

2.4.5-Repn-CPDs-loglinear

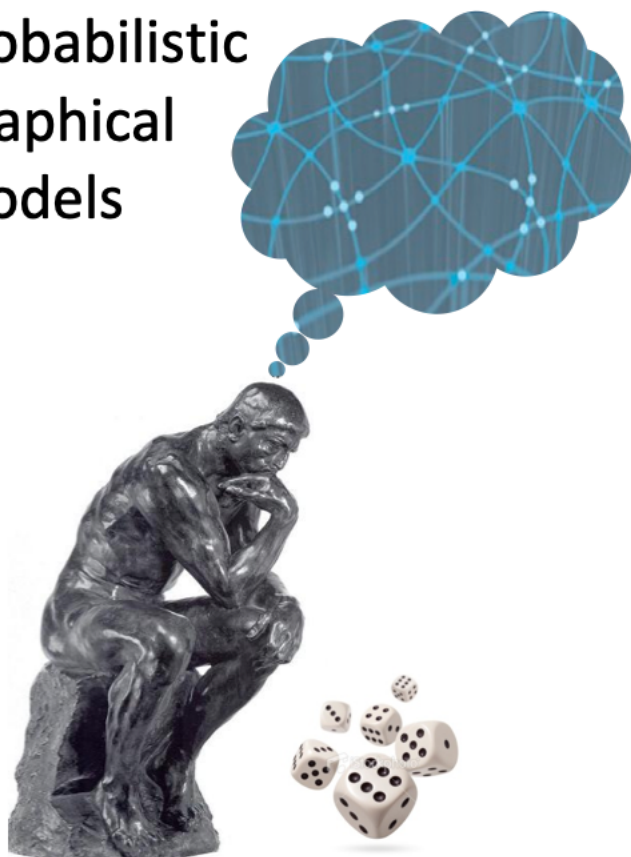
Tuesday, September 10, 2019

5:02 PM



2.4.5-Repn-
CPDs...

Probabilistic
Graphical
Models



Representation

Local Structure

Log-Linear

Models

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Log-Linear Representation

$$\tilde{P} = \prod_i \phi_i(D_i) \rightarrow \tilde{P} = \exp \left(- \sum_j \overset{\text{coeff}}{\underbrace{w_j}_{f_j}} f_j \right)$$

$$\tilde{P} = \prod_j \exp(-w_j f_j)$$

- Each feature f_j has a scope D_j
- Different features can have same

Representing Table Factors

$$\phi(X_1, X_2) = \begin{pmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{pmatrix}$$

