# Operations Management I

## Aggregate Planning (총괄계획)

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# Aggregate Planning

Introduction
Inputs and Objectives, Alternatives, and Planning Strategies
Planning Process
Approaches

- Definition
- Why

- Spreadsheets
- Linear programming models (Transportation models)

What is planning?

Krajewski and Ritzman, 2005, **Operations Management**, Prentice Hall. **(Chapter 14)**
Hopp and Spearman, 2008, **Factory Physics**, McGraw Hill. **(Section 16)**

# Aggregate Planning

## Introduction

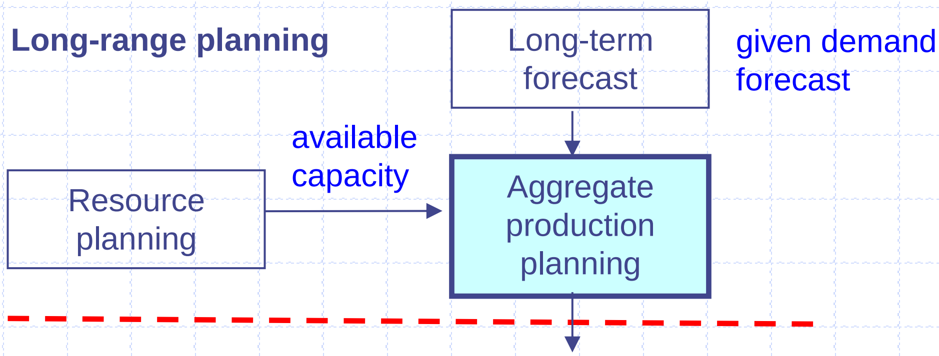
Aggregate Planning – Overview (1)

- Definition

  Process of planning the quantity and timing the output
  over the long or intermediate range (over one to three years)

  ← Respond to irregular market demands
  by effectively utilizing the organization's resources
  (adjusting production rate, employment, inventory, and other controllable variables)

✓ Staffing plan (for service firms)
✓ Production plan (for manufacturing firms)



# Aggregate Planning

## Introduction

Aggregate Planning – Overview (2)

- Aggregation

  Preparing production and staffing plans by aggregating or grouping similar services or products

  ←------ ✓ Accuracy of forecasting
  ✓ Simplification (reduce complexity)

✓ Product family

A group of customers, services, or products that have similar demand requirements and common processes, labor, and materials requirements

e.g., Manufacturers of bicycles that produce 12 different models
- Two groups: mountain bikes and road bikes

✓ Aggregated units (총괄단위)

- Number of customers
- Dollars
- Standard hours
- Gallons, tons
- Production quantity (without regard to product types)

# Aggregate Planning

## Introduction

Aggregate Planning – Overview (3)

- Why
  - ✓ Staffing (인력)

    Recruiting (and training) and eliminating workers
    (long-term production plan to decide how many and what type of workers to add and when to bring them on-line in order to meet production needs)

    Workforce planning

    close relation

  - ✓ Procurement (자재수급)
    - ➢ Contracts with suppliers
    - ➢ Long procurement lead time, etc.
  - ✓ Subcontracting (외주)

    Contracts with subcontractors
    (entire components, specific operations, etc.)

