Operations Management I

Forecasting

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Forecasting

Introduction
Methods
Measure and Control of Forecast Errors

- Definition and Classification
- Demand Forecasting – Roles
- Demand Characteristics
- Laws of Forecasting

- Overview
- Qualitative Methods
- Quantitative Methods

Hopp and Spearman, 2008, **Factory Physics**, McGraw Hill. **(Section 13.3)**
Krajewski and Ritzman, 2005, **Operations Management**, Prentice Hall. **(Chapter 13)**

Forecasting

◈ **Introduction**

Forecasting – Overview

- Definition

  Prediction of future events used for planning purpose

  ◄---- Generate expectations of the future in order to evaluate alternative policies

- Classification

  ✓ Economic forecasting ◄---- Economics

  Forecast the future of the economy

  e.g., rate of economic growth, unemployment rate, price index, etc.

  ✓ Technological forecasting ◄---- Management of Technology

  Forecast the development and progress of technology

  e.g., solar energy, nuclear energy

  ✓ Demand forecasting ◄---- Production/Operations Management

  Forecast the demand of products or services
  (for decision making of enterprise)

Focus

Forecasting

Introduction

Demand Forecasting

- Roles

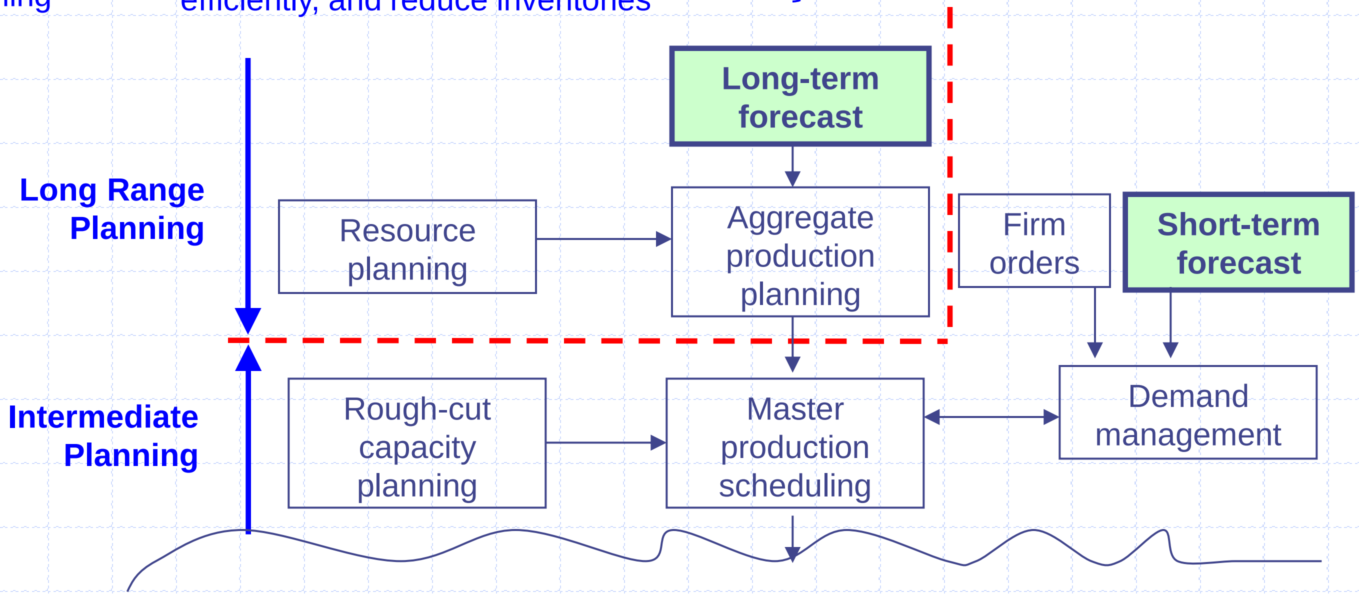
Basic data for various decision making of enterprises and aid in

- ✓ Determining what resources are needed and acquiring additional resources
- ✓ Scheduling existing resources

Accurate forecasts allow schedulers to increase customer satisfaction, reduce customer response time, use capacity efficiently, and reduce inventories

- ➢ Process design
- ➢ Capacity determination

- ➢ Production planning
- ➢ Scheduling



Forecasting

Introduction

Characteristics of Demands

Time series (시계열)

Set of observations of a variable over time (stochastic process: {X(t)), t ≥ 0})

- Five basic patterns of most demand time series
  - ✓ Horizontal (level)
    Fluctuation of data around a constant mean
  - ✓ Trend (추세)
    Systematic increase or decrease in the mean of the series over time
  - ✓ Seasonal (계절변동)
    Repeatable pattern of increase or decrease in demand, depending on the time of day, week, month or season
  - ✓ Cyclical (순환변동)
    Less predictable gradual increase or decrease over longer periods of time (years or decades)
    - ✓ Business cycle: recovery, prosperity, recession, depression
    - ✓ Product life cycle: introduction, growth, maturity, decline
  - ✓ Random (우연변동)
    Un predictable variation in demand due to chance and unusual occurrences

Forecasting

- **Introduction**

Laws of Forecasting

- Forecasts are always wrong !

