Math 255 - Homework 4

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Due in class, Friday April 19

Problem 1

Lohr textbook ch. 3 exercise 5.

(a) Selected scholars in American Council of Learned Societies in seven disciplines who answered to the survey.

(b)

```
> Nh <- c(9100, 1950, 5500, 10850, 2100, 5500, 9000)
> N <- sum(Nh)
> p.hats <- c(0.37, 0.23, 0.23, 0.29, 0.19, 0.43, 0.41)
> phat.str.1 <- sum(Nh/N * p.hats)
> phat.str.1
[1] 0.3336591
>
> nh <- c(636, 451, 481, 611, 493, 575, 588)
> var.str <- sum((Nh/N)^2 * (1 - nh/Nh) * p.hats * (1 - p.hats)/(nh - 1))
> se.str.1 <- sqrt(var.str)
> se.str.1
[1] 0.007903364
```

$$\begin{split} \hat{p}_{str} &= \sum_{h=1}^{7} \frac{N_h}{N} \hat{p}_h = \frac{9100}{44000} \cdot 0.37 + \dots + \frac{9000}{44000} \cdot 0.41 = 0.3336591 \\ SE[\hat{p}_{str}] &= \sqrt{\sum_{h=1}^{7} (\frac{N_h}{N})^2 (1 - \frac{n_h}{N_h}) \frac{\hat{p}(1 - \hat{p})}{n_h - 1}} \\ &= \sqrt{(\frac{9100}{44000})^2 (1 - \frac{636}{9100}) \frac{0.37(1 - 0.37)}{635} + \dots + (\frac{9000}{44000})^2 (1 - \frac{588}{9000}) \frac{0.41(1 - 0.41)}{587}} = 0.0079034 \end{split}$$

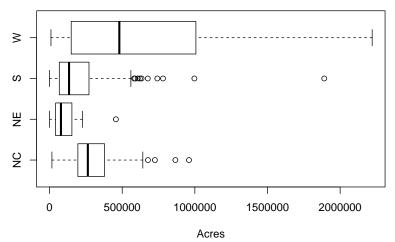
Problem 2

Lohr textbook ch. 3 exercise 9.

```
> agstrat$N <- recode(agstrat$region, NC = 1054, NE = 220, S = 1382,
     W = 422
> agstrat %>% group_by(region) %>% summarize(min(N), max(N))
# A tibble: 4 x 3
  region `min(N)` `max(N)`
  <fct>
          <dbl>
                     <dbl>
1 NC
            1054
                     1054
2 NE
             220
                      220
3 S
            1382
                      1382
              422
                      422
4 W
> agstrat <- agstrat %>% group_by(region) %>% mutate(n = n())
> agstrat %>% group_by(region) %>% summarize(min(n), max(n)) # check
# A tibble: 4 x 3
```

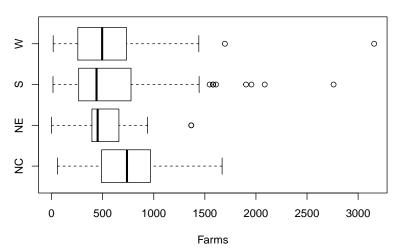
```
region `min(n)` `max(n)`
  <fct>
            <dbl>
                     <dbl>
1 NC
              103
                       103
2 NE
               21
                        21
3 S
              135
                       135
4 W
               41
                        41
> agstrat$wts <- agstrat$N/agstrat$n
> agstrat %>% group_by(region) %>% summarize(min(wts), max(wts)) #check
# A tibble: 4 x 3
  region `min(wts) `max(wts)`
  <fct>
              <dbl>
                         <dbl>
1 NC
               10.2
                          10.2
2 NE
               10.5
                          10.5
3 S
               10.2
                          10.2
4 W
               10.3
                          10.3
> design.strat <- svydesign(id = ~1, fpc = ~N, weights = ~wts,</pre>
      strata = ~region, data = agstrat)
(a)
> boxplot(acres87 ~ region, data = agstrat, horizontal = TRUE,
+ main = "Number of acres devoted to farms, 1987", xlab = "Acres")
```

