

# Chapter 5 Bottom-Up Parsing

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## **Bottom-up parsing**





#### Bottom-up parsing

#### Definition

 Parsing an input string of tokens by tracing out the steps in a rightmost derivation.

#### Categories

- LR(0) parsing
- LR(1) parsing : powerful but complex
- SLR(1) parsing : Simple LR(1)
- LALR(1) parsing: Lookahead LR(1)
  - More powerful than SLR(1), and less complex than LR(1)

## **Bottom-up parsing**





#### Outline

- Initialization
  - The stack is empty.
- Iteration of the followings until the stack is empty.
  - Shift a terminal from the front of the input to the top of the stack.
  - Reduce a string a at the top of the stack to a nonterminal A, given the BNF choice A  $\rightarrow$  a.
- If the stack contains the start symbol and the input is empty, accept.
- Shift-reduce parser

## Example: table 5.1





$$S' \rightarrow S$$
  
 $S \rightarrow (S) S \mid \varepsilon$ 

	Parsing stack	Input	Action
1	\$	()\$	shift
2	\$ (	)\$	reduce $S \rightarrow \varepsilon$
3	\$ ( S	)\$	shift
4	\$ (S)	\$	reduce $S \rightarrow \varepsilon$
5	\$ (S) S	\$	reduce $S \rightarrow (S) S$
6	\$ S	\$	reduce $S' \to S$
7	\$ S'	\$	accept

## Example: table 5.2





$$E' \rightarrow E$$
  
 $E \rightarrow E + n \mid n$ 

			C
	Parsing stack	Input	Action
1	\$	n + n\$	shift
2	\$ <b>n</b>	+ <b>n</b> \$	reduce $E \rightarrow \mathbf{n}$
3	\$E	+ n \$	shift
4	\$E +	<b>n</b> \$	shift
5	$\$E+\mathbf{n}$	\$	reduce $E \rightarrow E + \mathbf{n}$
6	\$ E	\$	reduce $E' \to E$
7	\$ E`	\$	accept

## **Bottom-up Parsing**



2nst

- Bottom-up parsers have less difficulty than top-down parsers with lookahead.
- However, a bottom-up parser may need to look deeper into the stack.
- Sometimes, the next token in the input may also need to be consulted as a lookahead.
- A shift-reduce parser traces out a rightmost derivation of the input string, but the steps of the derivation occur in reverse order.

## **Bottom-up Parsing**





- Right sentential form
  - Each of the intermediate strings of terminals and nonterminals in the rightmost derivation.
- Viable prefix
  - sequence of symbols on the parsing stack
- Handle
  - string in the right sentential form + production
- Determining the next handle in a parse is the main task of a shiftreduce parser.
  - The string at the top of the stack matches the right-hand side of a production.
  - Reduction occur when the resulting string is a right sentential form.
    - Step 3 of Table 5.1.

#### Finite automata of LR(0) items and LR(0) parsing



- LR(0) item (item for short)
- Initial item
- Complete item
- augmented grammar
- Closure items
- Kernel items



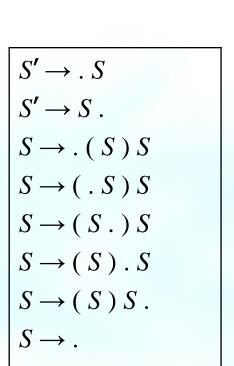
## P.202 example 5.3





$$S' \to S$$

$$S \to (S) S | \varepsilon$$





#### Finite automata of LR(0) items and LR(0) parsing





- LR(0) item (item for short)
- Initial item
- Complete item
- augmented grammar
- Closure items
- Kernel items



## P.202 example 5.4







$$E' \rightarrow . E$$

$$E' \to E$$
.

