Facilities planning

Material handling system

Overview

- 25% of employee, 55% of factory space, 15~70% of total cost of a manufacturing cost are related to material handling.
- Material handling is a means by which total manufacturing costs are reduced through more efficient material flow control, lower inventories and improved safety.
- Definition of material handling
  - Material handling is the art and science of moving, storing, protecting and controlling material.
  - Material handling means proving the right amount of the right material, in the right condition, at the right place, in the right orientation, in the right sequence, at the right time, and for the right cost, by the right methods.

Overview

- right amount: Right amount is what is needed not what is anticipated.
- right material: Automatic identification is the key.
- right condition: Right condition is the state in which the customer desires to receive the material.
- right place: Right place is the point of use rather than intermediate location.
- right orientation: Positioning the material for ease of handling.
- right sequence: Right sequence of manufacturing or distribution activities for efficiency.
- right time: on-time delivery
- right cost: Right cost is not necessary the lowest cost. The appropriate goal of material handling system design is to design the most efficient systems at the most reasonable cost.
- right methods: Using more than one method is generally the right thing to do

Unit load design

- Definition of unit load: a number of items, or bulk material which is arranged and restrained so that the load can be stored and picked up and moved between two locations as a single object (Tanchoco et al.).
- The size of the unit load can range from a single part carried by a person, to each carton moved through a conveyor system, to a number of cartons on a pallet moved by fork lift trucks, to a number intermodal (integrated) containers moved by rail across states.
- Large unit loads
  - may require bigger and heavier equipment, wider aisles and higher floor load capacities.
  - increase WIP since items have to accumulate to full unit load size before the container or pallet is moved.
  - A major advantage is fewer moves.

Unit load design

- Small loads
  - increase the transportation requirements but can potentially reduce WIP.
  - often require simple material handling methods such as push carts.
  - have impact on job completion time

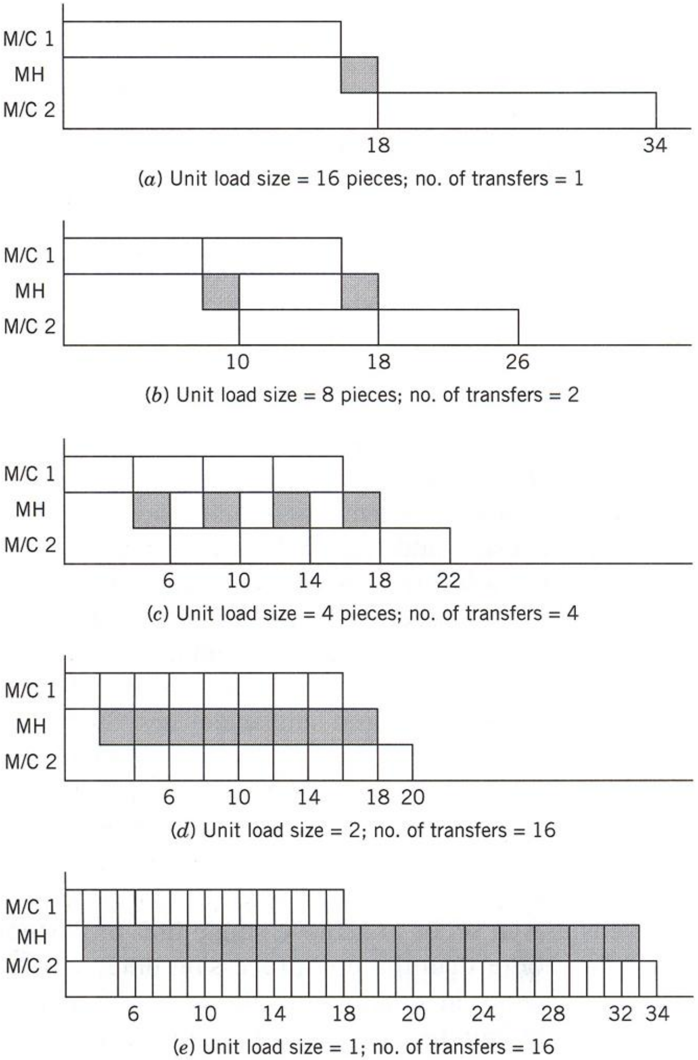
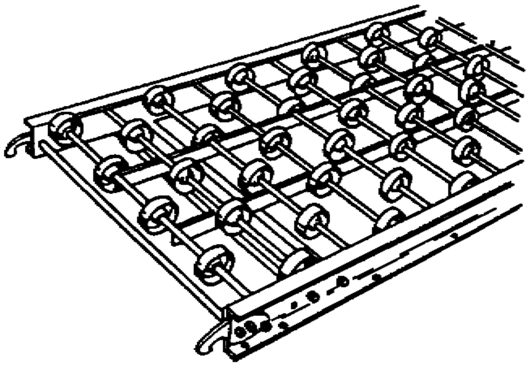


Figure 5.3 Effects of unit load size on job completion times.

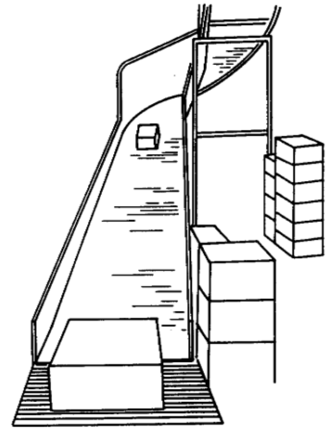
Material handling equipment

- Containers and unitizing equipment
  - Containers
  - Unitizers
- Material transport equipment
  - Conveyors
  - Industrial vehicles: AGV (Automated Guided Vehicle)
  - Monorails, hoists and cranes
- Storage and retrieval equipment
  - Unit load storage and retrieval: AS/RS (Automated Storage and Retrieval System)
  - Small load storage and retrieval: AS/RS
- Automatic data collection and communication equipment
  - Automatic identification and recognition
  - Automatic paperless communication

