Imports

In [1]:
```
import os

import pandas as pd
pd.options.mode.chained_assignment = None

import datetime as dt
import numpy as np
import statsmodels.api as sm

import matplotlib.pyplot as plt
import matplotlib.dates as mdates
import seaborn as sns
plt.style.context(("seaborn", "ggplot"))
os.chdir('../')
```

In [2]:
```
assert os.getcwd() == 'C:\\Users\\Garrett\\OneDrive\\Projects\\TempPred'
```

Goal: Predict peak-load on Thursday June 18, 2020 and hour of peak-load

In [3]:
```
def plot_dual_tseries(data_df, ts_1, ts_2, ts_1_title, ts_2_title, plot_title):

    """
    ================================================================
    Description:
        - Convenience function for generating consistent dual plots of time series
    --------------
    Inputs:
        - data_df: Dataframe containing the two time series to be plotted
        - ts_1, ts_2: column names from data_df which denote the respective series
        - ts_1_title, ts_2_title: strings to be used as the respective y-axis labels
        - plot_title: string giving the overall plot name
    Outputs:
        - None; plot generated on call
    ---------------
    Notes:
        - data_df index should be set as a Datetime index prior to being passed into the function
            - Should add an assert statement here in future?
            - Question is how to handle different freq.
    ================================================================

    """

    fig1, ax1 = plt.subplots(figsize = (20, 10))

    ax1.plot(ts_1, data = data_df, color = 'blue', linewidth = 2)
    ax1.tick_params(axis = 'y', labelcolor = 'blue', labelsize = 15)
```

```
    locator = mdates.AutoDateLocator(minticks=36)
    ax1.xaxis.set_major_locator(locator)
    ax1.xaxis.set_tick_params(rotation = 45, labelsize = 15)
    ax1.set_xlim(data_df.index[0], data_df.index[-1])
    ax1.set_ylabel(ts_1_title, fontsize = 20, color = 'blue')

    ax2 = ax1.twinx()
    ax2.plot(ts_2, data = data_df, color = 'red', linewidth = 2)
    ax2.tick_params(axis = 'y', labelcolor = 'red', labelsize = 15)
    ax2.set_ylabel(ts_2_title, fontsize = 20, color = 'red')

    fig1.suptitle(plot_title, fontsize = 25)

    fig1.tight_layout()

def plot_annual_loadprofile(data_df, month_num):

    """
    ================================================================
    Description:
        - Generate scatterplot of load vs. temperature annually, colored by peak vs. off-peak periods
    --------------
    Inputs:
        - data_df: Dataframe containing load and temperature data
        - month_num: integer corresponding to the month in question
    Outputs:
        - None; plot generated by call
    ---------------
    Notes:
        - Would refine by altering subplot titles, possibly legend as well
    ================================================================

    """
    month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec']

    df_tmp = data_df.loc[df['mth'] == month_num]
    month_name = month_names[month_num-1] #Because list is 0 indexed, but month number in df starts at

    plt.figure(figsize = (20,10))
    sns.set_context('talk')
    g1 = sns.relplot(data = df_tmp, x = 'temp', y = 'load',
                     hue = 'on_peak', palette = 'deep', col = 'yr',
                    height = 8, aspect=.75)

    g1.set_axis_labels('Temperature (F)', "Load (MW)")
    plt.suptitle(month_name+' Load vs Temperature by Year, Load Type', fontsize = 20)
    plt.tight_layout()

def plot_monthly_loadprofile(data_df):

    """
    ================================================================
    Description:
        - Generate scatterplot of load vs. temperature monthly, colored by peak vs. off-peak periods
    --------------
    Inputs:
        - data_df: Dataframe containing load and temperature data
    Outputs:
        - None; plot generated by call
    ---------------
    Notes:
```

```
    - Need to move legend in future
    ================================================================

    """
    plt.figure(figsize = (20,10))
    month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec']

    sns.set_context('talk')
    g2 = sns.relplot(data = data_df, x = 'temp', y = 'load',
                     hue = 'on_peak', palette = 'deep', col = 'mth', col_wrap = 4,
                    height = 8, aspect=.75)

    for n in g2.axes_dict.keys():
        tmp = g2.axes_dict[n]
        mth = month_names[n-1]
        tmp.set_title(mth, fontsize = 20)

    g2.set_axis_labels('Temperature (F)', "Load (MW)")
    plt.suptitle('Load vs Temperature by Month, Load Type', fontsize = 20)
    plt.tight_layout()

def plot_dayofweek_loadprofile(data_df, month_num):

    """
    ================================================================
    Description:
        - Generate scatterplot and violinplots of load vs. temperature daily
    --------------
    Inputs:
        - data_df: Dataframe containing load and temperature data
        - month_num: integer corresponding to the month in question
    Outputs:
        - None; plot generated by call
    --------------
    Notes:
        - Need to move legend in future
    ================================================================

    """
    month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec']
    day_names = ['Monday', 'Tuesday', 'Wednesday', 'Thursday', 'Friday', 'Saturday', 'Sunday']

    df_tmp = data_df.loc[df['mth'] == month_num]
    month_name = month_names[month_num-1] #Because list is 0 indexed, but month number in df starts at


    plt.figure(figsize = (20,10))
    sns.set_context('talk')
    g3 = sns.relplot(data = df_tmp, x = 'temp', y = 'load',
                     hue = 'on_peak', palette = 'deep', col = 'dow',
                    height = 8, aspect=.75, col_wrap = 4)

    for n in range(len(g3.axes_dict.keys())):
        g3.axes_dict[n].set_title(day_names[n], fontsize = 20)

    g3.set_axis_labels('Temperature (F)', "Load (MW)")
    plt.suptitle(month_name+' Load vs Temperature by Day of Week, Load Type', fontsize = 20)
    plt.tight_layout()

    plt.figure(figsize = (20,10))
    sns.set_context('talk')
```

```
    g4 = sns.violinplot(data = df_tmp, x = 'dow', y = 'load', hue = 'on_peak', split = True, inner = 'quart')

    plt.suptitle(month_name+' Load Distribution by Day of Week, Load Type', fontsize = 20)
    g4.set_xlabel(None)
    g4.set_ylabel("Load (MW)")
    g4.set_xticklabels(day_names)
    plt.tight_layout()

def plot_hourly_loadprofile(data_df, month_num):

    """
    ================================================================
    Description:
        - Generate scatterplot and violinplots of load vs. temperature hourly
    --------------
    Inputs:
        - data_df: Dataframe containing load and temperature data
        - month_num: integer corresponding to the month in question
    Outputs:
        - None; plot generated by call
    ---------------
    Notes:
        - Need to move legend in future
    ================================================================

    """
    month_names = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec']
    df_tmp = data_df.loc[df['mth'] == month_num]
    month_name = month_names[month_num-1] #Because list is 0 indexed, but month number in df starts at

    plt.figure(figsize = (20,10))
    sns.set_context('talk')
    g1 = sns.violinplot(data = df_tmp, x = 'hr', y = 'load', inner = 'quart')

    plt.suptitle(month_name+' Load Distribution by Hour, Load Type', fontsize = 20)
    g1.set_xlabel(None)
    g1.set_ylabel("Load (MW)")

    plt.tight_layout()

# If I were moving this code forward and operationalizing it, I would likely opt to create a wrapper function
# to the effect of .plot_loadprofile(data_df, month_num, profile_type) and aggregate above fuctions
# or create a class, but I think breaking out individually is helpful to better flow with the markdown discussion
```

In [4]:

```
def gen_peak_id(df, ts_col, hol_df, hol_col):

    """
    ================================================================
    Description:
        - Function for parsing load into on-peak and off-peak, accounting for business days and that year's holic
        - On-peak is defined as hours ending 7-22 on non-NERC holiday business days, and off-peak is all othe
    --------------
    Inputs:
        - df: Pandas Dataframe containing the load data and dates
        - ts_col: name of the column corresponding to the date (hourly index)
        - hol_df: Dataframe containing the holiday dates for the timespand covered by the data
        - hol_col: name of the column in hol_df dataframe containing the dates
    Outputs:
        - df_tmp: copy of input Dataframe, enhanced with an indicator variable for on-peak periods
```

```
    ----------
    Notes:
        - Occasionally Excel hourly data reads in strangely (23:59 instead of 00:00 for example), so we round to
        - Referenced against https://www.energygps.com/HomeTools/PowerCalendar
    ================================================================

    """
    dt_id = pd.DatetimeIndex(df[ts_col].round('H'), normalize = True)
    hday_idx = pd.DatetimeIndex(hol_df[hol_col], normalize=True)

    df_tmp = df.copy()
    df_tmp['ts'], df_tmp['yr'], df_tmp['mth'], df_tmp['dom'], df_tmp['dow'], df_tmp['hr'] = dt_id, dt_id.year, dt_id.m

    df_tmp['nerc_hol'] = np.nan

    df_tmp.loc[dt_id.isin(hday_idx), 'nerc_hol'] = 1
    df_tmp['nerc_hol'].ffill(limit = 23, inplace = True)
    df_tmp['nerc_hol'].fillna(0, inplace = True)

    peak_mask = (df_tmp['nerc_hol'] == 0) & (df_tmp['dow'] < 5) & ((df_tmp['hr'] > 6) & (df_tmp['hr']<23))

    df_tmp.loc[peak_mask, 'on_peak'] = 1
    df_tmp.loc[~peak_mask, 'on_peak'] = 0

    return df_tmp.drop(columns = 'nerc_hol')
```

