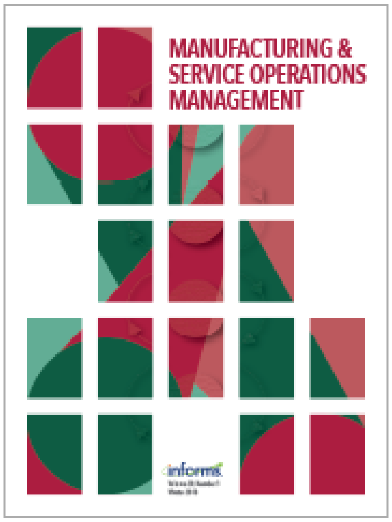

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Allocating Inventory Risk in Retail Supply Chains: Risk Aversion, Information Asymmetry, and Outside Opportunity

Chengfan Hou,[a] Mengshi Lu[b,*]

[a] School of Economics and Management, Tongji University, Shanghai 200092, China; [b] Mitchell E. Daniels, Jr. School of Business, Purdue University, West Lafayette, Indiana 47907

*Corresponding author

Contact: houchengfan@tongji.edu.cn, https://orcid.org/0000-0002-8174-3054 (CH); mengshilu@purdue.edu, https://orcid.org/0000-0002-5961-722X (ML)

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Abstract. ***Problem definition***: Recent global crises have caused unprecedented economic uncertainty and intensified retailers' concerns over inventory risks. Mitigating inventory risks and incentivizing retailer orders is critical to managing retail supply chains and restoring their norms after severe impacts. We study the allocation of inventory risk using contracts in a retail supply chain with a risk-neutral manufacturer and a risk-averse retailer. We consider two factors that affect the effectiveness of contracting: (1) asymmetric risk aversion information—retailers' attitudes are typically diverse and unknown to the manufacturer, and (2) uncertain outside opportunity—retailers typically face a volatile external business environment. ***Methodology/results***: With a game-theoretic model that captures the interaction among risk aversion, information asymmetry, and outside opportunity, we derive the contracting equilibrium under two widely adopted risk allocation schemes—push (i.e., the retailer bears the inventory risk) and pull (i.e., the manufacturer bears the inventory risk) contracts. Contrary to the conventional wisdom that pull contracts are more effective in risk mitigation, we show that push contracts may induce larger expected order quantities and achieve the highest supply chain efficiency due to the interaction of asymmetric risk aversion information and risky outside opportunities. We also find that the manufacturer may obtain higher profits with push contracts when both the heterogeneity in the retailer's risk attitude and the risk of the outside opportunity are sufficiently high. In addition, when the risk of the outside opportunity is in a medium range, the push contract allows the manufacturer to fully eliminate the information rent and achieve the supply chain's first-best outcomes. We further evaluate the effects of product profitability and demand uncertainty and generalize the retailer's risk measure to any coherent risk measure. ***Managerial implications***: Our analysis highlights the importance of modeling asymmetric risk aversion information and risky outside opportunities in analyzing supply chain contracting. When considering these practical factors, allocating more inventory risks to a risk-averse retailer may be better than a risk-neutral manufacturer. Our results provide novel insights into the selection of proper contract types for managing inventory risks in retail supply chains.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/msom.2022.0624.

Keywords: supply chain risk management • contracts • risk aversion • information asymmetry

1. Introduction

The outbreak of the COVID-19 pandemic and its economic fallout have exacerbated inventory risks in retail supply chains. A recent survey shows that during the pandemic, firms' perceived sales uncertainty doubled (Altig et al. 2020). Concerned with volatile demand and inflated costs, retailers are more hesitant about acquiring inventory (Coldrick and Jacobs 2021). U.S. retailers' inventory-to-sales ratio decreased by more than 25% during the pandemic (U.S. Census Bureau 2022). According to a recent survey, 47% of U.S. retailers think the problem of excess stock is a major concern for their business, and 53% in this situation admit there will be "dangerous" ramifications for business if they fail to sell off the excess stock (In Business Magazine 2022). Such risk aversion is even more pronounced for small- and medium-sized retailers who contribute significantly to economies and societies worldwide but are less tolerant of risks than large companies (Ma et al. 2021, McKinsey & Company 2022).

Retailers' concerns are further escalated by unprecedented economic uncertainty. A survey by the Federal Reserve Bank of Atlanta (2023) shows that business uncertainty, measured by sales revenue and employment, almost doubled at the beginning of the pandemic. The global economic policy uncertainty index increased from 53 in 2000 to 435 in April 2020 (Economic Policy Uncertainty 2023). Relieving small- and medium-sized retailers of excessive inventory risks and hence *incentivizing them to increase their inventory levels* are vital to restoring supply chain norms and boosting economic recovery.

Supply chain contracting is an effective means for mitigating retailers' inventory risks. Different forms of contracts can be used to adjust the allocation of inventory risks between the manufacturer and the retailer. Many studies in the supply chain management literature have advocated using pull contracts to replace traditional push contracts. Unlike push contracts, under which retailers bear most of the supply chain's risk, pull contracts can transfer the inventory risk to the upstream manufacturer and alleviate the retailer's risk exposure. Indeed, in a standard retail supply chain setting (i.e., a risk-neutral manufacturer sells to a risk-neutral retailer who faces random consumer demand), compared with push contracts, pull contracts can induce larger order quantities from the retailer and can improve both the manufacturer's expected profit and the entire supply chain's efficiency (Cachon 2004).

However, in practice, the choice between push and pull contracts is more complicated than conventional wisdom suggests. Various situations where push contracts may outperform pull contracts have been identified in the supply chain management literature. (Please refer to the literature review in Section 2.) In this paper, we focus on two issues that are ubiquitous and consequential in practice but have not been considered by existing studies that compare push and pull contracts for retail supply chains.

- *Information Asymmetry in Risk Aversion.* It is well known that firms are risk-averse when choosing inventory levels (Gurnani et al. 2014, Page 2015). However, the retailer's degree of risk aversion may depend on their cash flow, cost of financial distress, senior managers' personal risk tolerance, and many other factors (Bickel 2006, Lovallo et al. 2020). These factors exhibit significant disparity between firms, and most of them are a firm's private information. Therefore, retailers' degree of risk aversion is heterogeneous and cannot be perfectly estimated by outside entities. Most existing studies that compare push and pull contracts either consider a risk-neutral retailer or assume the retailer's degree of risk aversion is known. To bridge the gap between existing research and practice, it is critical to consider risk-averse retailers whose exact risk attitude is unknown to the manufacturer.

- *Risks Associated with Outside Opportunities.* In supply chain contracting, the retailer's potential gain from outside opportunities determines the minimum payoff the retailer is willing to accept from the contract. Such outside opportunities are typically risk-ridden, which has become even more pronounced in recent years because of unprecedented global economic uncertainty. The risk associated with outside opportunities has a significant impact on the effectiveness of supply chain contracting. However, most existing studies that compare pull and push contracts either assume the outside opportunity has a deterministic payoff or consider the expected payoff from a risk-neutral perspective. Furthermore, the impact of the outside risk is complicated by the presence of information asymmetry in the retailer's risk attitude. Their interaction creates the retailer's incentive to understate or overstate its private information, known as countervailing incentives (Lewis and Sappington 1989). It is important to model both the risk of outside opportunities and the interaction between such risks and asymmetric risk aversion information.

To tackle these issues, we consider a supply chain with a risk-neutral manufacturer and a risk-averse retailer. This setting represents a common retail scenario with large manufacturers and small- and medium-sized retailers. For instance, in Canada, the number of small- and medium-sized retailers is twice as many as that of small- and medium-sized manufacturers (StatCan 2022). In these scenarios, retailers are more sensitive and vulnerable to risks, and their risk aversion is more pronounced than manufacturers. Besides, such a model setup is widely adopted in the literature (Agrawal and Seshadri 2000; Gan et al. 2004, 2005; Wang and Webster 2007; Xiao and Yang 2008; Ma et al. 2012; Li et al. 2014).

The manufacturer offers a menu of push or pull contracts for the retailer to choose from. The manufacturer may exclude trading with certain retailer types by making contract terms unacceptable to them, which we refer to as the *shutdown* policy, following the economics literature in principal-agent models (Laffont and Martimort 2009). This policy is also known as the *cutoff* policy. It is usually implemented before entering into long-term business partnerships and shown to be a valuable screening device (Baron and Myerson 1982, Ha 2001, Laffont and Martimort 2009, Çakanyildirim et al. 2012). The risk-averse retailer chooses a contract from the menu to maximize the conditional value-at-risk (CVaR) of its profit. Although the retailer knows the exact value of the degree of risk aversion, the manufacturer has imperfect information and only knows its distribution. The retailer's outside opportunity may generate a random payoff, which reflects the intrinsic uncertainty/risk in the macroenvironment. To compare the performance of push and pull contracts, we focus on *preorder* and *consignment* contracts, which correspond to two extreme cases of supply chain risk allocation and represent the most common types of push and pull contracts in practice, respectively (Wang et al. 2004, 2014; Dong and Zhu 2007; Granot and Yin 2008; Lai et al. 2009; Davis et al. 2014; Kouvelis et al. 2023; Xiao and Kouvelis 2023). The manufacturer seeks to design the optimal contract menu within each contract type to maximize its expected profit.

1.1. Contribution and Insights

To the best of our knowledge, this paper is the first to compare push and pull contracts under the impact of asymmetric risk aversion information and risky outside opportunities. By analyzing the interaction between these two factors, our study generates several important insights.

First, our study identifies an important yet previously unexplored case where push contracts outperform pull contracts in retail supply chain risk mitigation. Although conventional wisdom suggests pull contracts can induce larger order quantities than push contracts (Cachon 2004), one central research question in the supply chain contracting literature is when push contracts perform better than pull contracts. Several papers have shown the superiority of push contracts (or combinations of push and pull contracts) under various supply chain settings, such as reactive ordering (Dong and Zhu 2007), financial constraints (Lai et al. 2009), competitive bidding (Li and Scheller-Wolf 2011), and risk-averse manufacturers (Yang et al. 2018). Our study contributes to this important stream of research by showing that push contracts can induce a larger expected order quantity than pull contracts and achieve the highest supply chain efficiency even under a more conventional supply chain setting (e.g., a risk-neutral manufacturer and a risk-averse retailer with a single ordering opportunity) when considering asymmetric risk-aversion information and risky outside opportunities, which are ubiquitous in practice.

Second, we investigate the manufacturer's preference over push and pull contracts under asymmetric risk-aversion information and risky outside opportunities. We find that push contracts can lead to a higher manufacturer's expected profit than pull contracts when both the heterogeneity in the retailer's risk attitude and the risk of the outside opportunity are sufficiently high. Moreover, the push contract may allow the manufacturer to fully eliminate the information rent due to the lack of information on the retailer's risk attitude and achieve the supply chain's first-best outcomes when the risk associated with the retailer's outside opportunity is in a medium range.

