Electrical Engineering

HW 5 – Chapter 6, Solution

<1>

$$\frac{v_{out}}{v_{in}}(j\omega)=\frac{R_2 \| R_3 + j\omega L + \dfrac{1}{j\omega(C_1+C_2)}}{R_1 + R_2 \| R_3 + j\omega L + \dfrac{1}{j\omega(C_1+C_2)}} = \frac{1-0.0008\omega^2 + j(0.05)\omega}{1-0.0008\omega^2 + j(0.11)\omega}$$

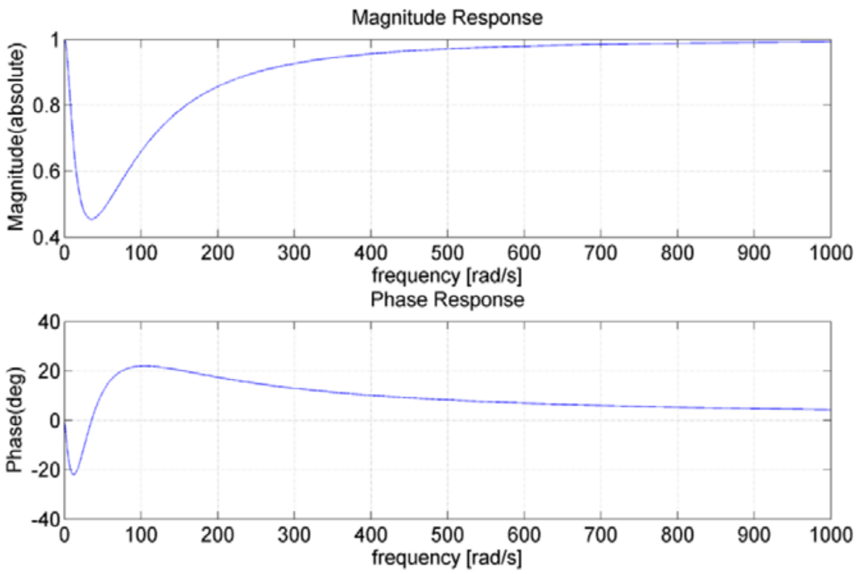
a)

$$\left|\frac{v_{out}}{v_{in}}(j\omega)\right| = \frac{\sqrt{(1-\omega^2 0.0008)^2 + ((0.05)\omega)^2}}{\sqrt{(1-\omega^2 0.0008)^2 + ((0.11)\omega)^2}}$$

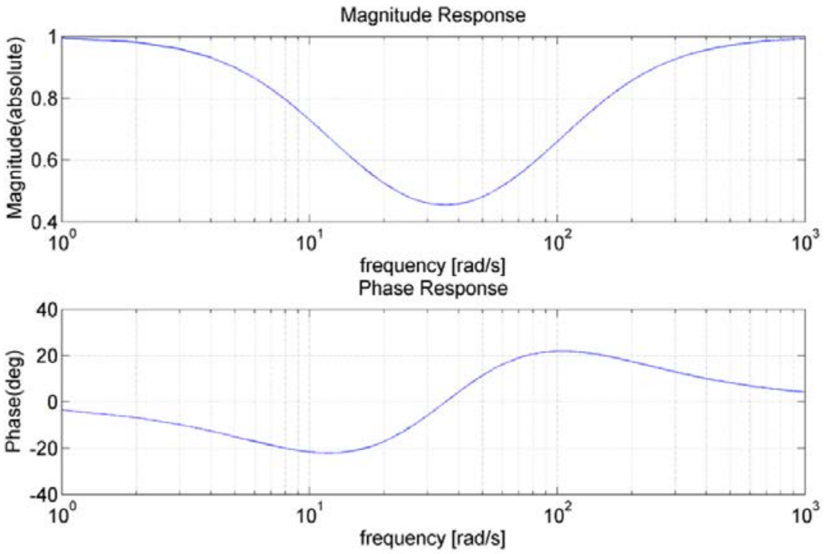
$$\angle\frac{v_{out}}{v_{in}}(j\omega) = \arctan\left(\frac{(0.05)\omega}{1-\omega^2 0.0008}\right) - \arctan\left(\frac{(0.11)\omega}{1-\omega^2 0.0008}\right)$$

