Operations Management I

Inventory Management (4)

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Inventory Management

Stochastic Inventory Models

- Overview
- Continuous Review Models
- Periodic Review Model

Hopp and Spearman, 2008, **Factory Physics**, McGraw Hill. **(Chapter 2)**

Krajewski and Ritzman, 2005, **Operations Management**, Prentice Hall. **(Chapter 15)**

Inventory Management

◈ Stochastic Inventory Models

Overview (1)

- Stochastic elements in inventory systems
  - ✓ Stochastic demand ←----- Demand is considered to be a random variable.
    - ➢ Two basic approaches
      - ▪ Model demand as if it were deterministic, and modify the solution to account for randomness
        ←----- appropriate when planning horizon is long
      - ▪ Explicitly represent randomness in the model
  - ✓ Stochastic lead time ←----- Lead time is considered to be a random variable.
    - ➢ Model lead time considering randomness

Inventory Management

Stochastic Inventory Models

Overview (2)

- Models – Overview
  - ✓ Single-period model
    - ➢ News boy problem ◄--------- ordering quantity
  - ✓ Multi-period models
    - ➢ Continuous review models
      - Base stock model ◄--------- reorder point
      - (Q, r) model ◄--------- ordering quantity and reorder point (approximation)
    - ➢ Periodic review model
      - (R, T) model (order up to R policy)

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Stochastic Inventory Models

Single-Period Model – Newsboy Problem (1)

- Example (Manufacturer of Christmas lights)
  - ✓ Demand is somewhat unpredictable and occurs in such a short burst prior to Christmas.
  - ✓ If inventory is not on the shelves, sales are lost. The cost of collecting unsold inventory and holding it until next year is too high to make year-to-year storage an attractive option. Instead, any unsold sets of lights are sold after Christmans at a steep discount.

Key question

- ✓ How many lights to produce must be made prior to the holiday season?

◄----- One replenishment (order quantity)

◄----- Related information to consider

- ➢ Anticipated demand
- ➢ Cost of producing (ordering) too much (overage cost)
- ➢ Cost of producing (ordering) too little (shortage cost)

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Stochastic Inventory Models

Single-Period Model – Newsboy Problem (2)

- Problem description
  - ✓ Situation

    A news vendor purchase newspapers at the begining of the day, sells a random amount, and then must discard any leftovers

  - ✓ Assumptions
    - Products are separable. ◄----- single product type
    - Planning is done for a single period. (Neglect future periods)
    - Demand is random. ◄----- random variable with known probability distribution
    - Deliveries are made in advance of demand. ◄----- All stock ordered or produced is available to meet demand.
    - Costs of shortage and leftover are linear.

  - ✓ Decision variable

    Order quantity ◄------ No constraint on the amount of purchase

  - ✓ Objective

    Minimizing the sum of underage (shortage) and overage (leftover) costs

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Parameters

$E(X)=\mu$ → X: demand (in units) during a period, a random variable

$Var(X)=\sigma^2$

G(x): = P(X ≤ x) cumulative distribution function of demand (continuous or dicrete)

g(x): density function of demand, g(x) = dG(x)/dx

c_o: cost per unit leftover after demand is realized (overage) ($/unit)

c_s: cost per unit of shortage ($/unit)

Decision variable

Q: order quantity

Stochastic Inventory Models

Single-Period Model – Newsboy Problem (3)

- Expected total cost
  - ✓ Expected units over

    Number of units of overage if *Q* units are on hand at the beginning of the period

$$\int_0^\infty \max\{Q-x,0\}g(x)dx \longleftarrow \text{Units over} = \max\{0, Q-X\}$$

$$= \int_0^Q (Q-x)g(x)dx \qquad = \begin{cases} Q-X, & \text{if } Q \ge X \\ 0, & \text{otherwise} \end{cases}$$

