Kuis 12 Interpolasi

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Gambarkan fungsi berikut interval $[-1, 1]$:

$$f(x) = \frac{1}{1+10x^2}$$

Siapkan $n = 3, 5, 9, 15$ titik t_i yang jaraknya sama pada interval $[-1, 1]$. Untuk masing-masing n, buat polynomial berderajat $n - 1$ yang melaluinya menggunakan metode interpolasi yang terbaik menurut Anda dan gambarkan grafiknya.

Kode newton untuk mencari nilai koefisien $a_0, \ldots, a_{n-1}$:

```
function [a] = newton(t, y)
  n = length(t);
  A = zeros(n);
  A(:,1)=ones(n,1);

  for j=2:n
    for i=j:n
      temp = t(i)-t(1);
      for k=2:j-1
        temp = temp * (t(i) - t(k));
      endfor
      A(i, j) = temp;
    endfor
  endfor
  a = segitigaBawah(A,y);
```

Disini kita membuat matriks A sebesar nxn. Nilai kolom pertama dari matriks tersebut adalah 1 semua, karena koefisien a_0 pasti 1. Matriks A adalah matriks segitiga bawah. Setelah itu, loop di atas digunakan untuk mengisi matriks segitiga bawah. Setelah matriks segitiga bawah A ditemukan, maka kita cari solusinya menggunakan metode segitigaBawah.

- $n = 3$, berarti $t_0 = -1,\ t_1 = 0,\ t_2 = 1$

  $p(t) = a_0 \cdot 1 + a_1 t + a_2 t^2$

  $p(t) = a_0 \cdot 1 + a_1(t+1) + a_2(t+1)(t-0)$

  $f(-1) = \frac{1}{1+10(-1)^2} = \frac{1}{11}$

  $f(0) = \frac{1}{1+10(0)^2} = 1$

  $f(1) = \frac{1}{1+10(1)^2} = \frac{1}{11}$

  didapat 3 titik (t, y)

  $(-1, \frac{1}{11}),\ (0, 1),\ (1, \frac{1}{11})$

  $p(-1) = a_0 = \frac{1}{11}$

  $p(0) = a_0 + a_1 = 1$

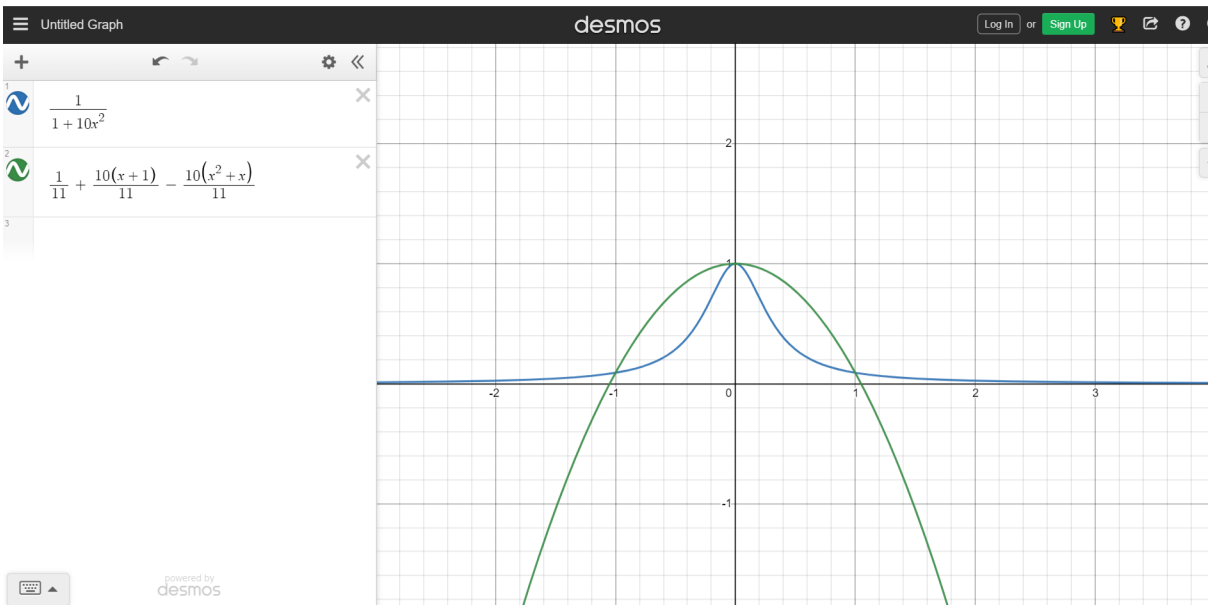
  $p(1) = a_0 + 2a_1 + 2a_2 = \frac{1}{11}$