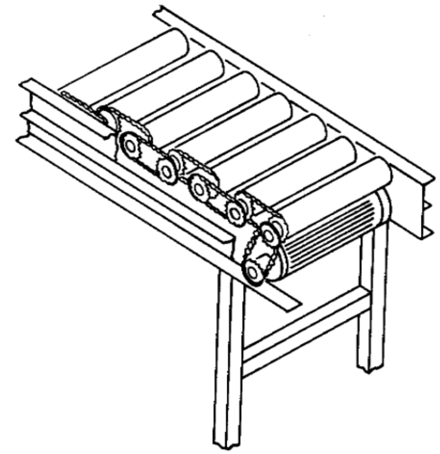
conveyors



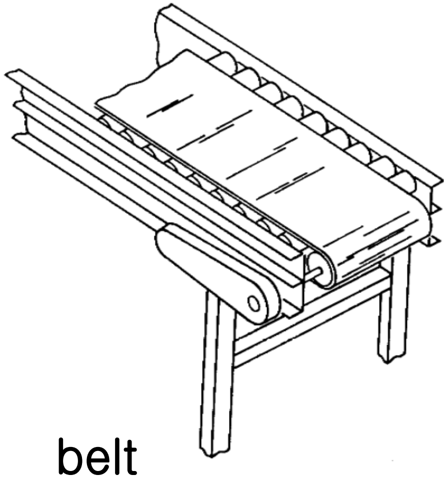
wheel



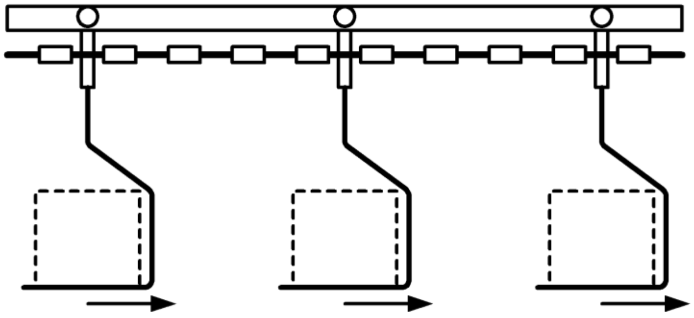
chute



roller

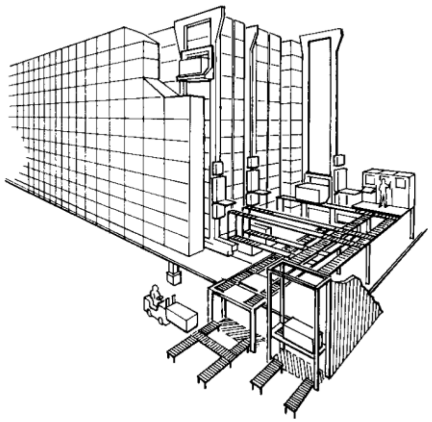


belt



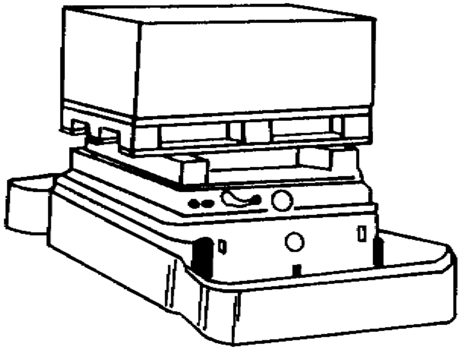
trolley

AGV, AS/RS



unit load AS/RS

carousel



unit load AGV

Trolley conveyor

- Conveyor layout–Muth

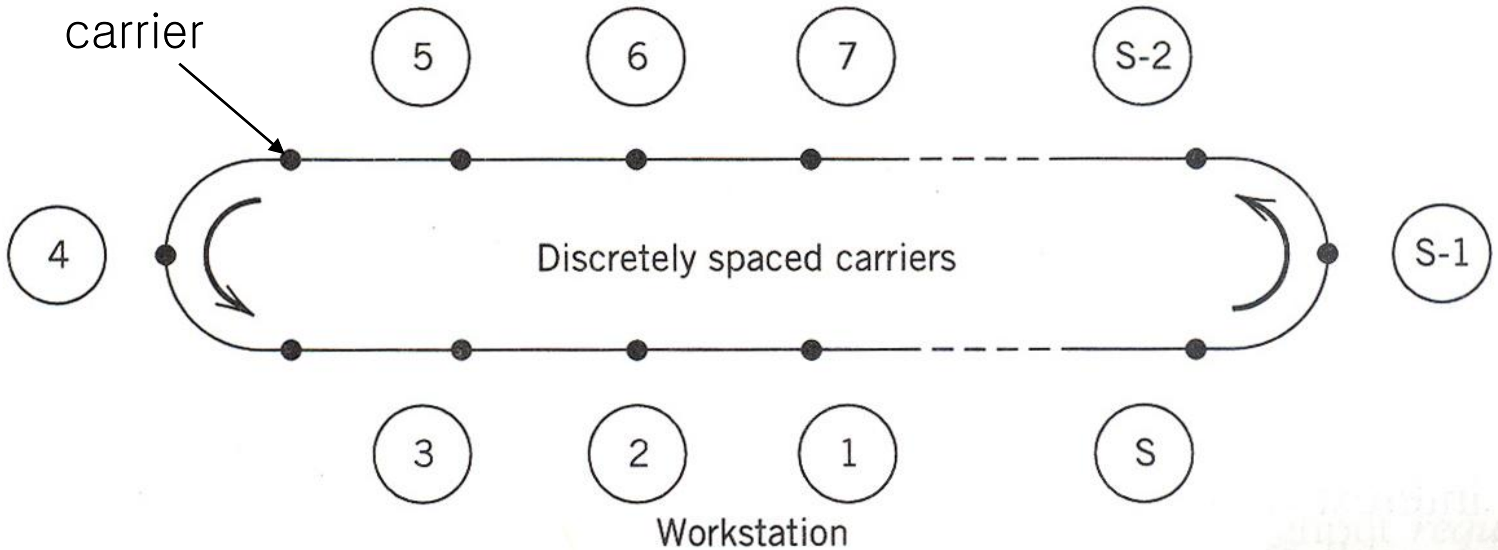
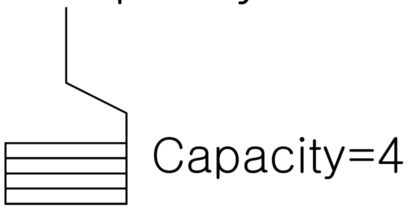


Figure 10.19 Conveyor layout considered by Muth [48].

- Given the number of carriers and the number of stations, we want to determine the capacity of carriers.



