Week 6 – First-level fMRI data analysis

L06-01. HRF modeling – Details of example methods

Byeol, Hongji, and Jungwoo

2 April 2021

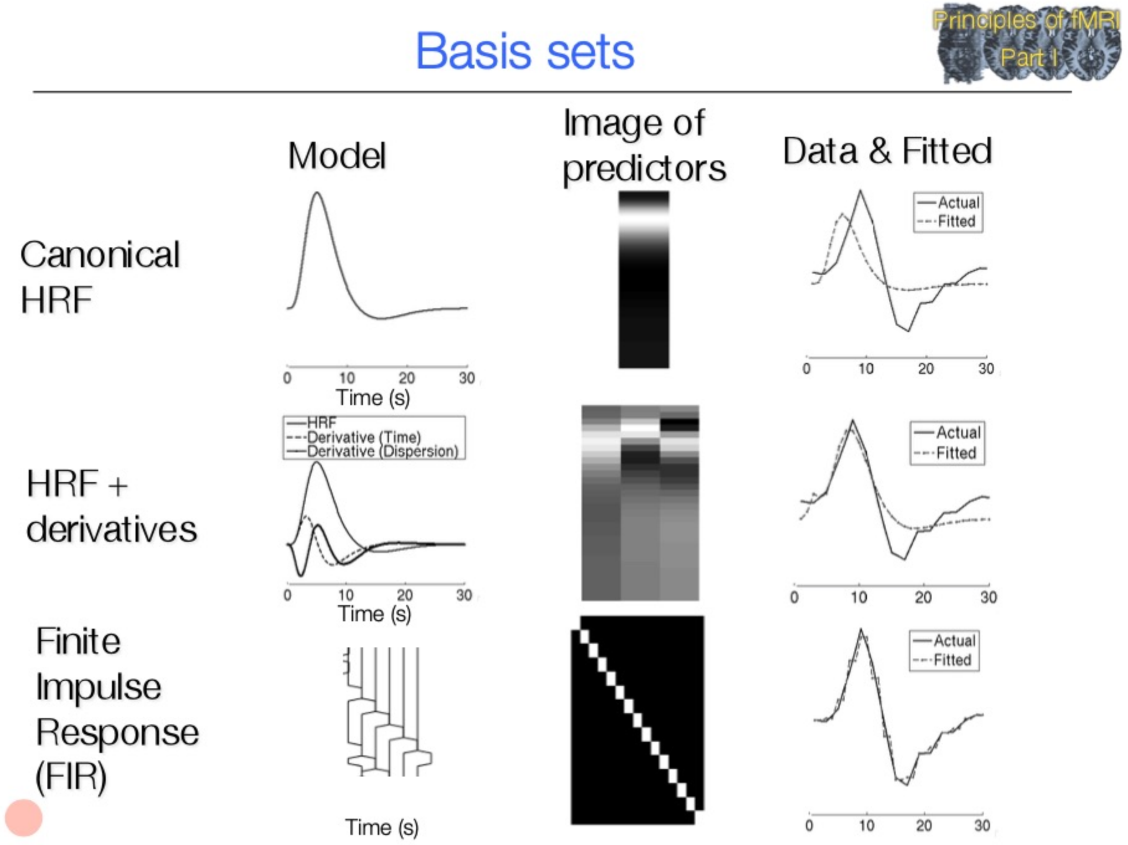
HRF modeling

Examples…

1) FIR model → Already Discussed!

2) Inverse Logit

3) HRF & derivatives



Contents credit: Tor Wager

HRF modeling

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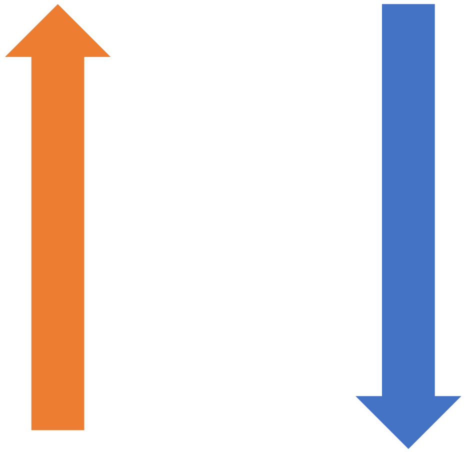
Higher variance
(require more parameters)

