# Predicting Mortgage Yield using Regression Analysis

Group 42

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## 1 Introduction

The study of A. H. Schaaf, 1966, "Regional Differences in Mortgage Financing Costs" investigates the existence and causes of regional differences in mortgage financing costs in the United States. These differences in mortgage yields have decreased in the early 20th century, however they remained stable after World War 2. The paper explores two main explanations for this phenomenon:

**1.** differences in investment value due to risk, terms, and liquidity
**2.** market imperfections such as legal barriers and information gaps.

The data used in this study comes from the Federal Home Loan Bank Board, which contains interest rates and fees in 18 SMSAs (Standard Metropolitan Statistical Areas). The findings suggest that distance from major financial centers, risk levels, and local demand for savings significantly affect mortgage yields. However, market structure and overall savings levels play a lesser role.

The aim of this report is to analyze the data and develop a predictive model to predict Mortgage Yield (`mortYld`) based on 8 explanatory variables:

- **smsa:** Standard Metropolitan Statistical Areas (18) → Name of the city/region.
- **mortYld:** Mortgage Yield, in % → The percentage return on a mortgage.
- **X1:** Loan-to-Mortgage Ratio, in % → High values indicate low down payments.
- **X2:** Distance from Boston, in miles → Measures regional proximity to financial centers.
- **X3:** Savings per new unit built, in $ → Indicator of regional credit demand.
- **X4:** Savings per capita, in $ → Measures local savings levels (credit supply).
- **X5:** Population increase, 1950-1960, in % → Proxy for housing demand growth.
- **X6:** Percentage of first mortgages from inter-regional banks, in % → Indicator of external financing reliance.

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## 2 Exploratory Data Analysis (EDA)

### 2.1 Load Data and Libraries

Table 1: First few rows of the dataset

| smsa | mortYld | X1 | X2 | X3 | X4 | X5 | X6 |
|---|---|---|---|---|---|---|---|
| Los Angeles-Long Bea | 6.17 | 78.1 | 3042 | 91.3 | 1738.1 | 45.5 | 33.1 |
| Denver | 6.06 | 77.0 | 1997 | 84.1 | 1110.4 | 51.8 | 21.9 |
| San Francisco-Oaklan | 6.04 | 75.7 | 3162 | 129.3 | 1738.1 | 24.0 | 46.0 |
| Dallas-Fort Worth | 6.04 | 77.4 | 1821 | 41.2 | 778.4 | 45.7 | 51.3 |
| Miami | 6.02 | 77.4 | 1542 | 119.1 | 1136.7 | 88.9 | 18.7 |
| Atlanta | 6.02 | 73.6 | 1074 | 32.3 | 582.9 | 39.9 | 26.6 |

```
##     smsa mortYld       X1       X2       X3       X4       X5       X6
##        0        0        0        0        0        0        0        0
```

Here is a display of the data, on the first few rows of the dataset. It contains mortgage yield (mortYld) as the dependent variable and six variables (X1 to X6). smsa represents the Standard Metropolitan Statistical Area which is the name of the city/region. We can observe that all data are numerical values and there is no missing value for each region.

