Week 14 Dimensionality Reduction

L14-00 Overview

Dong Hee Lee
Master's Student

Where We Are Now

Multivariate analyses

Exploratory techniques

- Identify robust patterns of covarying neural activity
  - **Principal Component Analysis (PCA)**
  - **Independent Component Analysis (ICA)**
  - **Non-Negative Matrix Factorization (NNMF)**

Dong Hee

- Possibly relating these patterns to design variables and/or behavior
  - **Canonical Correlation Analysis (CCA)**
  - **Partial Least Squares (PLS)**

Jungwoo

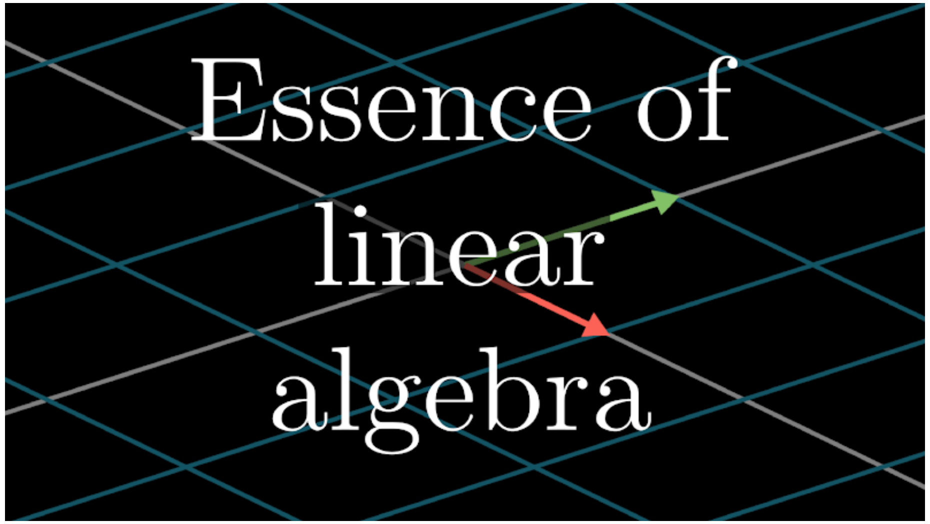
Confirmatory techniques

- An explicit model of regional interactions is formulated and tested to see whether it fits the data and/or whether it fits the observed data better than alternative models
  - **Structural Equation Modeling (SEM)**
  - **Dynamic Causal Modeling (DCM)**

(McIntosh, A. R., & Mišić, B., 2013)