In [5]:

```
class LoadModel:
    def __init__(self, data_df, y_col, x_cols):
        """
        ================================================================
        Description:
            - This class is used to specify, fit, and evaluate the various alternative regression models utilized in th
            - The overall class consists of 2 function calls that handle most of the work:
                - LoadModel.fit_model(): allows user to specify type of regression (OLS or Robust) as well as whet
                - LoadModel.plot_modelfit(): leverages hourly hues to plot the residuals of the regression vs. tempe
        ----------
        Inputs:
            - data_df: Dataframe containing the data used to build the model
            - y_col: string indicating what column to use as a dependent variable
            - x_cols: list or string indicating what column(s) to use as an independent variable
        Outputs:
            - None
        ----------
        Notes:
            - None
        ================================================================

        """

        self.data = data_df
        self.y = data_df[[y_col]]
        self.x = sm.add_constant(data_df[x_cols], prepend = False)

    def fit_model(self, model_type, error_type = None):
        """
        ================================================================
        Inputs:
            - model_type: string indicating whether the model should be fit using OLS or a Robust Regression
            - error_type: string indicating if model fit should use heteroscedasticity aware standard errors
```

```
            - Note this should only be used if model_type = OLS; could add an error catch here to handle this
        Outputs:
            - model_summary: Dataframe containing the statsmodels regression output
        ================================================================

        """
        if model_type == 'OLS':
            if error_type is not None:
                self.model = sm.OLS(self.y, self.x).fit(cov_type = error_type)
            else:
                self.model = sm.OLS(self.y, self.x).fit()

        if model_type == 'Robust':
            self.model = sm.RLM(self.y, self.x).fit()

        tmp_ = pd.DataFrame([self.model.fittedvalues, self.model.resid], index = ['yhat', 'e']).T
        self.output_df = pd.merge(self.data, tmp_, left_index = True, right_index = True)

        return self.model.summary2()

    def plot_modelfit(self, model_name):
        """
        ================================================================
        Inputs:
            - model_name: string to title plot and identify model
        Outputs:
            - None; plot generated by call
        ================================================================

        """
        sns.set_context('talk')
        sns.pairplot(data = self.output_df[['load', 'temp', 'e', 'hr']], hue = 'hr', height =5, aspect =2, palette = 'rocke
        plt.suptitle(model_name)
        plt.tight_layout()
```

Data Processing

- Below I conduct some general data pre-processing, especially regarding peak vs. off-peak load
- We need (for latter questions especially) a way to designate on and off peak load
- I pulled NERC Holidays and added them to the assessment on the NERC_HOL sheet
  - Holiday dates for 2016-2019 were generated using https://www.energygps.com/HomeTools/PowerCalendar
  - I checked total peak load hours using sums across month and year against the above link

In [6]:

```
df_raw = pd.read_excel('Data/Clean/TempLoadData.xlsx',
                       sheet_name='TempLoad',
                       usecols=['Hour Beginning','Temp (F)', 'Load (MW)'],
                       skiprows=1, parse_dates=True).rename(columns = {'Hour Beginning':'ts', 'Temp (F)':'temp', 'Lo

nerc_hol = pd.read_excel('Data/Clean/TempLoadData.xlsx',
                         sheet_name='NERC_HOL',
                         parse_dates=True).iloc[:-3]
```

In [7]:
```
df = gen_peak_id(df_raw, 'ts', nerc_hol, 'HolidayDate')
df.head()
```

Out[7]:

	ts	temp	load	yr	mth	dom	dow	hr	on_peak
0	2016-09-01 00:00:00	79.0	131.210245	2016	9	1	3	0	0.0
1	2016-09-01 01:00:00	79.0	123.057346	2016	9	1	3	1	0.0
2	2016-09-01 02:00:00	78.0	117.498482	2016	9	1	3	2	0.0
3	2016-09-01 03:00:00	77.0	113.287505	2016	9	1	3	3	0.0
4	2016-09-01 04:00:00	77.0	111.564578	2016	9	1	3	4	0.0

Exploratory Data Analysis

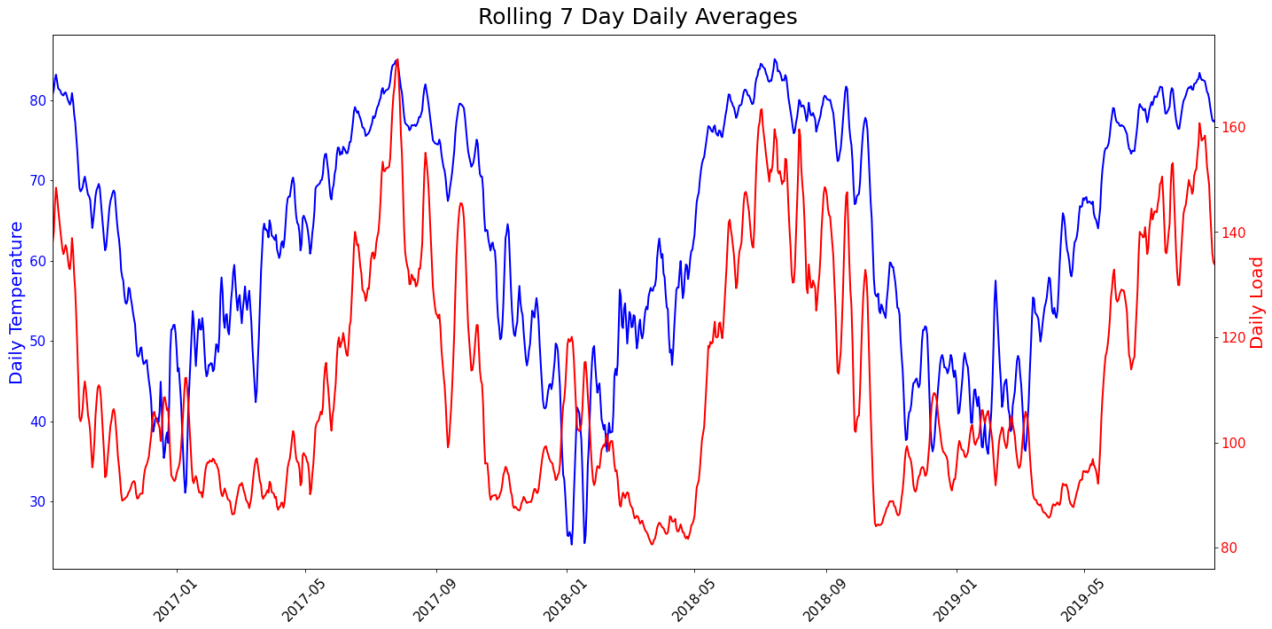
- I view EDA as equivalent to the axe sharpening in the alledged Lincoln quote "If I had eight hours to chop down a tree, I'd spend six sharpening my axe."
- Furthermore, I firmly believe if you cannot plot (or simulate) the process you are analyzing then you cannot properly model it; you don't understand it well enough

In [8]:
```
df_daily_7davg = df.set_index('ts').resample('D').mean().rolling(7).mean().dropna()
    # Generate the daily average load, then create a rolling 7-day average to cut down on the noise for
    # visualization's sake. Dropna() is called to get rid of first 6 days
df_daily_7davg.head()
```

Out[8]:

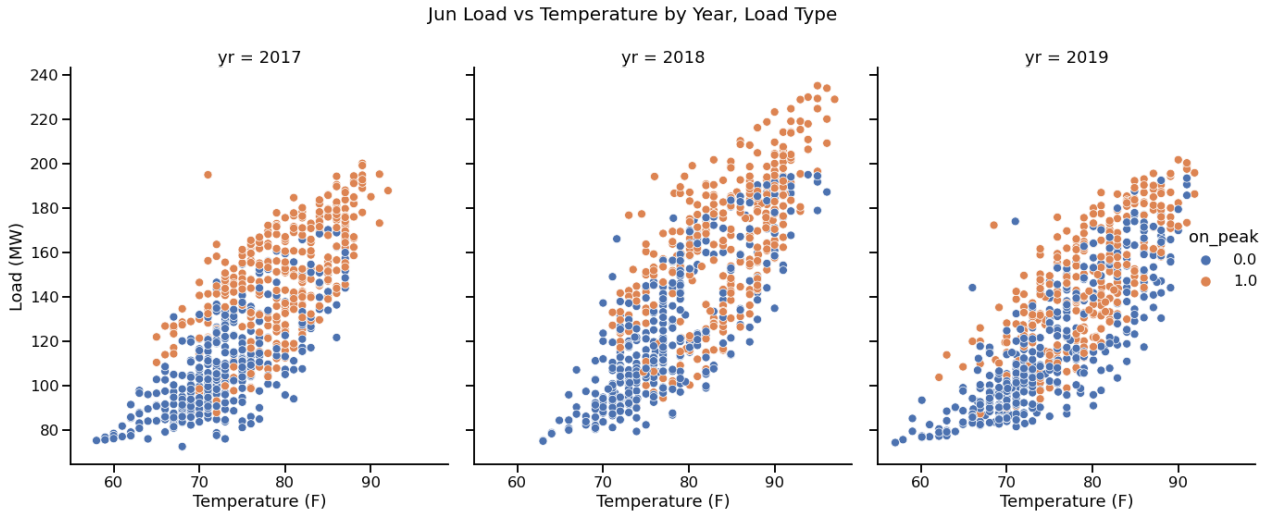
ts	temp	load	yr	mth	dom	dow	hr	on_peak
2016-09-07	80.779762	138.181312	2016.0	9.0	4.0	3.0	11.5	0.380952
2016-09-08	81.345238	140.156468	2016.0	9.0	5.0	3.0	11.5	0.380952
2016-09-09	82.541667	145.302565	2016.0	9.0	6.0	3.0	11.5	0.380952
2016-09-10	83.160714	148.414973	2016.0	9.0	7.0	3.0	11.5	0.380952
2016-09-11	82.357143	146.854949	2016.0	9.0	8.0	3.0	11.5	0.380952

In [9]:
```
plot_dual_tseries(df_daily_7davg, ts_1='temp', ts_2='load',
                  ts_1_title='Daily Temperature', ts_2_title='Daily Load',
                  plot_title='Rolling 7 Day Daily Averages')
```