    Aggregate planning

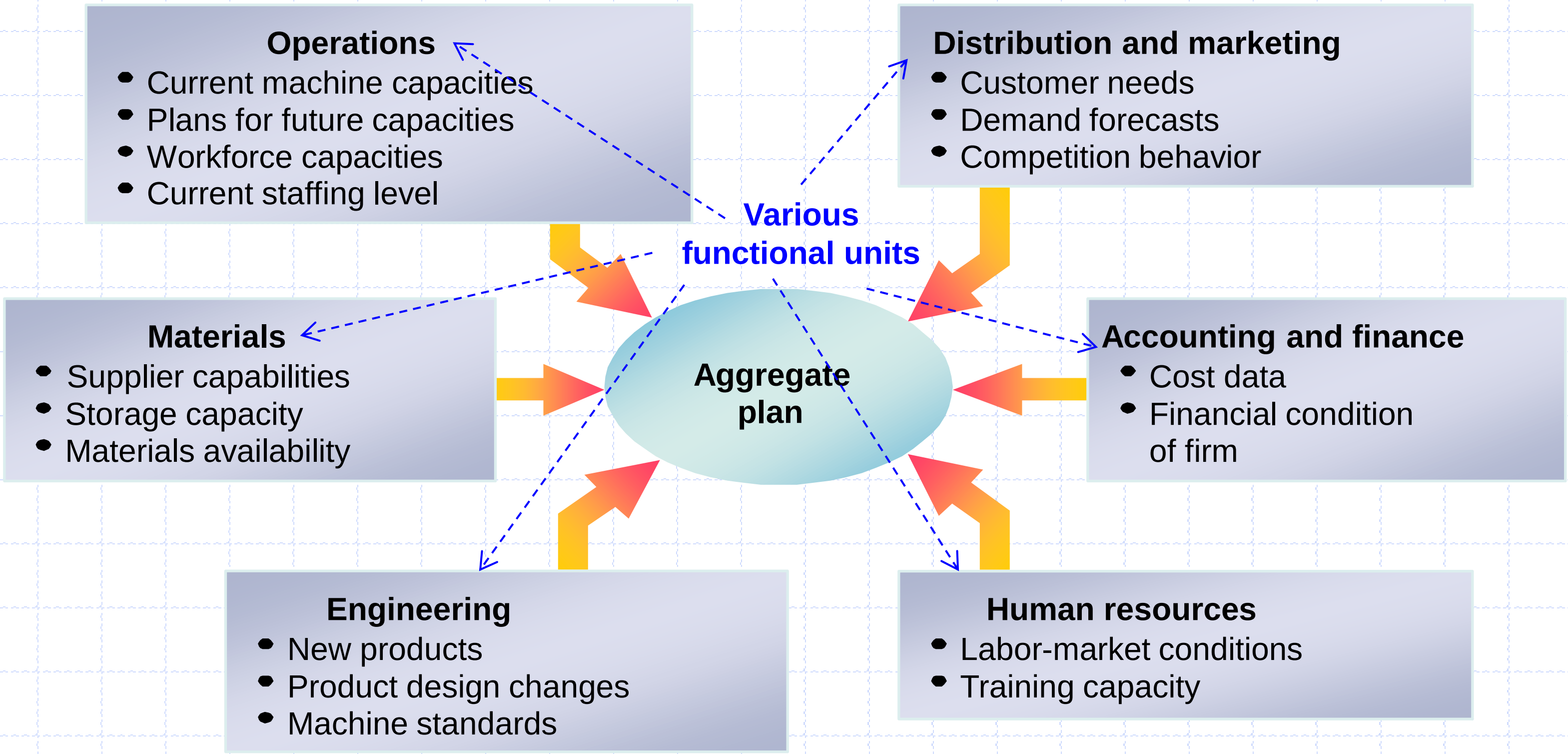
  - ✓ Marketing (마케팅)

    Marketing personnel's decisions on which products to promote
    (on the basis of both a demand forecast and knowledge of which products have tight capacity and which do not)

# Aggregate Planning

◆ **Inputs, Objectives, Alternatives, and Planning Strategies**

Inputs



# Aggregate Planning

## Inputs, Objectives, Alternatives, and Planning Strategies

Typical Objectives

- Minimize costs/Maximize profits
- Maximize customer service

  Improving delivery time and on-time delivery may require additional workforce, machine capacity, or inventory resources
- Minimize inventory investment
- Minimize changes in production rates

  Frequent changes in production rates can cause difficulties in coordinating the supply of materials and require production line rebalancing
- Minimize changes in workforce levels
- Maximize utilization of plant and equipment

Balancing the (conflicting) objectives to arrive at an acceptable aggregate plan involves consideration of various alternatives.

# Aggregate Planning

- **Inputs, Objectives, Alternatives, and Planning Strategies**

Alternatives

- Reactive alternatives

  Actions that can be taken to cope with demand requirements

  - ✓ Workforce adjustment: hiring or laying-off employees
  - ✓ Anticipation inventory: Inventory used to absorb uneven rates of demand or supply
  - ✓ Workforce utilization: overtime or undertime
  - ✓ Vacation schedules
  - ✓ Subcontractors
  - ✓ Backlog, backorders and stockout

- Aggressive alternatives

  Actions that attempt to modify demand and consequently, resource requirements (typically by marketing managers)

  - ✓ Complementary products or services
  - ✓ Creative pricing

    e.g., two for the price of one (1+1)

# Aggregate Planning

- **Inputs, Objectives, Alternatives, and Planning Strategies**

  Planning Strategies (1)

  - Chase strategy

    A strategy that matches demand during the planning horizon by varying either workforce level or output rate

    - ✓ Varying workforce level to match demand (capacity strategy)

      Hiring or laying-offs to keep the workforce's regular time capacity equal to demand

      - ➢ No inventory investment, no over time or under time, etc.
      - ➢ Expense of continually adjusting workforce levels, loss of productivity and quality, etc.

    - ✓ Varying output rate to match demand (utilization strategy)

      Additional reactive alternatives beyond changing the workforce level (undertime, overtime, vacation, subcontracting, etc.)

# Aggregate Planning

## Inputs, Objectives, Alternatives, and Planning Strategies

Planning Strategies (2)

- Level strategy

  A strategy that maintains a constant workforce level or constant output rate during the planning horizon

  ✓ Constant workforce level

  Not hiring or laying-offs workers while

  - Building up anticipation inventories or backorders
  - Using undertime in slack periods or overtime in peak periods,
  - Using subcontracting, etc.