  - ✓ Expected units short

    Number of units of shortage if *Q* units are on hand at the beginning of the period

$$\int_0^\infty \max\{x-Q,0\}g(x)dx \longleftarrow \text{Units short} = \max\{0, X-Q\}$$

$$= \int_Q^\infty (x-Q)g(x)dx \qquad = \begin{cases} X-Q, & \text{if } X \ge Q \\ 0, & \text{otherwise} \end{cases}$$

$$Y(Q) = \underbrace{\boxed{c_o \int_0^Q (Q-x)g(x)dx}}_{\text{Expected overage cost}} + \boxed{c_s \int_Q^\infty (x-Q)g(x)dx} \longleftarrow \text{expected shortage cost}$$

$$Y(Q) = c_o \int_0^Q (Q-x) g(x) dx + c_s \int_Q^\infty (x-Q) g(x) dx$$

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Leibnitz's rule

$$\frac{\partial}{\partial Q} \int_{a_1(Q)}^{a_2(Q)} f(x,Q) dx$$

$$= \int_{a_1(Q)}^{a_2(Q)} \frac{\partial}{\partial Q} [f(x,Q)] dx + f(a_2(Q),Q) \frac{da_2(Q)}{dQ} - f(a_1(Q),Q) \frac{da_1(Q)}{dQ}$$

Stochastic Inventory Models

Single-Period Model – Newsboy Problem (4)

- Optimal solution

Find the value of Q that minimizes the expected total cost Y(Q)

$$\frac{dY(Q)}{dQ} = c_0 \cdot \int_0^Q 1 \cdot g(x) dx + c_s \int_Q^\infty (-1) \cdot g(x) dx$$

$$= c_0 \cdot G(Q) - c_s \cdot [1 - G(Q)] = 0$$

$$\int_Q^\infty g(x) dx = 1 - \int_0^Q g(x) dx = 1 - G(Q)$$

✓ Convexity

$$\frac{d^2 Y(Q)}{dQ^2} = c_0 \cdot g(Q) - c_s \cdot (-g(Q)) = c_0 \cdot g(Q) + c_s \cdot g(Q) > 0$$

$$\boxed{G(Q^*) = \frac{c_s}{c_0 + c_s}}$$

Q* should be chosen such that the probability of having enough stock to meet demand is c_s / (c_o+c_s).

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Stochastic Inventory Models

Single-Period Model – Newsboy Problem (5)

- Optimal solution

$$G(Q^*) = \frac{c_s}{c_0 + c_s}$$

✓ When G is normal distribution

cumulative distribution function of the standard normal distribution

$$G(Q^*) = \Phi\left(\frac{Q^* - \mu}{\sigma}\right) = \frac{c_s}{c_0 + c_s}$$

$$\frac{Q^* - \mu}{\sigma} = z \quad \Rightarrow \quad Q^* = \mu + z \cdot \sigma$$

z is the value in the standard normal table for which $\Phi(z) = c_s / (c_o + c_s)$.

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Stochastic Inventory Models

Single-Period Model – Newsboy Problem (6)

- Example (Manufacturer of Christmas lights)
  - ✓ Suppose that a set of lights costs \$1 to make and distibute and sells for \$2.
  - ✓ Any sets not sold by Christmas will be discounted to \$0.5.
  - ✓ Suppose that demand has been forecasted to be 10,000 units with a standard deviation of 1,000 units and normal distibution is a reasonable representation of demand.

Solution

- ✓ Unit overage cost c_o = \$(1 – 0.5) = \$0.5
- ✓ Unit shortage cost c_s = \$(2 – 1) = \$1

$$G(Q^*) = \frac{c_s}{c_0 + c_s} = \frac{1}{0.5+1} \approx 0.67$$

$$\Phi(0.44) = 0.67$$

$$\dashrightarrow \quad Q^* = \mu + z \cdot \sigma = 10000 + (0.44)1000 = 10440$$

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Stochastic Inventory Models

Single-Period Model – Newsboy Problem (7)

- Insights
  - ✓ In an environment of uncertain demand, the appropriate production or order quantity depends on both the distribution of demand and the relative costs of overproducing versus underproducing

$$G(Q^*) = \frac{c_s}{c_0 + c_s}$$

  - ✓ If demand is normally distibuted, then increasing the variability (standard deviation) of demand will
    - increase the production or order quantity if $c_s / (c_o+c_s) > 0.5$ and
    - decrease the production or order quantity if $c_s / (c_o+c_s) < 0.5$

$$\frac{c_s}{c_0 + c_s} > 0.5 \rightarrow z > 0 \rightarrow Q^* = \mu + z \cdot \sigma \uparrow \text{ as } \sigma \text{ increases}$$

$$\frac{c_s}{c_0 + c_s} < 0.5 \rightarrow z < 0 \rightarrow Q^* = \mu + z \cdot \sigma \downarrow \text{ as } \sigma \text{ increases}$$

