# Trees

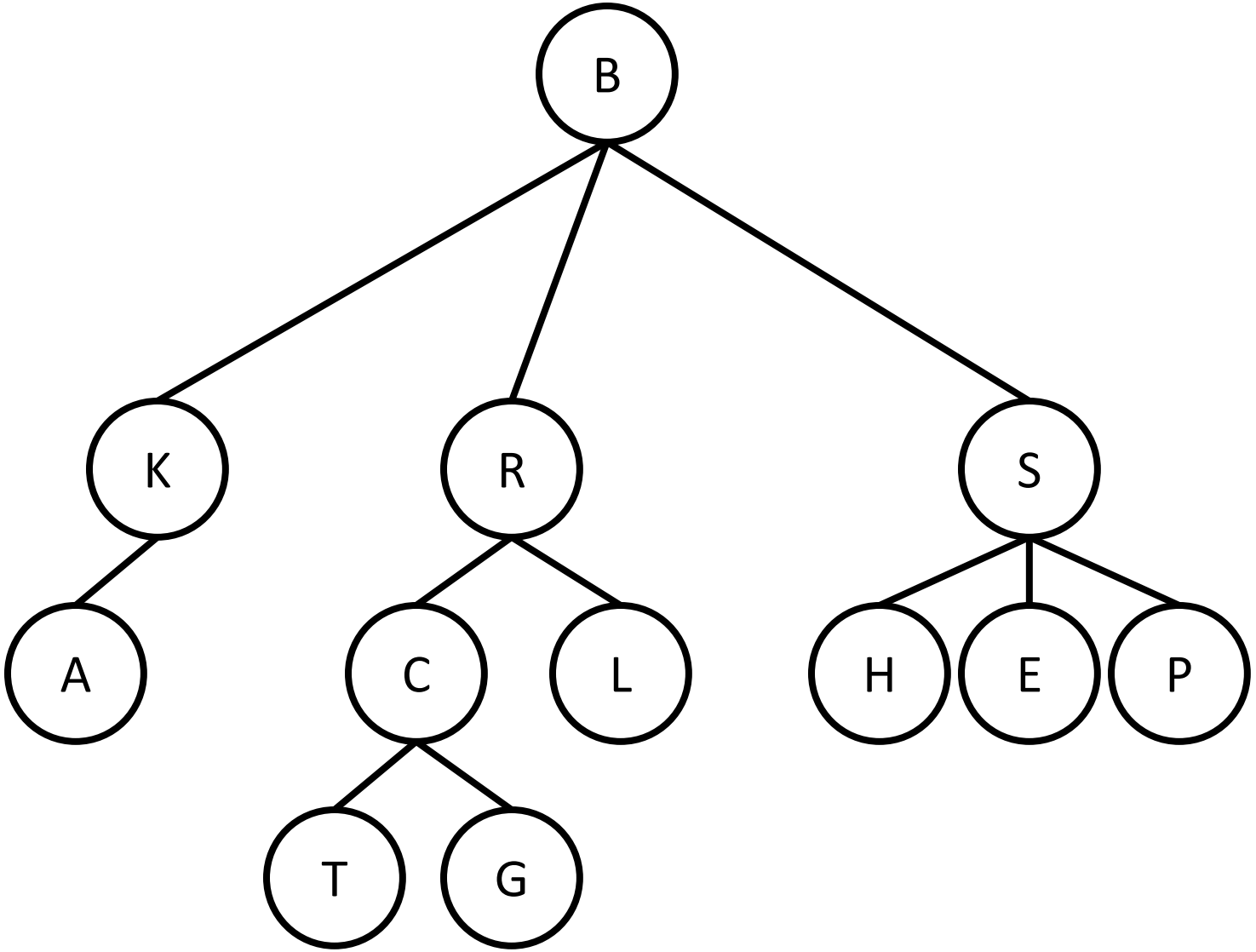



# A New Math Type: ***tree***

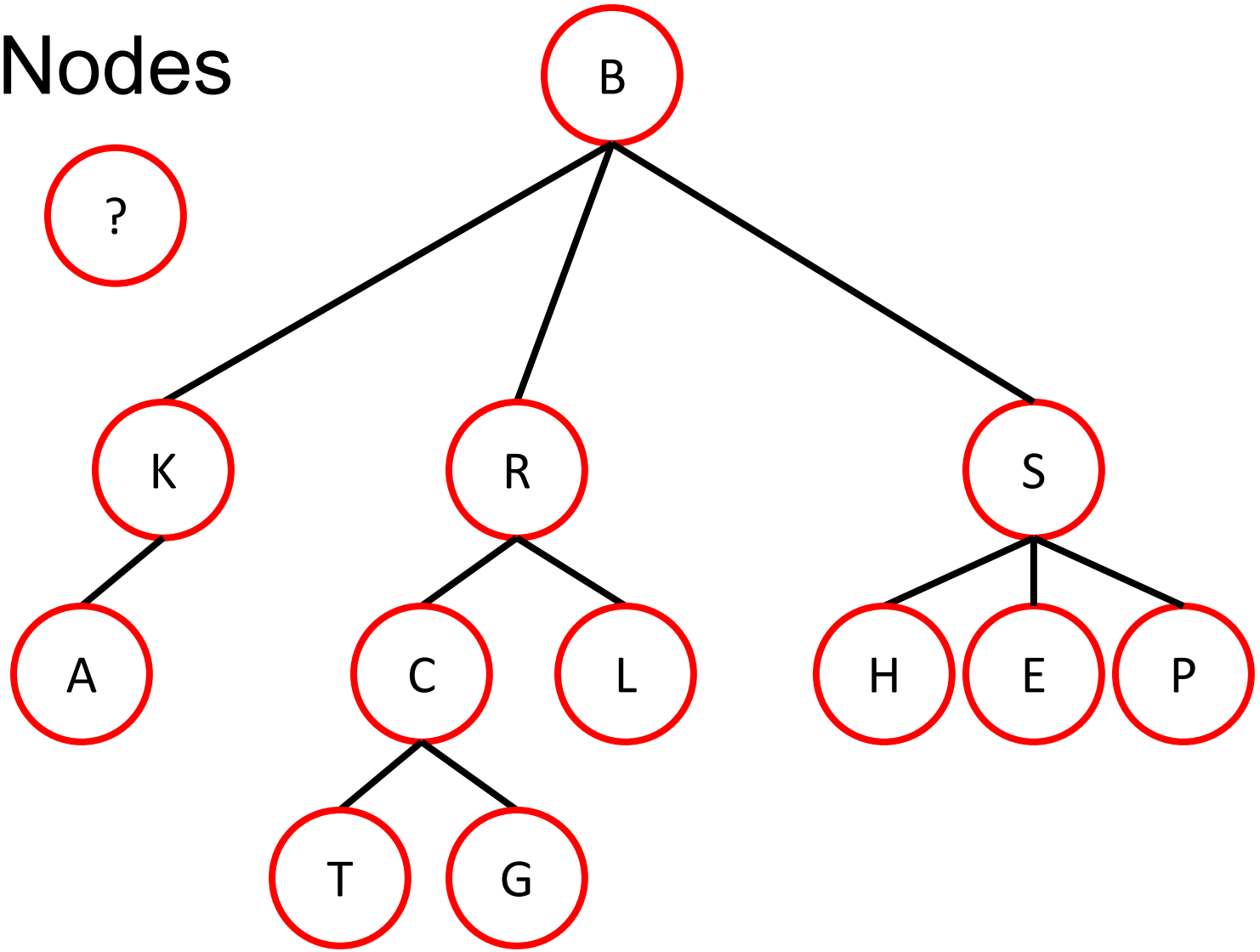
- A ubiquitous concept in computing is that of a ***tree***
  - Often we are interested in a ***binary tree***, a special case of a tree in which each *node* has at most two *children*
- An *informal* introduction ("node"?, "children"?) follows, using pictures rather than any new mathematical notation