Number of acres devoted to farms, 1987

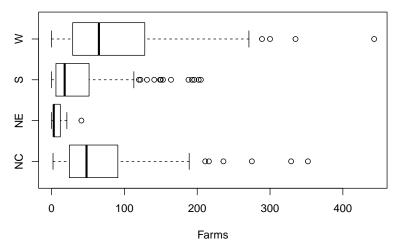


```
> boxplot(farms92 ~ region, data = agstrat, horizontal = TRUE,
+ main = "Number of farms, 1992", xlab = "Farms")
```

Number of farms, 1992

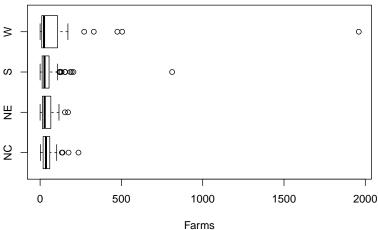


Number of farms with 1000 acres or more, 1992



```
> confint(svymean(~largef92, design.strat), df = degf(design.strat))  2.5 \% 97.5 \%  largef92 49.69636 63.69954  \bar{y}_{ssr} = 56.593 < \bar{y}_{str} = 56.698 \text{ (approximately the same)}   SE[\bar{y}_{ssr}] = 3.9904 > SE[\bar{y}_{str}] = 3.5577   CI_{ssr} = (48.77239, 64.41428) \text{ is wider than } CI_{str} = (49.69636, 63.69954).  (d)  > \text{boxplot(smallf92 ~ region, data = agstrat, horizontal = TRUE, }    + \text{main = "Number of farms with 9 acres or fewer, 1992", xlab = "Farms") }
```

Number of farms with 9 acres or fewer, 1992



```
> svymean(~largef92, design.strat)
            mean
largef92 56.698 3.5577
> confint(svymean(~largef92, design.strat), df = degf(design.strat))
             2.5 %
                     97.5 %
largef92 49.69636 63.69954
> svymean(~smallf92, design.strat)
                      SE
            mean
smallf92 56.863 7.2014
> confint(svymean(~smallf92, design.strat), df = degf(design.strat))
             2.5 % 97.5 %
smallf92 42.69033 71.03526
\bar{y}_{ssr} = 46.823 < \bar{y}_{str} = 56.863
SE[\bar{y}_{ssr}] = 3.6375 < SE[\bar{y}_{str}] = 7.2014
CI_{ssr} = (39.69387, 53.95279) is narrower than CI_{str} = (42.69033, 71.03526).
```

Problem 3

Lohr textbook ch. 3 exercise 15.

(a)

Advantage: proportional allocation provides the most precise result when within variance of all strata are similar.

Disadvantage: If the within variance varies on strata, proportional allocation is not the best way to produce the most precise result (optimal allocation is better than proportional allocation in this case).

(b)

$$\begin{split} \bar{y}_U &= \sum_{h=low}^{upper} \frac{N_h}{N} \bar{y}_h = \frac{190}{1408} \cdot 3.925 + \frac{407}{1408} \cdot 3.938 + \frac{811}{1408} \cdot 3.942 = 3.9385497 \\ SE[\bar{y}_U] &= \sqrt{\sum_{h=low}^{upper} (\frac{N_h}{N})^2 (1 - \frac{n_h}{N_h}) \frac{s_h^2}{n_h}} \\ &= \sqrt{(\frac{190}{1408})^2 \cdot (1 - \frac{21}{190}) \frac{0.037^2}{21} + (\frac{407}{1408})^2 \cdot (1 - \frac{14}{407}) \frac{0.053^2}{14} + (\frac{811}{1408})^2 \cdot (1 - \frac{22}{811}) \frac{0.070^2}{22}} \\ 95\% \text{ CI: } \bar{y}_U \pm q t_{0.975, df=57-3} \cdot SE[\bar{y}_U] = (3.9196859, 3.9574136) \end{split}$$

(c)

We can answer this answer by estimating the difference in means of log prices among the three strata.