$$E \rightarrow .E + \mathbf{n}$$

$$E \rightarrow E + \mathbf{n}$$

$$E \rightarrow E + . \mathbf{n}$$

$$E' \rightarrow . E$$

$$E' \rightarrow E .$$

$$E \rightarrow . E + \mathbf{n}$$

$$E \rightarrow E . + \mathbf{n}$$

$$E \rightarrow E + . \mathbf{n}$$

$$E \rightarrow E + . \mathbf{n}$$

$$E \rightarrow . \mathbf{n}$$

$$E \rightarrow . \mathbf{n}$$

$$E \rightarrow . \mathbf{n}$$

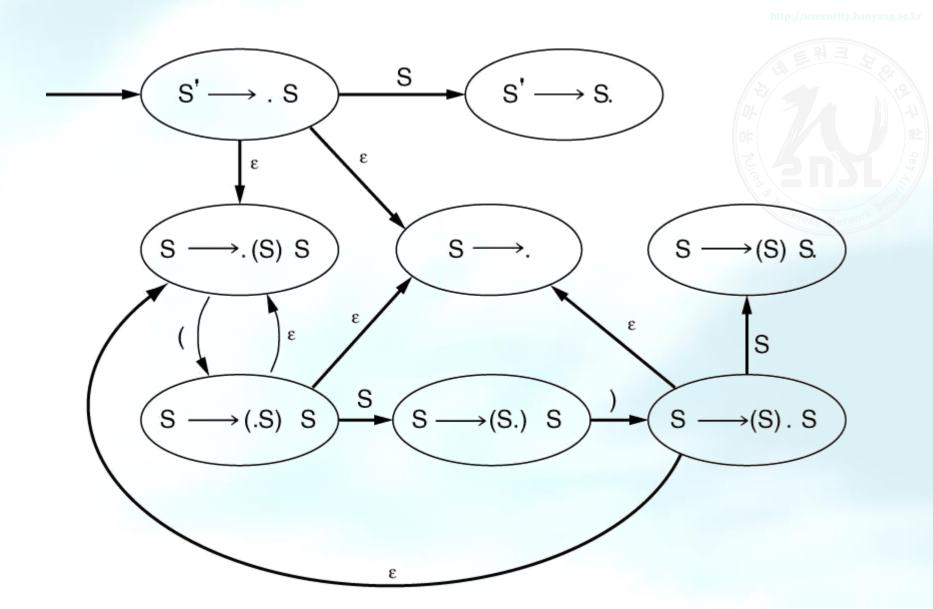
$$E \rightarrow .$$
 n

$$E \rightarrow \mathbf{n}$$

## P.204 figure 5.1



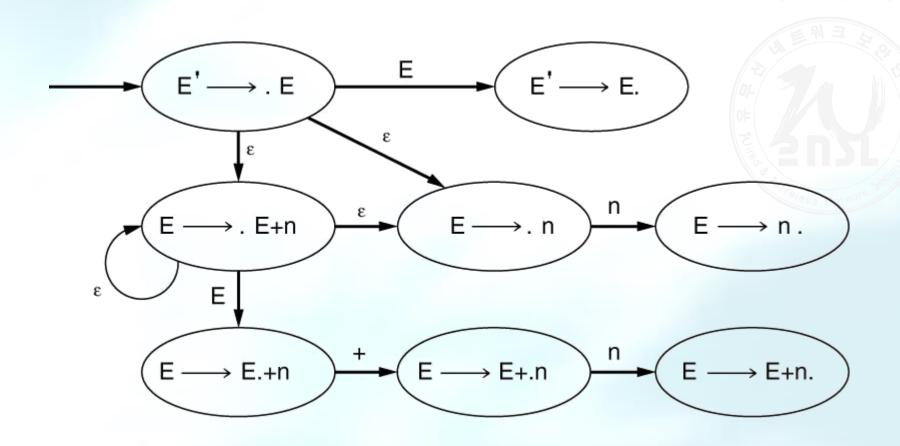




## P.204 figure 5.2



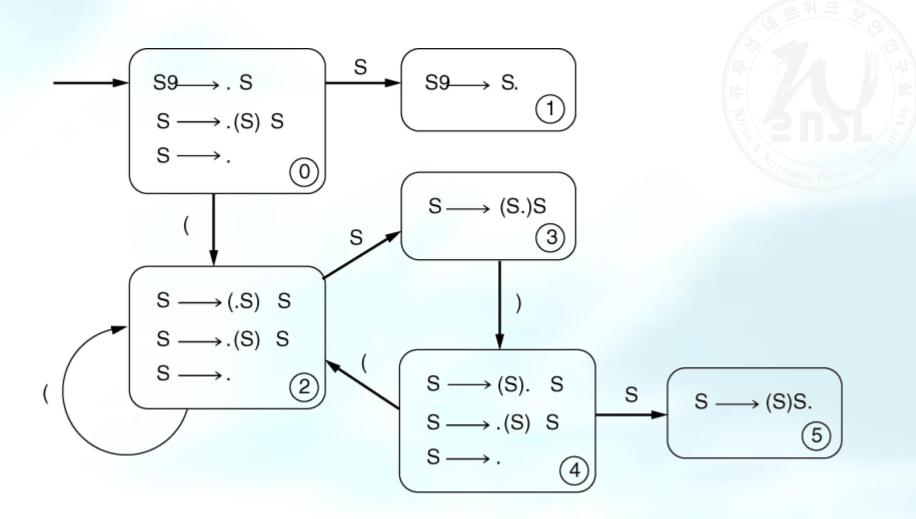




## P.205 figure 5.3



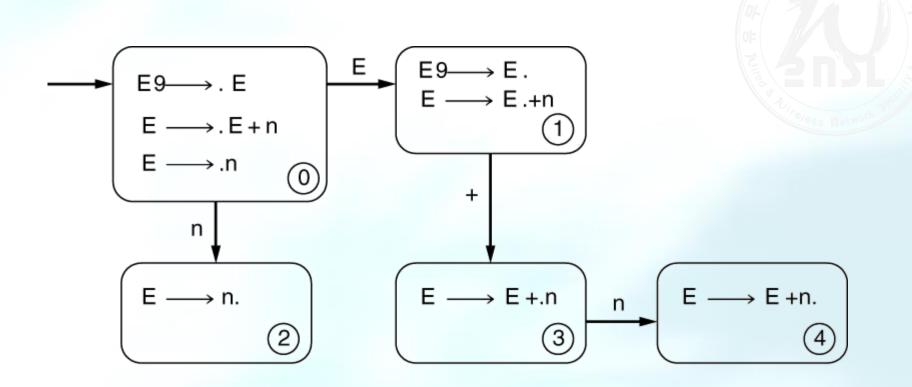




## P.205 figure 5.4







#### Finite automata of LR(0) items and LR(0) parsing





- LR(0) item (item for short)
- Initial item
- Complete item
- augmented grammar
- Closure items
- Kernel items
  - Only kernel items need to be specified to completely characterize the DFA

# The LR(0) Parsing







Parsing stack	Input
\$ 0 <b>n</b> 2	Rest of InputString \$

## The LR(0) Parsing



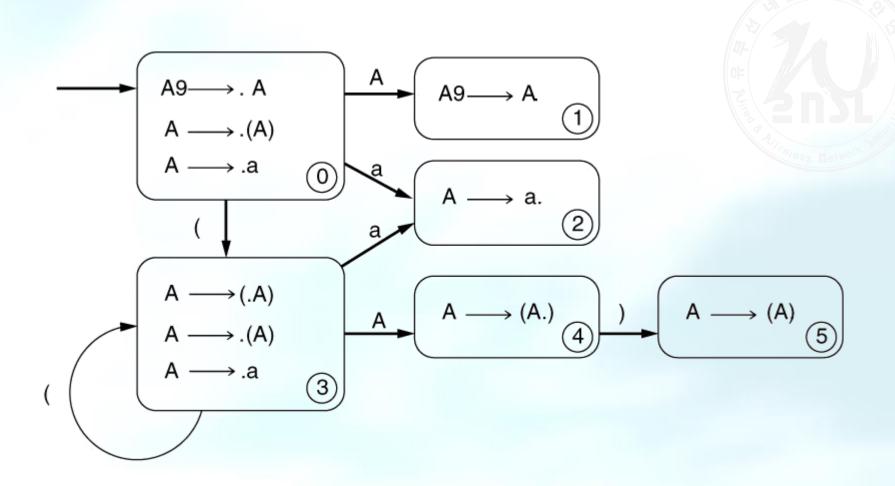


- A grammar is said to be an LR(0) grammar if a state has a complete item, then it can contain no other items.