# Recursive Structure

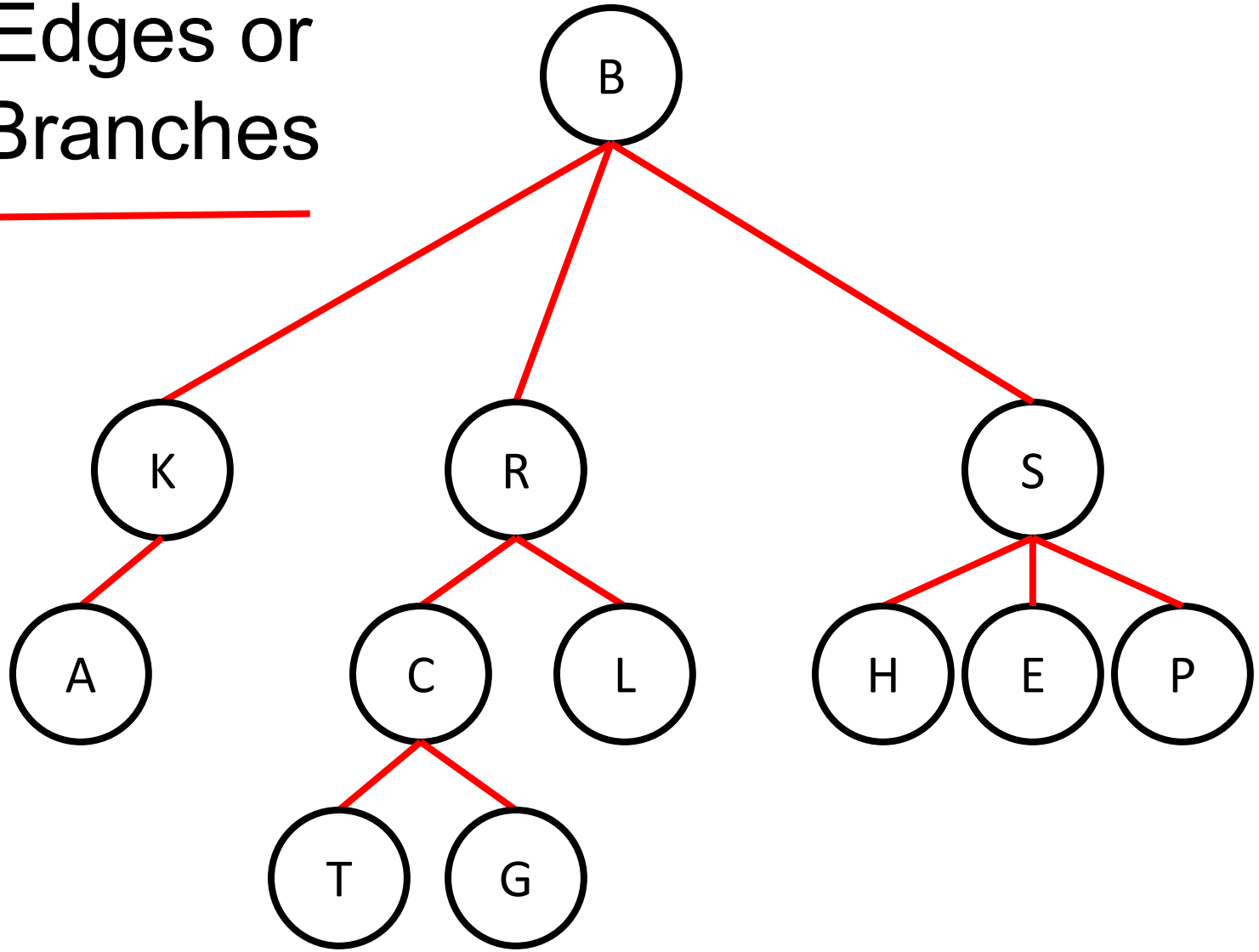
- A ***tree*** is made up of:
  - A ***root node***
  - A string of zero or more ***child nodes*** of the root, each of which is the root of its own tree
- Since a tree may contain other trees, its structure is ***recursive***
- Note: the following explanation of trees is adequate for present purposes but is not technically complete; details later…

