machine learning conference

Deep State

Deep State Space Models for Time Series Forecasting

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Why Forecasting

Forecasting in a Nutshell

- Predict the future behaviour of a time series given its past

$$z_1, z_2, \ldots, z_T \implies P(z_{T+1}, z_{T+2}, \ldots z_{T+\tau})$$

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$$\text{best action} = \underset{a}{\operatorname{argmin}} \operatorname{cost}(a, z_{T+1}, z_{T+2}, \ldots z_{T+\tau})$$

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- If I knew the future ...

$$\text{best action} = \operatorname*{argmin}_a \text{cost}(a, z_{T+1}, z_{T+2}, \dots z_{T+\tau})$$

- If I have a calibrated forecast $P(z_{T+1}, z_{T+2}, \dots z_{T+\tau})$ ...

$$\text{best action} = \operatorname*{argmin}_a \mathrm{E}_{\mathrm{P}}[\text{cost}(a, z_{T+1}, z_{T+2}, \dots z_{T+\tau})]$$

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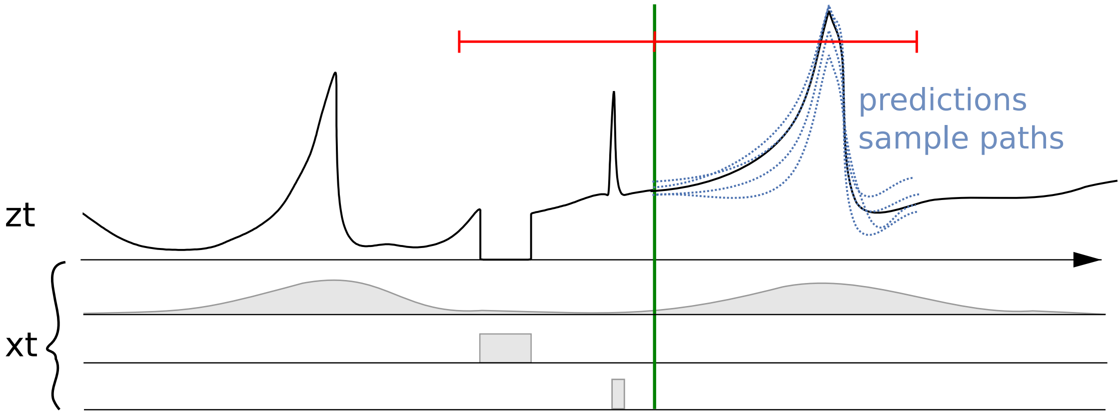
- Forecasts can be leveraged for optimising business problems in retail, AWS, logistics, professional services, etc.

