Electrical Engineering

HW 3 – Chapter 4, Solution

<1>

a)

$$i_C(t) = C\frac{dv_C(t)}{dt} = 200\times10^{-6}\frac{dv_C(t)}{dt} = 2\cdot10^{-4}\frac{dv_C(t)}{dt} = 0.088\cos(20t+\frac{\pi}{6})A$$

b)

$$i_C(t) = -40\cdot200\cdot10^{-6}\cdot90\cdot\cos(90t+\pi/2)A = -0.72\cos(90t+\pi/2-\pi)\,\mathbf{A} =$$
$$= 0.72\cos(90t-\pi/2)A$$

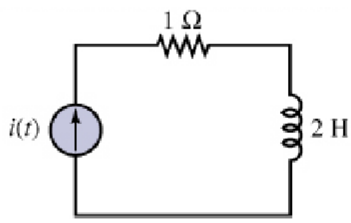
c)

$$i_C(t) = 200\cdot10^{-6}\cdot28\cdot15\left[-\sin\left(15t+\frac{\pi}{8}\right)\right] = -8.4\cdot10^{-2}\sin\left(15t+\frac{\pi}{8}\right)\mathbf{A}$$

d)

$$i_C(t) = 200\cdot10^{-6}\cdot45\cdot120\left[\cos\left(120t+\frac{\pi}{4}\right)\right] = 1.08\cdot\cos\left(120t+\frac{\pi}{4}\right)A$$

<2>



The magnetic energy stored in an inductor may be found from, (Eq. 4.16):

$$w_L(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}(2)i^2(t) = i^2(t)$$

For $-\infty < t < 0$,

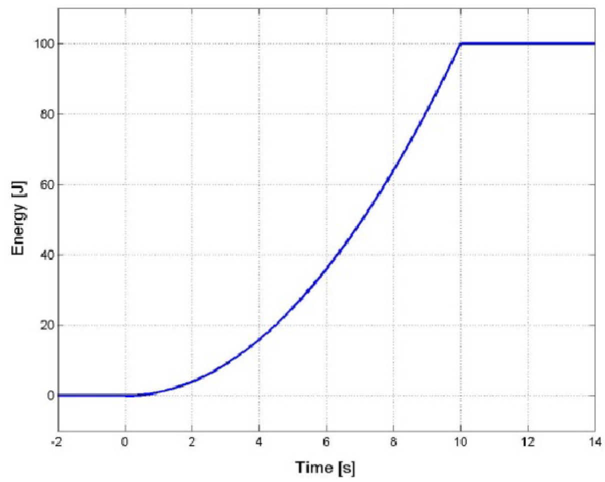
$$w_L(t) = 0$$

For $0 \le t < 10\,s$

$$w_L(t) = t^2\ \mathbf{J}$$

For $10\ s \le t < +\infty$

$$w_L(t) = 100\ \mathrm{J}$$



<3>

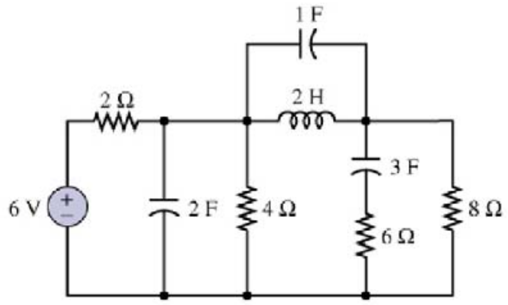
Since the voltage waveform is piecewise continuous, the integration can be performed over each continuous segment. Where not indicated t is supposed to be expressed in seconds.

$$i_L(t=30\ \mu s)=\frac{1}{L}\int_{-\infty}^{30\ \mu s} v_L(\tau)d\tau = i_L(0)+\frac{1}{L}\int_{0}^{20\ \mu s} v_L(\tau)d\tau+\frac{1}{L}\int_{20\ \mu s}^{30\ \mu s} v_L(\tau)d\tau =$$

$$=i_L(0)+\frac{1}{L}\left[\frac{3}{3}\tau^3\ \frac{V}{s^2}\right]_0^{20\ \mu s}+\frac{1}{L}[1.2\tau\ \mathbf{nV}]_{20\ \mu s}^{30\ \mu s}=0+\frac{1}{16\ \mu\mathbf{H}}\cdot 1\ \frac{\mathbf{V}}{\mathbf{s^2}}\cdot\left((20\ \mu\mathbf{s})^3-0\right)+$$

$$+\frac{1}{16\ \mu\mathbf{H}}\cdot(1.2\ \mathbf{nV})\cdot(30\ \mu\mathbf{s}-20\ \mu\mathbf{s})=1.250\ \mathbf{nA}$$

<4>



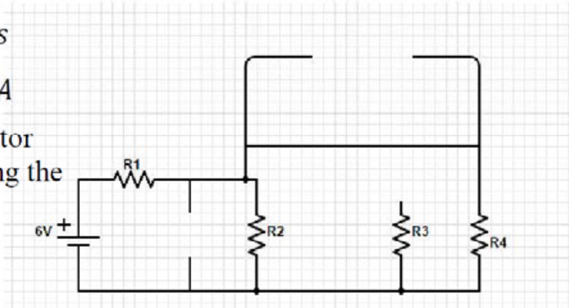
Analysis:

The first step is to redraw the circuit and place a break where there is a capacitor and a wire where there is an inductor.