• Low v. Middle

```
> diff.1 <- 3.925 - 3.938
>
> se.1 <- sqrt(sum((1 - nh[1:2]/Nh[1:2]) * sh[1:2]^2/nh[1:2]))
> se.1
[1] 0.01563599
>
> ci.3.1 <- diff.1 - se.1 * qnorm(c(0.975, 0.025)) # what df should I use?</pre>
```

$$\bar{y}_{low} - \bar{y}_{middle} \pm qt_{0.975, df=} \cdot \sqrt{Var[\bar{y}_{low}] + Var[\bar{y}_{middle}]} = -0.013 \pm 1.96 \cdot \sqrt{0.000057985 + 0.000185}$$

95% CI: (-0.043646, 0.017646)

We are 95% confident that the log price of low income stratum is -0.043646 to 0.017646 higher than middle income stratum in average. We can't conclude that the log price of low income stratum is different from that of middle income stratum.

• Middle v. Upper

```
> diff.2 <- 3.938 - 3.942
>
> se.2 <- sqrt(sum((1 - nh[2:3]/Nh[2:3]) * sh[2:3]^2/nh[2:3]))
> se.2
[1] 0.02007945
```

```
> ci.3.2 <- diff.2 - se.2 * qnorm(c(0.975, 0.025)) # what df should I use?
> ci.3.2
[1] -0.04335501 0.03535501
```

$$\bar{y}_{middle} - \bar{y}_{upper} \pm q t_{0.975, \ df=} \cdot \sqrt{Var[\bar{y}_{middle}] + Var[\bar{y}_{upper}]} = -0.004 \pm 1.96 \cdot \sqrt{0.0001864991 + 0.0002166853}$$

95% CI: (-0.043355, 0.035355)

We are 95% confident that the log price of middle income stratum is -0.043355 to 0.035355 higher than upper income stratum in average. We can't conclude that the log price of middle income stratum is different from that of upper income stratum.

• High v. low

$$\bar{y}_{low} - \bar{y}_{upper} \pm q t_{0.975, \ df=} \cdot \sqrt{Var[\bar{y}_{low}] + Var[\bar{y}_{upper}]} = -0.017 \pm 1.96 \cdot \sqrt{0.000057985 + 0.0002166853}$$

95% CI: (-0.0494829, 0.0154829)

We are 95% confident that the log price of low income stratum is -0.0494829 to 0.0154829 higher than upper income stratum in average. We can't conclude that the log price of low income stratum is different from that of upper income stratum.

Problem 4

Lohr textbook ch. 3 exercise 16. (Data in SDaA.)

```
> otters$N <- recode(otters$habitat, `1` = 89, `2` = 61, `3` = 40,
      ^{4} = 47
> otters %>% group_by(habitat) %>% summarize(min(N), max(N))
# A tibble: 4 x 3
  habitat `min(N)` `max(N)`
    <int>
             <dbl>
                      <dbl>
                          89
1
        1
                89
2
        2
                61
                          61
3
        3
                40
                          40
4
        4
                47
                          47
> otters <- otters %>% group by(habitat) %>% mutate(n = n())
> otters %>% group_by(habitat) %>% summarize(min(n), max(n)) # check
# A tibble: 4 x 3
```

```
habitat `min(n) `max(n)`
    <int>
              <dbl>
                        <dbl>
                 19
                           19
1
        1
2
         2
                 20
                           20
3
         3
                 22
                           22
4
         4
                 21
                           21
> otters$wts <- otters$N/otters$n
> otters %>% group_by(habitat) %>% summarize(min(wts), max(wts)) #check
# A tibble: 4 x 3
  habitat `min(wts)` `max(wts)`
    <int>
                <dbl>
                             <dbl>
1
         1
                 4.68
                             4.68
2
         2
                 3.05
                             3.05
3
        3
                 1.82
                             1.82
4
         4
                 2.24
                             2.24
> design4.strat <- svydesign(id = ~1, fpc = ~N, weights = ~wts,
      strata = ~habitat, data = otters)
(a)
> svytotal(~holts, design4.strat)
       total
holts 984.71 73.921
\hat{t}_{str} = 984.71
SE[\hat{t}_{str}] = 73.921
```