  Perfect prediction of future is not possible.

  ← - - - - - We should strive make decisions as robust as possible
  with respect to errors in the forecast. (due to approximate estimate)

- Detailed forecasts are worse than aggregate forecasts !

  Aggregate forecasts are more accurate than detailed forecasts (variability pooling)

  e.g., Two-tier forecasting system
  - ✓ Aggregate – product family level
  - ✓ Detailed – product level

- The further into the future, the less reliable the forecast will be !

  The further out one goes, the greater the potential for qualitative changes that completely invalidate whatever forecasting approach we use.

Forecasting

Methods

Overview

- Qualitative methods (정성적 방법)
  - ✓ Develop future scenarios by using the expertise of people, rather than precise mathematical models

    ←----- Appropriate for the situations where there are no historical data or data are inaccurate

- Quantitative methods (정량적 방법)
  - ✓ Causal forecasting (인과형 모형)

    Predict a future parameter as a function of other parameters

    e.g., Demand = f (interest rate, growth in GNP, etc.)

  - ✓ Time-series forecasting (시계열 분석법)

    Predict a future parameter as a function of past values of that parameter

    ←----- 기본가정: 과거의 수요패턴이 미래에도 지속됨

Long-term forecasting

Mod-term forecasting

Short-term forecasting

Forecasting

- **Methods**

Qualitative Methods (1)

- Sales-force estimates

  Forecasts compiled from the people estimates of future demands made periodically by members of a company's sales force rather than precise mathematical models

  ←------ Disadvantages

  ✓ Individual biases of the salesperson
  ✓ Underestimation (for looking good)

- Executive opinion

  Forecasts arrived from summarizing the opinions, experience, and technical knowledge of one or more managers

  ←------ Used to modify an existing sales forecast to account for unusual circumstances

- Historical analogy

  Forecasts based on the data for similar previous products

Forecasting

Methods

Qualitative Methods (2)

- Market research
  - ✓ Systematic method to determine external consumer interest in a service or product by creating and testing hypotheses through data-gathering surveys
    - ① Designing a questionnaire
    - ② Deciding how to administer the surveys (telephone, mailing, personal interviews, etc.)
    - ③ Analyzing the information (judgment, statistical tools, etc.)
- Delphi method
  - ✓ Process of gaining consensus from a group of experts while maintaining their anonymity
    - ① Experts are queried about some future subjects (usually in written form)
    - ② Responses are tabulated and returned to the panel of experts, who reconsider the responses again
    - ③ The above two processes are repeated several times until consensus is reached or the respondents have stabilized their answers.

Forecasting

Methods

Quantitative Methods – Causal Forecasting (1)

- Basic

  Explain the behavior of an uncertain future demand
  in terms of other observable or predictable parameters

  e.g., demand of a new fast-food outlet at a given location

  Predictable parameters
  - ✓ population within some distance of the location
  - ✓ number of competitor fast-food restaurants within some distance of the location

- Mathematical expression
  - ✓ Estimation of function f
    - Regression analysis
      - ✓ Simple (n =1) and Multiple (n >1)
      - ✓ Linear and Nonlinear

$$\underbrace{Y}_{\text{Demand (dependent variable)}} = f\,\underbrace{(X_1, X_2, \ldots X_n)}_{\text{Parameters (independent variables)}}$$

Forecasting

Methods

Quantitative Methods – Causal Forecasting (2)

- Simple linear regression analysis
  - ✓ Model

$$Y_j = \beta_0 + \beta_1 X_j + \varepsilon_j \qquad j = 1,2,\ldots,n$$

Estimate β0, β1 using n observed values

Y_j	jth observed value of Y
X_j	jth observed value of X
β_0, β_1	regression coefficients of population
ε_j	jth random error term

$$\varepsilon_j \sim IN(0, \sigma^2) \quad j = 1,2,\ldots,n$$

$$\dashrightarrow \quad Y_j \sim IN(\beta_0 + \beta_1 X_j, \sigma^2) \quad j = 1,2,\ldots,n$$

➢ Population regression line

$$E(Y_j) = \beta_0 + \beta_1 X_j$$

➢ Sample regression line

$$\hat{E}(Y_j) = \hat{\beta}_0 + \hat{\beta}_1 X_j$$

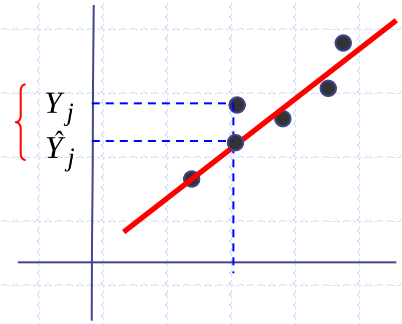
$$\longrightarrow \boxed{\hat{Y}_j = b_0 + b_1 X_j}$$

b_0, b_1 estimated values using the observed data

Residual

Difference between the observed value and the estimated value

$$e_j = Y_j - \hat{Y}_j = Y_j - b_0 - b_1 X_j$$



Forecasting

- **Methods**

Quantitative Methods – Causal Forecasting (3)