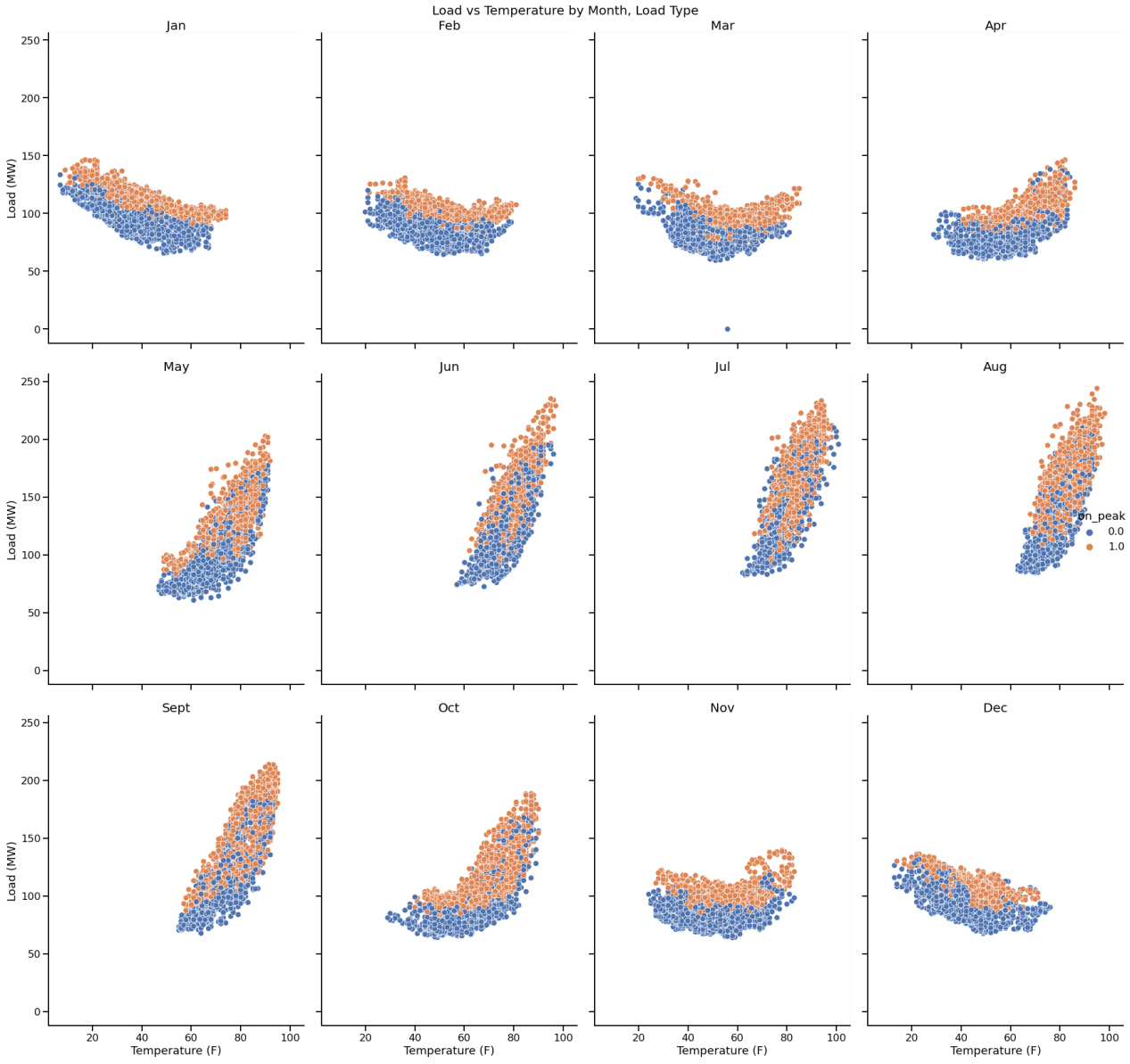
In [10]:
```
plot_annual_loadprofile(df, 6)
```

<Figure size 1440x720 with 0 Axes>



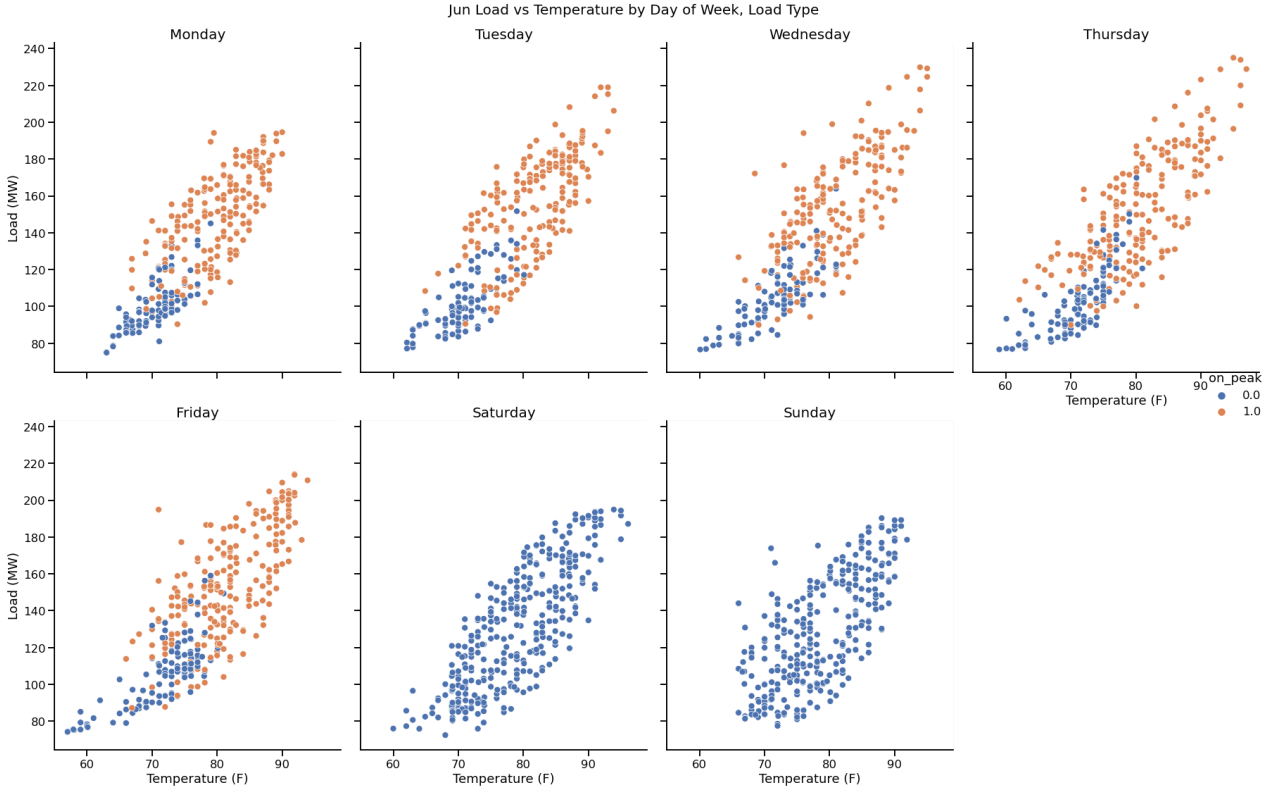
In [11]:
```
plot_monthly_loadprofile(df)
```