  ✓ Constant output rate

- Mixed strategy

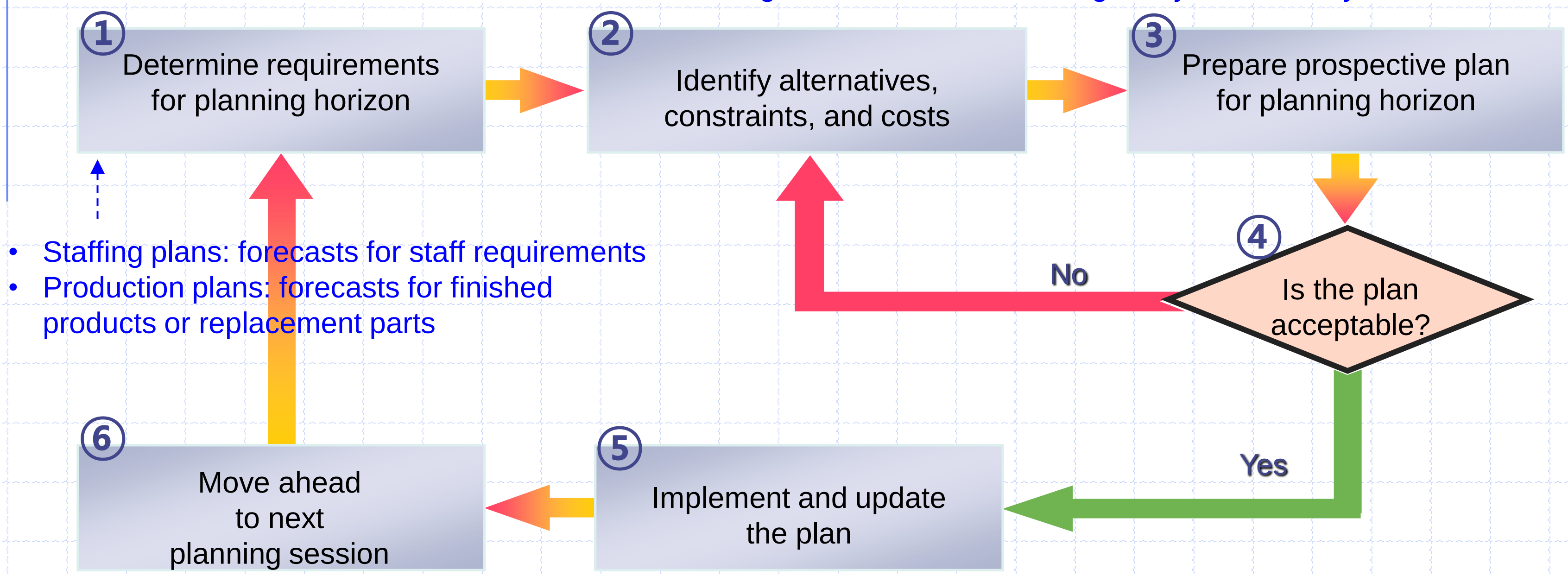
  A strategy that considers and implements a fuller range of reactive alternatives and goes beyond a pure chase or level strategy

# Aggregate Planning

## Planning Process

Overview

- Alternatives: chase, level or mixed
- Constraints: physical limitations or managerial policies
- Costs: regular time, overtime, hiring or layoff, inventory , backorders, etc.



# Aggregate Planning

- **Approaches – Spreadsheets**

Example – Staffing plans in a distribution center (1)

- Data

✓ Demands: workforce requirements

| Time period | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Requirements | 6 | 12 | 18 | 15 | 13 | 14 | 78 |

➢ Time period = 2 months
➢ Each part-time employee has the maximum regular time of 20 hours per week
➢ Current employment = 10 part-time clerks

✓ Constraints

➢ No more than 10 new hires in any period
➢ No backorders are permitted.
➢ Overtime can not exceed 20% of regular-time capacity. (1.2(20) = 24 hours per week)

✓ Costs

➢ Regular-time wage = 2,000 $/period at 20 hours per week
➢ Overtime wages = 150% of regular-time
➢ Hiring = 1,000 $/person
➢ Layoffs = 500 $ /person

| Time period | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Requirements | 6 | 12 | 18 | 15 | 13 | 14 | 78 |

- Staff size (that minimizes the amount of overtime)
  - ✓ Peak requirement = 18
  
    $w \times 1.2 = 18 \rightarrow w = 18/1.2 = 15$ employees

# Aggregate Planning

## Approaches – Spreadsheets

Example – Staffing plans in a distribution center (2)

- Solution
  - ✓ Level strategy with overtime and undertime

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Requirement | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Workforce level | 15 | 15 | 15 | 15 | 15 | 15 | 90 |
| Undertime | 9 | 3 | 0 | 0 | 2 | 1 | 15 |
| Overtime | 0 | 0 | 3 | 0 | 0 | 0 | 3 |
| Productive time | 6 | 12 | 15 | 15 | 13 | 14 | 75 |
| Hires | 5 | 0 | 0 | 0 | 0 | 0 | 5 |
| Layoffs | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **Costs** | 1 | 2 | 3 | 4 | 5 | 6 | Totals |
| Productive time | $12,000 | 24,000 | 30,000 | 30,000 | 26,000 | 28,000 | $150,000 |
| Undertime | $0 | 0 | 0 | 0 | 0 | 0 | $0 |
| Overtime | $0 | 0 | 9,000 | 0 | 0 | 0 | $9,000 |
| Hires | $5,000 | 0 | 0 | 0 | 0 | 0 | $5,000 |
| Layoffs | $0 | 0 | 0 | 0 | 0 | 0 | $0 |
| Total cost | $17,000 | 24,000 | 39,000 | 30,000 | 26,000 | 28,000 | $164,000 |

