

2. EXPONENTIAL: LOG-LINEAR

$$\log(y) = \beta_0 + \beta_1 x + u$$

WHY IS THIS EXPONENTIAL?

RECALL THE EXPONENTIAL GROWTH/DECAY FORMULA:

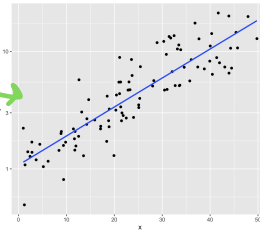
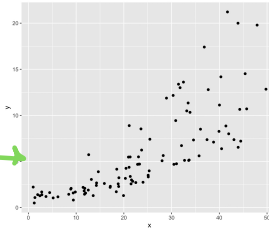
$$y = \underset{\substack{\uparrow \\ \text{INITIAL} \\ \text{AMOUNT}}}{a} (1 + \underset{\substack{\uparrow \\ \text{GROWTH} \\ \text{RATE}}}{r})^{\underset{\substack{\leftarrow \text{\# TIME STEPS} \\ \text{ELAPSED}}}{t}}$$

TAKE LOGS:

$$\log(y) = \underbrace{\log(a)}_{\beta_0} + \underbrace{t}_{x} \underbrace{\log(1+r)}_{\beta_1}$$

$$\log(y) = \beta_0 + \beta_1 x$$

```
data_exponential %>%  
  ggplot(aes(x = x, y = y)) +  
  geom_point()  
  
data_exponential %>%  
  ggplot(aes(x = x, y = y)) +  
  geom_point() +  
  geom_smooth(method = lm, se = F) +  
  scale_y_log10()  
  
data_exponential %>%  
  lm(log(y) ~ x, data = .) %>%  
  broom::tidy()
```



term	estimate	std.error	statistic	p.value
(Intercept)	0.0761	0.0855	0.891	3.75e-1
x	0.0568	0.00315	18.0	6.61e-33

$\hat{\beta}_0 = .0761$: WHEN $x = 0$,
 $\log(y)$ IS EXPECTED TO BE .0761

$$\hat{\beta}_1 = \log(1 + \hat{r}) = .0568$$

$$\exp(\log(1 + \hat{r})) = \exp(.0568)$$

$$1 + \hat{r} \approx 1.0568$$

$$\hat{r} = .0568$$
 GROWTH RATE

WHEN x INCREASES BY 1 UNIT,
 y IS EXPECTED TO INCREASE
BY 5.68%.