- Simple linear regression analysis
  - ✓ Estimating b_0 and b_1
    - ➢ Least square method

Obtain b_0, b_1 that minimize the sum of squares of residuals

$$Q = \sum_{j=1}^{n} (Y_j - \hat{Y}_j)^2 = \sum_{j=1}^{n} (Y_j - b_0 - b_1 X_j)^2$$

Error sum of squares

$$SS_E = \sum (Y_j - \hat{Y}_j)^2$$
$$= S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$$

$$\boxed{\begin{aligned} \frac{\partial Q}{\partial b_0} &= -2\sum_{j=1}^{n}(Y_j - b_0 - b_1 X_j) = 0 \\ \frac{\partial Q}{\partial b_1} &= -2\sum_{j=1}^{n}(Y_j - b_0 - b_1 X_j)X_j = 0 \end{aligned}}$$

normal equations

$$\sum Y_j = nb_0 + b_1 \sum X_j$$
$$\sum X_j Y_j = b_0 \sum X_j + b_1 \sum X_j^2$$

$$\boxed{\begin{aligned} b_1 &= \frac{\Sigma X_j Y_j - \Sigma X_j \Sigma Y_j / n}{\Sigma X_j^2 - (\Sigma X_j)^2 / n} = \frac{\Sigma (X_j - \overline{X})(Y_j - \overline{Y})}{\Sigma (X_j - \overline{X})^2} = \frac{S_{xy}}{S_{xx}} \\ b_0 &= \frac{1}{n}(\Sigma Y_j - b_1 \Sigma X_j) = \overline{Y} - b_1 \overline{X} \end{aligned}}$$

Forecasting

Methods

Quantitative Methods – Causal Forecasting (4)

- Simple linear regression analysis

✓ Coefficient of determination (R^2)

Value to measure how the estimated line explains the observed values

$$R^2 = 1 - \frac{\sum (Y_j - \hat{Y}_j)^2}{\sum (Y_j - \overline{Y})^2} = 1 - \frac{\sum e_j^2}{\sum (Y_j - \overline{Y})^2} = 1 - \frac{SS_E}{S_{yy}}$$

Error sum of squares

$$SS_E = \sum (Y_j - \hat{Y}_j)^2$$
$$= S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$$

Perfect explanation

$$R^2 = 1 \leftarrow \hat{Y}_j = Y_j \quad j = 1,2,\ldots,n$$

No explanation

$$R^2 = 0 \leftarrow \hat{Y}_j = \overline{Y} \quad j = 1,2,\ldots,n$$

Forecasting

Methods

Quantitative Methods – Causal Forecasting (5)

- Simple linear regression analysis

Example: Mr. Forest's Cookies

An emerging cookie store conjectured that the success of a store is strongly influenced by the number of people who live within five miles of it.

Data

Store	Population	Sales
1	50	200
2	25	50
3	14	210
4	76	240
5	88	400
6	35	200
7	85	410
8	110	500
9	95	610
10	21	120
11	30	190
12	44	180

Regression line

$$b_0 = \overline{Y} - b_1 \overline{X}$$

Sales = 27.38 + 4.43 $\times$ Population

$$b_1 = \frac{S_{xy}}{S_{xx}}$$

e.g., if population = 60, sales = 293.18

Coefficient of determination (R^2)

$R^2 = 0.7706$

$$R^2 = 1 - \frac{SS_E}{S_{yy}}$$

Forecasting

Methods

Quantitative Methods – Causal Forecasting (6)

- Other regression models

  ✓ Multiple (linear) regression model

$$Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \cdots + \beta_k X_{kj} + \varepsilon_j \qquad j = 1,2,\ldots,n$$

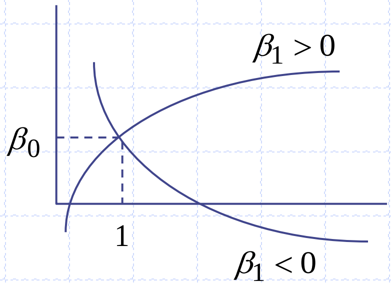
$$\varepsilon_j \sim IN(0, \sigma^2)$$

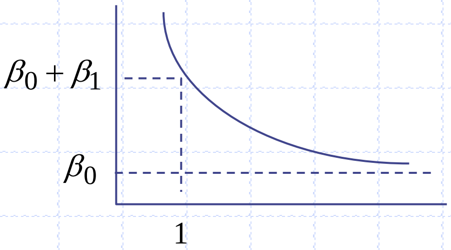
  ✓ Nonlinear regression models

e.g., transformation of X

$$X'_j = \ln X_j \qquad Y_j = \beta_0 + \beta_1 \ln X_j + \varepsilon_j \qquad Y_j = \beta_0 + \beta_1 X'_j + \varepsilon_j$$

$$Y_j = \beta_0 + \beta_1 \frac{1}{X_j} + \varepsilon_j \qquad Y_j = \beta_0 + \beta_1 X'_j + \varepsilon_j \qquad X'_j = \frac{1}{X_j}$$



