Week 10 - Predictive modeling

L10-02. Predictive modeling – Regression

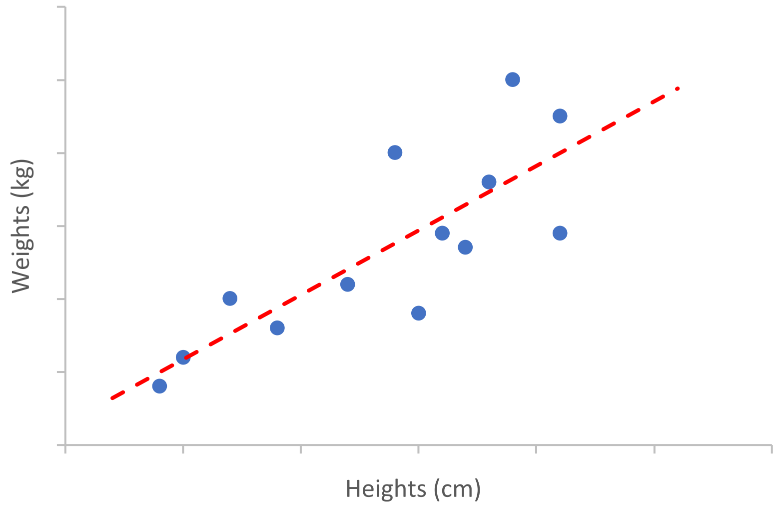
Soo Ahn Lee
Ph.D. student

30 April 2021

Simple linear regression

Linear regression: find a linear relationship between the independent variable and the dependent variable

(ex) linear relationship between the height and the weight

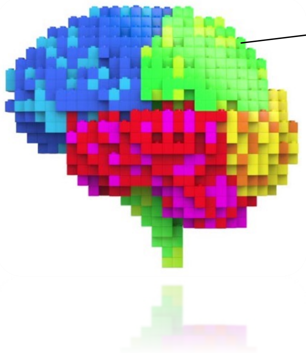


x: independent variable, predictor
$\hat{y}$: dependent variable, outcome
β: coefficients (β_0: intercept, β_1: slope)

$$\hat{y} = \beta_0 + \beta_1 \cdot x$$

Known as 'univariate' regression

Multivariate regression



one voxel = one predictor

When it becomes multivariate …

$$\hat{y} = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot {x_2}^2 + \beta_3 \cdot \sqrt{x_3} + \ \ldots$$

$$= ???$$

Curse of dimensionality!

Low interpretability

However, some of voxels may be dependent to other voxels

Moreover, we may not need all of these voxels as predictors

Dimension reduction (PCA, SVD, etc.)

Regularization methods for linear models

Multivariate predictive model example

Neurological Pain Signature (Wager et al., 2013, NEJM)

The NEW ENGLAND JOURNAL of MEDICINE

ORIGINAL ARTICLE

An fMRI-Based Neurologic Signature of Physical Pain

Tor D. Wager, Ph.D., Lauren Y. Atlas, Ph.D., Martin A. Lindquist, Ph.D., Mathieu Roy, Ph.D., Choong-Wan Woo, M.A., and Ethan Kross, Ph.D.

ABSTRACT

BACKGROUND

Persistent pain is measured by means of self-report, the sole reliance on which hampers diagnosis and treatment. Functional magnetic resonance imaging (fMRI) holds promise for identifying objective measures of pain, but brain measures that are sensitive and specific to physical pain have not yet been identified.