- Definition
  - **O** ...
  - If state s contains any complete item (an item of the form A → γ.), then the action is to reduce by the rule A → γ.
  - 0
- shift-reduce conflict
- reduce-reduce conflict

## P.208 figure 5.5







## P.209 table 5.3





	Parsing stack	Input	Action	
1	\$ 0	((a))\$	shift	17-
2	\$0(3	(a))\$	shift	IIOF1
3	\$0(3(3	a))\$	shift	a to
4	\$0(3(3a2	))\$	reduce $A \rightarrow a$	
5	\$0(3(3A4	))\$	shift	
6	\$0(3(3A4)5	)\$	reduce $A \rightarrow (A)$	
7	\$0(3A4	) \$	shift	
8	\$0(3A4)5	\$	reduce $A \rightarrow (A)$	
9	\$ 0 A 1	\$	accept	

Table-driven method!

## P.209 table 5.4





State	Action	Rule		Input		Goto
			(	a	)	A
0	shift		3	2		1
1	reduce	$A \to A$				
2	reduce	$A \rightarrow a$				
3	shift		3	2		4
4	shift				5	
5	reduce	$A \to (A)$				

## The SLR(1) Parsing





- Uses the DFA of sets of LR(0) items.
- shift-reduce and reduce-reduce conflicts can be minimized by consulting Follow sets.

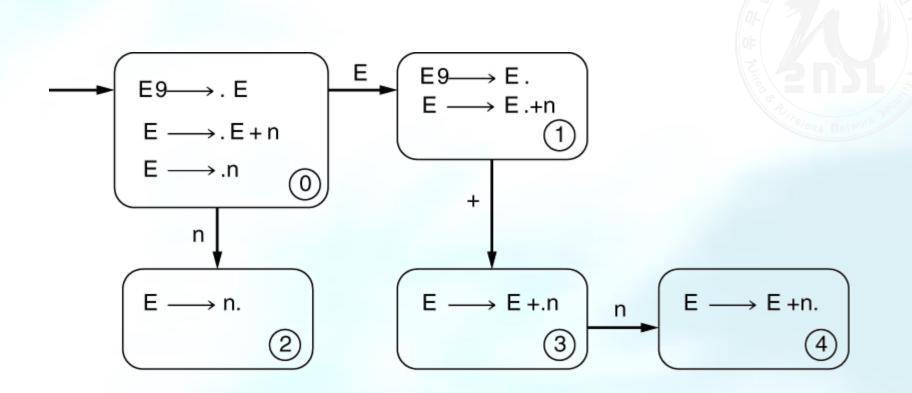
#### Definition

- **U** ...
- If state s contains the complete item A → γ., and the next token in the input string is in Follow(A), then the action is to reduce by the rule A → γ.
- 0 ...