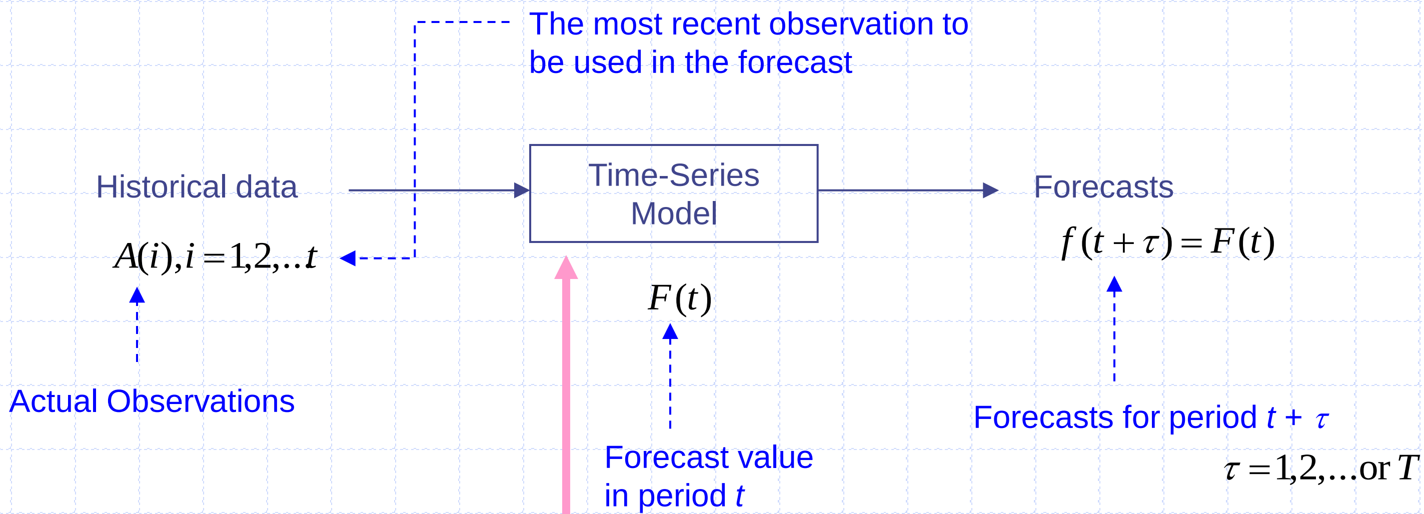


Forecasting

◈ Methods

Quantitative Methods – Time-Series Forecasting (1)

- Basic structure



✓ Methods

➢ Moving average (without and with trend)
➢ Exponential smoothing (without and with trend)

Winters method: exponential smoothing with seasonality

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (2)

- Last period demand method

Moving average

Compromise between last period demand and arithmetic average

$$F(t) = A(t)$$

$$\longrightarrow f(t+1) = F(t) = A(t)$$

Good in trend
Not good in seasonal and random effects

- Arithmetic average method

$$F(t) = \frac{1}{t}\sum_{i=1}^{t} A(i)$$

$$\longrightarrow f(t+1) = F(t) = \frac{1}{t}\sum_{i=1}^{t} A(i)$$

Good in smoothing random effects
Not good in trend and seasonal effect

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (3)

- Moving average methods
  - ✓ Simple moving average – no trend
    - Compute the forecast for the next period as the average of the last m observations (User chooses m, m = parameter)
    - Method

Model

$$A(t) = a + \boxed{e_t}$$

random variation

$$E(e_t) = 0$$

$$Var(e_t) = \sigma^2$$

$$F(t) = \frac{A(t) + A(t-1) + \ldots + A(t-m+1)}{m} = \frac{\sum_{i=t-m+1}^{t} A(i)}{m}$$

$$f(t+\tau) = F(t) \qquad \tau = 1,2,\ldots \text{ or } T$$

Compromise between last period demand and arithmetic average

$$F(t+1) = \frac{A(t+1) + A(t) + \ldots + A(t-m+2)}{m} = \frac{\sum_{i=t-m+2}^{t+1} A(i)}{m}$$

$$f(t+1+\tau) = F(t+1) \qquad \tau = 1,2,\ldots \text{ or } T$$

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (4)

- Moving average methods

✓ Simple moving average – no trend

Example (τ = 1)

Month t	Demand $A(t)$	Forecast m = 3	Forecast m = 5
1	10		
2	12		
3	12		
4	11	11.33	
5	15	11.67	
6	14	12.67	12.0
7	18	13.33	12.8
8	22	15.67	14.0