Inventory Management

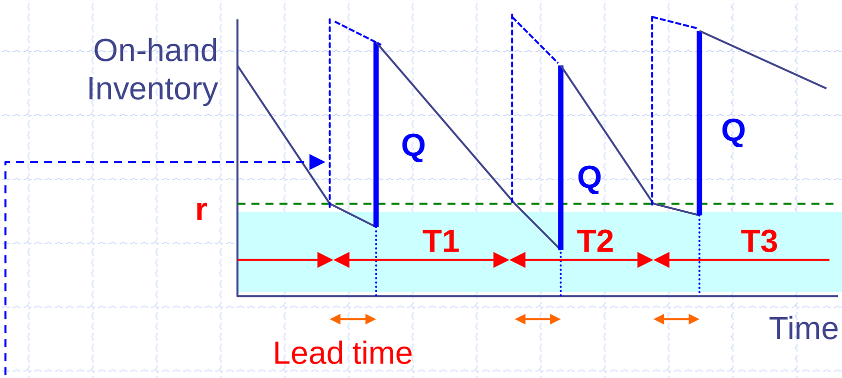
Stochastic Inventory Models

Continuous Review Models (1)

- Overview

  Tracks the remaining inventory of an item each time a withdrawal is made to determine whether it is time to order

  ✓ Reorder point (ROP) system
  ✓ Fixed order quantity system



✓ Method

Order the fixed quantity (Q) if the inventory level reaches the reorder point (r)

✓ Decision variables: Q and r

Inventory position
= net inventory + replenishment orders
= (on-hand inventory – backorders) + replenishment orders

physical inventory in stock
(always positive)

- Models

  ✓ Base stock Model (Q = 1)
  ✓ (Q, r) Model (Q > 1)

Inventory Management

- **Stochastic Inventory Models**

Continuous Review Models (1)

- Base stock model
  - ✓ Example (refrigerator store)
    - Because space is limited and the manufacturer makes frequent deliveries, the store finds it practical to order replacement refrigerators each time one is sold.

      ←--- A system that places purchase orders automatically whenever a sale is made. (no ordering (setup) cost)
    - But because the manufacturer is slow to fill replenishment orders, the store must carry some stock in order to meet customer demands promptly.

      ←--- Replenishment lead time > 0

→ Key question
- ✓ How much stock to carry?

←--- Base-stock system
(Special case of (Q, r) model (Q = 1))

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Stochastic Inventory Models

Continuous Review Models (2)

- Base stock model

✓ Assumptions

- A system that places purchase orders automatically whenever a sale is made. ← No ordering (setup) cost
- Products can be analyzed individually. ← single product type
- Demand is uncertain and occurs one at a time
- Unfilled demand is backordered.
- Replenishment lead times are fixed and known.

← Backorder may occur during lead time

✓ Problem description

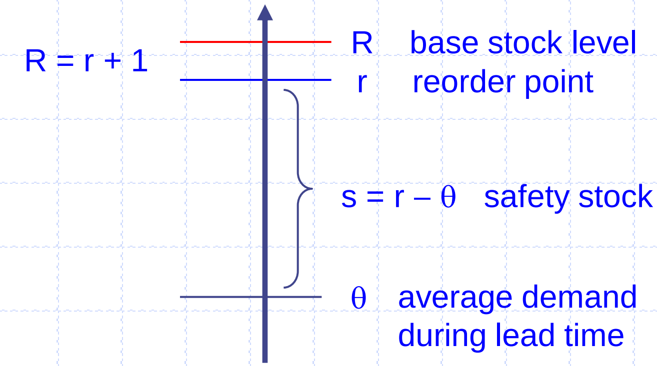
- Decision variable

  Number of stocks to carry
  (base stock level in terms of inventory position)

- Objective

  Minimizing the sum of inventory and backorder costs

Inventory position = r + 1 = R (constant)



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Stochastic Inventory Models

Continuous Review Models (3)

- Base stock model
  - ✓ Notation
    - l replenishment lead time (constant)
    - X demand during the replenishment lead time (random variable)
      - $g(x) = P\{X = x\}$ probability density function (of demand during lead time)
      - $G(x) = P(X \le x)$ cumulative distribution function
      - $\theta = E(X)$ average demand during lead time
      - $\sigma^2 = Var(X)$ variance of demand during lead time
    - h cost to carry one unit of inventory for one year (\$/unit·year)
    - b cost to carry one unit of backorder for one year (\$/unit·year)
    - r reorder point (units), inventory level that triggers a replenishment order
  - ✓ Decision variables
    - R base stock level (units), $R = r + 1$
    - s safety stock level (units), $s = r - \theta$

Inventory position
(not on-hand inventory)

How much stock to carry?