Trolley conveyor

- Example

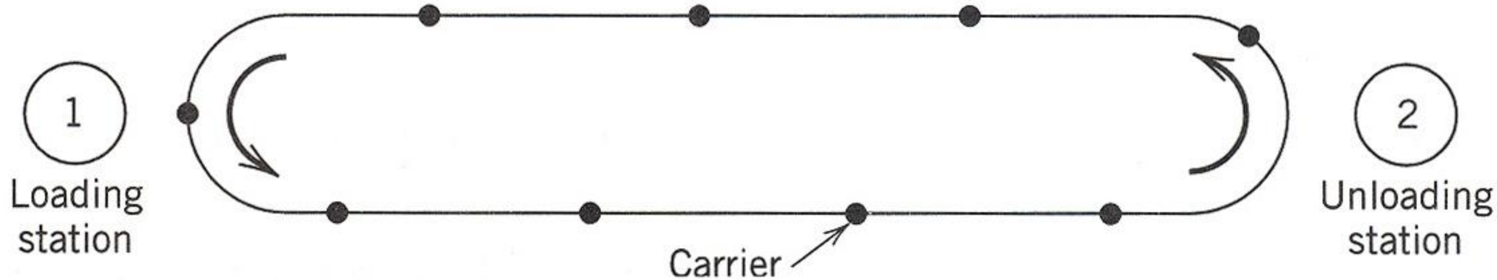


Figure 10.20 Conveyor layout for Example 10.30.

Trolley conveyor

- There are s stations located around the conveyor
numbered in reverse sequence to the rotation of the conveyor.
- Each station can perform loading and/or unloading of multiple items simultaneously.
- There are k carriers equally spaced around conveyor.
- The passage of a carrier by a workstation establishes the increment of time used to define material loading and unloading sequence.
- Station 1 is used as a reference point in defining time,
consequently carrier n becomes carrier n+k immediately after passing station 1.

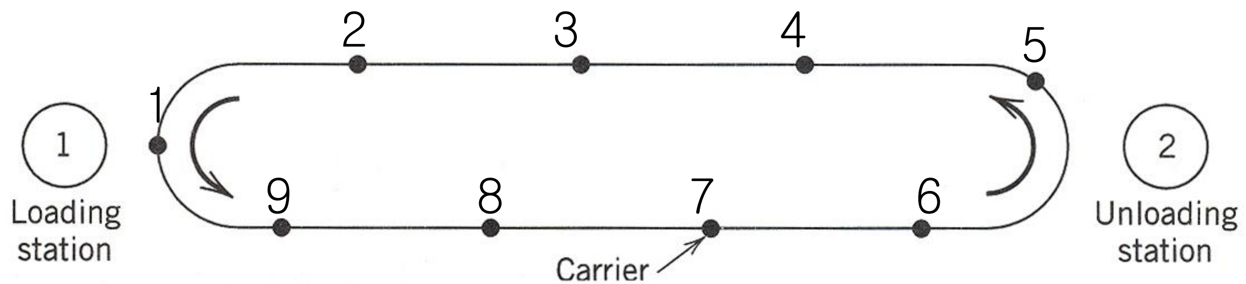


Figure 10.20 Conveyor layout for Example 10.30.

Trolley conveyor

- $f_i(n)$: the amount of material loaded on carrier n by station i when it passes the station. Negative value of $f_i(n)$ implies unloading.
- $\{f_i(n)\}$:sequence of $f_i(n)$ for i where $|\{f_i(n)\}|=p$

$\{f_i(n)\}=\{f_i(n+p)\}$:sequence of $f_i(n)$ is periodic with period p.

ex)p=7,$\{f_1(n)\}=\{1,1,2,2,2,1,1\}$, $\{f_1(n+7)\}=\{1,1,2,2,2,1,1\}$

- $H_i(n)$:the amount of material carried by carrier n immediately after passing station i.
- $\sum_{i=1}^{s}\sum_{n=1}^{p} f_i(n)=0$: all material loaded on the conveyor must be unloaded.

ex)p=7,$\{f_1(n)\}=\{1,1,2,2,2,1,1\}$,$\{f_2(n)\}=\{0,0,0,0,0,-5,-5\}$,

- $F_1(n)=\sum_{i=1}^{s} f_i(n)$:total material loaded on carrier n.

ex)$\{F_1(n)\}=\{1,1,2,2,2,-4,-4\}$

Trolley conveyor

- Theoretical results

-Blocking occurs at a loading station because arriving carriers are already full.

-Starving occurs at a unloading station because arriving carriers contain insufficient amount of materials.

-The conveyor is compatible if the conveyor can opeate over an infinite period of time without blocking and starving i.e., steady-state.

-$\frac{k}{p}$ cannot be an integer for steady-state operations. p is prime number.

-Let r=k mod p, where k mod p means the remainder of the division of k by p.

Trolley conveyor

- Solution procedure

1. Calculate $H_1^*(n)$ using the following recursive equation.

$H_1^*(n)=H_1^*(n\text{-}r)+F_1(n)$ where $H_1^*(1)=0$

Note that the visiting sequece is 1->s->s-1 thus $H_1^*(1)=0$.

2. Given $H_i^*(n)$, calculate $H_{i+1}^*(n)=H_i^*(n)\text{-}f_i(n)$

Note that the visiting sequence of carrier n is i+1->i.

Thus $H_{i+1}^*(n)+f_i(n)=H_i^*(n)$.

3. Given $\{H_i^*(n)\}$ for i=1,..,s, Let

$c=\min_{i,n} H_i^*(n)$

4. The desired solution is given by

$H_i(n)=H_i^*(n)\text{-}c$

5. The required capacity per carrier is

$B=\max_{i,n} H_i(n)$

Example 10.30

$\{f_1(n)\}=\{1,1,2,2,2,1,1\},\{f_2(n)\}=\{0,0,0,0,0,-5,-5\},\{F_1(n)\}=\{1,1,2,2,2,-4,-4\}$

