Chapter 4
Top-Down Parsing

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Top-down parsing

- Top-down parsing
  - Definition
    - Parsing an input string of tokens by tracing out the steps in a leftmost derivation.
  - Categories
    - Backtracking parsers
      - Powerful but slow
    - Predictive parsers
      - Using one or more lookahead tokens
      - Recursive-descent parsing
      - LL(1) parsing.

Recursive-descent parsing

- Recursive descent parsing
  - Each procedure is generated for each grammar rule.
- Expression grammar
  - *exp* → *exp addop term* | *term*
  - *addop* → + / −
  - *term* → *term mulop factor* / *factor*
  - *mulop* → *
  - *factor* → (*exp*) / ***number***

5 procedures are required.

Recursive-descent parsing

- *factor* → (*exp*) | ***number***
- *terminal: match*()
- *nonterminal: function*

```
procedure factor
begin
   case token of
   (:   match( ( ) ;
        exp ;
        match( ) ) ;
    number :
        match(number) ;
    else error ;
    end case ;
end factor ;
```

Recursive-descent parsing

```
procedure match( expectedToken) ;
begin
  if token = expectedToken then
    getToken ;
  else
    error ;
  end if ;
end match ;
```

- *compare match*(**(**) *with match*(**)**) in **procedure** *factor*.

Recursive-descent parsing

- *if-stmt* → **if (** *exp* **)** *statement*
  | **if (** *exp* **)** *statement* **else** *statement*
- **EBNF**
  - *if-stmt* → **if (** *exp* **)** *statement* [**else** *statement*]

```
procedure ifStmt ;
begin
  match ( if ) ;
  match ( ( ) ;
  exp ;
  match ( ) ) ;
  statement ;
  if token = else then
    match ( else ) ;
    statement ;
  end if ;
end ifStmt ;
```

Recursive-descent parsing

- *exp* → *exp addop term* | *term*

```
procedure exp ;
begin
case token of
  ?: exp ;
     addop ;
     term;
  ?: term;
end case
end exp ;
```

Recursive-descent parsing

- *exp* → *exp addop term* | *term*
- ***EBNF***
  - *exp* → *term* { *addop term* }

```
procedure exp ;
begin
  term ;
  while token = + or token = - do
    match (token) ;
    term ;
  end while ;
end exp ;
```

Recursive-descent parsing

- *term* → *term mulop factor* | *factor*

- ***EBNF***
  - *term* → *factor* { *mulop factor* }

procedure *term* ;
begin
  factor **;**
  while *token* **= * do**
    match (*token*) **;**
    factor **;**
  end while ;
end *term* **;**

Recursive-descent parsing

- *Left associativity is conserved.*
  - A simple integer arithmetic
    - *exp* → *term* { *addop term* }

```
function exp : integer ;
var temp : integer ;
begin
  temp := term ;
  while token = + or token = - do
    case token of
     + : match ( + ) ;
          temp := temp + term ;
      - : match ( - ) ;
          temp := temp – term ;
     end case ;
  end while ;
  return temp ;
end exp ;
```

- A simple calculator
  - p. 148-149

Recursive-descent parsing

- Syntax tree generation

```
function exp : syntaxTree ;
var temp, newtemp : syntaxTree ;
begin
  temp := term ;
  while token = + or token = - do
    case token of
     + : match (+) ;
         newtemp : = makeOpNode( + ) ;
         leftChild(newtemp) := temp ;
         rightChild(newtemp) := term ;
         temp := newtemp ;
     - : match (-) ;
         newtemp := makeOpNode( - ) ;
         leftChild(newtemp) := temp ;
         rightChild(newtemp) := term ;
         temp := newtemp ;
    end case ;
  end while ;
  return temp ;
end exp ;
```

Recursive-descent parsing

- Syntax tree generation

```
function exp : syntaxTree ;
var temp, newtemp : syntaxTree ;
begin
  temp := term ;
  while token = + or token = - do
    newtemp : = makeOpNode(token) ;
    match (token) ;
    leftChild(newtemp) := temp ;
    rightChild(newtemp) := term ;
    temp := newtemp ;
  end while ;
  return temp ;
end exp ;
```

Recursive-descent parsing

- Syntax tree generation

```
function ifStatement : syntaxTree
var temp : syntaxTree ;
begin
  match( if ) ;
  match( ( ) ;
  temp := makeStmtNode( if ) ;
  testChild(temp) := exp ;
  match ( ) ) ;
  thenChild(temp) := statement ;
  if token = else then
    match(else) ;
    elseChild(temp) := statement ;
  else
    elseChild(temp) := nil ;
  end if ;
end ifStatement ;
```

Recursive-descent parsing

- **Difficulties**
  - BNF → EBNF is not easy.
  - A → α | β …
    - The first sets should be determined.