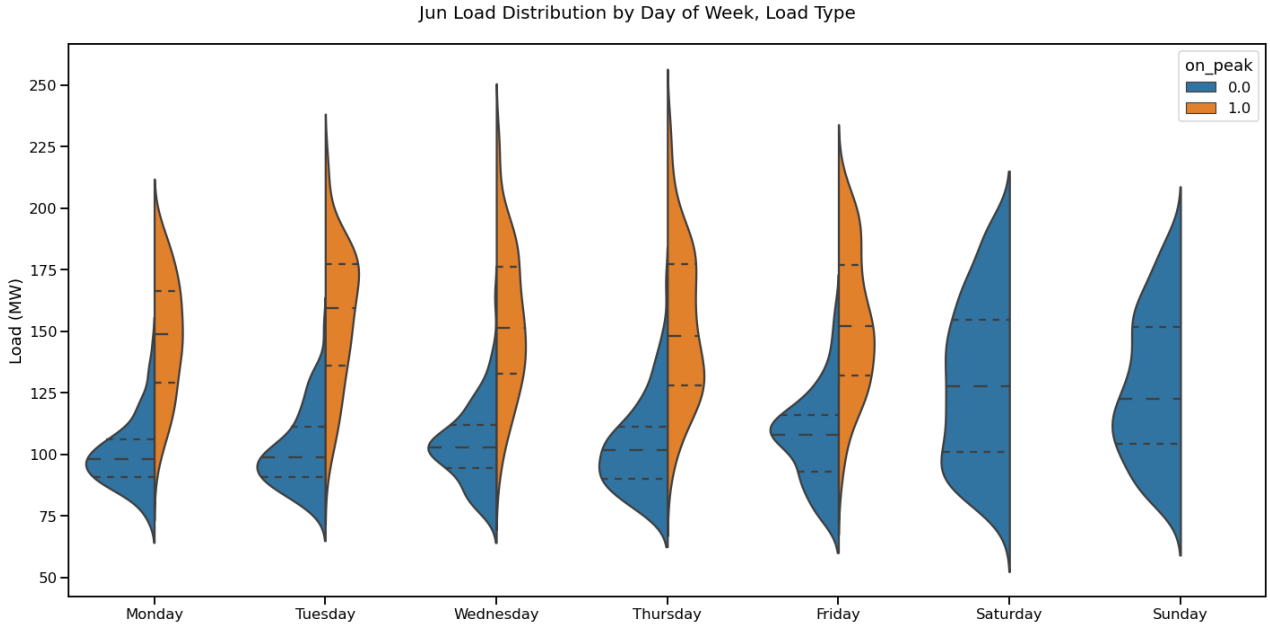
<Figure size 1440x720 with 0 Axes>



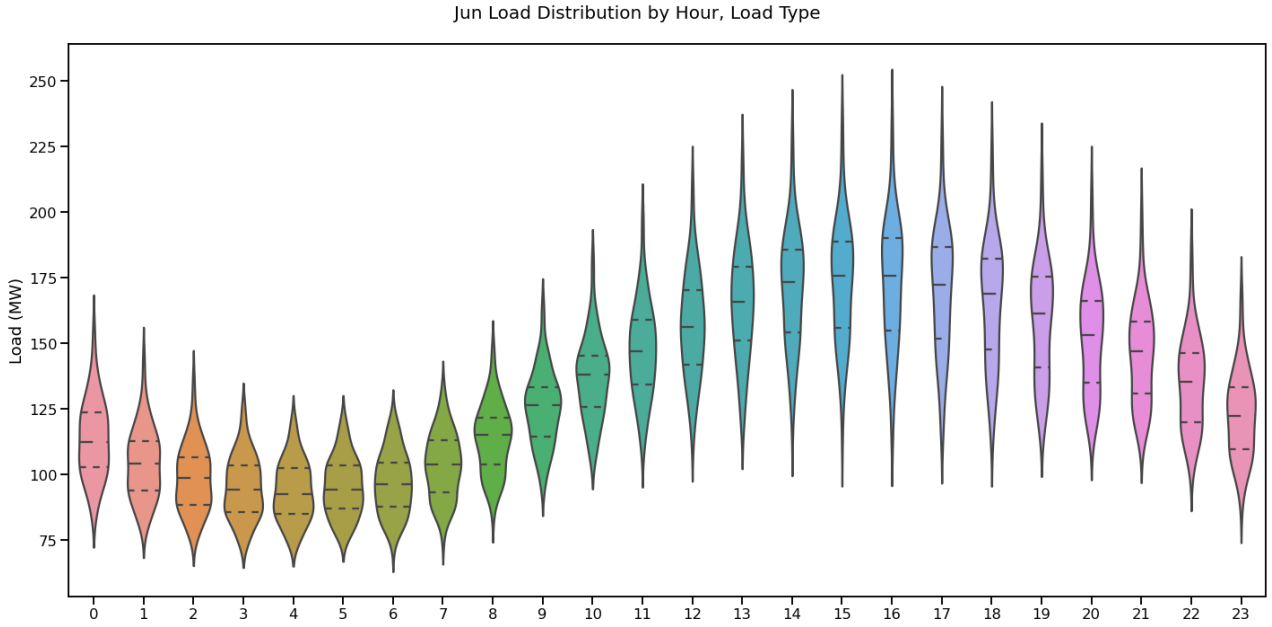
In [12]:
```
plot_dayofweek_loadprofile(df, 6)
```

<Figure size 1440x720 with 0 Axes>



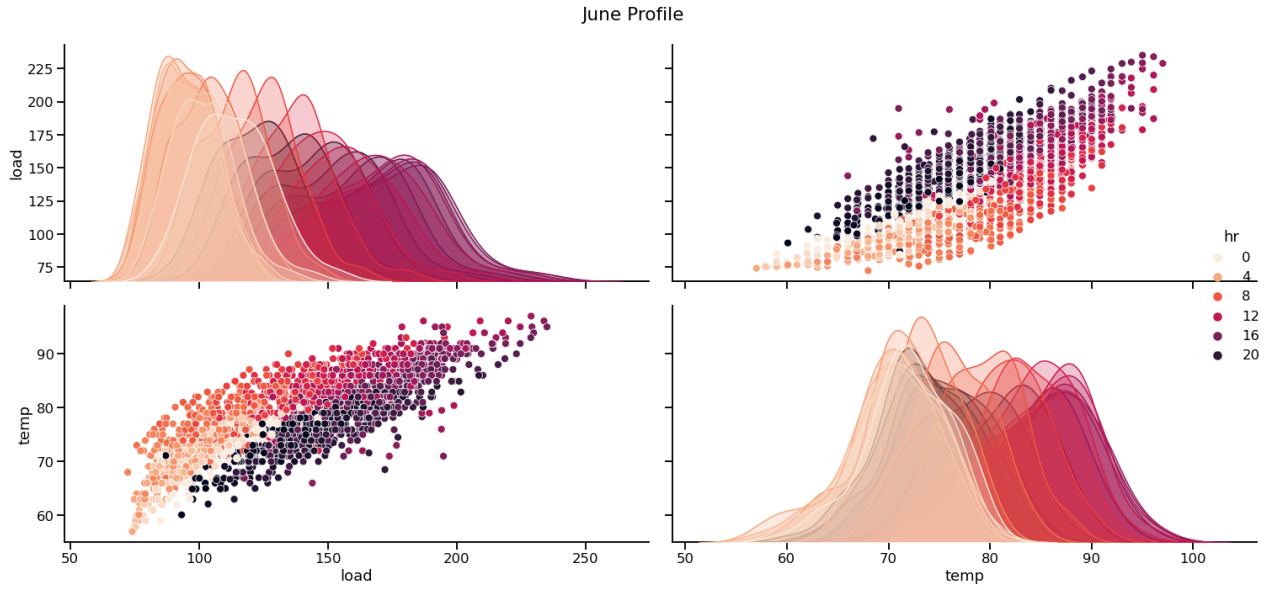


In [13]:
```
plot_hourly_loadprofile(df, 6)
```



In [14]:

```
df_j = df.loc[df['mth'] == 6] #only June obs.
sns.set_context('talk')
sns.pairplot(data = df_j[['load', 'temp', 'hr']], hue = 'hr', height =5, aspect =2, palette = 'rocket_r')
plt.suptitle('June Profile')
plt.tight_layout()
```



- Visualizing the load-temperature relationship annually, there does not appear to be significant variation while the opposite is true when viewing by month, day of week, and hour
  - Monthly load variation likely stems from natural seasonality while daily and hourly variation likely stem from the workweek and a workday/temperature cycle, respectively
- The hourly load profile displays a wave pattern which could be modeled with either a polynomial regression or sine wave for short term predictions
  - However, given we are forecasting at a longer-term horizon, I opted for a more structural model using dummy variables and interactions
  - Additionally, while fat tails (see next bullet) should give us some pause when using linear regression, polynomial regression is notoriously fragile when extrapolating out of sample

- Visualizing load by day of week and peak vs. off-peak, we can see both the mean and (more importantly) the distribution of load change markedly
  - Also note that our forecast day, a Thursday, displays the fattest tails
- Looking at the workweek, Monday's profile also differs markedly and thus I use Tuesday-Friday data to fit the model

Modeling

In [15]:
```
mask_1 = (df_j['dow'] >0) & (df_j['dow'] < 5)
reg_df = df_j.loc[mask_1].drop(columns = ['yr', 'mth', 'dom', 'dow']).reset_index(drop = True)
reg_df.head()
```

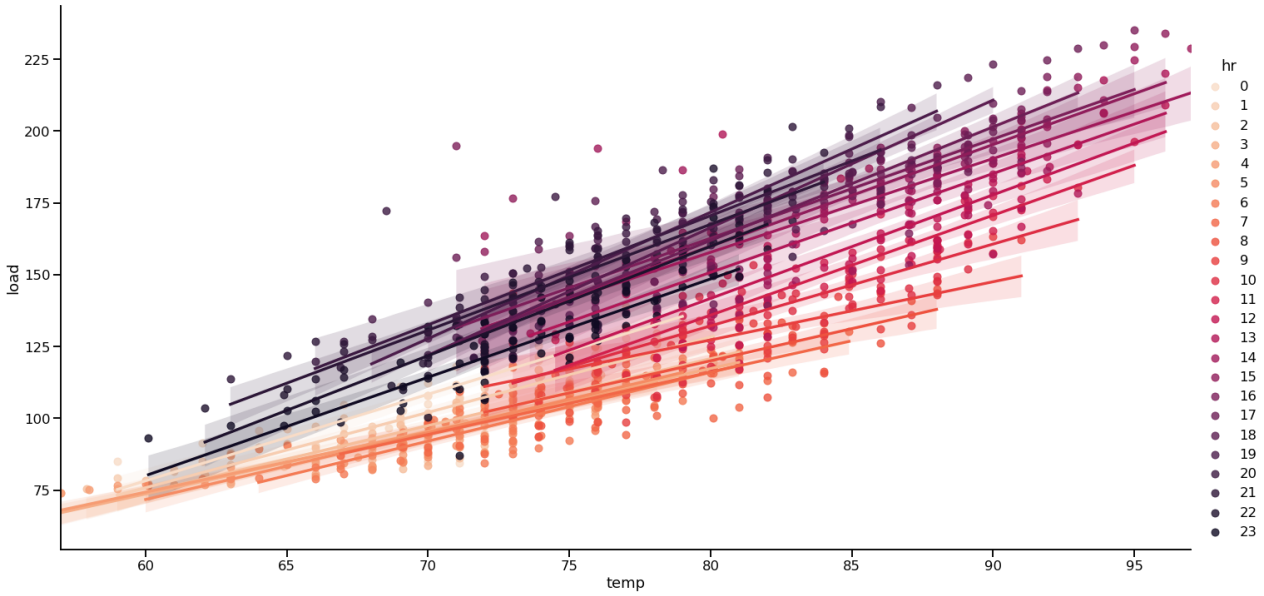
Out[15]:

	ts	temp	load	hr	on_peak
0	2017-06-01 00:00:00	70.0	108.028450	0	0.0
1	2017-06-01 01:00:00	69.0	100.454888	1	0.0
2	2017-06-01 02:00:00	68.0	94.941490	2	0.0
3	2017-06-01 03:00:00	67.0	91.426984	3	0.0
4	2017-06-01 04:00:00	68.0	90.396514	4	0.0

- The below plot provides a hint to what we find as we iterate through models--the optimal model will likely require hour-specific intercepts AND slopes

In [16]:
```
sns.lmplot(y = 'load', x = 'temp', hue = 'hr', data = reg_df, height = 10, aspect = 2, palette = 'rocket_r')
```

Out[16]:
<seaborn.axisgrid.FacetGrid at 0x25a84e002b0>



- We face two modeling choices

1. Whether to simply model differences in base load (intercepts) or also differences in load sensitivity (interactions) with the dummy variables
2. What to use as dummy variables—peak vs. off-peak or hour dummies

- I opted for hourly dummies as there is an hourly pattern in the load distribution we would overlook if aggregated to just peak and off-peak dummies
- All the models discussed below are fit to data from the Tues-Fri observations from June and utilize heteroscedasticity robust standard errors
  - Looking at the workweek using the daily visualizations in the prior section, Monday's profile also differs markedly and weekends are known to differ markedly as well