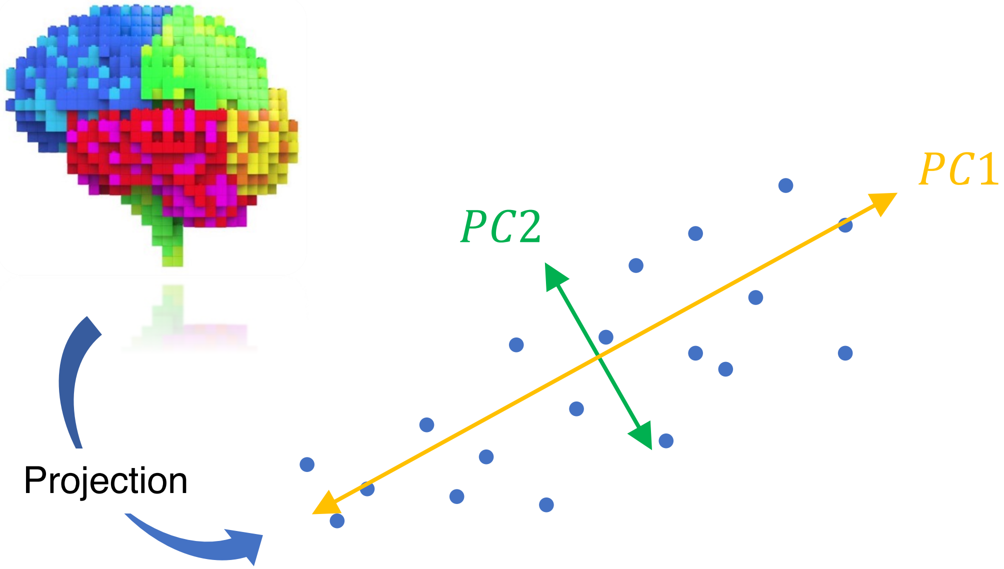
METHODS

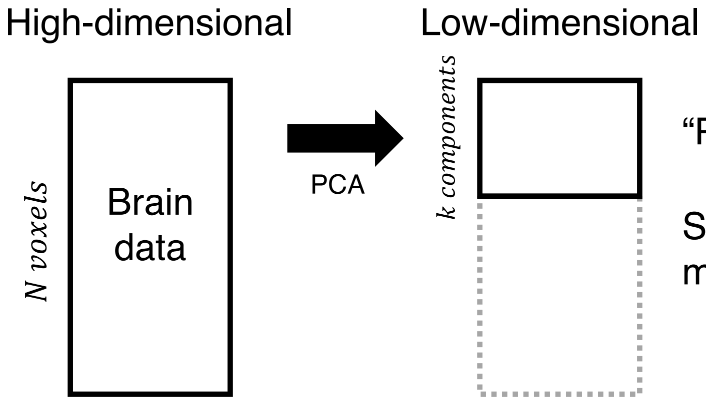
Multivariate predictive model example

Neurological Pain Signature (Wager et al., 2013, NEJM)

LASSO-PCR? ➔ Regression using principal components regularized by L1 norm

PCA for dimension reduction





“Parsimonious” model

Still preserve
most of the variance!

Multivariate predictive model example

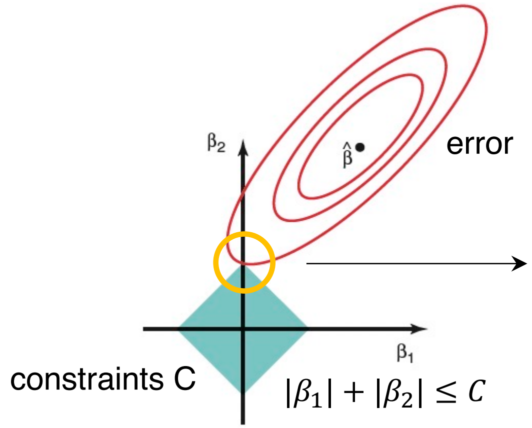
Neurological Pain Signature (Wager et al., 2013, NEJM)

LASSO (least absolute shrinkage and selection operator)

: nonlinearly constrains to make some parameters as 0 → make a simple, interpretable model

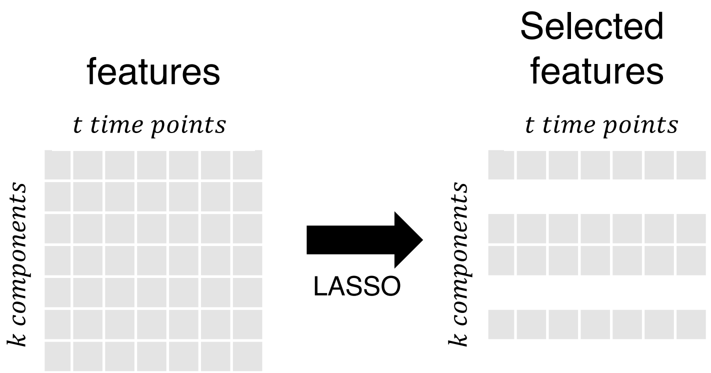
$$y: f(x, \beta)$$

$$minimize\ (y_{actual} - y_{estimated})^2\ subject\ to\ \sum_{i=1}^{p} |\beta_i| \leq C$$



some features should become to be 0

Cf.) ISLR 7th edition



Useful materials for machine learning:

Winner of the 2014 Eric Ziegel award from Technometrics.