The circuit can be simplified to find the Req and total current:

$$R_{eq} = 4.66\ Ohms$$

$$I_t = \frac{6}{4.66} = 1.29A$$



There is no current through R3, therefore, the 2F and the 3F capacitor have the same voltage across them. This can be found by calculating the voltage drop across R1:

$$\frac{6-V_1}{2} = 1.29A$$

$$V_1 = 3.43V$$

Plug this voltage into the equation for energy storage in a capacitor to get the energy stored in the 2F and the 3F capacitor:

$$\boldsymbol{W_{C2F} = \frac{1}{2} * 2F * (3.43V)^2 = 11.76J}$$

$$\boldsymbol{W_{C3F} = \frac{1}{2} * 3F * (3.43V)^2 = 17.63J}$$

The 1F capacitor is connected to the same node with both of its leads. This means there is no voltage across it and that its energy stored is:

$$\boldsymbol{W_{C1F} = 0}$$

The inductor requires the use of current division to find the current through the 8 Ohm resistor:

$$I_8 = \frac{R_2}{R_2 + R_4} * I_t = 0.43A$$

This current is also the current through the inductor. Plug it into the energy storage of an inductor equation:

$$\boldsymbol{W_{L2H} = \frac{1}{2} * 2H * (0.43A)^2 = 0.18}$$

<5>

a) $V(jw) = 155\angle\text{-}25°$ V

b) $V(jw) = 5\angle\text{-}130°$ V

c) $I(jw) = 10\angle 63° + 15\angle\text{-}42° = (4.54 + j8.91) + (11.15 - j\,10.04) = 15.69 - j1.13 = 15.73\angle\text{-}4.12°$ A

d) $I(jw) = 460\angle\text{-}25° - 220\angle 75° = (416.90 - j194.40) - (56.94\text{-}j212.50) = 359.96 + j18.10 = 360.4\angle 2.88°$ A

<6>

$$X_L = \omega L = 800\Omega \Rightarrow Z_L = +j \cdot X_L = +j \cdot 800\Omega$$

$$X_C = \frac{1}{\omega C} = 3571\Omega \Rightarrow Z_C = -j \cdot X_C = -j \cdot 3571\Omega$$

$$Z_{eq1} = Z_{R2} + Z_L = R_2 + jX_L = 500 + j \cdot 800\Omega = 943.4\angle 60\Omega$$

$$Z_{eq2} = \frac{Z_{eq1} \cdot R_1}{Z_{eq1} + R_1} = 440 + j219.8$$

$$Z_{eq} = \frac{Z_{eq2} \cdot Z_C}{Z_{eq2} + Z_C} = 540.9\angle 198.8$$

<7>

$$X_L = 8\Omega, X_C = 16\Omega$$

$$R_2 \parallel C \Rightarrow Z_{eq1} = \frac{Z_{R_2} \cdot Z_C}{Z_{R_2} + Z_C} = 8 - j8$$

$$Z_{eq2} = Z_{eq1} + Z_L = 8$$

Using divider current law:

$$\overline{I_L} = \overline{I} \cdot \frac{Z_{R1}}{Z_{R1} + Z_{eq2}} = 20\frac{8}{8+8} = 10$$

$$\overline{V_{R_2}} = \overline{I_L} Z_{eq1} = 80 - j80 \Rightarrow V_{R_2}(t) = 113.13 \cdot \cos(533.33t - 0.79)$$

<8>

a) $$Z_L = j\omega L = j1000\ \frac{\textbf{rad}}{\textbf{s}} \cdot 10\ \textbf{mH} = j10\ \Omega,$$

The equivalent impedance is:

$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_S = \frac{(j10)1000}{j10 + 1000} + 500 = 500 + \frac{j10^3}{100 + j} = 500.1 + j9.999\ \Omega$$

The equivalent Thèvenin voltage is: $\mathbf{V}_T = \mathbf{V}_S = 10\angle 0^o\ \mathbf{V}$

b) $$Z_L = j\omega L = j10^6\ \frac{\textbf{rad}}{\textbf{s}} \cdot 10\ \textbf{mH} = j10^4\ \Omega,$$

The equivalent impedance is:

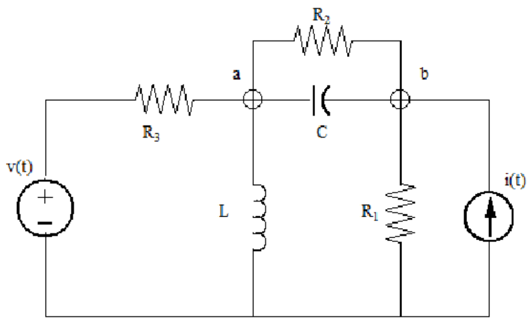
$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_S = \frac{\left(j10^4\right)1000}{j10^4 + 1000} + 500 = 500 + \frac{j10^4}{1 + j10} = 1490.1 + j99.01\ \Omega$$

The equivalent Thèvenin voltage is: $\mathbf{V}_T = \mathbf{V}_S = 10\angle 0^o\ \mathbf{V}$

<9>

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{300 \cdot 900 \cdot 10^{-6}} = -j3.7\Omega\ ,$$

$$Z_L = j\omega L = j300 \cdot 0.3 = j90\Omega$$



Applying KCL at node 1, we have:

$$-I + \frac{V_2}{R_1} + \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_1}{Z_C} = 0$$

Applying KCL at node 2, we have

$$\frac{V_1 - V_2}{R_2} + \frac{V_1}{Z_L} + \frac{V_1 - V_2}{Z_C} + \frac{V_1 - V}{R_3} = 0$$

Therefore, solving the linear system :

$$\begin{cases} V_1 = 4.25e^{j42.91} \Rightarrow v_1(t) = 4.25\cos(300t + 42.91°)V \\ V_2 = 2.94e^{j57.27} \Rightarrow v_2(t) = 2.94\cos(300t + 57.37°)V \end{cases}$$