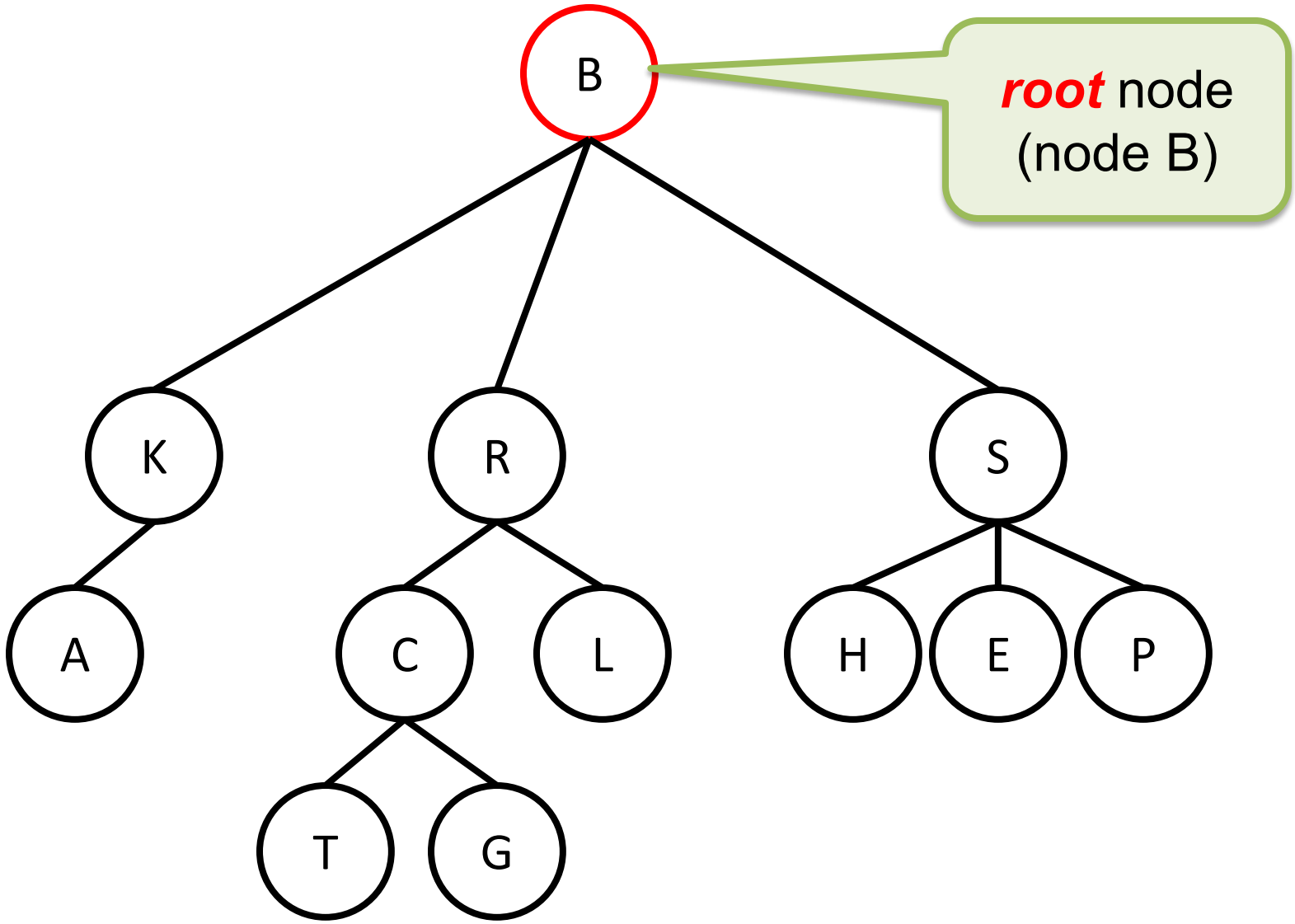

B
K
R
S
A
C
L
H
E
P
T
G

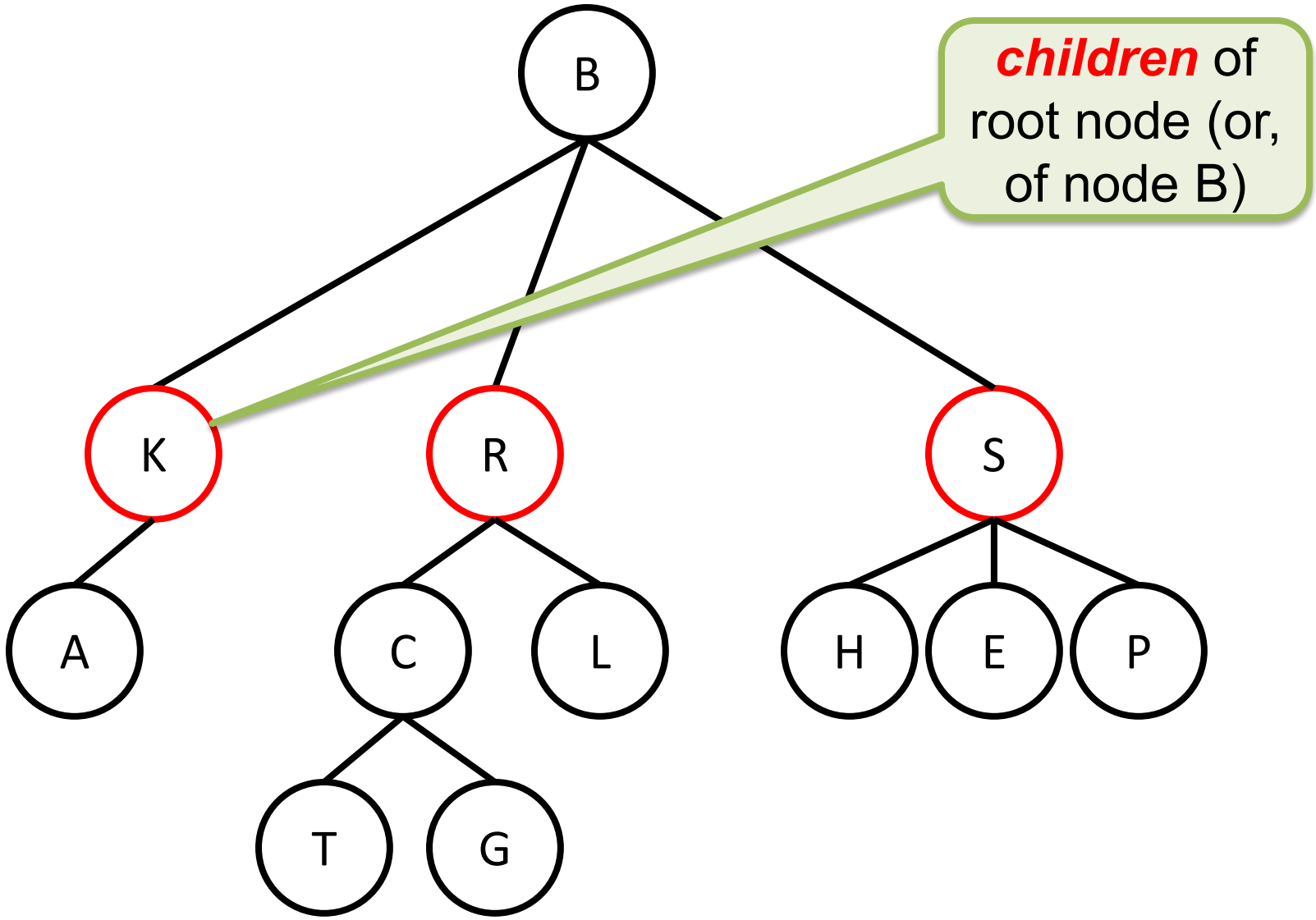
# Nodes

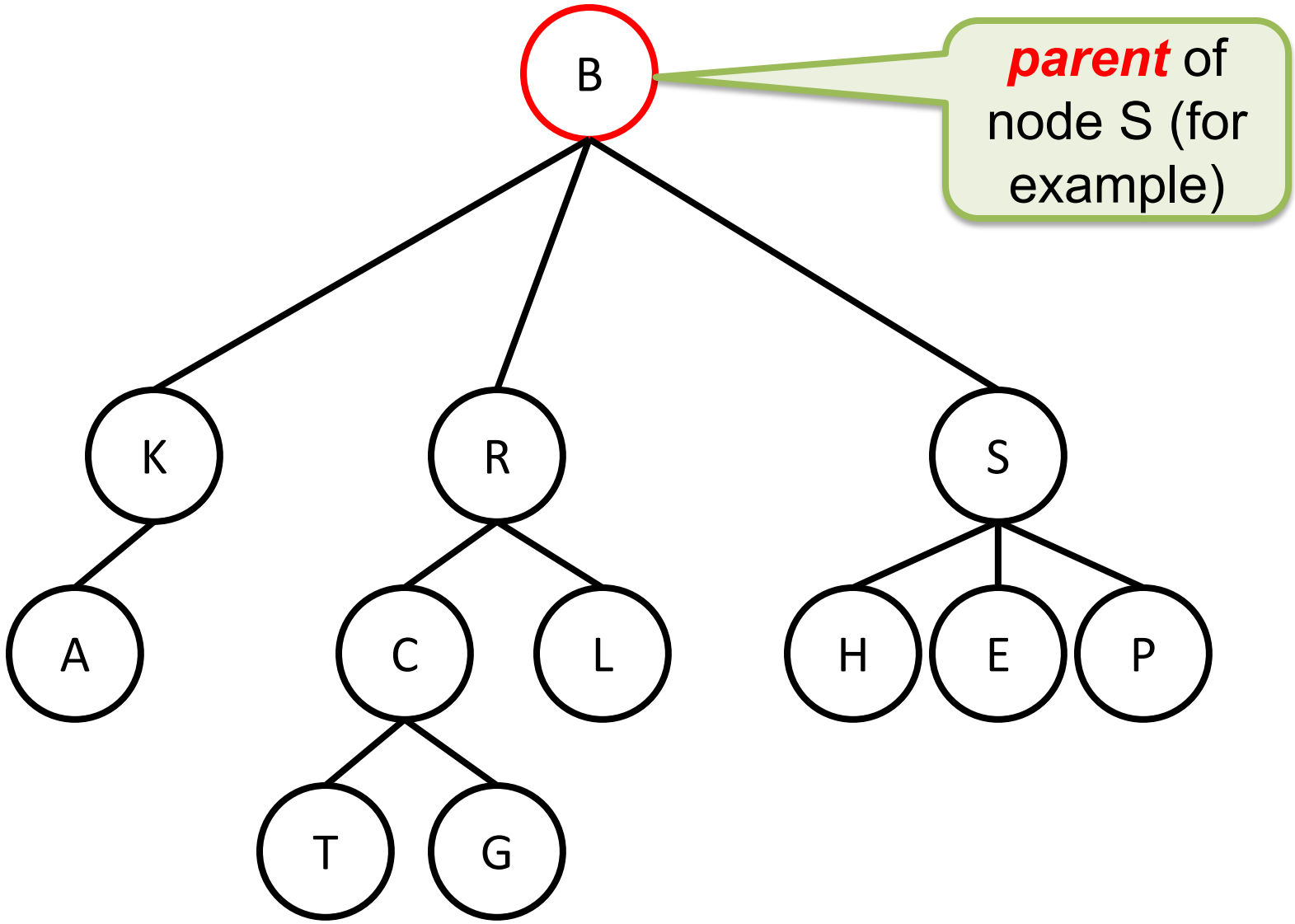


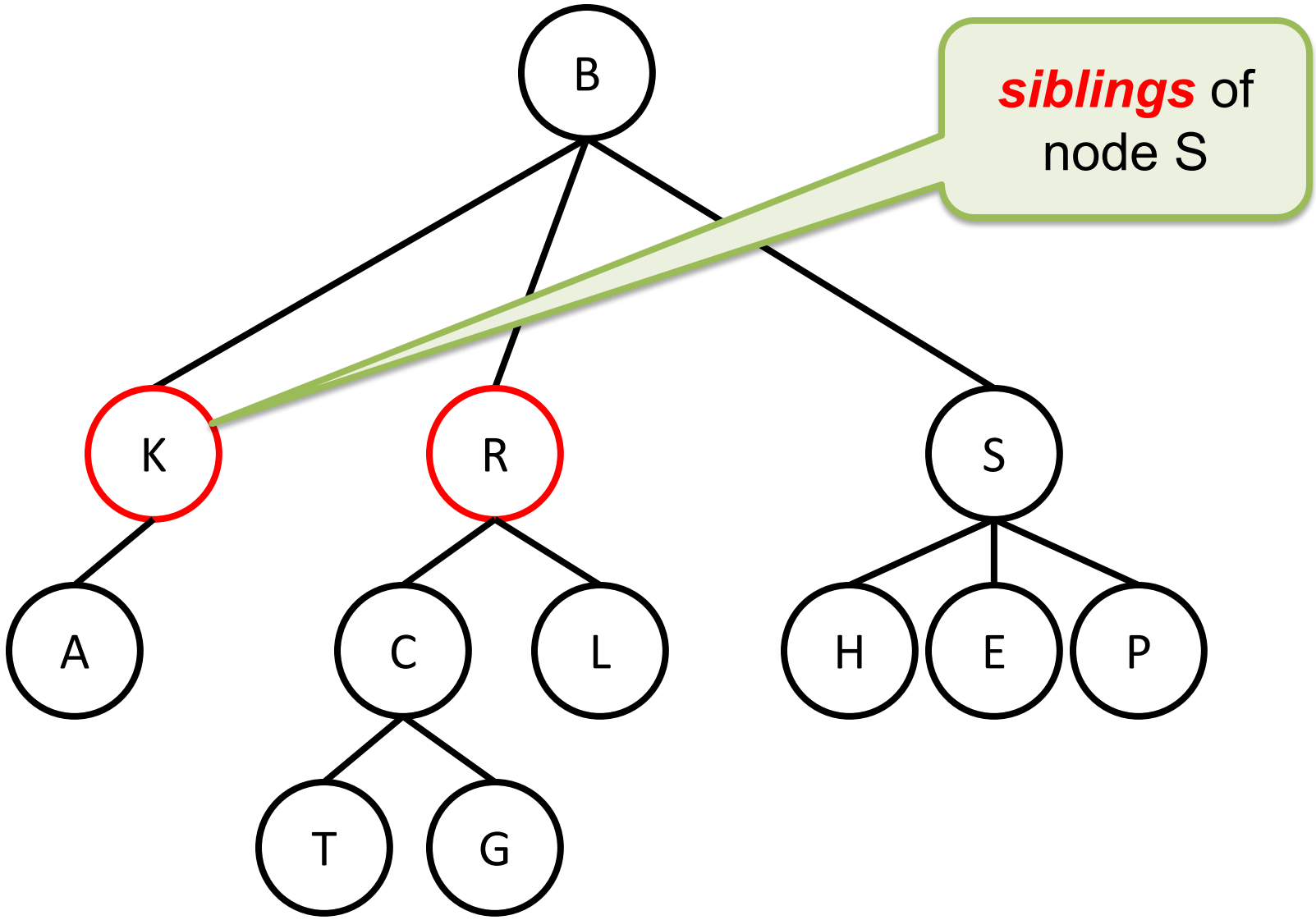
# Edges or Branches

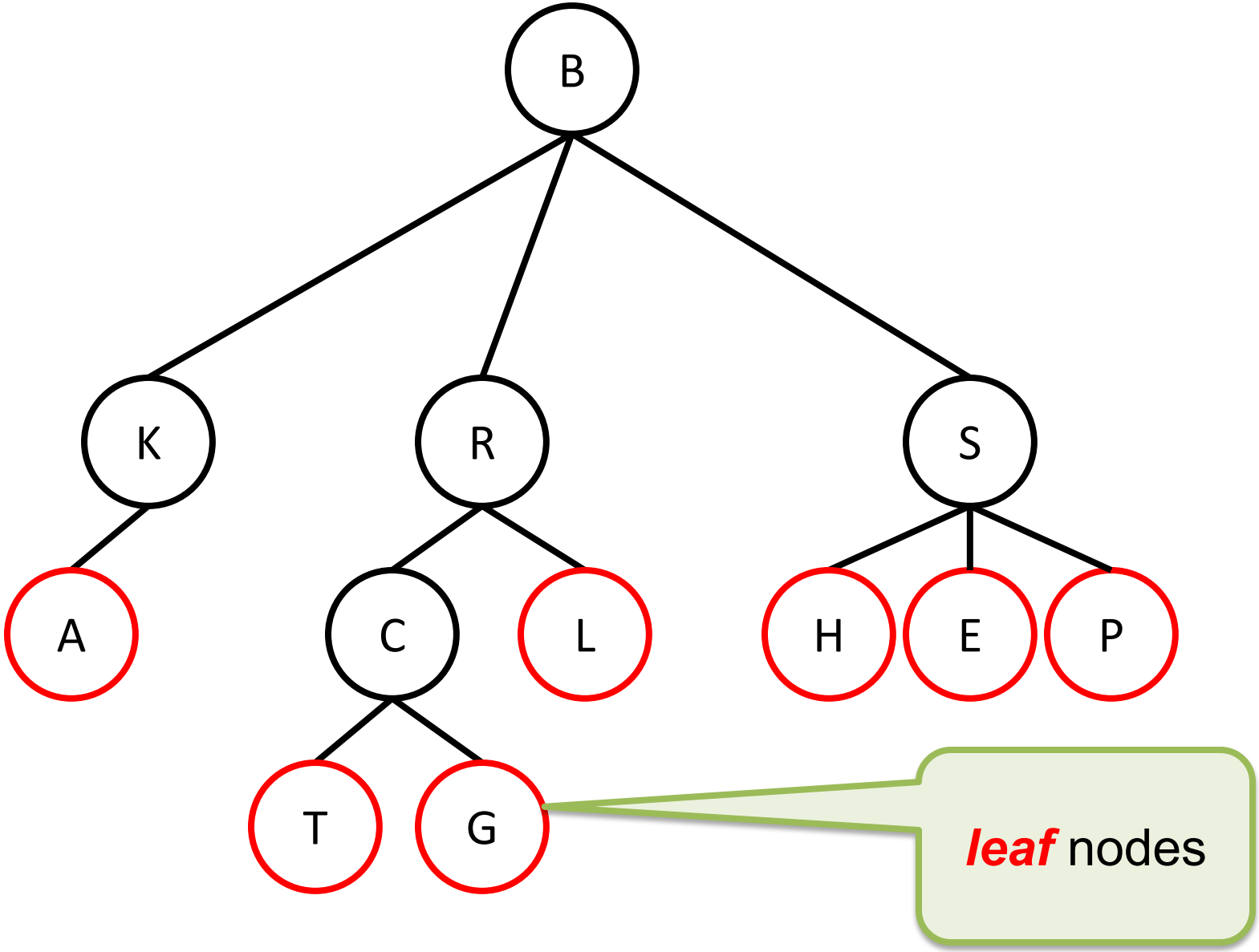


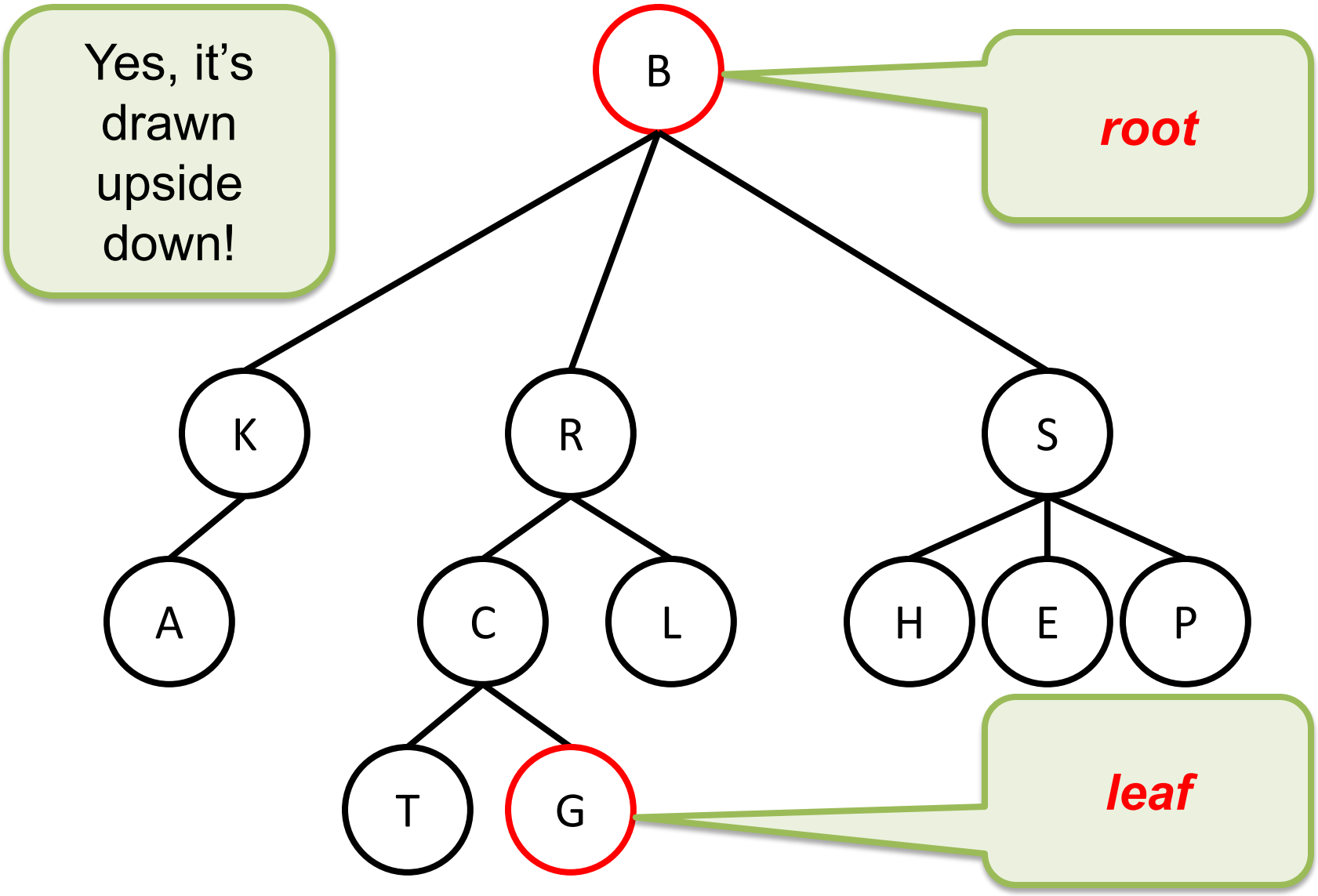