LL(1) Parsing

- **gets its name as follows**
  - **"L" : process input from left to right**
  - **"L" : leftmost derivation**
  - **"1" : only one symbol for lookahead**
- **The basic example of LL(1) parsing**
  - grammar
    - $S \rightarrow (S)\ S \mid \varepsilon$
  - Input string
    - ( )

$$
\begin{aligned}
S &\Rightarrow (S)S \\
  &\Rightarrow (\ )S \\
  &\Rightarrow (\ )
\end{aligned}
$$

LL(1) Parsing

- **The basic example of LL(1) parsing**
  - grammar
    - $S \rightarrow (S)\ S \mid \varepsilon$
  - Input string
    - ( )

Parsing stack	Input	Action
$\$ S$	( ) \$	$S \rightarrow (\ S\ )\ S$
$\$ S\ )\ S\ ($	( ) \$	match
$\$ S\ )\ S$	) \$	$S \rightarrow \varepsilon$
$\$ S\ )$	) \$	match
$\$ S$	\$	$S \rightarrow \varepsilon$
\$	\$	accept

- $S \Rightarrow (S)S$
  $\Rightarrow (\ )S$
  $\Rightarrow (\ )$

LL(1) Parsing

- **Outline**
  - **Initialization**
    - Put the start symbol in the stack
  - **Iteration of the followings until the stack is empty.**
    - If a **nonterminal** is at the stack top,
      - replace the nonterminal using a grammar rule.
    - If a **token** is at the stack top,
      - match.
  - **If the stack is empty,**
    - if the input string is empty, accept.
    - otherwise, reject.

LL(1) Parsing Table

- **LL(1) parsing table**
  - A two-dimensional array indexed by nonterminals and terminals.
  - It contains production choices to use at the appropriate parsing step.

$M[N, T]$	**(**	**)**	**$**
S	$S \rightarrow (S) S$	$S \rightarrow \varepsilon$	$S \rightarrow \varepsilon$

- Once a parsing table is given, LL(1) parsing is simple.
  - Figure 4.2

LL(1) Parsing Table

- **LL(1) parsing table generation**
  - A table entry $M[A, \boldsymbol{a}]$ has every grammar rule $A \rightarrow \alpha$
    - if there is a derivation $\alpha \Rightarrow^* \boldsymbol{a}\beta$ or
    - if there is a derivation $\alpha \Rightarrow^* \boldsymbol{\varepsilon}$ and $S \Rightarrow^* \beta A\boldsymbol{a}\gamma$ for start symbol S.

***M*[*N*, *T*]**	**(**	**)**	**$**
S	$S \rightarrow (S) S$	$S \rightarrow \varepsilon$	$S \rightarrow \varepsilon$

LL(1) Grammar

- **LL(1) grammar**
  - The LL(1) parsing table has **at most one production in each entry**.
- **An LL(1) grammar cannot be ambiguous.**

$M[N, T]$	(	)	$
S	$S \rightarrow (S) S$	$S \rightarrow \varepsilon$	$S \rightarrow \varepsilon$

Disambiguating rule

statement → *if-stmt* | ***other***

if-stmt → **if (** *exp* **)** *statement else-part*

else-part → **else** *statement* | *ε*

exp → **0** | **1**

***M*[*N*,*T*]**	**if**	***other***	**else**	**0**	**1**	**$**
statement	*statement* → *if-stmt*	*statement* → ***other***				
if-stmt	*if-stmt* → **if (** *exp* **)** *statement else-part*					
else-part			*else-part* → **else** *statement*			*else-part* → *ε*
exp				*exp*→0	*exp*→1	

