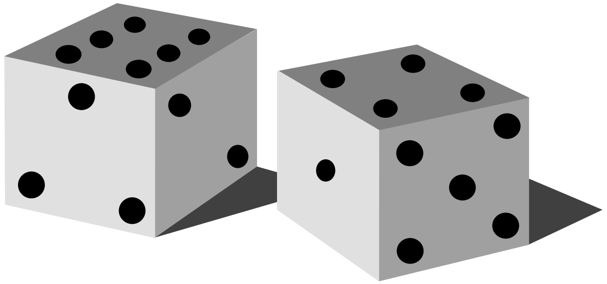

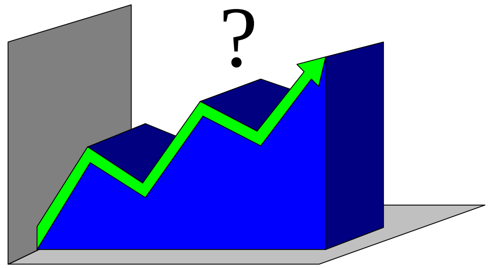

MOE/MOR

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Statistics Review

- Many statistics results are established under the assumptions of *independent and identically normally distributed* data.
- When we have R i.i.d. normal data,
  - S^2 is an unbiased estimator of σ^2
  - An $(1-\alpha)$X100% confidence interval is

sample mean $\pm$ $t_{(1-\alpha/2,R-1)}$* sqrt(var of sample mean).

- In industrial or service simulations, this is not true. For example, there is dependence between consecutive customers' waiting times.
- However, usually our performance measures are averages (average waiting time, average # waiting, average # servers busy….)
- Thus, we make multiple replications to achieve approximately i.i.d. normal data

Basic observations

- Rep 1 ________________ Y_1
- Rep 2 ________________ Y_2
- Rep 3 ________________ Y_3
- Rep 4 ________________ Y_4
- Rep 5 ________________ Y_5
- Note that Y's are within-replication averages. For example, Yi is an average of waiting times of all customers in the ith replication.

- Central Limit Theorem: averages of many observations are approximately normally distributed.
- We use different PRN for each replication. Thus, Y's are independent.
- All replications are from the simulation model of a system. Thus, Y's are identical.

→ **Y's are i.i.d. normally distributed.**

Expectation vs. Sample Mean

- μ vs. $\overline{Y}$
  - μ: average of a population, constant
  - $\overline{Y}$: average of sample, random variable
  - When sample size is small, sample mean is not a good representative value for the population.
  - As sample size increases, sample mean becomes more accurate but increasing sample size is sometimes expensive (more time & cost)
  - Thus, we need balance between accuracy and sample size (in our case, # of replications)
  - First, we need a measure of accuracy of sample mean.

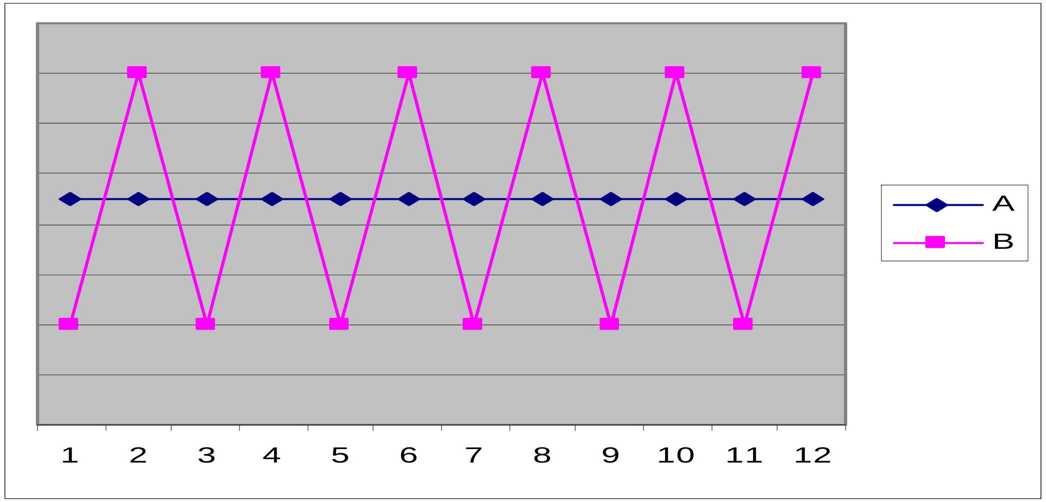
Variance vs. Sample Var

- σ^2 vs. S^2 (Measure of Risk)
  - σ^2: variance of a population
  - σ: how far from expected value
  - S^2: sample variance. Variance of sample
  - S: standard deviation → how far from sample mean

Example

- There are two mobile phone service companies, A and B. They have the same number of employees, same technology, same customer's in-and-out processing cost and same products with the same pricing. Also two companies had the same *average* number of customers last year. However, A had positive profit while B had negative profit last year. B wants to find a good explanation for this.

Why?



- $S_A \cong 0$, $S_B >> 0$. B had more cost for processing customer's in and out.
- Average itself doesn't tell that much. We need S!!!

S: Measures of Risk

- Standard Deviation and Quantile
- Large S: an observation can be very far from sample mean (High Risk)
- Small S: an observation is likely to be close to sample mean (Low Risk).
- The Std Deviation can be interpreted as the *average deviation of reality from the Mean*. Unlike Variance, it is in the same units as the output.