# Aggregate Planning

- **Approaches – Spreadsheets**

Example – Staffing plans in a distribution center (3)

- Solution
  - ✓ Chase strategy with hiring and layoffs

| | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Requirement | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Workforce level | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Undertime | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Overtime | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Productive time | 6 | 12 | 18 | 15 | 13 | 14 | 78 |
| Hires | 0 | 6 | 6 | 0 | 0 | 1 | 13 |
| Layoffs | 4 | 0 | 0 | 3 | 2 | 0 | 9 |
| **Costs** | 1 | 2 | 3 | 4 | 5 | 6 | Totals |
| Productive time | $12,000 | 24,000 | 36,000 | 30,000 | 26,000 | 28,000 | $156,000 |
| Undertime | $0 | 0 | 0 | 0 | 0 | 0 | $0 |
| Overtime | $0 | 0 | 0 | 0 | 0 | 0 | $0 |
| Hires | $0 | 6,000 | 6,000 | 0 | 0 | 1,000 | $13,000 |
| Layoffs | $2,000 | 0 | 0 | 1,500 | 1,000 | 0 | $4,500 |
| Total cost | $14,000 | 30,000 | 42,000 | 31,500 | 27,000 | 29,000 | $173,500 |

# Aggregate Planning

◈ **Approaches – Linear Programming Models**

Overview

- Basic aggregate planning

  Single product type and single resource

- Extensions

  ✓ Product mix planning

  Multiple product types and multiple resources (Basic model and extensions)

  ✓ Combined aggregate and workforce planning

  Aggregate planning + workforce planning

- Linear programming models
- Transportation models

# Aggregate Planning

◈ **Approaches – Linear Programming Models**

Basic Aggregate Planning (1)

- Situation

  ✓ Single product type (in aggregate unit)

  ✓ Single resource
  (consider the entire plant as a one resource)

  ←----- Given capacity in each period (from resource planning)

  ✓ Dynamic demand in multi-period ←---------------------- Demands represent customer orders
  (planning horizon with discrete planning periods) that are due at the end of the period

  ←----- ✓ Given demand forecast (from long-term forecasting)
  ✓ Backlogging not allowed.

  ✓ Deterministic
  (neglect randomness and yield loss)

  ⇒ Problem of determining production quantity, selling quantity, and inventory level in each period for the objective of maximizing profit.

# Aggregate Planning

## Approaches – Linear Programming Models

Basic Aggregate Planning (2)

- Overall problem description
  - ✓ Decision variables
    - Production quantity in each period
    - Selling quantity in each period
    - Inventory level in each period
  - ✓ Objective
    - Maximizing profit (= revenue – inventory holding cost)
  - ✓ Constraints
    - Limitation of sales amount
    - Capacity constraints
    - Inventory balance constraints, etc.

$X_t$ quantity produced during period $t$ (units)
(available to satisfy demand at the end of period $t$)

$S_t$ quantity sold during period $t$ (units)
(units produced in t are available for sale in t and thereafter)

$I_t$ inventory at the end of period $t$ (units)

$r$ revenue per unit of product sold ($/unit)

$h$ cost to hold one unit of inventory f
or one period ($/unit·period)

$d_t$ demand in period $t$ (units)
(aggregated unit)

$c_t$ capacity in period $t$ (units/period)

Capacity: units produced in each period

# Aggregate Planning

## Approaches – Linear Programming Models

Basic Aggregate Planning (3)

- Linear programming model

Maximize $\sum_{t=1}^{T}(rS_t - hI_t)$ ◄----- revenue – inventory carrying cost

subject to

$S_t \le d_t$ for all t = 1, 2, … T ◄----- limits sales to demand

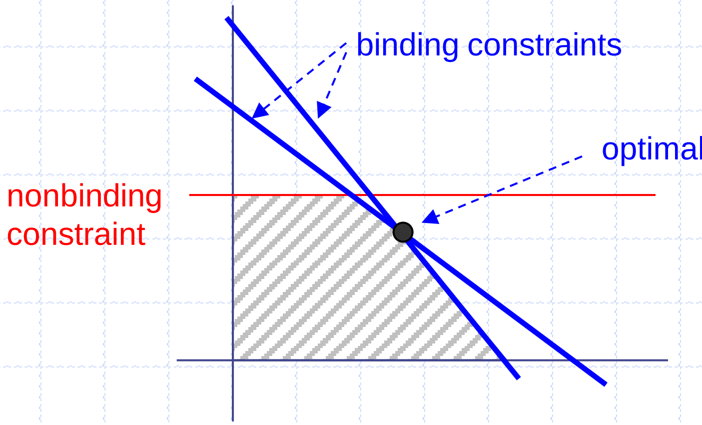
$X_t \le c_t$ for all t = 1, 2, … T ◄----- capacity constraints