The plots obtained using Matlab are shown below:

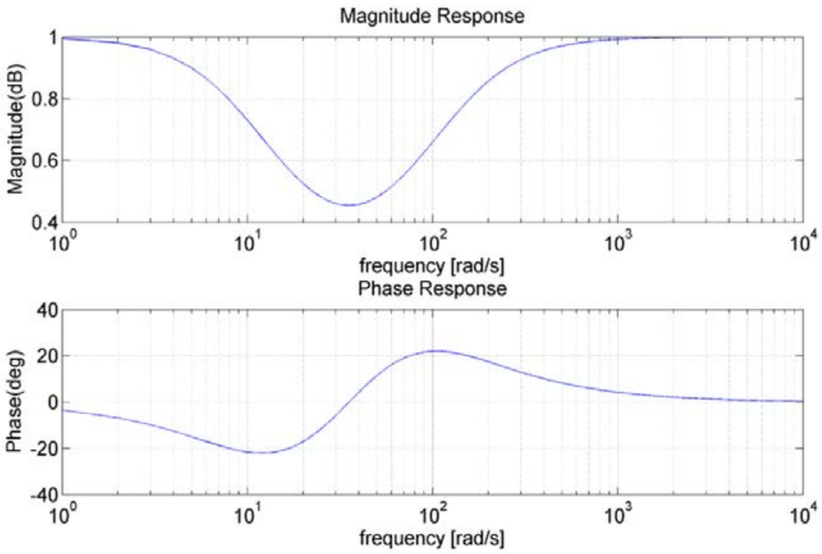
b)



c)


Magnitude Response
Magnitude(absolute)
frequency [rad/s]
Phase Response
Phase(deg)
frequency [rad/s]

d)


Magnitude Response
Magnitude(dB)
frequency [rad/s]
Phase Response
Phase(deg)
frequency [rad/s]

<2>

Assume:

The values of the resistors and of the capacitor in the circuit of Figure P6.12:

$R_1 = 16\,\Omega \quad R_2 = 16\,\Omega \quad C = 0.47\ \mu\text{F}$

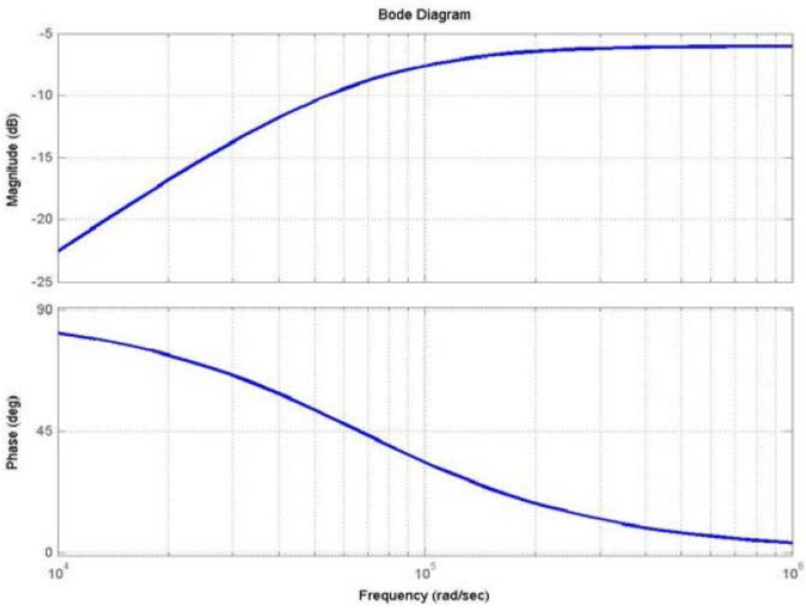
Analysis:

a)

$$VD: \quad V_o = V_i \frac{Z_{R2}}{Z_{R1} + Z_C + Z_{R2}} = V_i \frac{R_2}{R_1 + \frac{1}{j\omega C} + R_2}$$

$$H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{R_2}{R_1 + R_2} \frac{1}{1 - j\frac{1}{\omega C[\,R_1 + R_2\,]}}$$

b)