  $a_0 = \frac{1}{11},\ a_1 = \frac{10}{11},\ a_2 = \frac{-10}{11}$

  didapat persamaan

  $p(t) = \frac{1}{11} + \frac{10}{11}(t+1) - \frac{10}{11}(t^2 + t)$

  Screenshot grafik:



Penjelasan:

```
>> t =[-1 0 1]
t =

  -1    0    1

>>  y = f(t)
y =

   0.090909   1.000000   0.090909

>> newton(t,y)
ans =

   0.090909
   0.909091
  -0.909091
```

Grafik yang melewati titik $(-1, \frac{1}{11}),\ (0, 1),\ (1, \frac{1}{11})$ selain $f(x)$ adalah $p(t)$. Disini kita gunakan metode newton untuk mencari fungsi polynomial, khususnya koefisien-koefisien dari fungsi polynomial tersebut.

- $n = 5$, berarti $t_0 = -1,\ t_1 = \frac{-1}{2},\ t_2 = 0,\ t_3 = \frac{1}{2},\ t_4 = 1$

$$p(t) = a_0 \cdot 1 + a_1(t - t_0) + a_2(t - t_0)(t - t_1) + a_3(t - t_0)(t - t_1)(t - t_2) + a_4(t - t_0)($$

$$p(t) = a_0 \cdot 1 + a_1(t + 1) + a_2(t + 1)(t + \tfrac{1}{2}) + a_3(t + 1)(t + \tfrac{1}{2})(t - 0) + a_4(t + 1)(t$$

$$f(-1) = \frac{1}{1+10(-1)^2} = \frac{1}{11}$$

$$f(\frac{-1}{2}) = \frac{1}{1+10(\frac{-1}{2})^2} = \frac{2}{7}$$

$$f(0) = \frac{1}{1+10(0)^2} = 1$$

$$f(\frac{1}{2}) = \frac{1}{1+10(\frac{1}{2})^2} = \frac{2}{7}$$

$$f(1) = \frac{1}{1+10(1)^2} = \frac{1}{11}$$

Didapat :

$(-1, \frac{1}{11}), (-\frac{1}{2}, \frac{2}{7}), (0, 1), (\frac{1}{2}, \frac{2}{7}), (1, \frac{1}{11})$^

$$p(-1) = a_0 = \frac{1}{11}$$

$$p(-\tfrac{1}{2}) = a_0 + \tfrac{1}{2}a_1 = \tfrac{2}{7}$$

$$p(0) = a_0 + a_1 + \tfrac{1}{2}a_2 = 1$$

$$p(\tfrac{1}{2}) = a_0 + \tfrac{3}{2}a_1 + \tfrac{3}{2}a_2 + \tfrac{3}{4}a_3 = \tfrac{2}{7}$$

