Bayesian Inference Lighter and Faster

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What we'll see

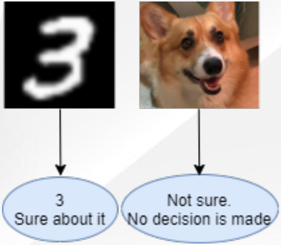
1. Uncertainty
2. Bayesian Networks
3. Existing Methods
4. Our Method
5. Experiments and Results

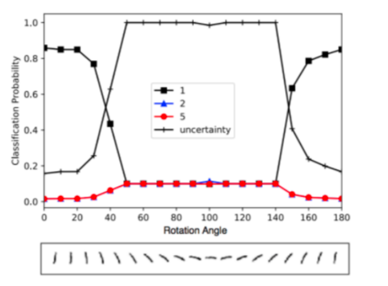
Model Uncertainty

Neural Networks achieving amazing results in many fields such as

- Computer Vision
- Natural Language Processing
- Medical Diagnosing

But suffer often from **Over confidence** in their predictions and specifically confidence in wrong once.
That's why we would like to have a measurement of uncertainty.





Bayesian Modeling

Bayesian Inference

Bayesian inference is a method of statistical inference in which Bayes' theorem is used to update the probability for a hypothesis as more evidence or information becomes available.

Start with a prior on the the model parameters:

$$X = \{x_1, x_2, ..., x_N\}, \quad Y = \{y_1, y_2, ..., y_N\}$$

$$p(y = d|x, w) = \frac{exp(f_d(x; w)}{\sum_{d'} exp(f_{d'}(x, w))}$$

From Bayes rule we get:

$$p(w|X, Y) = \frac{p(Y|X, w)p(w)}{p(Y|X)}$$

Bayesian Modeling

At inference time: Given new input x^* the output y^*:

$$p(y^*|x^*, X, Y) = \int p(y^*|x^*, w)p(w|X, Y)dw$$

A key component in posterior evaluation is the normaliser, also called model evidence:

$$p(Y|X) = \int p(Y|X, w)p(w)dw$$

Posterior Sampling Techniques

Calculating the Posterior analytically is infeasible.
Family of methods tries to sample from the posterior:

- Monte Carlo Markov chain - MCMC, HMC, NUTS
- Stochastic gradient Langevin dynamics

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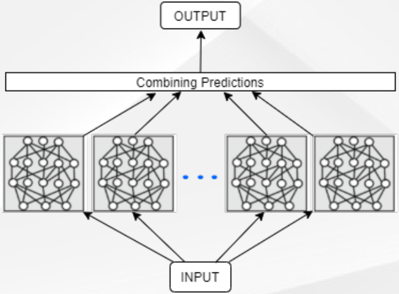
Such that in the end of training we have M Posterior Samples $\{W_1, W_2, \ldots, W_M\}$

At inference:

$$p(y^*|x^*, X, Y) = \frac{1}{Z}\sum_{i=1}^{M} p(y^*|x^*, W_i)$$

Difficulties with current methods

- **Time Complexity:**
  Predict Through each of the M models for final prediction
- **Memory Complexity:**
  At Inference time, hold in memory all M models parameters.



Our Method

The Idea: Model the predictive distribution directly

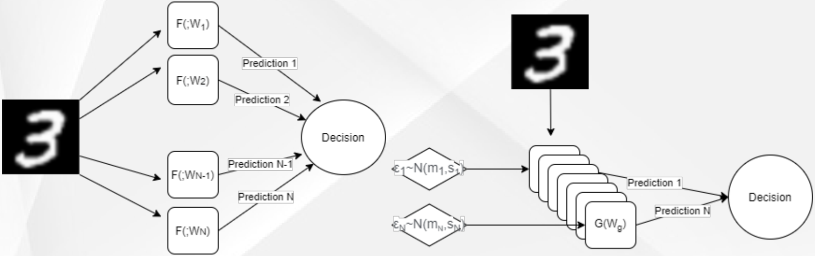


Figure: Left - Posterior samples used in inference. Right - Generative model that models the predictive distribution

Training Phase

Algorithm 1 Training

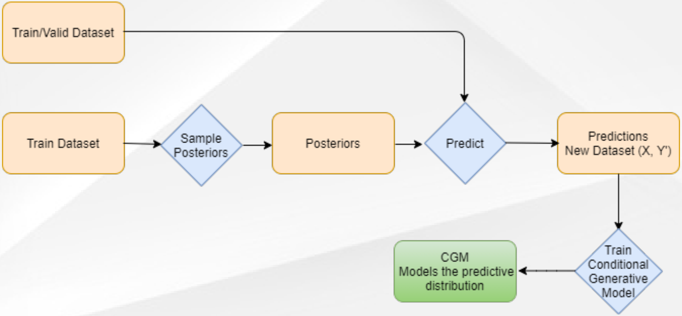
1: Given (X_{train}, Y_{train}) and Model $F(\theta)$