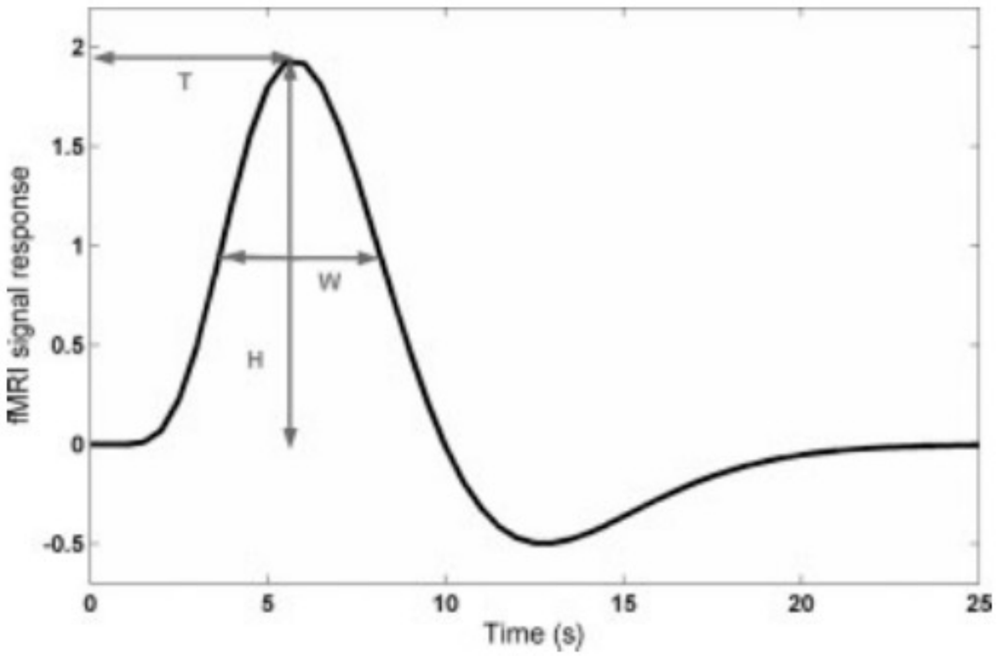


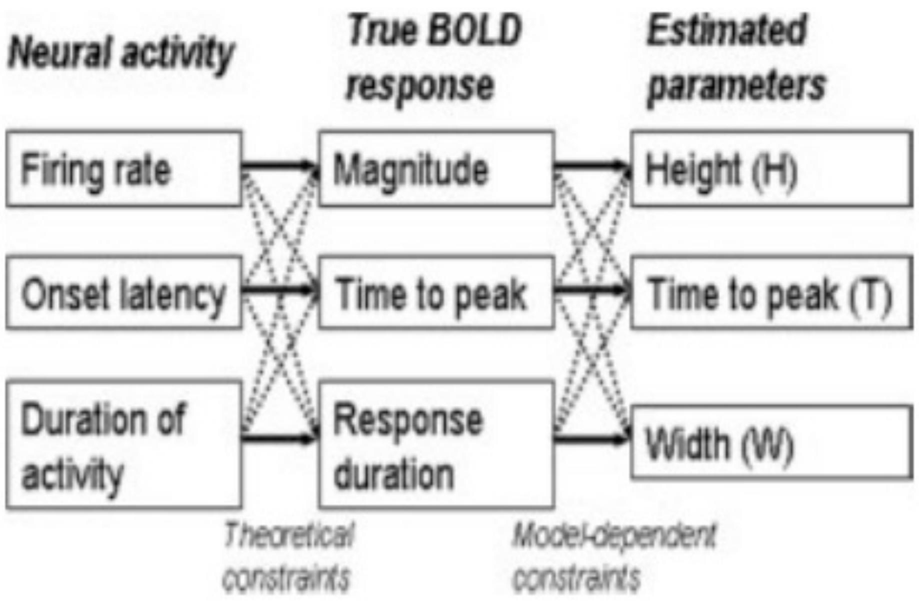
Higher bias
(require less parameters)

Inverse Logit

Validity and Power in Hemodynamic Response Modeling: A Comparison Study and a New Approach

Martin A. Lindquist[1*] and Tor D. Wager[2]