Since p=7,k=9, r=9 mod 7=2

1. $H_1^*(n)=H_1^*(n-2)+F_1(n)$ where $H_1^*(1)=0$

Consider the case of n=3 such that $H_1^*(3)=H_1^*(1)+F_1(3)=0+2=2$

Consider the case of n=5, $H_1^*(5)=H_1^*(3)+F_1(5)=2+2=4$

Consider the case of n=7, $H_1^*(7)=H_1^*(5)+F_1(7)=4-4=0$

Consider the case of n=9, $H_1^*(9)=H_1^*(7)+F_1(9)=H_1^*(7)+F_1(2)=0+1=1$

Since $H_1^*(9)=H_1^*(2)=1$

Consider the case of n=4, $H_1^*(4)=H_1^*(2)+F_1(4)=1+2=3$

Consider the case of n=6, $H_1^*(6)=H_1^*(4)+F_1(6)=3-4=-1$

Hence, $\{H_1^*(n)\}=\{0,1,2,3,4,-1,0\}$

Example 10.30

2. $H_2^*(n)=H_1^*(n)-f_1(n)$

$\{H_2^*(n)\}=\{-1,0,0,1,2,-2,-1\}$

3. $c=\min_{i,n} H_i^*(n)=\min\{0,1,2,3,4,-1,0,-1,0,0,1,2,-2,-1\}=-2$

4. $H_i(n)=H_i^*(n)-c$

$\{H_1(n)\}=\{2,3,4,5,6,1,2\},\ \{H_2(n)\}=\{1,2,2,3,4,0,1\}$

5. $B=\max_{i,n} H_i(n)=\max\{2,3,4,5,6,1,2,1,2,2,3,4,0,1\}=6$

Example 10.30

Table 10.11 *Values of* {$H_i(n)$} *for Various Values of* k *in Example 10.30*

	$k=8$		$k=9$		$k=10$		$k=11$		$k=12$		$k=13$	
n	$H_1(n)$	$H_2(n)$	$H_1(n)$	$H_2(n)$	$H_1(n)$	$H_2(n)$	$H_1(n)$	$H_2(n)$	$H_1(n)$	$H_2(n)$	$H_1(n)$	$H_2(n)$
1	2	1	2	1	5	4	4	3	6	5	9	8
2	3	2	3	2	2	1	7	6	5	4	8	7
3	5	3	4	2	5	3	5	3	5	3	7	5
4	7	5	5	3	7	5	3	1	4	2	5	3
5	9	7	6	4	4	2	6	4	3	1	3	1
6	5	4	1	0	1	0	3	2	2	1	1	0
7	1	0	2	1	3	2	1	0	1	0	5	4
	$B=9$		$B=6$		$B=7$		$B=7$		$B=6$		$B=9$	

Example 10.31

$p=5, k=7, r=7 \bmod 5=2$

$\{f_1(n)\}=\{0,0,2,3,1\}, \{f_2(n)\}=\{0,0,0,-2,-4\}, \{F_1(n)\}=\{0,0,2,1,-3\}$

AGV

- AGV system
  - inductive guidepath
  - local controller: controls a section of the path
  - central controller: assign transport tasks to vehicles
  - pickup and dropoff points
- Issues
  - design issues
    - determination of path layout
    - determination of the location of P, D
  - operational issues
    - determination of the number of vehicles
    - determination of the route that vehicles will take

Example

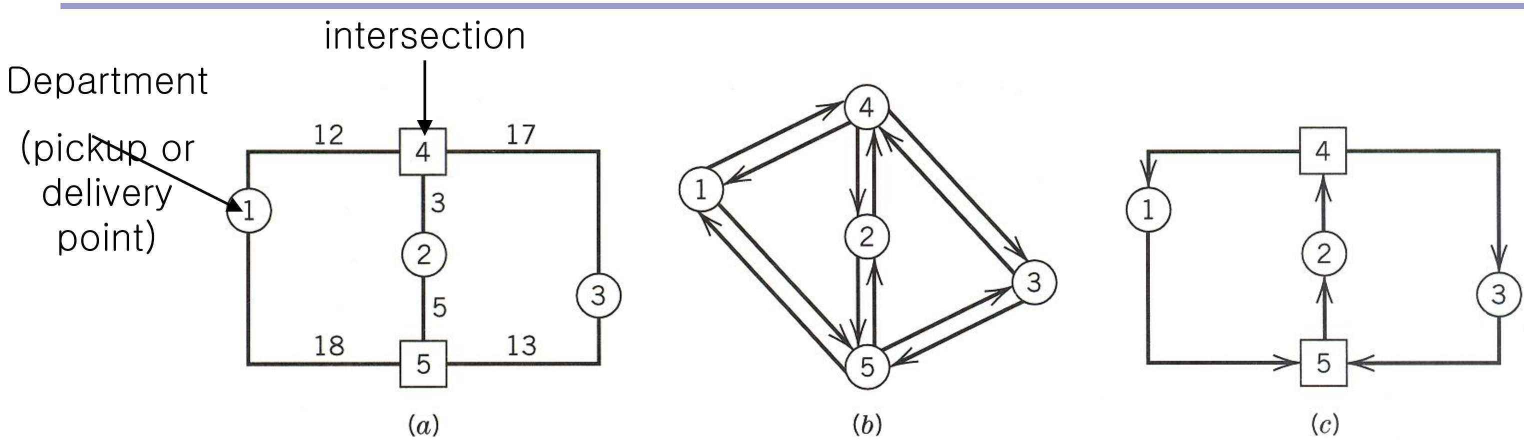


Figure 10.22 (*a*) The departmental layout, (*b*) the node-are network, and (*c*) the optimal flow path.

Table 10.19 *From-to Flow Chart*

	To		
From	1	2	3
1	—	10	15
2	20	—	ε
3	5	10	—

Path design-analytical model

node: pickup points, delivery points, aisle intersection points

arc: direction of flow between two adjacent nodes.

n:number of entries in the from-to chart

f_{lm}:flow intensity from pickup node l to delivery node m

d_{ij}:length of arc i-j (the distance from node i to an adjacent node j)

Y_{lm}:path length from pickup node l to delivery node m

$$X_{ijlm} = \begin{cases} \text{1:if arc i-j is included in the path from pickup node l} \\ \text{to delivery node m} \\ \text{0:otherwise} \end{cases}$$

$$Z_{ij} = \begin{cases} \text{1:if arc i-j is directed from node i to node j} \\ \text{0:otherwise} \end{cases}$$

Path design-analytical model

$$\min \sum_{l,m} f_{lm} Y_{lm}$$

st

$$(1) \sum_{i,j} X_{ijlm} d_{ij} = Y_{lm}, \forall l,m$$

Path length from l to m

$$(2) X_{ijlm} \le Z_{ij}, \forall l,m, \forall i,j$$

Arc from i to j cannot be included in the path from l to m unless it is directed so

$$(3) Z_{ij} + Z_{ji} \le 1, \forall i,j$$

Directions must be unidirecdtional

$$(4) \sum_{i} Z_{ij} \ge 1, \forall j$$

At least one input arc must exist for node j

$$(5) \sum_{k} Z_{jk} \ge 1, \forall j$$

At least one output arc must exist for node j

$$(6) \sum_{k} X_{lklm} = 1, \forall l,m$$

Exactly one output arc l->k exists on path from l to m

$$(7) \sum_{k} X_{kmlm} = 1, \forall l,m$$

Exactly one input arc k->m exists on path from l to m

$$(8) \sum_{i} X_{ijlm} = \sum_{k} X_{jklm}, \forall l,m, \forall j$$

Number of input arcs for node j on path from l to m, is equal to the number of output arcs