## P.205 figure 5.4







# P.212 table 5.5 – SLR(1) parsing table





State		Input				
	n	+	\$	E		
0	s2			1		
1		s3	accept			
2		$r(E \rightarrow n)$	$r(E \rightarrow n)$			
3	s4					
4		$r(E \rightarrow E + n)$	$r(E \rightarrow E + n)$			

## P.212 table 5.6





	Parsing stack	Input	Action	11- (1-)
1	\$ 0	n + n + n\$	shift 2	0F
2	\$ 0 <b>n</b> 2	+ n + n \$	reduce $E \rightarrow \mathbf{n}$	
3	\$ 0 E 1	+ n + n \$	shift 3	
4	\$ 0 E 1 + 3	n + n\$	shift 4	
5	$\$ 0 E 1 + 3 \mathbf{n} 4$	+ <b>n</b> \$	reduce $E \rightarrow E + \mathbf{n}$	
6	\$ 0 E 1	+ <b>n</b> \$	shift 3	
7	\$ 0 E 1 + 3	n \$	shift 4	
8	$\$ 0 E 1 + 3 \mathbf{n} 4$	\$	reduce $E \rightarrow E + \mathbf{n}$	
9	\$ 0 E 1	\$	accept	

# P.212 table 5.7 – SLR(1) parsing table





State		Goto		
	(	)	\$	S
0	s2	$r(S \rightarrow \varepsilon)$	$r(S \rightarrow \varepsilon)$	1
1			accept	
2	s2	$r(S \rightarrow \varepsilon)$	$r(S \rightarrow \varepsilon)$	3
3		s4		
4	s2	$r(S \rightarrow \varepsilon)$	$r(S \rightarrow \varepsilon)$	5
5		$r(S \to (S)S)$	$r(S \to (S)S)$	

## P.213 table 5.8





	Parsing stack	Input	Action
1	\$ 0	()()\$	shift 2
2	\$0(2	)()\$	reduce $S \rightarrow \varepsilon$
3	\$ 0 ( 2 <i>S</i> 3	)()\$	shift 4
4	\$0(253)4	()\$	shift 2
5	\$0(253)4(2	)\$	reduce $S \rightarrow \varepsilon$
6	\$0(253)4(253	)\$	shift 4
7	\$0(253)4(253)4	\$	reduce $S \rightarrow \varepsilon$
8	\$0(253)4(253)455	\$	reduce $S \rightarrow (S) S$
9	\$0(253)455	\$	reduce $S \rightarrow (S) S$
10	\$ 0 <i>S</i> 1	\$	accept

## **Disambiguating Rules for Parsing Conflicts**





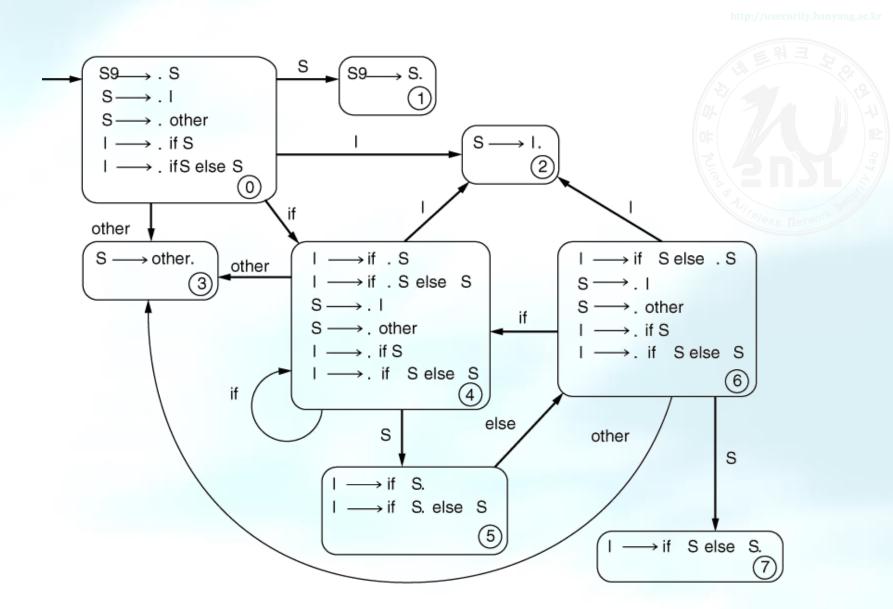
P.213 example 5.12

```
statement \rightarrow if\text{-}stmt \mid \textbf{other} if\text{-}stmt \rightarrow \textbf{if} \ (\ exp\ )\ statement \mid \textbf{if} \ (\ exp\ )\ statement\ \textbf{else}\ statement} exp \rightarrow 0 \mid 1
```

## P.214 figure 5.6







#### P.215 table 5.9





- The shift is preferred over the reduction
  - What if the reduction is preferred?