## 2.2 Univariate Analysis

### 2.2.1 Summary Statistics

Table 2: Summary Statistics of Variables

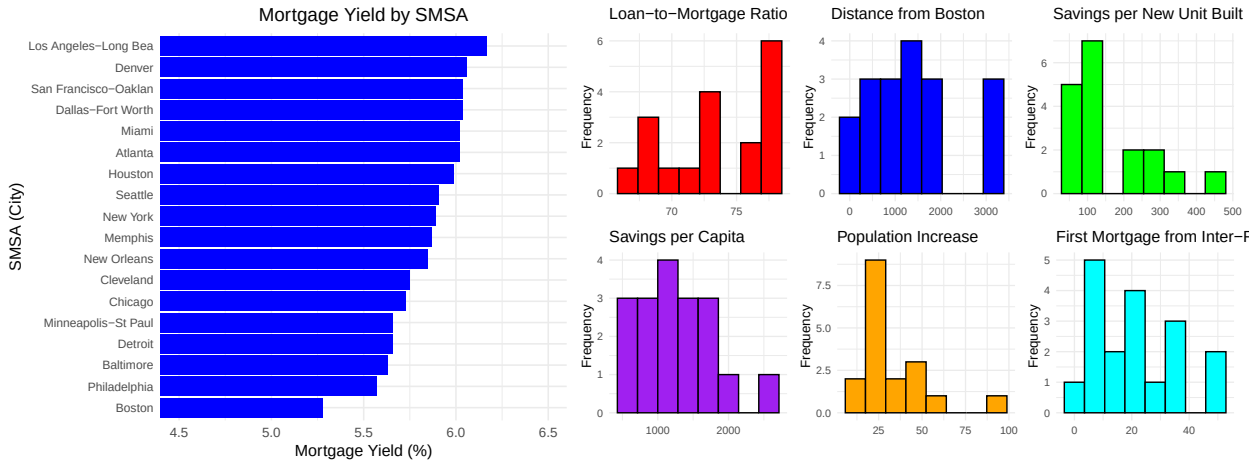
| mortYld | X1 | X2 | X3 | X4 | X5 | X6 |
|---|---|---|---|---|---|---|
| Min. :5.280 | Min. :67.00 | Min. : 0 | Min. : 32.3 | Min. : 582.9 | Min. : 7.50 | Min. : 2.0 |
| 1st Qu.:5.678 | 1st Qu.:70.03 | 1st Qu.: 648 | 1st Qu.: 85.9 | 1st Qu.: 792.9 | 1st Qu.:23.18 | 1st Qu.: 9 |
| Median :5.880 | Median :73.25 | Median :1364 | Median :122.2 | Median :1161.3 | Median :27.35 | Median :1 |
| Mean :5.841 | Mean :73.38 | Mean :1389 | Mean :159.8 | Mean :1245.9 | Mean :33.03 | Mean :20. |
| 3rd Qu.:6.020 | 3rd Qu.:77.22 | 3rd Qu.:1847 | 3rd Qu.:218.2 | 3rd Qu.:1556.6 | 3rd Qu.:44.10 | 3rd Qu.:3( |
| Max. :6.170 | Max. :78.10 | Max. :3162 | Max. :428.2 | Max. :2582.4 | Max. :88.90 | Max. :51. |

We can observe that each variable 18 observations corresponding to 18 different SMSAs. Through this summary, we can already observe mortgage yields don't vary much across regions. Most values are between 5.5% and 6.2%, suggesting relatively stable mortgage rates.

Loan-to-mortgage ratios (X1) are concentrated in between 70% and 78%, with low variance. Savings per New Unit Built (X3) are characterized by a mean bigger than the median, representing a right-skewed distribution and then large disparities in housing affordability across regions.

-> to complete

### 2.2.2 Graphical Representation




There is not a huge variation in mortgage yield across SMSAs, as most bars are at similar heights but if we focus in between 4 and 6%, we see regional differences exist in mortgage yields, possibly due to economic factors like savings, loan terms, and regional banking practices.

The histograms reveal the characteristics of the predictor variables.

The loan-to-mortgage ratio (X1) shows low variance with most values concentrated between 66% and 80%, possibly indicating limited variability across regions.

Distance from Boston (X2) displays a wide distribution, reflecting substantial geographic spread among SMSAs.

Savings per new unit built (X3) and savings per capita (X4) both exhibit right-skewed distributions, suggesting that a few cities have notably higher savings levels.

