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# Fall 2021 Precalc Lesson 7.2

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VOCAB  
adjugate  
determinate



Do now... Get out your notebook/binder. Write down the date and goal.

Be sure to...

1. read the text to the right
2. Identify what doesn't make sense to you in a bulleted list.
3. Let's say you want to know **how many patients can be given each dosage** and use up all the medicine. Explain how you would solve this problem.

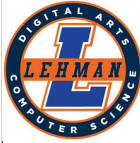
Dr. Galvez and Dr. Guillermo are testing a new experimental medicine (Precalodine) at Montefiore Hospital. The medicine is being given to a total of 100 patients. Patients take the medicine in a 1 liter solution, meaning that some percent of the solution is the medicine, and the rest is water. Patients are being given 2% or 4% solutions, and 2.5 liters of the medicine are on hand.

**class:** precalc **goal:** HDW use the adjugate and determinant to find the inverse of a  $2 \times 2$  matrix?

1. answers will vary, students might be unsure what is meant by a n% solution. explain on the board, with a drawing of a glass (some percent medicine rest water)
2. Answers will vary. the student could recognize that this can be transformed into the system of linear equations below:

$$\begin{aligned}x + y &= 100 \\ 0.02x + 0.04y &= 2.5\end{aligned}$$

where x represents the number of patients who take the 2% solution and y the rest. Then it can be solved in a variety of ways, including with matrix algebra.

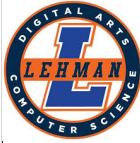


#### framing

- **what:** use the adjugate and determinant to find the inverse of a  $2 \times 2$  matrix
- **why:** Knowing the adjugate and determinant makes it really easy to find the inverse of a matrix. This makes it easy to solve systems of equations quickly, including in lots of real world applications.
- **where to:** Using adjugate and determinant to find the inverse of bigger square matrices.

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## Warm up: Stop 'n' Jot

Be sure to... Answer the question below in at least two complete sentences, in your notes. Be prepared to share out!

Why are the formulas below useful for solving systems of equations?

$$AX = B$$

$$X = A^{-1}B$$

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$AX = B$  is another way of writing a system of linear equations, where A is the coefficient matrix, B the solutions matrix and X the variables matrix. We typically want to figure out what variables satisfy all of the equations in a system, so our goal is to solve for X. The second formula tells us that if we can find the inverse of A we can multiply it by B to find X.

2 min. writing -> turn & talk -> share out.



Mini-lesson Be sure to copy the notes below in your notebook & ask questions!

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Last week we learned a formula for finding the inverse of a  $2 \times 2$  matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

### Vocabulary:

The determinant of a  $2 \times 2$  matrix A, written  $\det(A)$ :

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

The adjugate of a  $2 \times 2$  matrix A, written  $\text{adj}(A)$ :

$$\text{adj}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

We can now write:

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)}$$

**Practice problem:** for the matrix below (i) find the determinant and adjugate (ii) use the formula above to find the inverse. Be sure to... Show all work in your notebook!

$$X = \begin{bmatrix} 9 & 5 \\ 5 & 3 \end{bmatrix}$$

**class:** precalc **goal:** HDW use the adjugate and determinant to find the inverse of a  $2 \times 2$  matrix?

We'll learn more next week about how to find the determinant and adjugate for bigger matrices.

Why are the two Matrix inverse equations the same? because  $(1/x)*y = y/x$ .

Practice problem:

$$\det(X) = 9 \cdot 3 - 5 \cdot 5 = 27 - 25 = 2$$

$$\text{adj}(X) = \begin{bmatrix} 3 & -5 \\ -5 & 9 \end{bmatrix}$$

+HDW find the inverse?

$$X^{-1} = \text{adj}(X)/\det(X) = \begin{bmatrix} 3/2 & -5/2 \\ -5/2 & 9/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 & -2.5 \\ -2.5 & 4.5 \end{bmatrix}$$



Formula to find the inverse  
of matrix A:

$$A^{-1} = \frac{\text{adj}(A)}{\det(A)}$$

Independent work Be sure to:

1. Copy notes (if you came in late).
2. Do all work in your notebook.
3. Work silently for the first 4 minutes. After that you can check in with your neighbor.

The determinant of a  $2 \times 2$  matrix A, written

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

The adjugate of a  $2 \times 2$  matrix A, written  $\text{adj}(A)$ :

$$\text{adj}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

**class:** precalc **goal:** HDW use the adjugate and determinant to find the inverse of a  $2 \times 2$  matrix?

see answer key for solutions

pre-planned questions:

+How do you find the adjugate/determinant? use the formulas on the board!!!

+How can I use the adjugate/determinant to find the inverse?  $\text{adj}(A)/\det(A)$

+How do I convert to  $AX=B$ ? A is the coefficient matrix, X represents the variables and B the solutions

+Why can't I just use substitution? Because we're practicing using adjugates and determinates. It might seem silly here, but with bigger matrices it will be a LOT easier.

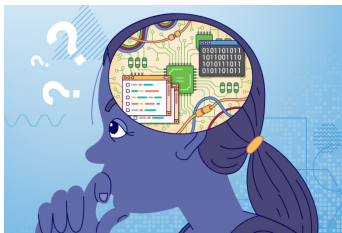


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## Reflection: Thinking about thinking

be sure to: Answer each question below with a complete sentence. Be prepared to share out!

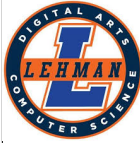
1. Why is it useful to be able to find the adjugate and determinant of a matrix?
2. What's one thing you'd like to understand better after today's lesson?



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last five minutes of class. share out.



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## wrapping up!

be sure to: read the directions below!



1. Make sure there isn't any litter near your workstation.
2. If you borrowed headphones, sign them back in.
3. **Make sure you are logged out of your computer!**
4. Remain in your seat until the bell rings.

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