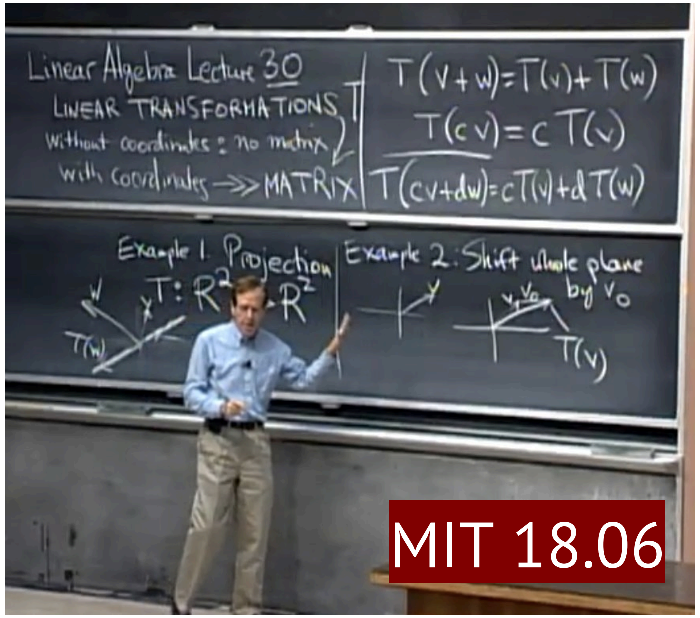
Brief Introduction of Linear Algebra

3Blue1Brown



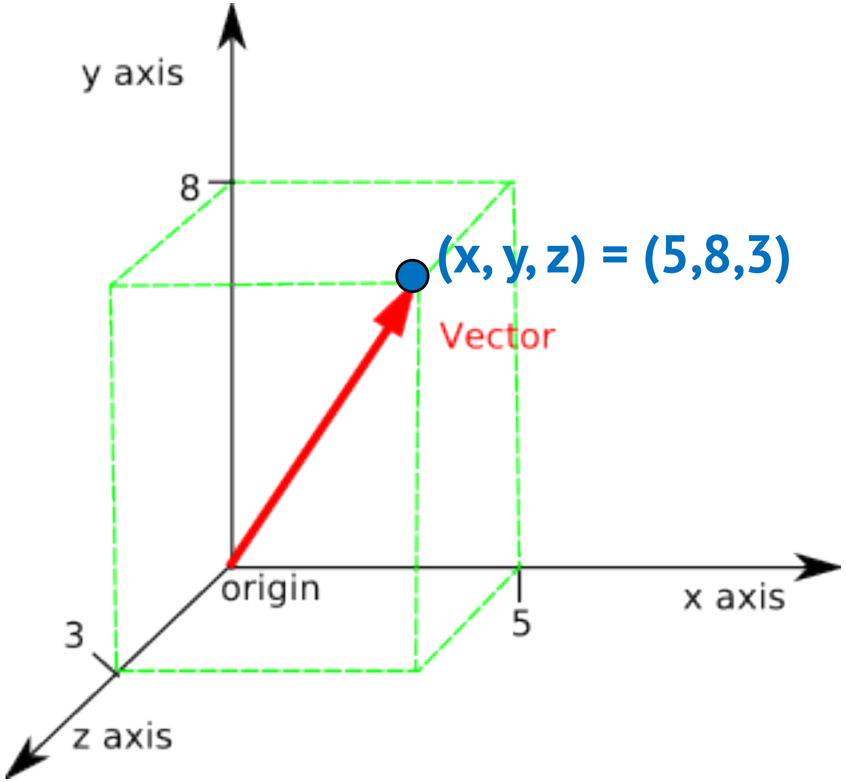
[Link](#)

Prof. Gilbert Strang



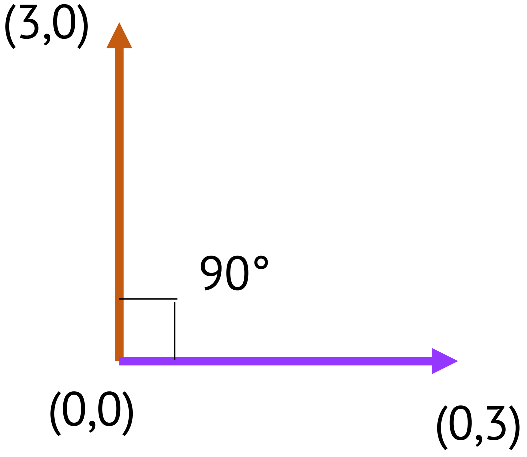
[Link](#)

■ Geometry Meaning of vectors



■ Orthogonality

Geometric interpretation

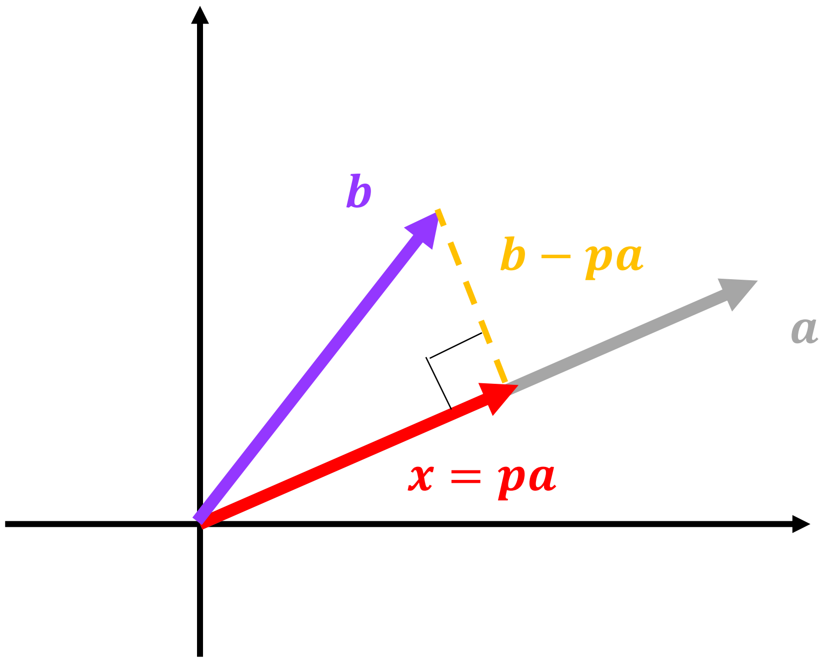


Algebraic interpretation

$$w^T v = 0$$

$$\begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = 0$$

■ Projection



$$(b - pa)^T a = 0 \implies b^T a - pa^T a = 0 \implies p = \frac{b^T a}{a^T a}$$