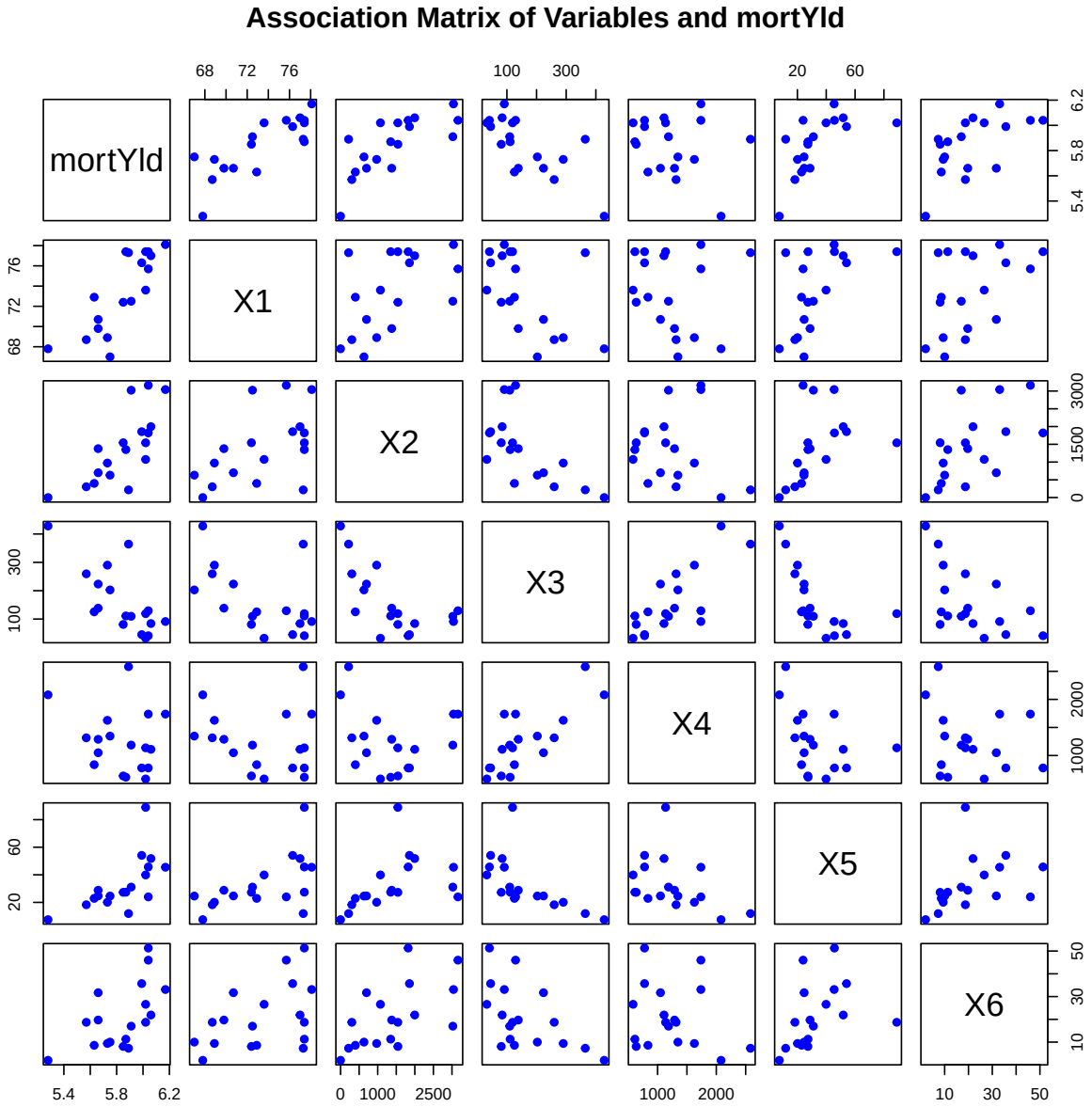
Population increase (X5) is also highly skewed with one major outlier, indicating that most cities had moderate growth, while a few experienced rapid expansion.

Finally, the percentage of first mortgages from inter-regional banks (X6) is also right-skewed, with most cities relying minimally on external financing and a few showing heavy dependence.

These patterns suggest that certain variables may benefit from transformation prior to regression modeling.

## 2.3 Bivariate Numerical Analysis

### 2.3.1 Association Analysis




The scatterplot matrix provides a quick visual assessment of linearity, strength of associations among predictors, and outlier detection. It complements numerical analyses like the correlation matrix and VIF.

We can visualizes bivariate relationships, how each variable relates to the others, especially to mortYld and assess if a relationship is linear, curved, or weak, positive or negative. We can also spot outliers or cities that don't follow the general trend.