Finding the reorder point r

- Finding the optimal base stock level R (= r +1)
- Finding safety stock level $s = r - \theta$

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Stochastic Inventory Models

Continuous Review Models (4)

- Base stock model

  ✓ Approach 1

  Specify the desired customer service level (fill rate) and find the smallest reorder point that attains it

  ➢ Service level (= fill rate)

  Fraction of demand that will be filled from stock

$$S(r) = 1 - P(X \geq r+1) = P(X \leq r+1) = G(r+1)$$

X demand during the replenishment lead time (continuous random variable)

Because the lead-time is a constant, all the other R – 1 items either in inventory or on order will be available to fill new demand before the order under consideration arrives.

If demand is normally distributed,

$$S(r) = G(r+1) = \Phi\left(\frac{r+1-\theta}{\sigma}\right)$$

Cumulative distribution function of the standard normal distribution

Step 1. Set the service level S(r)

Step 2. Obtain the reorder point *r*

- Base stock level R = r + 1
- Safety stock s = r – θ

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- **Stochastic Inventory Models**

Continuous Review Models (5)

- Base stock model
  - ✓ Approach 1
    - Example
      - Average demand for the refrigerator = 10 units/month.
      - Replenishment lead time = one month

← - - - Demand during lead time ~ Poisson distribution

Solution $\sigma = \sqrt{10} \cong 3.16$

✓ Fill rate ≥ 0.9

$$S(13) = G(14) = \Phi(\frac{14-10}{3.16}) = \Phi(1.26) = 0.896$$

$$S(14) = G(15) = \Phi(\frac{15-10}{3.16}) = \Phi(1.58) = 0.942 > 0.9$$

- - - → r = 14

- Base stock level (R) = r + 1 = 14 + 1 =15
- Safety stock level (s) = r - θ = 14 – 10 = 4

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Stochastic Inventory Models

Continuous Review Models (6)

- Base stock model

  ✓ Approach 2

  Formulate a cost function and find the reorder point that minimizes the cost function (under a base stock policy)

  ➢ Total cost = inventory holding cost + backorder cost

  - Backorder level

    number of orders at any time
    = number of demands during the lead time

    Backorder level when the number of orders is x

$$\text{Backorder level} = \begin{cases} 0 & \text{if} \quad x < r+1 \\ x - r - 1 & \text{if} \quad x \geq r+1 \end{cases}$$

Expected backorder level

$$B(r) = \int_{r+1}^{\infty} (x - r - 1) g(x) dx$$
$$= \int_{r}^{\infty} (x - r) g(x) dx$$

✓ If demand is normally distributed,

$$B(r) = (\theta - r) \cdot [1 - \Phi(z)] + \sigma \cdot \Phi(z)$$

$$z = \frac{r - \theta}{\sigma}$$

- On-hand inventory level

  on-hand inventory – backorders = r + 1 – X

Expected on-hand inventory level

$$I(r) = r + 1 - \theta + B(r)$$

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Stochastic Inventory Models

Continuous Review Models (7)

- Base stock model
  - Approach 2
    - Total cost function

$$Y(r) = h \cdot I(r) + b \cdot B(r)$$
$$= h \cdot (r + 1 - \theta + B(r)) + b \cdot B(r)$$
$$= h \cdot (r + 1 - \theta) + (b + h) \cdot B(r)$$