Model 1: $L_t = \alpha +\beta_1F_T $

- I began with a basic model, $L=a+β_1 Temp$
  - This has the advantage of letting us look at the residuals by hour
  - There is clearly clustering in the residuals of the basic model which supports the dummy variable intercepts
    - The pattern between residuals and temperature (which would give insight regarding interactions) is less clear when looking at the basic model residuals
  - The Adjusted R2 is 0.695

In [17]:
```
m1 = LoadModel(data_df = reg_df, y_col = 'load', x_cols = 'temp')
m1_results = m1.fit_model(model_type='OLS', error_type='HC1')
m1_results
```

```
C:\Users\Garrett\anaconda3\envs\tenaska_env\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all arguments of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

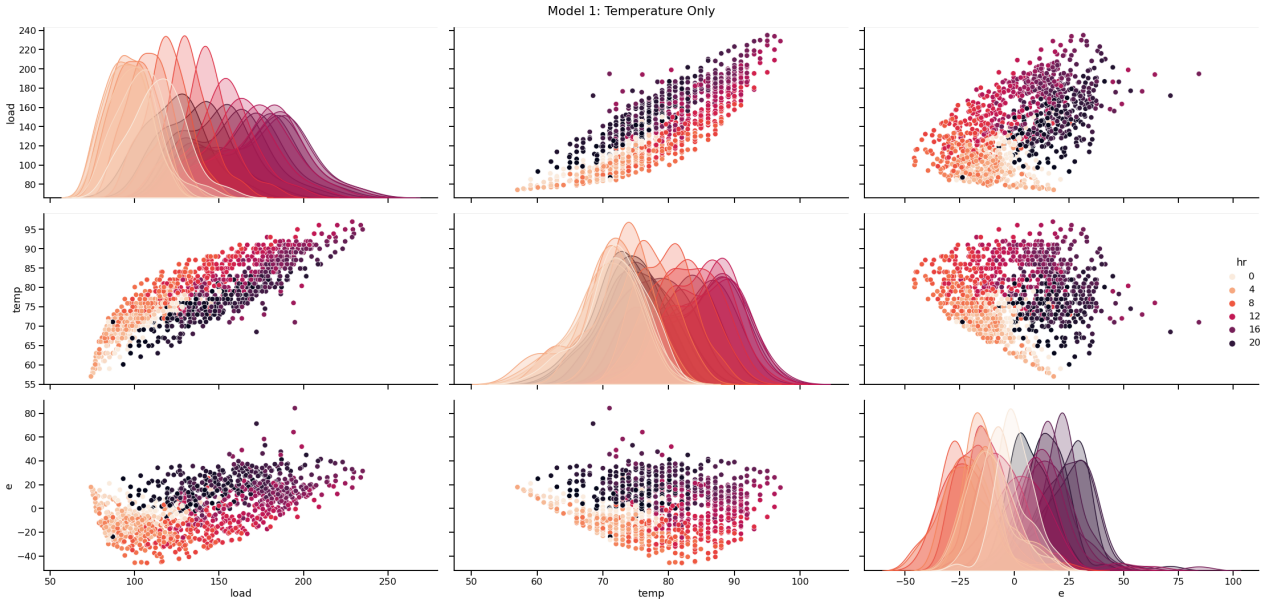
Out[17]:

Model:	OLS	Adj. R-squared:	0.695
Dependent Variable:	load	AIC:	10729.7419
Date:	2021-07-19 03:20	BIC:	10739.9617
No. Observations:	1224	Log-Likelihood:	-5362.9
Df Model:	1	F-statistic:	3644.
Df Residuals:	1222	Prob (F-statistic):	0.00
R-squared:	0.695	Scale:	374.88

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
temp	3.8760	0.0642	60.3631	0.0000	3.7501	4.0019
const	-164.6876	4.9825	-33.0534	0.0000	-174.4531	-154.9222

Omnibus:	19.673	Durbin-Watson:	0.195
Prob(Omnibus):	0.000	Jarque-Bera (JB):	18.354
Skew:	0.257	Prob(JB):	0.000
Kurtosis:	2.689	Condition No.:	813

In [18]:
```
m1.plot_modelfit('Model 1: Temperature Only')
```



Model 2: $L_t = \alpha +\sum_{i = 0}^{23} {\lambda_iH_i}+\beta_1F_T $

In [19]:
```
reg2_df = reg_df.copy()
hr_dummies = pd.get_dummies(reg2_df['hr'], drop_first=True, prefix = 'H')
reg2_df = reg2_df.merge(hr_dummies, left_index=True, right_index=True)
reg2_df.head()
```

Out[19]:

	ts	temp	load	hr	on_peak	H_1	H_2	H_3	H_4	H_5	...	H_14	H_15	H_16	H_17	I
0	2017-06-01 00:00:00	70.0	108.028450	0	0.0	0	0	0	0	0	...	0	0	0	0	
1	2017-06-01 01:00:00	69.0	100.454888	1	0.0	1	0	0	0	0	...	0	0	0	0	
2	2017-06-01 02:00:00	68.0	94.941490	2	0.0	0	1	0	0	0	...	0	0	0	0	
3	2017-06-01 03:00:00	67.0	91.426984	3	0.0	0	0	1	0	0	...	0	0	0	0	
4	2017-06-01 04:00:00	68.0	90.396514	4	0.0	0	0	0	1	0	...	0	0	0	0	

5 rows × 28 columns

In [20]:

