

## Example

- Use D2Q9 incompressible model.
- Grid is  $32 \times 16$  where  $\Delta x = 1.0/16.0 = 0.0625$ .
- Viscosity is  $\nu = 1.0$ . Relaxation parameter is  $\omega = 0.8$ .
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- Pressure is  $p = 1.1$  at left edge.
- Pressure is  $p = 1.0$  at right edge.
- Velocity is  $v = 0.0$  at top edge.
- Velocity is  $v = 0.0$  at bottom edge.

## Initial Conditions

- Pressure is  $p = 1.05$  at all interior points.
- Velocity is  $v = 0.0$  at all interior points.
- Distribution  $g^1 = g^0$  at all interior points.
- Calculate equilibrium  $g^0$  from  $v$  and  $p$ .

$$\begin{aligned} e_i &= \langle x_i, y_i \rangle \\ s_i(v) &= \hat{w}_i \left( 3 \frac{e_i \cdot v}{c} + 4.5 \frac{(e_i \cdot v)^2}{c^2} - 1.5 \frac{v \cdot v}{c^2} \right) \\ g_i^0 &= \{-4\sigma, \lambda, \lambda, \lambda, \lambda, \gamma, \gamma, \gamma, \gamma\} \frac{p}{c^2} + s_i(v) \end{aligned}$$

## Parameters

- $\Delta x = 0.0625$
- $\nu = 1.0$
- $\omega = 0.8$
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- $\Delta t = \frac{(\Delta x)^2}{\nu} \cdot \frac{2-\omega}{6\omega}$
- $c = \Delta x / \Delta t$
- $\sigma = 5/12$
- $\lambda = 1/3$
- $\gamma = 1/12$

## Main Loop

1. Collision.
2. Boundary conditions.
3. Streaming.
4. Update flow field.

- Repeat until velocity stops changing:

$$\frac{\sum (v - v_{old})^2}{\sum v^2} < 10^{-6}$$

## Main Loop

### 1. Collision.

- Calculate  $g^0$  from  $v$  and  $p$ , then  $g^2$  from  $g^1$  and  $g^0$ .

### 2. Boundary conditions.

- Specify values in  $g^2$ .

### 3. Streaming.

- Update  $g^1$  from  $g^2$ .

### 4. Update flow field.

- Calculate  $v$  and  $p$  from  $g^1$ .

## Collision

- We already know  $g^1$  and  $v$  and  $p$ .
- Calculate  $g^0$  from  $v$  and  $p$ .

$$e_i = \langle x_i, y_i \rangle$$
$$s_i(v) = \hat{w}_i \left( 3 \frac{e_i \cdot v}{c} + 4.5 \frac{(e_i \cdot v)^2}{c^2} - 1.5 \frac{v \cdot v}{c^2} \right)$$
$$g_i^0 = \{-4\sigma, \lambda, \lambda, \lambda, \lambda, \gamma, \gamma, \gamma, \gamma\} \frac{p}{c^2} + s_i(v)$$

- Then  $g^2 = (1 - \omega) \cdot g^1 + \omega \cdot g^0$ .
  - If  $\omega > 1$  then this is an overrelaxation.

## Boundary Conditions (1)

- Assume no internal walls for now.
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- At left and right edge where pressure  $p$  is proscribed:
  - Set  $v$  from grid and  $p$  as specified.
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- At top and bottom edge where velocity  $v$  is proscribed:
  - Set  $v$  as specified and  $p$  from grid.
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- Regardless of direction go horizontal vertical only.

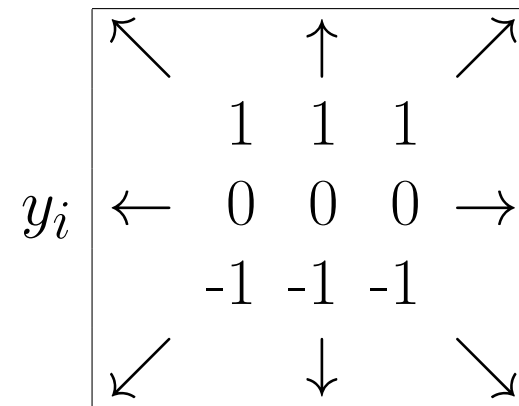
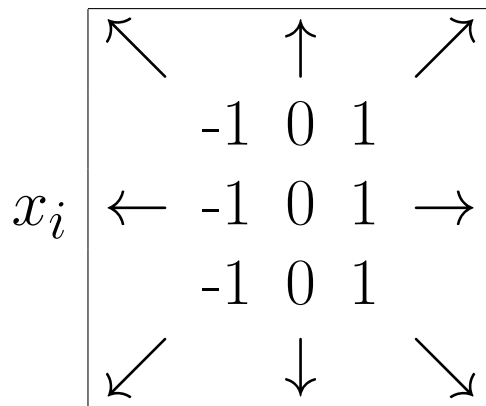
## Boundary Conditions (2)

- Using the identified  $v$  and  $p$ , then find  $g$ .
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- Set  $g^2 = g + (1 - \omega) \cdot (g^1 - g^0)$ .
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- Only required for directions that will stream *into* the field.
- 
- Trick
  - Put an extra row and column on each side of the grid.
  - Looping over actual field will access these extra cells.



## Streaming

- Set  $g^1$  from *directional* value of  $g^2$ , so that only  $g[0]$  stays put.
- Boundary cells only need values that stream *into* the field.



## Update Flow Field

- Velocity components sum over each  $g$ .

$$v_x = \sum_i (cx_i g_i)$$

$$v_y = \sum_i (cy_i g_i)$$

- Incompressible. Pressure also a sum.

$$p = \frac{c^2}{4\sigma} \cdot \left( \frac{1.5\hat{w}_0 (v \cdot v)}{c^2} + \sum_{i \neq 0} g_i \right)$$

## Collision (next loop)

- We now have new values for  $g^1$  and  $v$  and  $p$ .
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- So calculate a new equilibrium  $g^0$  from  $v$  and  $p$ .

$$e_i = \langle x_i, y_i \rangle$$

$$s_i(v) = \hat{w}_i \left( 3 \frac{e_i \cdot v}{c} + 4.5 \frac{(e_i \cdot v)^2}{c^2} - 1.5 \frac{v \cdot v}{c^2} \right)$$

$$g_i^0 = \{-4\sigma, \lambda, \lambda, \lambda, \lambda, \gamma, \gamma, \gamma, \gamma\} \frac{p}{c^2} + s_i(v)$$

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- Then a new  $g^2 = (1 - \omega) \cdot g^1 + \omega \cdot g^0$ .

## Visualize

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- Plot  $y$  versus  $v_x$  at horizontal midline, in Gnuplot: easy.
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- Plot  $|v|$  contours, in Matlab:

```
X=load('xfield.txt');  
Y=load('yfield.txt');  
V=load('vfield.txt');  
%  
contour(X,Y,V);  
colorbar;
```