$$f_{12}^{00} = \mathbf{1}\{X_1 = 0, X_2 = 0\}$$

$$f_{12}^{01} = \mathbf{1}\{X_1 = 0, X_2 = 1\}$$

$$f_{12}^{10} = \mathbf{1}\{X_1 = 1, X_2 = 0\}$$

$$f_{12}^{11} = \mathbf{1}\{X_1 = 1, X_2 = 1\}$$

General representation

TION

sublinear

$$\phi_j(D_j)$$

features

$$\phi_j(D_j)$$

factor

scope

tors

$$\{x_2 = 0\}$$

1

0

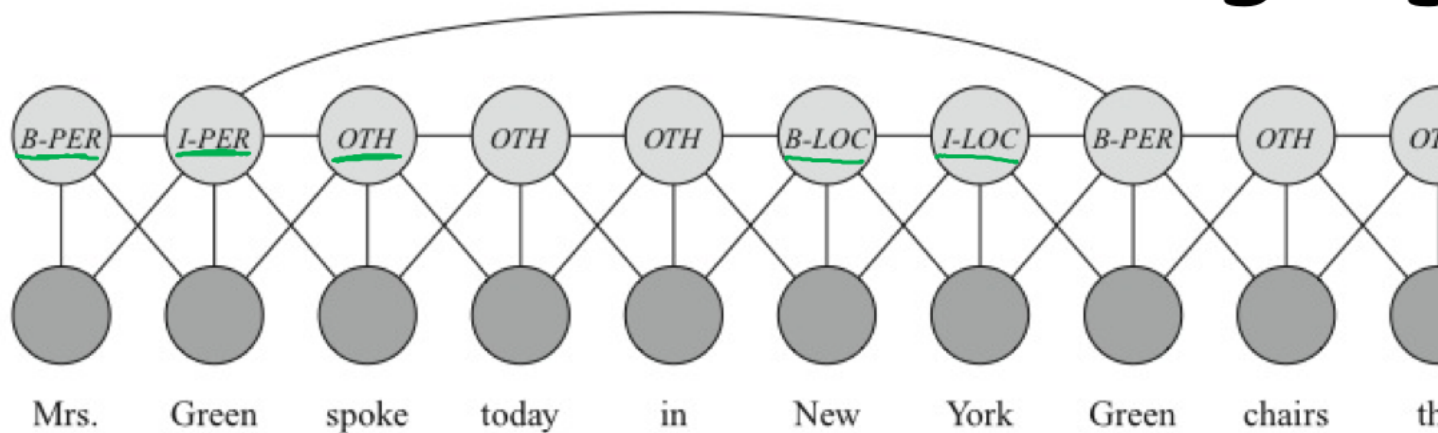
$$\{x_2 = 1\}$$
$$\{x_2 = 0\}$$
$$\{x_2 = 1\}$$

$$\phi(X_1, X_2) = \exp(- \sum_{kl} w_{kl} f_{ij}^{kl}(X_1, X_2))$$

$$w_{kl} = -\log a_{kl}$$

$\exp(-w_{00})$ when
 $\exp(-w_{01})$ when

Features for Language



Features: word capitalized, word in atlas or name list, previous word is "Mrs", next word is "Times", ...

$$f(x_i, x_{i+1}) = \begin{cases} 1 & \text{if } x_i = \text{person, } x_{i+1} \text{ is capitalized} \\ 1 & \text{if } x_i = \text{B-loc, } x_{i+1} \text{ appears in Atlas} \end{cases}$$

Ising Model

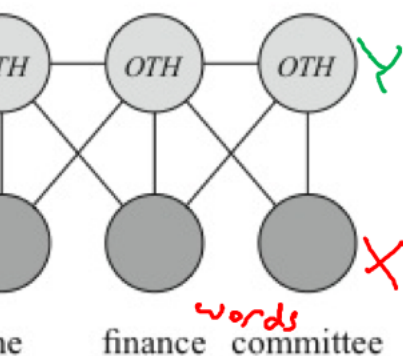
$$E(x_1, \dots, x_n) = - \sum_{i < j} w_{i,j} x_i x_j - \sum_i h_i x_i$$

$(x_1, x_2))$

$x_1=0, x_2=0$
 $x_1=0, x_2=1$

3
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previous word is

}

4
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wise

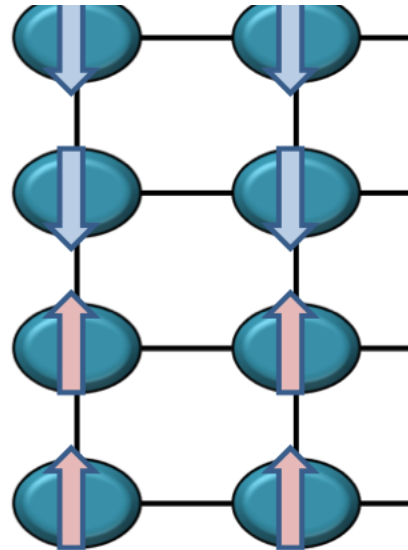
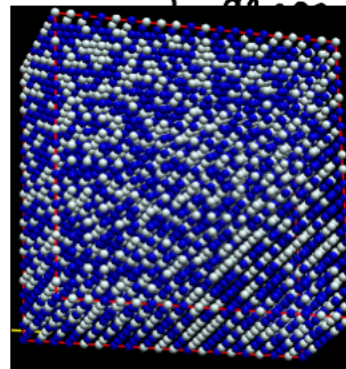
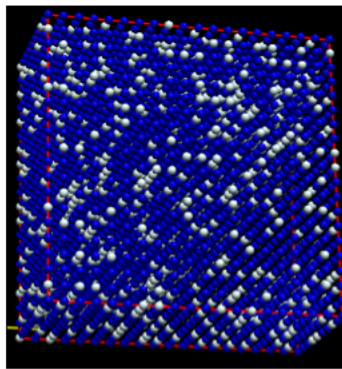
joint spins
 $u_i x_i$



$$\underline{x_i \in \{-1, +1\}} \quad f_{i,j}(X_i, X_j) = \underline{X_i \cdot X_j}$$