Parsing for if (0) if (1) *other* else *other*

Parsing stack	Input	Action
$ S	**i (0) i (1) o e o $**	$S \rightarrow I$
$ I	**i (0) i (1) o e o $**	$I \rightarrow$ **i (** E **)** $S\ L$
$ $L\ S$ **)** E **(i**	**i (0) i (1) o e o $**	match
$ $L\ S$ **)** E **(**	**(0) i (1) o e o $**	match
$ $L\ S$ **)** E	**0) i (1) o e o $**	$E \rightarrow$ **0**
$ $L\ S$ **) 0**	**0) i (1) o e o $**	match
$ $L\ S$ **)**	**) i (1) o e o $**	match
$ $L\ S$	**i (1) o e o $**	$S \rightarrow I$
$ $L\ I$	**i (1) o e o $**	$I \rightarrow$ **i (** E **)** $S\ L$
$ $L\ L\ S$ **)** E **(i**	**i (1) o e o $**	match

S = statement, *I = if-stmt*, *L=else-part*, *E=exp*, **i=if**, **e=else**, **o=*other***.

Parsing for if (0) if (1) *other* else *other*

Parsing stack	Input	Action
$ $L\ L\ S$ **)** E **(i**	**i (1) o e o $**	match
$ $L\ L\ S$ **)** E **(**	**(1) o e o $**	match
$ $L\ L\ S$ **)** E	**1) o e o $**	$E \rightarrow \mathbf{1}$
$ $L\ L\ S$ **) 1**	**1) o e o $**	match
$ $L\ L\ S$ **)**	**) o e o $**	match
$ $L\ L\ S$	**o e o $**	$S \rightarrow \mathbf{o}$
$ $L\ L$ **o**	**o e o $**	match
$ $L\ L$	**e o $**	$L \rightarrow \mathbf{e}\ S$
$ $L\ S$ **e**	**e o $**	match
$ $L\ S$	**o $**	$S \rightarrow \mathbf{o}$
$ L **o**	**o $**	match
$ L	**$**	$L \rightarrow \boldsymbol{\varepsilon}$
$	**$**	accept

Left Recursion Removal

- **Left recusion**
  - **Immediate left recursion**
    - *exp* → *exp addop term* | *term*
    - *exp* → *exp* + *term* | *exp* - *term* | *term*
  - **Indirect left recursion**
    - *A* → *Bb*| …
    - *B* → *Aa*| …

Left Recursion Removal

- **Simple immediate left recusion removal**

$$A \rightarrow A\alpha \mid \beta \quad \xRightarrow{\beta\alpha^*} \quad \begin{aligned} &A \rightarrow \beta A' \\ &A' \rightarrow \alpha A' \mid \varepsilon \end{aligned}$$

- **Example 4.1**
  - $exp \rightarrow exp\ addop\ term \mid term$
    - $A = exp$
    - $\alpha = addop\ term$
    - $\beta = term$

$$exp \rightarrow term\ exp'$$
$$exp' \rightarrow addop\ term\ exp' \mid \varepsilon$$

Left Recursion Removal

- **General immediate left recusion removal**

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots \mid A\alpha_n \mid \beta_1 \mid \beta_2 \mid \ldots \mid \beta_m$$

$$\Downarrow \quad (\beta_1 \mid \beta_2 \mid \ldots \mid \beta_m)(\alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n)^*$$

$$A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \ldots \mid \beta_m A'$$

$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \ldots \mid \alpha_n A' \mid \varepsilon$$

- **Example 4.2**
  - $exp \rightarrow exp + term \mid exp - term \mid term$
    - $A = exp,\ \alpha_1 = +\ term,\ \alpha_2 = -\ term,\ \beta = term$

$$exp \rightarrow term\ exp'$$

$$exp' \rightarrow +\ term\ exp' \mid -\ term\ exp' \mid \varepsilon$$

Left Recursion Removal

- **General left recursion removal (skip)**

for i := 1 **to** m **do**
  for j := 1 **to** i - 1 **do**
    replace each grammar rule choice of the form $A_i \rightarrow A_j\beta$ *by the rule*
      $A_i \rightarrow \alpha_1\beta \mid \alpha_2\beta \mid \dots \mid \alpha_k\beta$, *where* $A_j \rightarrow \alpha_1 \mid \alpha_2 \mid \dots \mid \alpha_k$ is
      the current rule for A_j