$I_t = I_{t-1} + X_t - S_t$ for all t = 1, 2, … T ◄----- Inventory balance constraint

$X_t, S_t, I_t \ge 0$ for all t = 1, 2, … T ◄----- nonnegativity constraints ($I_t \ge 0$: no backlogging)

- Demand < capacity (in each period) → trivial solution (no inventory)
- Demand > capacity (for some periods) → nontrivial solution (with inventory)

# Aggregate Planning




## Approaches – Linear Programming Models

Basic Aggregate Planning (4)

- General behavior
  - ✓ Changing the right hand side of nonbinding constraints by a small amount does not affect the optimal solution.
  - ✓ Increasing (decreasing) the right hand side of a binding constraint will increase (decrease) the objective by an amount equal to the shadow price times the size of increase (decrease), provided that the increase (decrease) is smaller than the allowable increase (decrease).
  - ✓ Changing the right hand sides beyond the allowable increase or decrease range have an indeterminate effect and must be evaluated by resolving the modified model.

# Aggregate Planning

## Approaches – Linear Programming Models

Product Mix Planning (1)

- Overview

  An extension of the basic aggregate planning to the one with multiple product types and multiple resources (others same as the basic aggregate planning)

  ✓ Product mix

  e.g., production quantities of products A, B, and C = (20, 30, 10)

  Floating bottlenecks

  The bottleneck (during a production period) depends on the mix of products during the period.

  - If the product mix is fixed, identify the bottleneck(s)
  - If the product mix is not fixed, adjust the mix in accordance with available capacity

# Aggregate Planning

## Approaches – Linear Programming Models

Product Mix Planning (2)

- Basic model – Overall problem description

✓ Decision variables

- Production quantity of each product in each period
- Selling quantity of each product in each period
- Inventory level of each product in each period

| | |
|---|---|
| $X_{it}$ | amount of product *i* produced in period *t* (units) |
| $S_{it}$ | amount of product *i* sold in period *t* (units) |
| $I_{it}$ | amount of inventory of product *i* at the end of period *t* (units) |

(i = 1, 2,…m, t = 1, 2,.., T)

✓ Objective

- Maximizing profit
  (= revenue – inventory holding cost)

| | |
|---|---|
| $r_i$ | revenue from one unit of product *i* ($/unit) |
| $h_i$ | inventory carrying cost of one unit of product *i* for one period ($/unit·period) |

✓ Constraints

- Limitation of sales amount
- Capacity constraints
- Inventory balance, etc.

| | |
|---|---|
| $\overline{d_{it}}$ | maximum demand of product *i* in period *t* (units) |
| $\underline{d_{it}}$ | minimum sales allowed of product *i* in period *t* (units) |
| $a_{ij}$ | time required on workstation *j* to produce one unit of product *i* (hr) |
| $c_{jt}$ | capacity of workstation *j* in period *t* (hr/period) (units consistent with those used to define $a_{ij}$) |

# Aggregate Planning

- Information from the LP model
  - ✓ Product mix ($X_{it}$)
  - ✓ Demand feasibility
    Determine whether a set of demand is capacity-feasible
  - ✓ Bottleneck locations
    Determine which workstations limit capacity in which periods
    (Bottlenecks require close management attention.)

$$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt}$$

$$\max_{j} \{\sum_{i=1}^{m} a_{ij} \cdot X_{it}\}$$

## Approaches – Linear Programming Models

Product Mix Planning (3)

- Basic model – Linear programming model

Maximize $\sum_{t=1}^{T}\sum_{i=1}^{m}(r_i S_{it} - h_i I_{it})$ ← revenue – inventory carrying cost

subject to

$\underline{d_{it}} \le S_{it} \le \overline{d_{it}}$ for all i, t ← lower and upper bounds on sales

$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt}$ for all j, t ← capacity constraint (for each workstation in each period)

$I_{it} = I_{i,t-1} + X_{it} - S_{it}$ for all i, t

$X_{it}, S_{it}, I_{it} \ge 0$ for all i, t ← non-negativity constraints ($I_{it} \ge 0$: no backlogging)

Inventory balance constraints
(multiproduct types version)

# Aggregate Planning

## Approaches – Linear Programming Models

Product Mix Planning (4)

- Example (single-period model)
  - ✓ Two product types, a single period, four workstations
  - ✓ Capacity of workstations = 2,400 minutes (5 days/week, 8 hours/day)
  - ✓ Other data

| Product | 1 | 2 |
|---|---|---|
| Selling price | \$90 | \$100 |
| Raw material cost | \$45 | \$40 |
| Maximum weekly sales | 100 | 50 |
| Minutes per unit on workstation A | 15 | 10 |
| Minutes per unit on workstation B | 15 | 35 |
| Minutes per unit on workstation C | 15 | 5 |
| Minutes per unit on workstation D | 25 | 14 |

$$r_1 = 90 - 45 = 45$$
$$r_2 = 100 - 40 = 60$$

Maximize $45X_1 + 60X_2$

subject to

maximum weekly sales
$$\begin{cases} X_1 \le 100 \\ X_2 \le 50 \end{cases}$$

capacity constraint of each workstation
$$\begin{cases} 15X_1 + 10X_2 \le 2400 \\ 15X_1 + 35X_2 \le 2400 \\ 15X_1 + 5X_2 \le 2400 \\ 25X_1 + 14X_2 \le 2400 \end{cases}$$

$$X_1, X_2 \ge 0$$

Optimal solution

$$X_1^* = 75.79$$
$$X_2^* = 36.09$$

Rounding-down

$$X_1^* = 75$$
$$X_2^* = 36$$

Objective value = \$535

Mix of two products

# Aggregate Planning

◈ **Approaches – Linear Programming Models**

Product Mix Planning (5)

