**ENE 3031 - Fall 2014**
**Homework 1**
**due Tuesday Sep/23**

1. Suppose that $X$ is a discrete random variable having probability function $\Pr(X = k) = ck$ for $k = 1, 2, 3$. Find the constant $c$, $\Pr(X \leq 2), \mathsf{E}[X]$, and $\mathsf{Var}(X)$.

2. Suppose that $X$ is a continuous random variable having probability function $\Pr(X = x) = cx$ for $1 \leq x \leq 2$. Find the constant $c$, $\Pr(X \geq 1), \mathsf{E}[X]$, and $\mathsf{Var}(X)$.

3. Suppose $X$ and $Y$ are jointly continuous random variables with

$$f(x,y) = \begin{cases} y - x, & \text{for } 0 < x < 1, 1 < y < 2, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

   (a) Compute $f_X(x)$ and $f_Y(y)$.
   (b) Are $X$ and $Y$ are independent?
   (c) Compute $F_X(x)$ and $F_Y(y)$.
   (d) Compute $\mathsf{E}[X]$, $\mathsf{Var}(X)$,$\mathsf{E}[Y]$, $\mathsf{Var}(Y)$, $\mathsf{Cov}(X,Y)$, and $\mathsf{Corr}(X,Y)$.

4. If $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with mean $\mu$ and variance $\sigma^2$, then compute $\mathsf{Cov}(\bar{X}, S^2)$ where $\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$ is the sample mean and $S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{(n-1)}$ is the sample variance. When will this covariance be equal to 0?

5. Suppose that the following 10 observations come from some distribution (not highly skewed) with unknown mean $\mu$.

$$7.3, 6.1, 3.8, 8.4, 6.9, 7.1, 5.3, 8.2, 4.9, 5.8$$

   Compute $\bar{X}, S^2$, and an approximate 95% confidence interval for $\mu$.

6. A random variable $X$ has the *memoryless property* if, for all $s, t > 0$,

$$\Pr(X > t + s | X > t) = \Pr(X > s). \tag{2}$$

   Show that the exponential distribution has the memoryless property.

7. A geometric distribution $X$ with parameter $p$ $(0 < p < 1)$ has the p.m.f.

$$f(x) = (1-p)^x p, \; x = 0, 1, 2, \ldots. \tag{3}$$

   Show that this distribution has the memoryless property.

8. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. Exp($\lambda$).

   (a) Find the m.g.f. of $X_i$.

   (b) Use m.g.f's to find the distribution of $Y = \sum_{i=1}^{n} X_i$.

   (c) Suppose $\lambda = 1$. Use the Central limit theorem to find the approximation of $\Pr(100 \leq \sum_{i=1}^{100} X_i \leq 110)$.

9. Generate 1000 pairs of i.i.d. Unif(0, 1)s, $(U_{1,1}, U_{2,1}), (U_{1,2}, U_{2,2}), \ldots, (U_{1,1000}, U_{2,1000})$. Set

$$X_i = \sqrt{-2\ln(U_{1,i})}\cos(2\pi U_{2,i}), \tag{4}$$
$$Y_i = \sqrt{-2\ln(U_{1,i})}\sin(2\pi U_{2,i}), \tag{5}$$

   for $i = 1, 2, \ldots, 1000$.

   (a) Make a histogram of the $X_i$'s. Comments?

   (b) Graph $X_i$ Vs. $Y_i$. Comment?

   (c) Make a histogram of $X_i/Y_i$. Comments?

   (d) Make a histogram of $X_i^2 + Y_i^2$. Comments?