- **Example 4.3**

$$A \rightarrow Ba \mid Aa \mid c$$
$$B \rightarrow Bb \mid Ab \mid d$$

$$\Longrightarrow$$

$$A \rightarrow BaA' \mid cA'$$
$$A' \rightarrow aA' \mid \varepsilon$$
$$B \rightarrow cA'bB' \mid dB'$$
$$B' \rightarrow bB' \mid aA'bB' \mid \varepsilon$$

Left Recursion Removal

- Simple arithmetic expression grammar with left recursion removed.

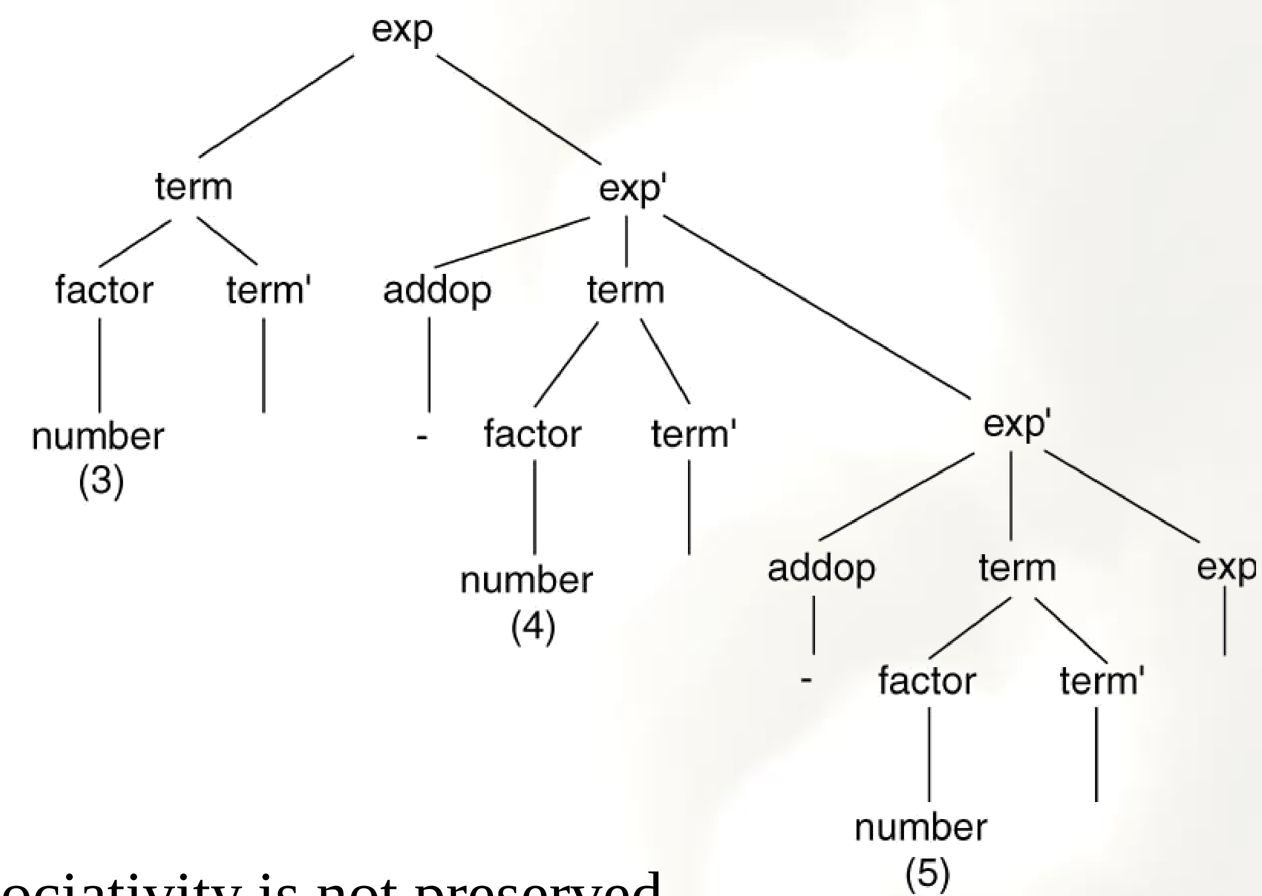
$$
\begin{aligned}
&exp \rightarrow term\ exp' \\
&exp' \rightarrow addop\ term\ exp' \mid \varepsilon \\
&addop \rightarrow + \mid - \\
&term \rightarrow factor\ term' \\
&term' \rightarrow mulop\ factor\ term' \mid \varepsilon \\
&mulop \rightarrow * \\
&factor \rightarrow (\ exp\ ) \mid \textbf{number}
\end{aligned}
$$