B
root node
(node B)
K
R
S
A
C
L
H
E
P
T
G

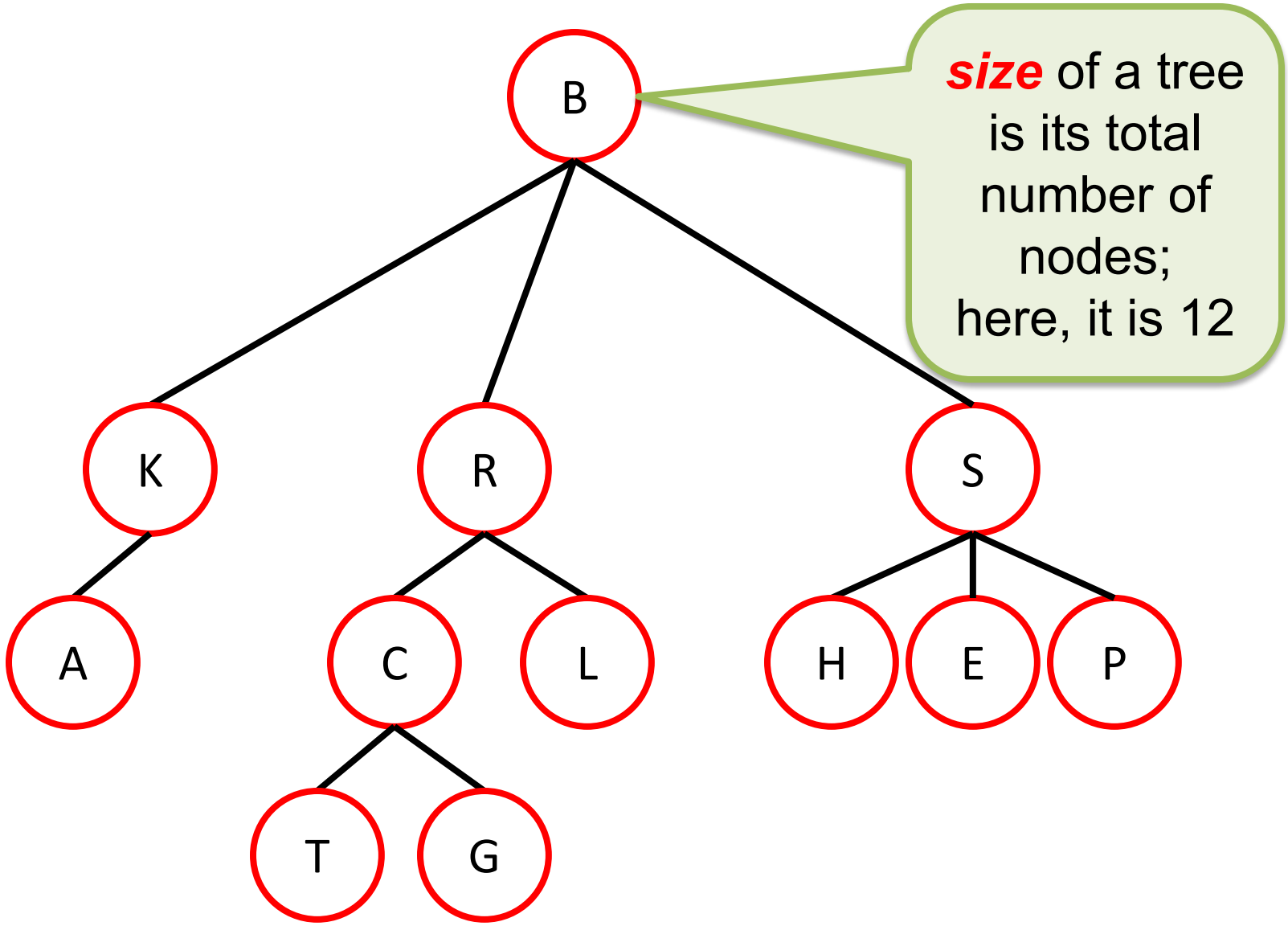

B
children of root node (or, of node B)
K
R
S
A
C
L
H
E
P
T
G


B
parent of node S (for example)
K
R
S
A
C
L
H
E
P
T
G


B
siblings of node S
K
R
S
A
C
L
H
E
P
T
G

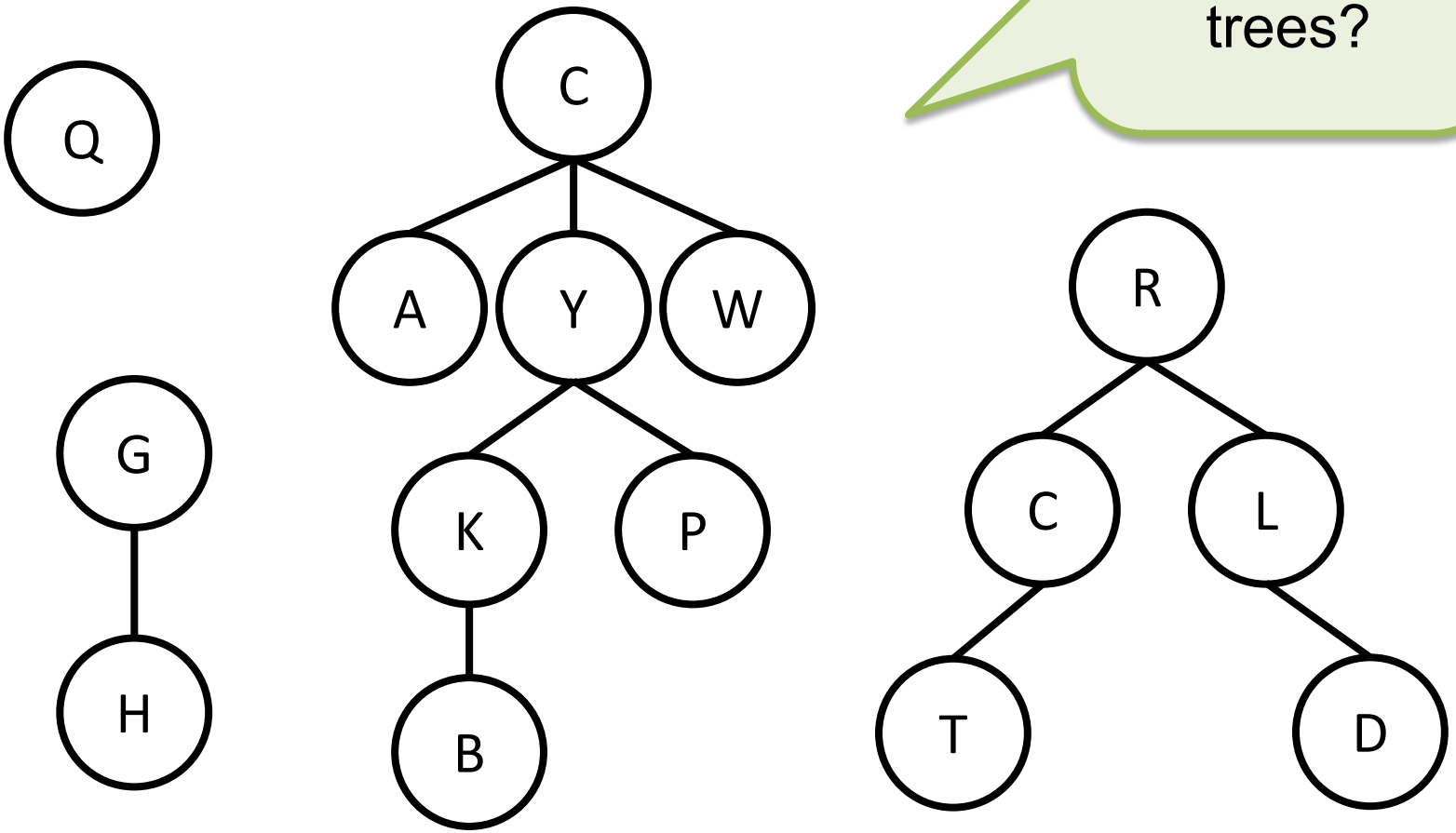