- Extensions

✓ Overview

Constraints of basic product mix planning

- Limitation of sales amount ← $\underline{d_{it}} \le S_{it} \le \overline{d_{it}}$
- Capacity constraints
- Inventory balance, etc.

$$\sum_{i=1}^{m} a_{ij} X_{it} \le c_{jt}$$

More constraints added to the basic model

- Other resource constraints
- Utilization matching
- Backorders
- Overtime
- Yield loss, etc.

$\}$ Extensions of the basic model

# Aggregate Planning

## Approaches – Linear Programming Models

Product Mix Planning (6)

- Extensions
  - ✓ Other resource constraints

    Resource constraints for others such as people, raw materials, transportation devices, etc.

  - ➢ General model

units of resource *j* required per unit of product *i*

number of units of resource *j* available in period *t*

$$\sum_{i=1}^{m} b_{ij} X_{it} \le k_{jt} \quad \text{for all } j, t$$

Example 1

An inspector mush check products 1, 2 and 3, which require 1, 2, and 1.5 hours, respectively. The inspector is available a total of 160 hours per month.

$$X_{1t} + 2X_{2t} + 1.5X_{3t} \le 160$$

If this constraint is binding in the optimal solution, the inspector is bottleneck. (Something should be done to remove this bottleneck.)

Example 2

A firm makes four different models of circuit board, all of which require one unit of a particular component.

$$X_{1t} + X_{2t} + X_{3t} + X_{4t} \le \boxed{k_t}$$

total number of components available in period *t*

# Aggregate Planning

- **Approaches – Linear Programming Models**

Product Mix Planning (7)

- Extensions
  - ✓ Utilization matching
    - ➢ Any source of randomness (machine failures, setups, errors in the scheduling process, etc.) can diminish utilization.
    - ➢ Idle time on machines

➡ Capacity adjustment is needed.

- ➢ Model

$$\sum_{i=1}^{m} a_{ij} X_{it} \leq \boxed{q}\, c_{jt} \qquad \text{for all } j, t$$

A constant representing setups, machine failure, worker breaks, etc. ($0 \leq q \leq 1$)

Estimation from historical data or simulation

# Aggregate Planning

$I_{it}^{+}$ (on-hand) Inventory of product *i* carried from period *t* to *t* + 1

$I_{it}^{-}$ number of backorders of product *i* carried from period *t* to *t* + 1

$\pi_i$ penalty to carry on unit of product *i* for one period of time (backorder cost)

## Approaches – Linear Programming Models

Product Mix Planning (8)

- Extensions
  - ✓ Backorders
    - ➢ Modeling

LP model for unrestricted variables

$$I_{it} = I_{it}^{+} - I_{it}^{-}$$

Basic model: no backorder, $I_{it} \geq 0$ (Demands should be met from inventory or lost.)

$$I_{it}^{+} \cdot I_{it}^{-} = 0$$

Maximize $\sum_{t=1}^{T} \sum_{i=1}^{m} (r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-})$ profit – inventory carrying cost – backorder cost

subject to

$\underline{d_{it}} \leq S_{it} \leq \overline{d_{it}}$ for all i, t — lower and upper bounds on sales

$\sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt}$ for all j, t — capacity constraints (for each workstation in each period)

Inventory balance constraints (multiproduct types)

$(I_{it}^{+} - I_{it}^{-}) = (I_{i,t-1}^{+} - I_{i,t-1}^{-}) + X_{it} - S_{it}$ for all i, t

$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-} \geq 0$ for all i, t

$$I_{it} = I_{i,t-1} + X_{it} - S_{it}$$

$$I_{it} = I_{it}^{+} - I_{it}^{-}$$

# Aggregate Planning

◈ **Approaches – Linear Programming Models**

Product Mix Planning (9)

- Extensions
  - ✓ Overtime
    - ➢ Increase capacity via the use of overtime
    - ➢ Model

Fixed capacity in the basic model and utilization matching

cost of one hour of overtime at workstation *j*

Maximize $\sum_{t=1}^{T}\sum_{i=1}^{m}(r_i S_{it} - h_i I_{it}^{+} - \pi_i I_{it}^{-}) - \sum_{t=1}^{T}\sum_{j=1}^{n} l'_j O_{jt}$

overtime at workstation *j* in period *t* (hours)

profit – inventory carrying cost
– backorder cost
– overtime cost

subject to

$\underline{d_{it}} \leq S_{it} \leq \overline{d_{it}}$ for all i, t

lower and upper bounds on sales

$\sum_{i=1}^{m} a_{ij} X_{it} \leq c_{jt} + O_{jt}$ for all j, t

capacity constraints with overtimes (for each workstation in each period)