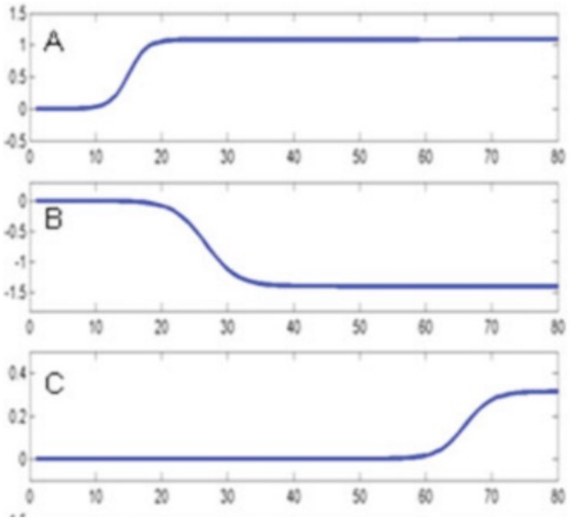


HRF modeling

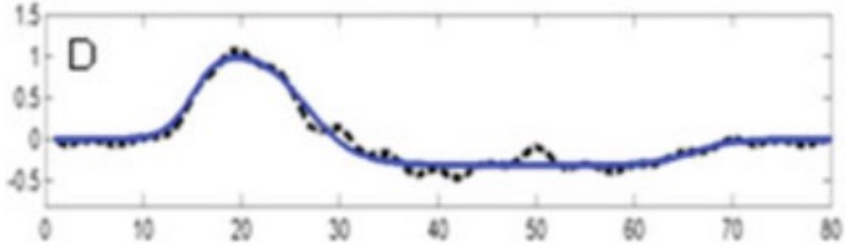
Inverse Logit

$$h(t|\theta) = \alpha_1 L((t - T_1)/D_1) + \alpha_2 L((t - T_2)/D_2) + \alpha_3 L((t - T_3)/D_3)$$

where $L(x) = \dfrac{1}{1+e^x}$, $\theta = \{\alpha, D, T\}$



Modeling 3 functions
to make 1 HRF function



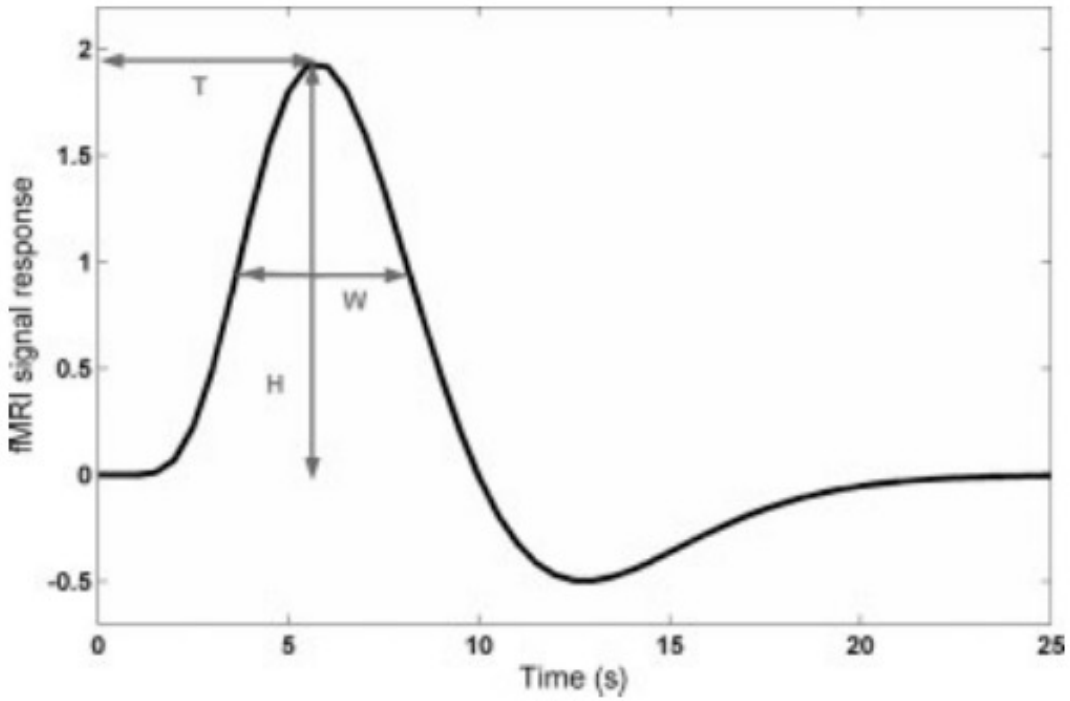
Require 7 parameters to determine!
(9 → 7 by constraints)