Path design-analytical model

(1)12x1412+18x1512+3x2412+5x2512+17x3412+13x3512+12x4112+18x5112+3x42
12+5x5212+17x4312+13x5312 – y12=0

(2)x1412 – z14 <= 0 x1413 – z14 <= 0 x1421 – z14 <= 0
x1423 – z14 <= 0 x1431 – z14 <= 0 x1432 – z14 <= 0

(3)z14+z41 <= 1

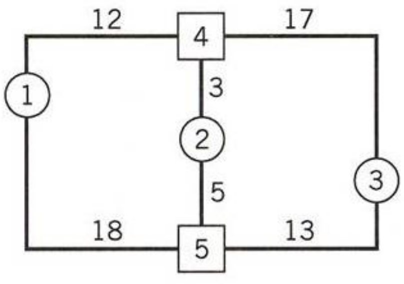
(4)z14+z24+z34 >= 1

(5)z41+z42+z43 >= 1

(6)x1412+x1512 = 1

(7)x4212+x5212 = 1

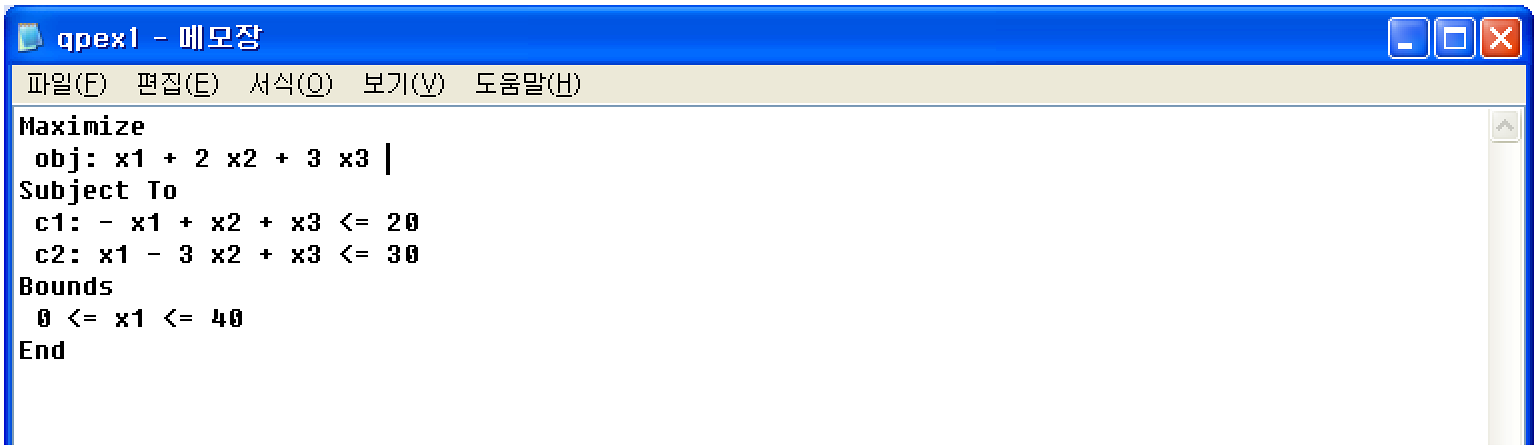
(8)x4312+x5312–x3412–x3512 = 0
x1412+x2412+x3412–x4112–x4212–x4312 = 0
x1512+x2512+x3512–x5112–x5212–x5312 = 0



(a)

z14 z15 z24 z25 z34 z35 z41 z42 z43 z51 z52 z53
x1412 x1512 x2412 x2512 x3412 x3512 x1413 x1513 x2413 x2513 x3413 x3513
x1421 x1521 x2421 x2521 x3421 x3521 x1423 x1523 x2423 x2523 x3423 x3523
x1431 x1531 x2431 x2531 x3431 x3531 x1432 x1532 x2432 x2532 x3432 x3532
x4112 x5112 x4212 x5212 x4312 x5312 x4113 x5113 x4213 x5213 x4313 x5313
x4121 x5121 x4221 x5221 x4321 x5321 x4123 x5123 x4223 x5223 x4323 x5323
x4131 x5131 x4231 x5231 x4331 x5331 x4132 x5132 x4232 x5232 x4332 x5332

CPLEX



- editor에서 확장자명이 lp인 파일을 작성(location.lp)
- C:₩ILOG₩CPLEX70₩bin₩msvc5₩stat_md

디렉토리에 lp화일을 저장

- 변수가 정수인 경우:

  Integers

  x1
- 변수가 0/1인 경우

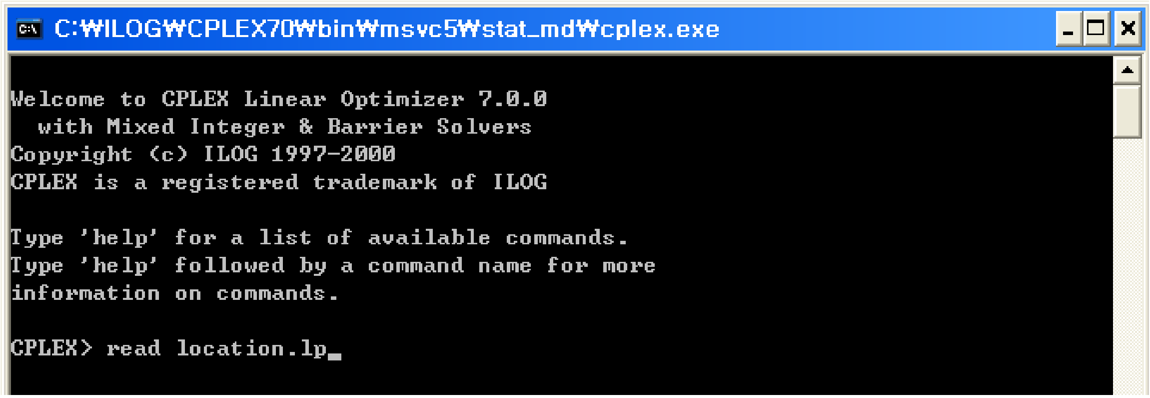
  Binaries

  x1
- 변수가 정수인 경우

  Generals

  x1

CPLEX



- C:₩ILOG₩CPLEX70₩bin₩msvc5₩stat_md로 이동해서 cplex70.exe를 실행
- read location.lp->lp화일을 읽어들임