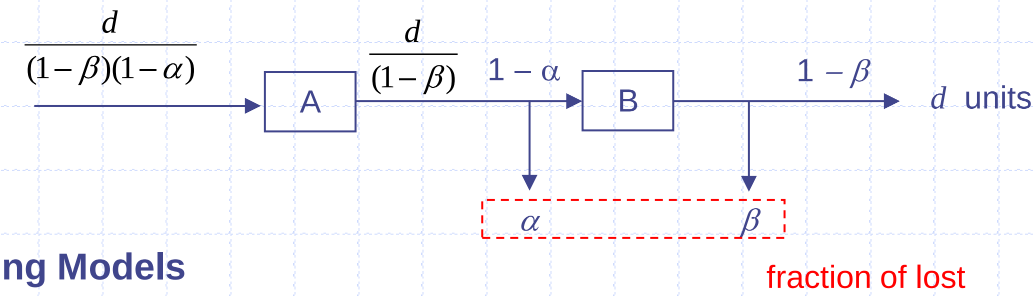
Inventory balance constraints (multiproduct types)

$(I_{it}^{+} - I_{it}^{-}) = (I_{i,t-1}^{+} - I_{i,t-1}^{-}) + X_{it} - S_{it}$ for all i, t

$X_{it}, S_{it}, I_{it}^{+}, I_{it}^{-} \geq 0$ for all i, t

# Aggregate Planning

Example




## Approaches – Linear Programming Models

Product Mix Planning (10)

- Extensions
  - ✓ Yield loss

    In systems where product is scrapped at various points in the line due to quality problems, we must release extra material into the system to compensate for these losses.

    - Model

Maximize $\sum_{t=1}^{T}\sum_{i=1}^{m}(r_i S_{it} - h_i I_{it})$ ← profit – inventory carrying cost

subject to

$\underline{d_{it}} \le S_{it} \le \overline{d_{it}}$ for all i, t ← lower and upper bounds on sales

$\sum_{i=1}^{m}\frac{a_{ij}\cdot X_{it}}{y_{ij}} \le c_{jt}$ for all j, t ← capacity constraints adjusted with yield loss

$y_{ij}$: cumulative yield from station *j* onward (including station *j*) for product *i*

Inventory balance constraints → $I_{it} = I_{i,t-1} + X_{it} - S_{it}$ for all i, t

$X_{it}, S_{it}, I_{it} \ge 0$ for all i, t

# Aggregate Planning

- **Approaches – Linear Programming Models**

Combined Aggregate and Workforce Planning (1)

- Overview

  Planning in systems where the capacity is subject to change
  (changing workforce size, overtime, subcontracting, inventory, etc.)

  ✓ Basic questions

  - How and when to resize the labor pool (workforce)
  - Whether to use overtime instead of workforce additions (overtime)
  - Whether to subcontract (subcontracting)
  - Whether to have inventory or backlog (inventory/backlogging)

  Basic strategies (for meeting demand)

  - Changes of man power: Hiring or firing
  - Changes of production rate: Overtime or undertime (idle time)
  - Inventory or backlog
  - Subcontracting

# Aggregate Planning

## Approaches – Linear Programming Models

Combined Aggregate and Workforce Planning (2)

- Situation
  - ✓ Single product type
  - ✓ No backlogging

Systems where product routings and processing times are either almost identical

- Decision variables
  - ✓ Amount of overtime in each period
    - $O_t$ amount produced during overtime in period *t* (units)
  - ✓ Amount of subcontracting in each period
    - $S_t$ amount of subcontracting in period t (units)
  - ✓ Workforce level (hiring and firing) in each period
    - $W_t$ workforce level in period *t* (units)
    - $H_t$ increase in workforce (hires) from period *t* – 1 to *t* (units)
    - $F_t$ decrease in workforce (fires) from period *t* – 1 to *t* (units)
  - ✓ Amount of inventory in each period
    - $I_t$ amount of inventory at the end of period *t* (units)

Amount of production in each period

$$X_t = W_t + O_t + S_t$$

total amount of output (regular, overtime, subcontract) in period *t* (units)

# Aggregate Planning

◈ **Approaches – Linear Programming Models**

Combined Aggregate and Workforce Planning (3)

- Objective

  Minimizing relevant costs

  $l$ cost of regular time ($/ unit) production cost (regular time/overtime)
  $l'$ cost of over time ($/unit)

  $e$ cost of increase workforce ($/unit) costs to change man power
  $e'$ cost of decrease workforce ($/unit)

  $h$ inventory carrying cost of one unit of product for one period ($/unit·period)

  $g$ subcontracting cost ($/unit)

- Constraints
  - Demand requirement
  - Workforce balance constraints
  - Inventory balance constraints
  - Limitations on overtime, hiring/firing, subcontracting, etc.

  b = allowable overtime (a proportion to the regular time)