						UF
State	Input			Go	oto	
	if	else	other	\$	S	I
0	s4		s3		1	2
1				accept		
2		r1		r1		
3		r2		r2		
4	s4		s3		5	2
5		s6		r3		
6	s4		s3		7	2
7		r4		r4		

## P.215 example 5.13





 There are a few situations in which SLR(1) parsing is not quite powerful enough.

```
stmt \rightarrow call\text{-}stmt \mid assign\text{-}stmt
call\text{-}stmt \rightarrow identifier
assign\text{-}stmt \rightarrow var := exp
var \rightarrow var [exp] \mid identifier
exp \rightarrow var \mid number
```

## Example 5.13





Rules

$$\circ$$
 S  $\rightarrow$  id | V := E

- $\circ$  V  $\rightarrow$  id
- $\bullet E \rightarrow V \mid n$
- A start state of the DFA
  - $\circ$  S'  $\rightarrow$  .S
  - $\circ$  S  $\rightarrow$  .id
  - $\circ$  S  $\rightarrow$  .V := E
  - $\circ$  V  $\rightarrow$  .id
- Conflict
  - $\circ$  S  $\rightarrow$  id.
  - $\bullet$  V  $\rightarrow$  id.

## **Definition of LR(1) transitions**





- Part 2
  - Given an LR(1) item [A → α.Bγ, a], where B is a nonterminal, there are ε-transitions to items [B → .β, b] for every production B → β and for every token b in First (γa).
- Examples

```
• A → (A) | a
```

• State 0: [A' → .A, \$]

$$\begin{bmatrix} \mathsf{A} \to & & \\ & \mathsf{A} \end{bmatrix}$$

• State 2: [A → ( . A ), \$]

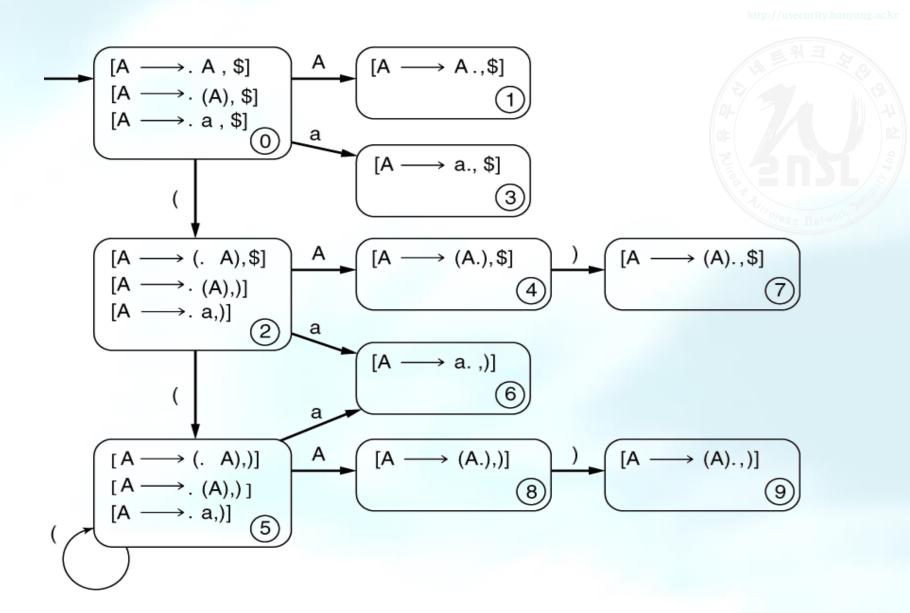
$$[\mathsf{A} \to \mathsf{I}]$$

$$[A \rightarrow ]$$

## P.220 figure 5.7 - LR(1)







## General LR(1) parsing algorithm



• ...

 If state s contains the complete LR(1) item [A → α., a], and the next token in the input string is a, then the action is to reduce by the rule  $A \rightarrow \alpha$ . ... The new state is computed as follows. Remove the string a and all of its corresponding states from the parsing stack. Correspondingly, back up in the DFA to the state from which the construction of α began. By construction, this state must contain an LR(1) item of the form [B  $\rightarrow \alpha$ .A $\beta$ , b]. Push A onto the stack, and push the state containing the item [B  $\rightarrow \alpha A.\beta$ , b].

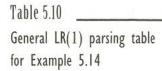
• ...

