PM520–Homework 1

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1 Introduction

Monte Carlo methods (or Monte Carlo experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; typically one runs simulations many times over in order to obtain the distribution of an unknown probabilistic entity. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to obtain a closed-form expression, or infeasible to apply a deterministic algorithm. Monte Carlo methods are mainly used in three distinct problem classes: optimization, numerical integration and generation of draws from a probability distribution.

The number π is a mathematical constant, the ratio of a circle's circumference to its diameter, commonly approximated as 3.14159. It is a commonly used mathematical constant in many areas. In the current homework, we propose to estimate π, using Monte Carlo methods. Such method counts the fraction of dots fall within a circle that inner-touches a square to estimate the probability that one dot falls within the circle. And the estimated π is four times of that fraction according to the following formula (1). We also plan to investigate the effect of factors, such as radius of the circle, number of trials and the seed number, which may impact the accuracy of the estimation.

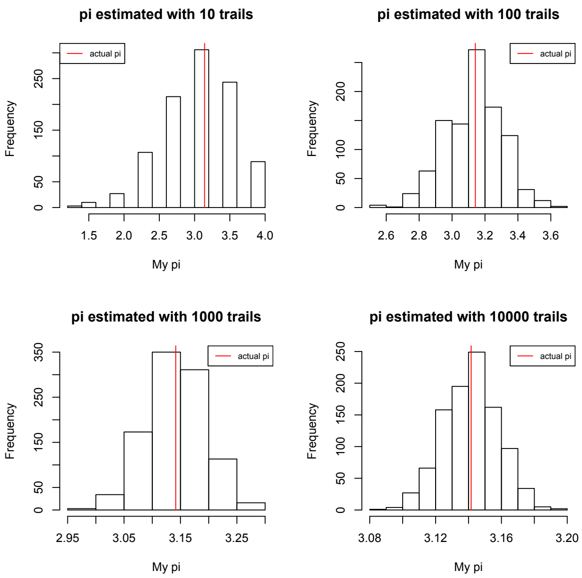
$$\begin{aligned} P(\text{a dot falls within the circle}) &= \frac{\pi * r^2}{(2r)^2} \\ P(\text{a dot falls within the circle}) &= \frac{\pi}{4} \\ \pi &= P(\text{a dot falls within the circle}) * 4 \end{aligned} \tag{1}$$

2 Method

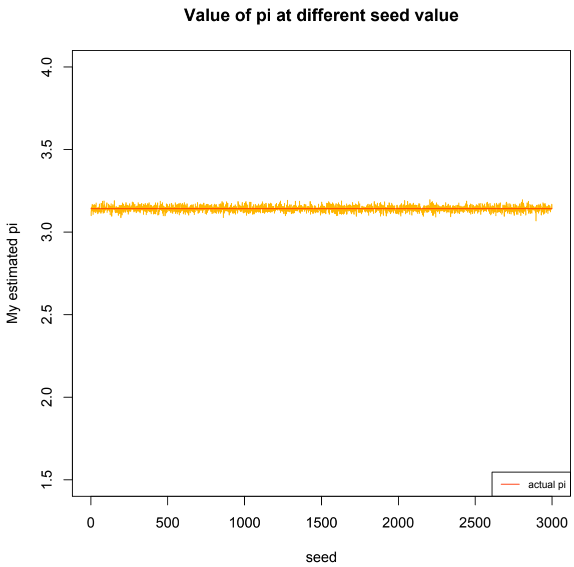
To estimate π, we randomly generated a set of points within a square and count how many points fall within the circle that inner-touches the square. The number of points randomly generated is denoted as "Number of Trails". The radius of the circle is denoted as "Radius". In addition, in most random functions, the random number generated is controlled by its seed. Here, we denote this as "Seed". To investigate the effect of these three variables on accuracy on estimating π, we generated π at different sets of parameters as follows. To investigate the influence of "Number of Trails", we estimated π at different "Number of Trails", including 10, 100, 1000 and 10000, each for 10000 times and examined their distribution. To investigate the influence of "Number of Trails" and "Radius" together, we estimated π at different combinations of "Number of Trails" and "Radius". For "Number of Trails", we selected 10, 100, 1000 and 10000 and for "Radius", we selected from 1 to 100. To investigate the influence of "Seed" together, we estimated π at different "Seed" from 1 to 3000 and examined the value of π generated.

3 Result

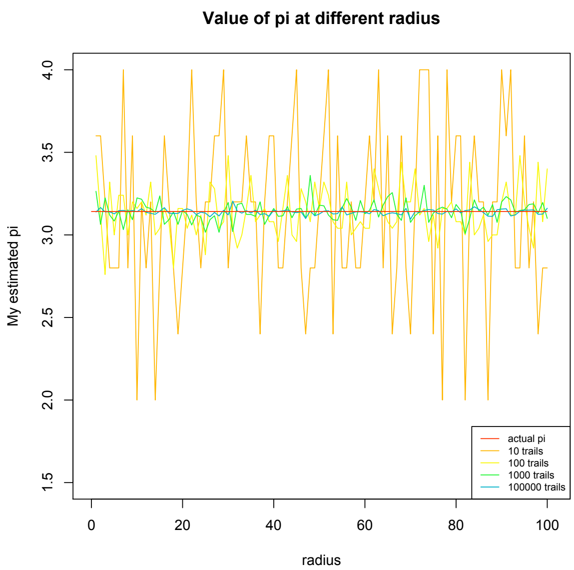
Following three figures demonstrated the effect of different factors on estimated π. From figure 1(a), we can see when we increase "Number of Trails" from 10, 100, 1000 to 10000, the estimated π gets more and more concentrated in the real value. From figure 1(b) we can see when we change "Seed" from 1 to 3000, even though we can observe certain fluctuation, the estimated π still distributes around the real value and that the variability does not increase with the increase of "Seed". From figure 1(c), we can see when we increase "Radius" from 1 to 100, the pattern of estimated π does not change at the same level in "Number of Trails". However, when we increase of trails, the variability of the estimated π decreases.



(a) Effect of Number of Trails on π



(b) Effect of Seed value on π



(c) Effect of Radius at different Number of Trails on π

Figure 1: The effect of different factors on estimated π

4 Conclusion

Here we applied Monte Carlo methods and estimated π by counting the faction of dots falling within a circle inner-touching a square. We found that increase "Number of Trails" reduces the variability of the estimated π, "Seed" has no obvious effect of estimated π. We also found that at the same level of "Number of Trails", "Radius" has no obvious effect on estimated π, but when we increase "Number of Trails", the variability reduces. Therefore, we recommend applying Monte Carlo method π estimation with large "Number of Trails" to ensure accuracy.