We can see that most of the plots are random dispersion, some are linear, and some are curved. X3 seems to be positively associated with X4 and negatively with X5. X2 and X3 seem exponentially

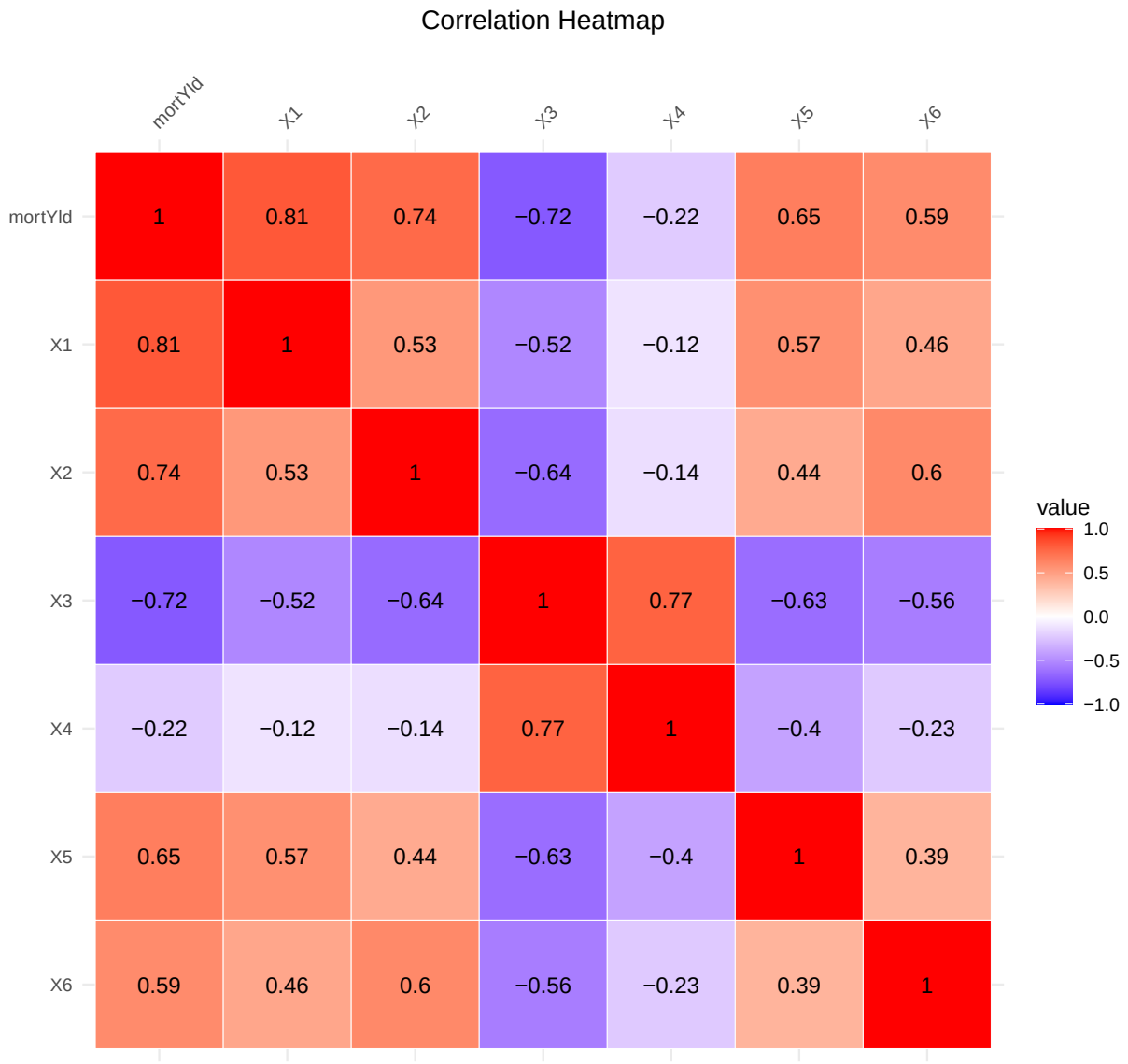
associated. mortYld and X5 seem to be associated in a squared root maner.

Let's take a closer look into the Association Matrix, regarding the relationship between Mortgage Yield (%) and the explanatory variables (x-axis), representing the first column in the precedent figure.