## P.222 table 5.10





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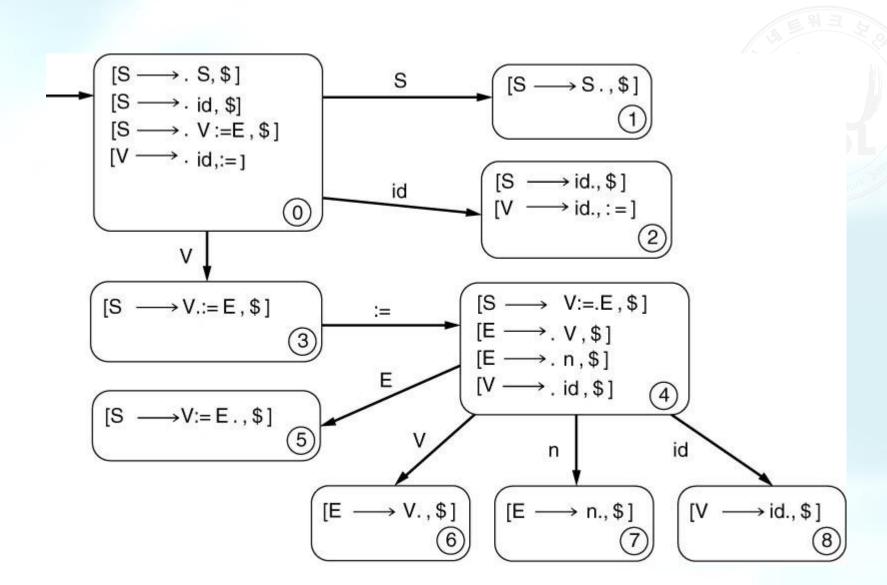


State	Input				
	(	a	)	\$	А
0	s2	s3	ber.		1
1	*140	1000		accept	
2	s5	s6			4
3		and the second		r2	
4		1.00	s7		
5	s5	s6			8
6		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	r2		
7				r1	•
8			s9		
9			r1		

# P.223 figure 5.8 – LR(1)







# LALR(1) Parsing



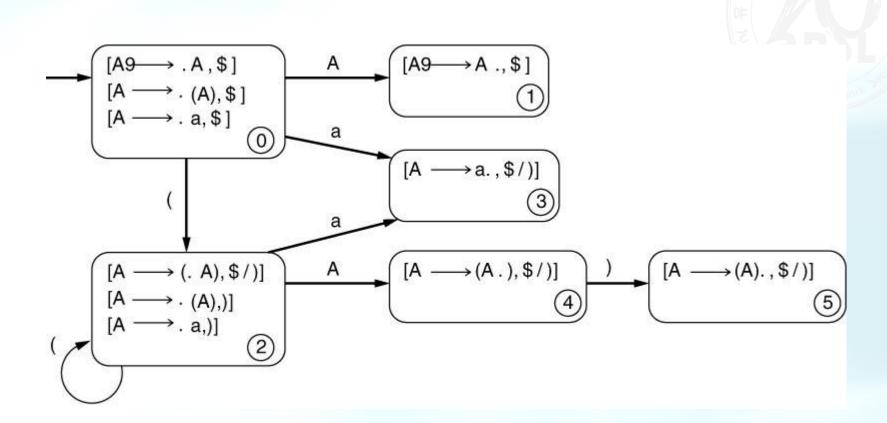


- The size of the DFA of sets of LR(1) items is due in part to the existence of many different states with different lookahead symbols
- FIRST PRINCIPLE OF LALR(1) PARSING
  - The core of a state of the DFA of LR(1) items is a set of the DFA of LR(0) items.
- SECOND PRINCIPLE OF LALR(1) PARSING
  - Given two states s1 and s2 of the DFA of LR(1) items that have the same core, suppose there is a transition on X from s1 to a state t1. Then there is also a transition on X from state s2 to a state t2, and the states t1 and t2 have the same core.
- LALR(1) item
  - $[A \rightarrow \alpha.\beta, a/b/c]$ .

# P.225 figure 5.9 - LALR(1)







#### Yacc





- A parser generator
- Yet another compiler-compiler
- Input
  - A specification of the syntax of a language
  - <filename>.y
- Output
  - A parse procedure for that language
  - y.tab.c, ytab.c or <filename>.tab.c

#### Yacc





A specification file

```
{definitions}
%%
{rules}
%%
{auxiliary routines}
```



#### **Yacc basics**





#### Definitions

• Information about the tokens, data types, and grammar rules.