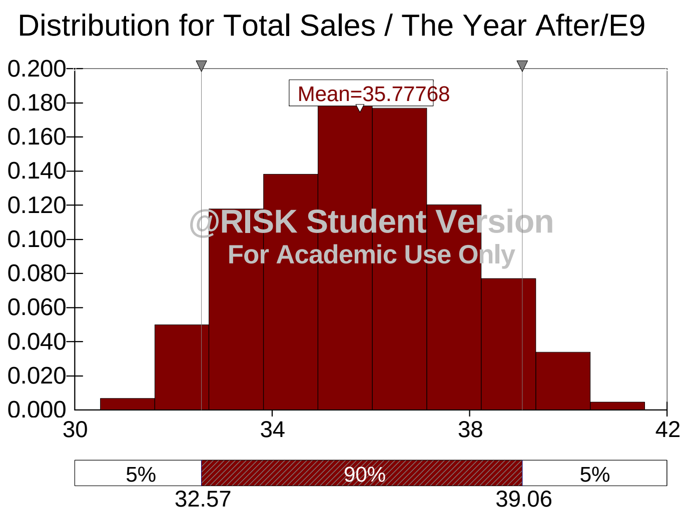
Percentiles: Measures of Risk

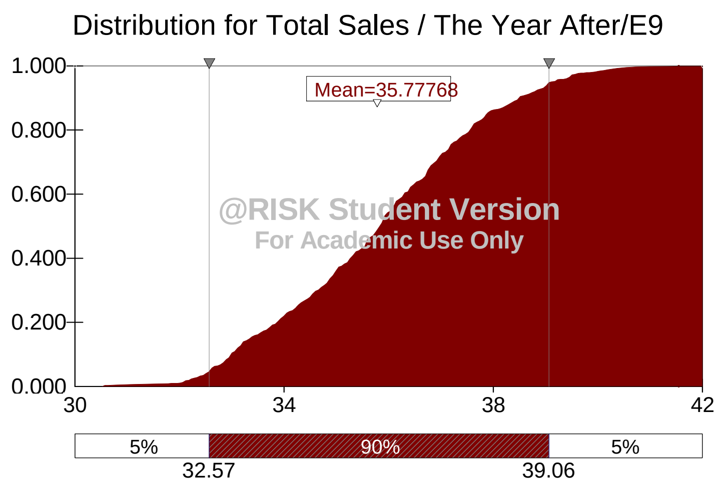
- A quantile (or percentile) is the inverse of a probability;it tells us what level of performance can be delivered with a prespecified probability.

This is given.

$$\Pr\{Y \le \gamma\} = p$$

Percentiles: Measures of Risk





Average total sale = 35.778

95% C.I. for mean total sale= [35.579, 35.977]

2.5% percentile of total sale = 32.014
97.5% percentile of total sale = 39.732

95% of data are between 32.014 and 39.732 thus there is 95% chance that my "actual" sales amount next year will be between 32.014 and 39.732. Also, with 95% confidence, I know that next year "expected" sales amount is between 35.579 and 35.977

Measure of Error or Accuracy

- Confidence interval or standard error
- A 95% C.I. for μ is

  sample mean $\pm$ $t_{(1-\alpha/2,R-1)} * S/\sqrt{R}$
- $S/\sqrt{R}$ is standard error
- Example:
  - 158.595 $\pm$ 30.56; [128.34, 189.46] : With 95% confidence, the true mean is somewhere between 128.34 and 189.46. Sample mean is not accurate at all.
  - 158.595 $\pm$ 0.245; [158.65, 159.14] : With 95% confidence, the true mean is somewhere between 158.65 and 159.14. μ must be around 159.
  - Rule of thumb for reporting sample mean: Report up to the first non-zero digit in ***standard error***. (e.g., For the first CI, s.e. $\approx$ 15.6 thus report average = 1.6×10^2. For the 2^{nd} CI, s.e. $\approx$ 0.13 thus report 158.9)

Measure of Error (con'd)

- Note that C.I. becomes narrower and $S/\sqrt{R}$ smaller as R increases. This makes sense: the more observations we get, the more accurate a sample mean is.
- Why is a C.I. important?
  - Getting accurate performance estimate is critical to compare alternatives.
  - We can get accurate performance estimates by controlling half-width of C.I. We do this through the number of replications, R.

Setting the Number of Iterations

- As the # of iterations increases, the error in the performance estimates (mean, std deviation, percentiles, etc.) decreases.
- The decrease is slow: to cut the error in half requires 4 times as many observations (error decreases as the square root of the number of iterations).

A Direct Approach

- Remember that a 95% confidence interval for the mean is
  Mean $\pm$ t*Std Deviation/$\sqrt{}$#iterations
- To estimate mean performance to $\pm$ H, first make a test simulation of at least 10 iterations, then make your “keeper” simulation with

  # iterations = [t*Std Deviation/H]^2

Example

- After a test simulation of 400, we got the following results for the average wait time.

- Average wait time: 35.77768 Half width: 0.1988462
- I'd like to estimate the wait time within ± 0.1 minute. First I need S.

  $0.1988462 = 1.96 * S/\sqrt{400} \rightarrow S = 2.029043$

  Then $1.96 \times S/\sqrt{R_{keeper}} = 0.1$

  $1.96 \times 2.029043/\sqrt{R_{keeper}} = 0.1 \rightarrow R_{keeper} = 1581.59.$

- Thus, we need to run 1582 replications to estimate the average wait time within ± 0.1 minute

Warning

- Don’t confuse C.I. with S or quantiles.