Characteristics

- Useful when there is only random effect (no trend)
- Selecting m
  - ✓ Large m → reduce the random effect (make the model more stable)
  - ✓ Small m → sensitive to demand changes

When there is trend

Underestimate parameters with an increasing trend

Overestimate parameters with a decreasing trend

Double moving average (with linear trend)

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (5)

- Moving average methods
  - ✓ Double moving average – with linear trend
    - ➢ Estimation of a and b

Model

linear trend

$$A(t) = a + \boxed{bt} + \boxed{e_t}$$

random variation

$$E(e_t) = 0$$

$$Var(e_t) = \sigma^2$$

$$M_t = \frac{1}{m}(A(t) + A(t-1) + \ldots + A(t-m+1))$$

$$\begin{aligned} E(M_t) &= \frac{1}{m}(E(A(t)) + E(A(t-1)) + \ldots + E(A(t-m+1))) \\ &= \frac{1}{m}((a+bt) + (a+b(t-1)) + \ldots + (a+b(t-m+1)) \\ &= a + bt - \frac{b}{2}(m-1) \end{aligned}$$

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (6)

- Moving average methods
  - ✓ Double moving average – with linear trend
    - ➢ Estimation of a and b

$$M_t^{[2]} = \frac{1}{m}(M_t + M_{t-1} + \ldots + M_{t-m+1})$$

$$\begin{aligned}
\rightarrow\quad E(M_t^{[2]}) &= \frac{1}{m}(E(M_t) + E(M_{t-1}) + \ldots + E(M_{t-m+1})) \\
&= \frac{1}{m}[(a + bt - \frac{b}{2}(m-1)) + (a + b(t-1) - \frac{b}{2}(m-1)) + \ldots + (a + b(t-m+1) - \frac{b}{2}(m-1))] \\
&= a + bt - \frac{b}{2}(m-1) - \frac{1}{m}(b + 2b + \ldots (m-1)b) \\
&= a + bt - b(m-1)
\end{aligned}$$

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (7)

- Moving average methods
  - ✓ Double moving average – with linear trend
    - Estimation of a and b

$$E(M_t) = a + bt - \frac{b}{2}(m-1) \qquad (1)$$

$$E(M_t^{[2]}) = a + bt - b(m-1) \qquad (2)$$

$$E(M_t) - E(M_t^{[2]}) = \frac{b}{2}\cdot(m-1)$$

$$b = \frac{2}{m-1}[E(M_t) - E(M_t^{[2]})] \quad \leftarrow [(2) – (1)] \times (2 / (m – 1))$$

$$2E(M_t) - E(M_t^{[2]}) = a + bt$$

$$a = (2E(M_t) - E(M_t^{[2]})) - bt \quad \leftarrow 2\times(1) – (2)$$

$$\hat{b} = \frac{2}{m-1}(M_t - M_t^{[2]})$$

$$\hat{a} = (2M_t - M_t^{[2]}) - \hat{b}\cdot t$$

Forecast in period $t + \tau$

$$f(t+\tau) = \hat{a} + \hat{b}(t+\tau) = (\hat{a} + \hat{b}t) + \hat{b}\tau$$

$$= (2M_t - M_t^{[2]}) + \frac{2}{m-1}(M_t - M_t^{[2]})\tau$$

$$\tau = 1,2,\ldots \text{or } T$$

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (8)

- Moving average methods

✓ Double moving average – with linear trend

$$f(t+\tau)=\hat{a}+\hat{b}(t+\tau)=(\hat{a}+\hat{b}t)+\hat{b}\tau$$

$$=(2M_t-M_t^{[2]})+\frac{2}{m-1}(M_t-M_t^{[2]})\tau$$

$$\tau=1 \longrightarrow \boxed{f(t+1)=(2M_t-M_t^{[2]})+\frac{2}{m-1}(M_t-M_t^{[2]})}$$

Example (m = 5)

Week	Demand	M_t	$M_t^{[2]}$
1	10		
2	12		
3	15		
4	14		
5	16	13.4	
6	19	15.2	
7	18	16.4	
8	21	17.6	
9	23	19.4	16.4
10	20	20.2	17.76
11			

One week ahead forecast (τ = 1)

$$f(10)=(2M_9-M_9^{[2]})+\frac{2}{5-1}(M_9-M_9^{[2]})$$

$$=(2\cdot 19.4-16.4)+0.5\cdot(19.4-16.4)$$

$$=23.9$$

$$f(11)=(2M_{10}-M_{10}^{[2]})+\frac{2}{5-1}(M_{10}-M_{10}^{[2]})$$

$$=(2\cdot 20.2-17.76)+0.5\cdot(20.2-17.76)$$

$$=23.86$$

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (9)