Left Recursion Removal

M(N, T)	(	**number**	)	+	-	*	$
exp	*exp* → *term exp′*	*exp* → *term exp′*					
exp′			*exp′* → ε	*exp′* → *addop term exp′*	*exp′* → *addop term exp′*		*exp′* → ε
addop				*addop* → +	*addop* → -		
term	*term* → *factor term′*	*term* → *factor term′*					
term′			*term′*→ ε	*term′* → ε	*term′* → ε	*term′* → *mulop factor term′*	*term′* → ε
mulop						*mulop* → *	
factor	*factor* → (*exp*)	*factor* → number					

Left Recursion Removal

- parse tree for the expression “**3 – 4 – 5**”



- Left associativity is not preserved.

Left Factoring

- Two grammar rules share a common prefix
  - $A \rightarrow \alpha\beta \mid \alpha\gamma$

- Left factoring
  - $A \rightarrow \alpha A'$
  - $A' \rightarrow \beta \mid \gamma$

Left Factoring

- **Examples**
  - *stmt-sequence* → *stmt* **;** *stmt-sequence* | *stmt*
  - *stmt* → **s**
  - *stmt-sequence* → *stmt stmt-seq′*
  - *stmt-seq′* → **;** *stmt-sequence* | ε
- **Left recursion removal**
  - *stmt-sequence* → *stmt-sequence* **;** *stmt* | *stmt*
  - *stmt* → **s**
  - *stmt-sequence* → *stmt stmt-seq′*
  - *stmt-seq′* → **;** *stmt stmt-seq′* | ε

Left Factoring

- **Examples**

  - *if-stmt* → **if (** *exp* **)** *statement* |
    if (*exp* **)** *statement* **else** *statement*

$$\Downarrow$$

  - *if-stmt* → **if (** *exp* **)** *statement else-part*
  - *else-part* → **else** *statement* | ε

Left Factoring

- **Examples**

  - *exp* → *term* + *exp* | *term*

  - *exp* → *term exp*′
  - *exp*′ → + *exp* | ε

  - *exp* → *term exp*′
  - *exp*′ → + *term exp*′ | ε

Left Factoring

- **Examples**
  - *statement* → *assign-stmt* | *call-stmt* | ***other***
  - *assign-stmt* → ***identifier*** **:=** *exp*
  - *call-stmt* → ***identifier*** **(** *exp-list* **)**

LL(1) grammar?

⇓

  - *statement* → ***identifier*** **:=** *exp* | ***identifier*** **(** *exp-list* **)** | ***other***

⇓

  - *statement* → ***identifier*** *statement′* | ***other***
  - *statement′* → **:=** *exp* | **(** *exp-list* **)**

First Sets

- The first set is defined on a grammar symbol X or a string $X_1X_2\ldots X_n$.
- **First(X)** for a grammar symbol X.
  - If X is a terminal or ε, First(X) = **{X}.**
  - If X is a nonterminal, for each grammar rule $X \rightarrow X_1X_2\ldots X_n$, First($X$) includes **First($X_1X_2\ldots X_n$).**
    - *exp* → *term addop exp* | *factor*
    - First(*exp*) = First(*term addop exp*) ∪ First(*factor*)

First Sets

- **First(X_1X_2…X_n)** for a string $X_1X_2\ldots X_n$.
  - If there are no ε-*productions*, First($X_1X_2\ldots X_n$) = **First(X_1).**
    - First(*exp addop term*) = First(*exp*)
    - First(**;** *stmt*) = First(**;**) ={**;**}
  - If there are some ε-*productions*,
    - First($X_1X_2\ldots X_n$) includes **First(X_1) - {ε}.**
    - If First(X_1) includes ε, First(X) also includes **First(X_2) - {ε}.**
    - If First(X_2) includes ε, First(X) also includes **First(X_3) - {ε}.**
    - …..
    - If all First(X_k)'s include ε, First(X) also includes ε.