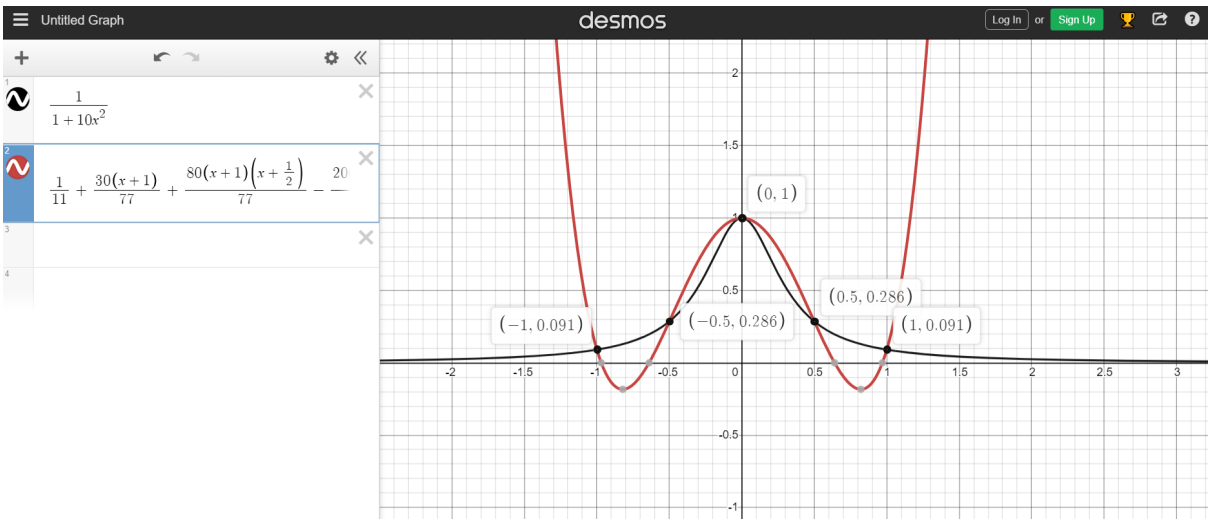
$$p(1) = a_0 + 2a_1 + 3a_2 + 3a_3 + \tfrac{3}{2}a_4 = \tfrac{1}{11}$$

$$a_0 = \tfrac{1}{11}\ a_1 = \tfrac{30}{77}\ a_2 = \tfrac{80}{77}\ a_3 = \tfrac{-200}{77}\ a_4 = \tfrac{200}{77}$$

didapat fungsi polinomial:

$$p(t) = \tfrac{1}{11} \cdot 1 + \tfrac{30}{77}(t + 1) + \tfrac{80}{77}(t + 1)(t + \tfrac{1}{2}) + \tfrac{-200}{77}(t + 1)(t + \tfrac{1}{2})(t - 0) + \tfrac{200}{77}($$

Screenshot grafik:



Penjelasan:

```
>> newton(t,y)
ans =

    0.090909
    0.389610
    1.038961
   -2.597403
    2.597403
```

Grafik yang melewati titik $(-1, \frac{1}{11}), (-\frac{1}{2}, \frac{2}{7}), (0, 1), (\frac{1}{2}, \frac{2}{7}), (1, \frac{1}{11})$ selain $f(x)$ adalah $p(t)$. Disini kita gunakan metode newton untuk mencari fungsi polinomial, khususnya koefisien-koefisien dari fungsi polinomial tersebut.

- $n = 9$, berarti

$$t_0 = -1,\ t_1 = \frac{-3}{4},\ t_2 = \frac{-1}{2},\ t_3 = \frac{-1}{4},\ t_4 = 0,\ t_5 = \frac{1}{4},\ t_6 = \frac{1}{2},\ t_7 = \frac{3}{4},\ t_8 = 1$$

