**ENE 3031 – Fall 2014**

**Homework2 Solution**

## #1.

The tool crib is modeled by an M/M/c queue ($\lambda = 1/4, \mu = 1/3, c = 1$ or 2). Given that attendants are paid \$6 per hour and mechanics are paid \$10 per hour,

$$\text{Mean cost per hour } = \$10c + \$15L$$

assuming that mechanics impose cost on the system while in the queue and in service.

CASE 1: one attendant - M/M/1 ($c = 1, \rho = \lambda/\mu = .75$)

$L = \rho/(1-\rho) = 3$ mechanics

Mean cost per hour = \$10(1) + \$15(3) = \$55 per hour.

CASE 2: two attendants - M/M/2 ($c = 2, \rho = \lambda/c\mu = .375$)

$$L = c\rho + \left[(c\rho)^{c+1}P_0\right] / \left[c(c!)(1-\rho)^2\right] = .8727,$$

where

$$P_0 = \left\{\left[\sum_{n=0}^{c-1}(c\rho)^n/n!\right] + \left[(c\rho)^c(1/c!)(1/(1-\rho))\right]\right\}^{-1} = .4545$$

Mean cost per hour = \$10(2) + \$15(.8727) = \$33.09 per hour

It would be advisable to have a second attendant because long run costs are reduced by \$21.91 per hour.

## #2.

A single landing strip airport is modeled by an M/M/1 queue ($\mu = 2/3$). The maximum arrival rate, $\lambda$, such that the average wait, $w_Q$, does not exceed three minutes is computed as follows:

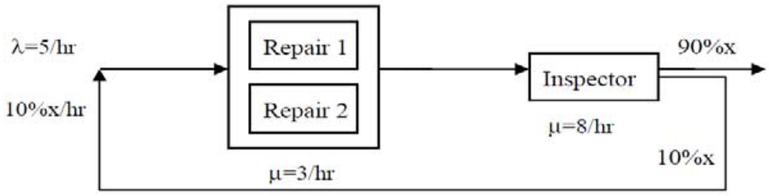
$$w_Q = \lambda/[\mu(\mu-\lambda)] \leq 3$$

or

$$\lambda = \mu/[1/\mu w_Q + 1] \leq .4444 \text{ airplanes per minute.}$$

Therefore, $\lambda_{\max} = .4444$ airplanes per minute.

## #3.




$x = \frac{\lambda}{1-10\%} = 5.556/hr$

At the repair station: $w = \frac{1}{\mu(1-\rho^2)} = \frac{1}{3(1-(\frac{5.556}{(2)(3)})^2)} = 2.34hr$

At the inspection station: $w = \frac{1}{8(1-\frac{5.556}{8})} = 0.41hr$

The maximum arrival rate the system can handle without adding personnel is: $\lambda = (2)(3)(90\%) = 5.4/hr$ because the utilization at the repair stations are much higher than that at the inspection station, which indicates the repair stations are the bottleneck of the system.

## #4.

The physical examination is modeled as an $M/G/1$ queue. The arrival rate is $\lambda = 1/60$ patient per minute. The mean service time is $15 + 15 + 15 = 45$ minutes, so the service rate is $\mu = 1/45$ patient per minute. Thus, $\rho = \lambda/\mu = 3/4$. The variance of the service time is $\sigma^2 = 15^2 + 15^2 + 15^2 = 675$ minutes, the sum of the variance of three exponentially distributed random variables, each with mean 15. Applying the formula for $L_Q$ for the $M/G/1$ queue we obtain

$$L_Q = \frac{\rho^2(1+\sigma^2\mu^2)}{2(1-\rho)} = 1\frac{1}{2} \text{ patients.}$$

## #5.

| Customer # | Ai | Ti | Wi | Si | Di |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 5 | 5 |
| 2 | 4 | 5 | 1 | 5 | 10 |
| 3 | 8 | 10 | 2 | 1 | 11 |
| 4 | 10 | 11 | 1 | 3 | 14 |
| 5 | 17 | 17 | 0 | 2 | 19 |
| 6 | 18 | 19 | 1 | 1 | 20 |
| 7 | 19 | 20 | 1 | 4 | 24 |
| 8 | 20 | 24 | 4 | 7 | 31 |
| 9 | 27 | 31 | 3 | 3 | 34 |
| 10 | 29 | 34 | 5 | 1 | 35 |

**Ave. Waiting Time = 1/10 *(0+1+2+1+0+1+1+4+3+5)**

**#6.**

**(a)**

$$\begin{aligned}
\hat{I}_n &= \frac{b-a}{n}\sum_{i=1}^{n} f(a+(b-a)U_i) \\
&= \frac{2}{100}\sum_{i=1}^{100} f(-1+2U_i) \\
&\qquad \text{(since } a=-1,\ b=1, \text{ and } n=100) \\
&= \frac{1}{50}\sum_{i=1}^{100} \frac{1}{\sqrt{2\pi}}\, e^{-(-1+2U_i)^2}.
\end{aligned}$$

**(b)**

$$\begin{aligned}
I &= \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} \exp\{-x^2/2\}\, dx \\
&= 2\Phi(1)-1 \\
&\qquad \text{(where } \Phi(\cdot) \text{ is the standard normal c.d.f.)} \\
&= 2(0.8413) \;=\; 0.6826. \qquad \Box
\end{aligned}$$