First Sets

- **Example**
  - *exp* → *exp addop term* | *term*
  - *addop* → + | -
  - *term* → *term mulop factor* | *factor*
  - *mulop* → *
  - *factor* → (*exp*) | ***number***

First(*exp*) = {
First(*addop*)
First(*term*) =
First(*mulop*)
First(*factor*) =

First Sets

- **Example**
  - *statement* → *if-stmt* | ***other***
  - *if-stmt* → **if (** *exp* **)** *statement else-part*
  - *else-part* → **else** *statement* | ε
  - *exp* → 0 | 1

First(*statement*) =
First(*if-stmt*) =
First(*else-part*) =
First(*exp*) =

First Sets

- **Example**
  - *stmt-sequence* → *stmt stmt-seq*′
  - *stmt-seq*′ → **;** *stmt-sequence* | ε
  - *stmt* → **s**

First(*stmt-sequence*) = {**s**}
First(*stmt-seq*′) = {**;**, ε}
First(*stmt*) = {**s**}

Follow Sets

- The follow set is defined on a nonterminal ***A***.

- **Follow(*A*)** for a nonterminal *A*.
  - If *A* is a start symbol, Follow(*A*) includes **$.**
  - If there is $B \rightarrow \alpha\, A\, \gamma$, Follow(*A*) includes **First(γ)-{ε}**.
  - If there is $B \rightarrow \alpha\, A\, \gamma$ such that $\varepsilon \in$ First(γ),
    Follow(*A*) includes **Follow(*B*)**.

Follow Sets

- **Example**
  - *exp* → *exp addop term* | *term*
  - *addop* → + | -
  - *term* → *term mulop factor* | *factor*
  - *mulop* → *
  - *factor* → **(***exp***)** | ***number***

First(*exp*) = {(, *number*}
First(*addop*) = {+,-}
First(*term*) = {(, *number*}
First(*mulop*) = {*}
First(*factor*) = {(, *number*}

Follow(*exp*) = {$,), +, -}
Follow(*addop*) = {(,*number*}
Follow(*term*) = {*,$,),+,-}
Follow(*mulop*) = {(, *number*}
Follow(*factor*) = {*,$,),+,-}

Follow Sets

- **Example**
  - *statement* → *if-stmt* | ***other***
  - *if-stmt* → **if (** *exp* **)** *statement else-part*
  - *else-part* → **else** *statement* | ε
  - *exp* → 0 | 1

First(*statement*) = {***other***,**if**}
First(*if-stmt*) = {**if**}
First(*else-part*) = {**else**,ε}
First(*exp*) = {**0,1**}

Follow(*statement*) = {$, **else**}
Follow(*if-stmt*) = {$, **else**}
Follow(*else-part*) = {$, **else**}
Follow(*exp*) = {**)**}

Follow Sets

- **Example**
  - *stmt-sequence* → *stmt stmt-seq′*
  - *stmt-seq′* → **;** *stmt-sequence* | ε
  - *stmt* → **s**

First(*stmt-sequence*) = {**s**}
First(*stmt-seq′*) = {**;**,ε}
First(*stmt*) = {**s**}

Follow(*stmt-sequence*) = {\$}
Follow(*stmt-seq′*) = {\$}
Follow(*stmt*) = {**;**,\$}

Constructing LL(1) Parsing Tables

- **LL(1) parsing table generation**
  - A table entry $M[A, \boldsymbol{a}]$ has every grammar rule $A \rightarrow \alpha$ for a nonterminal A and a terminal $\boldsymbol{a}$
    - if there is a derivation $\alpha \Rightarrow^* \boldsymbol{a}\beta$ or
    - if there is a derivation $\alpha \Rightarrow^* \boldsymbol{\varepsilon}$ and $S \Rightarrow^* \beta A\boldsymbol{a}\gamma$ for start symbol S.

