

# Physically Based Rendering

## FROM THEORY TO IMPLEMENTATION

This book was typeset with  $\text{\TeX}$ , using the  $\text{Zz}\text{\TeX}$  macro package on the Microsoft Windows XP platform. The main body of the text is set in Minion at 9.5/12, and the margin indices are set in Bitstream Letter Gothic 12 Pitch at 5.5/7. Chapter titles are set in East Bloc ICG Open and Univers Black. Cholla Sans Bold is used for other display headings.

The manuscript for this book is written in pyweb, a literate programming markup format of the authors' own design. This input format is based heavily on the noweb system developed by Norman Ramsey. The pyweb scripts simultaneously generate the  $\text{\TeX}$  input for the book as well as the source code of the pbrt system.

In addition, these scripts semi-automatically generate the code identifier cross-references that appears in the margin indices. Wherever possible, these indices are produced automatically by parsing the source code itself. Otherwise, usage and definition locations are marked explicitly in the pyweb input, and these special marks are removed before either the book or the code is generated. These scripts were originally written by the authors, and subsequently rewritten by Paul Anagnostopoulos in Gossip to integrate into the  $\text{Zz}\text{\TeX}$  package.

Overall, the book comprises over 90,000 lines of pyweb input, or nearly 5 megabytes of text. The cover image, example renderings, and chapter images were generated by pbrt, the software that is described in this book.

# Index of Notation

$p$	Point in 3D space	56
$\mathbf{v}$	Direction vector in 3D space	56
$p_x, v_x$	$x$ component of point / vector	57
$\mathbf{v} \cdot \mathbf{w}$	Dot product of vectors $\mathbf{v}$ and $\mathbf{w}$	60
$  \mathbf{v}  $	Length of vector $\mathbf{v}$	61
$\mathbf{v} \times \mathbf{w}$	Cross product of vectors $\mathbf{v}$ and $\mathbf{w}$	61
$\hat{\mathbf{v}}$	Normalized vector $\mathbf{v}$	62
$\mathbf{T}, \mathbf{M}$	Linear transformation ( $4 \times 4$ matrix)	74
$q$	Quaternion	92
$(u, v)$	2D parametric coordinates of point on surface	101
$\lambda$	Wavelength of light	261
$X(\lambda)/Y(\lambda)/Z(\lambda)$	XYZ spectral matching curves	270
$\Phi$	Radiant flux (power)	282
$E$	Irradiance (flux area density)	282
$M$	Radiant exitance (flux area density)	282
$\omega$	Normalized vector on the unit sphere	284
$I$	Radiant intensity (flux density per solid angle)	285
$L$	Radiance (flux density per area per solid angle)	285
$L_i(p, \omega)/L_o(p, \omega)$	Incident and exitant radiance functions	286
$Y$	Luminance	287
$\Omega$	Set of directions on the unit sphere	288
$\mathcal{H}^2(\mathbf{n})$	Hemisphere about surface normal $\mathbf{n}$	288
$f_r(\omega_i, \omega_o)$	BRDF (bidirectional reflectance distribution function)	295
$f(\omega_o, \omega_i)$	BSDF (bidirectional scattering distribution function)	296
$S^2$	Sphere of directions	296
$S(p_o, \omega_o, p_i, \omega_i)$	BSSRDF (bidirectional scattering-surface reflectance distribution function)	296
$F(\omega)$	Frequency-space representation of function $f(x)$	326
$\delta(x)$	Dirac delta distribution	327
$\text{III}_T(x)$	Shah function with period $T$	327
$\otimes$	Convolution operation	327
$\Pi_T(x)$	Box function with period $T$	329
$\Phi_b(n)$	Radical inverse function, base $b$	361
$\rho_{hd}(\omega)$	Hemispherical-directional reflectance	430
$\rho_{hh}$	Hemispherical-hemispherical reflectance	430
$\eta_i, \eta_o$	Index of refraction (incident and outgoing media)	432
$F_r$	Fresnel reflectance	435
$k$	Absorption coefficient	435
$\omega_h$	Half-angle vector	452
$D(\omega_h)$	Microfacet distribution function	452

# Index of Notation

$G(\omega_o, \omega_i)$	Microfacet geometric attenuation term	454
$(s, t)$	2D texture coordinates	514
$\sigma_a$	Absorption cross section	576
$L_{ve}(p, \omega)$	Emission in a participating medium	578
$\sigma_s$	Scattering coefficient	579
$\sigma_t$	Attenuation (absorption) coefficient	580
$T_r(p \rightarrow p')$	Beam transmittance	580
$\tau(p \rightarrow p')$	Optical thickness	581
$p(\omega \rightarrow \omega')$	Phase function	582
$S(p, \omega)$	Source term	582
$X, Y$	Random variables	638
$P(x)$	Cumulative distribution function (CDF)	639
$\xi$	Canonical uniform random variable	639
$p(x)$	Probability density function (PDF)	640
$E_p[f(x)]$	Expected value of a function over distribution $p$	640
$E[f(x)]$	Expected value of a function over uniform distribution	640
$V[f(x)]$	Variance	641
$T(X \rightarrow X')$	Transition function	653
$a(X \rightarrow X')$	Acceptance probability	653
$J_T$	Jacobian of multidimensional function	661
$p(y x)$	Conditional density function	662
$\epsilon[F]$	Efficiency of Monte Carlo estimator	679
$t(p, \omega)$	Ray casting function	752
$L(p' \rightarrow p)$	Exitant radiance from $p'$ to $p$	754
$f(p'' \rightarrow p' \rightarrow p)$	Three-point form of the BSDF	754
$G(p \leftrightarrow p')$	Geometric coupling term	754
$V(p \leftrightarrow p')$	Visibility term	754
$P(\bar{p}_i)$	Contribution of $i + 1$ vertex path	756
$T(\bar{p}_i)$	Throughput of $i + 1$ vertex path	757
$W(p, \omega)$	Importance function	760
$\alpha$	Particle weight	776, 798
$\sigma'_s$	Reduced scattering coefficient	902
$\sigma'_t$	Reduced extinction coefficient	902
$\phi(p)$	Fluence	904
$F_{dr}$	Diffuse Fresnel reflectance	905
$R_d(p_i, p_o)$	Diffuse subsurface reflectance	906
$Y_l^m(\theta, \phi), Y_l^m(\omega)$	Spherical harmonic basis function	932
$T(p, \omega)$	Transfer function	971