<3>

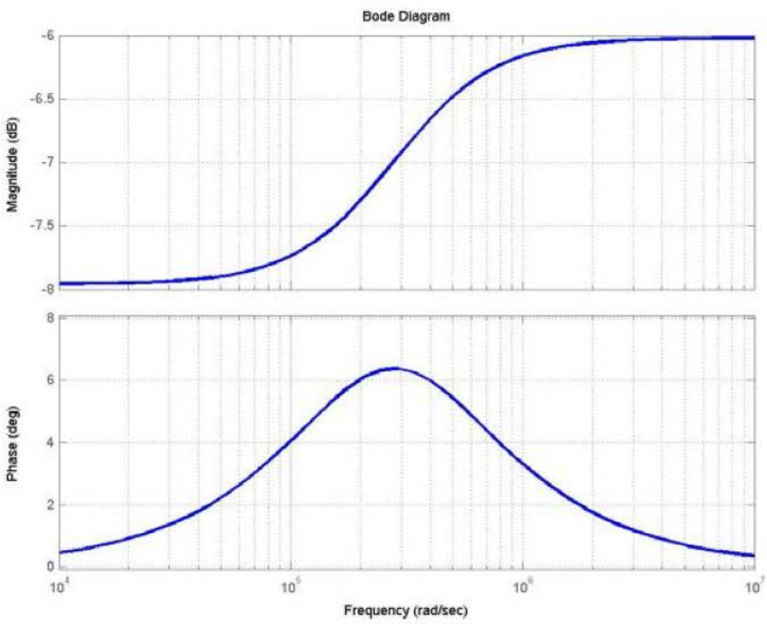
Using voltage division:

$$Z_{eq} = \frac{Z_{R2} Z_C}{Z_{R2} + Z_C} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \frac{j\omega C}{j\omega C} = \frac{R_2}{1 + j\,\omega R_2 C}$$

$$VD: \quad H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{RL}}{Z_{R1} + Z_{eq} + Z_{RL}} = \frac{R_L}{R_1 + \frac{R_2}{1 + j\omega R_2 C} + R_L} \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} =$$

$$= \frac{R_L\,[1 + j\,\omega R_2 C]}{R_1 + R_2 + R_L + j\,[R_1 + R_L]\,\omega R_2 C} = \frac{R_L}{R_1 + R_2 + R_L} \frac{1 + j\,\omega R_2 C}{1 + j\,\frac{[R_1 + R_L]\,\omega R_2 C}{R_1 + R_2 + R_L}}$$

Plotting the response in a Bode Plot:



<4>

The function in Figure P6.20 is an even function. Thus, we only need to compute the a_n coefficients. We can compute the Fourier series coefficient using the integrals in equations (6.20) and (6.21):

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_0^{T/2} 2\sin\left(\frac{2\pi}{T}t\right)dt = \frac{1}{2\pi}\left[-\cos\left(\frac{2\pi}{\mathrm{T}}t\right)\right]_0^{T/2} = \frac{1}{2\pi}\left[-\cos(\pi)+\cos(0)\right] = \frac{1}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos\left(n\frac{2\pi}{T}t\right)dt = \frac{2}{T}\int_0^T \sin\left(\frac{2\pi}{T}t\right)\cos\left(n\frac{2\pi}{T}t\right)dt = -\frac{\cos(n\pi)+1}{\pi\left(n^2-1\right)} = \begin{cases} -\dfrac{2}{\pi\left(n^2-1\right)} & (n \text{ even}) \\ 0 & (n \text{ odd}) \end{cases}$$