B
K
R
S
A
C
L
H
E
P
T
G
leaf nodes

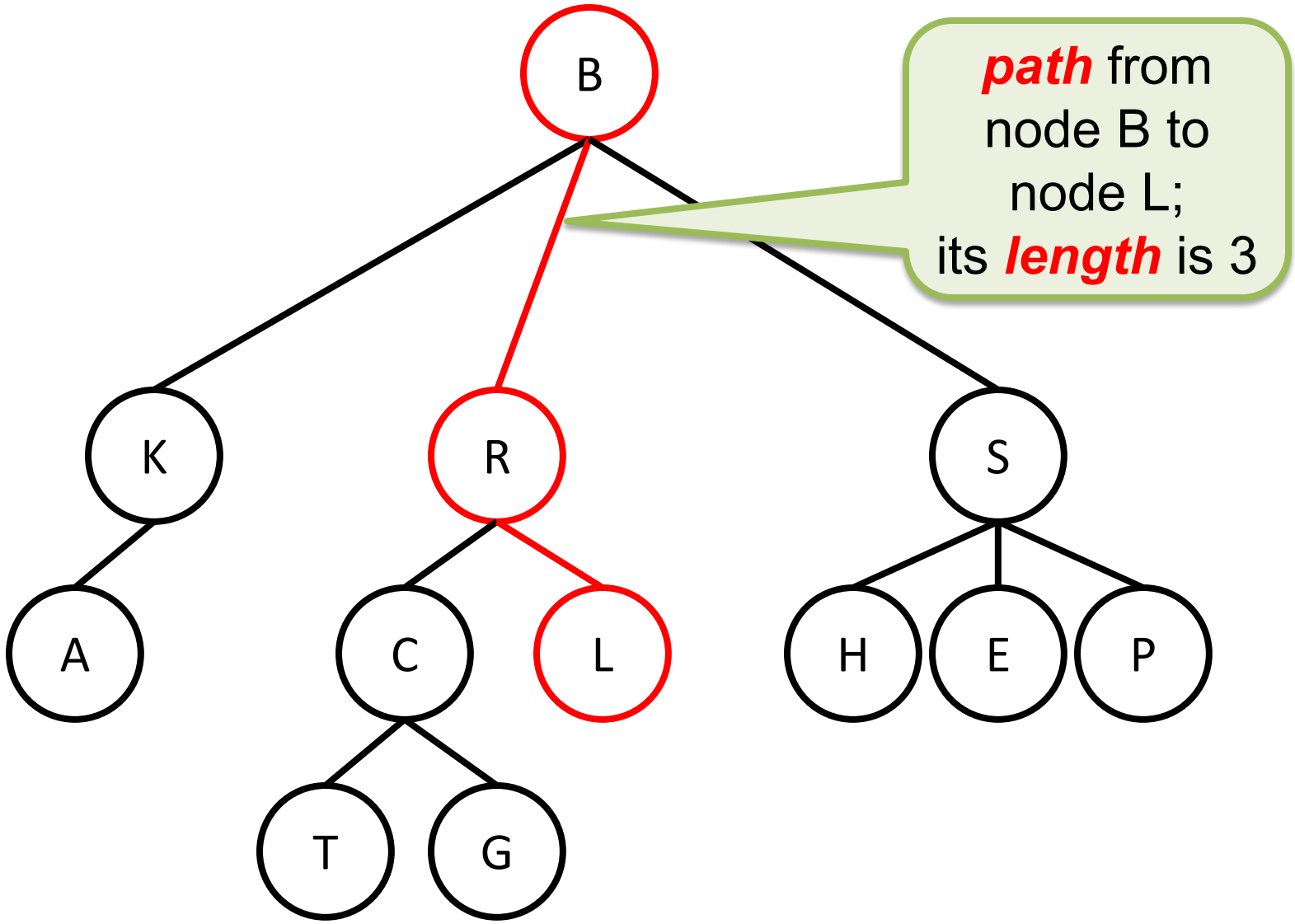

Yes, it's drawn upside down!
B
root
K
R
S
A
C
L
H
E
P
T
G
leaf

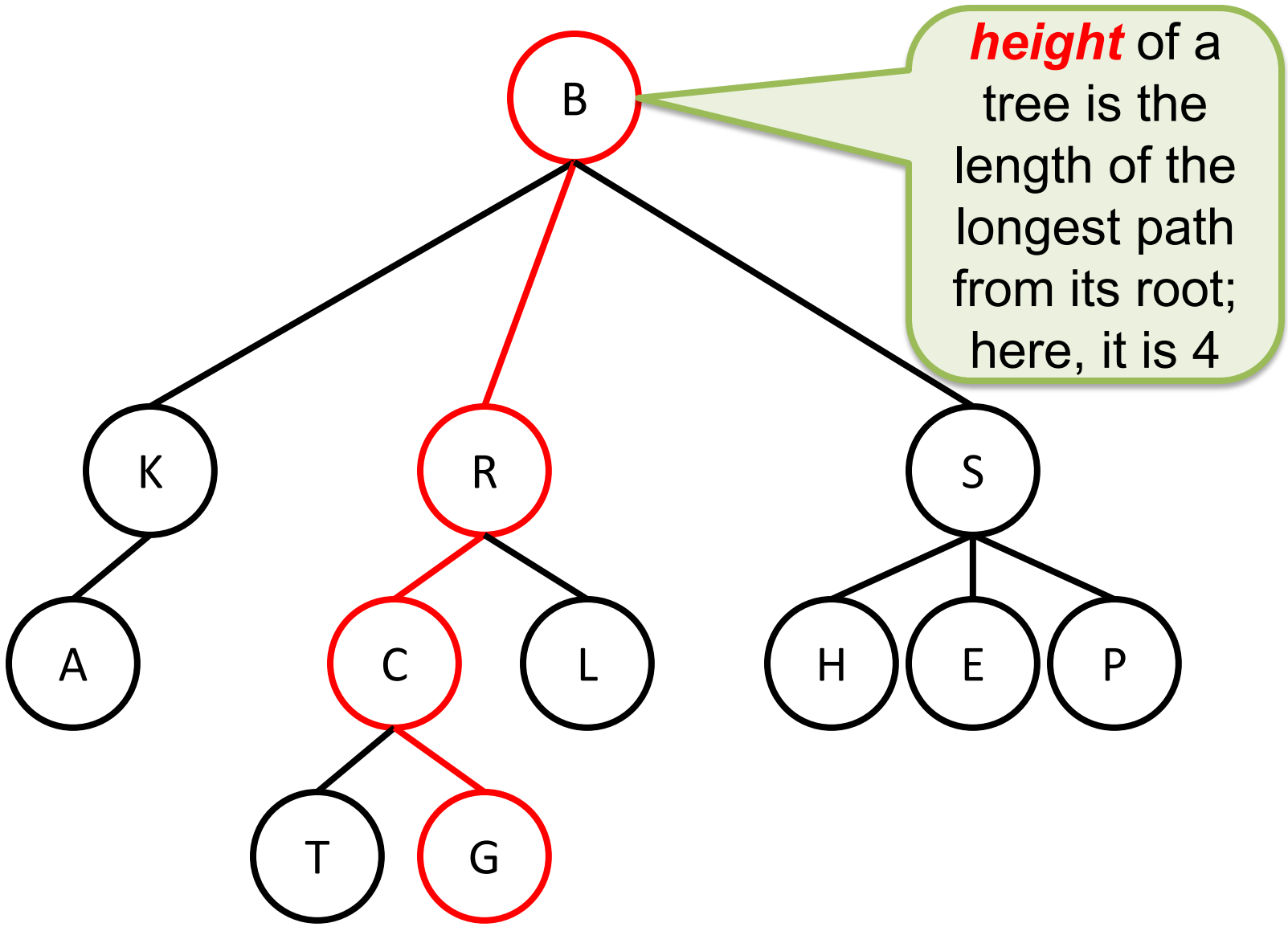

B
K
R
S
A
C
L
H
E
P
T
G
size of a tree is its total number of nodes; here, it is 12