CPLEX

```
C:₩ILOG₩CPLEX70₩bin₩msvc5₩stat_md₩cplex.exe
CPLEX>
CPLEX>
CPLEX>
CPLEX> display problem all
Maximize
 obj: 1675 x11 + 409 x12 + 685 x13 + 1630 x14 + 1160 x15 + 2800 x16 + 1460 x21
 + 1940 x22 + 970 x23 + 100 x24 + 495 x25 + 1200 x26 + 1925 x31 + 2400 x32
```

- display problem all->문제가 제대로 읽혀졌는지 확인

CPLEX

```
CPLEX> primopt
Tried aggregator 1 time.
No LP presolve or aggregator reductions.
Presolve time =    0.00 sec.

Iteration log . . .
Iteration:     1    Infeasibility =             0.000000
Iteration:     2    Objective     =         26535.000000

Primal - Optimal:  Objective =   2.6535000000e+004
Solution time =    0.00 sec.  Iterations = 2 (1)

CPLEX> _
```

- primopt-> primal simplex method로 문제를 풀라는 명령
- mipopt->mixed integer programming을 branch and bound로 풀라는 명령
- 오류가 없다면 iteration횟수와 objective value가 나옴

CPLEX

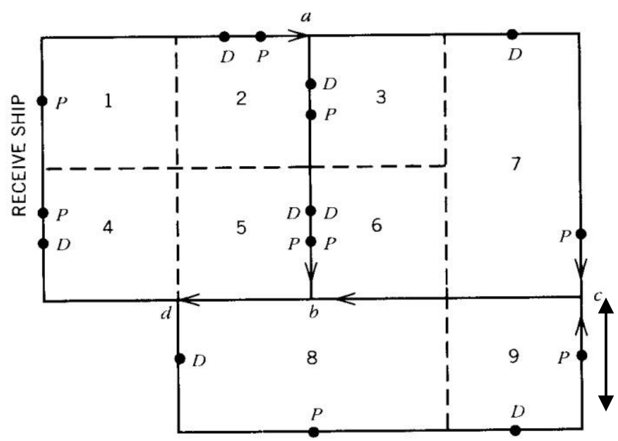
```
CPLEX> display solution variables -
Variable Name           Solution Value
x12                           8.000000
x13                           2.000000
x24                           6.000000
x36                          11.000000
x41                          10.000000
x43                          12.000000
x55                           7.000000
All other variables in the range 1-30 are zero.
CPLEX> _
```

- display solution variables – :변수의 값을 화면에 나타내라는 명령어

Pickup, delivery points design

- Design rule
  - Pickup stations should be downstream of delivery station(The vehicle should be able to drop off its load and then pick up a new load)
  - For each pickup point along a segment, total deliveries from the start of the segment to this pickup point should be at least as large as total pickups to this point in the segment(The goal is dual command operation for each segment transversal)
    - segment: any portion of the path from one intersection point to another
  - Place P and D points on low usage segments(This avoids blocking of vehicles attempting to bypass a P or D point)

Estimating vehicle requirement



(b) First path alternative

AGV path

50 ft

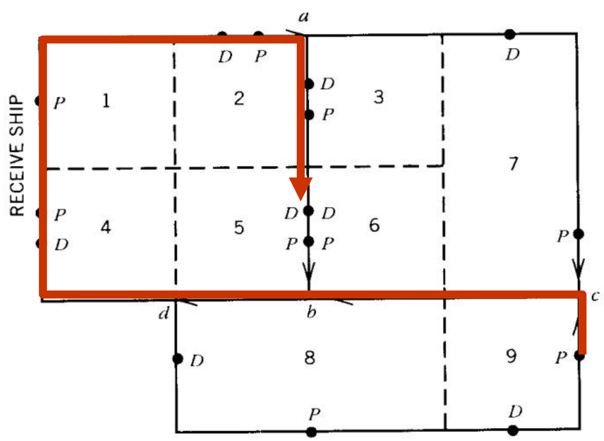
Department	D	P
1	(0,120)	(0,130)
2	(70,150)	(80,150)
3	(100,130)	(100,120)
4	(0,70)	(0,80)
5	(100,80)	(100,70)
6	(100,80)	(100,70)
7	(170,150)	(200,70)
8	(50,25)	(100,0)
9	(175,0)	(200,30)

D,P locations

Table 9.3 Interdepartmental AGV Flows

From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	40	25	30	10	10	20	5	10	150
2		—	40		30		10	10		90
3			—				50		10	60
4		5	10	—		10				25
5				100	—					100
6				60		—				60
7						40	—		40	80
8				10		5		—		15
9					60				—	60
Sum	0	45	75	200	100	65	80	15	60	640

Loaded travel distance



(b) First path alternative

Department	D	P
1	(0,120)	(0,130)
2	(70,150)	(80,150)
3	(100,130)	(100,120)
4	(0,70)	(0,80)
5	(100,80)	(100,70)
6	(100,80)	(100,70)
7	(170,150)	(200,70)
8	(50,25)	(100,0)
9	(175,0)	(200,30)

Loaded travel distance from department 9 to 6

=distance between P of department 9 to D of department 6

=20+200+100+100+70=490ft

Table 9.4 Loaded Travel Distances (Feet) for Alternative 1

From P to D	1	2	3	4	5	6	7	8	9
1	—	90	140	340	190	190	190	295	445
2	290	—	40	240	90	90	90	195	345
3	240	340	—	180	40	40	440	145	295
4	40	140	190	—	240	240	240	345	495
5	190	290	340	140	—	390	390	95	245
6	190	290	340	140	390	—	390	95	245
7	290	390	440	240	490	490	—	195	345
8	420	520	570	370	620	620	620	—	75
9	290	390	440	240	490	490	490	195	—

Loaded travel distance

Table 9.3 Interdepartmental AGV Flows

From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	40	25	30	10	10	20	5	10	150
2		—	40		30		10	10		90
3			—				50		10	60
4		5	10	—		10				25
5				100	—					100
6				60		—				60
7						40	—		40	80
8				10		5		—		15
9					60				—	60
Sum	0	45	75	200	100	65	80	15	60	640

Table 9.4 Loaded Travel Distances (Feet) for Alternative 1

From *P* to *D*	1	2	3	4	5	6	7	8	9
1	—	90	140	340	190	190	190	295	445
2	290	—	40	240	90	90	90	195	345
3	240	340	—	180	40	40	440	145	295
4	40	140	190	—	240	240	240	345	495
5	190	290	340	140	—	390	390	95	245
6	190	290	340	140	390	—	390	95	245
7	290	390	440	240	490	490	—	195	345
8	420	520	570	370	620	620	620	—	75
9	290	390	440	240	490	490	490	195	—