Thus, the Fourier series expansion of the function is:

$$x(t) = \frac{1}{\pi} - \sum_{n=1}^{\infty}\frac{\cos(n\pi)+1}{\pi\left(n^2-1\right)}\cos\left(n\frac{2\pi}{T}t\right)$$

<5>

The periodic function shown in Figure P6.21 can be defined as:

$$x(t) = \begin{cases} A & 0 \le t \le \dfrac{T}{4} \\ -A & T - \dfrac{T}{4} \le t \le T \end{cases}$$

The function in Figure P6.19 is an odd function. Thus, we only need to compute the b_n coefficients. We can compute the Fourier series coefficient using the integrals in equation (6.22):

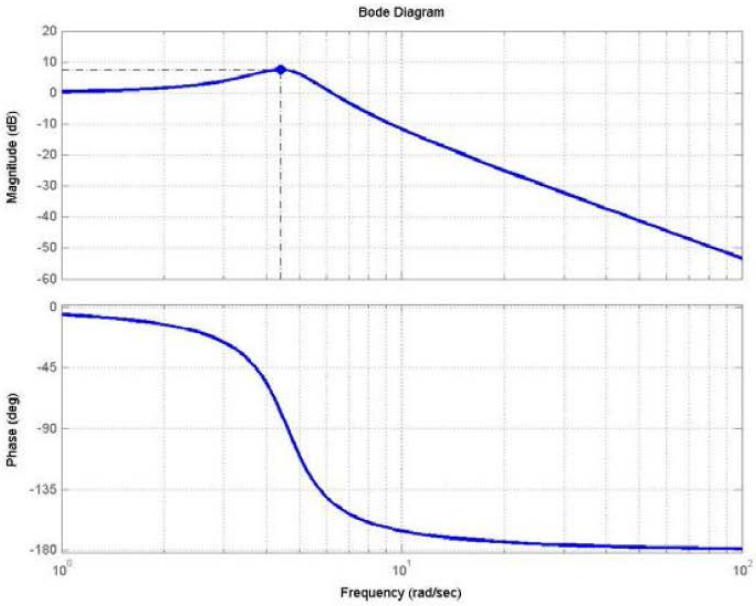
$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin\left(n\frac{2\pi}{T}t\right)dt = \frac{2}{T}\left(-\int_{-T/4}^{0} A\sin\left(n\frac{2\pi}{T}t\right)dt + \int_0^{T/4} A\sin\left(n\frac{2\pi}{T}t\right)dt\right) = \frac{2A}{n\pi}\left(1-\cos\left(n\frac{\pi}{2}\right)\right) = \frac{2A}{n\pi}$$

<6>

Taking the output as the voltage across the parallel R-C subcircuit,

$$\frac{V_o}{V_S}=\frac{1/LC}{(j\omega)^2+j\omega\frac{1}{RC}+\frac{1}{LC}}=\frac{3/64}{(j\omega)^2+j\omega 2+3/64}\left(=\frac{\omega_n^2\mu}{(j\omega)^2+j\omega(2\xi\omega_n)+\omega_n^2}\right)$$

The corresponding Bode diagrams are shown below:



In this circuit, as frequency increases, the impedance of the capacitor decreases and the impedance of the inductor increases. Both effects cause the magnitude of the output voltage to decrease so this is a 2nd order low pass filter.
The resonance frequency is,

$$\omega_n=\sqrt{\frac{1}{LC}}=\sqrt{\frac{64}{3}}\cong 4.6188 \text{ rad/s.}$$

The damping ratio is,

$$\zeta=\frac{1/RC}{2\omega_n}=\frac{\sqrt{3}}{8}\cong 0.2165$$

The quality factor is,

$$Q=\frac{1}{2\zeta}=\frac{4}{\sqrt{3}}\cong 2.3094$$

The bandwidth is,

$$B=\frac{\omega_n}{Q}=\frac{8}{\sqrt{3}}\frac{1}{4/\sqrt{3}}=2 \text{ rad/s.}$$

<7>

a)

$$As\ \omega \to 0: Z_L \to 0 \Rightarrow Short$$

$$Z_C \to \infty \Rightarrow Open$$

$$\Rightarrow H_v \to 0$$

$$As\ \omega \to \infty: Z_L \to \infty \Rightarrow Open$$

$$Z_C \to 0 \Rightarrow Short$$

$$\Rightarrow H_v = \frac{V_o}{V_i} \to \frac{R_2}{R_1 + R_2}$$

The filter is a high pass filter.

b) First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = (Z_{R1} + Z_C) \| (Z_{R2} \| Z_L)$$

and

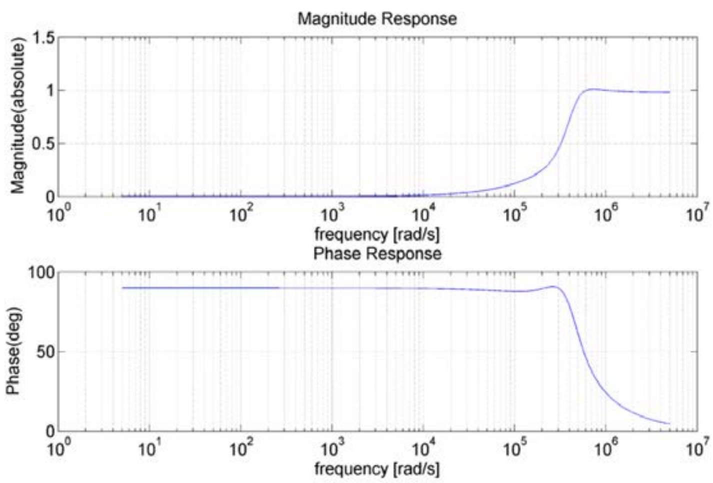
$$\mathbf{V}_{OC} = \frac{Z_{R2}}{Z_{R1} + Z_C + Z_{R2}} \mathbf{V}_{in}$$

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{OC}} = \frac{Z_C}{Z_T + Z_L}$$

Therefore,

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_1 + \frac{1}{j\omega C} + R_2} \cdot \frac{Z_L}{Z_T + Z_L}$$

Substituting the numerical values, the corresponding Bode diagrams are shown in the Figure.



<8>

(a)

$$Z_{eq} = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_L} + \frac{1}{Z_{R_L}}} = \frac{1}{j\omega C + \frac{1}{j\omega L} + \frac{1}{R_L}} = \frac{j\omega L R_L}{(j\omega)^2 LCR_L + j\omega L + R_L}$$

$$VD: \quad H_v(j\omega) = \frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_i(j\omega)} = \frac{Z_{eq}}{Z_{R_s} + Z_{eq}} = \frac{\frac{j\omega L R_L}{(j\omega)^2 LCR_L + j\omega L + R_L}}{R_S + \frac{j\omega L R_L}{(j\omega)^2 LCR_L + j\omega L + R_L}}$$

$$H_v(j\omega) = \frac{j\omega L R_L}{(j\omega)^2 LCR_S R_L + j\omega L(R_L + R_S) + R_S R_L} = \frac{1}{R_S}\,\frac{j\omega L}{(j\omega)^2 LC + j\omega L\left(\frac{R_L + R_S}{R_S R_L}\right) + 1}$$

b) The resonance frequency is $\omega_n = \sqrt{\frac{1}{LC}} \cong 1.4142$ Mrad/s.