```
m2 = LoadModel(data_df = reg2_df, y_col = 'load', x_cols = list(hr_dummies.columns)+['temp'])
m2_results = m2.fit_model(model_type='OLS', error_type='HC1')
m2_results
```

C:\Users\Garrett\anaconda3\envs\tenaska_env\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all arguments of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)

Out[20]:

Model:	OLS	Adj. R-squared:	0.915
Dependent Variable:	load	AIC:	9193.2466
Date:	2021-07-19 03:20	BIC:	9320.9936
No. Observations:	1224	Log-Likelihood:	-4571.6
Df Model:	24	F-statistic:	605.5
Df Residuals:	1199	Prob (F-statistic):	0.00
R-squared:	0.916	Scale:	104.87

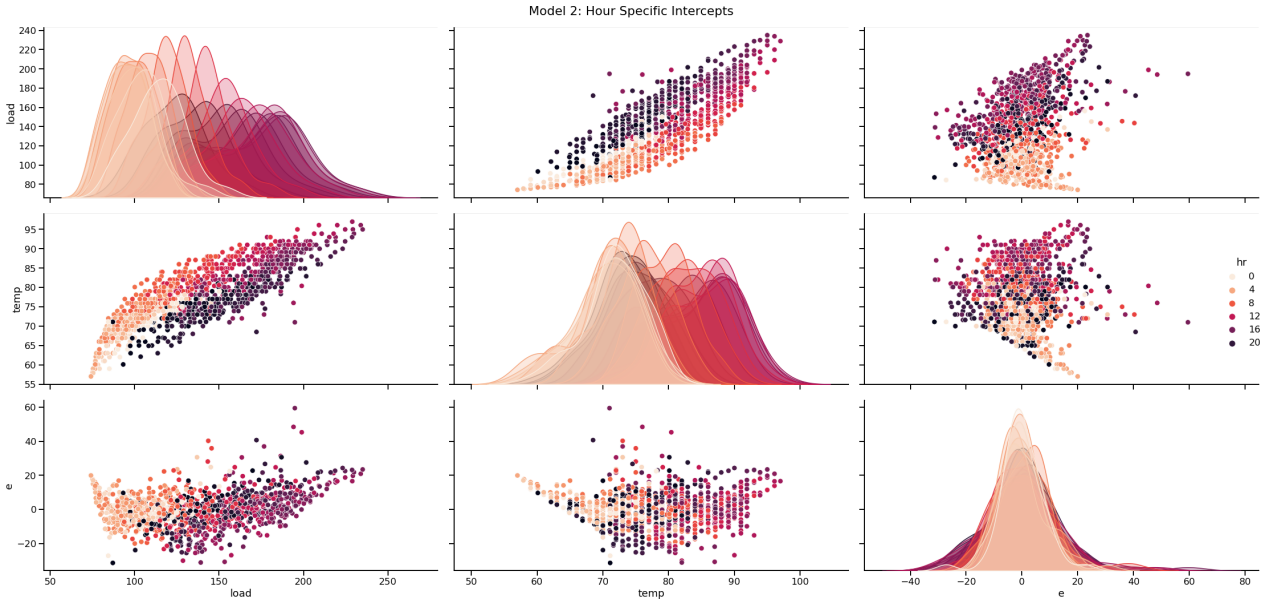
	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
H_1	-6.5369	1.4968	-4.3672	0.0000	-9.4705	-3.6032
H_2	-11.2749	1.5632	-7.2127	0.0000	-14.3387	-8.2111
H_3	-12.7532	1.5343	-8.3121	0.0000	-15.7604	-9.7460
H_4	-12.6012	1.5244	-8.2664	0.0000	-15.5889	-9.6135
H_5	-13.8413	1.3881	-9.9711	0.0000	-16.5620	-11.1206
H_6	-18.5411	1.4766	-12.5565	0.0000	-21.4351	-15.6470
H_7	-20.0111	1.6252	-12.3128	0.0000	-23.1965	-16.8257
H_8	-19.3028	1.7887	-10.7913	0.0000	-22.8086	-15.7969
H_9	-14.2521	2.1571	-6.6072	0.0000	-18.4799	-10.0243
H_10	-8.7280	1.9567	-4.4606	0.0000	-12.5630	-4.8930
H_11	-2.4433	1.9774	-1.2356	0.2166	-6.3190	1.4324
H_12	4.3396	2.1420	2.0259	0.0428	0.1413	8.5379
H_13	12.5552	2.4995	5.0232	0.0000	7.6564	17.4541
H_14	18.4991	2.6663	6.9382	0.0000	13.2733	23.7249
H_15	22.0461	2.4175	9.1193	0.0000	17.3079	26.7843
H_16	24.1415	2.6061	9.2633	0.0000	19.0336	29.2494
H_17	24.5153	2.0910	11.7240	0.0000	20.4170	28.6137
H_18	27.2946	1.9937	13.6907	0.0000	23.3871	31.2022
H_19	29.1463	2.0304	14.3552	0.0000	25.1668	33.1257
H_20	28.1307	2.1464	13.1058	0.0000	23.9238	32.3376
H_21	25.0117	1.7319	14.4416	0.0000	21.6172	28.4062
H_22	16.1874	1.6670	9.7103	0.0000	12.9201	19.4547

H_23	6.9243	1.6555	4.1825	0.0000	3.6795	10.1691
temp	3.1729	0.0812	39.0833	0.0000	3.0137	3.3320
const	-114.0204	5.9171	-19.2698	0.0000	-125.6177	-102.4232

Omnibus:	108.252	Durbin-Watson:	0.366
Prob(Omnibus):	0.000	Jarque-Bera (JB):	302.268
Skew:	0.460	Prob(JB):	0.000
Kurtosis:	5.254	Condition No.:	1945

- We could conduct a Lagrange Multiplier test to test Model 2 vs. Model 1, but given all the dummies are significant we know Model 2 offers a superior fit

In [21]:
```
m2.plot_modelfit('Model 2: Hour Specific Intercepts')
```



Model 3: $L = \alpha +\sum_{i = 0}^{23} {\lambda_iH_i}+\beta_1F +\sum_{i = 0}^{23} {\phi_iH_iF_t}$

- To test whether the intercepts and interactions are significant, I conducted a joint Likelihood Multiplier test of the intercept-interaction model vs. the intercept model
  - This stepwise process (thinking of our iteration from Model 1 to 2 to 3) is valid because the models are nested
- The LM test returns a p-value < 0.01, which supports the least restricted model
  - By least restricted I mean the model which includes both hourly intercepts and hourly interactions with temperature.
  - Thus, each hour has a distinct base load and sensitivity to temperature: $L=a+∑λ_i H_i+β_1 Temp+∑ϕ_i H_i Temp$

- Plotting the residuals, they also appear much closer to white noise which supports the additional terms

In [22]:
```
reg3_df = reg2_df.copy()
for n in range(1, 24):
    hr = 'H_'+str(n)
    reg3_df[hr+'*Temp'] = reg3_df[hr]*reg2_df['temp']
```

In [23]:
```
m3 = LoadModel(data_df = reg3_df, y_col = 'load', x_cols = list(reg3_df.columns[5:])+['temp'])
m3_results = m3.fit_model(model_type='OLS', error_type='HC1')
m3_results
```

C:\Users\Garrett\anaconda3\envs\tenaska_env\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all arguments of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)

Out[23]:

Model:	OLS	Adj. R-squared:	0.923
Dependent Variable:	load	AIC:	9082.0310
Date:	2021-07-19 03:20	BIC:	9327.3052
No. Observations:	1224	Log-Likelihood:	-4493.0
Df Model:	47	F-statistic:	503.3
Df Residuals:	1176	Prob (F-statistic):	0.00
R-squared:	0.926	Scale:	94.034

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
H_1	21.0204	22.3118	0.9421	0.3461	-22.7099	64.7508
H_2	37.0042	19.8630	1.8630	0.0625	-1.9265	75.9349
H_3	45.2223	19.5867	2.3088	0.0210	6.8331	83.6116
H_4	50.8637	19.6731	2.5854	0.0097	12.3052	89.4222
H_5	34.6025	19.7110	1.7555	0.0792	-4.0304	73.2354
H_6	26.8484	21.5464	1.2461	0.2127	-15.3817	69.0786
H_7	44.5183	24.0702	1.8495	0.0644	-2.6585	91.6950
H_8	43.6461	31.2883	1.3950	0.1630	-17.6779	104.9701
H_9	67.6933	41.2118	1.6426	0.1005	-13.0803	148.4669
H_10	6.8559	37.8498	0.1811	0.8563	-67.3282	81.0401
H_11	-39.7431	32.3525	-1.2284	0.2193	-103.1529	23.6666
H_12	-44.1712	34.3683	-1.2852	0.1987	-111.5317	23.1894
H_13	-21.9236	38.9662	-0.5626	0.5737	-98.2961	54.4488
H_14	-0.7227	49.6884	-0.0145	0.9884	-98.1101	96.6647
H_15	-20.7471	43.7202	-0.4745	0.6351	-106.4371	64.9429
H_16	-10.9402	51.2922	-0.2133	0.8311	-111.4711	89.5907

H_17	-51.6466	32.6981	-1.5795	0.1142	-115.7337	12.4405
H_18	-86.4031	26.0255	-3.3199	0.0009	-137.4121	-35.3941
H_19	-77.6422	29.0815	-2.6698	0.0076	-134.6409	-20.6435
H_20	-31.0055	37.8336	-0.8195	0.4125	-105.1579	43.1470
H_21	-24.7757	27.7453	-0.8930	0.3719	-79.1554	29.6040
H_22	-42.2078	28.2470	-1.4942	0.1351	-97.5708	13.1552
H_23	-22.8889	26.9547	-0.8492	0.3958	-75.7190	29.9413
H_1*Temp	-0.3904	0.3183	-1.2265	0.2200	-1.0142	0.2334
H_2*Temp	-0.6844	0.2829	-2.4195	0.0155	-1.2389	-0.1300
H_3*Temp	-0.8298	0.2782	-2.9830	0.0029	-1.3751	-0.2846
H_4*Temp	-0.9133	0.2792	-3.2704	0.0011	-1.4606	-0.3659
H_5*Temp	-0.6852	0.2782	-2.4628	0.0138	-1.2305	-0.1399
H_6*Temp	-0.6164	0.3004	-2.0519	0.0402	-1.2053	-0.0276
H_7*Temp	-0.8362	0.3284	-2.5461	0.0109	-1.4799	-0.1925
H_8*Temp	-0.7763	0.4056	-1.9137	0.0557	-1.5713	0.0187
H_9*Temp	-0.9863	0.5117	-1.9275	0.0539	-1.9892	0.0166
H_10*Temp	-0.1646	0.4657	-0.3535	0.7237	-1.0773	0.7481
H_11*Temp	0.4645	0.3980	1.1672	0.2431	-0.3155	1.2445
H_12*Temp	0.5926	0.4179	1.4182	0.1561	-0.2264	1.4116
H_13*Temp	0.4278	0.4667	0.9166	0.3593	-0.4870	1.3426
H_14*Temp	0.2502	0.5811	0.4306	0.6667	-0.8887	1.3892
H_15*Temp	0.5268	0.5195	1.0140	0.3106	-0.4914	1.5450
H_16*Temp	0.4389	0.6070	0.7230	0.4697	-0.7508	1.6286
H_17*Temp	0.9383	0.4064	2.3090	0.0209	0.1418	1.7348
H_18*Temp	1.4284	0.3397	4.2044	0.0000	0.7625	2.0942
H_19*Temp	1.3858	0.3848	3.6018	0.0003	0.6317	2.1400
H_20*Temp	0.7881	0.4998	1.5770	0.1148	-0.1914	1.7677
H_21*Temp	0.6723	0.3808	1.7654	0.0775	-0.0741	1.4186
H_22*Temp	0.7948	0.3895	2.0406	0.0413	0.0314	1.5583
H_23*Temp	0.4114	0.3736	1.1013	0.2708	-0.3208	1.1436
temp	3.0107	0.2378	12.6624	0.0000	2.5447	3.4767
const	-102.3596	16.8909	-6.0600	0.0000	-135.4652	-69.2539

Omnibus:	174.044	Durbin-Watson:	0.420
Prob(Omnibus):	0.000	Jarque-Bera (JB):	748.492

Skew:	0.606	Prob(JB):	0.000
Kurtosis:	6.634	Condition No.:	30560

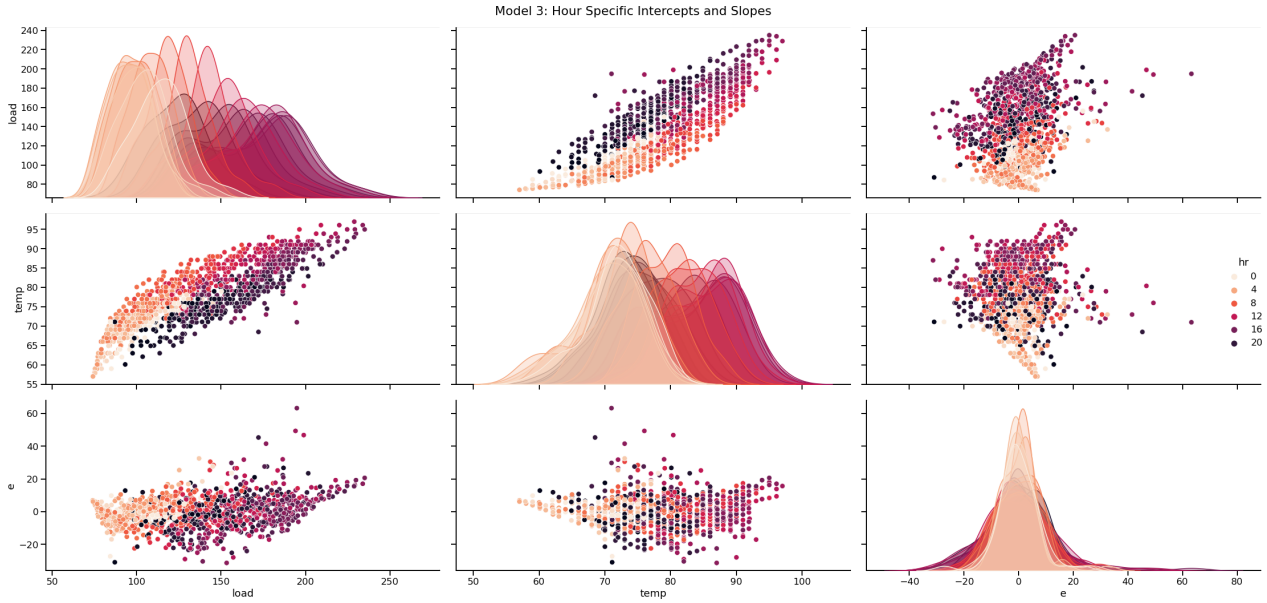
- Given the significance of the slope terms is mixed, I conduct a LM test of overall model significance vs. Model 2; the results below support Model 3

In [24]:
```
m3.model.compare_lm_test(m2.model)
```