Third, we further characterize the impact of product profitability and demand uncertainty on firms' utility. For push contracts, when the product becomes more (less) profitable or the demand becomes more (less) volatile, a less (more) risk-averse retailer obtains a higher utility because it has a stronger incentive to imitate the other type and therefore receives a higher rent from the manufacturer. However, for pull contracts, the retailer's utility remains constant when product profitability and demand uncertainty vary as the manufacturer fully bears their impact.

At last, we generalize the retailer's risk measure to *coherent* risk measures and find that our main result—the manufacturer may prefer the push contract to the pull contract when the risk of the outside opportunity is high—still holds.

The remainder of the paper is organized as follows. Section 2 reviews relevant literature; Section 3 presents our model setting and discusses the case in which the manufacturer has complete information on the retailer's risk attitude; Section 4 discusses the case in which the manufacturer has incomplete information on the retailer's risk attitude; Section 5 generalizes the retailer's risk measure from CVaR to any coherent risk measure; and Section 6 concludes the paper.

2. Literature Review

Our paper is related to several streams of literature on supply chain management.

2.1. Comparison Between Simple Push and Pull Contracts

Our study contributes to the literature regarding push and pull contracts and their comparison. In particular, we consider the consignment contract that is a typical form of pull contract and similar to a drop-shipping arrangement (Gan et al. 2010) and a vendor-managed inventory program (Nagarajan and Rajagopalan 2008). As mentioned previously, an important stream of research seeks to identify specific supply chain settings under which push contracts may outperform pull contracts. Dong and Zhu (2007) identified conditions under which push contracts achieve a higher supply chain efficiency than pull contracts when the retailer has ordering opportunities before and after demand realization. Lai et al. (2009) found that the combination of push and pull contracts may outperform the pure pull contract in the presence of financial constraints. Li and Scheller-Wolf (2011) considered a buyer who sources from multiple suppliers through a competitive bidding process. They found that push contracts can lead to higher buyer profits than pull contracts when the supplier number is large, and the demand level is high. Yang et al. (2018) showed that push contracts may induce higher order quantities than pull contracts when the manufacturer is sufficiently more risk-averse than the retailer.

Our work differs from this stream of literature in that our model captures asymmetric information in the retailer's risk attitude and the risk associated with outside opportunities. The interaction between these two factors makes the retailer's reservation utility contingent on their heterogeneous risk attitude. Such a contingency complicates the problem by creating countervailing incentives. Consequently, although we consider a more conventional supply chain setting, we still uncover conditions under which push contracts outperform pull contracts in mitigating supply chain risks.

Another feature of our study is that we focus on specific simple push and pull contracts that are common and easy to implement in practice instead of theoretically optimal contracts. Simple contracts not only have practical relevance but also exhibit fairly good performance under all but extreme scenarios, as supported by both theoretical models and behavioral studies (Kalkanci et al. 2011, Li and Scheller-Wolf 2011, Kayiş et al. 2013, Devlin et al. 2018).

2.2. Risk Aversion and Information Asymmetry

Our work focuses on a supply chain with a risk-neutral manufacturer and a risk-averse retailer. In the supply chain contracting literature, such a setting was studied by Agrawal and Seshadri (2000), Gan et al. (2004, 2005), and Yang et al. (2009). Furthermore, we assume that the retailer's risk aversion is modeled by the CVaR risk measure. Introduced by Rockafellar and Uryasev (2000), CVaR equals the expected value of some percentage of the worst-case scenarios. It is a *coherent risk measure* and has been shown to be a popular tool for managing risk (Sarykalin et al. 2008). For example, CVaR was widely adopted in inventory management (Chen et al. 2004, 2009), in pricing and production planning (Kouvelis et al. 2021), and in supply chain contracting (Chen et al. 2014, Yang et al. 2018). Besides, CVaR enjoys great theoretical generality. Kusuoka (2001) demonstrates that any law-invariant coherent risk measure can be represented in the form of integrals of CVaRs.

Information asymmetry has been widely studied in the supply chain management literature (Chen 2003). Asymmetric information in the Stackelberg follower's cost was studied by Corbett and Tang (1999), Ha (2001), Cachon and Zhang (2006), and Çakanyildirim et al. (2012), among others. Our study departs from the previous papers by modeling asymmetric information regarding the retailer's degree of risk aversion. Such information asymmetry affects the manufacturer's perception not only of the retailer's profit from their contracting but also of the profit from the random outside opportunity, which creates the countervailing incentive. Such a countervailing incentive makes it harder for the manufacturer to screen but may allow the manufacturer to fully eliminate the information rent and achieve the supply chain's first-best outcomes.

Agrawal and Seshadri (2000) studied a problem closely related to ours. In their model, a risk-neutral distributor (intermediary) serves multiple retailers with different degrees of risk aversion, and there is always ample inventory through emergency purchases. Chen and Seshadri (2006) studied the same model as Agrawal and Seshadri (2000) but with a continuum of retailers. Gan et al. (2005) addressed the issue of unknown risk aversion by showing that their proposed risk-sharing contracts are naturally incentive-compatible among retailers. Unlike these studies, our paper focuses on comparing different inventory risk allocation schemes (i.e., push or pull contracts) under the effect of asymmetric information and outside risks.

2.3. Type-Dependent Reservation Profit

Our work is also related to papers that consider type-dependent reservation profits. Such type dependence gives rise to countervailing incentives; that is, a firm may have the incentive to both understate or overstate its private information (Lewis and Sappington 1989). In operations management, Chakravarty and Zhang (2007) studied the optimal contracting problem in which two firms collaborate on capacity investment with asymmetric information on the capacity investment cost. Chen et al. (2012) analyzed the optimal procurement outsourcing strategy for an original equipment manufacturer who competes with the contract manufacturer (CM) in the end market and has asymmetric information on the CM's purchasing cost. Both Çakanyildirim et al. (2012) and Gan et al. (2019) analyzed a supply chain consisting of a supplier and a retailer with asymmetric information on the supplier's cost efficiency. They found that system efficiency can be achieved when the supplier's reservation profit matches their production cost differential. Feng et al. (2015) considered a dynamic game between a seller and a buyer with asymmetric information on the buyer's demand type. In their model, distinct demand types use distinct negotiation strategies and then lead to distinct reservation profits. Cao et al. (2023) studied an optimal contract design problem for a national brand manufacturer facing a retailer that may introduce its own store brand with private cost information. Their results showed that when the manufacturer's cost is above a certain threshold, it is optimal for the manufacturer to only do business with retailers with high store brand costs.

Our work differs from most of these studies in that we explicitly model the interaction between the retailer's private risk attitude and the risk of its outside opportunity. Our study uncovers the pivotal role of outside risks in the presence of countervailing incentives and generates several surprising results contrary to the conventional wisdom regarding the effectiveness of push contracts in mitigating retail supply chain risks.

3. Model Setting

We study a supply chain with one risk-neutral manufacturer and one risk-averse retailer. The manufacturer offers a contract (or menu of contracts) to the retailer to maximize its (expected) profit. The retailer then selects a contract to maximize the CVaR of its profit. Such a model has been widely studied in OM literature (see Cachon (2003) and references therein). Let $\beta \in (0, 1]$ denote the retailer's risk attitude. A smaller β means more risk-averse, and the case of $\beta = 1$ corresponds to risk neutrality. For $\beta \in (0, 1]$ and a random variable X representing profits or gains, the β-CVaR of X is defined as $\rho_\beta(X) = \sup_\eta\{\eta + \mathbb{E}[\min\{X-\eta, 0\}/\beta]\}$.

Let c be the unit production cost and s the fixed selling price. To avoid trivial cases, we assume $c < s$. Let D be the random demand supported on $[0, \infty)$ with cumulative distribution function (CDF) F_D and density f_D. The system-wide profit from order quantity q is $\phi(q) \triangleq s\min\{q, D\} - cq$.

3.1. Centralized Case

We first examine the centralized case in which a single firm with risk attitude β determines the order quantity to maximize the CVaR of the system-wide profit, defined as $\rho_\beta(\phi(q)) = s\rho_\beta(\min\{q, D\}) - cq$. For ease of exposition, let $S_\beta(q) \triangleq \rho_\beta(\min\{q, D\})$ denote the CVaR of sales when the order quantity is q. The *first-best* order quantity that maximizes the firm's CVaR of the system-wide profit is given by $q_\beta^{\mathrm{NW}} \triangleq \arg\max_q\{\rho_\beta(\phi(q))\} = F_D^{-1}(\beta(1-c/s))$. The corresponding CVaR of the system-wide profit is referred to as the first-best utility for a firm with risk attitude β.

3.2. Decentralized Case with Complete Information

To establish a benchmark, we next consider the decentralized case with complete information, where both the manufacturer and retailer know the exact value of the retailer's risk attitude β. The manufacturer can offer the retailer a take-it-or-leave-it contract to extract all the surplus. We assume the retailer has an outside opportunity with random potential profit Ψ. Without loss of generality, we assume $\Psi \geq 0$. The retailer will only accept the offer if the CVaR of its profit under this contract is greater than $\rho_\beta(\Psi)$. This condition is known as the *individual rationality* (IR) constraint. We analyze two types of contracts that correspond to two extreme risk allocations in the supply chain—the preorder contract and the consignment contract.

3.2.1. Preorder Contract. Under the preorder contract, the retailer pays the manufacturer a fixed payment t to order q units before the demand realization. The retailer retains all the revenue and bears all the risk. To separate from the consignment contract in terms of notations, we use "^" to denote results under the preorder contract. The manufacturer's profit is $\hat{\pi}^{\mathrm{M}}(t,q) = t - cq$, and the retailer's CVaR of profit is $\hat{\pi}_\beta^{\mathrm{R}}(t,q) = \rho_\beta(s\min\{q,D\} - t) = sS_\beta(q) - t$. Then, the manufacturer's problem is to choose t and q to maximize its profit while satisfying the retailer's IR constraint:

$$\begin{aligned} \max_{q\geq 0, t} \quad & t - cq \\ \text{s.t.} \quad & sS_\beta(q) - t \geq \rho_\beta(\Psi). \end{aligned}$$

The optimal contract can be verified to be $\hat{q}_\beta^* = q_\beta^{\mathrm{NW}} = F_D^{-1}(\beta(1-c/s))$ and $\hat{t}_\beta^* = sS_\beta(\hat{q}_\beta^*) - \rho_\beta(\Psi)$. The manufacturer's optimal profit (if the manufacturer offers a contract that the retailer will accept) is given by $\hat{\pi}_\beta^* \triangleq \hat{\pi}^{\mathrm{M}}(\hat{t}_\beta^*, \hat{q}_\beta^*) = sS_\beta(\hat{q}_\beta^*) - c\hat{q}_\beta^* - \rho_\beta(\Psi)$. The manufacturer will offer a contract if and only if (iff) $\hat{\pi}_\beta^* \geq 0$.