c) Half power frequencies (see the following d) for ς):

$$\omega_{1,2} = \omega_n\sqrt{1+\varsigma^2} \pm \varsigma\omega_n = 1.4142\cdot 10^6\sqrt{1+(0.1485)^2} \pm (0.1485)\cdot 1.4142\cdot 10^6 = (1.4297 \pm 0.21)\cdot 10^6 \text{ rad/sec}$$

So $\omega_2 = 1.2197\cdot 10^6$ rad/sec; $\omega_1 = 1.6397\cdot 10^6$ rad/sec;

d) The damping ratio is $\zeta = \frac{\omega_n}{2} L\left(\frac{R_L + R_S}{R_S R_L}\right) \cong 0.1485$. The quality factor is $Q = \frac{1}{2\zeta} \cong 3.3672$. The bandwidth is,

$$B = \frac{\omega_n}{Q} \cong 420 \text{ Krad/s.}$$

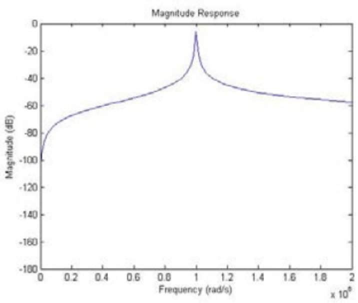
<9>

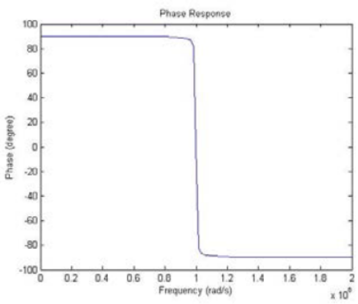
Analysis:

$$\frac{\mathbf{V}_{out}(j\omega)}{\mathbf{V}_s(j\omega)} = \frac{R_L \parallel Z_{ab}}{R_L \parallel Z_{ab} + R_S}$$

$$Z_{ab} = \frac{(j\omega L)\left(\frac{1}{j\omega C}\right)}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - LC\square^2}$$

$$R_L \| Z_{ab} = \frac{j\omega L R_L}{R_L - \omega^2 LCR_L + j\omega L} = \frac{j\omega L}{1 - \omega^2 LC + j\omega L/R_L}$$





$$\frac{\mathbf{V}_{out}(j\omega)}{\mathbf{V}_s(j\omega)} = \frac{j\omega L/R_S}{j\omega(L/R_S + L/R_L) + 1 - \omega^2 LC} = \frac{j\omega L/R_S}{j\omega(L/R_S + L/R_L) + 1 - \omega^2 LC}$$

<10>

(a) $$\frac{V_O(s)}{V_S(s)} = \frac{\frac{1}{C_2 s} \| R_L}{\frac{1}{C_2 s} \| R_L + sL + R_S \| \frac{1}{C_1 s}}$$

We have $$\frac{1}{C_2 s} \| R_L = \frac{R_L}{R_L C_2 s + 1}$$

Therefore,

$$\frac{V_O(s)}{V_S(s)} = \frac{\frac{R_L}{R_L C_2 s + 1}}{\frac{R_L}{R_L C_2 s + 1} + sL + \frac{R_S}{R_S C_1 s + 1}} = \frac{R_S C_1 s + 1}{s^3 K_1 + s^2 K_2 + s K_3 + K_4}$$

where

$$K_1 = C_2 R_S C_1 L$$

$$K_2 = C_2 L + C_1 L \frac{R_S}{R_L}$$

$$K_3 = R_S C_1 + R_S C_2 + \frac{L}{R_L}$$

$$K_4 = 1 + \frac{R_S}{R_L}$$

$$\frac{V_O(\omega)}{V_S(\omega)} = \frac{\frac{j\omega}{2000\pi} + 1}{\frac{-j\omega^3}{(2000\pi)^3} - \frac{2\omega^2}{(2000\pi)^2} + \frac{3j\omega}{2000\pi} + 2}$$

(b)