Constructing LL(1) Parsing Tables

- For a grammar rule A→ α,
  - for each token ***a*** in First(α), add it to the entry *M*[*A*, ***a***].
- If ε is in First(α),
  - for each element ***a*** of Follow(*A*), add A→ α to the entry *M*[*A*, ***a***].

- A grammar in BNF is LL(1) if the following conditions are satisfied.
  - For every production $A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$,
    $\text{First}(\alpha_i) \cap \text{First}(\alpha_j)$ is empty for all $i \neq j$.
  - For every nonterminal *A* such that First(*A*) contains ε,
    $\text{First}(A) \cap \text{Follow}(A)$ is empty.

Constructing LL(1) Parsing table

statement → *if-stmt* | ***other***
if-stmt → **if (** *exp* **)** *statement else-part*
else-part → **else** *statement* | ε
exp → **0** | **1**

Ft(*st..*) = {***other***, **if**}
Ft(*if-stmt*) = {**if**}
Ft(*else..*) = {**else**,ε}
Ft(*exp*) = {**0**,**1**}

Fw(*st..*) = {$, **else**}
Fw(*if-stmt*) = {$, **else**}
Fw(*else..*) = {$, **else**}
Fw(*exp*) = {)}

***M*[*N*,*T*]**	**if**	***other***	**else**	**0**	**1**	**$**
statement	*statement* → *if-stmt*	*statement* → ***other***				
if-stmt	*if-stmt* → **if (** *exp* **)** *statement else-part*					
else-part			*else-part* → **else** *statement* *else-part* → ε			*else-part* → ε
exp				*exp*→0	*exp*→1	

Constructing LL(1) Parsing Tables

- **Example**
  - *stmt-sequence* → *stmt stmt-seq′*
  - *stmt-seq′* → **;** *stmt-sequence* | *ε*
  - *stmt* → **s**

First(*stmt-sequence*) = {**s**}
First(*stmt-seq′*) = {**;**,*ε*}
First(*stmt*) = {**s**}

Follow(*stmt-sequence*) = {$}
Follow(*stmt-seq′*) = {$}
Follow(*stmt*) = {**;**,$}

***M*[*N*,*T*]**	**s**	**;**	**$**
stmt-sequence	*stmt-sequence* → *stmt stmt-seq′*		
stmt	*stmt* → **s**		
stmt-seq′		*stmt-seq′* → **;** *stmt-sequence*	*stmt-seq′* → *ε*

Constructing LL(1) Parsing table

- **Examples 4.15**

$exp \rightarrow term\ exp'$

$exp' \rightarrow addop\ term\ exp' \mid \varepsilon$

$addop \rightarrow + \mid -$

$term \rightarrow factor\ term'$

$term' \rightarrow mulop\ factor\ term' \mid \varepsilon$

$mulop \rightarrow *$

$factor \rightarrow (\ exp\ ) \mid \textbf{number}$

Ft(exp) = {(, $number$}
Ft(exp') = {+,-,ε}
Ft($addop$) = {+,-}
Ft($term$) = {(, $number$}
Ft($term'$) = {*,ε}
Ft($mulop$) = {*}
Ft($factor$) = {(, $number$}

Fw(exp) = {\$,)}
Fw(exp') = {\$,)}
Fw($addop$) = {(,$number$}
Fw($term$) = {\$,),+,-}
Fw($term'$) = {\$,),+,-}
Fw($mulop$) = {(,$number$}
Fw($factor$) = {\$,),+,-,*}

Constructing LL(1) Parsing table

$M(N, T)$	(	**number**	)	+	-	*	$
exp	*exp* → *term exp′*	*exp* → *term exp′*					
exp′			*exp′* → ε	*exp′* → *addop term exp′*	*exp′* → *addop term exp′*		*exp′* → ε
addop				*addop* → +	*addop* → -		
term	*term* → *factor term′*	*term* → *factor term′*					
term′			*term′* → ε	*term′* → ε	*term′* → ε	*term′* → *mulop factor term′*	*term′* → ε
mulop						*mulop* → *	
factor	*factor* → (*exp*)	*factor* → number					

Extending the Lookahead: LL(*k*) Parseres

- Lookahead *k* symbols
- The parsing table becomes much larger.
  - The number of columns increases exponentially with *k*.
- However, LL(*k*) parsing is not so powerful.
  - A grammar with left recursion is never LL(*k*) for any large *k*.