General Setup



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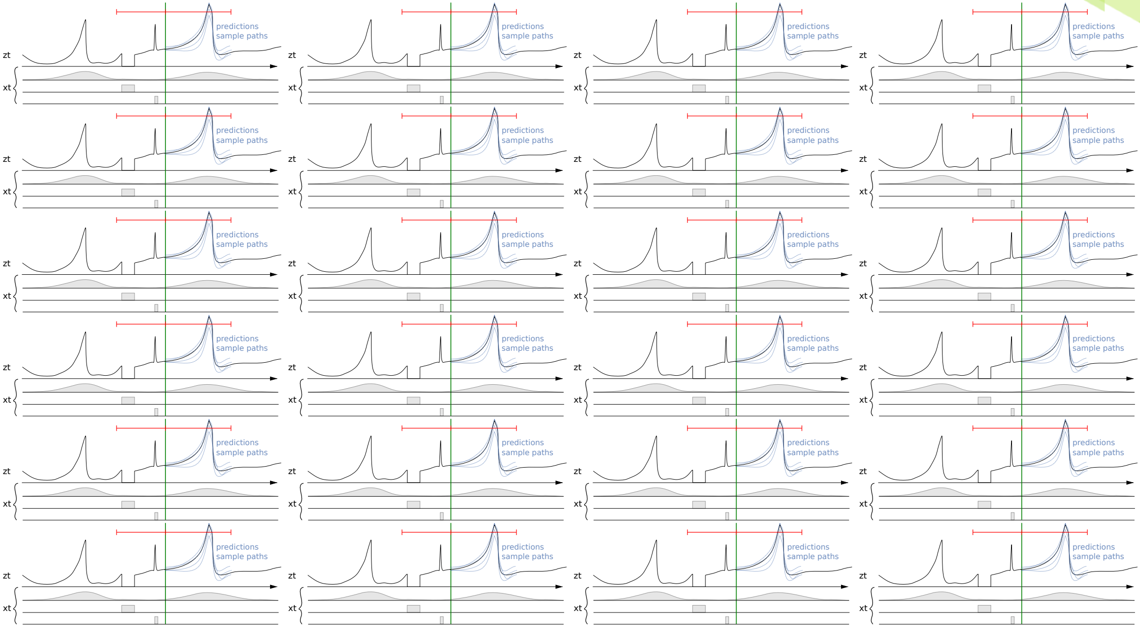
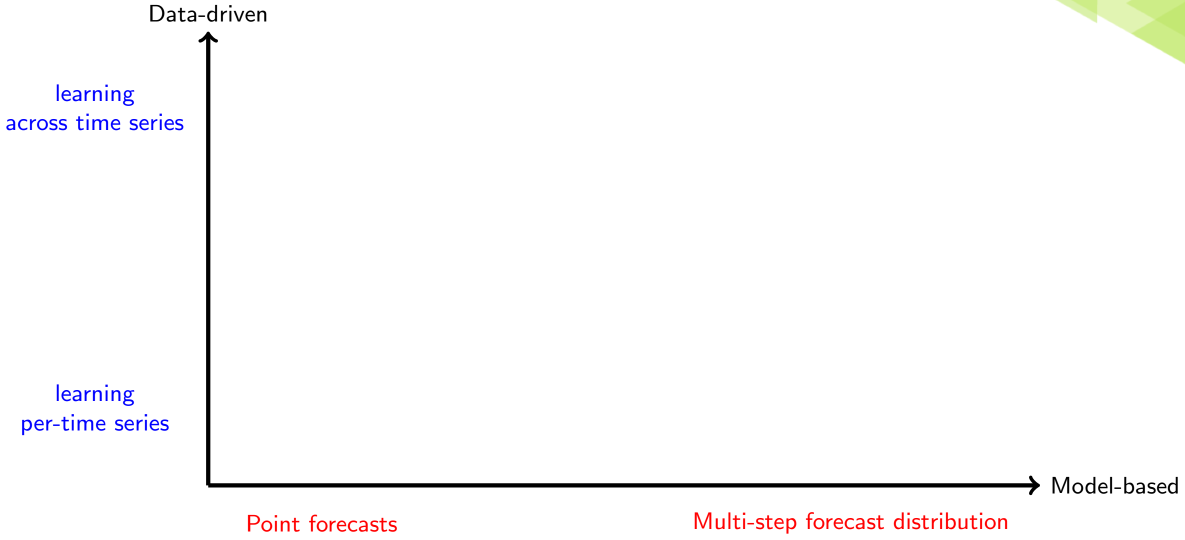


Forecasting Literature



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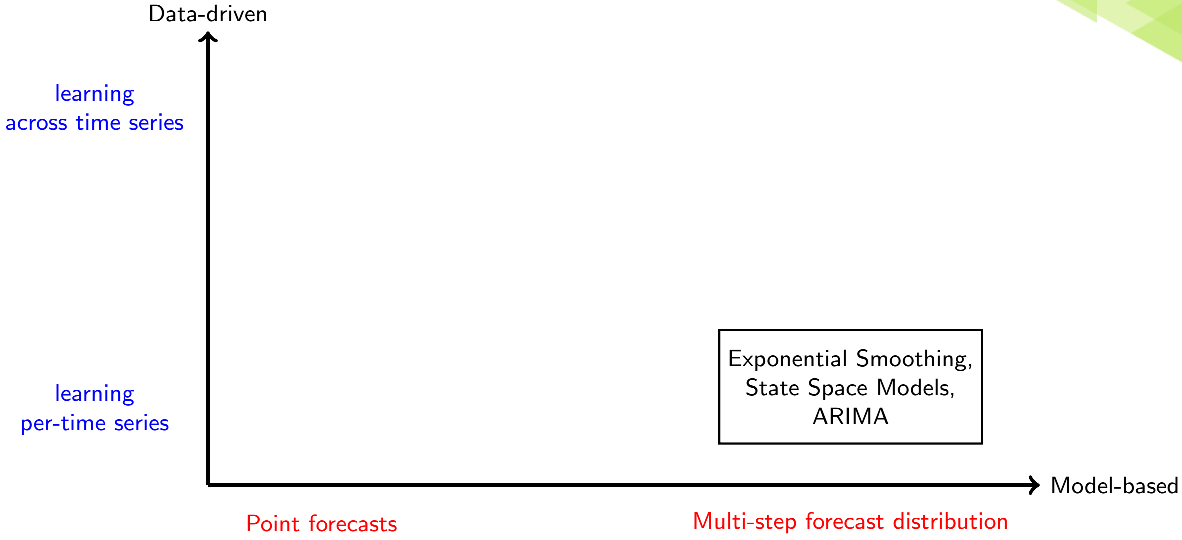