Optimal reorder point

$$\frac{dY(r)}{dr} = h + (h+b) \cdot \frac{dB(r)}{dr}$$
$$= h - (h+b) \cdot [1 - G(r+1)] = 0$$

$$\boxed{G(r^*+1) = \frac{b}{b+h}}$$

Leibnitz's rule

$$\frac{\partial}{\partial Q}\int_{a_1(Q)}^{a_2(Q)} f(x,Q)dx$$
$$= \int_{a_1(Q)}^{a_2(Q)} \frac{\partial}{\partial Q}[f(x,Q)]dx + f(a_2(Q),Q)\frac{da_2(Q)}{dQ} - f(a_1(Q),Q)\frac{da_1(Q)}{dQ}$$

$$\frac{dB(r)}{dr} = \frac{d}{dr}\int_{r+1}^{\infty}(x - r - 1)g(x)dx$$
$$= -\int_{r+1}^{\infty} g(x)dx = -[1 - G(r+1)]$$

If demand is normally distributed,

$$r^* + 1 = \theta + z \cdot \sigma$$

$$\Phi(z) = \frac{b}{b+h}$$

Inventory Management

- **Stochastic Inventory Models**

Continuous Review Models (8)

- Base stock model
  - ✓ Approach 2
    - ➢ Example
      - Average demand for the refrigerator = 10 units/month.
      - Replenishment lead time = one month

        ◄---- Demand during lead time (X) ~ normal with θ =10 and σ = 3.16
      - Inventory holding cost (h) = 15 \$/unit·month
      - Backorder cost (b) = 25 \$/unit·month

Solution

$$G(r^*+1)=\frac{b}{b+h}=\frac{25}{25+15}=0.625$$

$$\dashrightarrow \quad r^*+1=\theta+z\cdot\sigma=10+0.32(3.16)=11.01\approx 11$$

✓ Fill rate

$$S(10)=G(11)=\Phi\left(\frac{11-10}{3.16}\right)=\Phi(0.316)=0.62$$

low fill rate due to low backorde cost

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Stochastic Inventory Models

Continuous Review Models (9)

- Base stock model

  ✓ Insights

  - Reorder points control the probability of stockouts by establishing safety stock.

    fill rate (S(r)) = $1-P(X \geq r+1) = P(X \leq r+1) = G(r+1)$

    safety stock level (s) = $r - \theta$

  - The required base stock level that achieves a given fill rate is an increasing function of the mean and standard deviation of the demand during replenisment lead time (provided that unit backorder cost exceeds unit holding cost).

    $$R^* = r^* + 1 = \theta + z \cdot \sigma$$

    $$b > h \rightarrow \frac{b}{b+h} > 0.5 \rightarrow z > 0$$

  - The optimal fill rate is an increasing function of the backorder cost and a decreasing function of the holding cost.

    $$S(r^*) = G(r^*+1)$$

    $$G(r^*+1) = \frac{b}{b+h}$$

    If we fix the holding cost, we can use either a service constraint or a backorder cost to determine the appropriate base stock level.

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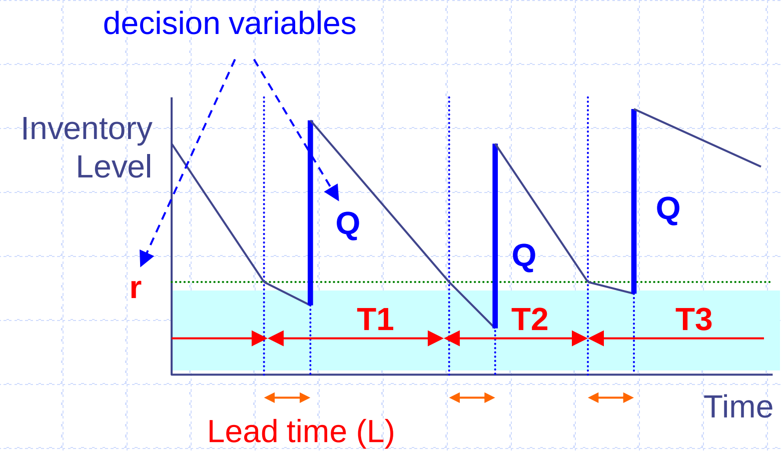

Stochastic Inventory Models

Continuous Review Models (10)

- (Q, r) Model
  - ✓ Method

    Order the fixed quantity (Q) if the inventory level reaches the reorder point (r)

    Generalization of the base-stock model (Q >1)
  - ✓ Models
    - Backorder cost approach

      min { fixed setup cost + backorder cost + holding cost }
      Q, r

      charge a penalty that is proportional to the length or time a customer order waits to be filled ($/unit·year)
    - Stockout cost approach

      min { fixed setup cost + stockout cost + holding cost }
      Q, r

      charge a cost each time a demand cannot be filled from stock ($/unit)