The study area is divided into stratum based on the predominant terrain type. In other words, some of the sections may exhibit characteristics of more than one classification (for example we can see Cliffs and Agriculture in certain section). So classifying such sections heavily relies on the researchers' judgement, possibly resulting in selection bias. Also, some of the dens can be either abandoned or belonged to other animals, implying that the study is not also free from measurement error.

Problem 5

(a) Households that are not listed in the county's telephone number have no chance to be sampled, so it is not free from selection bias. Also, the survey is not free from nonresponse issue as well.

(b)

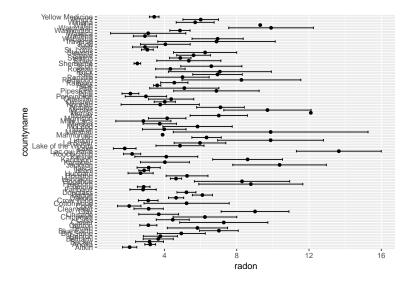
(b)

```
> radon <- read.csv("http://math.carleton.edu/kstclair/data/radon.csv")
> options(survey.lonely.psu = "remove")
> radon$wts <- radon$popsize/radon$sampsize
> head(radon %>% group_by(countyname) %>% summarize(min(wts), max(wts))) #check
# A tibble: 6 x 3
  countyname `min(wts)` `max(wts)`
  <fct>
                  <dbl>
                              <dbl>
1 Aitkin
                  1350
                              1350
2 Anoka
                  1261.
                              1261.
3 Becker
                  2750
                              2750
4 Beltrami
                  1643.
                              1643.
```

```
5 Benton
                   2375
                              2375
6 Big Stone
                   967.
                               967.
> design5.strat <- svydesign(id = ~1, fpc = ~popsize, weights = ~wts,</pre>
      strata = ~countyname, data = radon)
> svymean(~radon, design5.strat)
        mean
radon 4.8986 0.1543
\hat{\bar{y}}_{str} = 4.8986 \text{ pCi/L}
SE[\hat{y}_{str}] = 0.1543 \text{ pCi/L}
(c)
> htf4radon <- ifelse(radon$radon >= 4, 1, 0)
> update(design5.strat, htf4radon = htf4radon)
Stratified Independent Sampling design
update(design5.strat, htf4radon = htf4radon)
> svytotal(~htf4radon, design5.strat)
           total
                    SE
htf4radon 722781 28101
> confint(svytotal(~htf4radon, design5.strat), df = degf(design5.strat))
             2.5 % 97.5 %
htf4radon 667632.1 777930.4
t_{radon>4pCi/L} = 722781 households
95% CI: (667632.1, 777930.4) households
(d)
> head(radon %>% group_by(countyname) %>% arrange(sampsize))
# A tibble: 6 x 6
# Groups: countyname [5]
  countynum countyname sampsize popsize radon
      <int> <fct>
                   <int> <int> <dbl> <dbl>
         43 Mahnomen
                             1
                                   1600 3.9 1600
1
                                   3900 12.1 3900
2
         51 Murray
                              1
3
                                          9.3 2800
         84 Wilkin
                              1
                                   2800
4
         16 Cook
                               2
                                    1800
                                           2.7
                                                 900
                               2
5
         16 Cook
                                    1800
                                           1.4
                                                 900
                               2 7900 3.2 3950
         23 Fillmore
```

Mahomen, Murray, and Wilkin county only have 1 home sampled.