Figure 5.10

Yacc definition for a simple

calculator program

• Any C code that must go directly in the output file.

```
exp \rightarrow exp \ addop \ term \mid term

addop \rightarrow + \mid -

term \rightarrow term \ mulop \ factor \mid factor

mulop \rightarrow *

factor \rightarrow (exp) \mid number
```

```
%{
#include <stdio.h>
#include <ctype.h>
%token NUMBER
command : exp { printf("%d\n",$1);}
      ; /* allows printing of the result */
       : exp '+' term {$$ = $1 + $3;}
exp
        exp'-'term {$$ = $1 - $3;}
         term \{\$\$ = \$1;\}
term : term '*' factor {$$ = $1 * $3;}
         factor {$$ = $1;}
                             \{\$\$ = \$1;\}
             : NUMBER
 factor
                             \{\$\$ = \$2;\}
               '(' exp ')'
 %%
```

#### **Yacc basics**





#### **o**Rules

- Grammar rules in a modified BNF form
- Actions in C code (used in a reduction)

```
exp \rightarrow exp \ addop \ term \mid term

addop \rightarrow + \mid -

term \rightarrow term \ mulop \ factor \mid factor

mulop \rightarrow *

factor \rightarrow (exp) \mid number
```

```
%{
#include <stdio.h>
#include <ctype.h>
%}
%token NUMBER
command : exp { printf("%d\n",$1);}
      ; /* allows printing of the result */
       : exp '+' term {$$ = $1 + $3;}
exp
        exp'-'term {$$ = $1 - $3;}
         term \{\$\$ = \$1;\}
term : term '*' factor {$$ = $1 * $3;}
         factor {$$ = $1;}
                             \{\$\$ = \$1;\}
             : NUMBER
 factor
                             \{\$\$ = \$2;\}
               '(' exp ')'
 %%
```

Figure 5.10

Yacc definition for a simple

calculator program

#### **Yacc basics**





#### Auxiliary routines

 $exp \rightarrow exp \ addop \ term \mid term$   $addop \rightarrow + \mid$   $term \rightarrow term \ mulop \ factor \mid factor$   $mulop \rightarrow *$  $factor \rightarrow (exp) \mid number$ 

```
%%
main()
{ return yyparse();
int yylex(void)
{ int c;
  while((c = getchar()) == ' ');
  /* eliminates blanks */
  if ( isdigit(c) ) {
    ungetc(c,stdin);
     scanf("%d", &yylval);
     return(NUMBER);
   if (c == '\n') return 0;
   /* makes the parse stop */
   return(c);
 int yyerror(char * s)
 { fprintf(stderr, "%s\n",s);
   return 0;
 } /* allows for printing of an error message */
```





- -d (heaDer file)
- -v (verbose)
  - y.output
  - yacc –v calc.y

```
%token NUMBER
%%
command
ехр
                         term
                term
                      '*' factor
term
                factor
                NUMBER
factor
                '(' exp
```

# Yacc options – verbose option

reduce 1





```
state 0
      $accept : _command $end
                                 state 3
                                       exp: term_{\_} (4)
      NUMBER shift 5
      ( shift 6
       error
                                       * shift 9
      command goto 1
      exp goto 2
                                       . reduce 4
      term goto 3
      factor goto 4
                                 state 4
state 1
      $accept : command_$end
      $end accept
                                       . reduce 6
         error
                                state 5
state 2
      command : exp_
                        (1)
                                       factor : NUMBER_
      exp : exp_+ term
      exp : exp_- term
                                    . reduce 7
      + shift 7
      - shift 8
```





```
state 6
      factor : (_exp )
      NUMBER shift 5
      ( shift 6
        error
      exp goto 10
      term goto 3
      factor goto 4
state 7
      exp : exp +_term
             shift 5
      NUMBER
      ( shift 6
         error
      term goto 11
      factor goto 4
```

```
state 8
      exp : exp -_term
      NUMBER shift 5
      ( shift 6
        error
      term goto 12
      factor goto 4
state 9
      term : term *_factor
      NUMBER shift 5
      ( shift 6
         error
      factor goto 13
```





```
state 12
state 10
                                    exp : exp - term_
                                                          (3)
     exp:
            exp_+ term
                                    term : term_* factor
     exp : exp_- term
     factor: (exp_)
                                       shift 9
                                       reduce 3
        shift 7
        shift 8
        shift 14
                               state 13
                                    term : term * factor_ (5)
        error
state 11
                                    . reduce 5
                           (2)
     exp : exp + term_
                               state 14
     term : term_* factor
                                    factor: (exp)_
                                                          (8)
        shift 9
        reduce 2
                                    . reduce 8
8/127 terminals, 4/600 nonterminals
9/300 grammar rules, 15/1000 states
0 shift/reduce, 0 reduce/reduce conflicts reported
9/601 working sets used
memory: states, etc. 36/2000, parser 11/4000
9/601 distinct lookahead sets
6 extra closures
18 shift entries, 1 exceptions
8 goto entries
4 entries saved by goto default
Optimizer space used: input 50/2000, output 218/4000
218 table entries, 202 zero
maximum spread: 257, maximum offset: 43
```





