Math 255 - Homework 9

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Due in class, Wednesday May 29

Problem 1

$\pi_B = \pi_{BA} + \pi_{BC} + \pi_{BD} = [P(A_1)P(B_2|A_1) + P(B_1)P(A_2|B_1)] + [P(C_1)P(B_2|C_1) + P(B_1)P(C_2|B_1)] + [P(D_1)P(B_2|D_1) + P(B_1)P(D_2|B_1)] = \frac{1}{16}\frac{2}{16-1} + \frac{2}{16}\frac{1}{16-2} + \frac{3}{16}\frac{2}{16-3} + \frac{2}{16}\frac{3}{16-2} + \frac{10}{16}\frac{2}{16-10} + \frac{2}{16}\frac{10}{16-2} = 0.3705$

$\pi_D = \pi_{DA} + \pi_{DB} + \pi_{DC} = [P(A_1)P(D_2|A_1) + P(D_1)P(A_2|D_1)] + [P(B_1)P(D_2|B_1) + P(D_1)P(B_2|D_1)] + [P(C_1)P(D_2|C_1) + P(D_1)P(C_2|D_1)] = \frac{1}{16}\frac{10}{16-1} + \frac{10}{16}\frac{1}{16-10} + \frac{2}{16}\frac{10}{16-2} + \frac{10}{16}\frac{2}{16-10} + \frac{3}{16}\frac{10}{16-3} + \frac{10}{16}\frac{3}{16-10} = 0.9002$

$\pi_{BD} = P(D_1)P(B_2|D_1) + P(B_1)P(D_2|B_1) = \frac{10}{16}\frac{2}{16-10} + \frac{2}{16}\frac{10}{16-2} = 0.2976$

$t_B = 20$, $t_D = 245$

$\hat{t}_{HT} = \sum_{i \in S} \frac{t_i}{\pi_i} = \frac{20}{0.3705} + \frac{245}{0.9002} = 326.1428484$

$\hat{V}(\hat{t}_{HT}) = \frac{(1-0.9002)(245)^2}{0.9002^2} + \frac{(1-0.3705)(20)^2}{0.3705^2} + 2\frac{0.2976-(0.9002)(0.3705)}{0.2976}(\frac{245}{0.9002})(\frac{20}{0.3705}) = 5679.8011849$

$\hat{V}_{SYG}(\hat{t}_{HT}) = \frac{(0.9002)(0.3705)-0.2976}{0.2976}(\frac{245}{0.9002} - \frac{20}{0.3705})^2 = 5746.2613352$

$\hat{V}_{SYG}(\hat{t}_{HT})$ is slightly bigger than $\hat{V}(\hat{t}_{HT})$

Problem 2

Lohr textbook ch. 6 exercise 24(a).

$\pi_1 = \pi_{12} + \pi_{13} + \pi_{14} = 0.31 + 0.20 + 0.14 = 0.65$

$\pi_2 = \pi_{21} + \pi_{23} + \pi_{24} = 0.31 + 0.03 + 0.01 = 0.35$

$\pi_3 = \pi_{31} + \pi_{32} + \pi_{34} = 0.20 + 0.03 + 0.31 = 0.54$

$\pi_4 = \pi_{41} + \pi_{42} + \pi_{43} = 0.14 + 0.01 + 0.31 = 0.46$

Problem 3

```
> agpps <- read.csv("http://math.carleton.edu/kstclair/data/agpps.csv")
>
> probs <- as.matrix(agpps[, 10:24])
> diag(probs) <- agpps$pii
>
> agpps.design <- svydesign(id = ~1, fpc = ~pii, data = agpps,
+     pps = ppsmat(probs))
```

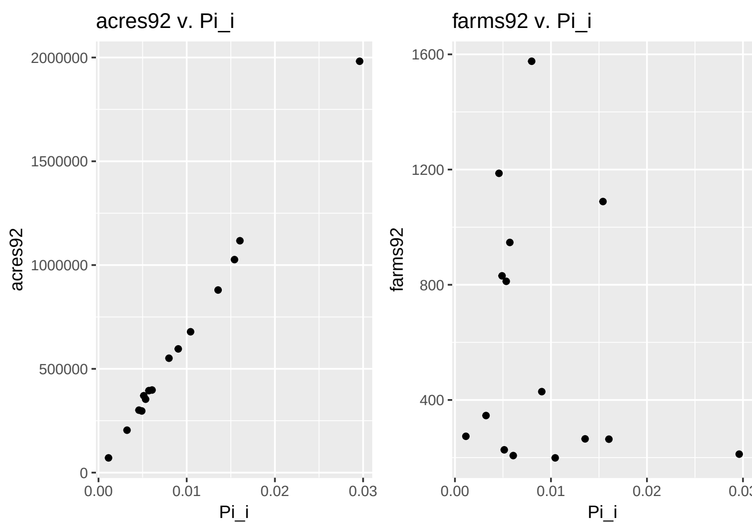
(a)

```
> svytotal(~farms92, agpps.design)
          total     SE
farms92 1549517 354030
```

$\hat{t}_{HT} = 1549517$, $SE[\hat{t}_{HT}] = 354030$

(b)

```
> agsrs$n <- nrow(agsrs)
> agsrs$N <- 3078
> agsrs$wts <- agsrs$N/agsrs$n
> agsrs.design <- svydesign(id = ~1, fpc = ~N, weights = ~wts,
+     data = agsrs)
> svytotal(~farms92, agsrs.design)
          total    SE
farms92 1843907 67908
>
> acres <- ggplot(agpps) + geom_point(aes(y = acres92, x = pii)) +
+     labs(x = "Pi_i", title = "acres92 v. Pi_i")
>
> farms <- ggplot(agpps) + geom_point(aes(y = farms92, x = pii)) +
+     labs(x = "Pi_i", title = "farms92 v. Pi_i")
>
> grid.arrange(acres, farms, ncol = 2)
```




```
>
> cor(agpps$acres92, agpps$pii)
[1] 0.9991585
> cor(agpps$farms92, agpps$pii)
[1] -0.2041094
```