$$x = pa = \frac{b^T a}{a^T a} a$$

If a is a unit vector, then

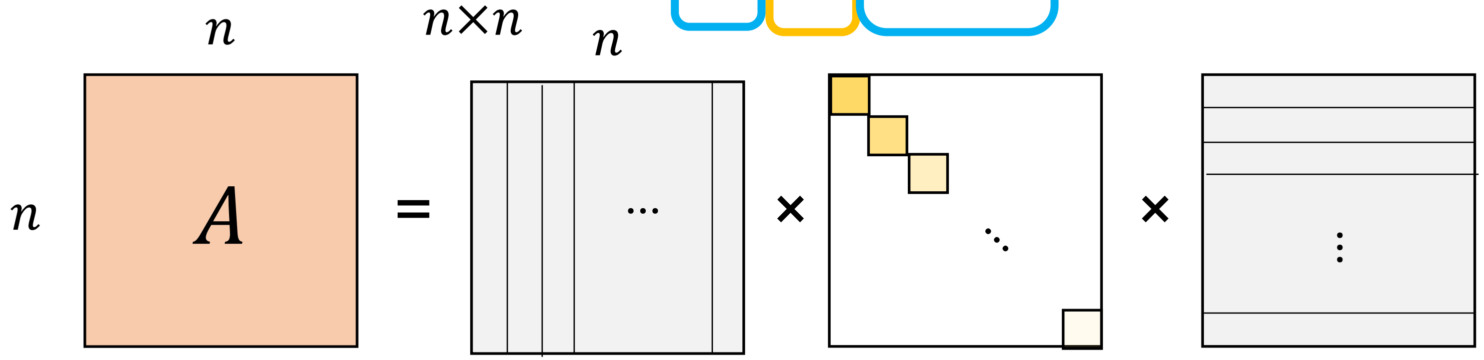
$$p = b^T a \implies x = pa = (b^T a)a$$

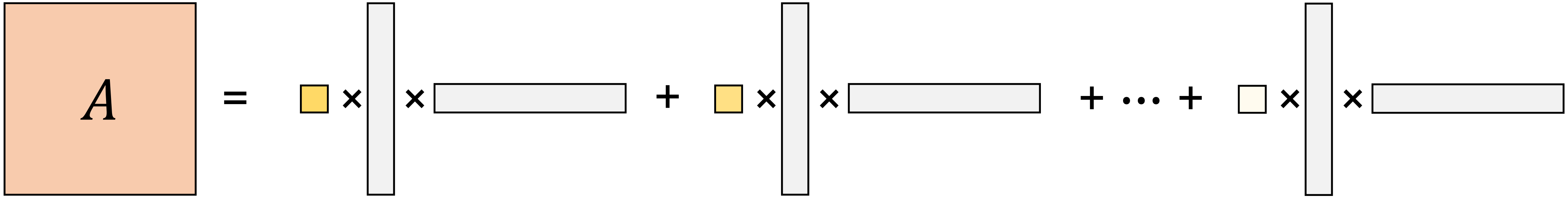
■ Eigendecomposition

Matrix Factorization

eigenvectors eigenvalue

$$\underset{n\times n}{\boldsymbol{A}} = \boxed{\boldsymbol{V}}\boxed{\boldsymbol{\Lambda}}\boxed{\boldsymbol{V}^{-1}}$$





Brief Introduction of Linear Algebra

■ Singular Value Decomposition (SVD)

