Electrical Engineering

HW 6– Chapter 7 Solution

<1>

(a) The power factor is

$$pf = \cos\theta = \frac{P}{V_{rms}I_{rms}} = \frac{800}{12\times 120} = 0.56$$

(b) The phase angle θ is

$$\theta = \arccos(0.56) = 56.25°$$

(c) The impedance Z is

$$Z = |Z|\angle\theta = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle 0°}{12\angle -56.25°} = 10\angle 56.25° = 5.56 + j8.31 \quad \Omega$$

(d) Obviously, the resistance is $5.56\,\Omega$

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a) $P = \frac{650}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\cos(10°) = 6401.25 \text{ W}$

$Q = \frac{650}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\sin(10°) = 1128.7 \text{ VAR}$ $\quad S = \frac{650}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\angle 10° = 6500\angle 10° \text{ VA}$

b) Use the same calculation as shown above, we can have

$P = 4599.3 \text{ W}$ $\quad Q = 4599.3 \text{ VAR}$ $\quad S = 6504.4\angle 45° \text{ VA}$

c) $P = 60 \text{ W}$ $\quad Q = 857.91 \text{ VAR}$ $\quad S = 860\angle 86° \text{ VA}$

d) $P = 401.22 \text{ W}$ $\quad Q = 260.56 \text{ VAR}$ $\quad S = 478.4\angle 33° \text{ VA}$

<3>

(a) $jX_L = j\omega L = j377\times 25.55\times 10^{-3} = j9.63 \ \Omega$

$$jX_C = \frac{1}{j\varpi C} = \frac{1}{j377\times 265\times 10^{-6}} = \text{-}j10.01\ \ \Omega$$

$$jX_L || jX_C = j255.64\ \ \Omega$$

The equivalent impedance Z is

$$Z = jX_L \,||\, jX_C + R = 10 + j255.64 = 255.83\angle 87.76°$$

The current in the circuit is

$$I = \frac{V_S}{Z} = \frac{120\angle 0\square}{255.5\angle 87.8\square} = 0.47\angle\text{-}87.76°$$

The real power P is

$$P = I^2R = 0.47^2 \times 10 = 2.20\ \ \text{W}$$

The reactive power Q is

$$Q = I^2X\ = 0.47^2 \times 255.64 = 56.24\ \text{VAR}$$

(b) $\quad jX_L = j\omega L = j314\times 25.55\times 10^{-3} = j8.02\ \ \Omega$

$$jX_C = \frac{1}{j\varpi C} = \frac{1}{j314\times 265\times 10^{-6}} = \text{-}j12.02\ \ \Omega$$

$$jX_L || jX_C = j24.13\ \ \Omega$$

The equivalent impedance Z is

$$Z = jX_L \,||\, jX_C + R = 10 + j24.13 = 26.12\angle 67.49°$$

The current in the circuit is

$$I = \frac{V_S}{Z} = \frac{120\angle 0\square}{26\angle 67.38\square} = 4.59\angle\text{-}67.49°$$

The real power P is

$$P = I^2R = 4.59^2 \times 10 = 211.01\ \ \text{W}$$

The reactive power Q is

$$Q = I^2X\ = 4.59^2 \times 24.13 = 509.25\ \ \text{VAR}$$

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The current is

$$I = \frac{120}{12 + j377 \times 20 \times 10^{-3}} = \frac{120}{14.17\angle 32.14^\circ} = 8.47\angle -32.14^\circ \quad \text{A}$$

(a) The average power dissipated in the load is $P_{av} = I^2 R = 8.47^2 \times 10 = 717.4 \ \text{W}$

(b) The power factor of the motor is $pf = \cos\ 32.14^\circ = 0.847$ lagging

(c) $\theta = \cos^{-1} 0.9 = 25.84^\circ$

$S_{NEW}\angle 25.84^\circ = 717.4\ \text{W} + j(Q_L - Q_C)$ $\quad S_{NEW} = 797.1$ $\qquad Q_{NEW} = 347.4$