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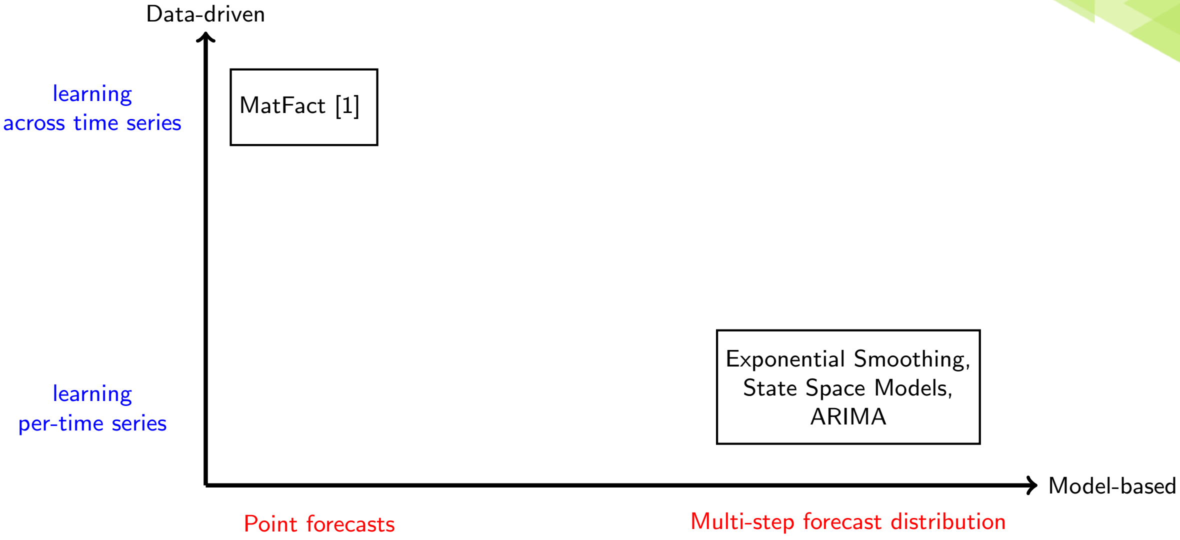

[1] Yu, Rao, Dhillon. Temporal Regularized Matrix Factorization for High-dimensional Time Series Prediction. NIPS 2016

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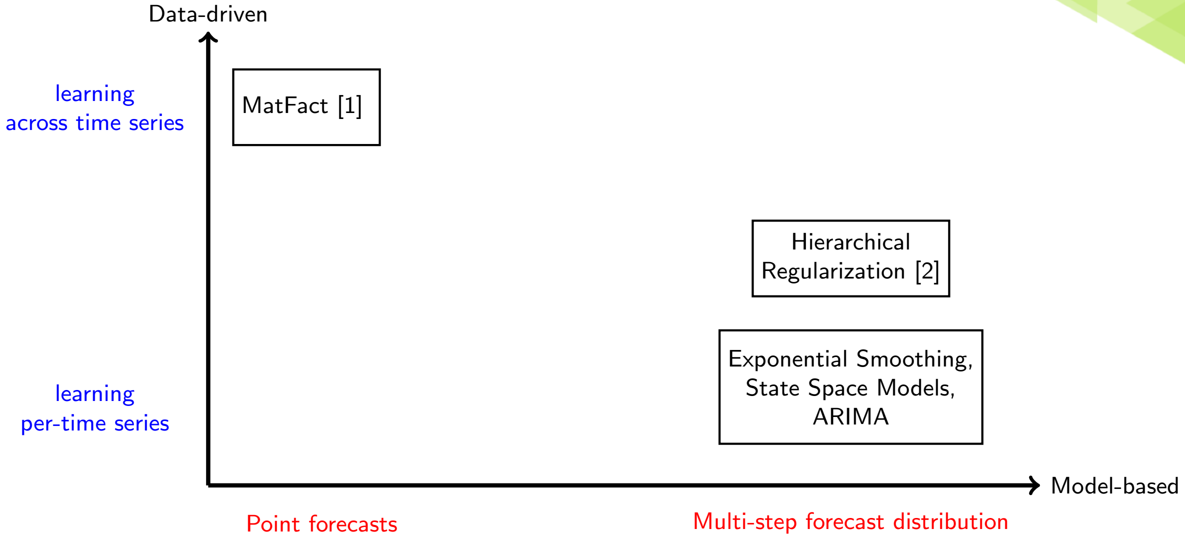