Implemented in canlab toolbox (https://github.com/canlab/CanlabCore/tree/master/CanlabCore/HRF_Est_Toolbox2)

HRF & Derivatives

Nonlinear Event-Related Responses in fMRI

Karl J. Friston, Oliver Josephs, Geraint Rees, Robert Turner



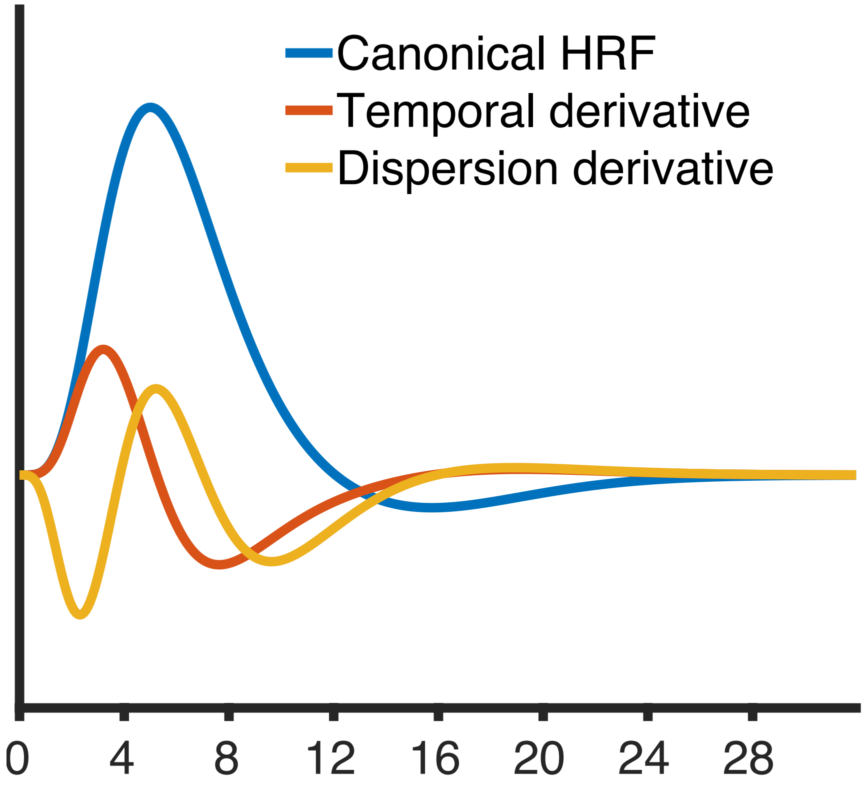
Purpose of using HRF & derivatives is same with using IL.

To capture…

- Height
- Time to peak
- Width

But in more direct way!

HRF & Derivatives



Single canonical HRF,
Time to peak & Width is *fixed*.
→ Only possible to capture Height diffrences.

