Fall 2021 Precalc
Lesson 3.1

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Do Now

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Be sure to…Get out your notebook/binder. Read the paragraph below carefully, then answer the questions below. Show all work and check your results!

1. Jean is trying to use Gaussian elimination to remove x in row 2. Do you agree that he did it right? Explain why or why not.

$$\begin{matrix} x + y + z = 3 \\ x - 2y + 4z = 5 \\ 3y + 4z = 5 \end{matrix} \rightarrow \begin{matrix} x + y + z = 3 \\ y + 5z = 5 \\ 3y + 4z = 5 \end{matrix}$$

2. What's the pivot here? What will be the pivot when Jean removes 3y?

class: precalc **goal:** HDW use Gaussian elimination to solve nonsquare systems of equations?

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1. No, Jean should multiply the first row by -1 and add it to row 2:

$$\begin{matrix} -x - y - z = -3 \\ +.\ x - 2y. + 4z = 5 \\ \hline -3y + 3z = 2 \end{matrix}$$

2. The pivot is positive 1. In the next step the pivot will be postive 1 (from y).

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B24 rules

Welcome to our new room, B24! Please read the information below:

1. When you come in, please find a seat at a desk (if one's available) or one of the six closest desks to the screen. ***Do not sit in the back of the classroom***. We'll conduct the do now and mini lesson from here.
2. When I dismiss you for independent work, find a sit at one of the computer workstations.
3. ***No food or drink by the computers.***
4. At the end of the period, you'll be directed to assemble for the exit ticket/debrief. Log out of your computer, and ***quietly*** return to a seat near the front.

class: precalc **goal:** HDW use Gaussian elimination to solve nonsquare systems of equations?

framing

- **what:** use Gaussian elimination to solve nonsquare systems of equations
- **why:** Gaussian elimination is a powerful method for solving systems of equations. It's what computers use.
- **where to:** matrices

Row-echelon form (review)

To solve systems with more than two variables, we want to transform the system into **row-echelon form:**

System of Three Linear Equations in Three Variables

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y + z = -2 \\ 2x - 5y + 5z = 17 \end{cases}$$

Equivalent System in Row-Echelon Form

$$\begin{cases} x - 2y + 3z = 9 \\ y + 4z = 7 \\ z = 2 \end{cases}$$

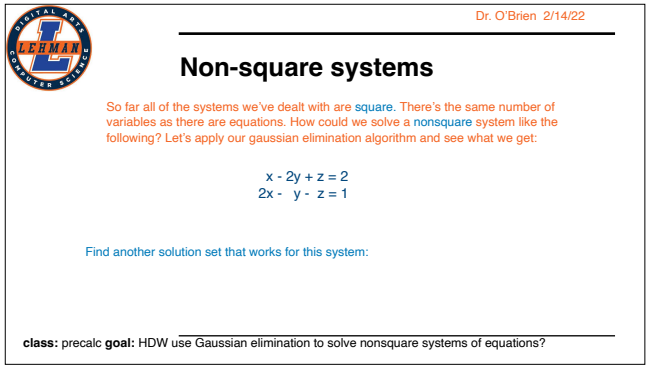
A system is in row-echelon form if it has a stair-step pattern and each equation has a leading coefficient of 1.

Row operations (review)

Gaussian elimination involves three **row operations**:

1. Exchange equations
2. Multiply one of the equations but some number (but not zero)
3. Add one equation to a multiple of another equation

Johann Gauss, the guy who came up with this algorithm



x - 2y + z = 2
2x - y - z = 1.

pivot: 1, coefficient: 2, multiplier: -2:

-2x + 4y. -2z = -4
2x. - y. -z = 1
———————————
3y. -3z = -3

system:
x - 2y + z = 2
3y - 3z = -3

Next let's solve for y in terms of z:
y - z = -1
y = z - 1
Let's solve for z in terms of z:

x - 2(z - 1) + z = 2

x - 2z + 2 + z = 2
x - z + 2 = 2
x - z = 0
x = z

We can pick any random real number for z and find x and y. Let z = 3

x = 3
y = 3 - 1 = 2
z = 3

In general we can say that the solution is

x = a
y = a - 1
z = a

where a is any real number.

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Non-square systems

On your own, try to find a solution set for the system below. Be sure to try solving it on your own. Be prepared to share out!

$$\begin{aligned} x - 2y + 5z &= 2 \\ 4x \quad - \quad z &= 0 \end{aligned}$$

Find another solution set that works for this system:

class: precalc **goal:** HDW use Gaussian elimination to solve nonsquare systems of equations?

$$\begin{aligned} x - 2y + 5z &= 2 \\ 4x \quad - \quad z &= 0 \end{aligned}$$

$$\begin{aligned} -4x. + 8y - 20z &= -8 \\ 4x \quad - \quad 4z &= 0 \end{aligned}$$

$$8y. \; -24z = -8$$

$$y. - 3z = 1$$

$$y = 3z + 1$$

$$x - 2(3z + 1) + 5z = 2$$
$$x - 6z - 2 + 6z = 2$$
$$x. \; -z - 2 = 2$$
$$x - z = 4$$
$$x = z + 4$$

$$z = a$$
$$x = a + 4$$
$$y = 3a + 1$$

where a is any real number

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Independent work

Today we'll be working on Pset #2.

1. Solve the nonsquare system below:

$$\begin{aligned} 2x - 3y + z &= -2 \\ -4x + 9y \quad &= 7 \end{aligned}$$

2. Solve the nonsquare system below:

$$\begin{aligned} x + 2y + 3z &= 4 \\ -x - 2y + z &= 5 \\ 2x + 8y + z &= -2 \end{aligned}$$

3. Tired of doing precalc homework, you borrowed $775 to build an app that solves systems of linear equations automatically. You borrowed some money at 8% interest, some at 9%, and some at 10%. How much did you borrow at each rate, given that annual interest is $67.5 and the amount of money borrowed at 8% is four times the amount borrowed at 10%?

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1.
pivot: 2, elimination coefficient: -4, multiplier: 2

$$\begin{aligned} 4x - 6y + 2z &= -4 \\ -4x + 9y. \quad &= 7 \end{aligned}$$

$$3y. \; +2z = 3$$

$$y + 2/3 \, z = 1$$

$y = 1 - 2/3z$

$$4x - 6(1 - 2/3z) + 2z = -4$$

$$4x. \; -6 + 4z + 2z = -4$$

$$4x + 6z. = 2$$

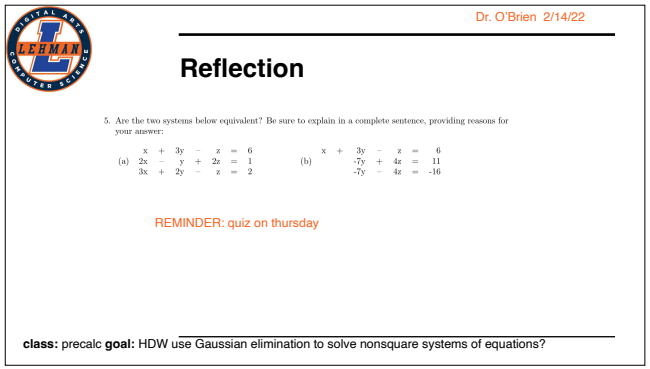
2x + 3z = 1

x = 0.5 - 1.5z

z = a, where a is any real.

2. This system will appear to end up with a zero pivot in the second row, if students don't perform a row exchange (see ipad for detailed solution).

3.



possible exit ticket.

See answer key for detailed solution