The result shows that the manufacturer's optimal order quantity equals the first-best order quantity for the firm with risk attitude β. It is larger when the retailer is less risk-averse (i.e., β is larger) and irrelevant to the retailer's outside opportunity Ψ. From the system-wide perspective, the channel utility (i.e., the manufacturer's profit plus the retailer's CVaR of profit) is larger when the retailer is less risk-averse. Moreover, for any given β, the manufacturer's optimal profit increases as the risk associated with Ψ increases (i.e., as $\rho_\beta(\Psi)$ decreases). This result is straightforward because the riskier the outside opportunity is, the easier it is for the manufacturer to satisfy the retailer's IR constraint, bringing it larger profits.

The monotonicity of the manufacturer's optimal profit with respect to the retailer's risk attitude β, however, is not as straightforward. Let us define D^c as a random variable that follows the distribution of $D|D \leq F_D^{-1}(1-c/s)$; that is, D^c follows the conditional demand distribution when demand is less than or equal to the first-best order quantity of a risk-neutral firm. Our next result shows that the monotonicity of the manufacturer's optimal profit with respect to β depends on the risk preference between the outside opportunity and the potential profit from D^c. The risk preference is defined in the sense of the *location independent riskier (LIR) order* (Jewitt 1989), formally stated as follows.

Definition 1. A random variable Y is said to be larger than a random variable X in the LIR order, denoted $X \leq_{\mathrm{lir}} Y$, if for any real number a and for all increasing concave functions h and v, such that v and $u(\cdot) = h(v(\cdot))$ are integrable with respect to the distribution of X and Y, we have

$$\mathbb{E}[v(Y)] \leq \mathbb{E}[v(X-a)] \Rightarrow \mathbb{E}[u(Y)] \leq \mathbb{E}[u(X-a)].$$

To provide an intuitive interpretation of Definition 1, consider v to be the utility function of a risk-averse agent and X and Y to be two random assets. If the agent is willing to pay the amount a for the replacement of Y by X, then so is another agent with utility function u that is (Arrow-Pratt) more risk-averse than the first (Hu et al. 2006). The LIR order is equivalent to comparing the weights of two random variables in the lower tail. Such comparison is independent of a random variable's location, linear in scale, and preserved under convolution (Jewitt 1989, Hu et al. 2006). The LIR order and the *excess wealth order* (Shaked and Shanthikumar 1998) are closely related. These two orders have important theoretical implications and wide applications in operations management, finance, economics, and statistics. Please refer to Shaked and Shanthikumar (2007), Kochar et al. (2002), and Chateauneuf et al. (2004) for more details. We derive the condition under which the manufacturer's optimal profit is monotone in the retailer's risk attitude β. (Throughout the paper, we use increasing and decreasing in the weak sense unless otherwise stated.)

Proposition 1. *The manufacturer's optimal profit is increasing (decreasing) in β if and only if $\Psi \leq_{\text{lir}} (\geq_{\text{lir}})(s-c)D^c$.*

Given the deterministic outside opportunity, Yang et al. (2018) showed that the less risk-averse the retailer, the larger the manufacturer's profit. Proposition 1 is consistent with their results and further suggests that as long as the retailer's outside opportunity is not too risky (i.e., it is location independent less risky than the potential profit from D^c), the manufacturer benefits from a lower risk attitude of the retailer. When the outside opportunity is sufficiently risky, a higher risk attitude leads to a significant reduction in the retailer's utility from the outside opportunity. This effect outweighs the decrease in channel utility and thus benefits the manufacturer. Note that Ψ and $(s-c)D^c$ may not be LIR ordered. In that case, the manufacturer's optimal profit is not monotone in β.

3.2.2. Consignment Contract. Under the consignment contract, the manufacturer pays the retailer a fixed payment t to sell q units before demand realization. The manufacturer receives all the revenue and bears all the risk. To separate from the preorder contract in terms of notations, we use "~" to denote results under the consignment contract. The manufacturer's expected profit is $\tilde{\pi}^{M}(t,q) = sS_1(q) - cq - t$, and the retailer's utility is $\tilde{\pi}^{R}_{\beta}(t,q) = t$. The manufacturer's optimization problem under the consignment contract is given by

$$\begin{aligned} \max_{q\geq 0,\, t} \quad & sS_1(q) - cq - t \\ \text{s.t.} \quad & t \geq \rho_\beta(\Psi). \end{aligned}$$

The optimal consignment contract is given by $\tilde{q}^*_\beta = q_1^{\text{NW}} = F_D^{-1}(1-c/s)$ and $\tilde{t}^*_\beta = \rho_\beta(\Psi)$. Then, the manufacturer's optimal profit is $\tilde{\pi}^*_\beta \triangleq \tilde{\pi}^{M}(\tilde{t}^*_\beta, \tilde{q}^*_\beta) = sS_1(q_1^{\text{NW}}) - cq_1^{\text{NW}} - \rho_\beta(\Psi)$. The manufacturer will offer the optimal consignment contract iff $\tilde{\pi}^*_\beta \geq 0$.

The previous result implies that the optimal quantity and the channel utility under the consignment contract are equal to those of the risk-neutral centralized case and independent of the retailer's risk attitude β. Similar to the case under the preorder contract, the manufacturer's optimal expected profit under the consignment contract is increasing in the risk associated with Ψ for any given β. Distinct from the case under the preorder contract, the manufacturer's optimal expected profit under the consignment contract always decreases in β for any given Ψ distribution.

From the manufacturer's perspective, the consignment contract strictly dominates the preorder contract unless the retailer is risk-neutral. In addition, the difference in the manufacturer's (expected) profit under the consignment contract and that under the preorder contract increases in the disparity between the retailer's and manufacturer's risk attitudes. Therefore, when the retailer's risk attitude is publicly known, the more risk-averse the retailer is, the more preferable the consignment contract is for the manufacturer. This result, however, may not hold when the manufacturer does not know the exact value of the retailer's risk attitude.

4. Decentralized Case with Asymmetric Risk Aversion Information

In reality, there is asymmetric information regarding the retailer's risk attitude β. The retailer knows the exact value of β, but the manufacturer does not. Instead, the manufacturer has a prior belief about the retailer's risk attitude that $\beta = \beta_1$ with probability $\nu \in (0,1)$ and $\beta = \beta_2$ with probability $1-\nu$, where $0 < \beta_2 < \beta_1 \leq 1$. We refer to the retailer with risk attitude β_1 as the less risk-averse type and the retailer with risk attitude β_2 as the more risk-averse type. Following the vast literature considering asymmetric information between the manufacturer and retailer (Dasgupta and Spulber 1990, Chen 2007, Hu and Qi 2018), we assume that the manufacturer offers a menu of contracts to the retailer to maximize its expected profit, and the retailer chooses one from the menu. The manufacturer can focus on incentive-compatible direct mechanisms according to the revelation principle.

4.1. Preorder Contract

Under the preorder contract with the shutdown policy, three options are available to the manufacturer: trading with both types, denoted as P(1, 2); trading only with the less risk-averse retailer, denoted as P(1); and trading only with the more risk-averse retailer, denoted as P(2).

- *P(1, 2): The manufacturer trades with both types.* The manufacturer designs a menu of contracts, (t_1, q_1) for the less risk-averse retailer and (t_2, q_2) for the more risk-averse retailer, to maximize its perceived expected profit, as given by

$$\begin{aligned} \max_{q_1\geq 0,\, q_2\geq 0,\, t_1,\, t_2} \quad & \nu(t_1 - cq_1) + (1-\nu)(t_2 - cq_2) \\ \text{s.t.} \quad & sS_{\beta_1}(q_1) - t_1 \geq \rho_{\beta_1}(\Psi), \\ & sS_{\beta_2}(q_2) - t_2 \geq \rho_{\beta_2}(\Psi), \\ & sS_{\beta_1}(q_1) - t_1 \geq sS_{\beta_1}(q_2) - t_2, \\ & sS_{\beta_2}(q_2) - t_2 \geq sS_{\beta_2}(q_1) - t_1. \end{aligned}$$

The first two constraints impose IR constraints for each retailer type. The last two constraints ensure that each retailer type will voluntarily choose the contract designed for him, referred to as the incentive compatibility (IC) constraint. This setting is consistent with quantity-payment schedules in practice. We can see that it is a special case of the setting where the manufacturer offers a menu of wholesale prices and order quantities, and the retailer chooses an order quantity from the menu. Suppose the retailer's possible risk attitude falls into a continuum, which, unfortunately, seems intractable under our problem context. In that case, the menu will specify the wholesale price as a function of the order quantity. In our case, because the retailer's risk attitude has two possible values, the manufacturer offers two wholesale price and order quantity pairs for the retailer to choose from.

We define $\Delta S(q) = S_{\beta_1}(q) - S_{\beta_2}(q)$ and $\Delta\rho(\Psi) = \rho_{\beta_1}(\Psi) - \rho_{\beta_2}(\Psi)$. Let $U_i = sS_{\beta_i}(q_i) - t_i$ represent the CVaR of the type-β_i retailer's profit for $i \in \{1,2\}$. By applying a change of variables, we reformulate the manufacturer's optimization problem as

$$\mathrm{P}(1,2): \max_{q_1\ge 0, q_2\ge 0, U_1, U_2} \nu(sS_{\beta_1}(q_1) - cq_1 - U_1) + (1-\nu)(sS_{\beta_2}(q_2) - cq_2 - U_2) \tag{1}$$

$$\text{s.t.}\quad U_1 \ge \rho_{\beta_1}(\Psi), \tag{IR-1}$$

$$U_2 \ge \rho_{\beta_2}(\Psi), \tag{IR-2}$$

$$U_1 \ge U_2 + s\Delta S(q_2), \tag{IC-1}$$

$$U_2 \ge U_1 - s\Delta S(q_1). \tag{IC-2}$$

- *P(1): The manufacturer only trades with the less risk-averse retailer.* To let the more risk-averse retailer voluntarily choose not to participate, the manufacturer's optimization problem becomes

$$\mathrm{P}(1): \max_{q_1, U_1} \nu(sS_{\beta_1}(q_1) - cq_1 - U_1) \tag{2}$$

$$\text{s.t.}\quad U_1 - s\Delta S(q_1) \le \rho_{\beta_2}(\Psi), \tag{EX-2}$$

$$\text{(IR-1)}.$$

Constraint (EX-2) is equivalent to $sS_{\beta_2}(q_1) - t_1 \le \rho_{\beta_2}(\Psi)$ and ensures the more risk-averse retailer obtains a lower utility than the outside opportunity and voluntarily chooses not to participate.

- *P(2): The manufacturer only trades with the more risk-averse retailer.* The manufacturer's optimization problem is

$$\mathrm{P}(2): \max_{q_2, U_2} (1-\nu)(sS_{\beta_2}(q_2) - cq_2 - U_2) \tag{3}$$

$$\text{s.t.}\quad U_2 + s\Delta S(q_2) \le \rho_{\beta_1}(\Psi), \tag{EX-1}$$

$$\text{(IR-2)}.$$

Analogous to Constraint (EX-2) in P(1), Constraint (EX-1) ensures the less risk-averse retailer will not participate.