Simply add…
Temporal Derivatives & Dispersion derivatives

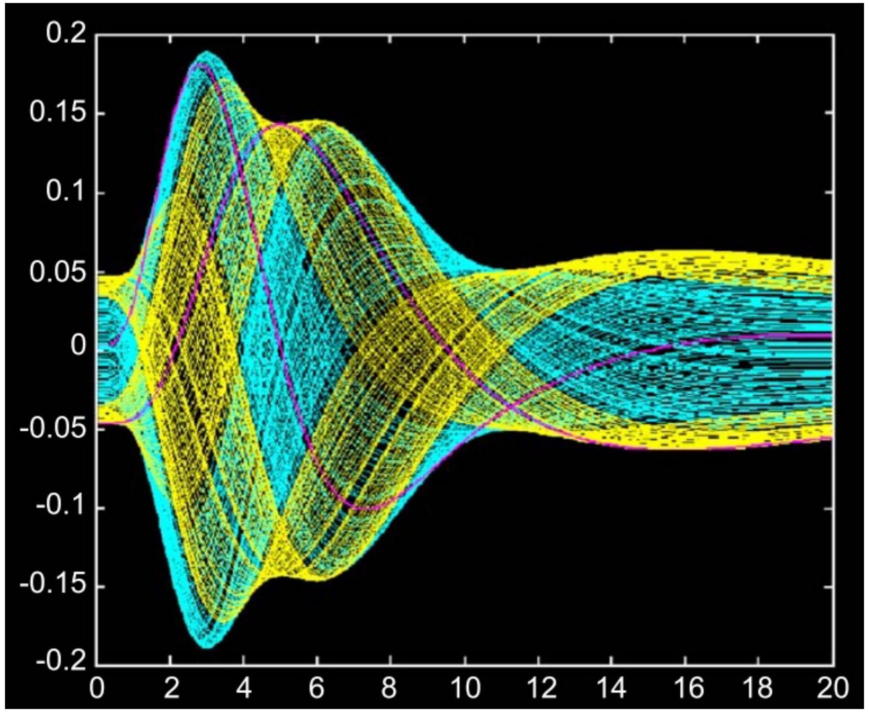
Temporal derivatives : Capture variation of time to peak.
Dispersion derivatives : Capture Variation of width.

Possible to use only two of them. (Usually HRF & Temporal derivs)

Temporal derivative is literally derivative of canonical HRF …
Dispersion derivate … comes from Volterra kernel of HRF …!
(https://en.wikipedia.org/wiki/Volterra_series)

HRF & Derivatives

Then… what would happen?



Calhoun et al. Neuroimage. 2003

Some possible ranges of shape that could be modelled!

Lot more flexible than using single HRF!

HRF & Derivatives

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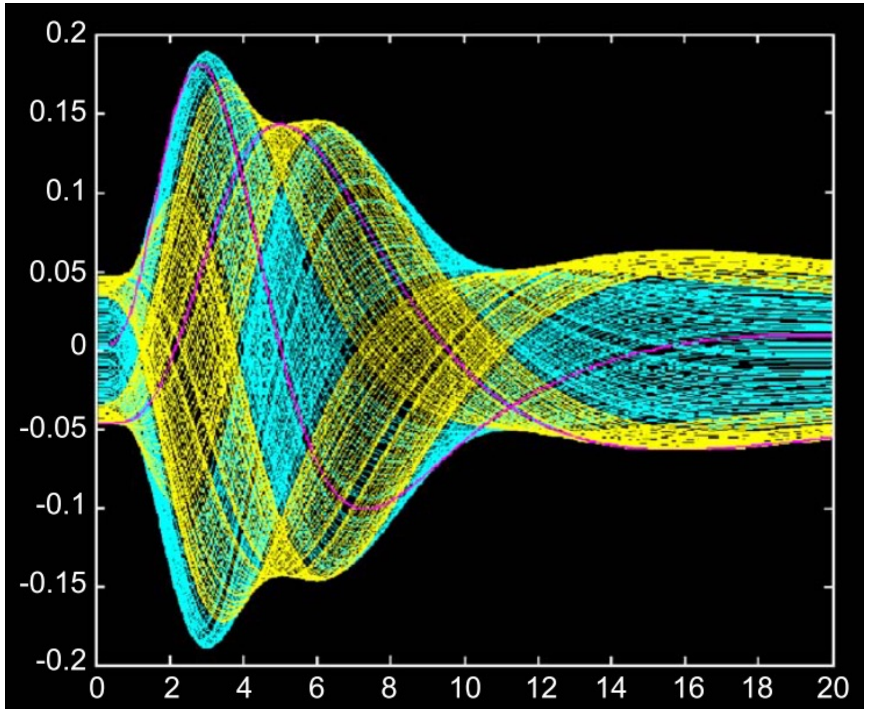
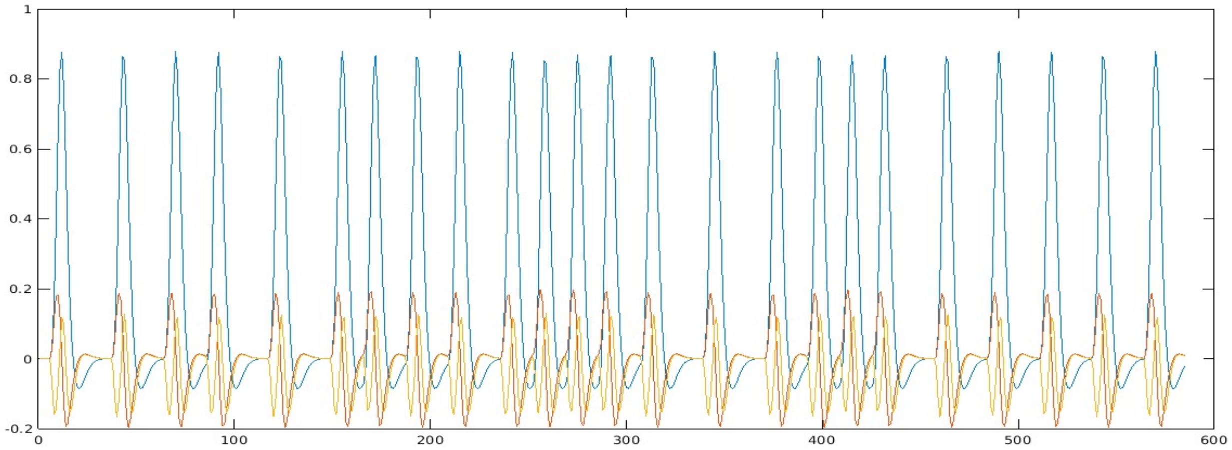