Error Recovery

- **Recognizer**
  - **A parser to check a program is syntactically correct or not.**
- **Error detection**
  - **Determine the location where an error has occurred as closely as possible.**
- **Error correction**
  - **Try to parse as much of the code as possible.**
  - **Avoid the error cascade problem**
  - **Avoid infinite loops on errors without consuming any input.**

Error Recovery in Recursive-Descent Parsers

- **Errors**
  - **Insertion**
  - **Deletion**
  - **Change**

- **Error recovery in recursive-descent parsers.**
  - **Panic mode**
    - **Provide each recursive procedure with an extra parameter consisting of a set of synchronizing tokens.**
    - **If an error is encountered, the parser scans ahead, throwing away tokens until one of the synchronizing set of tokens is seen.**
    - **Synchronizing tokens: Follow sets and First sets.**

Error Recovery in LL(1) Parsers

- **Error recovery in LL(1) parsers**
  - **Error occurs when the input token is not in First(A) where A is at the top of the stack.**
  - **Panic mode can be used.**
    - **Additional stack is needed to keep the synchset parameters.**
      - **because LL(1) parsing is not recursive.**

Error Recovery in LL(1) Parsers

- **Build the synchronizing tokens into the LL(1) parsing table.**
  - **Pop**
    - **Pop A from the stack**
  - **Scan**
    - **Successively pop tokens from the input until a token is seen for which we can restart the parse.**
  - **Push**
    - **Push a new nonterminal onto the stack.**

Error Recovery in LL(1) Parsers

***M*(*N*, *T*)**	**(**	***number***	**)**	**+**	**-**	*****	**$**
exp	*exp* → *term exp′*	*exp* → *term exp′*	pop	scan	scan	scan	pop
exp′	scan	scan	*exp′* → ε	*exp′* → *addop term exp′*	*exp′* → *addop term exp′*	scan	*exp′* → ε
addop	pop	pop	scan	*addop* → +	*addop* → -	scan	pop
term	*term* → *factor term′*	*term* → *factor term′*	pop	pop	pop	scan	pop
term′	scan	scan	*term′*→ ε	*term′* → ε	*term′* → ε	*term′* → *mulop factor term′*	*term′* → ε
mulop	pop	pop	scan	scan	scan	*mulop* → *	pop
factor	*factor* → (*exp*)	*factor* → ***number***	pop	pop	pop	pop	pop

Error Recovery in LL(1) Parsers

Parsing stack	Input	Action
$ $E'T')E'T$	***) $**	scan (error)
$ $E'T')E'T$	**) $**	pop (error)
$ $E'T')E'$	**) $**	$\boldsymbol{E'} \rightarrow \varepsilon$
$ $E'T')$	**) $**	match
$ $E'T'$	**$**	$\boldsymbol{T'} \rightarrow \varepsilon$
$ E'	**$**	$\boldsymbol{E'} \rightarrow \varepsilon$
$	**$**	accept

Syntax Tree Construction in LL(1) Parsing

Parsing stack	Input	Action	Value stack
$\$ E$	3 + 4 + 5 $	$E \rightarrow n\, E'$	$
$\$ E'\, n$	3 + 4 + 5 $	match / push	$
$\$ E'$	+ 4 + 5 $	$E' \rightarrow +\, n\, \#\, E'$	3 $
$\$ E'\, \#\, n\, +$	+ 4 + 5 $	match	3 $
$\$ E'\, \#\, n$	4 + 5 $	match / push	3 $
$\$ E'\, \#$	+ 5 $	addstack	4 3 $
$\$ E'$	+ 5 $	$E' \rightarrow +\, n\, \#\, E'$	7 $
$\$ E'\, \#\, n\, +$	+ 5 $	match	7 $
$\$ E'\, \#\, n$	5 $	match / push	7 $
$\$ E'\, \#$	$	addstack	5 7 $
$\$ E'$	$	$E' \rightarrow \varepsilon$	12 $
$	$	accept	12 $