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A setup or purchase order cost per replenishment ($)
D expected demand per year (in units)
h cost to carry one unit of inventory for one year ($/unit·year)
L replenishment lead time (days) – constant
X demand during replenishment lead time (random variable)

$$\theta = E(X) = \frac{D \cdot L}{365}$$

$$\sigma^2 = Var(X)$$

Stochastic Inventory Models

Continuous Review Models (11)

- (Q, r) Model
  - ✓ Backorder cost approach – Approximate solution
    - ➢ Order quantity (Q) – EOQ

$$Q^* = \sqrt{\frac{2AD}{h}}$$

annual unit holding cost ($/unit·year)

    - ➢ Reorder point (r) – Base stock model

$$G(r^*) = \frac{b}{b+h}$$

annual unit backorder cost ($/unit·year)

$$X \sim N(\theta, \sigma^2)$$

$$r^* = \theta + z \cdot \sigma$$

Safety stock: s = r* – θ

$$\Phi(z) = \frac{b}{b+h}$$

Inventory Management

A	setup or purchase order cost per replenishment ($)
D	expected demand per year (in units)
h	cost to carry one unit of inventory for one year ($/unit·year)
L	replenishment lead time (days) – constant
X	demand during replenishment lead time (random variable)

$$\theta = E(X) = \frac{D \cdot L}{365}$$

$$\sigma^2 = Var(X)$$

Stochastic Inventory Models

Continuous Review Models (12)

- (Q, r) Model
  - ✓ Stockout cost approach – Approximate solution
    - Order quantity (Q) – EOQ

$$Q^* = \sqrt{\frac{2AD}{h}}$$

    - Reorder point (r)

cost per stockout ($/unit)

$$G(r^*) = \frac{k \cdot D}{k \cdot D + h \cdot Q}$$

$$X \sim N(\theta, \sigma^2)$$

$$r^* = \theta + z \cdot \sigma$$

Safety stock: s = r* – θ

$$\Phi(z) = \frac{k \cdot D}{k \cdot D + h \cdot Q}$$

Inventory Management

Stochastic Inventory Models

Continuous Review Models (13)

- (Q, r) Model
  - ✓ Example
    - Solution
      - Order quantity

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(15)(14)}{30}} = 3.7 \approx 4$$

- Reorder point

Backorder model $\frac{b}{b+h} = \frac{100}{100+30} = 0.769$

$$r^* = \theta + z \cdot \sigma = 1.726 + 0.736 \cdot \sqrt{1.726} = 2.693 \approx 3$$

$$\Phi(0.736) = 0.769$$

Stockout model $\frac{k \cdot D}{k \cdot D + h \cdot Q} = \frac{40(14)}{40(14) + 30(4)} = 0.824$

$$r^* = \theta + z \cdot \sigma = 1.726 + 0.929 \cdot \sqrt{1.726} = 2.946 \approx 3$$

$$\Phi(0.929) = 0.824$$

- Annual demand (D) = 14 units
- Unit cost of the part (c) = \$150
- Inventory holding cost (h) = 0.2(150) = 30 \$/unit·year (Interest rate (i) = 20 %)
- Replenishment lead time (L) = 45 days
  → Average demand during a replenishment lead time (θ) = (14/365) × 45 = 1.726
- Cost to place a purchase order (A) = \$15
- Backorder cost (b) = 100 (\$/unit·year)
- Cost per stockout event (k) = 40 (\$/unit)

Demand during lead time ~ Poisson distribution

Approximate Poisson by normal

$$X \sim N(1.726, 1.726)$$

Inventory Management

Stochastic Inventory Models

Continuous Review Models (14)

- (Q, r) Model
  - ✓ Setting reorder point (r) using customer service level

reorder point (r) = average demand during lead time (θ) + safety stock (s)

- Average demand during lead time (θ)