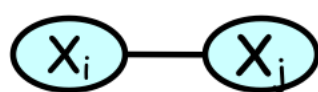
$$P(\mathbf{X}) \propto e^{-\frac{1}{T} E(\mathbf{X})} \quad \frac{w_{ij}}{T} \rightarrow 0$$

T grows



Metric MRFs

- All X_i take values in label space V

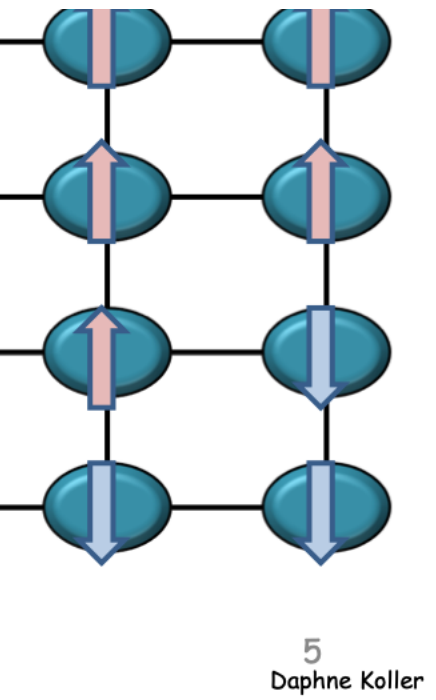


want X_i and X_j to take "similar" values

- Distance function μ : $V \times V \rightarrow \mathbb{R}^+$

- metric
- Reflexivity: $\mu(v, v) = 0$ for all v
 - Symmetry: $\mu(v_1, v_2) = \mu(v_2, v_1)$ for all v
 - Triangle inequality: $\mu(v_1, v_2) \leq \mu(v_1, v_3) + \mu(v_3, v_2)$ for all v_1, v_2, v_3



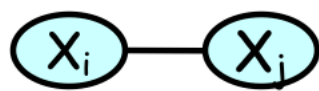


$$v_1, v_2 \left. \vphantom{\begin{matrix} v_1, v_2 \\ \end{matrix}} \right\} \text{semi metric}$$

$$) + \mu(v_3, v_2)$$

Metric MRFs

- All X_i take values in label space V



want X_i and X_j to take "similar" values

- Distance function $\mu : V \times V \rightarrow \mathbb{R}$

$$f_{i,j}(X_i, X_j) = \mu(X_i, X_j)$$

$$\exp(-w_{ij} f_{ij}(X_i, X_j)) \quad w_{ij} > 0$$

values of X_i and X_j far in μ



lower probability

lower
higher
lower

Metric MRF Example

$$\mu(v_k, v_l) = \begin{cases} 0 & v_k = v_l \\ 1 & \text{otherwise} \end{cases}$$

$|v_k - v_l|$

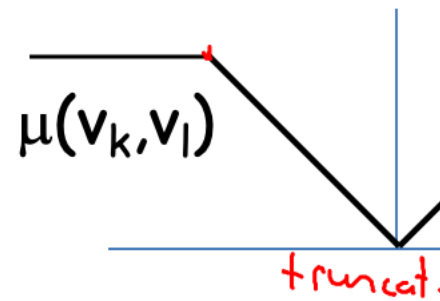
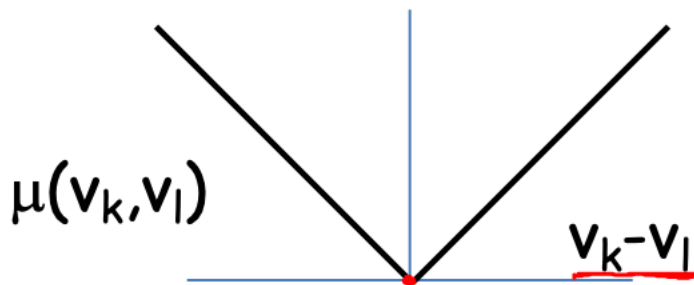
$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

