factor levels is the discrete change from the base level.
```

Appendix

Average Treatment Effect on the Treated

```
. tmeffect, tmeffect(att)

Treatment effect                                     Number of obs     =      1,352
Model VCE     : OIM

Expression    : normal(graduate=1|program=1) - normal(graduate=1|program=0)
Effect        : Average treatment effect on the treated
dydx w.r.t.   : 1.program

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         att |   .1033448   .0489003     2.11   0.035     .0075019    .1991877
------------------------------------------------------------------------------
```

Appendix

Average Treatment Effect on the Conditional Probability

```
. tmeffect, tmeffect(atc)

Treatment effect                                   Number of obs     =      2,500
Model VCE     : OIM

Expression    : Pr(graduate=1|program=1)-Pr(graduate=1|program=0), predict(pcond1)-
> ict(pcond10)
Effect        : Average treatment effect on conditional probability
dydx w.r.t.   : 1.program

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         atc |   .2765848   .0164366    16.83   0.000     .2443696    .3087999
------------------------------------------------------------------------------
```