A critical factor in characterizing the optimal solution of the above three problems is $\Delta\rho(\Psi) = \rho_{\beta_1}(\Psi) - \rho_{\beta_2}(\Psi)$, which is the difference in the profit of the outside opportunity under the CVaR criterion with respect to different levels of the risk attitude (i.e., β_1 and β_2). The value of $\Delta\rho(\Psi)$ reflects a joint effect of the risk of outside opportunities and the disparity between risk attitudes. When risk attitudes β_1 and β_2 are fixed, $\Delta\rho(\Psi)$ measures the risk associated with the outside opportunity and indicates the market uncertainty. This is because we can verify that the value of $\Delta\rho(\Psi)$ increases as Ψ becomes greater in the sense of the LIR order given the value of β_1 and β_2. (Please refer to Section EC.2 in the Online Appendix for a rigorous proof.) Therefore, throughout the paper, we refer to $\Delta\rho(\Psi)$ as the risk associated with the outside opportunity under the condition that values of β_1 and β_2 are fixed.

Our analysis shows that the countervailing incentive arises in these problems; that is, both retailer types may have the incentive to misreport their types depending on the risk associated with the outside opportunity. As shown in Table 1, when the risk associated with the outside opportunity is low (respectively, high), the less (respectively, more) risk-averse retailer has the incentive to understate (respectively, overstate) its type. Then, the manufacturer has to distort the corresponding order quantity and may grant some information rent to mitigate such an incentive. Interestingly, when the risk associated with the outside opportunity is moderate, the misreporting incentive for both types vanishes. We next characterize the solution to Problems P(1,2), P(1), and P(2) and the manufacturer's optimal choice among options P(1, 2), P(1), and P(2), and not contracting at all under various degree of the risk associated with outside opportunities.

Table 1. Overview of the Impact of Outside Opportunity Risk on Preorder Contracts

		Risk associated with the outside opportunity, $\Delta\rho(\Psi)$		
Impact	Risk aversion type	Low	Medium	High
Retailer's mimicking incentive	Less	✓	✗	✗
	More	✗	✗	✓
Order quantity distortion	Less	✗	✗	✓
	More	✓	✗	✗
Information rent	Less	✓	✗	✗
	More	✗	✗	✓

4.1.1. Low or Low-to-Medium Risk Associated with Outside Opportunities. Consider first the case in which the risk associated with the outside opportunity is relatively low (i.e., $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$). The following proposition characterizes the optimal contract menu. Let $\{(\hat{q}_1^{P(1,2)}, \hat{U}_1^{P(1,2)}), (\hat{q}_2^{P(1,2)}, \hat{U}_2^{P(1,2)})\}$ denote the optimal contract for Problem P(1,2), $(\hat{q}_1^{P(1)}, \hat{U}_1^{P(1)})$ and $(\hat{q}_2^{P(2)}, \hat{U}_2^{P(2)})$ denote the optimal contracts for Problems P(1) and P(2), respectively.

Proposition 2. *When $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$, the optimal preorder contract menu is given in Table 2, where $\hat{\beta}_2 = (1/\beta_2 + (\nu/(1-\nu))(1/\beta_2 - 1/\beta_1))^{-1}$, $\hat{q}_2^{P(1,2):L} \triangleq F_D^{-1}(\hat{\beta}_2(s-c)/s)$, and $\hat{q}_2^{P(1,2):LM} \triangleq (\Delta S)^{-1}(\Delta\rho(\Psi)/s)$.*

When the risk associated with the outside opportunity is relatively low (i.e., $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$), the *less* risk-averse retailer has a strong incentive to mimic the more risk-averse one. To mitigate such a misreporting incentive, Proposition 2 shows that the manufacturer must distort downward the order quantity for the more risk-averse retailer and make the contract less attractive to the less risk-averse retailer as long as it wants to trade with the more risk-averse retailer, such as in options P(1, 2) and P(2). Besides, the *more* risk-averse retailer does not have the incentive to mimic the less risk-averse retailer. If the manufacturer trades only with the less risk-averse retailer (i.e., P(1)), it can exclude the more risk-averse retailer while achieving the first-best contract for the less risk-averse retailer.

To illustrate, Figure 1 shows the effect of the risk of the outside opportunity on the optimal order quantity. In this figure, two dot-dash lines represent the optimal order quantity for the more risk-averse retailer, with the line with triangle markers for P(2) and the line without markers for P(1,2), and two solid lines represent the optimal order quantity for the less risk-averse retailer, with the line with circle markers for P(1) and the line without markers for P(1,2). When $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$, two dot-dash lines fall below the corresponding first-best order quantity $\hat{q}^*_{\beta_2}$, whereas two solid lines remain flat and coincide with the corresponding first-best order quantity $\hat{q}^*_{\beta_1}$. We defer the discussion of the case where $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_2})$ or with the consignment contract to Sections 4.1.2, 4.1.3, and 4.2.

Moreover, when the risk is sufficiently low (i.e., $\Delta\rho(\Psi) \le s\Delta S(\hat{q}_2^{P(1,2):L})$) and the manufacturer trades with both retailer types (i.e., P(1, 2)), the less risk-averse retailer obtains a utility higher than that from the outside opportunity (i.e., $\hat{U}_1^{P(1,2)} \ge \rho_{\beta_1}(\Psi)$). This result means that the manufacturer further needs to leave an information rent to the less risk-averse retailer to eliminate the misreporting incentive. The deterministic outside opportunity corresponds to the special case with $\Delta\rho(\Psi) = 0$. In this case, only the more risk-averse retailer has the incentive to misreport its type. The order quantity for the more risk-averse retailer is lower than the first-best one, and the less risk-averse retailer extracts an information rent.

We let P denote the option among P(1,2), P(1), and P(2) that generates the highest profit for the manufacturer, and let 0 denote not contracting at all. Proposition 3 characterizes the manufacturer's optimal trading choice.

Proposition 3. *Suppose $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$ and $P \ge 0$ (i.e., $\rho_{\beta_1}(\Psi) < \rho_{\beta_1}(\phi(\hat{q}^*_{\beta_1}))$). The following holds.*

(a) *When $\Delta\rho(\Psi) \le s\Delta S(\hat{q}_2^{P(1,2):L})$, $P = P(1)$ if $\rho_{\beta_2}(\Psi) - \nu\rho_{\beta_1}(\Psi) > \rho_{\beta_2}(\phi(\hat{q}_2^{P(1,2):L})) - \nu\rho_{\beta_1}(\phi(\hat{q}_2^{P(1,2):L}))$ and $P = P(1,2)$ otherwise.*

(b) *When $s\Delta S(\hat{q}_2^{P(1,2):L}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$, $P = P(1)$ if $\rho_{\beta_2}(\Psi) > \rho_{\beta_2}(\phi(\hat{q}_2^{P(1,2):LM}))$ and $P = P(1,2)$ otherwise.*

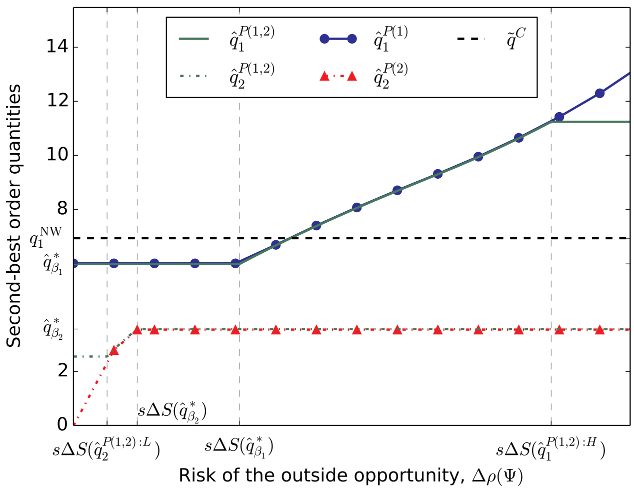
We find that the manufacturer will trade with the preorder contract iff the first-best utility for the less risk-averse retailer is higher than the retailer's utility from the outside opportunity (i.e., $\rho_{\beta_1}(\phi(\hat{q}^*_{\beta_1})) > \rho_{\beta_1}(\Psi)$). In addition, the manufacturer can never be strictly better off by trading only with the more risk-averse retailer.

When the risk associated with the outside opportunity is sufficiently low (i.e., $\Delta\rho(\Psi) \le s\Delta S(\hat{q}_2^{P(1,2):L})$), Proposition 3(a) suggests that the manufacturer will trade only with the less risk-averse retailer iff its profit from the

Table 2. Optimal Preorder Contract When $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$

Range of $\Delta\rho(\Psi)$	P(1,2)	P(1)	P(2)
Low: $[0, s\Delta S(\hat{q}_2^{P(1,2):L})]$	$\hat{q}_1^{P(1,2)} = \hat{q}^*_{\beta_1}$ $\hat{U}_1^{P(1,2)} = \rho_{\beta_2}(\Psi) + s\Delta S(\hat{q}_2^{P(1,2):L})$ $\hat{q}_2^{P(1,2)} = \hat{q}_2^{P(1,2):L}$ $\hat{U}_2^{P(1,2)} = \rho_{\beta_2}(\Psi)$	$\hat{q}_1^{P(1)} = \hat{q}^*_{\beta_1}$ $\hat{U}_1^{P(1)} = \rho_{\beta_1}(\Psi)$	$\hat{q}_2^{P(2)} = \Delta S^{-1}(\Delta\rho(\Psi)/s)$ $\hat{U}_2^{P(2)} = \rho_{\beta_2}(\Psi)$
Low-to-Medium: $[s\Delta S(\hat{q}_2^{P(1,2):L}), s\Delta S(\hat{q}^*_{\beta_2})]$	$\hat{q}_1^{P(1,2)} = \hat{q}^*_{\beta_1}$ $\hat{U}_1^{P(1,2)} = \rho_{\beta_1}(\Psi)$ $\hat{q}_2^{P(1,2)} = \hat{q}_2^{P(1,2):LM}$ $\hat{U}_2^{P(1,2)} = \rho_{\beta_2}(\Psi)$		

Figure 1. (Color online) Impact of the Risk Associated with Outside Opportunities, $\Delta\rho(\Psi)$, on the Second-Best Order Quantity



Notes. $\beta_1 = 0.9$, $\beta_2 = 0.6$, $\nu = 0.5$, $s = 2$, $c = 1$, and $\rho_{\beta_2}(\Psi) = 1$. Demand D follows the exponential distribution with $\mathbb{E}[D] = 10$.

more risk-averse retailer (i.e., $\rho_{\beta_2}(\phi(\hat{q}_2^{\mathrm{P}(1,2):\mathrm{L}})) - \rho_{\beta_2}(\Psi))$ is less than $\nu(\rho_{\beta_1}(\phi(\hat{q}_2^{\mathrm{P}(1,2):\mathrm{L}})) - \rho_{\beta_1}(\Psi))$. The manufacturer may exclude the more risk-averse retailer even if it could make a positive profit with him in Problem P(1,2). This is because of the information rent the manufacturer forfeits to the less risk-averse retailer when trading with both retailer types.

When the risk associated with the outside opportunity is low-to-medium (i.e., $s\Delta S(\hat{q}_2^{\mathrm{P}(1,2):\mathrm{L}}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$), Proposition 3(b) suggests that the manufacturer will trade only with the less risk-averse retailer iff it cannot profit from the more risk-averse retailer in Problem P(1,2).