# Your Turn!

What's the ***size*** of each of these 4 trees?

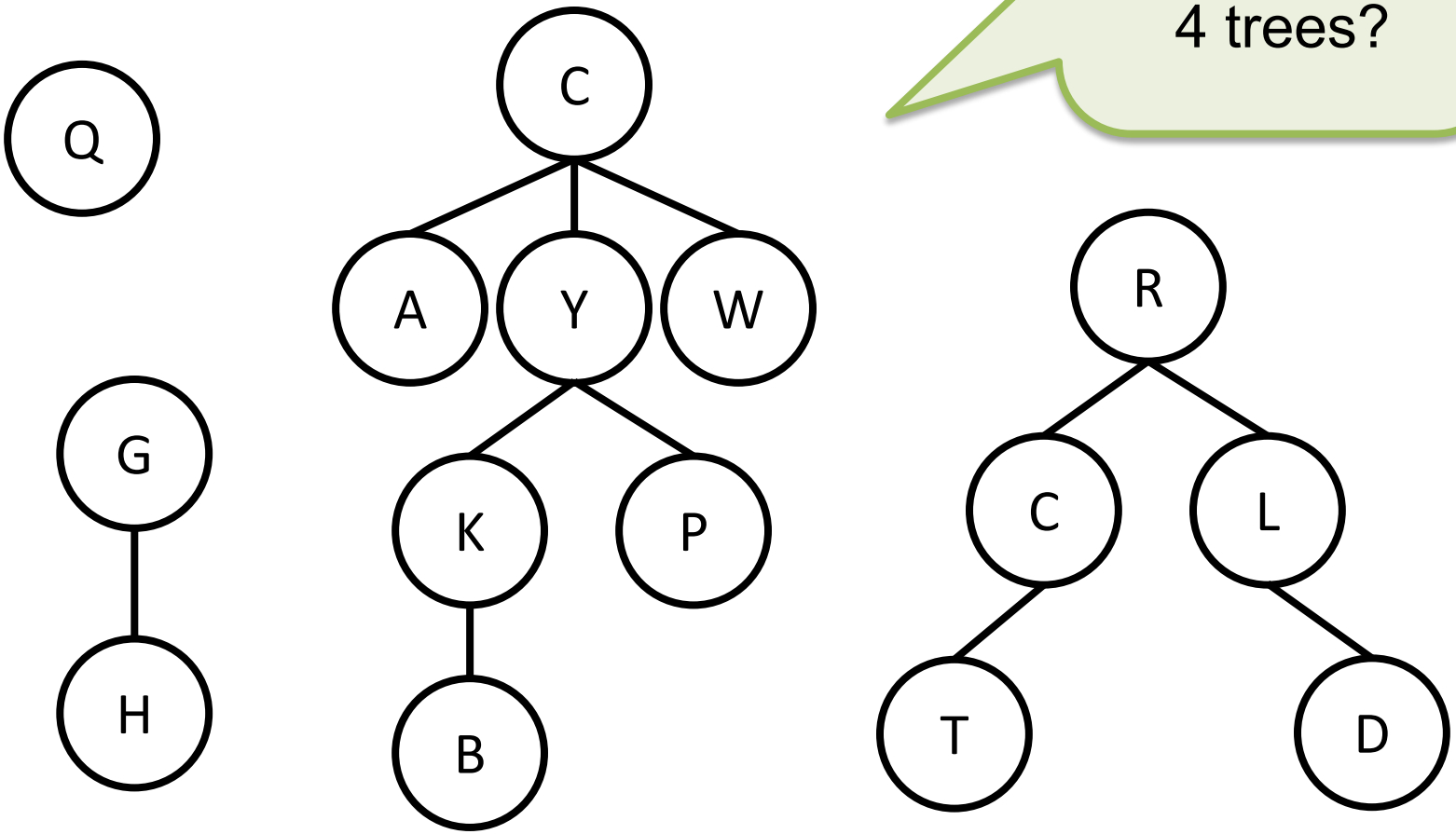



B
path from node B to node L; its length is 3
K
R
S
A
C
L
H
E
P
T
G

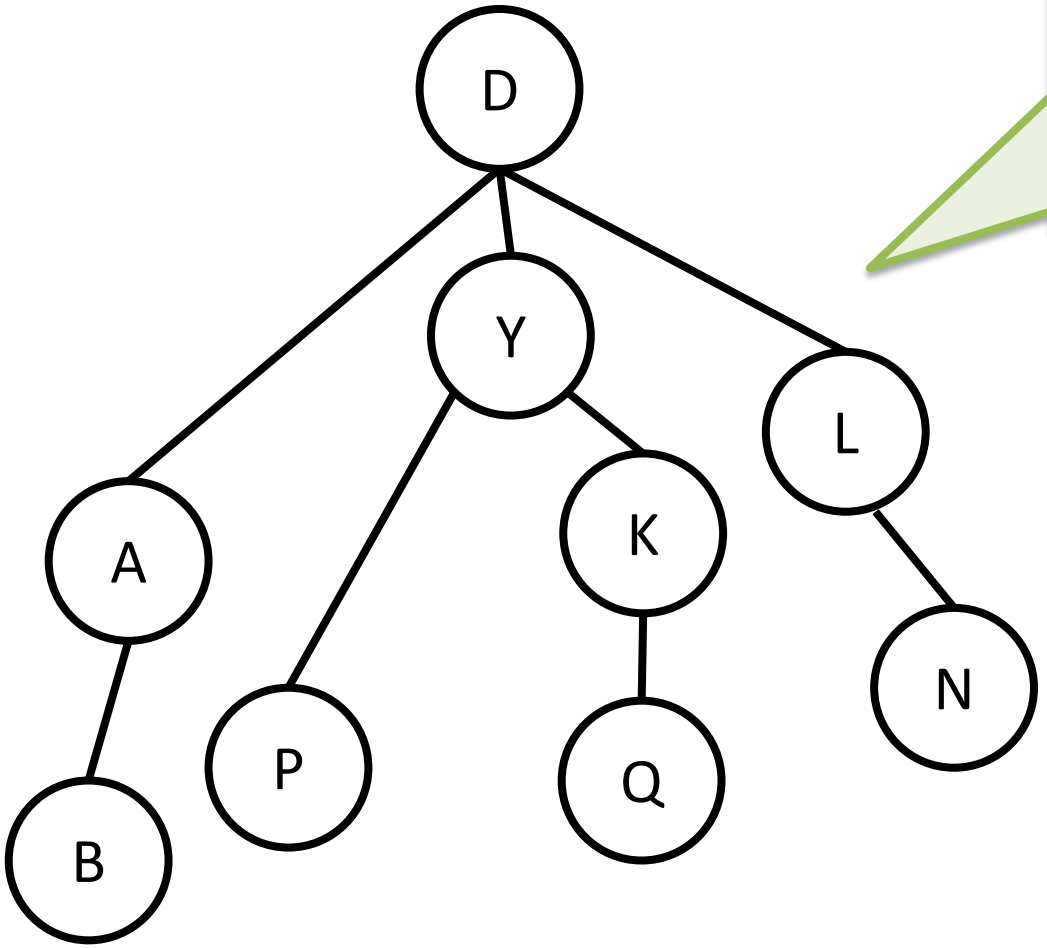

B
K
R
S
A
C
L
H
E
P
T
G
height of a tree is the length of the longest path from its root; here, it is 4

