ENE 3031 – Fall 2014

Homework3 Solution

#1.

$X_0 = 27,\ a = 8,\ c = 47,\ m = 100$

$X_1 = (8 \times 27 + 47) \text{mod } 100 = 63,\quad R_1 = 63/100 = .63$

$X_2 = (8 \times 63 + 47) \text{mod } 100 = 51,\quad R_2 = 51/100 = .51$

$X_3 = (8 \times 51 + 47) \text{mod } 100 = 55,\quad R_3 = 55/100 = .55$

#2.

Let ten intervals be defined each from $(10i-9)$ to $(10i)$ where $i = 1, 2, \ldots, 10$. By counting the numbers that fall within each interval and comparing this to the expected value for each interval, $E_i = 10$, the following table is generated:

Interval	O_i	$(O_i - E_i)^2/E_i$
(01-10)	9	0.1
(11-20)	9	0.1
(21-30)	9	0.1
(31-40)	6	1.6
(41-50)	17	4.9
(51-60)	5	2.5
(61-70)	10	0.0
(71-80)	12	0.4
(81-90)	7	0.9
(91-00)	16	3.6
	100	14.2= χ_0^2

From Table A.6, $\chi^2_{.05,9} = 16.9$. Since $\chi_0^2 < \chi_{.05,9}$, then the null hypothesis of no difference between the sample distribution and the uniform distribution is not rejected.

#3.

	Case (a)	Case (b)	Case (c)	Case (d)
i	X_i	X_i	X_i	X_i
0	7	8	7	8
1	13	8	1	8
2	15		7	8
3	5			
4	7			

Inferences:

Maximum period, $p = 4$, occurs when X_0 is odd and $a = 3 + 8k$ where $k = 1$. Even seeds have the minimal possible period regardless of a.

#4.

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N=20

# of runs (A) = 12

E(A) = 2*20/3 = 40/3 = 13

Var(A) = 16*20-29/90 ≈ 3.23333

Z0 = (12-13)/sqrt(3.23333) ≈ -0.55613

If alpha = 0.10, then 0.55613 < 1.645 and fail to reject H0.

If alpha = 0.05, then 0.55613 < 1.96 and also fail to reject H0.

Thus, they are approximately independent.

#5.

Step 1.

$$\text{cdf} = F(x) = \begin{cases} 1 - x + x^2/4, & 2 \leq x < 3 \\ x - x^2/12 - 2, & 3 < x \leq 6 \end{cases}$$

Step 2. Set $F(X) = R$ on $2 \leq X \leq 6$

Step 3. Solve for X to obtain

$$X = \begin{cases} 2 + 2\sqrt{2} & 0 \leq R \leq 1/4 \\ 6 - 2\sqrt{3 - 3R} & 1/4 < R \leq 1 \end{cases}$$

The true mean is $(a + b + c)/3 = (2 + 3 + 6)/3 = 11/3$.