- Exponential smoothing methods
  - ✓ Exponential smoothing – no trend
    - Compute the forecast for the next period as a weighted average of the most recent observation and the previous smoothed estimate (User chooses the weight α, α = parameter)
    - Method

$$
\begin{aligned}
F(t) &= \alpha\cdot A(t) + (1-\alpha)\cdot F(t-1)\\
&= \alpha\cdot A(t) + (1-\alpha)\cdot[\alpha\cdot A(t-1) + (1-\alpha)\cdot F(t-2)]\\
&= \alpha\cdot A(t) + \alpha\cdot(1-\alpha)\cdot A(t-1) + (1-\alpha)^2\cdot F(t-2)\\
&= \ldots\\
&= \boxed{\alpha\cdot A(t) + \alpha\cdot(1-\alpha)\cdot A(t-1) + \alpha\cdot(1-\alpha)^2\cdot A(t-2) + \ldots + \alpha\cdot(1-\alpha)^{t-1}\cdot A(1) + (1-\alpha)^t\cdot F(0)}
\end{aligned}
$$

Large α $\rightarrow$ higher weight to recent demands (sensitive to demand changes)

Small α $\rightarrow$ high smoothing effect

smoothing constant

$0 \le \alpha \le 1$

Forecast error in period *t*

$$
\begin{aligned}
F(t) &= \boxed{\alpha\cdot} A(t) + (1-\alpha)\cdot F(t-1)\\
&= \boxed{F(t-1)} + \alpha\cdot\boxed{[A(t) - F(t-1)]}
\end{aligned}
$$

Forecasted value in period *t* – 1

$\longrightarrow \quad f(t+\tau) = F(t) \qquad \tau = 1,2,\ldots \text{ or } T$

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (10)

- Exponential smoothing methods
  - ✓ Exponential smoothing – no trend

Example

F(0) = A(1) = 10 and τ = 1

$\alpha = 0.6$

$$f(2) = F(1) = 0.6 \cdot A(1) + (1-0.6) \cdot F(0) = 0.6 \cdot 10 + 0.4 \cdot 10 = 10$$

$$f(3) = F(2) = 0.6 \cdot A(2) + (1-0.6) \cdot F(1) = 0.6 \cdot 12 + 0.4 \cdot 10 = 11.2$$

$\alpha = 0.2$

$$f(2) = F(1) = 0.2 \cdot A(1) + (1-0.2) \cdot F(0) = 0.2 \cdot 10 + 0.8 \cdot 10 = 10$$

$$f(3) = F(2) = 0.2 \cdot A(2) + (1-0.2) \cdot F(1) = 0.2 \cdot 12 + 0.8 \cdot 10 = 10.40$$

Month	Demand	Forecast	
t	$A(t)$	α = 0.2	α = 0.6
1	10	-	-
2	12	10.00	10.00
3	12	10.40	11.20
4	11	10.72	11.68
5	15	10.78	11.27
6	14	11.62	13.51
7	18	12.10	13.80
8	22	13.28	16.32

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (11)

- Exponential smoothing methods
  - ✓ Exponential smoothing – with linear trend
    - ➢ Update a smoothed estimate *F*(*t*) and a smoothed trend *T*(*t*) each time a new observation becomes available
    - ➢ Method

$$F(t) = \alpha \cdot A(t) + (1-\alpha)\cdot[F(t-1)+T(t-1)]$$

$$T(t) = \beta \cdot [F(t) - F(t-1)] + (1-\beta)\cdot T(t-1)$$

smoothing constants

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$\longrightarrow \quad f(t+\tau) = F(t) + \tau \cdot T(t)$$

$$\tau = 1,2,\ldots \text{ or } T$$

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (12)

- Exponential smoothing methods
  - ✓ Exponential smoothing – with linear trend

Example (α = 0.2, β = 0.2)

$$f(2) = F(1) + 1 \cdot T(1) = 10 + 1 \cdot 0 = 10$$

F(0) = A(1) = 10, T(1) = 0 and τ = 1

Month t	Demand $A(t)$	Smoothed Estimate $F(t)$	Smoothed Trend $T(t)$	Forecast $f(t)$
1	10	10.00	0	
2	12	**10.40**	**0.08**	10.00
3				**10.48**

$$\left.\begin{aligned} F(2) &= 0.2 \cdot A(2) + 0.8 \cdot (F(1) + T(1)) \\ &= 0.2 \cdot 12 + 0.8 \cdot (10 + 0) = 10.4 \\ T(2) &= 0.2 \cdot (F(2) - F(1)) + 0.8 \cdot T(1) \\ &= 0.2 \cdot (10.40 - 10.00) + 0.8 \cdot 0 \\ &= 0.08 \end{aligned}\right\}$$

$$f(3) = F(2) + 1 \cdot T(2) = 10.40 + 0.08 = 10.48$$

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (13)