(e)



(f)

```
> head(svyby.out %>% arrange(se))
       countyname
                      radon
         Mahnomen 3.900000 0.0000000
1
2
           Murray 12.100000 0.0000000
           Wilkin 9.300000 0.0000000
3
4
        Sherburne
                  2.500000 0.1855169
5
             Pope
                  3.600000 0.1999565
6 Yellow Medicine 3.433333 0.2665797
> head(svyby.out %>% arrange(desc(se)))
  countyname
                radon
    Marshall 9.877778 5.389193
2
     Redwood 8.260000 3.288240
3
    Traverse 6.880000 3.249807
4
    Lincoln 9.875000 2.930659
    Freeborn 8.780000 2.879234
6
        Rock 7.066667 2.849216
```

The county with the largest SE for estimating the mean randon level of homes is Marshall county. The county with the smallest SE for estimating the mean randon level of homes is Sherburne county.

```
> radon %>% filter(countyname %in% c("Marshall", "Sherburne")) %>%
      group_by(countyname) %>% summarize(n = n(), sd = sd(radon))
# A tibble: 2 x 3
  countyname
                       sd
  <fct>
             <int>
                    <dbl>
1 Marshall
                 9 16.2
2 Sherburne
                 9 0.557
> 16.2/0.557
[1] 29.08438
> 5.389193/0.1855169
[1] 29.04961
```

$$SE[\hat{\bar{y}}] = \sqrt{(1 - \frac{n_h}{N_h})} \frac{s_h}{\sqrt{n_h}}$$

SE is proportional to sample standard deviation and inversely proportional to square root of sample size. 9 homes are sampled from both counties, but the sample standard deviation of Marshall county is almost 30 times that of Sherburne county, which is close to the ratio of standard errors between the two counties (let's ignore fpc as population size is big). In sum, significant difference in sample standard deviation between the two counties (while the number of the sample collected are the same) is biggest reason why the SE's of these counties are either biggest or smallest.

Problem 6

Revisit problem 2 above. Compute and interpret the design effect using the survey package for each of four estimates computed for exercise 9. Which estimate has the smallest DEff and which has the largest? Use the EDA (graphs) you produced for problem 2 to explain why these variables have the smallest and largest DEff.

```
> svymean(~acres87, design.strat, deff = T)
                        DEff
          mean
                   SE
acres87 298547 16293 0.8008
> svymean(~farms92, design.strat, deff = T)
           mean
                     SE
                          DEff
farms92 637.164 24.278 0.9751
> svymean(~largef92, design.strat, deff = T)
                      SE DEff
            mean
                  3.5577 0.865
largef92 56.6980
> svymean(~smallf92, design.strat, deff = T)
                      SE
                           DEff
            mean
smallf92 56.8628 7.2014 0.9789
```

$$Var(\hat{t}_{str}) = (1 - \frac{n}{N}) \frac{N}{n} (SSW + \sum_{h} s_{h}^{2})$$

$$Var(\hat{t}_{ssr}) = (1 - \frac{n}{N}) N^{2} \frac{SSB + SSW}{n(N-1)}$$

$$DEff = \frac{Var(\hat{t}_{str})}{Var(\hat{t}_{ssr})} = \frac{(1 - \frac{n}{N}) \frac{N}{n} (SSW + \sum_{h} s_{h}^{2})}{(1 - \frac{n}{N}) N^{2} \frac{SSB + SSW}{n(N-1)}} = \frac{SSW + \sum_{h} s_{h}^{2}}{\frac{N(SSB + SSW)}{N-1}} = \frac{SSW + \sum_{h} s_{h}^{2}}{NS^{2}}$$

The equation above suggests that DEff gets bigger as the proportion of SSW + $\sum_h s_h^2$ over SST (approximately, as SST = $(N-1)S^2$) gets bigger. In other words, DEff gets larger if the relative portion of SSB in SST gets smaller (SSW + SSB = SST).