**1. Loan-to-Mortgage Ratio:** As this ratio increases, the Mortgage Yield increases. This suggests a positive correlation, and that higher loan-to-mortgage ratios (more borrowed money relative to the property value) are associated with higher mortgage yields.
**2. Distance from Boston:** There is a positive correlation. Boston represents a major financial center with surplus capital. Regions further from Boston might have higher yields.
**3. Savings per New Unit Built :** There is a negative correlation. This indicates that areas with more savings dedicated to new construction have better access to local financing, resulting in lower mortgage yields.
**4. Savings per Capita:** The relationship is less clear but appears to be a weak negative correlation or a random dispersion.
**5. Population Increase:** There is a positive association which can be seen as a square-root relationship. High population growth may imply higher demand for housing, increasing mortgage yields due to heightened competition for available funds. We can observe a potential outlier at the right side of the plot.
**6. First Mortgage from Inter-Regional Banks:** No clear trend. It seems like the reliance on external financing (measured by the percentage of first mortgages from inter-regional banks) does not significantly influence mortgage yields.

**To resume:**
- X1, X2 and X5 seem to be the most influential variables positively correlated with Mortgage Yield.
- X3 is the most influential negative variable.
- X4 X6 variables show weak relationships with mortgage yields.
- These observations support the findings of Schaaf (1966) that distance from financial centers, risk factors, and local demand for savings contribute to yield variations.

### 2.3.2 Correlation analysis




X3 and X4 are strongly correlated (0.77).

X2 and X3 have a high negative correlation (-0.64).

X3 is also strongly negatively correlated with X5 at -0.63.

X1, X2 and X5 have a high positive correlation with mortYld, and X3 a strong negative one. In consequence, we can confirm our previous statements about the scatter-plots.

We can then think about removing one of the highly correlated predictors, if multicollinearity affects the regression model.

These correlations tell only about the if the variables are linearly associated. A low value doesn't

mean that the variables are not correlated in another way.

```
cor(residuals(lm(df$X2 ~ df$X3)), residuals(lm(df$X6 ~ df$X3)))
```

```
## [1] 0.3760938
```

We can also see that X2 is only weakly positively correlated (r = 0.21) to X6 after controlling for X3; compare this to the much higher simple correlation (r = 0.60). In other words, much of the apparent correlation between X2 and X6 can be explained by their mutual positive correlation with X3.

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# 3 Model Fitting

In this analysis, all predictors are continuous variables and each observation corresponds to a unique SMSA. Since the dataset contains no grouping or categorical factors with unequal group sizes, this is a standard multiple regression model with one observation per row. Therefore, the design is not factorial and does not involve unbalanced group structures. As a result, the order in which predictors are entered into the lm() function does not influence the coefficient estimates, F-tests, or model interpretation.

## 3.1 Pairwise Simple Regressions

Table 3: Simple Linear Regressions: $R^2$ and p-values

| Predictor | R_squared | p_value |
|---|---|---|
| X1 | 0.654 | 0.0000 |
| X2 | 0.546 | 0.0005 |
| X3 | 0.517 | 0.0008 |
| X4 | 0.049 | 0.3763 |
| X5 | 0.419 | 0.0037 |
| X6 | 0.346 | 0.0103 |

The table summarizes the strength of individual linear relationships between each predictor (X1–X6) and the mortgage yield using simple linear regression.

- **X1** has the **strongest linear association** with mortgage yield,

explaining approximately **65.4% of its variance** and is highly significant ($p < 0.001$)

- **X2** and **X3** also show strong and significant associations ($R^2$ =

0.546 and 0.517, respectively).

- **X5** and **X6** show moderate yet significant associations ($R^2$ =

0.419 and 0.346).

- **X4 (Savings per Capita)** does **not** show a significant

relationship with mortgage yield ($R^2 = 0.049$, $p = 0.3763$), suggesting it may not be a strong individual predictor.

This preliminary analysis indicates that variables X1, X2, and X3 are the most promising candidates for predicting mortgage yield in a multivariate model.