- Exponential smoothing methods
  - ✓ Exponential smoothing – with linear trend

Example (α = 0.2, β = 0.2)

$$F(3) = 0.2 \cdot A(3) + 0.8 \cdot (F(2) + T(2))$$
$$= 0.2 \cdot 12 + 0.8 \cdot (10.4 + 0.08) = 10.78$$

$$T(3) = 0.2 \cdot (F(3) - F(2)) + 0.8 \cdot T(2)$$
$$= 0.2 \cdot (10.78 - 10.40) + 0.8 \cdot 0.08$$
$$= 0.14$$

F(0) = A(1) = 10, T(1) = 0 and τ = 1

Month t	Demand $A(t)$	Smoothed Estimate $F(t)$	Smoothed Trend $T(t)$	Forecast $f(t)$
1	10	10.00	0	
2	12	10.40	0.08	10.00
3	12	**10.78**	**0.14**	10.48
4				**10.92**

$$f(4) = F(3) + 1 \cdot T(3)$$
$$= 10.78 + 0.14 = 10.92$$

Forecasting

linear trend

Model

$$A(t) = (a + bt) \cdot c_t + e_t$$

random variation

$E(e_t) = 0$

$Var(e_t) = \sigma^2$

seasonality

Methods

Quantitative Methods – Time-Series Forecasting (14)

- Exponential smoothing methods
  - ✓ Winters method – with linear trend and seasonality
    - ➢ Procedure

Step 1. Estimate a multiplicative seasonality factor $(c(i))$

$$c(i) = \frac{A(i)}{(\sum_{t=1}^{N} A(t)/N)}$$

demand during period *i*

average demand during a given number of periods

Step 2. Compute the seasonality adjusted forecast

$$F(t) = \alpha \cdot \frac{A(t)}{c(t-N)} + (1-\alpha)\cdot[F(t-1)+T(t-1)]$$

normalizes all the observations relative to the average

$$T(t) = \beta \cdot [F(t) - F(t-1)] + (1-\beta)\cdot T(t-1)$$

places the smoothed estimate and trend in units of average (non-seasonal) demand

$$c(t) = \gamma \cdot \frac{A(t)}{F(t)} + (1-\gamma)\cdot c(t-N)$$

update the seasonality factor as a weight of this season's ratio of actual demand to smoothed estimate

smoothing constants

$0 \le \alpha \le 1$

$0 \le \beta \le 1$

$0 \le \gamma \le 1$

$$\longrightarrow f(t+\tau) = [F(t) + \tau \cdot T(t)] \cdot c(t+\tau-N)$$

multiplicative seasonality factor

Forecasting

- **Methods**

Quantitative Methods – Time-Series Forecasting (15)

- Exponential smoothing methods

✓ Winters method – with linear trend and seasonality

Example ($\alpha = \beta = \gamma = 0.1$) – Step 1

Year	Month t	Demand $A(t)$	Seasonal Factor $c(t)$
1997	1	4	**0.480**
	2	2	**0.240**
	3	5	**0.600**
	4	8	**0.960**
	5	11	**1.320**
	6	13	**1.560**
	7	18	**2.160**
	8	15	**1.800**
	9	9	**1.080**
	10	6	**0.720**
	11	5	**0.600**
	12	4	**0.480**

$$\frac{4}{(100/12)} = \frac{48}{100}$$

$$c(i) = \frac{A(i)}{(\sum_{t=1}^{N} A(t)/N)}$$

total demand = 100

Forecasting

Year	Month t	Demand $A(t)$	**Seasonal Factor $c(t)$**
1997	1	4	**0.480**

$$f(13) = [F(12) + 1 \cdot T(12)] \cdot c(12+1-12)$$
$$= [8.33 + 1 \cdot 0.00] \cdot 0.480 = 4.00$$

$$\boxed{f(t+\tau) = [F(t) + \tau \cdot T(t)] \cdot c(t+\tau-N)}$$

Methods

Quantitative Methods – Time-Series Forecasting (16)

- Exponential smoothing methods
  - ✓ Winters method – with linear trend and seasonality

Example ($\alpha = \beta = \gamma = 0.1$) – Step 2

$$F(12) = (4+2+5+...+5+4)/12 = 8.33$$

Year	Month t	Demand $A(t)$	Smoothed Estimate $F(t)$	Smoothed Trend $T(t)$	Seasonal Factor $c(t)$	Forecast $f(t)$
1997	12	4	8.33	0.00		
1998	13	5	**8.54**	**0.02**	**0.491**	**4.00**

$$F(13) = \alpha \cdot \frac{A(13)}{c(1)} + (1-\alpha) \cdot [F(12) + T(12)]$$
$$= 0.1 \cdot \frac{5}{0.480} + 0.9 \cdot (8.33 + 0.00) = 8.54$$

$$\boxed{F(t) = \alpha \cdot \frac{A(t)}{c(t-N)} + (1-\alpha) \cdot [F(t-1) + T(t-1)]}$$

$$c(13) = \gamma \cdot \frac{A(13)}{F(13)} + (1-\gamma) \cdot c(1)$$
$$= 0.1 \cdot \frac{5}{8.54} + 0.9 \cdot (0.480) = 0.491$$

$$\boxed{c(t) = \gamma \cdot \frac{A(t)}{F(t)} + (1-\gamma) \cdot c(t-N)}$$

$$T(13) = \beta \cdot [F(13) - F(12)] + (1-\beta) \cdot T(12)$$
$$= 0.1 \cdot [8.54 - 8.33] + 0.9 \cdot 0 = 0.02$$

$$\boxed{T(t) = \beta \cdot [F(t) - F(t-1)] + (1-\beta) \cdot T(t-1)}$$

Forecasting

Methods

Quantitative Methods – Time-Series Forecasting (17)