4.1.2. Medium-to-High or High Risk Associated with Outside Opportunities.

Next, consider the case in which the risk associated with the outside opportunity is relatively high (i.e., $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_1})$). The analysis of this case is very similar to the previous case except that now the *more* risk-averse retailer has a strong incentive to mimic the less risk-averse one. We summarize the optimal contract menu in the following proposition for the sake of completeness and leave the rest of the result and analysis in Section EC.1 of the Online Appendix.

Proposition 4. *When $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_1})$, the optimal preorder contract menu is given in Table 3, where* $\hat{\beta}_1 = (1/\beta_1 - ((1-\nu)/\nu)(1/\max\{\beta_2, (1-\nu c/s)\beta_1\} - 1/\beta_1))^{-1}$, $\hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}} = F_D^{-1}(\hat{\beta}_1(s-c)/s)$, *and* $\hat{q}_1^{\mathrm{P}(1,2):\mathrm{MH}} = (\Delta S)^{-1}(\Delta\rho(\Psi)/s)$.

4.1.3. Medium Risk Associated with Outside Opportunities.

At last, consider the case in which the risk associated with the outside opportunity is in an intermediate range (i.e., $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$). This intermediate range of the risk associated with the outside opportunity (i.e., $[s\Delta S(\hat{q}^*_{\beta_2}), s\Delta S(\hat{q}^*_{\beta_1})]$) is nonempty. The nonemptiness can be verified by showing the strict supermodularity of $S_\beta(q)$ with respect to risk attitude β and order quantity q. We find that the manufacturer offers the first-best contract to either type and makes the retailer extract no information rent under all trading options—P(1,2), P(1), and P(2). This result means that the manufacturer achieves a result as if it has complete information, which is summarized in the following proposition.

Proposition 5. *When $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$, the optimal preorder contract menu is given in Table 4.*

Regardless of the trading choice, the manufacturer can achieve the same result under complete information and restore information transparency when the risk associated with the outside opportunity is medium. This is because the misreporting incentive of either type is less pronounced, making it easier for the manufacturer to either satisfy the retailer's IC constraint or exclude a

Table 3. Optimal Preorder Contract When $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_1})$

Range of $\Delta\rho(\Psi)$	P(1,2)	P(1)	P(2)
Medium-to-High: $[s\Delta S(\hat{q}^*_{\beta_1}), s\Delta S(\hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}})]$	$\hat{q}_1^{\mathrm{P}(1,2)} = \hat{q}_1^{\mathrm{P}(1,2):\mathrm{MH}}$ $\hat{U}_1^{\mathrm{P}(1,2)} = \rho_{\beta_1}(\Psi)$ $\hat{q}_2^{\mathrm{P}(1,2)} = \hat{q}^*_{\beta_2}$ $\hat{U}_2^{\mathrm{P}(1,2)} = \rho_{\beta_2}(\Psi)$	If $\Delta\rho(\Psi) > s\Delta S(F_D^{-1}(\beta_1))$, P(1) is infeasible. Otherwise, $\hat{q}_1^{\mathrm{P}(1)} = (\Delta S)^{-1}(\Delta\rho(\Psi)/s)$, $\hat{U}_1^{\mathrm{P}(1)} = \rho_{\beta_1}(\Psi)$	$\hat{q}_2^{\mathrm{P}(2)} = \hat{q}^*_{\beta_2}$ $\hat{U}_2^{\mathrm{P}(2)} = \rho_{\beta_2}(\Psi)$
High: $[s\Delta S(\hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}}), \infty)$	$\hat{q}_1^{\mathrm{P}(1,2)} = \hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}}$ $\hat{U}_1^{\mathrm{P}(1,2)} = \rho_{\beta_1}(\Psi)$ $\hat{q}_2^{\mathrm{P}(1,2)} = \hat{q}^*_{\beta_2}$ $\hat{U}_2^{\mathrm{P}(1,2)} = \rho_{\beta_1}(\Psi) - s\Delta S(\hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}})$		

Table 4. Optimal Preorder Contract When $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$

Range of $\Delta\rho(\Psi)$	P(1,2)	P(1)	P(2)
Medium: $[s\Delta S(\hat{q}^*_{\beta_2}), s\Delta S(\hat{q}^*_{\beta_1})]$	$\hat{q}_1^{\mathrm{P}(1,2)} = \hat{q}^*_{\beta_1}$ $\hat{U}_1^{\mathrm{P}(1,2)} = \rho_{\beta_1}(\Psi)$ $\hat{q}_2^{\mathrm{P}(1,2)} = \hat{q}^*_{\beta_2}$ $\hat{U}_2^{\mathrm{P}(1,2)} = \rho_{\beta_2}(\Psi)$	$\hat{q}_1^{\mathrm{P}(1)} = \hat{q}^*_{\beta_1}$ $\hat{U}_1^{\mathrm{P}(1)} = \rho_{\beta_1}(\Psi)$	$\hat{q}_2^{\mathrm{P}(2)} = \hat{q}^*_{\beta_2}$ $\hat{U}_2^{\mathrm{P}(2)} = \rho_{\beta_2}(\Psi)$

retailer type from trading. To illustrate, we depict the feasible region of (U_1, U_2) in Problem P(1,2) in Figure 2. In this figure, the retailer's IC constraints (i.e., (IC-1) and (IC-2)) are voluntarily satisfied given tight IRs and first-best order quantities derived in Section 3.2.1.

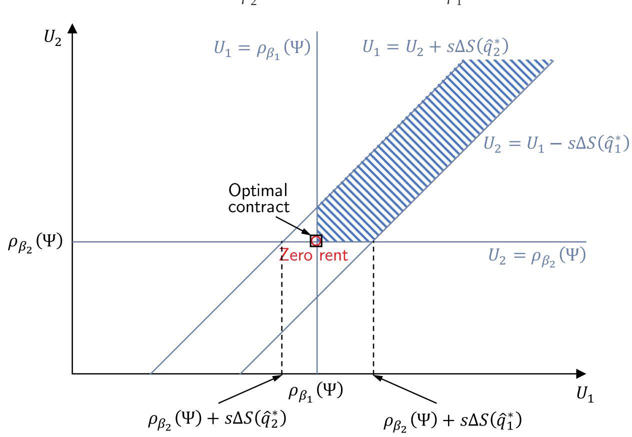
We characterize the manufacturer's trading strategy in the following proposition.

Proposition 6. *Suppose* $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$ *and* $P \ge 0$ *(i.e., either* $\rho_{\beta_1}(\Psi) < \rho_{\beta_1}(\phi(\hat{q}^*_{\beta_1}))$ *or* $\rho_{\beta_2}(\Psi) < \rho_{\beta_2}(\phi(\hat{q}^*_{\beta_2}))$ *is true). If* $\rho_{\beta_2}(\Psi) > \rho_{\beta_2}(\phi(\hat{q}^*_{\beta_2}))$, $P = P(1)$; *if* $\rho_{\beta_1}(\Psi) > \rho_{\beta_1}(\phi(\hat{q}^*_{\beta_1}))$, $P = P(2)$; *otherwise,* $P = P(1,2)$.

When the risk associated with the outside opportunity is in an intermediate range (i.e., $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$), whether the manufacturer should trade with a certain type of retailer depends on whether it could obtain a positive profit from that retailer in Problem P(1,2). This condition is also equivalent to whether it could obtain a positive profit from that retailer under the complete information setting.

Combining the results in Propositions 2, 4, and 5, we find that $\hat{q}_1^{\mathrm{P}(1,2)} \ne \hat{q}_2^{\mathrm{P}(1,2)}$. In other words, the equilibrium is *separable*; that is, the manufacturer is able to distinguish the retailer type after executing the preorder contract if trading with both types.

Figure 2. (Color online) Feasible Region of (U_1, U_2) in Problem P(1,2) When $s\Delta S(\hat{q}^*_{\beta_2}) \le \Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_1})$



Notes. Circles specify the value of (U_1, U_2) such that retailers extract zero rent. Squares specify the optimal value of (U_1, U_2). The dashed area specifies the feasible region of (U_1, U_2) under the optimal order quantity $(\hat{q}_1^{\mathrm{P}(1,2)}, \hat{q}_2^{\mathrm{P}(1,2)})$.

To further illustrate the equilibrium result under the preorder contract, we characterize the monotonicity of the order quantity, expected information rent, and relative channel efficiency loss with respect to the risk of outside opportunities (i.e., $\Delta\rho(\Psi)$) in the following corollary. In particular, the expected information rent is given by $\nu(\hat{U}_1^{\mathrm{P}(1,2)} - \rho_{\beta_1}(\Psi)) + (1-\nu)(\hat{U}_2^{\mathrm{P}(1,2)} - \rho_{\beta_2}(\Psi))$ under P(1,2), $\nu(\hat{U}_1^{\mathrm{P}(1)} - \rho_{\beta_1}(\Psi))$ under P(1), and $(1-\nu)\,(\hat{U}_2^{\mathrm{P}(2)} - \rho_{\beta_2}(\Psi))$ under P(2). The relative channel efficiency loss is measured by the relative decrease in the channel utility under asymmetric information compared with that under complete information. As the manufacturer's trading strategy cannot be fully characterized by the value of $\Delta\rho(\Psi)$, we can only analyze its effect given the trading option (i.e., P(1,2), P(1), and P(2)) instead of analyzing its effect on the optimal trading strategy (i.e., P). Whenever we refer to P(1), the underlying condition is $\Delta\rho(\Psi) \le s\Delta S(F_D^{-1}(\beta_1))$ to ensure P(1) is feasible.

