ENE 3031 – Fall 2014

Homework4 Solution

#1.

Use Inequality (8.14) to conclude that, for R given, X will assume the value x in $R_X = \{1, 2, 3, 4\}$ provided

$$F(x-1) = \frac{(x-1)x(2x-1)}{180} < R \leq \frac{x(x+1)(2x+1)}{180} = F(x)$$

By direct computation, $F(1) = 6/180 = .033$, $F(2) = 30/180 = .167$, $F(3) = 42/180 = .233$, $F(4) = 1$. Thus, X can be generated by the table look-up procedure using the following table:

x	1	2	3	4
$F(x)$	.033	.167	.233	1

$$\begin{aligned} R_1 &= 0.83 \longrightarrow X = 4 \\ R_2 &= 0.24 \longrightarrow X = 4 \\ R_3 &= 0.57 \longrightarrow X = 4 \end{aligned}$$

#2.

The mean is $(1/p) - 1 = 2.5$, so $p = 2/7$. By Equation (9.21),

$$X = \lceil -2.97 \ln(1-R) - 1 \rceil$$

Equation (9.21) R is a random number.

$$\left\lceil \frac{\boldsymbol{ln(1-R)}}{\boldsymbol{ln(1-p)}} \right\rceil - \mathbf{1}$$

#3.

Generate $X = 8[-\ln R]^{4/3}$

If $X \leq 5$, set $Y = X$.

Otherwise, set $Y = 5$.

(Note: for Equation 8.6, it is permissible to replace $1 - R$ by R.)

#4.

Step 1: Set $n = 0$

Step 2: Generate R

Step 3: If $R \leq p$, set $X = n$, and go to step 4.

If $R > p$, increment n by 1 and return to step 2.

Step 4: If more geometric variates are needed, return to step 1.

#5.

Let us take g to be $g(x) = 1$ for $0 \leq x \leq 1$; that is the desity of $U(0, 1)$. Thus

$$\max_x \frac{f(x)}{g(x)} = 3 = c$$

So we'll use $\frac{f(x)}{cg(x)} = x^2$ in the above algorithm

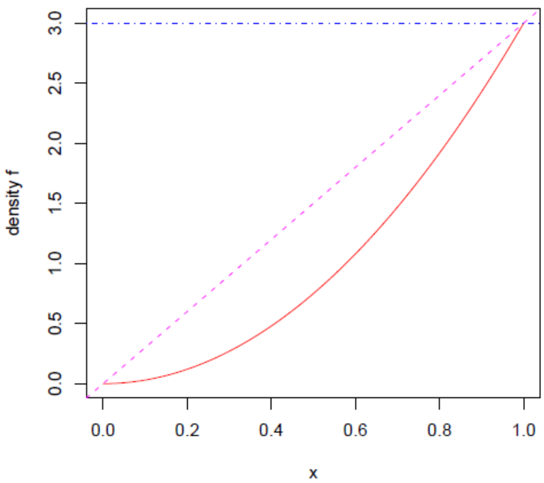
Algorithm:

1) Generate two uniform random variables u_1 and u_2 from $U(0, 1)$.
2) If $u_2 \leq u_1^2$ accept u_1 as the random variable from f

   else go to step 1).

By looking at the shapes of f and g we can guess that this is not a terribly efficent algorithm.

Let us try to find a better majorizing function for f



Instead of using $q(x) = 3$ (the blue line), use $q(x) = 3x$ (the magenta line)

To find g we need to find the normalizing constant k such that $k\int_0^1 3x dx = 1$

Easy to show $k = 2/3$ so that $g(x) = 2x$ for $0 \le x \le 1$ and

$$\max_x \frac{f(x)}{g(x)} = 3/2 = c$$

This majorizing function reduces the rejection rate by 50%

To implement the rejection method we need to generate from g

Note that the cdf of is easy to derive. (Show $F(t) = t^2$); thus to generate from this distribution using the inverse method is simple: Generate u from $U(0,1)$ and set $X = +\sqrt{u}$.

New Algorithm:

1) Generate two uniform random variables u_1 and u_2 from $U(0,1)$.

2) If $u_2 \le \frac{3\sqrt{u_1}}{2}$ accept u_1 as the random variable from f

   else go to step 1).

#6.

ai =(x(i)-x(i-1))/(1/10)

i	LB $(x_{(i-1)})$	UB$(x_{(i)})$	Prob	Cum.Prob	a_i
1	0	0.54	0.1	0.1	5.4
2	0.54	0.76	0.1	0.2	2.2
3	0.76	1.01	0.1	0.3	2.5
4	1.01	1.32	0.1	0.4	3.1
5	1.32	2.44	0.1	0.5	11.2
6	2.44	3.26	0.1	0.6	8.2
7	3.26	3.29	0.1	0.7	0.3
8	3.29	4.65	0.1	0.8	13.6
9	4.65	5.42	0.1	0.9	7.7
10	5.42	5.84	0.1	1	4.2

$X = x_{(i-1)} + a_i(U-(i-1)/10)$

U1 = 0.24 -> X1=0.86 (i=3)

U2=0.35 -> X1 = 1.165 (i=4)