```
C:\Users\Garrett\anaconda3\envs\tenaska_env\lib\site-packages\statsmodels\regression\linear_model.py:2086: FutureWarning: Support for multi-dimensional indexing (e.g. `obj[:, None]`) is deprecated and will be removed in a future version.  Convert to a numpy array before indexing instead.
  scores = wexog * wresid[:, None]
```

Out[24]:
```
(142.29949120575606, 3.4713934404480327e-19, 23.0)
```

In [25]:
```
m3.plot_modelfit('Model 3: Hour Specific Intercepts and Slopes')
```



Model 3A: $L = \alpha +\sum_{i = 0}^{23} {\lambda_iH_i}+\beta_1F +\sum_{i = 0}^{23} {\phi_iH_iF_t}$ Using Robust Regression

- Given the presence of outliers, especially on our prediction day of Thursday, I chose to implement a robust regression version of Model 3 for the final prediction
  - This minimizes the mean absolute deviation instead of squared deviations
  - MAD is more robust in the presence of fat tails
- I arrived at the predictions by averaging model fitted values across all Thursdays in June

In [26]:
```
robust_m3 = LoadModel(data_df = reg3_df, y_col = 'load', x_cols = list(reg3_df.columns[5:])+['temp'])
robust_m3_results = robust_m3.fit_model(model_type='Robust')
robust_m3_results
```

```
C:\Users\Garrett\anaconda3\envs\tenaska_env\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWar
```

ning: In a future version of pandas all arguments of concat except for the argument 'objs' will be keyword-only
x = pd.concat(x[::order], 1)

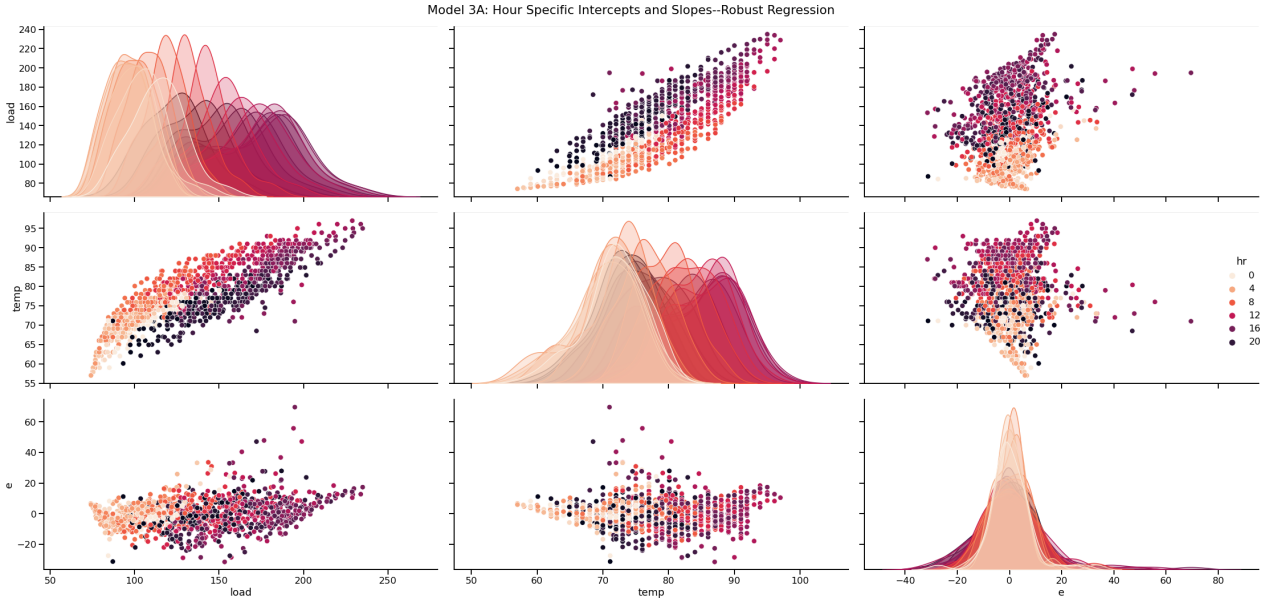
Out[26]:

Model:	RLM	Df Residuals:	1176
Dependent Variable:	load	Norm:	HuberT
Date:	2021-07-19 03:20	Scale Est.:	mad
No. Observations:	1224	Cov. Type:	H1
Df Model:	47	Scale:	7.9209

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
H_1	20.8267	25.6809	0.8110	0.4174	-29.5070	71.1604
H_2	33.2344	25.2795	1.3147	0.1886	-16.3124	82.7813
H_3	39.6250	25.0577	1.5813	0.1138	-9.4873	88.7373
H_4	44.3473	24.9521	1.7773	0.0755	-4.5580	93.2526
H_5	26.5021	26.4874	1.0006	0.3170	-25.4122	78.4164
H_6	18.3708	28.6680	0.6408	0.5216	-37.8174	74.5591
H_7	31.1214	29.0137	1.0726	0.2834	-25.7444	87.9871
H_8	24.3217	30.4256	0.7994	0.4241	-35.3115	83.9548
H_9	36.0314	28.8069	1.2508	0.2110	-20.4290	92.4918
H_10	-7.7902	29.4797	-0.2643	0.7916	-65.5693	49.9889
H_11	-47.6050	29.4130	-1.6185	0.1056	-105.2534	10.0434
H_12	-69.4663	28.1258	-2.4698	0.0135	-124.5918	-14.3408
H_13	-38.3628	27.7047	-1.3847	0.1661	-92.6631	15.9374
H_14	-57.7402	26.3350	-2.1925	0.0283	-109.3559	-6.1246
H_15	-69.2570	25.8005	-2.6843	0.0073	-119.8250	-18.6891
H_16	-54.3457	25.2407	-2.1531	0.0313	-103.8166	-4.8747
H_17	-77.3517	25.5575	-3.0266	0.0025	-127.4434	-27.2599
H_18	-97.5068	26.2954	-3.7081	0.0002	-149.0448	-45.9687
H_19	-89.4892	26.8753	-3.3298	0.0009	-142.1638	-36.8147
H_20	-60.6430	26.8429	-2.2592	0.0239	-113.2540	-8.0319
H_21	-25.2572	26.0641	-0.9690	0.3325	-76.3418	25.8274
H_22	-37.9109	26.7977	-1.4147	0.1572	-90.4335	14.6117
H_23	-22.3002	26.5371	-0.8403	0.4007	-74.3119	29.7116
H_1*Temp	-0.3945	0.3585	-1.1002	0.2712	-1.0972	0.3083
H_2*Temp	-0.6401	0.3533	-1.8121	0.0700	-1.3325	0.0522
H_3*Temp	-0.7588	0.3515	-2.1590	0.0309	-1.4477	-0.0699
H_4*Temp	-0.8267	0.3507	-2.3572	0.0184	-1.5142	-0.1393

H_5*Temp	-0.5742	0.3701	-1.5513	0.1208	-1.2997	0.1513
H_6*Temp	-0.4975	0.3937	-1.2637	0.2064	-1.2691	0.2741
H_7*Temp	-0.6503	0.3892	-1.6709	0.0947	-1.4132	0.1125
H_8*Temp	-0.5229	0.3976	-1.3152	0.1884	-1.3022	0.2564
H_9*Temp	-0.5923	0.3731	-1.5874	0.1124	-1.3235	0.1390
H_10*Temp	0.0234	0.3763	0.0621	0.9505	-0.7142	0.7610
H_11*Temp	0.5746	0.3723	1.5434	0.1227	-0.1551	1.3042
H_12*Temp	0.8981	0.3566	2.5189	0.0118	0.1993	1.5970
H_13*Temp	0.6406	0.3515	1.8226	0.0684	-0.0483	1.3296
H_14*Temp	0.9228	0.3370	2.7380	0.0062	0.2622	1.5834
H_15*Temp	1.1069	0.3320	3.3336	0.0009	0.4561	1.7577
H_16*Temp	0.9599	0.3272	2.9339	0.0033	0.3187	1.6012
H_17*Temp	1.2547	0.3330	3.7682	0.0002	0.6021	1.9073
H_18*Temp	1.5797	0.3458	4.5682	0.0000	0.9019	2.2575
H_19*Temp	1.5561	0.3587	4.3385	0.0000	0.8531	2.2590
H_20*Temp	1.1891	0.3626	3.2794	0.0010	0.4785	1.8998
H_21*Temp	0.6908	0.3550	1.9460	0.0517	-0.0050	1.3867
H_22*Temp	0.7386	0.3668	2.0134	0.0441	0.0196	1.4575
H_23*Temp	0.4059	0.3659	1.1092	0.2673	-0.3113	1.1231
temp	2.8892	0.2580	11.2005	0.0000	2.3836	3.3948
const	-93.6152	18.5900	-5.0358	0.0000	-130.0509	-57.1795

In [27]:
```
robust_m3.plot_modelfit('Model 3A: Hour Specific Intercepts and Slopes--Robust Regression')
```



In [28]:
```
robust_m3_results
```

Out[28]:

Model:	RLM	Df Residuals:	1176
Dependent Variable:	load	Norm:	HuberT
Date:	2021-07-19 03:20	Scale Est.:	mad
No. Observations:	1224	Cov. Type:	H1
Df Model:	47	Scale:	7.9209

	Coef.	Std.Err.	z	P>|z|	[0.025	0.975]
H_1	20.8267	25.6809	0.8110	0.4174	-29.5070	71.1604
H_2	33.2344	25.2795	1.3147	0.1886	-16.3124	82.7813
H_3	39.6250	25.0577	1.5813	0.1138	-9.4873	88.7373
H_4	44.3473	24.9521	1.7773	0.0755	-4.5580	93.2526
H_5	26.5021	26.4874	1.0006	0.3170	-25.4122	78.4164
H_6	18.3708	28.6680	0.6408	0.5216	-37.8174	74.5591
H_7	31.1214	29.0137	1.0726	0.2834	-25.7444	87.9871
H_8	24.3217	30.4256	0.7994	0.4241	-35.3115	83.9548
H_9	36.0314	28.8069	1.2508	0.2110	-20.4290	92.4918
H_10	-7.7902	29.4797	-0.2643	0.7916	-65.5693	49.9889
H_11	-47.6050	29.4130	-1.6185	0.1056	-105.2534	10.0434
H_12	-69.4663	28.1258	-2.4698	0.0135	-124.5918	-14.3408
H_13	-38.3628	27.7047	-1.3847	0.1661	-92.6631	15.9374
H_14	-57.7402	26.3350	-2.1925	0.0283	-109.3559	-6.1246
H_15	-69.2570	25.8005	-2.6843	0.0073	-119.8250	-18.6891
H_16	-54.3457	25.2407	-2.1531	0.0313	-103.8166	-4.8747
H_17	-77.3517	25.5575	-3.0266	0.0025	-127.4434	-27.2599
H_18	-97.5068	26.2954	-3.7081	0.0002	-149.0448	-45.9687
H_19	-89.4892	26.8753	-3.3298	0.0009	-142.1638	-36.8147
H_20	-60.6430	26.8429	-2.2592	0.0239	-113.2540	-8.0319
H_21	-25.2572	26.0641	-0.9690	0.3325	-76.3418	25.8274
H_22	-37.9109	26.7977	-1.4147	0.1572	-90.4335	14.6117
H_23	-22.3002	26.5371	-0.8403	0.4007	-74.3119	29.7116
H_1*Temp	-0.3945	0.3585	-1.1002	0.2712	-1.0972	0.3083
H_2*Temp	-0.6401	0.3533	-1.8121	0.0700	-1.3325	0.0522
H_3*Temp	-0.7588	0.3515	-2.1590	0.0309	-1.4477	-0.0699
H_4*Temp	-0.8267	0.3507	-2.3572	0.0184	-1.5142	-0.1393

H_5*Temp	-0.5742	0.3701	-1.5513	0.1208	-1.2997	0.1513
H_6*Temp	-0.4975	0.3937	-1.2637	0.2064	-1.2691	0.2741
H_7*Temp	-0.6503	0.3892	-1.6709	0.0947	-1.4132	0.1125
H_8*Temp	-0.5229	0.3976	-1.3152	0.1884	-1.3022	0.2564
H_9*Temp	-0.5923	0.3731	-1.5874	0.1124	-1.3235	0.1390
H_10*Temp	0.0234	0.3763	0.0621	0.9505	-0.7142	0.7610
H_11*Temp	0.5746	0.3723	1.5434	0.1227	-0.1551	1.3042
H_12*Temp	0.8981	0.3566	2.5189	0.0118	0.1993	1.5970
H_13*Temp	0.6406	0.3515	1.8226	0.0684	-0.0483	1.3296
H_14*Temp	0.9228	0.3370	2.7380	0.0062	0.2622	1.5834
H_15*Temp	1.1069	0.3320	3.3336	0.0009	0.4561	1.7577
H_16*Temp	0.9599	0.3272	2.9339	0.0033	0.3187	1.6012
H_17*Temp	1.2547	0.3330	3.7682	0.0002	0.6021	1.9073
H_18*Temp	1.5797	0.3458	4.5682	0.0000	0.9019	2.2575
H_19*Temp	1.5561	0.3587	4.3385	0.0000	0.8531	2.2590
H_20*Temp	1.1891	0.3626	3.2794	0.0010	0.4785	1.8998
H_21*Temp	0.6908	0.3550	1.9460	0.0517	-0.0050	1.3867
H_22*Temp	0.7386	0.3668	2.0134	0.0441	0.0196	1.4575
H_23*Temp	0.4059	0.3659	1.1092	0.2673	-0.3113	1.1231
temp	2.8892	0.2580	11.2005	0.0000	2.3836	3.3948
const	-93.6152	18.5900	-5.0358	0.0000	-130.0509	-57.1795

Prediction

In [29]:
```
pred_df = robust_m3.output_df.merge(df_j[['ts','dow']], left_on='ts', right_on = 'ts')
pred_df = pred_df[['dow', 'hr', 'yhat']].groupby(['dow', 'hr']).mean().reset_index()
pred_df.loc[pred_df['dow'] == 3].max()
```

Out[29]:
```
dow      3.000000
hr      23.000000
yhat   175.067255
dtype: float64
```

Prediction: Peak load—175MW at Hour 16

- This is consistent with the density bivariate plot of max load and max load hour
- I think this is the optimal prediction given the data, but we should be relatively more skeptical forecasting a Thursday given it shows the fattest tails of the weekdays
- Similar logic holds for Hour 16 load

Discussion:

- Robust regression will help mitigate the impact but will not cure it. However, the residuals look relatively Gaussian.
- Given weather is typically modeled as a normal distribution, I would seek to improve the model by looking for alternative drivers behind the fat tails.
- The forecast would also improve as the forecast horizon approaches.

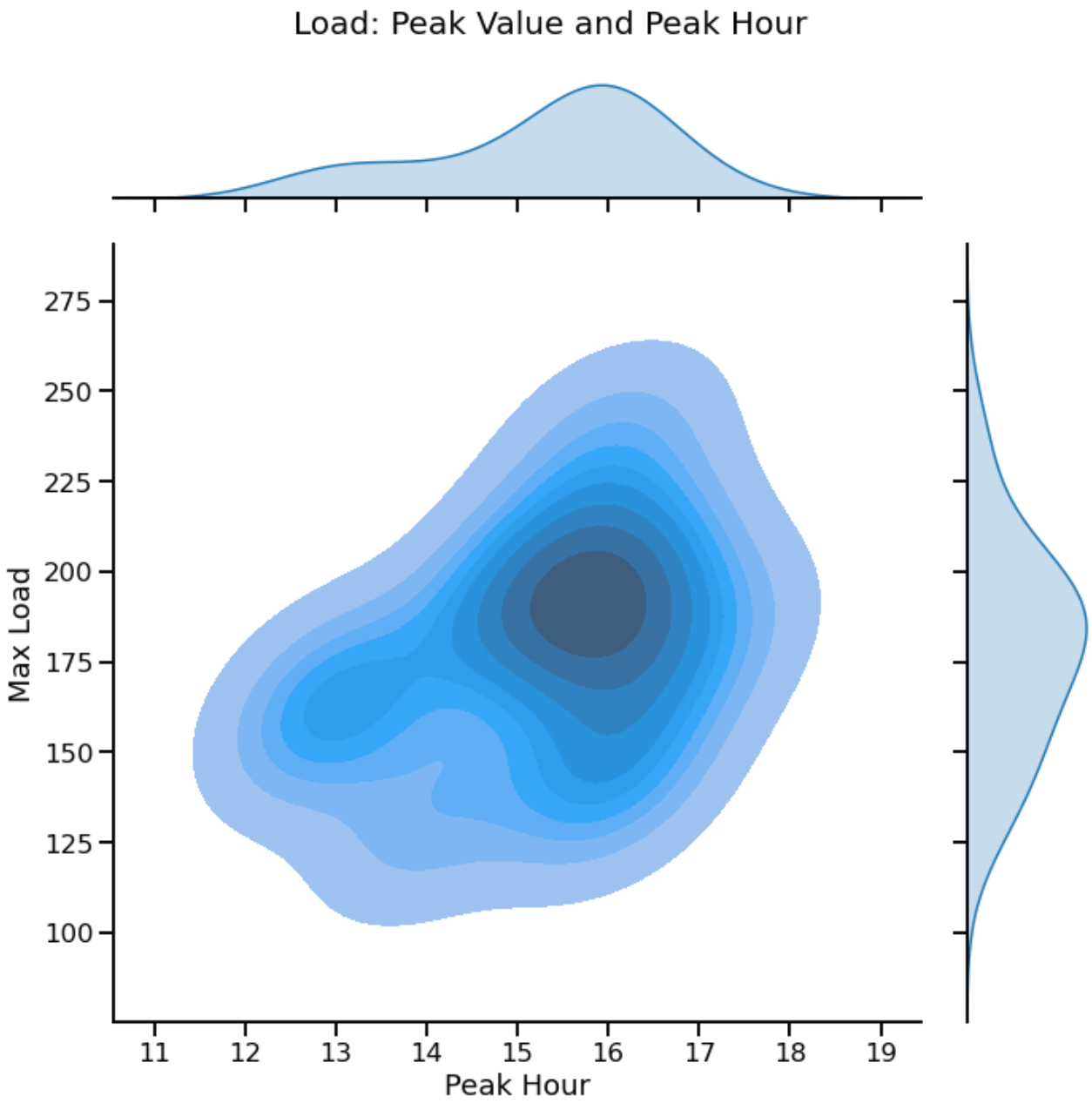
In [30]:

```
df_jth = df_j.loc[df_j['dow'] == 3]
df_jth['ts'] = pd.DatetimeIndex(df_jth['ts'], normalize=True)
df_jth.set_index('ts', inplace = True)

june_thurs = ['2017-06-01', '2017-06-08', '2017-06-15', '2017-06-22','2017-06-29',
              '2018-06-07', '2018-06-14', '2018-06-21','2018-06-28',
              '2019-06-06', '2019-06-13', '2019-06-20','2019-06-27']

df_jth_maxdf = pd.DataFrame([[df_jth.loc[dt, 'load'].idxmax().hour, df_jth.loc[dt, 'load'].max(),
                              df_jth.loc[dt, 'temp'].idxmax().hour, df_jth.loc[dt, 'temp'].max()] for dt in june_thurs],
                            columns=['maxlload_hr', 'maxload', 'maxtemp_hr', 'maxtemp'], index = june_thurs)

sns.set_context('talk')
g2 = sns.jointplot(x = 'maxlload_hr', y = 'maxload', data = df_jth_maxdf,
                   kind = 'kde', fill = True, height = 10,  palette = 'rocket_r')
g2.fig.suptitle('Load: Peak Value and Peak Hour', fontsize = 20)
g2.set_axis_labels('Peak Hour', 'Max Load')
g2.fig.tight_layout()
```


Load: Peak Value and Peak Hour
Max Load
275
250
225
200
175
150
125
100
11
12
13
14
15
16
17
18
19
Peak Hour