Corollary 1. *Under the preorder contract with* $\rho_{\beta_2}(\phi(\hat{q}_2^*)) > 0$, *when the risk of the outside opportunity (i.e.,* $\Delta\rho(\Psi)$*) increases,*

(i) *The second-best order quantities increase.*

(ii) *The expected information rent*

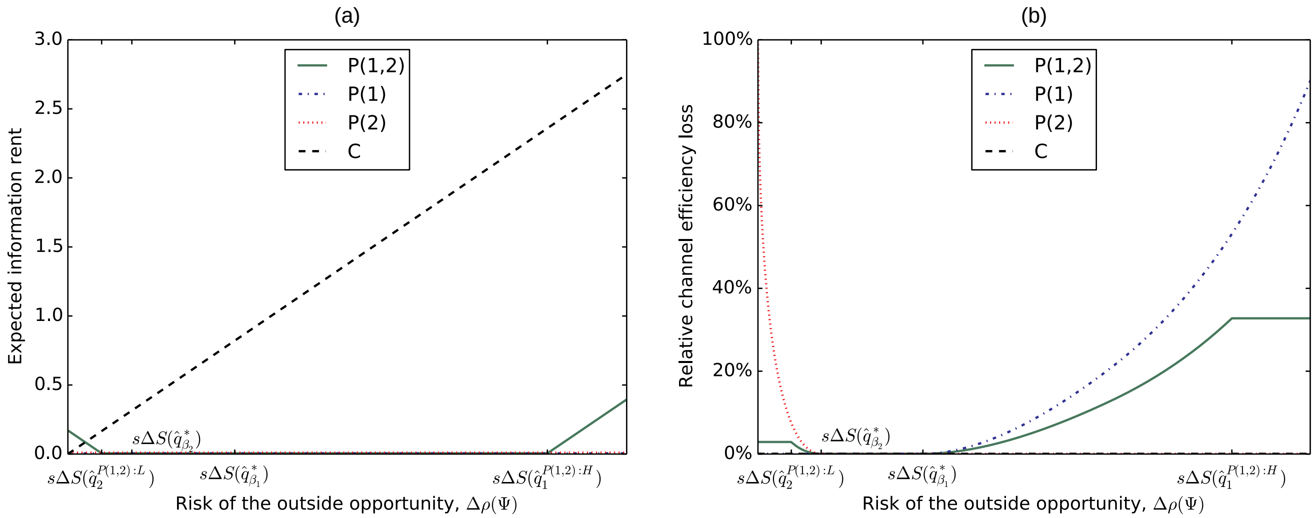
- *P(1,2): decreases when* $\Delta\rho(\Psi) < s\Delta S(\hat{q}_2^{\mathrm{P}(1,2):\mathrm{L}})$, *increases when* $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}_1^{\mathrm{P}(1,2):\mathrm{H}})$, *and remains zero otherwise;*
- *P(1) and P(2): is always zero.*

(iii) *The relative channel efficiency loss*

- *P(1,2): decreases when* $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$, *increases when* $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_1})$, *and remains zero otherwise;*
- *P(1): increases when* $\Delta\rho(\Psi) \ge s\Delta S(\hat{q}^*_{\beta_1})$ *and remains zero otherwise;*
- *P(2): decreases when* $\Delta\rho(\Psi) \le s\Delta S(\hat{q}^*_{\beta_2})$ *and remains zero otherwise.*

When the outside retail market becomes increasingly volatile because of unforeseen events, such as a pandemic, Corollary 1 suggests that the order quantity under the preorder contract increases, as shown in Figure 1. To illustrate, we further depict the effect of the risk of outside opportunities on the expected information rent and relative channel efficiency loss in Figure 3. (We defer the discussion for the consignment contract to Section 4.2.)

Figure 3. (Color online) Impact of the Risk Associated with Outside Opportunities, $\Delta\rho(\Psi)$



Notes. (a) Expected information rent. (b) Relative channel efficiency loss. $\beta_1 = 0.9$, $\beta_2 = 0.6$, $\nu = 0.5$, $s = 2$, and $c = 1$. Demand D follows the exponential distribution with $\mathbb{E}[D] = 10$. Lines for cases P(1) and P(2) in (a) and the line for C in (b) always overlap with the horizontal axis.

Our result shows that only if the risk associated with the outside opportunity is in a medium range, the manufacturer can eliminate the information rent and relative channel efficiency loss when trading with both types. When the risk associated with the outside opportunity deviates away from this medium range, both the information rent and relative channel efficiency loss increase. This result is because when the outside market is sufficiently volatile and becomes more volatile (or is sufficiently stable and becomes more stable), the more (or less) risk-averse retailer has a stronger incentive to mimic the other one. As a result, the manufacturer has to distort the order quantity under complete information, compromise the channel efficiency, and leave more rent to mitigate such an incentive. Interestingly, from 2018 to 2020, the net profit margin of the U.S. retail industry increased from 2.9% to 5.9%, whereas the U.S. economic uncertainty index increased from 163 to 334 (Economic Policy Uncertainty 2023, Forrester 2023). This is consistent with our result that the retail channel may benefit from increased outside risks.

4.2. Consignment Contract

If the manufacturer wants to trade with both retailer types, it is not able to distinguish the retailer type by executing the consignment contract and can only achieve a *pooling* equilibrium. We use C to denote trading with both types using the consignment contract, where the manufacturer's problem is given by

$$\mathrm{C}: \max_{q,t} \quad sS_1(q) - cq - t \tag{4}$$

$$\text{s.t.} \quad t \geq \rho_{\beta_i}(\Psi), \; i = 1, 2. \tag{5}$$

Constraint (5) ensures that the IR constraint is satisfied for both retailer types. The manufacturer has absolute power in determining both the transfer payment and order quantity under the consignment contract. This setup is plausible because the manufacturer bears all the inventory risk, and the retailer's payoff is irrelevant to the order quantity. It is also consistent with vendor-managed inventory (VMI) programs in practice where the manufacturer makes the inventory decision on behalf of retailers (Cachon 2004, Wang et al. 2004, Chen and Seshadri 2006).

Because of the pooling nature of the consignment contract, we only consider trading with both types. The retailer's utility under the consignment contract is equal to the transfer payment and independent of the order quantity. Therefore, the manufacturer cannot trade only with the less risk-averse retailer as the transfer payment acceptable to the less risk-averse retailer is also acceptable to the more risk-averse one. It is possible for the manufacturer to trade only with the more risk-averse retailer. However, trading only with the more risk-averse retailer under the consignment contract generates the same optimal order quantity as trading with both. It does not affect our main result regarding the order quantity comparison between preorder and consignment contracts and, therefore, is omitted for brevity.

The optimal consignment contract under C is characterized in the following proposition. The manufacturer's expected profit then is $\tilde{\pi}^M(\tilde{t}^C, \tilde{q}^C) = sS_1(q_1^{NW}) - cq_1^{NW} - \rho_{\beta_1}(\Psi)$.

Proposition 7. *The optimal consignment contract is* $(\tilde{q}^C, \tilde{t}^C) = (q_1^{NW}, \rho_{\beta_1}(\Psi))$.

The order quantity under the consignment contract coincides with the first-best order quantity under complete information and therefore is irrelevant to the risk of the outside opportunity, as the dashed line in Figure 1 depicts. The relative channel efficiency loss is then given by $\rho_1(\phi(\tilde{q}^*)) - \rho_1(\phi(\tilde{q}^{\text{C}})) = 0$. Therefore, asymmetric information regarding the retailer's risk attitude does not cause any efficiency loss under the consignment contract, as shown in Figure 3(b).

Moreover, with the consignment contract, the less risk-averse retailer always obtains zero information rent, whereas the more risk-averse retailer always extracts positive information rent. The expected information rent expropriated by the retailer is then given by $(1-\nu)(\tilde{t}^{\text{C}} - \rho_{\beta_2}(\Psi)) = (1-\nu)\Delta\rho(\Psi)$ and increases in the risk of the outside opportunity, as the dash line in Figure 3(a) shows.

With the consignment contract, we find that the manufacturer will trade with both types iff the first-best utility for the risk-neutral firm is higher than the less risk-averse retailer's utility from the outside opportunity, as stated in Proposition 8.

Proposition 8. *We have* $C > 0$ *iff* $\rho_{\beta_1}(\Psi) < \rho_1(\phi(q_1^{\text{NW}}))$.

4.3. Preorder vs. Consignment Contract

In this section, we seek to understand which contract induces a higher expected order quantity and how the manufacturer chooses between the preorder and consignment contract.

4.3.1. Expected Order Quantity. We find that the expected order quantity under the preorder contract could be higher than that under the consignment contract. Let $\eta(\beta_2) = \{\eta' \in [1, s/(s-c)] \mid \nu F_D^{-1}(\eta'(1-c/s)) + (1-\nu)F_D^{-1}(\beta_2(1-c/s)) = F_D^{-1}(1-c/s)\}$, whose existence can be easily verified. The value of $\eta(\beta_2)$ represents the threshold of the less risk-averse retailer's *virtual risk attitude* $\hat{\beta}_1$ to ensure the possibility of having the expected order quantity under the preorder contract larger than that under the consignment contract.

Proposition 9. *The expected order quantity under the preorder contract is (strictly) higher than that under the consignment contract if and only if*

1. *The disparity between two possible risk attitudes is sufficiently large; that is,* $\beta_1 > \max\{\eta(\beta_2)(s-c)/(s-\nu c), \beta_2/(1-\nu+\eta(\beta_2)\nu\beta_2)\}$,
2. *The risk of outside opportunities is sufficiently high; that is,* $\Delta\rho(\Psi) > s\Delta S(F_D^{-1}(\eta(\beta_2)(1-c/s)))$ *and*
3. *The manufacturer trades with both types under the preorder contract (please refer to Proposition* EC.1 *in the Online Appendix for the exact condition under which* $P = P(1,2)$ *when* $\Delta\rho(\Psi) \geq s\Delta S(\hat{q}^*_{\beta_1})$*).*

With the high risk associated with outside opportunities in the volatile global economic environment since the pandemic, the previous result indicates the preorder contract has the potential to induce a higher expected order quantity than the consignment contract. Equivalently, making retailers bear more inventory risk may bring up inventory levels of small- and medium-sized retailers, raise product availability to consumers, and boost consumption.

With a risk-neutral manufacturer, our result contrasts with the conventional finding that the pull contract yields a higher order quantity than the push contract. We find that the push contract may lead to a higher order quantity when the retailer's possible risk attitudes are sufficiently dispersed, and the risk of outside opportunities is sufficiently high. This result is due to the manufacturer's incomplete information about the retailer's risk attitude and intention to screen the retailer's risk attitude by distorting the order quantity designed for the less risk-averse retailer upward. Yang et al. (2018) also show that the push contract may lead to a higher order quantity than the pull contract. Unlike our setting, this paper does not consider private risk attitudes but assumes a risk-averse manufacturer and finds that the result holds when the manufacturer is sufficiently more risk-averse than the retailer. By modeling the interaction between the retailer's private risk attitude and the outside risk, we show that even if the manufacturer is risk-neutral, allocating more inventory risk to the risk-averse retailer may still increase the order quantity.

4.3.2. Manufacturer's Profit. Let $\succ$ and $\prec$ ($\succeq$ and $\preceq$) denote the manufacturer's preference. For example, $C \succ (\succeq)\ P$ means that the manufacturer's expected profit under the consignment contract is strictly (weakly) larger than that under the preorder contract. The manufacturer's preference is characterized as follows.