### 3.2 Null Model vs Full Model Comparison

```
## Analysis of Variance Table
##
## Model 1: mortYld ~ 1
## Model 2: mortYld ~ X1 + X2 + X3 + X4 + X5 + X6
##   Res.Df     RSS Df Sum of Sq      F    Pr(>F)
## 1     17 0.84858
## 2     11 0.10980  6   0.73877 12.335 0.0002523 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The ANOVA comparison between the null model and the full model reveals that the full model, which includes the predictors, significantly improves the model fit. The null model (intercept-only) does not explain much of the variation in mortgage yield.

The full model, provides a better explanation of the mortgage yield, as shown by the significant F-statistic and the p-value. This indicates that at least one of the predictors is significantly related to mortgage yield, and the explanatory variables are useful for improving the model.

```
##
## Call:
## lm(formula = mortYld ~ X1 + X2 + X3 + X4 + X5 + X6, data = df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.145030 -0.017814  0.001474  0.034316  0.134565
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.285e+00  6.682e-01   6.413 4.99e-05 ***
## X1           2.033e-02  9.308e-03   2.184   0.0515 .
## X2           1.359e-05  4.692e-05   0.290   0.7775
## X3          -1.584e-03  7.532e-04  -2.103   0.0593 .
## X4           2.017e-04  1.124e-04   1.794   0.1002
## X5           1.283e-03  1.765e-03   0.727   0.4826
## X6           2.357e-04  2.302e-03   0.102   0.9203
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.09991 on 11 degrees of freedom
## Multiple R-squared:  0.8706, Adjusted R-squared:    0.8
## F-statistic: 12.33 on 6 and 11 DF,  p-value: 0.0002523
```

The model explains approximately **87%** of the variance in mortgage yield, and after adjusting for the number of predictors, **80%** is still explained. This is a strong fit. The overall model is statistically significant, with a very low p-value. Once again, it means that at least one predictor contributes

significantly to explaining the variation in mortYld. The intercept appears to be really significant to fit the model.

However, most variables do not show statistically significant individual contributions. Only X1 and X3 show strong significance ($p \approx 0.05$). Other variables, X2, X5 and X6, do not show strong individual effects. This suggests that a reduced model may be more interpretable.

### 3.3 Make stepwise regression to select the best model

```
## Start:  AIC=-77.79
## mortYld ~ X1 + X2 + X3 + X4 + X5 + X6
##
##        Df Sum of Sq     RSS     AIC
## - X6    1  0.000105 0.10991 -79.773
## - X2    1  0.000837 0.11064 -79.653
## - X5    1  0.005271 0.11507 -78.946
## <none>              0.10980 -77.790
## - X4    1  0.032141 0.14194 -75.168
## - X3    1  0.044144 0.15395 -73.707
## - X1    1  0.047593 0.15740 -73.308
##
## Step:  AIC=-79.77
## mortYld ~ X1 + X2 + X3 + X4 + X5
##
##        Df Sum of Sq     RSS     AIC
## - X2    1  0.000971 0.11088 -81.614
## - X5    1  0.005259 0.11517 -80.931
## <none>              0.10991 -79.773
## + X6    1  0.000105 0.10980 -77.790
## - X4    1  0.033056 0.14297 -77.040
## - X3    1  0.046942 0.15685 -75.371
## - X1    1  0.048227 0.15813 -75.224
##
## Step:  AIC=-81.61
## mortYld ~ X1 + X3 + X4 + X5
##
##        Df Sum of Sq     RSS     AIC
## - X5    1  0.005047 0.11593 -82.813
## <none>              0.11088 -81.614
## + X2    1  0.000971 0.10991 -79.773
## + X6    1  0.000238 0.11064 -79.653
## - X1    1  0.047325 0.15820 -77.216
## - X4    1  0.076985 0.18786 -74.123
## - X3    1  0.139427 0.25031 -68.958
##
## Step:  AIC=-82.81
## mortYld ~ X1 + X3 + X4
##
##        Df Sum of Sq     RSS     AIC
```

```
## <none>              0.11593 -82.813
## + X5    1  0.005047 0.11088 -81.614
## + X2    1  0.000759 0.11517 -80.931
## + X6    1  0.000202 0.11572 -80.845
## - X1    1  0.065583 0.18151 -76.743
## - X4    1  0.075826 0.19175 -75.755
## - X3    1  0.164703 0.28063 -68.900
```