[2] N. Chapados. Effective bayesian modeling of groups of related count time series. ICML 2014

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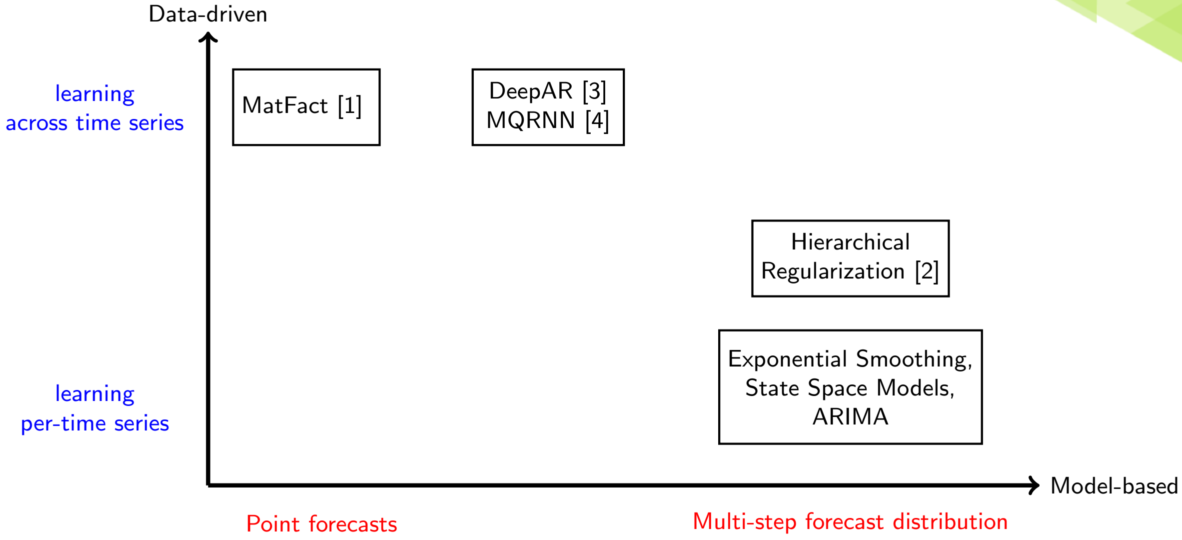
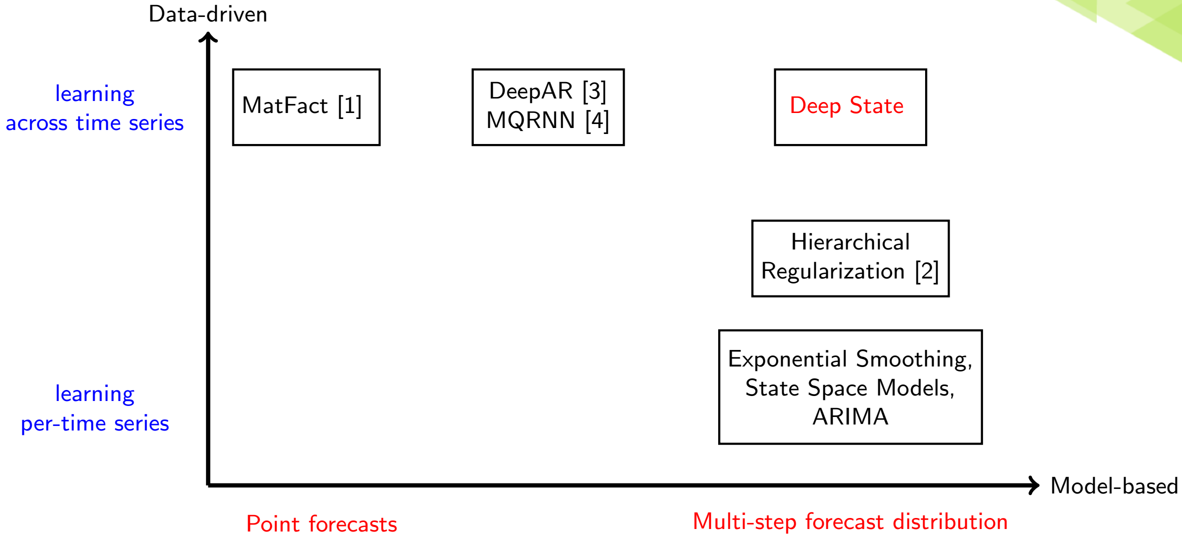