# Your Turn!

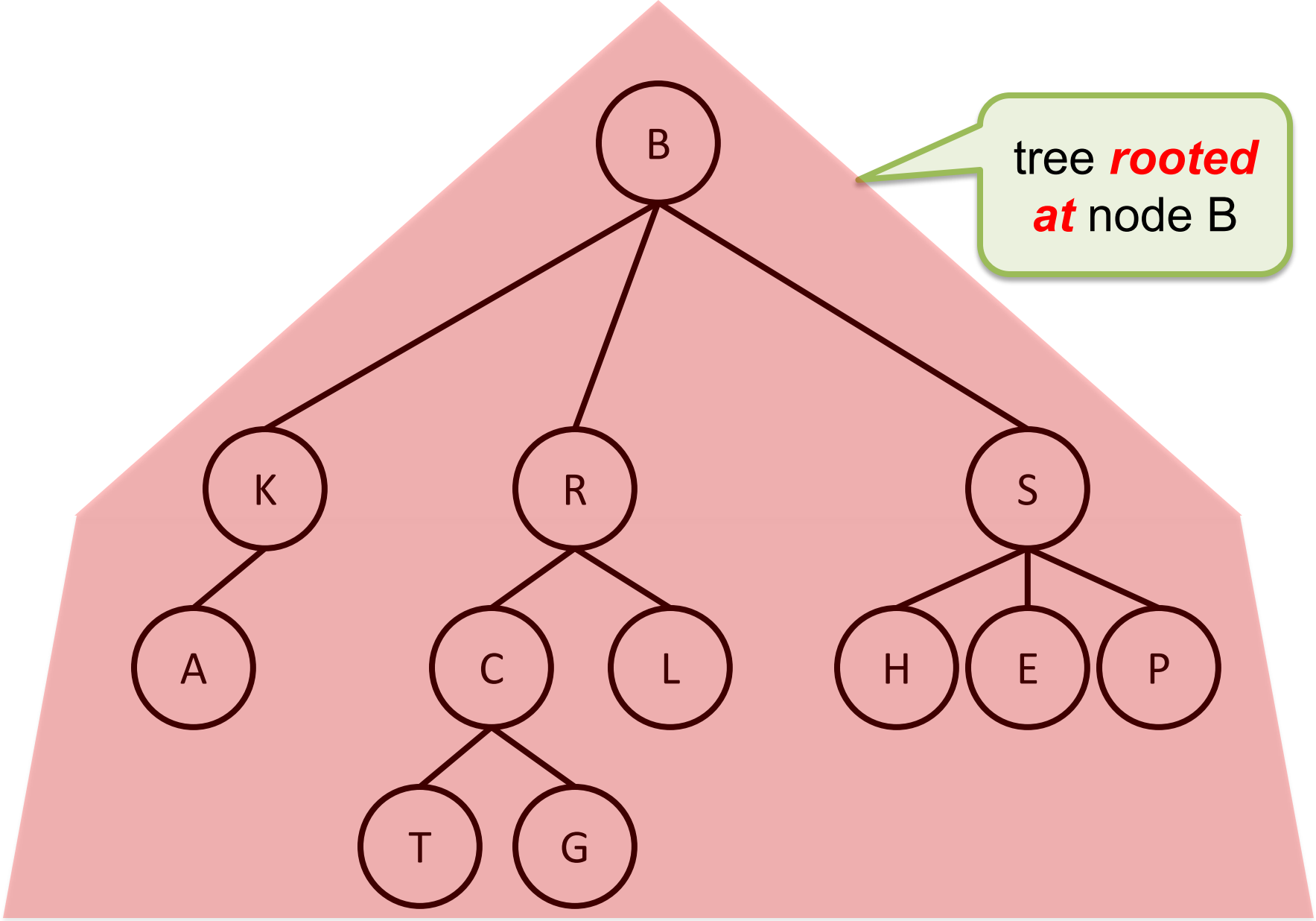
What's the ***height*** of each of these 4 trees?

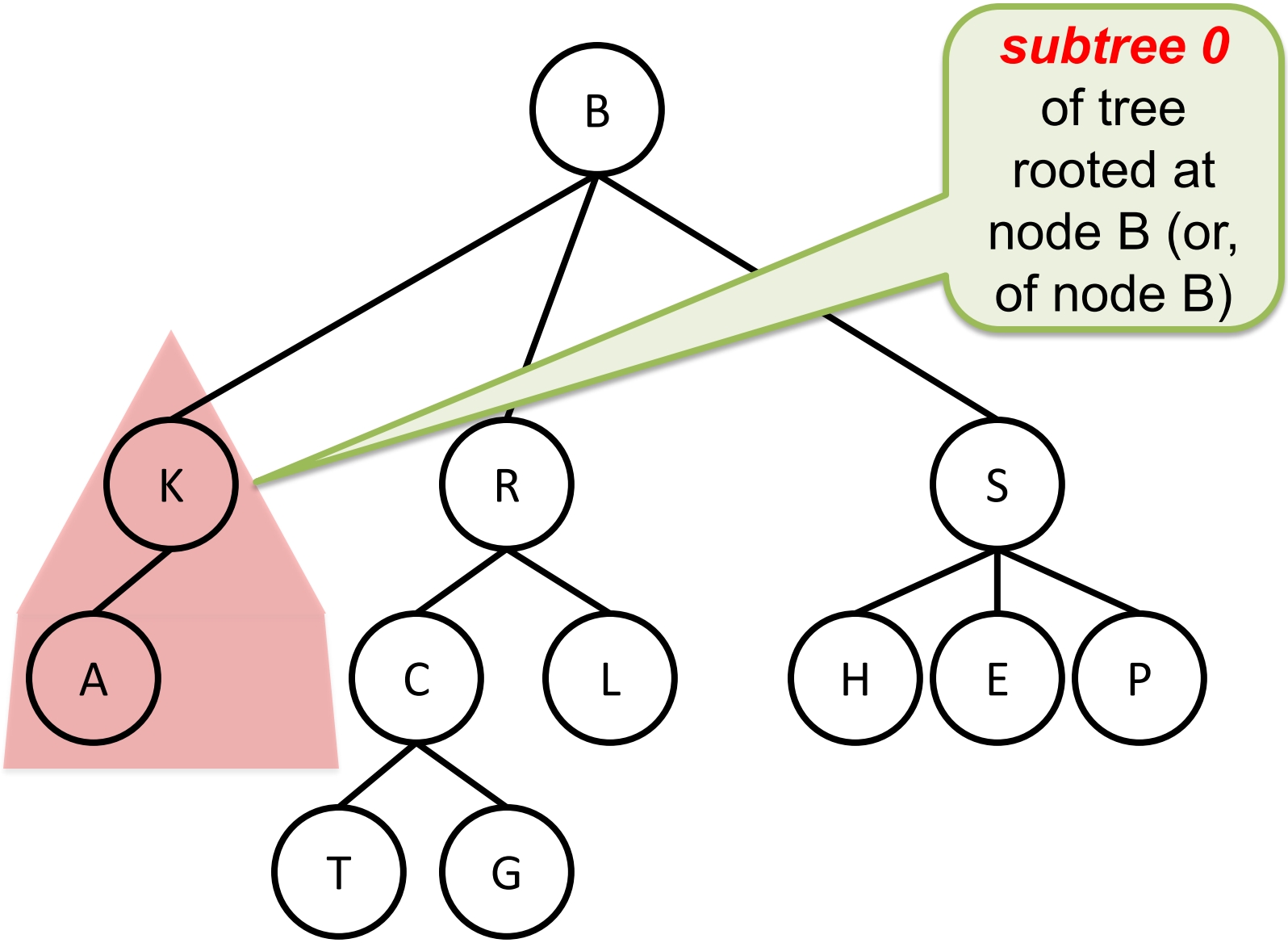


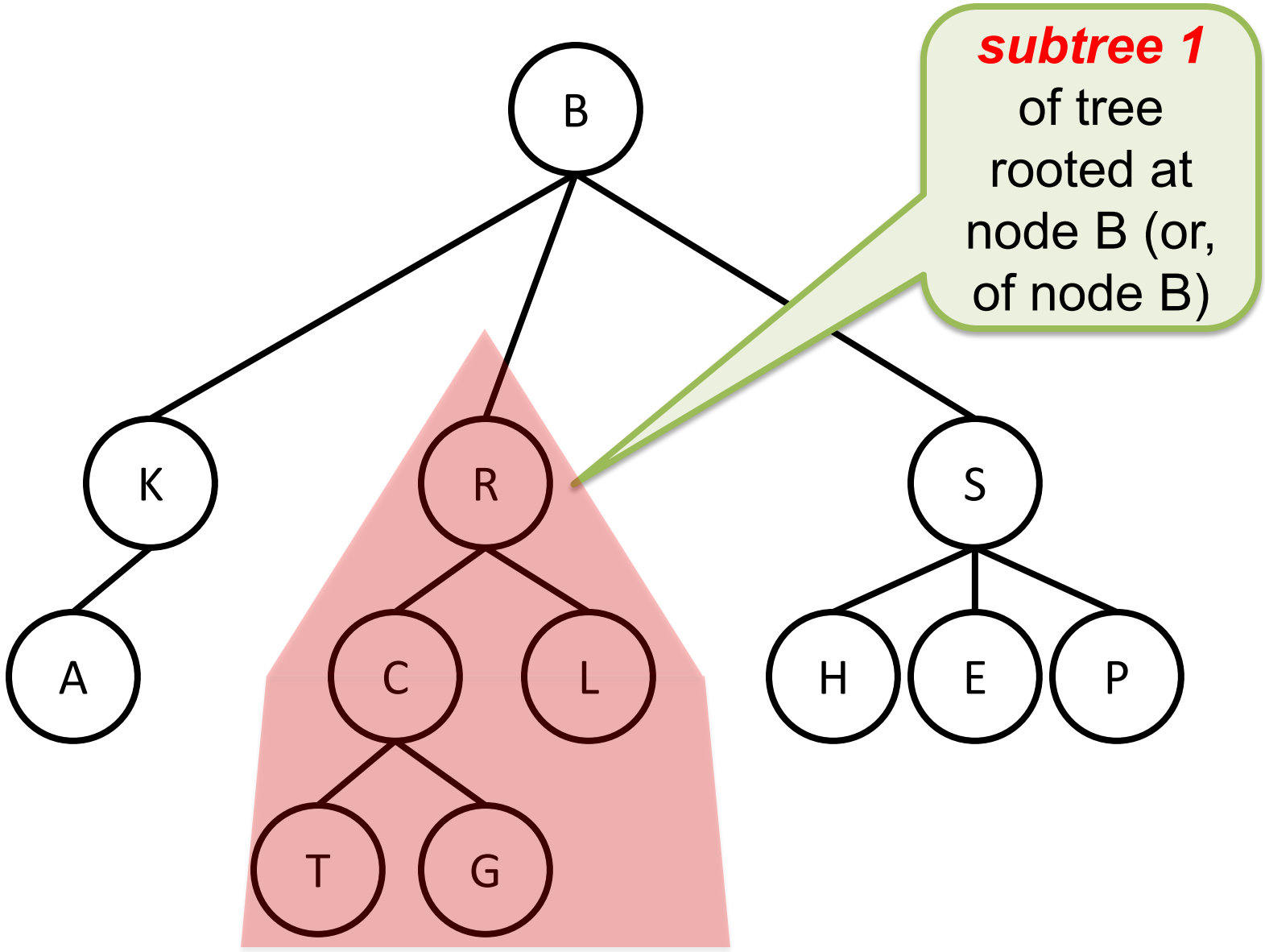
# A Tricky One?