Proposition 10. *The following holds when both the preorder contract (i.e., P) and the consignment contract (i.e., C) can generate a positive profit for the manufacturer.*

(1) *If* $P = P(1,2)$, *there are two thresholds of* β_1, $\underline{\beta_1}(\beta_2)$ *and* $\overline{\beta_1}(\beta_2)$, *which satisfy* $\beta_2 < \underline{\beta_1}(\beta_2) \leq \overline{\beta_1}(\beta_2) < \overline{1}$ *and are increasing in* β_2, *such that*

(i) *If* $\beta_2 < \beta_1 \leq \underline{\beta_1}(\beta_2)$, $C \succeq P$ *regardless of the risk of outside opportunities;*

(ii) *If* $\underline{\beta_1}(\beta_2) < \beta_1 \leq \overline{\beta_1}(\beta_2)$, *there exists a threshold* $\Delta_1 \in [s\Delta S(\hat{q}^*_{\beta_1}), s\Delta S(\hat{q}_1^{\text{P}(1,2):\text{H}})]$ *such that* $C \succeq P$ *when* $\Delta\rho(\Psi) \leq \Delta_1$ *and* $C \prec P$ *otherwise;*

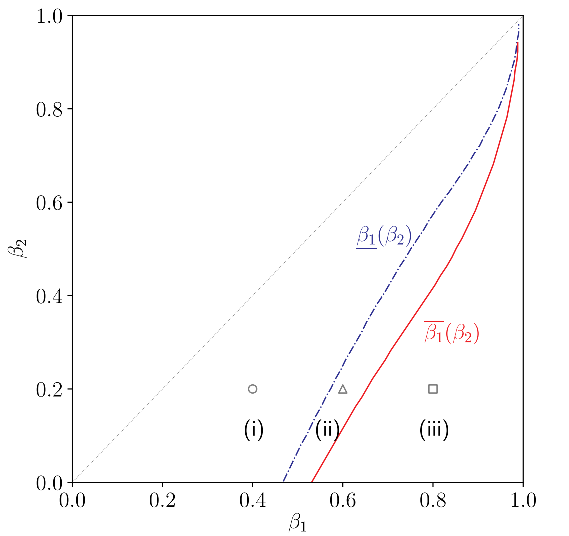
(iii) *If* $\overline{\beta_1}(\beta_2) < \beta_1 \leq 1$, *there exists a threshold* $\Delta_2 \in [s\Delta S(\hat{q}^*_{\beta_2}), s\Delta S(\hat{q}^*_{\beta_1})]$ *such that* $C \succeq P$ *when* $\Delta\rho(\Psi) \leq \Delta_2$ *and* $C \prec P$ *otherwise.*

(2) *If* $P = P(1)$, $C \succeq P$.

(3) *If* $P = P(2)$, $C \succeq P$ *iff* $\rho_{\beta_1}(\Psi) - (1-\nu)\rho_{\beta_2}(\Psi) \leq \rho_1(\phi(q_1^{\text{NW}})) - (1-\nu)\rho_{\beta_2}(\phi(\hat{q}^*_{\beta_2}))$.

Consider the case where the manufacturer trades with both types. Proposition 10(1) indicates that the manufacturer's preference between preorder and consignment contracts depends on both the disparity between two possible risk attitudes and the risk of outside opportunities. To explain this result, Figure 4 plots two thresholds

Figure 4. (Color online) Threshold Values $\underline{\beta_1}(\beta_2)$ and $\overline{\beta_1}(\beta_2)$



Notes. $s = 2.5$, $c = 0.5$, and $\nu = 0.5$. Both demand D and the profit of the outside opportunity Ψ follow the exponential distribution with $\mathbb{E}[D] = 10$. The downward-right side of the dashed diagonal line represents the feasible region defined by the constraint $\beta_1 > \beta_2$.

$\underline{\beta_1}(\beta_2)$ and $\overline{\beta_1}(\beta_2)$ and the resulting cases (i), (ii), and (iii). We only consider the downward-right side of the dotted diagonal line, which represents the feasible region defined by the constraint $\beta_1 > \beta_2$.

Compared with the preorder contract, the consignment contract always offers the first-best order quantity under complete information and therefore is more advantageous in terms of efficiency. It, however, needs to grant retailers higher information rent because it results in a pooling equilibrium. When the disparity between two risk attitudes is small (i.e., $\beta_2 < \beta_1 \leq \underline{\beta_1}(\beta_2)$, region (i) in Figure 4), the effect of efficiency dominates the effect of the larger information rent that the retailer receives, and the manufacturer prefers the consignment contract over the preorder contract.

When the disparity between two risk attitudes is large (i.e., $\underline{\beta_1}(\beta_2) < \beta_1 \leq 1$, regions (ii) and (iii) in Figure 4), the effect of the larger information rent becomes more pronounced. The magnitude of such an effect and the manufacturer's preference depend on the risk associated with the outside opportunity. If the risk is below a certain threshold, the effect of the larger information rent cannot offset the effect of efficiency, and the manufacturer still prefers the consignment contract. However, if the risk is above the threshold, the effect of the larger information rent dominates the effect of efficiency, and the manufacturer prefers the preorder contract. Regardless of the disparity between the two risk attitudes, the manufacturer always prefers the consignment contract over the preorder contract when the risk associated with the outside opportunity is relatively low (i.e., $\Delta\rho(\Psi) \leq s\Delta S(\hat{q}^*_{\beta_2})$).

Of particular interest is the case in which the disparity between two risk attitudes is sufficiently large (i.e., $\overline{\beta_1}(\beta_2) < \beta_1 \leq 1$) and the risk of the outside opportunity lies in an intermediate range (i.e., $\Delta\rho(\Psi) \in [\Delta_2, s\Delta S(\hat{q}^*_{\beta_1})]$). In this case, Proposition 5 already shows that the preorder contract induces the first-best order quantity and achieves zero information rent. Proposition 10(1)(iii) further indicates that the preorder contract can also bring a higher profit to the manufacturer than the consignment contract in this case.

Proposition 10(2) shows that the manufacturer never prefers trading only with the less risk-averse retailer using the preorder contract (i.e., P(1)). However, the manufacturer may prefer trading only with the more risk-averse retailer using the preorder contract when the risk of the outside opportunity is sufficiently high. The exact condition for this to happen is characterized in Proposition 10(3).

4.4. Comparative Statics

To provide further managerial insights, we discuss the effects of some key model specifications. In particular, we analyze the impact of the unit production cost and demand uncertainty on the contract results in Sections 4.4.1 and 4.4.2, respectively. As the manufacturer's trading strategy cannot be fully characterized by either the unit production cost or demand uncertainty, we can only analyze their effects given the trading option (i.e., P(1, 2), P(1), and P(2)) instead of analyzing its effect on the optimal trading strategy (i.e., P).

4.4.1. Unit Production Cost. We evaluate the effect of product profitability, as measured by the unit production cost c, on the contract order quantities and the firms' utility.

Proposition 11. *Under the preorder contract with either trading strategy, when the unit production cost c increases,*

(i) *The second-best order quantities decrease;*

(ii) *The manufacturer's profit strictly decreases, the less risk-averse retailer's utility decreases, and the more risk-averse retailer's utility increases.*

Under the consignment contract, when the unit production cost c increases,

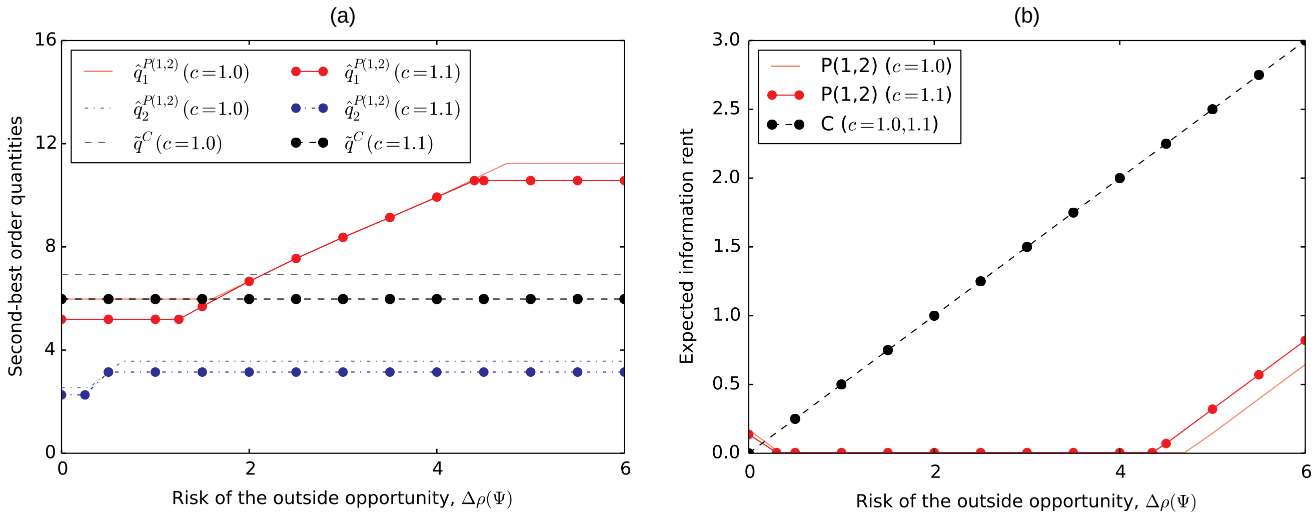
(iii) *The second-best order quantities strictly decrease;*

(iv) *The manufacturer's profit strictly decreases, and the retailer's utility remains constant.*

When the product is less profitable (i.e., the unit production cost c increases), under either contract, the contract order quantity decreases, as shown in Figure 5(a). (As the qualitative results with P(1) and P(2) are similar to those with P(1,2), Figure 5 does not plot the relevant results with P(1) and P(2) for clarity.) This is because the overage cost (i.e., the loss of the marginal profit due to overstocking) increases. In addition, the manufacturer's profit strictly decreases because of the reduced profit margin.

Under the preorder contract, whether a retailer's utility increases or decreases in the unit production cost,

Figure 5. (Color online) Impact of the Unit Production Cost, c



Notes. (a) Second-best order quantity. (b) Expected information rent. $\beta_1 = 0.9$, $\beta_2 = 0.6$, $\nu = 0.5$, and $s = 2$. Demand D follows the exponential distribution with $\mathbb{E}[D] = 10$.

however, depends on its type. The less risk-averse retailer's utility decreases in the unit production cost. The less risk-averse retailer earns an information rent when the risk of outside opportunities is low. In this case, a larger unit production cost reduces the misreporting incentive. Therefore, the manufacturer pays a smaller information rent. In contrast, the more risk-averse retailer's utility increases in the unit production cost. The more risk-averse retailer earns an information rent when the risk of outside opportunities is high. In this case, a larger unit production cost intensifies the more risk-averse retailer's misreporting incentive. Therefore, the manufacturer pays a larger information rent. The previous discussion reveals that the effect of the unit production cost on the information rent is not monotone and depends on the risk of outside opportunities. Under the consignment contract, the retailer's utility and information rent remain constant in c regardless of the retailer's type. The result regarding information rent is illustrated in Figure 5(b).

4.4.2. Demand Uncertainty.

We next study the effect of the increased demand uncertainty.

Proposition 12. *If the manufacturer trades with both retailer types with the preorder contract (i.e., P(1,2)), when the demand becomes more uncertain in the sense of the LIR order, the less risk-averse retailer's utility increases and the more risk-averse retailer's utility decreases when $\beta_2 \geq (1 - \nu c/s)\beta_1$. If the manufacturer trades with only one retailer type with the preorder contract (i.e., P(1) or P(2)) or uses the consignment contract, the retailer's utility remains constant when the demand becomes more uncertain.*

If the manufacturer trades with both retailer types with the preorder contract, the increased demand uncertainty may have the opposite effect on different retailers' profits. Specifically, when the demand becomes more uncertain, the less risk-averse retailer obtains a higher profit, whereas the more risk-averse retailer may obtain a lower profit. This is because in these cases, the increased demand uncertainty intensifies the less risk-averse retailer's misreporting incentive while it reduces the more risk-averse retailer's misreporting incentive. As a result, the manufacturer pays a larger information rent to the less risk-averse retailer and a smaller information rent to the more risk-averse retailer.