distance
n
fisher probability
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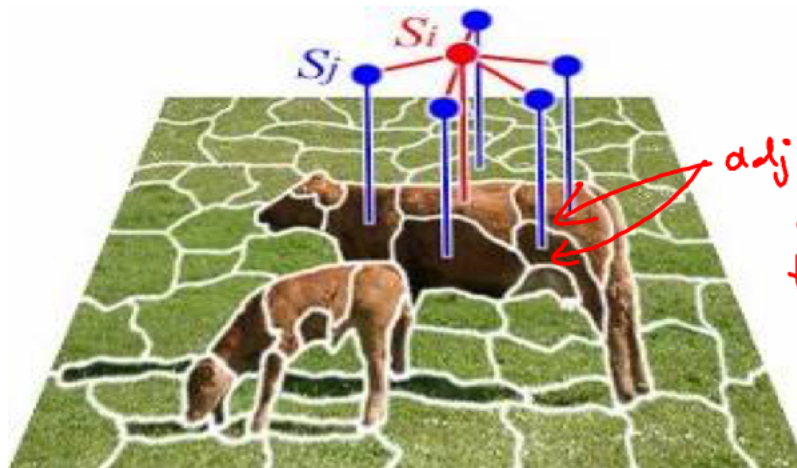
$\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$



Metric MRF: Segmentation

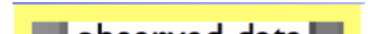
$$\mu(v_k, v_l) = \begin{cases} 0 & v_k = v_l \\ 1 & \text{otherwise} \end{cases}$$

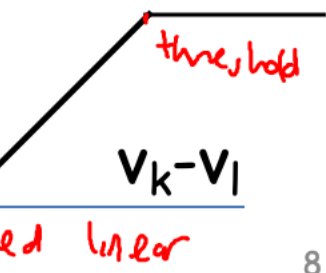
$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$



adjacent
suppixels
take same class

Metric MRF: Denoising





8
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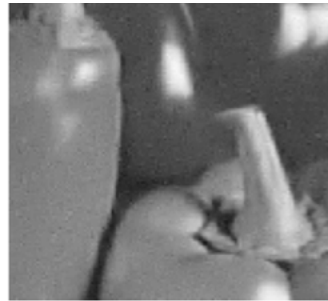
tion

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

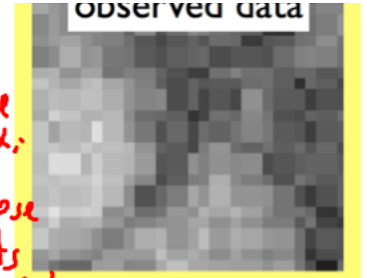
9
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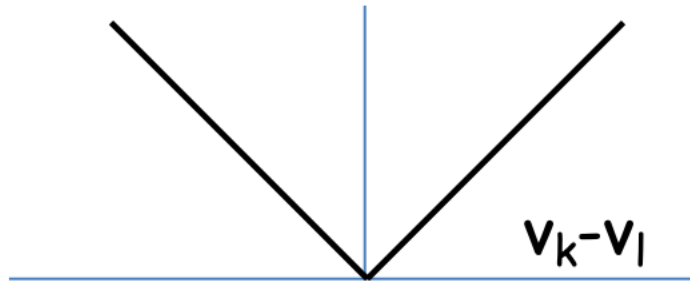


v_i close to x_i
 v_i close to its neighbors

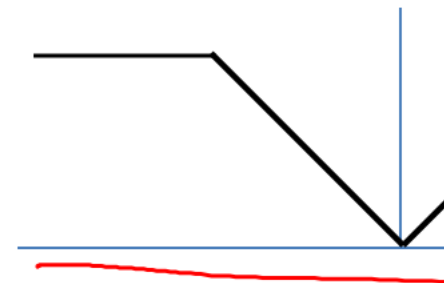


$\mu(v_k, v_l) = |v_k - v_l|$

\times noisy pixels
 \checkmark clean pixels



$\mu(v_k, v_l) = \max$



Similar idea for stereo reconstruction

reconstructed image



$$\kappa(|v_k - v_l|, d)$$