[3] Flunkert, Salinas, Gasthaus. Probabilistic Forecasting with Autoregressive Recurrent Networks. arXiv 2017

[4] Wen, Torkkola, Narayanaswamy. A Multi-Horizon Quantile Recurrent Forecaster. arXiv 2017

Forecasting Literature



State Space Model

- Generative model for observations: $(z_1, \ldots, z_T)$

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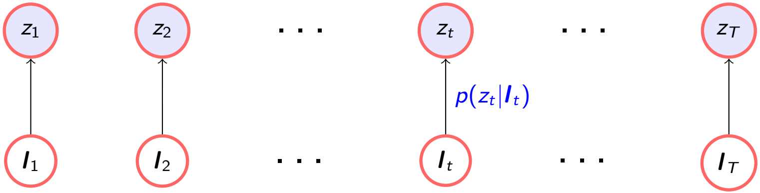

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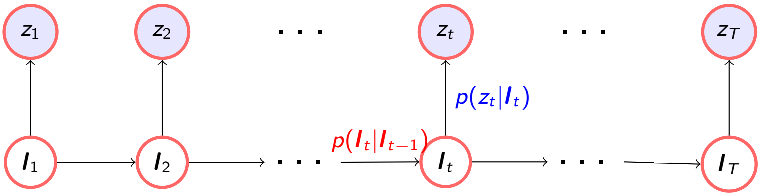

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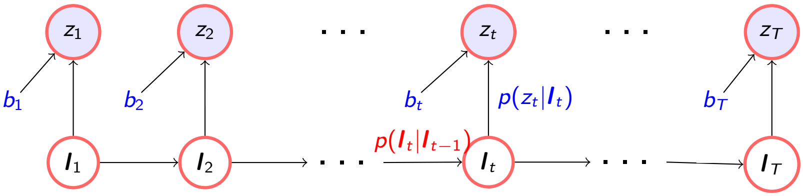

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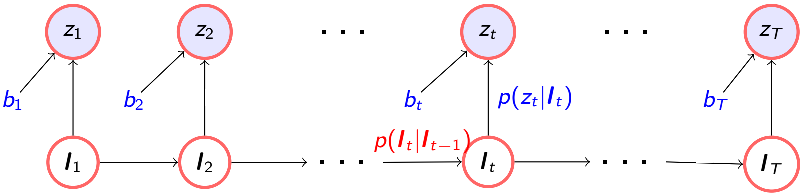

- Linear Gaussian Model

$$\boldsymbol{l}_t = \boldsymbol{l}_{t-1} + g_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1) \qquad \text{(state transistion)}$$

$$z_t = b_t + \boldsymbol{a}_t{}^T \boldsymbol{l}_{t-1} + \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1) \qquad \text{(measurements)}$$

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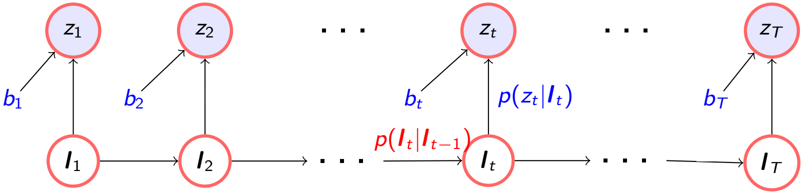

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- Free parameters: $\Theta = \{b_t, \boldsymbol{a}_t, \boldsymbol{g}_t, \sigma_t | \ \forall t\} \cup \{\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0\}$

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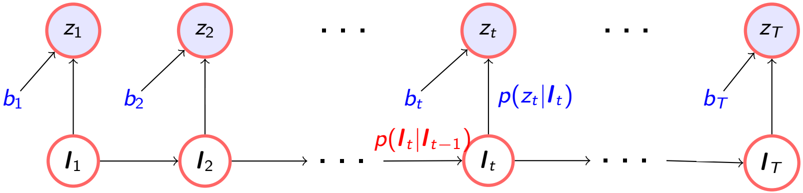

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- Free parameters: $\Theta = \{b_t, \boldsymbol{a}_t, \boldsymbol{g}_t, \sigma_t | \; \forall t\} \cup \{\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0\}$
- Θ learned using maximum likelihood principle

$$\Theta^* = \operatorname*{argmax}_{\Theta} p_{\mathrm{SS}}(z_1, \ldots, z_T|\Theta)$$

