Parcial 2 - Modelación Experimental.

Nombre: Santiago Cartagena Agudelo

Código: 201910025101

Pregunta 1 (2.5 Puntos)

Obtener las expresiones matemáticas para el cálculo del a) Gradiente y b) Hessiano del siguiente modelo:

OE 121 → $na = 1$, $nb = 2$, $nk = 1$.

Solución

Recordemos la estructura OE:

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})} \cdot u(t) + e(t).$$

a) Gradiente:

$$A(q^{-1}) = 1 + a_1 q^{-1}$$

$$B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2}$$

$$y(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1}} u(t) + e(t)$$

→ Continua página siguiente

$$\hat{y}(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1}} u(t)$$

$$\varepsilon(t) = y(t) - \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1}} u(t).$$

* Se va a usar $V = \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t).$

→ $\hat{a}_1(k+1) = \hat{a}_1(k) - a_k \frac{\partial V}{\partial \hat{a}_1}$

$$\frac{\partial V}{\partial a_1} = \frac{2}{N} \sum_{t=1}^{N} \varepsilon(t) \frac{\partial \varepsilon}{\partial a_1} = \frac{2}{N} \sum_{t=1}^{N} \varepsilon(t) \cdot \hat{y}_F(t-1)$$

$$\frac{\partial \varepsilon}{\partial a_1} = \frac{b_1 q^{-1} + b_2 q^{-2}}{(1 + a_1 q^{-1})^2} \cdot q^{-1} u(t).$$

$$= \frac{q^{-1}}{1 + a_1 q^{-1}} \hat{y}(t) = \frac{\hat{y}(t-1)}{1 + a_1 q^{-1}} = \hat{y}_F(t-1)$$

→ $\hat{b}_1(k+1) = \hat{b}_1(k) - \alpha_k \frac{\partial V}{\partial \hat{b}_1}$

$$\frac{\partial V}{\partial b_1} = \frac{2}{N} \sum_{t=1}^{N} \varepsilon(t) \frac{\partial \varepsilon}{\partial b_1} = -\frac{2}{N} \sum_{t=1}^{N} \varepsilon(t) \cdot u_F(t-1)$$

$$\frac{\partial \varepsilon}{\partial b_1} = \frac{-q^{-1}}{1 + a_1 q^{-1}} u(t) = \frac{-u(t-1)}{1 + a_1 q^{-1}} = -u_F(t-1)$$

→ Continúa página siguiente

$$\rightarrow \hat{b}_2(k+1) = \hat{b}_2(k) - \alpha_k \frac{\partial V}{\partial \hat{b}_2}$$

$$\frac{\partial V}{\partial b_2} = \frac{2}{N}\sum_{t=1}^{N} \varepsilon(t)\frac{\partial \varepsilon}{\partial b_2} = -\frac{2}{N}\sum_{t=1}^{N} \varepsilon(t)\,u_F(t-2)$$

$$\frac{\partial \varepsilon}{\partial b_2} = \frac{-q^{-2}}{1+a_1q^{-1}}\,u(t) = \frac{-u(t-2)}{1+a_1q^{-1}} = -u_F(t-2)$$

b) Hessiano.

$$\begin{bmatrix} \frac{\partial^2 V}{\partial a_1^2} & \frac{\partial^2 V}{\partial a_1 \partial b_1} & \frac{\partial^2 V}{\partial a_1 \partial b_2} \\ \frac{\partial^2 V}{\partial b_1 \partial a_1} & \frac{\partial^2 V}{\partial b_1^2} & \frac{\partial^2 V}{\partial b_1 \partial b_2} \\ \frac{\partial^2 V}{\partial b_2 \partial a_1} & \frac{\partial^2 V}{\partial b_2 \partial b_1} & \frac{\partial^2 V}{\partial b_2^2} \end{bmatrix}$$

$$* \ \frac{\partial^2 V}{\partial a_1^2} = \left(\sum_{t=1}^{N} \hat{y}_F^2(t-1)\right)\frac{2}{N}$$

$$* \ \frac{\partial^2 V}{\partial b_1^2} = \frac{2}{N}\sum_{t=1}^{N} u_F^2(t-1)$$

$$* \ \frac{\partial^2 V}{\partial b_2^2} = \frac{2}{N}\sum_{t=1}^{N} u_F^2(t-2)$$

→ Continúa siguiente página.

* $$\frac{\partial^2 V}{\partial a_1 \partial b_1} = -\frac{2}{N} \sum_{t=1}^{N} \hat{y}_f(t-1)\, u_F(t-1)$$

* $$\frac{\partial^2 V}{\partial a_1 \partial b_2} = -\frac{2}{N} \sum_{t=1}^{N} \hat{y}_F(t-1)\, u_F(t-2)$$

* $$\frac{\partial^2 V}{\partial b_1 \partial b_2} = \frac{2}{N} \sum_{t=1}^{N} u_f(t-1)\, u_F(t-2)$$