Electrical Engineering

HW 2 – Chapter 3, Solution

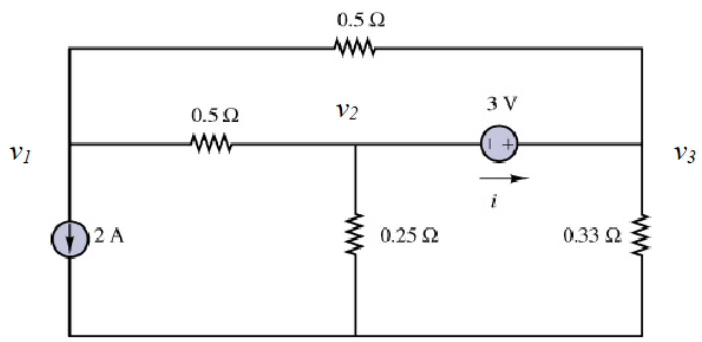
<1>

Solution:

Known quantities:
Circuit shown in Figure P3.4

Find:
Current through the voltage source.

Analysis:
Label the nodes, v_1, v_2, and v_3 as shown.



At node 1:

$$\frac{v_1 - v_2}{0.5} + \frac{v_1 - v_3}{0.5} = -2 \qquad (1)$$

At node 2:

$$\frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \qquad (2)$$

At node 3:

$$\frac{v_3 - v_1}{0.5} + \frac{v_3}{0.33} - i = 0 \qquad (3)$$

Further, we know that $v_3 = v_2 + 3$. Now we can eliminate either v_2 or v_3 from the equations, and be left with three equations in three unknowns:

$$\frac{v_1 - v_2}{0.5} + \frac{v_1 - (v_2 + 3)}{0.5} = -2 \qquad (1)$$

$$\frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \qquad (2)$$

$$\frac{(v_2 + 3) - v_1}{0.5} + \frac{(v_2 + 3)}{0.33} - i = 0 \qquad (3)$$

Solving the three equations we compute

$$i = 8.310\text{A}$$

<2>

Use KCL at all the nodes:

$$\frac{V_1 - V_a}{R_1} - I_1 - \frac{V_a - V_b}{R_2} = 0 \; \boldsymbol{Node\ 1}$$

$$\frac{V_a - V_b}{R_2} + I_1 - \frac{V_b}{R_3} - \frac{V_b - -V_2}{R_4} = 0 \; \boldsymbol{Node\ 2}$$

Substitute known values and collect coefficients:

$$-0.35 * V_a + 0.25 * V_b = 1.8 \; \textbf{Node 1}$$

$$0.25 * V_a - \frac{7}{12} * V_b = -1.33 \; \textbf{Node 2}$$

Solve:

$$\boldsymbol{V_a = -5.06V \;\; V_b = 0.117V}$$

<3>

Analysis:

Mesh #1 $\quad 20i_1 + 15i_1 + 10(i_1 - i_2) = 0$

Mesh #2 $\quad -50 + 10(i_2 - i_1) + 40i_2 + 10i_2 = 0$

Therefore,

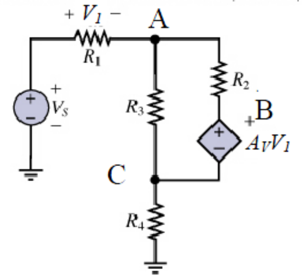
$I_1 = 0.1923$ A and $I_2 = 0.865$ A ,

$v_{10\Omega} = 10(i_2 - i_1) = 6.73$ V

<4>

Solution:

Known quantities:



Circuit shown in Figure P3.24

$V_S = 5\text{ V} \qquad A_V = 70 \qquad R_1 = 2.2\text{ k}\Omega$

$R_2 = 1.8\text{ k}\Omega \qquad R_3 = 6.8\text{ k}\Omega \qquad R_4 = 220\ \Omega$

Find:

The voltage across R_4 using node voltage analysis.

Analysis:

Node analysis is not a method of choice because the dependent source is [1] a voltage source and [2] a floating source. Both factors cause difficulties in a node analysis. A ground is specified. There are three unknown node voltages (labeled A, B, & C in the figure above), one of which is the voltage across R_4. The dependent source will introduce two additional unknowns, the current through the dependent source, I_{DS} and the controlling voltage (across R_1) that is not a node voltage. Therefore 5 equations are required:

$$[1]\text{KCL: } \frac{V_A - V_S}{R_1} + \frac{V_A - V_C}{R_3} + \frac{V_A - V_B}{R_2} = 0$$

$$[2]\text{KCL: } \frac{V_B - V_A}{R_2} - I_{DS} = 0$$

$$[3]\text{KCL: } \frac{V_C - V_A}{R_3} + I_{DS} + \frac{V_C}{R_4} = 0$$

$$[4]\text{KVL}: -V_S + V_1 + V_A = 0$$

$$[5]\text{KVL}: -V_C - A_V V_1 + V_B = 0$$

Solving these five equations simultaneously we find::

$V_C = V_4 = 8.8\text{mV}$

We also find:

$V_A = 4.91\text{V}$

$V_B = 6.14\text{V}$

$V_1 = 87.6\text{mV, and}$

$I_{DS} = 681\mu\text{A}$

<5>

Using the mesh current analysis:

$$-V_1 - i_1 * R_1 - (i_1 - i_2) * R_5 - V_5 - (i_1 - i_3) * R_6 + V_6 = 0 \textbf{ Mesh 1}$$

$$-(i_2 - i_3) * R_4 + V_4 + V_5 - (i_2 - i_1) * R_5 + V_2 - i_2 * R_2 = 0 \textbf{ Mesh 2}$$

$$V_3 - V_6 - (i_3 - i_1) * R_6 - V_4 - (i_3 - i_2) * R_4 - i_3 * R_3 = 0 \textbf{ Mesh 3}$$