Total loaded travel distance
=40(90)+25(140)+,…,+60(490)
=159925ft

Empty travel distance between departments

t_{ij} : the shortest travel time from D of department i to P of department j

v_{ij} : the number of loads moved from department i to j per period

I_i : the desired initial number of vehicles at department i

E_i : the desired ending number of vehicles at department i

x_{ij} : the number of empty moves from department i to j

$$\text{minimize} \quad \sum_{i} \sum_{j} t_{ij} x_{ij}$$

$$\text{st} \quad E_i = I_i + \sum_{j} v_{ji} - \sum_{j} v_{ij} + \sum_{j} x_{ji} - \sum_{j} x_{ij}, \forall i$$

$\rightarrow$ vehicle conservation constraints

Empty travel distance between departments

Remark:In an optimal solution, at most one of the sums $\sum_j x_{ij}$ and $\sum_j x_{ji}$

is positive for each i,

Proof)want to prove if a solution has some i,j,k,such that $x_{ij} > 0$ and $x_{ki} > 0$

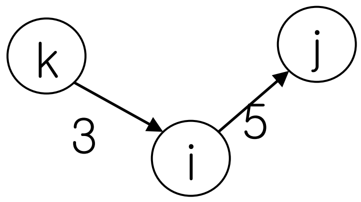
then the solution is not an optimal solution.

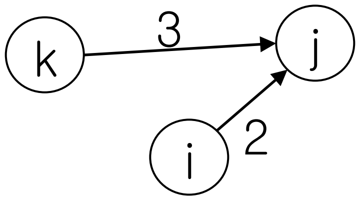
Increase $x_{kj} = \min(x_{ij}, x_{ki})$ then objective function value is reduced by

$(t_{ki} + t_{ij})x_{kj} - t_{kj}x_{kj} = (t_{ki} + t_{ij} - t_{kj})x_{kj}$

If triangular inequality holds, that is $t_{ki} + t_{ij} - t_{kj} \geq 0$,

the solution is improved and the solution is not optimal





Empty travel distance between departments

$$\text{If } \sum_j x_{ij} > 0 \text{ then } \sum_j x_{ji} = 0$$

$$E_i = I_i + \sum_j v_{ji} - \sum_j v_{ij} + \sum_j x_{ji} - \sum_j x_{ij}, \forall i \text{ becomes}$$

$$\sum_j x_{ij} = I_i - E_i + \sum_j v_{ji} - \sum_j v_{ij} \text{ and let } I_i - E_i + \sum_j v_{ji} - \sum_j v_{ij} = g_i$$

$$\text{that is } \sum_j x_{ij} = g_i > 0$$

$$\text{Else if } \sum_j x_{ji} > 0 \text{ then} \sum_j x_{ij} = 0$$

vehicle conservation constraints become

$$\sum_j x_{ji} = -(I_i - E_i + \sum_j v_{ji} - \sum_j v_{ij})$$

$$\text{that is } \sum_j x_{ji} = -g_i > 0$$

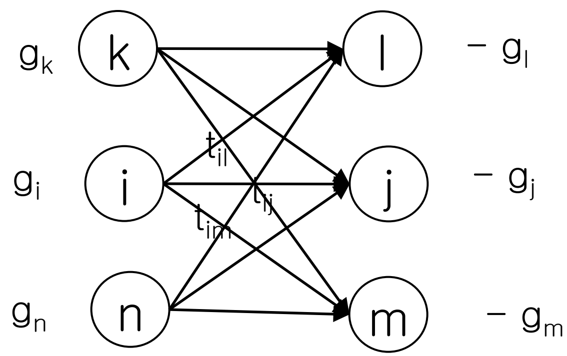
Empty travel distance between departments

Finding an optimal empty vehicle move is equivalant to solving the following transportation problem

$$\text{minimize} \sum_{i}\sum_{j} t_{ij}x_{ij}$$

$$\text{st} \sum_{j} x_{ij} = g_i \text{ for all i such that } g_i > 0$$

$$\sum_{j} x_{ji} = -g_i \text{ for all i such that } g_i < 0$$



Example

Table 9.3 Interdepartmental AGV Flows

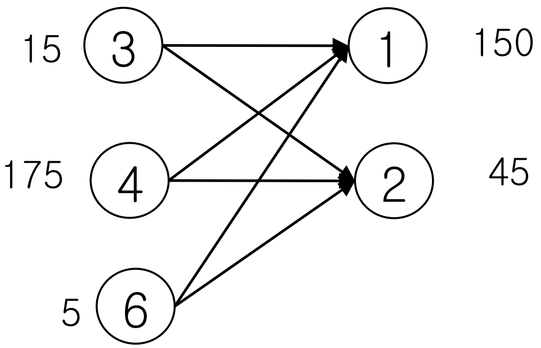
From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	40	25	30	10	10	20	5	10	150
2		—	40		30		10	10		90
3			—				50		10	60
4		5	10	—		10				25
5				100	—					100
6				60		—				60
7						40	—		40	80
8				10		5		—		15
9					60				—	60
Sum	0	45	75	200	100	65	80	15	60	640

Suppose that there is no desired accumulation or depletion of vehicles for any station, that is $I_i = E_i = 0$

$$g_1 = I_1 - E_1 + \sum_j v_{j1} - \sum_j v_{1j} = 0 - 0 + 0 - 150 = -150$$

$$g_2 = -45, g_3 = 15, g_4 = 175, g_6 = 5$$

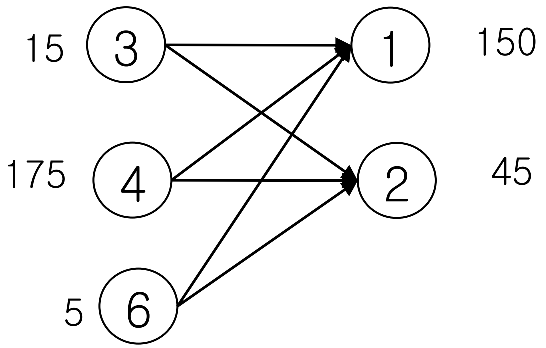
$$g_5 = g_7 = g_8 = g_9 = 0$$



Example

$$\min\ 260x_{31} + 360x_{32} + 60x_{41} + 160x_{42} + 210x_{61} + 310x_{62}$$

$$\begin{aligned} \text{st}\quad & x_{31} + x_{32} = 15 \\ & x_{41} + x_{42} = 175 \\ & x_{61} + x_{62} = 5 \\ & x_{31} + x_{41} + x_{61} = 150 \\ & x_{32} + x_{42} + x_{61} = 45 \end{aligned}$$