acres87 has the smallest DEff, and smallf92 has the largest DEff. EDB illustrates that the relatitve portion of SSB on SST in the case of acres87 is big because we can observe huge difference in values among different strata; however, relative portion of SSB on SST is small in smallf92 as we only see small difference in values among different strata.

```
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

> smallf.aov <- aov(smallf92 ~ region, data = agstrat)

> anova(smallf.aov)
Analysis of Variance Table

Response: smallf92

Df Sum Sq Mean Sq F value Pr(>F)
region 3 217384 72461 4.2593 0.005769 **
Residuals 296 5035679 17012

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The ANOVA output also supports the claim that the relative size of SSB to SST in acres87 case is significantly bigger than smallf92 case. Approximately, SSB/SST in acres87 case is $\frac{6.9528e + 12}{(6.9528e + 12) + (2.5937e + 13)} = 0.2113968$ but only $\frac{217384}{217384 + 5035679} = 0.04138233$ in smallf92 case.

Problem 7

Lohr textbook ch. 3 exercise 35 parts (a)-(d).

(a)

```
> pop <- read.csv("http://math.carleton.edu/kstclair/data/baseball.csv",
     header = FALSE, na.strings = c("NA", " ", "."))
> names(pop) <- c("team", "league", "player", "salary", "POS",
      "G", "GS", "InnOuts", "PO", "A", "E", "DP", "PB", "GB", "AB",
      "R", "H", "SecB", "ThiB", "HR", "RBI", "SB", "CS", "BB",
      "SO", "IBB", "HPB", "SH", "SF", "GIDP")
> table(pop$team) # all roughly the same size populations
ANA ARI ATL BAL BOS CHA CHN CIN CLE COL DET FLO HOU KCA LAN MIL MIN MON
26 28 28 25 27 26 29 27 28 27 26 26
                                                25 27 24 25 25 28
NYA NYN OAK PHI PIT SDN SEA SFN SLN TBA TEX TOR
29 26 27 25 27 26 27 28 26 26 27 26
> pop$logsal <- log(pop$salary)</pre>
> pop$N <- recode(pop$team, ANA = 26, ARI = 28, ATL = 28, BAL = 25,
     BOS = 27, CHA = 26, CHN = 29, CIN = 27, CLE = 28, COL = 27,
     DET = 26, FLO = 26, HOU = 25, KCA = 27, LAN = 24, MIL = 25,
     MIN = 25, MON = 28, NYA = 29, NYN = 26, OAK = 27, PHI = 25,
     PIT = 27, SDN = 26, SEA = 27, SFN = 28, SLN = 26, TBA = 26,
     TEX = 27, TOR = 26)
> head(pop %>% group_by(team) %>% summarize(min(N), max(N)))
# A tibble: 6 x 3
 team `min(N)` `max(N)`
  <fct>
          <dbl>
                    <dbl>
1 ANA
             26
                       26
2 ARI
              28
                       28
3 ATL
             28
                       28
```

