**ENE 3031 - Fall 2014**
**Homework 5**
**due Friday Nov/14**

1. Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed (i.i.d.) random samples. Check whether estimators, $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X}_n)^2$ are unbiased for mean and variance of $X_i$ or not and explain why.

2. Suppose that a random sample of size $n$, $X_1, X_2, \ldots, X_n$, has been taken and that the observations are assumed to come from a Weibull distribution (its pdf is below). Explain how to get estimates of $\alpha$ and $\beta$ using MLE.

$$f(x;\alpha,\beta) = \begin{cases} \frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], & x \geq \nu; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

3. The following data are generated randomly from a Weibull distribution $\nu = 0$:

| | | | |
|---|---|---|---|
| 7.936 | 5.224 | 3.937 | 6.513 |
| 4.599 | 7.563 | 7.172 | 5.132 |
| 5.259 | 2.759 | 4.278 | 2.696 |
| 6.212 | 2.407 | 1.857 | 5.002 |
| 4.612 | 2.003 | 6.908 | 3.326 |

   Compute the maximum-likelihood estimators $\hat{\alpha}$ and $\hat{\beta}$ (This exercise requires a computer program, like Excel, C, Java, but do not use any “input analyzer”, like ExpertFit).

4. The table in the next page lists $n = 219$ interarrival time $X_i$ (in minute) between cars $i$ and $i + 1$ for $i = 1, 2, \ldots, 219$ in front of a bank. Suggest a distribution and estimate its parameter(s). Then check if your model fits the data. (This exercise requires a computer program, like Excel, C, Java, but do not use any “input analyzer”, like ExpertFit).