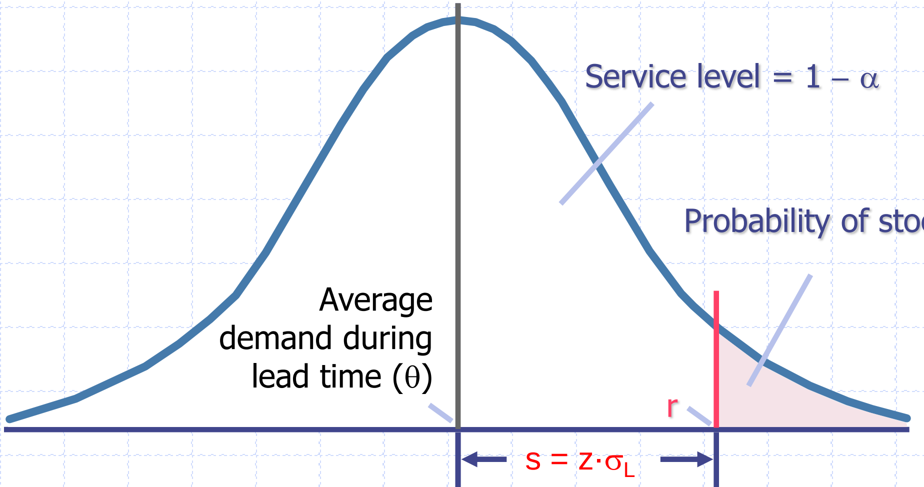
$$\theta = E(X) = \frac{D \cdot L}{365}$$

X demand during replenishment lead time (random variable)
L replenishment lead time (days) – constant
D expected demand per year (in units)

- Safety stock (s)

$$s = z \cdot \sigma$$

$$\Phi(z) = 1 - \alpha$$



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Stochastic Inventory Models

Continuous Review Models (15)

- (Q, r) Model
  - ✓ Insights
    - Increasing the average annual demand *D* tends to increase the order quantity *Q*.

$$Q^* = \sqrt{\frac{2AD}{h}}$$

- Increasing the average demand θ during a replenishmnt lead time tends to increase the reorder point *r*.
- Increasing the variability σ of the demand process tends to increase the reorder point *r*.

$$r^* = \theta + z \cdot \sigma$$

✓ High demand or long replenishment lead times tend to require more inventory in stock.

✓ Highly variable demand process typically requires more safety stock as protection against stockouts than does a very stable demand process.

- Increasing the holding cost *h* will tend to decrease both the replenishment quantity *Q* and reorder point *r*.

$$Q^* = \sqrt{\frac{2AD}{h}} \qquad G(r^*) = \frac{b}{b+h} \qquad G(r^*) = \frac{k \cdot D}{k \cdot D + h \cdot Q}$$

Inventory Management

L replenishment lead time in number of periods (discrete random variable)

$l = E(L)$ $\sigma_L^2 = Var(L)$

D_t demand on period *t* in units (random variable)

D_t has the same distribution for each period

$d = E[D_t]$ $\sigma_D^2 = Var[D_t]$

X demand during replenishment lead time (random variable)

Stochastic Inventory Models

Continuous Review Models (16)

- (Q, r) Model

✓ Modeling lead time variability — fixed lead time → variable lead time

Compute the formula for σ (standard deviation of demand during lead time) that considers the lead time variability
(Others same as those of the (Q, r) model)

The primary effect of the additional variability is to inflate the standard deviation σ of the demand during the replenishment lead time

✓ Demand during the variable lead time (L)

$$X = \sum_{t=1}^{L} D_t$$

$$E[X] = E[L]E[D_t] = l \cdot d = \theta$$

L, D_t: random variables

$$\mathrm{Var}[X] = E[L] \cdot \mathrm{Var}[D_t] + E[D_t]^2 \cdot \mathrm{Var}[L] = l \cdot \sigma_D^2 + d^2 \cdot \sigma_L^2$$

Formula for σ that considers the lead time variability

$$\sigma = \sqrt{\mathrm{Var}[X]} = \sqrt{l \cdot \sigma_D^2 + d^2 \cdot \sigma_L^2}$$

When demand is Poisson

$$\sigma = \sqrt{l \cdot d + d^2 \cdot \sigma_L^2} = \sqrt{\theta + \boxed{d^2 \cdot \sigma_L^2}}$$

Additional term compared with the case of constant lead time

Inventory Management

◈ **Stochastic Inventory Models**

Continuous Review Models (17)

- (Q, r) Model

✓ Modeling lead time variability

➢ Example (base stock model)