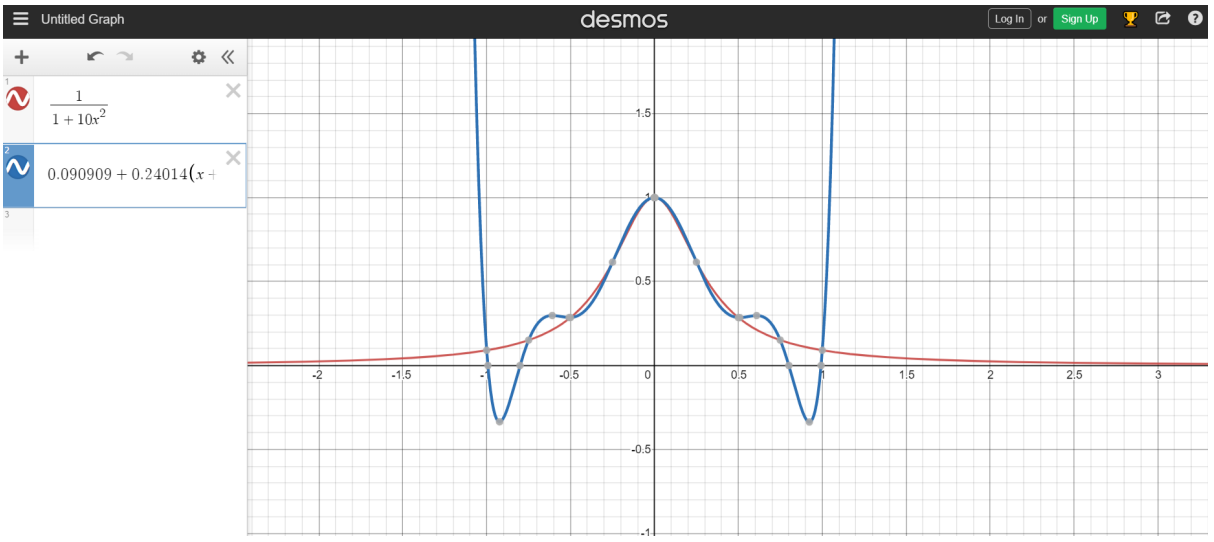
$$p(t) = a_0 \cdot 1 + a_1(t+1) + a_2(t+1)(t+\tfrac{3}{4}) + a_3(t+1)(t+\tfrac{3}{4})(t+\tfrac{1}{2}) + a_4(t+1)$$

$$+ a_5(t+1)(t+\tfrac{3}{4})(t+\tfrac{1}{2})(t+\tfrac{1}{4})(t-0) + a_6(t+1)(t+\tfrac{3}{4})(t+\tfrac{1}{2})(t+\tfrac{1}{4})(t-0)(t$$

$$+ a_7(t+1)(t+\tfrac{3}{4})(t+\tfrac{1}{2})(t+\tfrac{1}{4})(t-0)(t-\tfrac{1}{4})(t-\tfrac{1}{2})$$

$$+ a_8(t+1)(t+\tfrac{3}{4})(t+\tfrac{1}{2})(t+\tfrac{1}{4})(t-0)(t-\tfrac{1}{4})(t-\tfrac{1}{2})(t-\tfrac{3}{4})$$

Screenshot grafik:



Penjelasan:

```
>> t = [-1:(2/(9-1)):1]
t =

  -1.0000  -0.7500  -0.5000  -0.2500        0   0.2500   0.5000   0.7500   1.0000

>> y = f(t)
y =

   0.090909   0.150943   0.285714   0.615385   1.000000   0.615385   0.285714   0.150943   0.090909

>> newton(t,y)
ans =

   9.0909e-02
   2.4014e-01
   5.9789e-01
   1.2817e+00
  -2.7746e+00
  -3.6190e+00
   1.5682e+01
  -2.4127e+01
   2.4127e+01
```

Grafik yang melewati titik tersebut selain $f(x)$ adalah $p(t)$. Disini kita gunakan metode newton untuk mencari fungsi polinomial, khususnya koefisien-koefisien dari fungsi polinomial tersebut.