2: Sample M posteriors $\{\theta_1, \ldots, \theta_M\}$

3: Predict on X_{valid}: $\forall i \in \{1, \ldots, N\}, \forall x_j \in X_{valid} : Y_i = F(x_{;i})$

4: Using $\{(X_i, Y_i^j)\}_{j=1,\ldots,N}^{i=1,\ldots,M}$ Train Conditional Generative Model(CGM) Such that

$$(G(\epsilon_1; X), \ldots, G(\epsilon_M; X)) \sim (F(X; \theta_1), \ldots, F(X; \theta_M))$$



Inference Phase

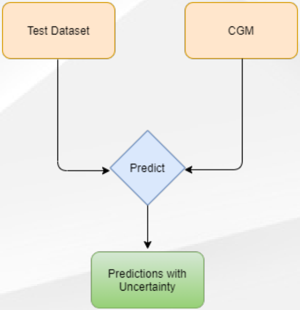
Algorithm 2 Inference

1: Given X_{test}

2: Predict using CGM:

$$Y = argmax(G(\epsilon_1; X), ..., G(\epsilon_M; X))$$

$$Unceratainty = Histogram(G(\epsilon_1; X), ..., G(\epsilon_M; X))$$



Get into details

How do we achieve

$$(G(\epsilon_1; X), \ldots, G(\epsilon_N; X)) \sim (F(X; \theta_1), \ldots, F(X; \theta_N))$$

Trying the following approaches:

- MMD for probability matching:
  CGM learns to model the predictive distribution.
- Mapping each posterior sampled model to a learned prior in the CGM:
  Based on Wasserstein Auto Encoder, each posterior is mapped to a Normal distribution, which ϵ is sampled from

Experiments and Results

Experiments

Metrics used for comparison:

- Accuracy
- Accuracy without uncertain examples
- Confidence on wrong predictions
- Calibration
- Uncertainty count

The following results are on MNIST dataset and a two layer neural network.

Accuracy and Accuracy Filtered

	Posteriors	N(0,1) with T
Accuracy	0.9204999804496765	0.9350000023841858

	Posteriors	N(0,1) with T	N(0,1) without T
0.6	0.9645625948905945	0.9605963826179504	0.959280788898468
0.7	0.9708284735679626	0.971381425857544	0.9714442491531372
0.8	0.9841370582580566	0.9825682640075684	0.9825783967971802
0.9	0.9927588701248169	0.9905242919921875	0.9905242919921875
0.95	0.9991063475608826	0.9916724562644958	0.9916724562644958
0.98	0.9990825653076172	0.995230495929718	0.995230495929718
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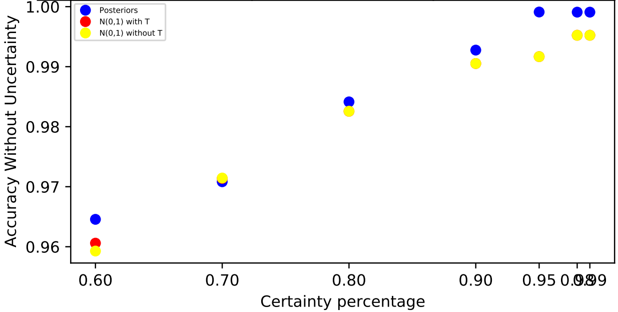
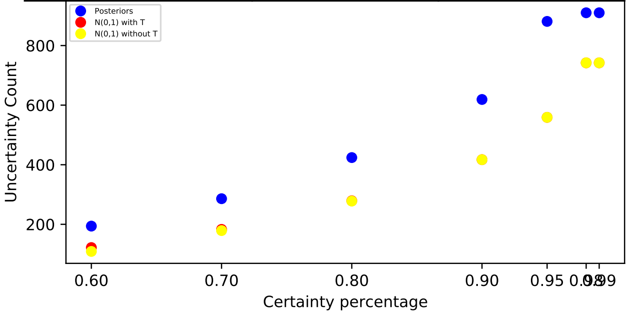


Uncertainty Count

	Posteriors	N(0,1) with T	N(0,1) without T
0.6	194	122	109
0.7	286	183	179
0.8	424	279	278
0.9	619	417	417
0.95	881	559	559
0.98	910	742	742
0.99	910	742	742



Calibration

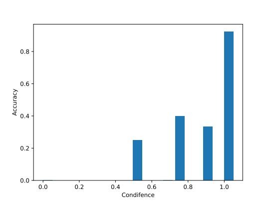


Figure: Generator - Before T

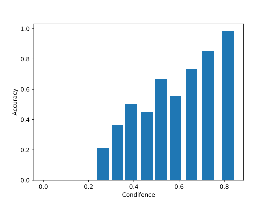


Figure: Generator - After T

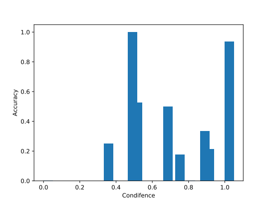


Figure: Posteriors - Before T

Questions?