HW6 Solution

1. It was assumed that orders could be partially fulfilled before backlogging occurred.

   (a) For the (50,30) policy, the average monthly cost over 100 months, $\bar{Y}_{r\cdot}$, for replication r ($r = 1,2,3,4$), is given by

   $\bar{Y}_{1\cdot} = \$233.71, \bar{Y}_{2\cdot} = \$226.36, \bar{Y}_{3\cdot} = \$225.78, \bar{Y}_{4\cdot} = \$241.06.$

   By Equation (12.39), the point estimate is

   $\bar{Y}_{\cdot\cdot} = \$231.73$ and by Equation (12.40), $S^2 = (\$7.19)^2$.

   An approximate 90% confidence interval is given by

   $\$231.73 \pm t_{0.05,3}(\$7.19)/\sqrt{4}, (t_{0.05,3} = 2.353)$ or $[\$223.27, \$240.19]$

   (b) The minimum number of replications is given by

   $$R = \min\{R > R_0 : t_{\alpha/2,R-1}S_0/\sqrt{R} \leq \$5\} = 8$$

   where $R_0 = 4, \alpha = 0.10, S_0 = \$7.19$ and $\epsilon = \$5$.

   The calculation proceeds as follows:
   $R \geq (z_{.05}S_0/\epsilon)^2 = [1.645(7.19)/5]^2 = 5.60$

R	6	7	8
$t_{.05,R-1}$	1.94	1.90	1.86
$t_{.05,R-1}S_0/\epsilon^2$	7.78	7.46	7.15

   Thus, four additional replications are needed.

2. (a) The following estimates were obtained for the long-run monthly cost on each replication.

   $\bar{Y}_{1\cdot} = \$412.11, \bar{Y}_{2\cdot} = \$437.60, \bar{Y}_{3\cdot} = \$411.26, \bar{Y}_{4\cdot} = \$455.75, \bar{Y}_{\cdot\cdot} = \$429.18, S = \$21.52$

   An approximate 90% c.i. for long-run mean monthly cost is given by

   $\$429.18 \pm 2.353(\$21.52)/\sqrt{4}$, or

   $[\$403.86, \$454.50]$

   (b) With $R_0 = 4, \alpha = 0.10, S_0 = \$21.52$, and $\epsilon = \$25$ the number of replications needed is

   $$\min\{R \geq R_0 : t_{\alpha/2,R-1}S/\sqrt{R} < \$25\} = 5$$

   Thus, one additional replication is needed to achieve an accuracy of $\epsilon = \$25$.
   To achieve an accuracy of $\epsilon = \$5$, the total number of replications needed is

   $$\min\{R \geq R_0 : t_{.05,R-1}S_0/\sqrt{R} < 5\} = 53.$$

   The calculations for $\epsilon = \$5$ are as follows:

   $$R \geq [z_{.05}S_0/\epsilon]^2 = [1.645(21.52)/5]^2 = 50.12$$

R	51	52	53
$t_{.05,R-1}$	1.675	1.674	1.674
$[t_{.05,R-1}S_0/\epsilon]^2$	52.9	52.9	52.9

   Therefore, for $\epsilon = \$5$, the number of additional replications is $53 - 4 = 49$.

3. Ten initial replications were made. The estimated profit is \$98.06 with a standard deviation of $S_0 =$ \$12.95.

For $\alpha = 0.10$ and absolute precision of $\epsilon = \$5.00$, the sample size is given by

$$\min\{R \geq 10 : t_{\alpha/2,R-1}(12.95)/\sqrt{R} < \$5\}$$

R	$t_{\alpha/2,R-1}S_0/\sqrt{R}$
19	5.15
20	5.01
21	4.87

Thus, 21 replications are needed. Based on 21 replications the estimated profit is:

$$\bar{Y} = \$96.38, S = \$13.16$$

and a 90% c.i. is given by

$$\$96.38 \pm t_{.05,20}S/\sqrt{21}$$

or $\$96.38 \pm \$4.94$.

If $\epsilon = \$0.50$ and $\alpha = 0.10$, then the sample size needed is approximately 1815.

4. (a)

$$\begin{aligned} Y_j = \frac{1}{2000}\int_{(j-1)2000}^{j(2000)} L_Q(t)dt &= \frac{1}{2000}\int_{(j-1)2000}^{j(2000)-1000} L_Q(t)dt + \frac{1}{2000}\int_{j(2000)-1000}^{j(2000)} L_Q(t)dt \\ &= \frac{1}{2}\left(\frac{1}{1000}\int_{(j-1)2000}^{j(2000)-1000} L_Q(t)dt + \frac{1}{1000}\int_{j(2000)-1000}^{j(2000)} L_Q(t)dt\right) \end{aligned}$$