- $n = 15$, berarti

  terdapat 15 titik t :

  $$t_0 = -1,\ t_1 = \frac{-6}{7},\ t_2 = \frac{-5}{7},\ t_3 = \frac{-4}{7},\ t_4 = \frac{-3}{7},\ t_5 = \frac{-2}{7},\ t_6 = \frac{-1}{7},$$

  $$t_7 = 0,\ t_8 = \frac{1}{7},\ t_9 = \frac{2}{7},\ t_{10} = \frac{3}{7},\ t_{11} = \frac{4}{7},\ t_{12} = \frac{5}{7},\ t_{13} = \frac{6}{7},\ t_{14} = 1$$

  Bentuk polynom jika $n = 15$:

  $$p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots + a_{14} t^{14}$$

  $$p(t) = a_0 \cdot 1 + a_1(t+1) + a_2(t+1)(t+\tfrac{6}{7}) + a_3(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7}) + a_4(t+1)$$

  $$+ a_5(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7}) + a_6(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7}$$

  $$+ a_7(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})$$

  $$+ a_8(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)$$

  $$+ a_9(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})$$

  $$+ a_{10}(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})(t-\tfrac{2}{7})$$

  $$+ a_{11}(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})(t-\tfrac{2}{7})(t-\tfrac{3}{7}$$

  $$+ a_{12}(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})(t-\tfrac{2}{7})(t-\tfrac{3}{7}$$

  $$+ a_{13}(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})(t-\tfrac{2}{7})(t-\tfrac{3}{7}$$

  $$+ a_{14}(t+1)(t+\tfrac{6}{7})(t+\tfrac{5}{7})(t+\tfrac{4}{7})\ (t+\tfrac{3}{7})(t+\tfrac{2}{7})(t+\tfrac{1}{7})(t-0)(t-\tfrac{1}{7})(t-\tfrac{2}{7})(t-\tfrac{3}{7}$$

  cari titik y:

  $$f(-1) = \frac{1}{11}$$

$f(\frac{-6}{7}) = \frac{49}{409}$

$f(\frac{-5}{7}) = \frac{49}{299}$

$f(\frac{-4}{7}) = \frac{49}{209}$

$f(\frac{-3}{7}) = \frac{49}{139}$

$f(\frac{-2}{7}) = \frac{49}{89}$

$f(\frac{-1}{7}) = \frac{49}{59}$

$f(0) = \frac{1}{1+10(0)^2} = 1$

$f(\frac{1}{7}) = \frac{49}{59}$

$f(\frac{2}{7}) = \frac{49}{89}$

$f(\frac{3}{7}) = \frac{49}{139}$

$f(\frac{4}{7}) = \frac{49}{209}$

$f(\frac{5}{7}) = \frac{49}{299}$

$f(\frac{6}{7}) = \frac{49}{409}$

$f(1) = \frac{1}{11}$

Jadi didapatkan titik (t, y):

$(-1, \frac{1}{11}), (\frac{-6}{7}, \frac{49}{409}), (\frac{-5}{7}, \frac{49}{299}), (\frac{-4}{7}, \frac{49}{209}), (\frac{-3}{7}, \frac{49}{139}), (\frac{-2}{7}, \frac{49}{89}), (\frac{-1}{7}, \frac{49}{59}), (0, 1), (\frac{1}{7}, \frac{49}{59}), (-$

$(\frac{4}{7}, \frac{49}{209}), (\frac{5}{7}, \frac{49}{299}), (\frac{6}{7}, \frac{49}{409}), (1, \frac{1}{11})$

cari nilai koefisien a dari polinomial $p(t)$

$$p(-1) = a_0 = \frac{1}{11}$$

$$p(\frac{-6}{7}) = a_0 + \frac{1}{7}a_1 = \frac{49}{409}$$

$$p(\frac{-5}{7}) = a_0 + \frac{2}{7}a_1 + \frac{2}{49}a_2 = \frac{49}{299}$$

$$p(\frac{-4}{7}) = a_0 + \frac{3}{7}a_1 + \frac{6}{49}a_2 + \frac{6}{343}a_3 = \frac{49}{209}$$