$Q_L = 450.7\ \text{VAR}$ $\quad Q_C = 103.3\ \text{VAR}$

$$Q_C = \frac{V^2}{X_C} = \frac{120^2}{X_C} = 103.3$$

$X_C = 139.4\ \Omega$ $\qquad C = \frac{1}{\omega X_C} = 19$“F

<5>

a) $\omega = 5, Z_T = R \| Z_{L2} + Z_C + Z_{L1} = 4 + j5\Omega, \tilde{I} = \frac{15}{\sqrt{2} \cdot 4} = 26517.\text{A}$

$P = \tilde{I}^2 R = 28.12\text{W}, Q = \tilde{I}^2 X = 35.15\text{VAR}$

b) $\omega = 15, Z_T = R \| Z_{L2} + Z_C + Z_{L1} = 60 + j44.33\Omega, \tilde{I} = \frac{15}{\sqrt{2} \cdot 60} = 0.1768.\text{A}$

$P = \tilde{I}^2 R = 1.8755\text{W}, Q = \tilde{I}^2 X = 1.3857\text{VAR}$

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First, we compute the load impedance:

$$Z_L = (R + jX_L) \| jX_C = 0.38 - j8.88$$

Then, we compute the load current

$$\tilde{I}_L = \frac{\tilde{V}_L}{Z_L} = 0.4 + j10.1 \text{ A}$$

and the complex power:

$$S = \tilde{V}_L \cdot \tilde{I}_L^* = 36 - j909 \text{ W}$$

Therefore

$$P_{av} = 36W$$
$$Q = -909VAR$$

For computing the reactance needed for the power factor correction, we compute the complex power without capacitor. First, we compute the load impedance:

$$Z_L = (R + jX_L) = 25 + j70$$

Then, we compute the load current

$$\tilde{I}_L = \frac{\tilde{V}_L}{Z_L} = 0.4 - j1.14 \text{ A}$$

and the complex power:

$$S = \tilde{V}_L \cdot \tilde{I}_L^* = 36.64 + j102.63 \text{ W}$$

Therefore

$$Q = 102.63VAR$$

So, we need to contribute a negative reactive power equal to -102.63. This requires a negative reactance and then a capacitor with $Q_C = -102.63VAR$

$$X_C = \frac{\left|\tilde{V}_L\right|^2}{Q_C} = -\frac{90^2}{-102.63} = -78.92\Omega$$

$$C = -\frac{1}{\omega X_C} = 3.36 \cdot 10^{-5} F$$

<7>

(a) The equivalent resistance seen by the voltage source:

To find the equivalent resistance seen by the voltage source, use equation 7.32:

$$\boldsymbol{Z_1} = \frac{1}{N^2}\boldsymbol{Z_2}$$

In this case, the load impedance, $\boldsymbol{Z_2}$**,** is the output resistance R_O, which has the impedance:

$$\boldsymbol{Z_2} = R_O = 12\ \Omega = 12\angle 0$$

Plug $\boldsymbol{Z_2}$ into equation 7.32:

$$\boldsymbol{Z_1} = \frac{1}{5^2} * 12\angle 0$$

$$= 0.48\angle 0\ \Omega$$

Final answer: $\boldsymbol{Z_1 = 0.48\angle 0\ \Omega}$

(b) The power P_{source} supplied by the voltage source:

From equation 7.31, we know that ideal transformers conserve power. Therefore:

$$\boldsymbol{S_1 = S_2}$$

Using this property, we may solve for the source power by computing the real power at the primary terminal using equation 7.12:

$$P_{source} = P_{avg} = \frac{\widetilde{\boldsymbol{V}}^2}{|\boldsymbol{Z}|}\cos(\theta_Z)$$

where $\widetilde{\boldsymbol{V}} = \widetilde{\boldsymbol{V}}_{\boldsymbol{g}}$ and $\boldsymbol{Z} = \boldsymbol{Z_S} + \boldsymbol{Z_1}$**.** Plug these values into equation 7.12:

$$P_{source} = \frac{80}{(0.48 + 2)} * \cos(0)$$

$$= 168.67\ W$$

Final answer: $\boldsymbol{P_{source} = 168.67\ \text{W}}$

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$$\mathbf{R_{Seq} = R_0}$$

Assume R_{Seq} is load impedance seen by the source and $R_{Seq} = n^2 R_0$

For maximum power delivery $R_{Seq} = R_{in}$

$$n = \sqrt{\frac{R_{in}}{R_0}} = \sqrt{12} = 3.46$$

<9>

$$\tilde{\mathbf{I}}_n = \tilde{\mathbf{I}}_{an} + \tilde{\mathbf{I}}_{bn} + \tilde{\mathbf{I}}_{cn} = 22\angle 0^\circ + 10\angle 120^\circ + 15\angle 45^\circ = 22 + 5 + j8.66 + 10.6 + 10.6j =$$
$$= 37.6 + j19.26 = 42.24\angle 27.12^\circ \mathrm{A}$$

<10>

Current in each wire:

Note that the phase voltages of the wye network are balanced by equations 7.49 and 7.47. Therefore:

$$\tilde{\boldsymbol{V}}_{an} = \tilde{\boldsymbol{V}}_R = 110\angle 0\ V\ rms$$

$$\tilde{\boldsymbol{V}}_{bn} = \tilde{\boldsymbol{V}}_B = 110\angle\, 4\pi/3\, V\ rms$$

$$\tilde{\boldsymbol{V}}_{cn} = \tilde{\boldsymbol{V}}_W = 110\angle\, 2\pi/3\ \ V\ rms$$

However, the wye load configuration is not balanced, so the current in the neutral line will be non-zero. Refer to Figure 7.49 for a simplified representation of the circuit. Using KCL, we can write the neutral line current as:

$$\tilde{\boldsymbol{I}}_N = \tilde{\boldsymbol{I}}_R + \tilde{\boldsymbol{I}}_W + \tilde{\boldsymbol{I}}_B$$

Now, calculate each of these currents using superposition:

$$\tilde{\boldsymbol{I}}_R = \frac{\tilde{\boldsymbol{V}}_{an}}{\boldsymbol{Z}_a}$$

where $\boldsymbol{Z}_a$ is R. Plug in known values:

$$\tilde{\boldsymbol{I}}_R = \frac{110\angle 0}{50\angle 0}$$

$$= 2\ A$$

Solve for $\tilde{\boldsymbol{I}}_B$:

$$\tilde{\boldsymbol{I}}_B = \frac{\tilde{\boldsymbol{V}}_{bn}}{\boldsymbol{Z}_b}$$

$$= \frac{110\angle 4\pi/3}{45.24\angle \pi/2}$$

$$= 2.43\angle \frac{5\pi}{6}$$

$$= -2.10 + j1.22\ A$$

Solve for $\tilde{\boldsymbol{I}}_W$:

$$\tilde{\boldsymbol{I}}_W = \frac{\tilde{\boldsymbol{V}}_{cn}}{\boldsymbol{Z}_c}$$

$$= \frac{110\angle 2\pi/3}{19.94\angle(-\pi/2)}$$

$$= 5.52\angle 7\pi/6$$

$$= -4.78 - j2.76\ A$$

Add up the components to solve to the neutral line current:

$$\tilde{\boldsymbol{I}}_N = 2 - 2.10 + j1.22\ - 4.78 - j2.76$$

$$= -4.88 - j1.54\ A$$

Final answer: $\tilde{\boldsymbol{I}}_R = 2\ A,\ \tilde{\boldsymbol{I}}_B = -2.10 + j1.22\ A, \tilde{\boldsymbol{I}}_W = -4.78 - j2.76\ A,\ \tilde{\boldsymbol{I}}_N = -4.88 - j1.54\ A$

Real power:

The real power consumed will be the power in branch R, because it is the only branch with a real load. Furthermore, the voltage and load in this branch have only real components. Therefore, the real power is:

$$P = 110 * 2$$

$$= 220\ W$$

Final answer: $\boldsymbol{P} = \mathbf{220}\ \ \mathbf{W}$