Collect coefficients:

$$-15 * i_1 + 4 * i_2 + 3 * i_3 = 5 \textbf{ Mesh 1}$$

$$4 * i_1 - 9 * i_2 + 2 * i_3 = -7 \textbf{ Mesh 2}$$

$$3 * i_1 + 2 * i_2 - 10 * i_3 = \mp 3 \textbf{ Mesh 3}$$

Solve set of equations:

$$i_1 = -213\text{mA}$$
$$i_2 = 630\text{mA}$$
$$i_3 = -238\text{mA}$$

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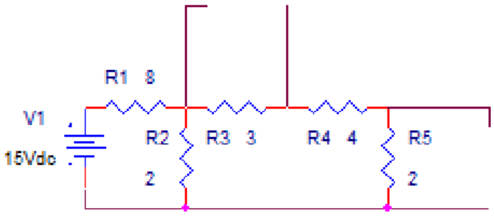
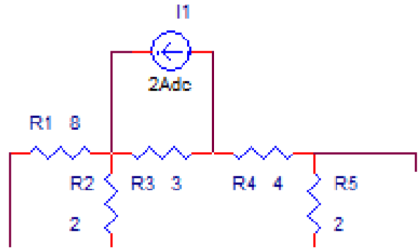
Using the superposition principle:

Case 1: Keep V_1 and remove (open) the current sources. Redraw the circuit.



$$i_0' = \frac{V_1}{R_1 + [R_2 \,||\, (R_3 + R_4 + R_5)]} = 1.557\text{A}$$

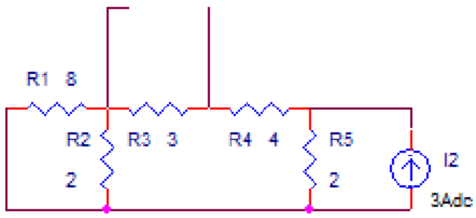
Case 2: Keep I_1 and remove (short) the voltage source and (open) the other current source. Redraw the circuit.



$$i_x = I_1 \cdot \frac{R_3}{R_3 + R_4 + R_5 + (R_1 \,||\, R_2)} = 0.566\text{A}$$

$$i_0'' = -i_x \cdot \frac{R_2}{R_1 + R_2} = -0.113\text{A}$$

Case 3: Keep I_2 and remove (short) the voltage source and (open) the othe



current source. Redraw the circuit.

Using divider current:

$$i_y = I_2 \cdot \frac{R_5}{R_4 + R_5 + R_3 + (R_1 \,||\, R_2)} = 0.566\text{A}$$

$$i_0''' = -i_y \cdot \frac{R_2}{R_2 + R_1} = -0.113A$$

Finally $i_0 = i_0' + i_0'' + i_0''' = 1.33\text{A}$

<7>

Remove the 3Ω resistor.
For the Thévenin resistance, remove (short) the voltage source and calculate the equivalent resistance..

$$R_{TH} = 1\Omega + 4\Omega \| 5\Omega = 3.22\Omega$$

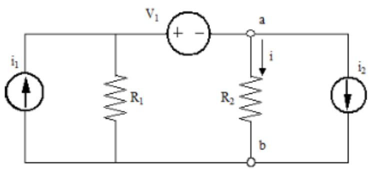
$$R_{TH} = 1 + \frac{1}{\frac{1}{5} + \frac{1}{4}} = 1 + \frac{20}{9} = \frac{29}{9}\,\Omega = 3.222\,\Omega$$

For the Thévenin voltage, use the divider

$$V_{TH} = \left(\frac{4}{4+5}\right) 36 = 16\ \text{V}$$

<8>

Zero all the sources (short the voltage sources and open the current sources).
The equivalent resistance, R_{TH}, is equal to R_1 $R_{TH} = 3\Omega$.



Short R_3. The current through this short circuit is the Norton equivalent current.

Using the superposition method to find I_N

Keep i_1, short v_1 and open i_2:

$$i'_{SC} = 10\text{A}$$

Keep i_2, short v_1 and open i_1:

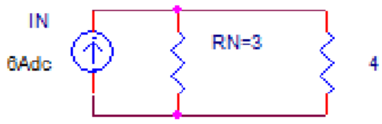
$$i''_{SC} = -2\text{A}$$

Keep v_1, open both i_1 and i_2:

$$i'''_{SC} = -\frac{6}{3}\text{A} = -2\text{A}$$

Finally: $i_{SC} = i'_{SC} + i''_{SC} + i'''_{SC} = 6\text{A} = I_N$

Using current division :



$$i = 6 \cdot \frac{3}{3+4} = 2.6\text{A}$$

<9>

Known quantities:

The values of the voltage and of the resistor in the equivalent circuit of Figure P3.73: $V_T = 10\text{V}; R_T = 2\Omega$

Assumptions:

Assume the conditions for maximum power transfer exist.

Find:

a. The value of R_0.

b. The power developed in R_0.

c. The efficiency of the circuit, that is the ratio of power absorbed by the load to power supplied by the source.

Analysis:

a. For maximum power transfer: $R_0 = R_T = 2\Omega$

b. *VD*: $$V_{R_0} = \frac{V_T R_0}{R_T + R_0} = \frac{(10)(2)}{2+2} = 5\text{V}$$

$$P_{R_0} = \frac{V_{R_0}^2}{R_0} = \frac{(5)^2}{2} = 12.5\text{W}$$

c.

$$P_{V_T} = \frac{V_T^2}{R_{total}} = \frac{(10)^2}{4} = 25$$

$$\eta = \frac{P_0}{P_{V_T}} = 0.5 = 50\%$$