Calhoun et al. Neuroimage. 2003

Some possible ranges of shape that could be modelled!

Lot more flexible than using single HRF!

When using in actual regressors… it looks like



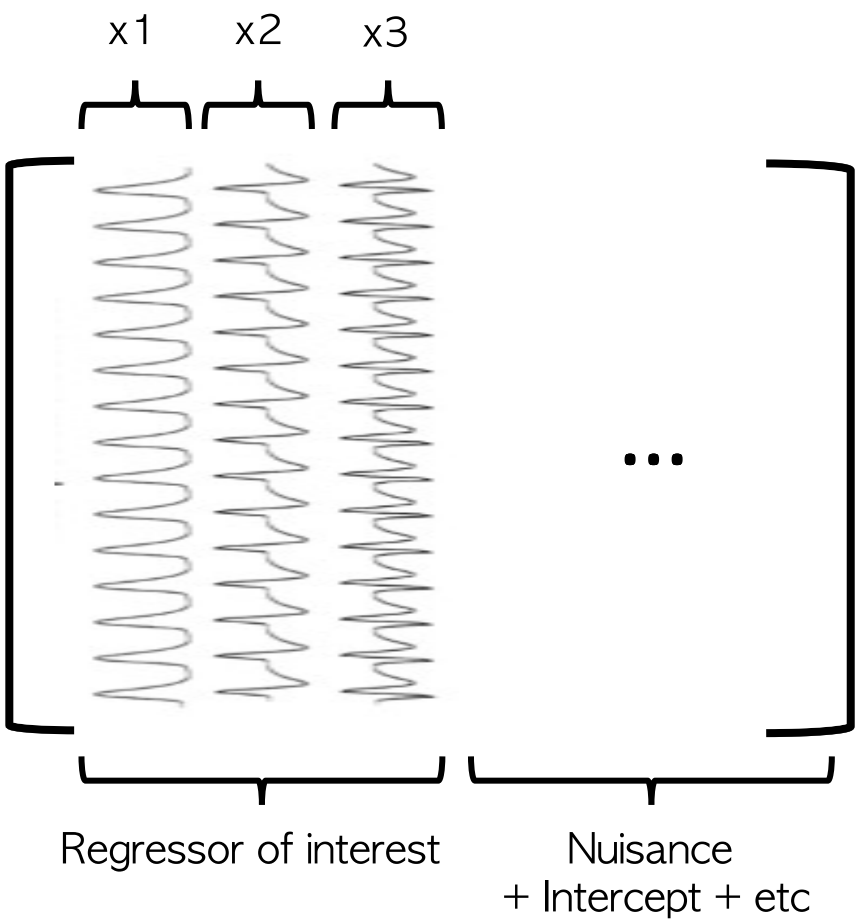
HRF & Derivatives

Then…how do we make single GLM beta out of 3 regressors?

x1 : Canonical   x2 : Temporal derivs.   x3 : Dispersion derivs.