```
4 BAL
5 BOS
             27
                      27
6 CHA
             26
                      26
> set.seed(30) # put your favorite large integer here
> baseball.strat <- pop %>% group_by(team) %>% sample_n(size = 5) %>%
      ungroup()
> str(baseball.strat)
Classes 'tbl_df', 'tbl' and 'data.frame':
                                           150 obs. of 32 variables:
        : Factor w/ 30 levels "ANA", "ARI", "ATL", ...: 1 1 1 1 1 2 2 2 2 2 ...
 $ league : Factor w/ 2 levels "AL", "NL": 1 1 1 1 1 2 2 2 2 2 ...
 $ player : Factor w/ 791 levels "aardsda0","abbotpa0",..: 154 291 252 270 198 118 669 125 725 115 ...
 $ salary : int 375000 575000 9900000 301500 5750000 335000 500000 2750000 325000 325750 ...
         : Factor w/ 9 levels "1B", "2B", "3B", ...: 5 3 3 7 7 9 7 1 7 7 ...
         : int 108 46 58 5 1 154 27 20 17 69 ...
 $ G
$ GS
         : int 27 22 19 0 33 125 18 1 0 0 ...
 $ InnOuts: int 743 640 495 263 625 3297 362 27 54 152 ...
                75 26 11 2 16 141 8 7 0 3 ...
         : int
         : int 1 46 27 5 24 383 21 1 4 10 ...
 $ A
         : int 0 10 2 0 0 15 2 0 0 0 ...
 $ E
         : int 1 2 2 1 1 61 0 0 0 1 ...
$ DP
 $ PB
         : int NA NA NA NA NA NA NA NA NA ...
 $ GB
         : int 108 46 58 5 1 154 27 20 17 69 ...
 $ AB
         : int 285 114 207 0 2 564 31 27 0 1 ...
 $ R
         : int 41 10 47 0 0 56 0 1 0 0 ...
 $ H
         : int 79 23 52 0 0 148 4 3 0 0 ...
 $ SecB
        : int 11 5 11 0 0 31 0 0 0 0 ...
$ ThiB
        : int 4010070000...
                7 4 18 0 0 4 0 0 0 0 ...
$ HR
         : int
         : int 34 13 42 0 0 49 1 1 0 0 ...
 $ RBI
$ SB
         : int 18 1 2 0 0 3 0 0 0 0 ...
         : int 3 1 3 0 0 3 0 0 0 0 ...
$ CS
 $ BB
         : int 46 7 31 0 0 31 0 1 0 0 ...
 $ SO
         : int 54 30 52 0 0 59 14 5 0 1 ...
         : int 2 0 3 0 0 2 0 0 0 0 ...
         : int 0030020000...
$ HPB
 $ SH
                1 0 0 0 0 12 1 0 0 0 ...
         : int
$ SF
         : int 5 0 1 0 0 4 0 0 0 0 ...
        : int 2 3 6 0 0 11 0 0 0 0 ...
$ GIDP
$ logsal : num
                12.8 13.3 16.1 12.6 15.6
       : num 26 26 26 26 26 28 28 28 28 28 ...
```

I used 5 samples from each strata using SRS (number of sample is constant (5) because population size is roughly same across the strata).

(b)

```
> confint(svymean(~logsal, design7.strat), df = degf(design7.strat))
          2.5 %
                 97.5 %
logsal 13.70935 14.05757
\hat{logsal} = 13.883
95% CI: (13.70935, 14.05757)
(c)
> pitcher <- ifelse(baseball.strat$POS == "P", 1, 0)
> update(design7.strat, pitcher = pitcher)
Stratified Independent Sampling design
update(design7.strat, pitcher = pitcher)
> svymean(~pitcher, design7.strat)
           mean
                    SE
pitcher 0.44918 0.0364
> confint(svymean(~pitcher, design7.strat), df = degf(design7.strat))
            2.5 %
                     97.5 %
pitcher 0.3771607 0.5212082
\hat{p}_{pitcher} = 0.44918
95% CI: (0.3771607, 0.5212082)
(d)
> knitr::kable(data.frame(logsal = c("Mean", "SE", "CI"), ssr = c(13.982,
+ 0.095, "(13.79421 14.16963)"), str = c(13.883, 0.0879, "(13.70935, 14.05757)")))
```

logsal	ssr	str
Mean	13.982	13.883
SE	0.095	0.0879
CI	(13.79421 14.16963)	(13.70935, 14.05757)

```
> knitr::kable(data.frame(pitcher = c("Mean", "SE", "CI"), ssr = c(0.493333,
+ 0.0369, "(0.420412591, 0.56625408)"), str = c(0.44918, 0.0364,
+ "(0.3771607, 0.5212082)")))
```

pitcher	ssr	str
Mean SE	0.493333 0.0369	0.44918 0.0364
CI	(0.420412591, 0.56625408)	(0.3771607, 0.5212082)

The estimates from stratified sampling is smaller than those from SSR. Also the SE's are smaller than SSR. So, the CI's from stratified sampling is narrower than those from SSR.