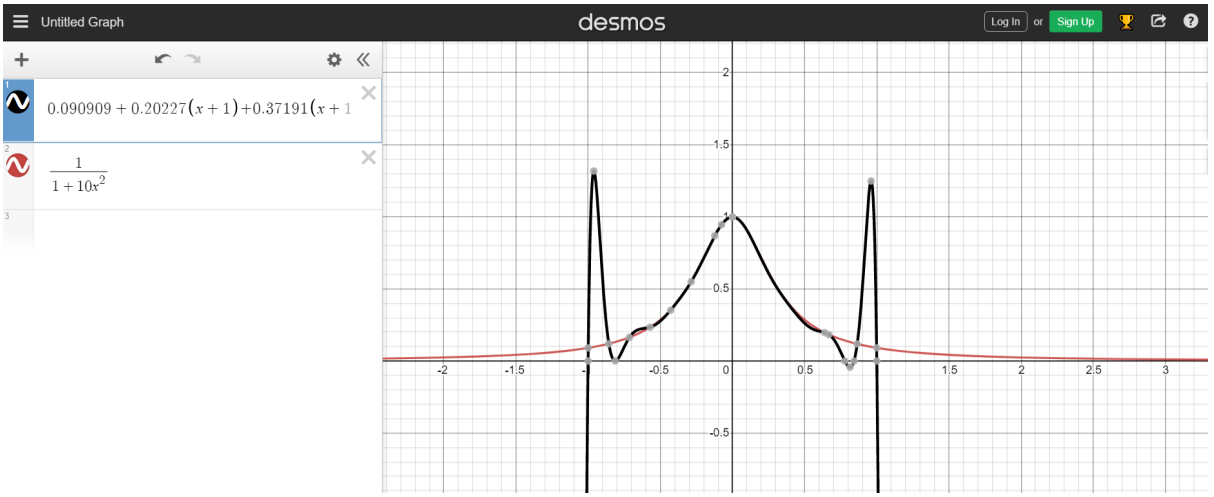
$$p(\frac{-3}{7}) = a_0 + \frac{4}{7}a_1 + \frac{12}{49}a_2 + \frac{24}{343}a_3 + \frac{24}{2401}a_4 = \frac{49}{139}$$

$$p(\frac{-2}{7}) = a_0 + \frac{5}{7}a_1 + \frac{20}{49}a_2 + \frac{60}{343}a_3 + \frac{120}{2401}a_4 + \frac{120}{16807}a_5 = \frac{49}{89}$$

$$p(\frac{-1}{7}) = a_0 + \frac{6}{7}a_1 + \frac{30}{49}a_2 + \frac{120}{343}a_3 + \frac{360}{2401}a_4 + \frac{720}{16807}a_5 + \frac{720}{117649}a_6 = \frac{49}{59}$$

....

Grafik:



Penjelasan:

setelah menggunakan metode newton (kode ada di halaman paling awal)

didapatkan koefisien sebagai berikut.

```
>> newton(t,y)
ans =

   9.0909e-02
   2.0227e-01
   3.7191e-01
   6.4685e-01
   9.6921e-01
   2.5020e-01
  -7.1588e+00
  -1.2729e+01
   7.4556e+01
  -1.0584e+02
   3.6773e+00
   2.3952e+02
  -5.1070e+02
   6.7451e+02
  -6.7451e+02
```

Kemudian didapat fungsi polinomial $p(t)$yang melewati titik

$(-1, \frac{1}{11}), (\frac{-6}{7}, \frac{49}{409}), (\frac{-5}{7}, \frac{49}{299}), (\frac{-4}{7}, \frac{49}{209}), (\frac{-3}{7}, \frac{49}{139}), (\frac{-2}{7}, \frac{49}{89}), (\frac{-1}{7}, \frac{49}{59}), (0, 1), (\frac{1}{7}, \frac{49}{59}), (\frac{4}{7}, \frac{49}{209}), (\frac{5}{7}, \frac{49}{299}), (\frac{6}{7}, \frac{49}{409}), (1, \frac{1}{11})$.

Kesimpulan: Kami disini menggunakan metode interpolasi newton untuk mencari polinomial berderajat n-1, karena metode newton menghasilkan matriks koefisien berbentuk segitiga bawah yang bisa di solve dalam O(n^2). Sehingga biaya dan proses komputasinya lebih murah dan efisien dari metode lain.