$SE[\hat{t}_{srs}] = 67908 < SE[\hat{t}_{HT}] = 354030$
When π_i is positively related with the response variable (t_i), PPS can enhance the precision relative to SRS (in that π_i is constructed based on `acres87`, and as `acres87` is positively associated with `acres92`, pps reduces the variability in a similar mechanism as the ratio estimates using auxiliary variable). The scatterplot suggests that there is a strong positive correlation (r = 0.9991585) between π_i and `acres92` but not with `farms92` (r = -0.2041094). PPS estimate may produce imprecision when π_i is small. In the case of `acres92`, PPS estimate enhances the precision relative to SRS because the precision gained from the positive association between π_i and t_i surpasses the imprecision caused from small π_i, whereas pps estimate of `farms92` rather produces more imprecise result due to the absence of a strong positve relationship between π_i and t_i.

(e)

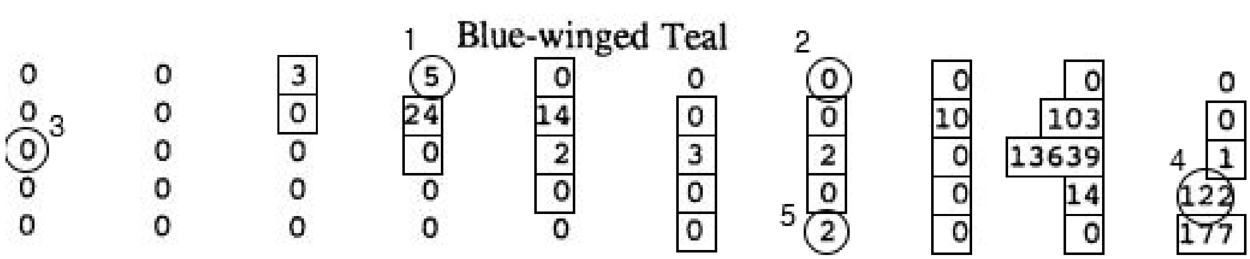
```
> probs.wolf <- matrix(c(0.8998871, 0.899871, 0.5075547, 0.899871,
+     0.9802246, 0.5715625, 0.5075547, 0.5715625, 0.5904), nrow = 3,
+     byrow = T)
> pi.wolf <- c(0.8998871, 0.9802246, 0.5904)
> y.wolf <- c(1, 2, 2)
> wolves <- data.frame(y.wolf, pi.wolf, probs.wolf)
> wolves
  y.wolf   pi.wolf        X1        X2        X3
1      1 0.8998871 0.8998871 0.8998710 0.5075547
2      2 0.9802246 0.8998710 0.9802246 0.5715625
3      2 0.5904000 0.5075547 0.5715625 0.5904000
> wolves.design <- svydesign(id = ~1, fpc = ~pi.wolf, data = wolves,
+     pps = ppsmat(probs.wolf))
> svytotal(~y.wolf, wolves.design)
        total     SE
y.wolf 6.5391 2.1144
```

$SE[\hat{t}_{HT}] = 2.1144$

Problem 5

(a)

```
> set.seed(70)
> initial <- sample(1:50, 5, replace = F)
> initial
[1]  4 47 31  7 40
>
> include_graphics("BlueWing.png")
```



Network 1: {3,4,14,15,25,26,27}
Network 2: {7}
Network 3: {11}
Network 4: {18,19,29,30,39,40,50}
Network 5: {47}

(b)

$y_1^* = 3 + 5 + 24 + 14 + 2 + 3 + 2 = 53,\ x_1^* = 7$
$y_2^* = 0,\ x_2^* = 1$
$y_3^* = 0,\ x_3^* = 1$
$y_4^* = 10 + 103 + 13639 + 1 + 14 + 122 + 177 = 14066,\ x_4^* = 7$

$y_5^* = 2$, $x_5^* = 1$

```
> bluewinged_data <- data.frame(y_net = c(53, 0, 0, 14066, 2),
+     x_net = c(7, 1, 1, 7, 1))
>
> n1 <- 5
> N <- 50
> bluewinged_data$pi_single <- 1 - choose(N - bluewinged_data$x_net,
+     n1)/choose(N, n1)
>
> jnt_fun <- function(xj, x = bluewinged_data$x_net, N = 50, n1 = 5) {
+     1 - choose(N - xj, n1)/choose(N, n1) - choose(N - x, n1)/choose(N,
+         n1) + choose(N - xj - x, n1)/choose(N, n1)
+ }
>
> jnt_mat <- matrix(c(jnt_fun(bluewinged_data$x_net[1]), jnt_fun(bluewinged_data$x_net[2]),
+     jnt_fun(bluewinged_data$x_net[3]), jnt_fun(bluewinged_data$x_net[4]),
+     jnt_fun(bluewinged_data$x_net[5])), byrow = TRUE, nrow = 5)
>
> diag(jnt_mat) <- bluewinged_data$pi_single
>
> bluewinged_design <- svydesign(id = ~1, fpc = ~pi_single, pps = ppsmat(jnt_mat),
+     data = bluewinged_data)
>
> svytotal(~y_net, bluewinged_design)
      total    SE
y_net 25894 17355
> confint(svytotal(~y_net, bluewinged_design), df = 4)
          2.5 %   97.5 %
y_net -22290.54 74078.95
```

$SE[\hat{t}_{HT}] = 17355$
95% CI: (-22290.54, 74078.95)
True t = 14121
The interval contains the true t value.