Deep State Space Model in a Nutshell

Similar features → similar state space models (modulo scale)

$$\Theta_t = \Psi(\mathbf{x}_{1:t}, \Phi), \quad \forall t$$

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Generative Model:

LSTM

$$\Theta_t = \theta(\mathbf{h}_t)$$

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$$\mathbf{z} \sim p_{\mathrm{SS}}(z_{1:T} | \{\Theta_t\})$$

$\Theta_1, \Theta_2, \ldots, \Theta_T$

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Deep State - Training

- Loss: negative log-likelihood of the data under our model given features

observations $z_{1:T_i}^{(i)}$

$\mathbf{x}_1^{(i)}$ $\cdots$ $\mathbf{x}_t^{(i)}$ $\cdots$ $\mathbf{x}_{T_i}^{(i)}$ features

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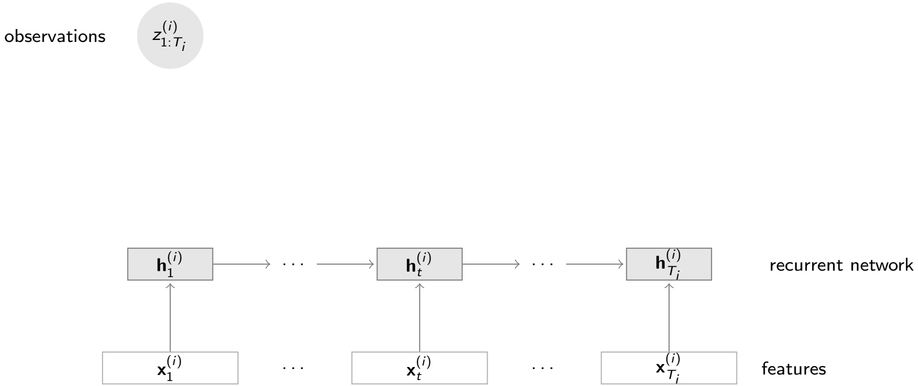

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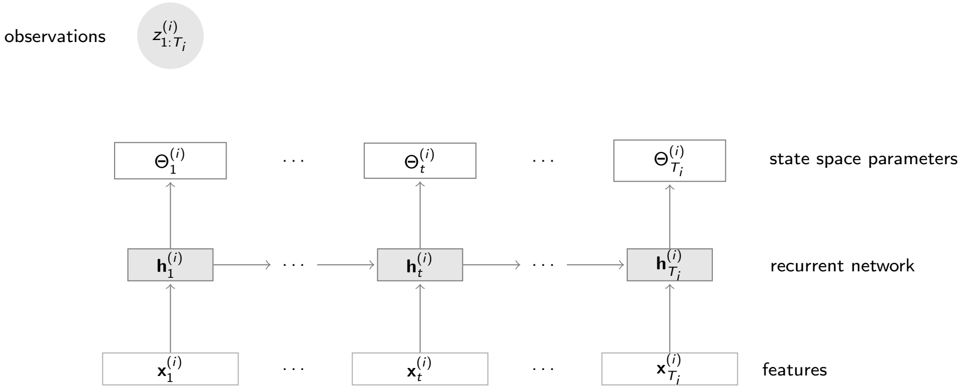

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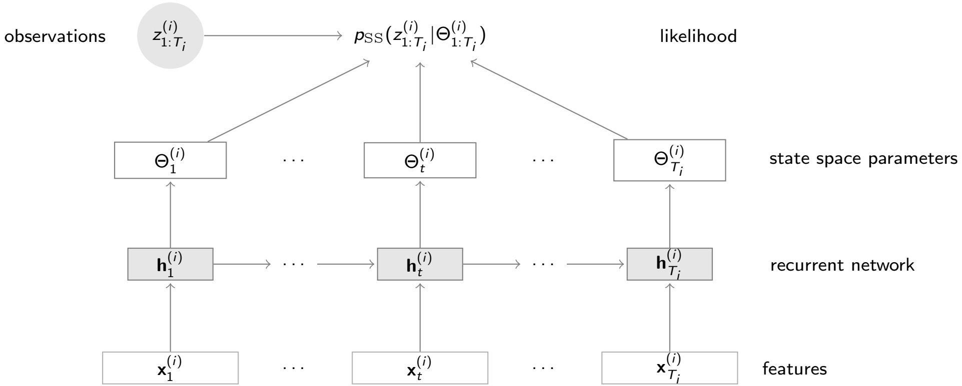