Just using first canonical HRF as beta?
→ Same with other two basis functions as nuisance variables

Just linear sum of three betas?
→ Could be biased because of the sign of each betas.
(e.g. 5, -2, -3 would result in zero…)



Design Matrix

HRF & Derivatives

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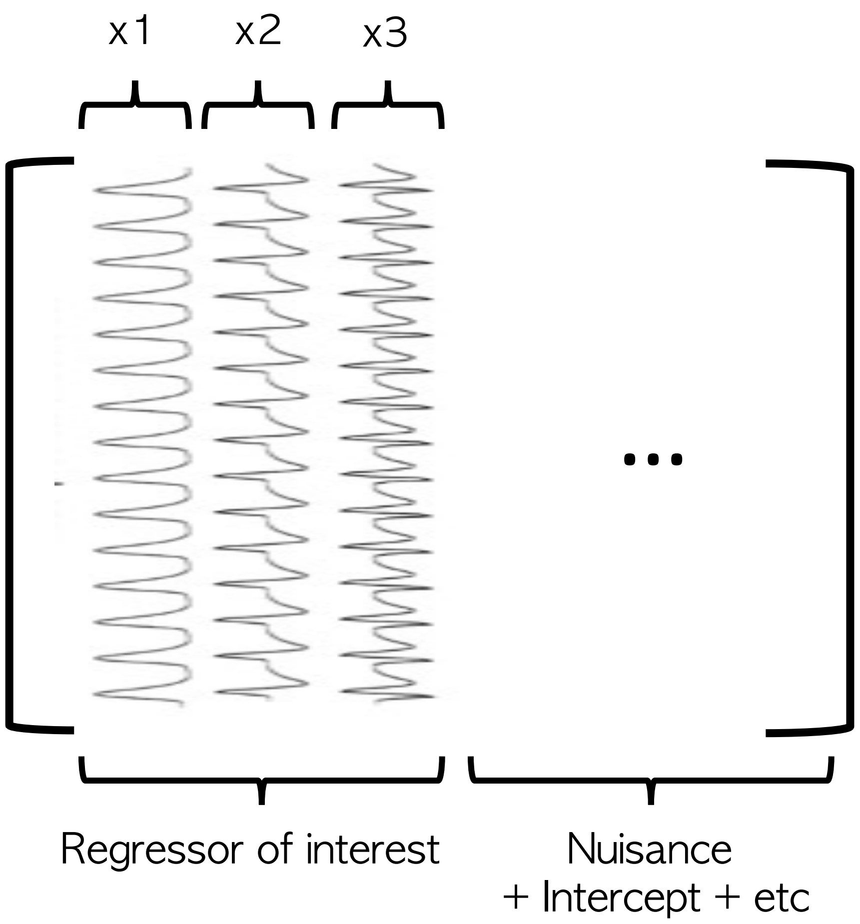
Then How…?



Design Matrix

HRF & Derivatives

Derivative boost

$$y_t = \hat{\beta}_0 + \hat{\beta}_1 x_t + \hat{\beta}_2 \frac{\partial x_t}{\partial t} + \varepsilon_t$$

Calhoun et al. Neuroimage. 2003

HRF & Derivatives

Derivative boost

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$$\sum_t^N \left(\frac{\partial x_t}{\partial t} \right)^2 = 1, \quad \sum_t^N x_t^2 = 1, \quad \text{and} \sum_t^N x_t \frac{\partial x_t}{\partial t} = 0$$

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$$y_t = \alpha\left(\frac{\hat{\beta}_1}{\alpha} x_t + \frac{\hat{\beta}_2}{\alpha} \frac{\partial x_t}{\partial t}\right) + \hat{\beta}_0 + \varepsilon_t$$

α is the amplitude that takes both term into accout!

Calhoun et al. Neuroimage. 2003

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$$\operatorname{sign}\left(\hat{\beta}_1\right) \sqrt{\hat{\beta}_1^2 + \hat{\beta}_2^2}$$

=> Your estimated beta!!

Calhoun et al. Neuroimage. 2003

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$\Rightarrow$ Your estimated beta!!
(If you are using 3 basis fuctions)

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$\Rightarrow$ Your estimated beta!!
(If you are using 3 basis fuctions)

Caution when using…The original assumptions are hard to satisfy.