Table 5.11

Parsing table corresponding to the Yacc output of Figure 5.12

State	Input							Goto			
	NUMBER	(	+	-	*	)	\$	command	ехр	term	factor
0	s5	s6			M. A	ALL :		1	2	3	4
1	•.						accept				
2	r1	r1	s7	s8	r1	r1	r1				
3	r4	r4	r4	r4	s9	r4	r4				
4	r6	r6	r6	r6	r6	r6	r6				
5	r7	r7	r7	r7	r7	r7	r7				
6	s5	s6			150	- 27			10	3	4
7	s5	s6								11	4
8	s5	s6			74					12	4
9	s5	s6									13
10			s7	s8		s14		H. S. L.			
11	r2	r2	r2	r2	s9	r2	r2				
12	r3	r3	r3	r3	s9	r3	r3			18	
13	r5	r5	r5	r5	r5	r5	r5				
14	r8	r8	r8	r8	r8	r8	r8				

# Parsing Conflicts and Disambiguating Rules





- shift/reduce conflict
  - shift > reduce
  - Table 5.9 (p. 215) for example 5.12 (p. 213)
- reduce/reduce conflict
  - The reduction rule listed first is preferred.

#### **Reduce-Reduce Conflict**





Figure 5.13

Yacc output file for the grammar of Example 5.18

state 0

\$accept : \_S \$end

- a shift 4
- . error
- S goto 1
- A goto 2
- B goto 3

state 1

\$accept : S\_\$end

\$end accept

. error

state 2

S: A\_ (1)

. reduce 1

state 3

S: B\_ (2)

. reduce 2

4: reduce/reduce conflict (red'ns 3 and 4 ) on \$end state 4

$$A : a_{-}(3)$$

. reduce 3

Rule not reduced: B: a

3/127 terminals, 3/600 nonterminals 5/300 grammar rules, 5/1000 states 0 shift/reduce, 1 reduce/reduce conflicts reported ...

$$S \to A \mid B$$
$$A \to a$$

$$B \rightarrow a$$

# Operator precedence and associativity





#### Figure 5.14

Yacc specification for a simple calculator with ambiguous grammar and precedence and associativity rules for operators

```
%{
#include <stdio.h>
#include <ctype.h>
%}
%token NUMBER
%left '+' '-'
%left '*'
%%
                      { printf("%d\n",$1);}
command : exp
                         \{\$\$ = \$1;\}
ехр
         NUMBER
                         \{\$\$ = \$1 + \$3;\}
                         \{\$\$ = \$1 - \$3;\}
                  exp
                  exp
                         \{\$\$ = \$1 * \$3;\}
         exp
             exp ')'
                         \{\$\$ = \$2;\}
%%
   auxiliary procedure declarations as in Figure 5.10
```

# **Tracing the Execution of** a Yacc Parser





#define YYDEBUG 1

Figure 5.15 Tracing output using vydebug for the Yacc parser generated by Figure 5.10, given the input 2+3

Starting parse Entering state 0 Input: 2+3 Next token is NUMBER Shifting token NUMBER, Entering state 5 Reducing via rule 7, NUMBER -> factor state stack now 0 Entering state 4 Reducing via rule 6, factor -> term state stack now 0 Entering state 3 Next token is '+' Reducing via rule 4, term -> exp state stack now 0 Entering state 2 Next token is '+' Shifting token '+', Entering state 7 Next token is NUMBER Shifting token NUMBER, Entering state 5 Reducing via rule 7, NUMBER -> factor state stack now 0 2 7 Entering state 4 Reducing via rule 6, factor -> term state stack now 0 2 7 Entering state 11 Now at end of input. Reducing via rule 2, exp '+' term -> exp state stack now 0 Entering state 2 Now at end of input. Reducing via rule 1, exp -> command state stack now 0 Entering state 1 Now at end of input.

# **Arbitrary Value Types in Yacc**





```
%{
  #define YYSTYPE double
%}
```



### **Arbitrary Value Types in Yacc**





%token NUMBER

```
exp \rightarrow exp \ addop \ term \mid term
addop \rightarrow + / -
```

#### **Embedded Actions in Yacc**





```
decl
      : type { current_type = $1; }
        var_list
type
        INT { $$ = INT_TYPE; }
        FLOAT { $$ = FLOAT_TYPE; }
var_list : var_list ',' ID
            { setType(tokenString,current_type);}
           ID
              setType(tokenString,current_type);}
```

### **Embedded Actions in Yacc**





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Yacc internal names and definition mechanisms	Yacc internal name	Meaning/Use			
denintion mechanisms	y.tab.c y.tab.h yyparse yylval	Yacc output file name Yacc-generated header file containing token definitions Yacc parsing routine value of current token in stack user-defined error message printer used by Yacc Yacc error pseudotoken procedure that resets parser after error contains the lookahead token that caused an erro preprocessor symbol that defines the value type of the parsing stack variable which, if set by the user to 1, causes the generation of runtime information on parsing actions  Meaning/Use  defines token preprocessor symbols defines the start nonterminal symbol defines a union YYSTYPE, allowing values of different types on parser stack defines the variant union type returned by a symbol defines the associativity and precedence (by position) of operators			
	yyerror error yyerrok yychar YYSTYPE yydebug				
	Yacc definition mechanism				
D.	%token %start %union				
	%type				
	%left %right %nonassoc				