- Kalman filtering in the case of linear Gaussian model

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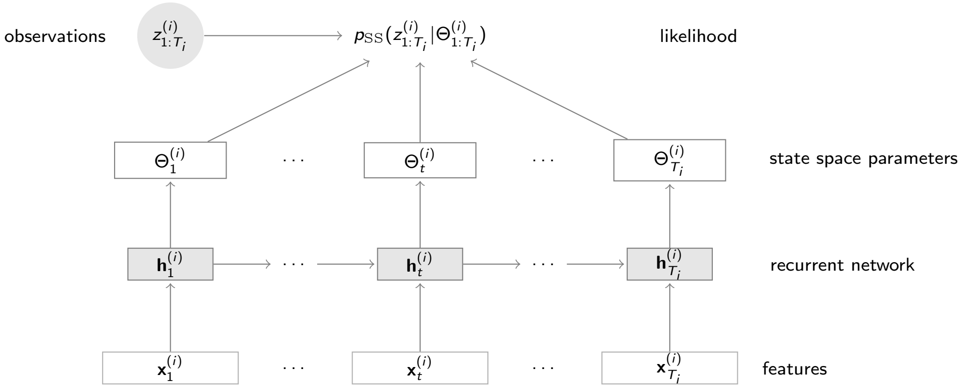
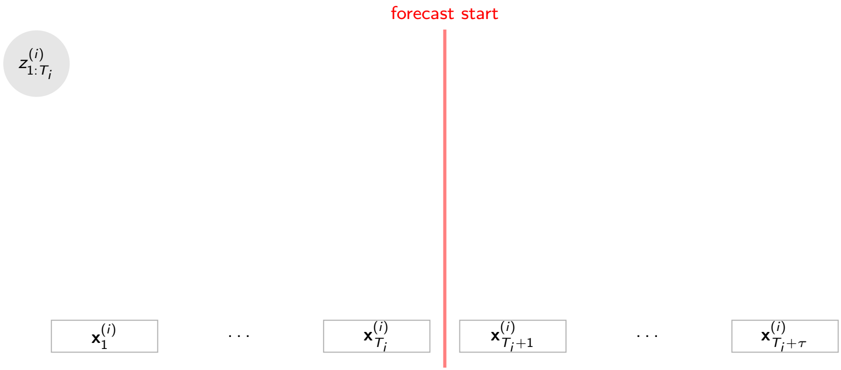


- Kalman filtering in the case of linear Gaussian model
- Robust to outliers and can handle missing data (z_{t-1} is not fed back as feature for time step t)

Deep State - Prediction

Test time series (possibly new/unseen) with features in both training and prediction ranges



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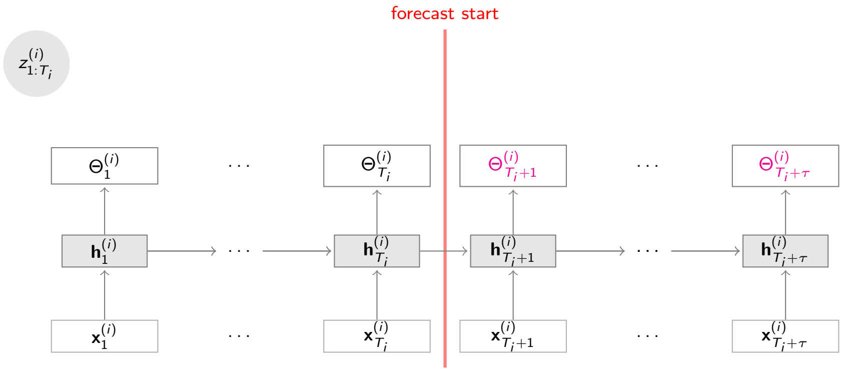

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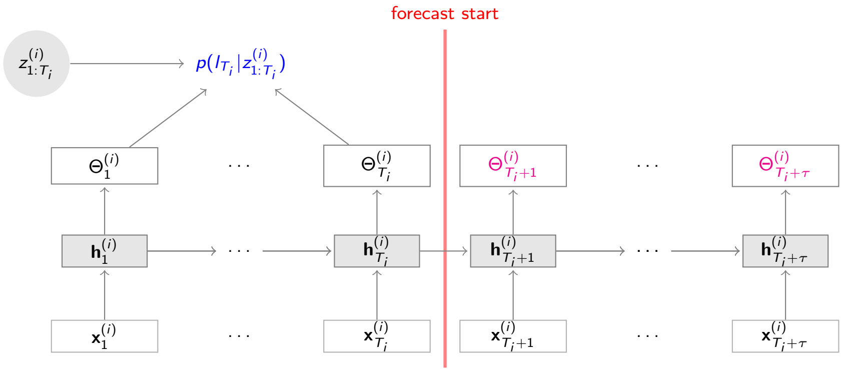