Calhoun et al. Neuroimage. 2003

HRF modeling

Derivative boost (Actual code)

First, get three regressors of interest.

```
canlab_spm_fmri_model_job(…, ‘hrf’, 1, 1 …)
```

```
'hrf', <1|0 time derivatives flag>, <1|0 dispersion derivatives flag>
   SPM can add time and/or dispersion derivatives to HRF convolutions
   e.g., canlab_spm_fmri_model_job(..., 'hrf', 1, 0,...)
```

1, 1 indicates adding both temporal and dispersion derivatives…

After getting beta using SPM, you have go into the directory that contains beta maps.

which looks like…

Then use the following function!

```
apply_derivative_boost(varargin)
```

There are more conditions that should be considered, so see details of what the code is actually doing!

https://github.com/CPernet => Here also contains tools for boosting! (spmup_hrf_boost.m)

HRF & Derivatives

Is it good enough…?

In part 1…

Advantages

- Less biased and allows more variance.
- Possible to test hypothesis about parameters of HRF.

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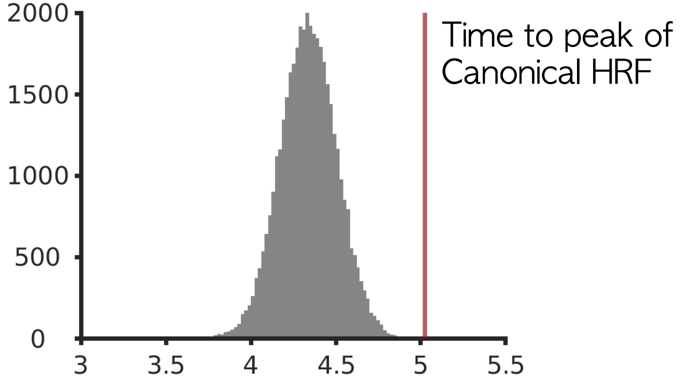

Time to peak map
of CBV signal

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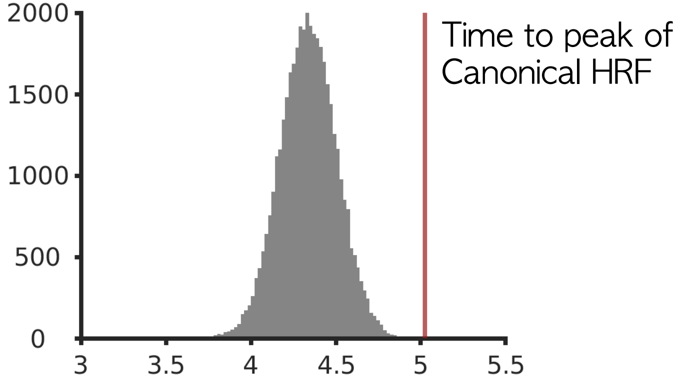

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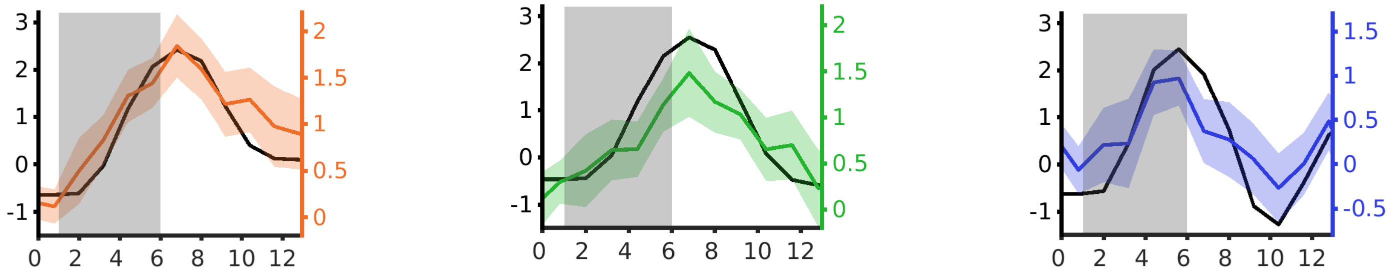

Colored line : Actual CBV percent changes in different regions of the Monkey brain

Black line: Fitted signal using 3 basis functions

HRF & Derivatives

Implication…

Might not use in BOLD, Human study (unless it is 7T layer imaging…)
=> As seen in part 1, Canonical HRF usually works fine.

Keep it as an index for your future projects!

Cocoan 101

https://cocoanlab.github.io