As the scale and scope of data collection continue to increase across virtually all fields, statistical learning has become a critical toolkit for anyone who wishes to understand data. *An Introduction to Statistical Learning* provides a broad and less technical treatment of key topics in statistical learning. Each chapter includes an R lab. This book is appropriate for anyone who wishes to use contemporary tools for data analysis.

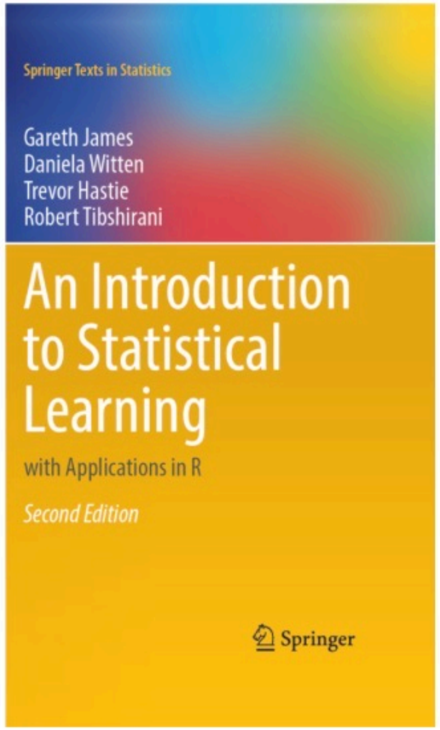
The book has been translated into Chinese, Italian, Japanese, Korean, Mongolian, Russian and Vietnamese.

The First Edition topics include:

- Sparse methods for classification and regression
- Decision trees
- Boosting
- Support vector machines
- Clustering

The Second Edition adds:

- Deep learning
- Survival analysis
- Multiple testing
- Naive Bayes and generalized linear models
- Bayesian additive regression trees
- Matrix completion



Lectures: https://www.youtube.com/watch?v=5N9V07EIfIg&list=PLOg0ngHtcqbPTIZzRHA2ocQZqB1D_qZ5V

Multivariate predictive model example

Neurological Pain Signature (Wager et al., 2013)

Training & validation: leave-one-subject-out cross-validation

Training set

(N -1)

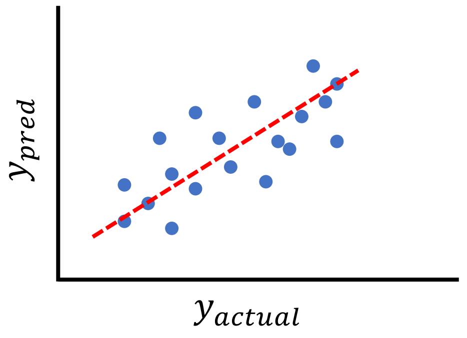
Pain report
(N-1)

LASSO-PCR

Weight map

$Weights$

X N times



Validation set
k[th] subject

$Predictor$

$n\ voxels$

$\cdot$

Weight map

$n\ voxels$

$=$

R_{pred}

$$Prediction\ error = \sum (y_{actual} - y_{pred})$$

$$Prediction\ r = corr(\overrightarrow{y_{actual}}, \overrightarrow{y_{pred}})$$

CANlab toolbox

You can do this by using predict.m function implemented in CanlabCore toolbox!

```
[~, stats] = predict(dat_sweet_roi, 'algorithm_name', 'cv_lassopcr', 'lasso_num', 5, 'nfolds', 5, 'error_type', 'mse', 'numcomponents', 10);
```

Descriptions:

```
%   **cv_lassopcr:**
%        Cross-val LASSO-PCR; can enter 'lasso_num' followed by components to retain by shrinkage
%        NOTE: can enter 'EstimateParams' to use shrankage
%        lasso method based on the estimated optimal lambda
%        that minimizes the mean squared error (MSE) of nested
%        cross-validation models. Output of nested cv model is
%        saved in stats.other_output_cv{:,3}. Output includes
%        'Lambda' parameter and min MSE value.
```