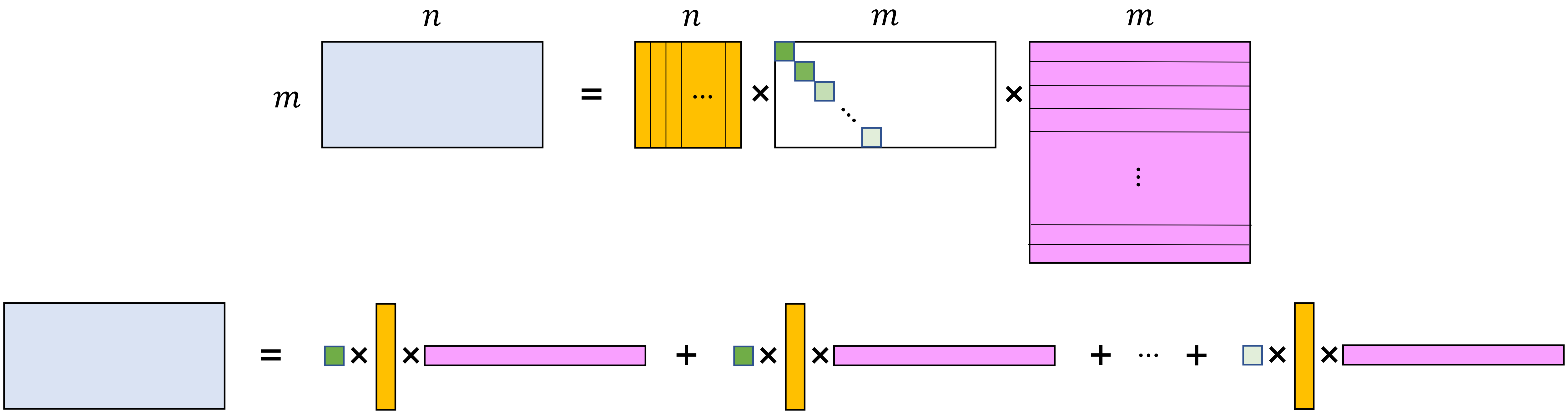
Matrix Factorization

Left singular vectors

Singular values

Right singular vectors

$$\underset{m\times n}{\boldsymbol{X}} = \boxed{\boldsymbol{U}}\,\boxed{\boldsymbol{\Sigma}}\,\boxed{\boldsymbol{V}^{\boldsymbol{T}}}$$

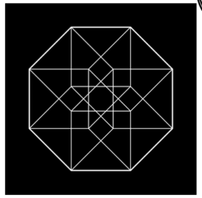


The Necessity of Dimensionality Reduction

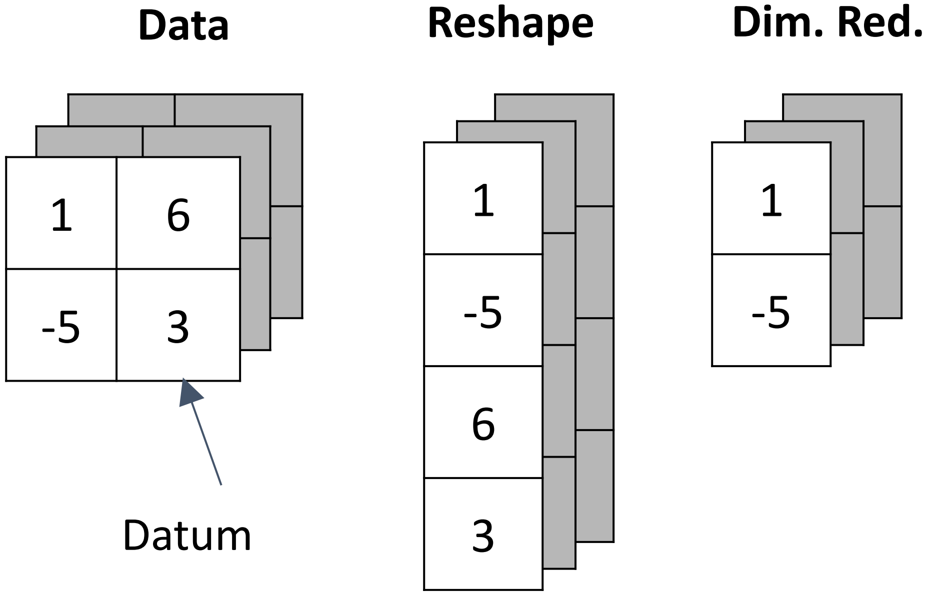
Each datum is a vector with *m* values a.k.a. dimensions

m = # pixels (256^2)

m = # voxels (10^5)