- Given the posterior of the final state and state space parameters, one can obtain the forecast distribution

$$P(z_{T_i+1}, z_{T_i+2}, \ldots, z_{T_i+\tau} | z_1, z_2, \ldots, z_{T_i})$$

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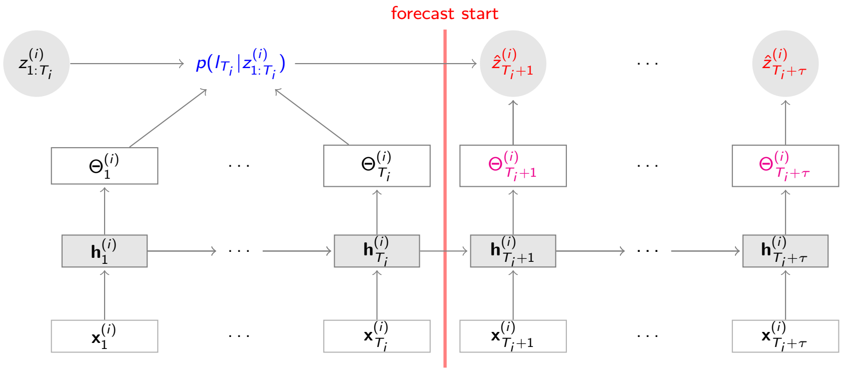

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- Analytical form or samples (sampling here is much faster than in DeepAR)

Experiments

Data Efficiency:

- **Public datasets:**
  - `traffic`: 963 hourly time series measuring occupancy rates of car lanes
  - `electricity`: 370 hourly time series measuring electricity consumption of different consumers
- **Forecast horizon:** 7 days
- **Metrics:** P50QL, P90QL: normalized quantile regression loss

	2-weeks		3-weeks		Full-data	
	P50QL	P90QL	P50QL	P90QL	P50QL	P90QL
`DeepAR`	0.177	0.153	0.126	0.096	0.132	0.104
`Deep State`	**0.126**	**0.069**	**0.125**	**0.068**	**0.124**	**0.07**

Results on `traffic` dataset

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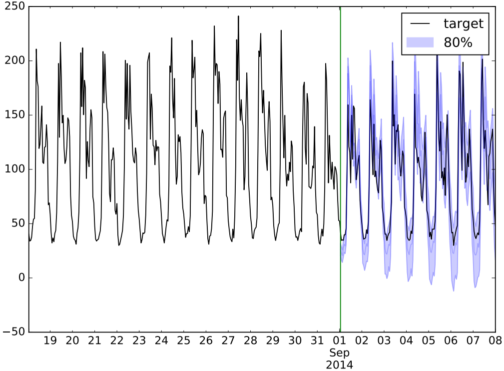
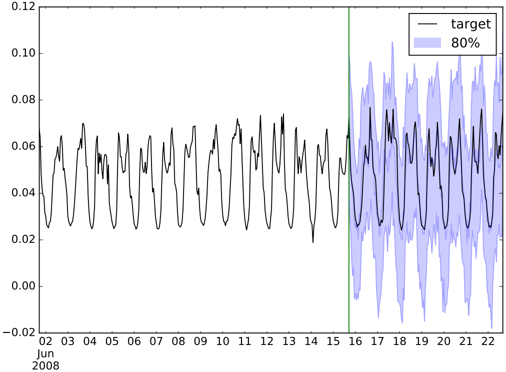
Results on `electricity` dataset

Example Predictions



electricity



traffic

Experiments

More quantitative experiments

- **Public datasets:**
  - `traffic`: 963 hourly time series measuring occupancy rates of car lanes
  - `electricity`: 370 hourly time series measuring electricity consumption of different consumers
- **Forecast horizon:** 24 hours
- **Metrics:** P50QL/P90QL (Matrix factorization method only produces point forecasts)

	`electricity`	`traffic`
`MatFact`	0.16/-	0.20/-
`DeepAR`	0.079/**0.0415**	**0.128**/0.0961
`Deep State`	**0.076**/0.0418	0.14/**0.0957**

Average P50QL/P90QL for rolling day prediction for seven days

Conclusion

Deep State

- New approach by marrying state space models with deep recurrent neural networks
- Explicitly incorporates structural assumptions and hence data-efficient
- More robust to outliers and can handle missing data
- Learns a joint global model and can forecast time series with little history (cold start problem)

Further directions

- Extensions to other likelihoods