# Aggregate Planning

- **Approaches – Linear Programming Models**

Combined Aggregate and Workforce Planning (3)

- Linear programming model

Minimize $\sum_{t=1}^{T} (\underbrace{l \cdot W_t + l' \cdot O_t}_{\text{costs for regular and overtimes}} + g \cdot S_t + \underbrace{e \cdot H_t + e' \cdot F_t}_{\text{hiring and firing costs}} + h \cdot I_t)$

subject to

$W_t = W_{t-1} + H_t - F_t$ for all t ← workforce balance constraints

$W_T$ = required workforce level at the end of period $t$

$0 \le H_t \le$ maximum allowable hiring level for all t

$0 \le F_t \le$ maximum allowable firing level for all t

$X_t = W_t + O_t + S_t$ for all t

$0 \le O_t \le b \cdot W_t$ for all t

$0 \le S_t \le$ maximum allowable subcontracting quantity for all $t$

$I_t = I_{t-1} + X_t - d_t$ for all t ← Inventory balance constraints

Nonnegative constraints

# Aggregate Planning

## Approaches – Linear Programming Models

Transportation Models (1)

- Situation
  - ✓ Single product type
  - ✓ Constant workforce level
    - Production sources: regular, overtime, and subcontracting
- Model
  - ✓ Decision variables
    - ➢ Amount of regular time production in each period
    - ➢ Amount of overtime production in each period
    - ➢ Amount of subcontracting in each period
  - ✓ Objective
    - ➢ Minimizing relevant costs
      - ✓ Production cost for each source
      - ✓ Inventory/backorder costs
      - ✓ Subcontracting costs
  - ✓ Constraints
    - ➢ Available capacity for each production source

Transportation model

# Aggregate Planning

## Approaches – Transportation Models

Transportation Models (2)

- Transportation tableau
  - ✓ 3-period model

| | |
|---|---|
| $l$ | regular time cost (\$/unit) |
| $l'$ | over time cost (\$/unit) |
| $g$ | subcontracting cost (\$/unit) |
| $h$ | inventory holding cost (\$/unit) |

| Supply (units from) | | Demands (units for) | | | Final Inventory | Unused capacity | Total Capacity Available |
|---|---|---|---|---|---|---|---|
| | | Period 1 | Period 2 | Period 3 | | | |
| Initial inventory | | 0 | $h$ | $2h$ | $3h$ | 0 | $I_0$ |
| Period 1 | Regular | $l$ | $l + h$ | $l + 2h$ | $l + 3h$ | 0 | $W_1$ |
| | Overtime | $l'$ | $l' + h$ | $l' + 2h$ | $l' + 3h$ | 0 | $O_1$ |
| | Subcontract | $g$ | $g + h$ | $g + 2h$ | $g + 3h$ | 0 | $S_1$ |
| Period 2 | Regular | | $l$ | $l + h$ | $l + 2h$ | 0 | $W_2$ |
| | Overtime | | $l'$ | $l' + h$ | $l' + 2h$ | 0 | $O_2$ |
| | Subcontract | | $g$ | $g + h$ | $g + 2h$ | 0 | $S_2$ |
| Period 3 | Regular | | | $l$ | $l + h$ | 0 | $W_3$ |
| | Overtime | | | $l'$ | $l' + h$ | 0 | $O_3$ |
| | Subcontract | | | $g$ | $g + h$ | 0 | $S_3$ |
| Demand | | $d_1$ | $d_2$ | $d_3$ | $I_3$ | | |

# Aggregate Planning

| Period | Demand | Production Capacity | |
|---|---|---|---|
| | | Regular | Overtime |
| 1 | 800 | 700 | 350 |
| 2 | 900 | 700 | 250 |
| 3 | 800 | 700 | 250 |
| 4 | 700 | 700 | 250 |

- Initial inventory = 50 (no backorder)
- Cost of regular time = 50
- Cost of over time = 75
- Inventory holding cost = 5

## Approaches – Transportation Models

Transportation Models (3)

- Transportation tableau
  - ✓ Example

| Supply (units from) | | Demands (units for) Period 1 | Period 2 | Period 3 | Period 4 | Final Inventory | Unused capacity | Total Capacity Available |
|---|---|---|---|---|---|---|---|---|
| Initial inventory | | 0 | 5 | 10 | 15 | 20 | 0 | 50 |
| Period 1 | Regular | 50 | 55 | 60 | 65 | 70 | 0 | 700 |
| | Overtime | 75 | 80 | 85 | 90 | 95 | 0 | 350 |
| Period 2 | Regular | M | 50 | 55 | 60 | 65 | 0 | 700 |
| | Overtime | M | 75 | 80 | 85 | 90 | 0 | 250 |
| Period 3 | Regular | M | M | 50 | 55 | 60 | 0 | 700 |
| | Overtime | M | M | 75 | 80 | 85 | 0 | 250 |
| Period 4 | Regular | M | M | M | 50 | 55 | 0 | 700 |
| | Overtime | M | M | M | 75 | 80 | 0 | 250 |
| Demand | | 800 | 900 | 800 | 700 | | 250 | 3,950 |