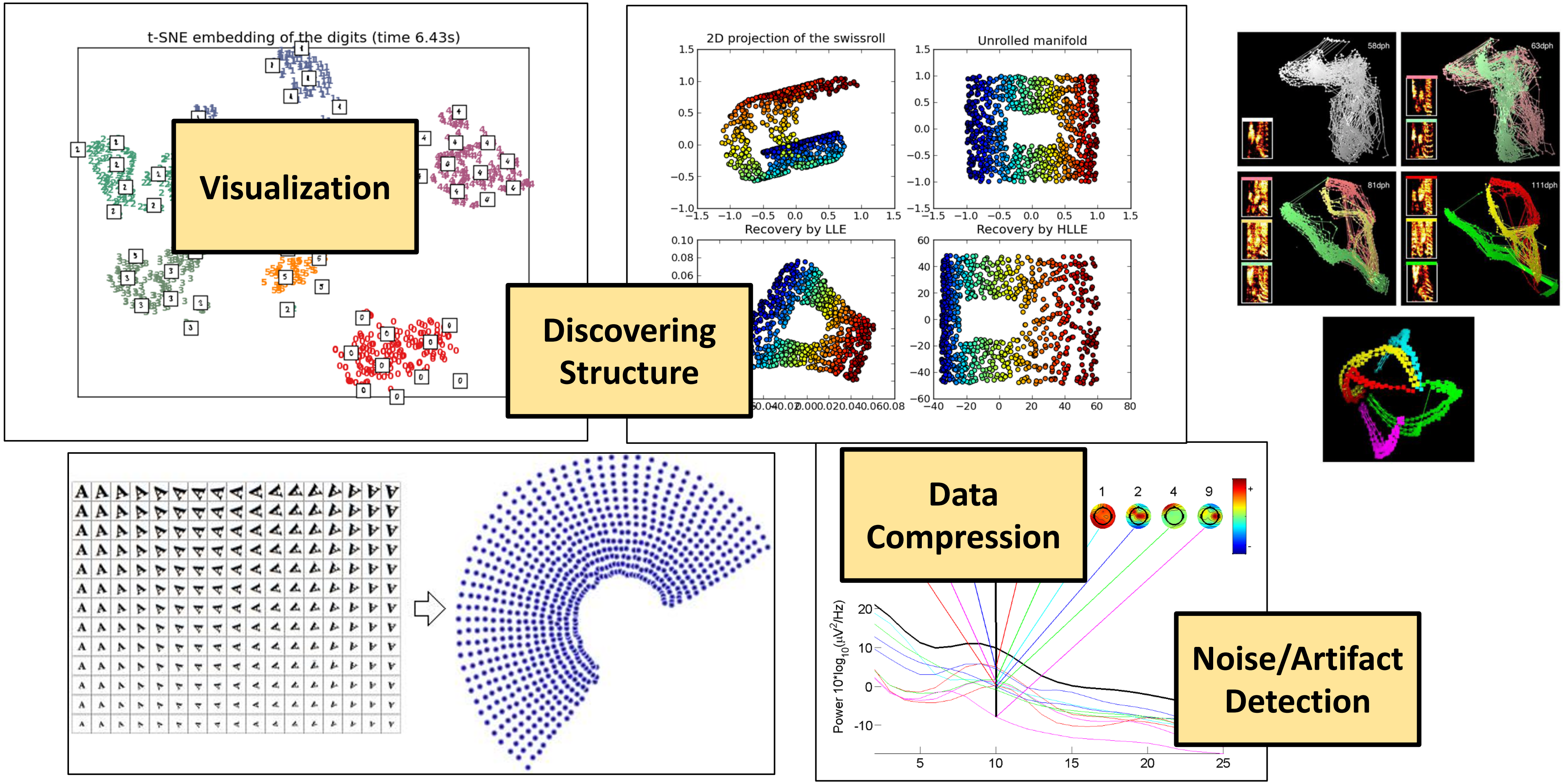
m = # features (??)



<u>Dimensionality Reduction</u>
A procedure that decreases a dataset's dimensions from *m* to *n*, *n* < *m*.

(Mackevicius & Ciccarelli, 2016)

The Necessity of Dimensionality Reduction



(Mackevicius & Ciccarelli, 2016)

Bonus Tip

😂 😂 😂

Warning!

- For non-mathematicians, this is going to be tougher than the previous material.
  - You may have to spend a long time studying the next two parts.
- If you are not used to thinking about hyper-planes in high-dimensional spaces, now is the time to learn.
- To deal with hyper-planes in a 14-dimensional space, visualize a 3-D space and say "fourteen" to yourself very loudly. Everyone does it.
  - But remember that going from 13-D to 14-D creates as much extra complexity as going from 2-D to 3-D.

(Hinton, 2013)

Reference

- McIntosh, A. R., & Mišić, B. (2013). Multivariate Statistical Analyses for Neuroimaging Data. *Annual Review of Psychology*, *64*(1), 499–525. https://doi.org/10.1146/annurev-psych-113011-143804
- Mackevicius, E. & Ciccarelli, G. (2016). Dimensionality Reduction I [PowerPoint Slides]. *Center For Brains, Minds, and Machines*. https://cbmm.mit.edu/learning-hub/tutorials/computational-tutorial/dimensionality-reduction-i
- Hinton, G. (2013). Neural Networks for Machine Learning. *Coursera*.