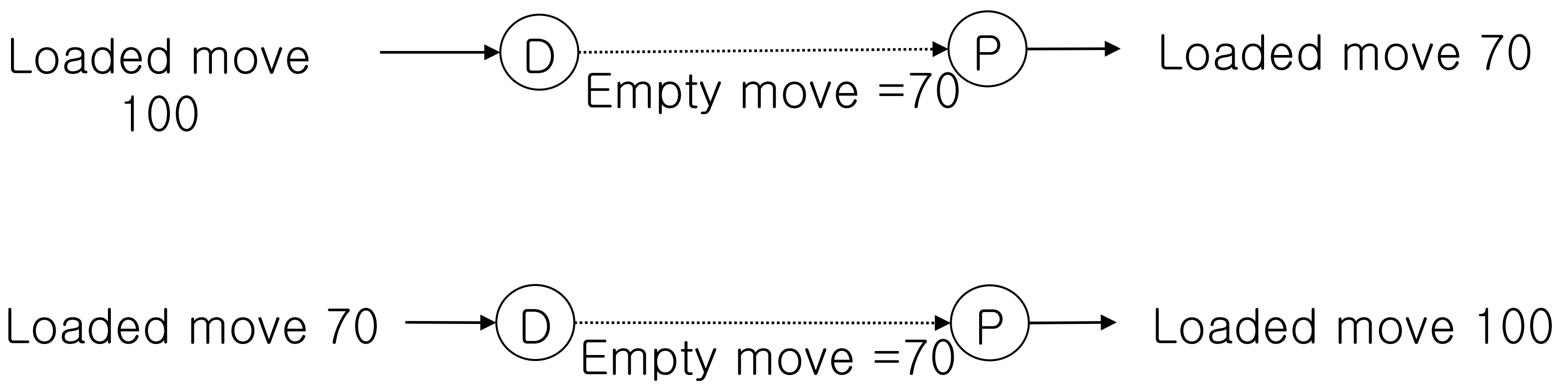
Optimal solution, $x_{31} = 15, x_{41} = 135, x_{42} = 40, x_{62} = 5$

with objective value $= 19950\text{ft}$

Empty travel distance within departments

- Transportation solution finds only lower bound on the empty move since it does not consider the empty move within department
- The empty move within department i

$$= \min\left(\sum_j v_{ji}, \sum_j v_{ij}\right)$$



Example

Table 9.3 Interdepartmental AGV Flows

From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	40	25	30	10	10	20	5	10	150
2		—	40		30		10	10		90
3			—				50		10	60
4		5	10	—		10				25
5				100	—					100
6				60		—				60
7						40	—		40	80
8				10		5		—		15
9					60				—	60
Sum	0	45	75	200	100	65	80	15	60	640

$$\text{Travel distance from D to P within department } i = (10,10,10,\ 10,10,10,1\ \ 10,75,55)$$

$$\text{Empty moves from D to P within department } i = (0,45,60,2\ \ 5,100,60,8\ \ 0,15,60)$$

$$\text{Total empty travel distance within department} = 10(0) + 10(45) + 10(60) +,\ldots,+ 55(60) = 16125$$

Example

Assume vehicles require 30 seconds per load or unload and travel at 5 ft./sec. Vehicles are estimated to be available 12 hours per period

$$
\begin{aligned}
\text{Total travel time} &= \text{loaded travel time} \\
&+ \text{empty travel time between departments} \\
&+ \text{empty travel time within departments} \\
&+ \text{loading/unloading time} \\
&= \frac{159925\text{ft}}{5\text{ft/sec}} + \frac{19950\text{ft}}{5\text{ft/sec}} + \frac{16125\text{ft}}{5\text{ft/sec}} + 640 \times 30\text{ sec} + 640 \times 30\text{ sec} \\
&= 21.54\text{ hour}
\end{aligned}
$$

$$\text{The number of vehicles required} = \left\lceil \frac{21.54}{12} \right\rceil = 2\text{ vehicles}$$

AGV operation

- Define a cycle as a path that starts at a P or D location and alternates from P to D locations, eventually returning to its starting point.
- The objective is to determine a set of cyclical routes and a frequency for each route so as to satisfy all of the loaded and unloaded moves determined in the previous section.

Example

Table 9.5a Total Interdepartmental AGV Flows

From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	40	25	30	10	10	20	5	10	150
2		—	40		30		10	10		90
3	15		—				50		10	75
4	135	45	10	—		10				200
5				100	—					100
6		5		60		—				65
7						40	—		40	80
8				10		5		—		15
9					60				—	60
Sum	150	90	75	200	100	65	80	15	60	835

Table 9.5b Reduced Interdepartmental AGV Flows

From-To	1	2	3	4	5	6	7	8	9	Sum
1	—	0	25	30	10	10	20	5	10	110
2		—	0		30		10	10		50
3	15		—				10		10	35
4	95	45	10	—		10				160
5				100	—					100
6		5		20		—				25
7						0	—		40	40
8				10		5		—		15
9					60				—	60
Sum	110	50	35	160	100	25	40	15	60	835

- Start with the largest number in the table, 135
- 4→1(135), 1→2(40), 2→3(40), 3→7(50), 7→6(40), 6→4(60)
- We can take at most 40 loads from this route
- Reduce 40 loads along the route from the table

Example

Table 9.6 Planned AGVS Routes	
Departmental Path	Number of Trips
4 →1 →2 →3 → 7→6 →4	40
5 →4 →1 →4 →1 →3 →1 →7 →9 →5	15
4 →1 →4 →1 →3 →7 →9 →5 →4	10
5 →4 →2 →5	30
5 →4 →1 →5	10
4 →1 →6 →4	10
9 →5 →4 →1 →9	10
9 →5 →4 →1 →4 →2 →7 →9	5
9 →5 →4 →3 →9	10
2 →8 →4 →1 →7 →9 →5 →4 →2	5
6 →4 →6	10
5 →4 →1 →8 →4 →2 →7 →9 →5	5
2 →8 →6 →2	5

Final AGV routes

- Assign trips to vehicles
- Try to distribute each route evenly over the period
- To avoid extra empty travel time, assign trips with the same starting point to the same vehicle

HW#2

- Consider the AGV path problem with data shown on figure 10.22 a) and table 10.19.

  a) Provide mathematical model to find optimal path.

  b) Provide screen captures of CPLEX for solution and the corresponding objective function value.

  c) Draw the optimal path.