- Exponential smoothing methods
  - ✓ Winters method – with linear trend and seasonality

Example ($\alpha = \beta = \gamma = 0.1$) – Step 2

Year	Month t	Demand $A(t)$	Seasonal Factor $c(t)$
1997	1	4	**0.480**
	2	2	**0.240**

$$f(14) = [F(13) + 1 \cdot T(13)] \cdot c(13+1-12)$$
$$= [8.54 + 1 \cdot 0.02] \cdot 0.240 = 2.05$$

Year	Month t	Demand $A(t)$	Smoothed Estimate $F(t)$	Smoothed Trend $T(t)$	Seasonal Factor $c(t)$	Forecast $f(t)$
1997	12	4	8.33	0.00		
1998	13	5	8.54	0.02	0.491	4.00
	14	4	**9.37**	**0.10**	**0.259**	**2.05**

$$F(14) = \alpha \cdot \frac{A(14)}{c(2)} + (1-\alpha) \cdot [F(13) + T(13)]$$
$$= 0.1 \cdot \frac{4}{0.240} + 0.9 \cdot (8.54 + 0.02) = 9.37$$

$$c(14) = \gamma \cdot \frac{A(14)}{F(14)} + (1-\gamma) \cdot c(2)$$
$$= 0.1 \cdot \frac{4}{9.37} + 0.9 \cdot (0.240) = 0.259$$

$$T(14) = \beta \cdot [F(14) - F(13)] + (1-\beta) \cdot T(13)$$
$$= 0.1 \cdot [9.37 - 8.54] + 0.9 \cdot 0.02 = 0.10$$

Forecasting

Forecast Errors

Measures of forecast error (over a long period of time) (1) ← useful when selecting parameters for time-series methods (trial and error)

- Measuring bias
  - ✓ Cumulative sum of forecast error (CFE)

$$CFE = \sum_{t=1}^{n} (A(t) - f(t))$$

forecast for period t

actual demand for period t

  - ✓ Bias (BIAS)

$$BIAS = \frac{\sum_{t=1}^{n} (A(t) - f(t))}{n}$$

- Measuring random errors
  - ✓ Mean square deviation (MSD)

$$MSD = \frac{\sum_{t=1}^{n} (A(t) - f(t))^2}{n}$$

  - ✓ Mean absolute deviation (MAD)

$$MAD = \frac{\sum_{t=1}^{n} |A(t) - f(t)|}{n}$$

  - ✓ Mean absolute percentage error

$$MAPE = \frac{(\sum_{t=1}^{n} |A(t) - f(t)| / A(t))100}{n}$$

Forecasting

◈ Forecast Errors

Measures of forecast error (over a long period of time) (2)

- Example

$$MAPE = \frac{(\sum_{t=1}^{8} |A(t) - f(t)| / A(t))100}{8} = 10.2\%$$

Month t	Demand, A(t)	Forecast, f(t)	Error A(t)- f(t)	Error Squared $(A(t)- f(t))^2$	Absolute Error |A(t)-f(t)|	Absolute Percent Error (|A(t)-f(t)|/A(t))(00)
1	200	225	-25	625	25	12.5%
2	240	220	20	400	20	8.3
3	300	285	15	225	15	5.0
4	270	290	–20	400	20	7.4
5	230	250	–20	400	20	8.7
6	260	240	20	400	20	7.7
7	210	250	–40	1600	40	19.0
8	275	240	35	1225	35	12.7
		Total	–15	5275	195	81.3%

$$CFE = \sum_{t=1}^{8} (A(t) - f(t)) = -15$$

$$BIAS = \frac{\sum_{t=1}^{8} (A(t) - f(t))}{8} = -1.875$$

$$MSD = \frac{\sum_{t=1}^{8} (A(t) - f(t))^2}{8} = 659.4$$

$$MAD = \frac{\sum_{t=1}^{8} |A(t) - f(t)|}{8} = 24.4$$

Forecasting

- **Forecast Errors**

Tracking Signals

A measure that indicates whether a method of forecasting is accurately predicting actual changes in demand

$$TS(t) = \frac{CFE}{MAD} = \frac{\sum_{i=1}^{t}(A(i) - f(i))}{MAD}$$

cumulative sum of forecast error

$$MAD = \frac{\sum_{i=1}^{t}\left| f(i) - A(i) \right|}{t}$$

Consider the change of
1) parameters
2) current forecasting method