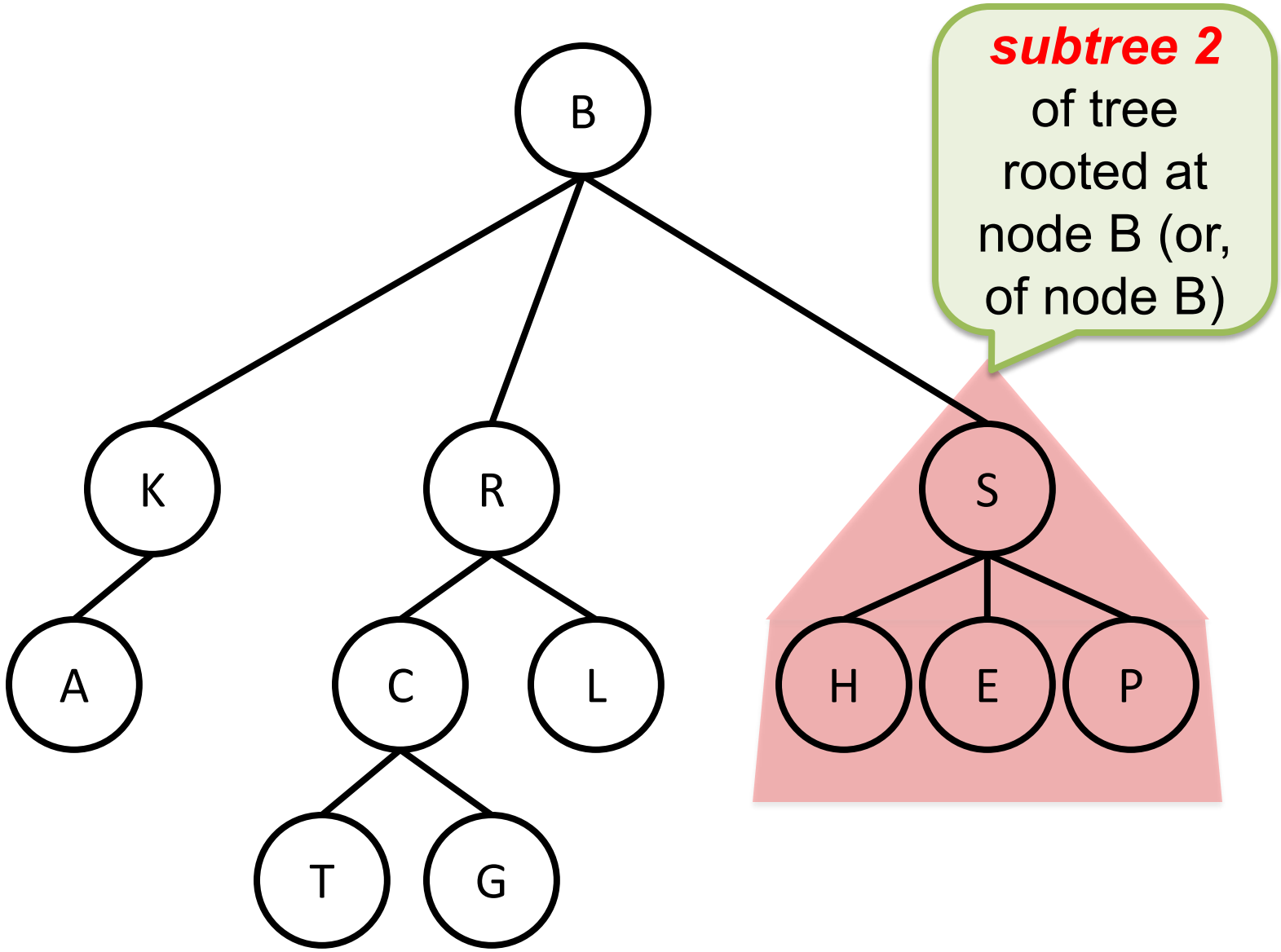


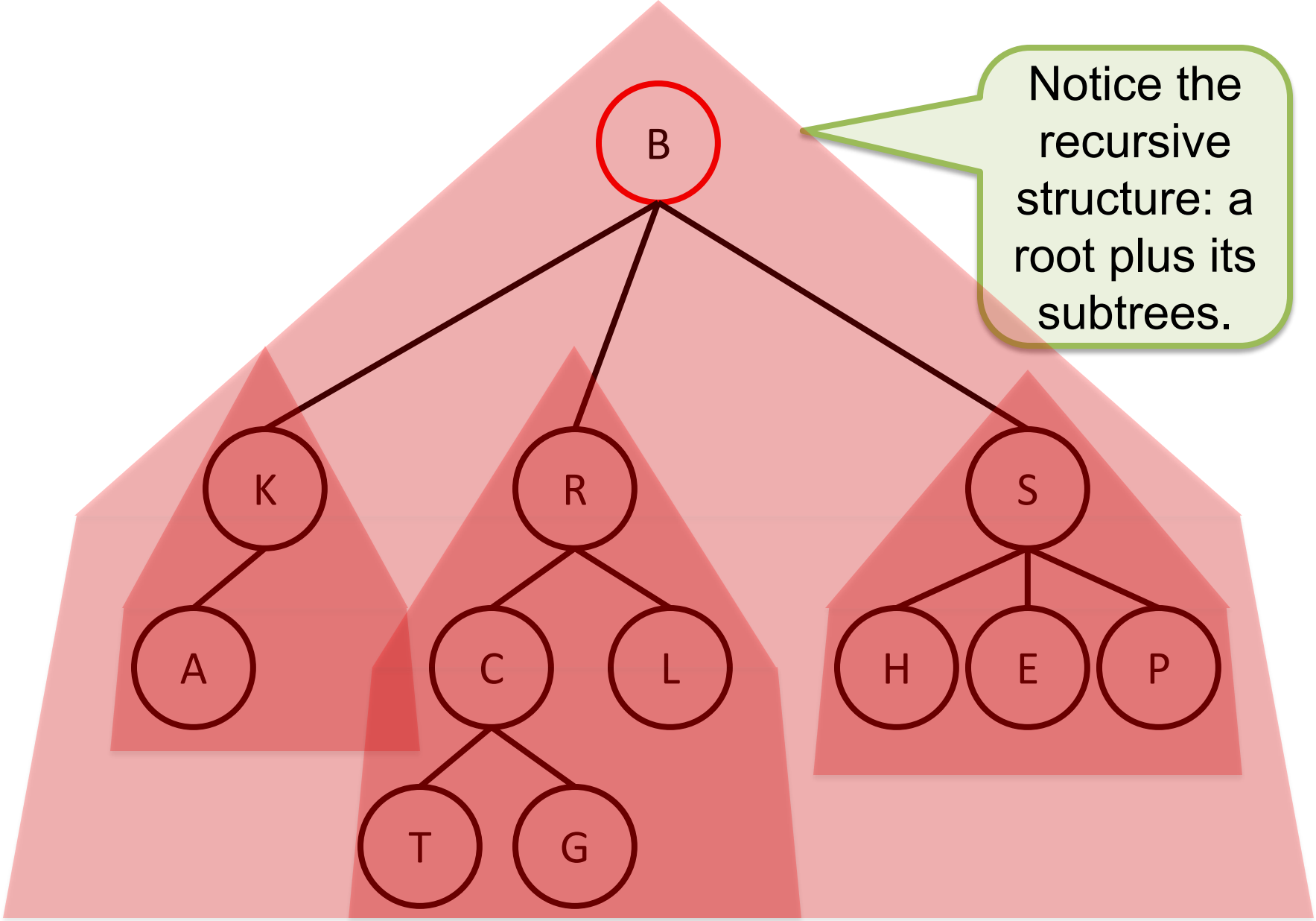

What's the ***height*** of this tree?


B
K
R
S
A
C
L
H
E
P
T
G
tree rooted at node B


subtree 0
of tree
rooted at
node B (or,
of node B)
B
K
R
S
A
C
L
H
E
P
T
G


subtree 1
of tree
rooted at
node B (or,
of node B)
B
K
R
S
A
C
L
H
E
P
T
G


subtree 2
of tree
rooted at
node B (or,
of node B)
B
K
R
S
A
C
L
H
E
P
T
G


B
K
R
S
A
C
L
H
E
P
T
G
Notice the recursive structure: a root plus its subtrees.

# Resources

- Wikipedia: Tree structure
  - http://en.wikipedia.org/wiki/Tree_structure