If the manufacturer trades with only one retailer type with the preorder contract, the retailer extracts no information rent. The retailer's utility then is the same as that under the outside opportunity and irrelevant to the demand uncertainty. Under the consignment contract, the retailer's utility is unaffected by the demand uncertainty because it is the manufacturer who shoulders the demand risk.

5. Extension to Coherent Risk Measures

In the previous analysis, we assume the retailer's risk measure is CVaR. This section shows that our result can be easily generalized to any coherent risk measure ρ, which is known by the retailer but unknown to the manufacturer. A risk measure is said to be coherent if it satisfies axioms of translation invariance, subadditivity, positive homogeneity, and monotonicity, which is consistent with the modern theory of economics and finance. Besides CVaRs, coherent risk measures include expectile, entropic value-at-risk, and mean-deviation and mean-upper-semideviation risk measures with certain parameters (Ahmadi-Javid 2012, Bellini and Di Bernardino 2017, Shapiro et al. 2021).

The manufacturer has a prior belief about the retailer's risk measure that $\rho = \rho_1$ with probability $\nu \in (0,1)$ and $\rho = \rho_2$ with probability $1-\nu$, where both ρ_1 and ρ_2 are coherent. Let $\rho_0(\cdot) = \mathbb{E}[\cdot]$ denote the risk measure for a risk-neutral firm. To make the analysis tractable, we use an equivalent representation of coherent risk measures in newsvendor problems (Ahmed et al. 2007), which is shown in the following lemma.

Lemma 1 (Ahmed et al. 2007). *Consider a random outcome X viewed as an element of a linear space $\mathcal{X}$ of measurable functions. Any coherent risk measure ρ is associated with a (convex) set $\mathcal{G}$ of probability measures, depending on the dual space to $\mathcal{X}$ such that the dual representation $\rho(X) = \sup_{\mathbb{P}\in\mathcal{G}}\mathbb{E}_{\mathbb{P}}[X]$ holds, and a CDF $\mathbb{Q} \in \mathcal{G}$ such that $\mathbb{Q}(x) = 0$ for any $x < 0$, and $\rho(\phi(q)) = \rho(s\min\{D,q\} - cq)$ and $S_\rho(q) \triangleq \rho(\min\{D,q\})$ can be written in the form $(s-c)q - s\int_0^q \mathbb{Q}(x)\mathrm{d}x$ and $q - \int_0^q \mathbb{Q}(x)\mathrm{d}x$,, respectively.*

By Lemma 1, we let $\mathbb{Q}_i$ denote CDFs associated with ρ_i $(i = \{0,1,2\})$ and satisfy the following condition, where $\mathbb{Q}_0 = F_D$.

Assumption 1. *Let $\mathbb{Q}_i^{-1}$ denote the right continuous inverse of $\mathbb{Q}_i$ $(i = \{0,1,2\})$. We assume* (1) $\mathbb{Q}_0^{-1}(t) = F_D^{-1}(t) > \mathbb{Q}_1^{-1}(t) > \mathbb{Q}_2^{-1}(t)$ *for any* $t \in (0,1)$, *and* (2) $\rho_1(\Psi) > \rho_2(\Psi)$.

In the centralized case, the first-best order quantity for a firm with risk measure ρ_i $(i \in \{0,1,2\})$ is given by $q_i^{\mathrm{NW}} \triangleq \mathbb{Q}_i^{-1}((s-c)/s)$. (With a slight abuse of notations but for notational simplicity, we redefine some notations (e.g., q_i^{NW}, $\hat{q}_i^*$, $\tilde{q}_i^*$) in this section with similar meanings as their use in previous sections as long as there is no confusion.) Therefore, Assumption 1(1) ensures that the retailer with risk measure ρ_1 is less (respectively, more) risk-averse and has a higher (respectively, lower) first-best order quantity in the centralized case than the retailer with risk measure ρ_2 (respectively, the risk-neutral retailer) regardless of values of s and c. Assumption 1(2) ensures that the retailer with risk measure ρ_1 is also less risk-averse than the retailer with risk measure ρ_2 in the face of the outside opportunity. We refer to the retailer with risk measure ρ_1 as the less risk-averse type and the retailer with risk attitude ρ_2 as the more risk-averse type.

Our base model with CVaR as the retailer's risk measure satisfies Assumption 1. To see this, consider the case where ρ_1 and ρ_2 are CVaRs with parameters β_1 and β_2, respectively. Their corresponding CDFs by Lemma 1 are $\mathbb{Q}_1(t) = \min\{F_D(t)/\beta_1, 1\}$ and $\mathbb{Q}_2(t) = \min\{F_D(t)/\beta_2, 1\}$. As $0 < \beta_2 < \beta_1 < 1$, $\mathbb{Q}_1$ and $\mathbb{Q}_2$ can be easily verified to satisfy Assumption 1.

Consider the decentralized case with complete information on ρ. Suppose $\rho = \rho_i$ $(i \in \{1,2\})$. We can verify that the optimal order quantity is $\hat{q}_i^* = q_i^{\mathrm{NW}}$ under the preorder contract and $\tilde{q}_i^* = q_0^{\mathrm{NW}}$ under the consignment contract, and retailers under both contracts extract no information rent.

In the decentralized case with asymmetric information, we derive the optimal contract terms with general coherent risk measures, which are presented in Section EC.5 of the Online Appendix and are omitted in the main text. Consistent with the base model, the manufacturer may achieve the first-best result and eliminate information rent when the risk associated with the outside opportunity is medium under the preorder contract. Under the consignment contract, the manufacturer achieves the first-best result but leaves information rent to the more risk-averse retailer.

Our main result that the expected order quantity under the preorder contract could be higher than that under the consignment contract still holds with coherent risk measures under a mild condition. To characterize the exact condition, let $\underline{q_1}(\mathbb{Q}_2) = \{q > 0 \mid \nu q + (1-\nu)\hat{q}_2^* = q_0^{\mathrm{NW}}\}$, whose existence can be easily verified. The value of $\underline{q_1}(\mathbb{Q}_2)$ represents the threshold of the less risk-averse retailer's order quantity to ensure the expected order quantity under the preorder contract higher than that under the consignment contract to be possible.

Proposition 13. *If there is a unique q that satisfies $\mathbb{Q}_1(q) - (1-\nu)\mathbb{Q}_2(q) = \nu(s-c)/s$, the expected order quantity under the preorder contract is (strictly) higher than that under the consignment contract if and only if*

1. *The disparity between the two risk measures is sufficiently large; that is, $\mathbb{Q}_1(\underline{q_1}(\mathbb{Q}_2)) \le (1-\nu)\mathbb{Q}_2(\underline{q_1}(\mathbb{Q}_2)) + \nu(s-c)/s$,*
2. *The risk of outside opportunities is sufficiently high; that is, $\Delta\rho(\Psi) > s\Delta S(\underline{q_1}(\mathbb{Q}_2))$, and*
3. *The manufacturer trades with both types under the preorder contract.*

The assumption that there is a unique q satisfying $\mathbb{Q}_1(q) - (1-\nu)\mathbb{Q}_2(q) = \nu(s-c)/s$ ensures the manufacturer's optimization problem is convex when the risk of outside opportunities is sufficiently high. A sufficient condition for this assumption is that $\mathbb{Q}_1(q) - (1-\nu)\mathbb{Q}_2(q)$ is monotone, which can be easily verified given the exact function form of $\mathbb{Q}_1(q)$ and $\mathbb{Q}_2(q)$.

Proposition 13 generalizes our results in Proposition 9. When the retailer uses coherent risk measures, Proposition 13 shows that the push contract can induce a higher expected order quantity than the pull contract when the two possible risk measures are sufficiently distinct, the outside opportunity is sufficiently risky, and the shutdown policy is not used.

6. Conclusion

Motivated by the exacerbated risk in retail supply chains, we analyze how to allocate inventory risk under imperfect information regarding the retailer's risk attitude and uncertain outside opportunities. We focus on push (i.e., preorder) and pull (i.e., consignment) contracts, the former of which allocates demand risk to the downstream retailer, whereas the latter allocates risk to the upstream manufacturer.

Our paper enriches the literature that compares the push and pull risk allocation schemes. We find that a push contract can lead to a higher order quantity than a pull contract. This result is not new to the literature, but the rationale behind it is new. Yang et al. (2018) identified the same phenomenon when the manufacturer is sufficiently more risk-averse than the retailer. We show that even if the manufacturer is risk-neutral such a result can still arise and is robust to the generalization of the retailer's risk measure from CVaR to any coherent risk measure because of the manufacturer's imperfect information on the retailer's risk attitude. To screen the retailer's risk attitude when the risk of the outside opportunity is high, the manufacturer has to inflate the order quantity for the less risk-averse retailer, which may lead to a higher order quantity than that under the pull contract. This result is particularly relevant for small- and medium-sized retailers who have diverse risk attitudes and are experiencing exacerbated risk since the outbreak of the COVID-19 pandemic. Contrary to conventional wisdom, our result suggests making such small- and medium-sized retailers bear more inventory risk may bring up inventory levels along the supply chain, raise product availability to consumers, and boost consumption.

In addition, we show that the push contract may lead to a higher manufacturer profit than the pull contract when the retailer's risk attitude is dispersed, and the outside opportunity is highly uncertain. An important implication is that the elevated risk brought by the COVID-19 pandemic may make the push contract not only more advantageous in terms of increasing the inventory level but also more attractive to the manufacturer. When the risk of the outside opportunity is in an intermediate range, the push contract can entirely eliminate the information rent. In contrast, this cannot be achieved by the pull contract as long as the outside opportunity is random. These results are robust with or without the shutdown policy.

Last, the impact of product profitability and demand uncertainty is identified. Under the push contract, the less (more) risk-averse retailer extracts more (less) rent when the product becomes more profitable or the demand becomes more uncertain. Under the pull contract, the utility of both retailer types is unaffected by product profitability or demand uncertainty as the manufacturer owns the inventory.

Our paper has several limitations. First, large manufacturers may also be risk-averse, and their risk-aversion level is not publicly known. In that scenario, the manufacturer must signal its own risk-aversion level while eliciting the retailer's private information. Although challenging, incorporating the manufacturer's private risk-aversion level helps us glean insights into how the risk attitude and information structure among firms in a supply chain affect the optimal contract choice and the manufacturer's tradeoff between signaling and screening. Besides, companies may use other common approaches to model their risk aversion in practice, such as assuming concave utility functions (Agrawal and Seshadri 2000), using mean-variance risk measures (Chen and Seshadri 2006), and imposing downside risk constraints (Gan et al. 2005). Analyzing these alternative modeling approaches can help us understand the effect of different risk models on the optimal contract choice.

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