- Demand for refrigerators ~ Poisson with mean 10 per month

  ----▸ Daily demand d = 10 / 30 = 1 / 3

- Holding cost (h) = 15
- Backorder cost (b) = 25

✓ Optimal base stock level

$$G(r^*+1) = \frac{b}{b+h} = \frac{25}{15+25} = 0.625$$

$$\Phi(0.32) = 0.625$$

$$\dashrightarrow \; R^* = r^*+1 = \theta + z \cdot \sigma = \theta + z \cdot \sqrt{\theta + d^2 \cdot \sigma_L^2}$$

$\sigma_L = 0$ ----▸ $R^* = 11.01$

$\sigma_L = 30$ ----▸ $R^* = 13.34$

Additional 2.33 units for achieving the same service level in the face of more variable demand

Inventory Management

◈ **Stochastic Inventory Models**

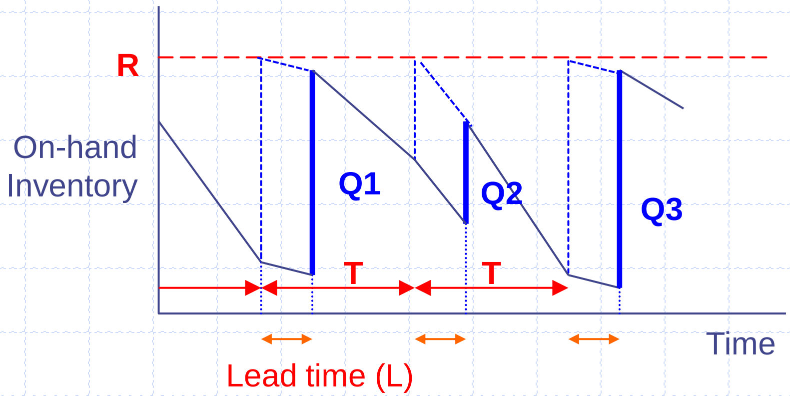
Periodic Review Model (1)

- (R, T) Model

✓ Method

Tracks the remaining inventory of an item periodically

- ➢ Order with the fixed interval T
- ➢ Ordering quantity is the difference between the target inventory level R and the inventory level at the ordering time

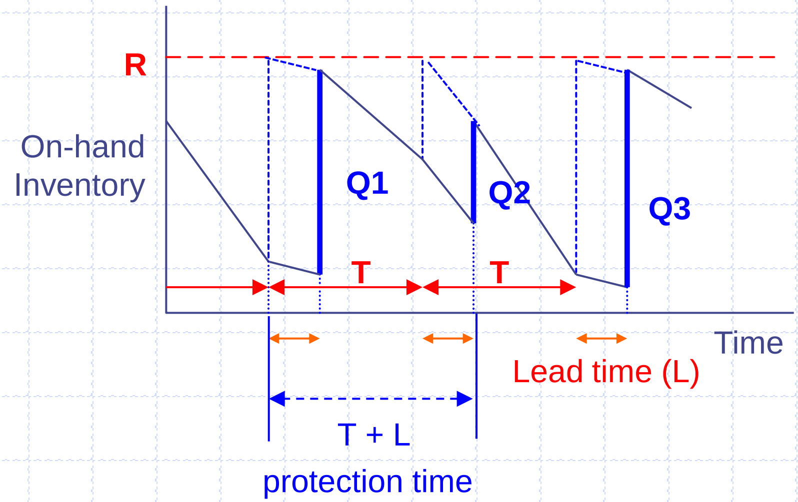


✓ Decision variables: R and T

✓ Characteristics

- ➢ Constant ordering cycle
- ➢ Variable ordering quantity

Inventory Management



- **Stochastic Inventory Models**

  Periodic Review Model (2)

  - (R, T) Model
    - ✓ Approximate solution
      - Time between reviews (T) – EOQ

$$T = \frac{Q}{D} = \frac{\sqrt{2AD/h}}{D} = \sqrt{\frac{2A}{hD}}$$

      - Target Inventory level (R)

$$R = D \cdot (T + L) + z \cdot \sigma$$

safety stock (s)

average demand during protection time (T + L)

σ = standard deviation of demand during protection time (T + L)

$$\Phi(z) = \boxed{1 - \alpha}$$

service level

A setup or purchase order cost per replenishment ($)
D expected demand per year (in units)
h cost to carry one unit of inventory for one year ($/unit·year)
L replenishment lead time (days) – constant