```
##
## Call:
## lm(formula = mortYld ~ X1 + X3 + X4, data = df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.172291 -0.018862  0.006091  0.040552  0.145956
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.223e+00  5.814e-01   7.263 4.14e-06 ***
## X1           2.229e-02  7.922e-03   2.814 0.013787 *
## X3          -1.863e-03  4.178e-04  -4.460 0.000539 ***
## X4           2.249e-04  7.433e-05   3.026 0.009070 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.091 on 14 degrees of freedom
## Multiple R-squared:  0.8634, Adjusted R-squared:  0.8341
## F-statistic: 29.49 on 3 and 14 DF,  p-value: 2.618e-06
```

(mettre en annexe ?)

The stepwise regression process identified X1, X3, and X4 as the most significant predictors of mortality yield, leading to the final model.

It's interesting to see that X4 appears among the 3 most significant predictoes although it shows the weakest correlation in the correlation matrix. Multiple regression measures the effect of each variable while holding all other constant. As X4 has very strong correlation with X3, holding X3 can make the unique contribution of X4 clearer.

The final model explains approximately **83.4% of the variance** in mortgage yield using only these three predictors.
The AIC isn't increased by a lot when keeping the other variables, which means that these are still statistically valid but not so useful. The final is simpler but still explain the data just as well or better.

The residual standard error (0.091) is low, and the overall model is highly significant, indicating a good fit.
We end up with : mortYld = 4.223 + 0.02229.X1 - 0.001863.X3 + 0.0002249.X4

Let's try a model with 2-way interactions.

```
##
## Call:
## lm(formula = mortYld ~ X1 + X3 + X4 + X1:X3 + X1:X4 + X3:X4,
##     data = df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.187649 -0.015613  0.007221  0.030851  0.155167
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.371e+00  2.033e+00   2.642   0.0229 *
## X1           6.912e-03  2.657e-02   0.260   0.7996
## X3          -1.040e-04  9.611e-03  -0.011   0.9916
## X4          -9.096e-04  2.454e-03  -0.371   0.7180
## X1:X3       -2.091e-05  1.322e-04  -0.158   0.8772
## X1:X4        1.497e-05  3.225e-05   0.464   0.6516
## X3:X4       -4.751e-08  4.540e-07  -0.105   0.9185
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1002 on 11 degrees of freedom
## Multiple R-squared:  0.8698, Adjusted R-squared:  0.7988
## F-statistic: 12.25 on 6 and 11 DF,  p-value: 0.0002604
```

The interactions increase the complexity of the model for an improvement that seems very small.

We decided not to include a 3-way interaction model in our analysis. Given the small sample size (18 observations), adding high-order interactions would significantly reduce degrees of freedom and increase the risk of overfitting. Moreover, 3-way interactions are often difficult to interpret meaningfully.

### 3.4 Model Comparison

Table 4: Comparison of Model Performance Metrics

| Model | R2 | Adj_R2 | AIC | Residual_SE | F_statistic |
|---|---|---|---|---|---|
| Full Model | 0.871 | 0.800 | -24.708 | 0.100 | 12.335 |
| Stepwise Model | 0.863 | 0.834 | -29.731 | 0.091 | 29.493 |
| 2-Way Interaction Model | 0.870 | 0.799 | -24.600 | 0.100 | 12.250 |

The **Stepwise Model** offers the best trade-off between simplicity and performance: It has the **lowest AIC**, indicating the best model fit among the three. Despite having a slightly lower $\mathbf{R^2}$ than the full model, it achieves the **highest Adjusted $\mathbf{R^2}$**.

It also has the **lowest residual standard error** and the **highest F-statistic**, confirming overall model significance and parsimony.

```
## Analysis of Variance Table
```

```
##
## Model 1: mortYld ~ X1 + X3 + X4
## Model 2: mortYld ~ X1 + X3 + X4 + X1:X3 + X1:X4 + X3:X4
##   Res.Df     RSS Df Sum of Sq      F Pr(>F)
## 1     14 0.11593
## 2     11 0.11046  3 0.0054608 0.1813 0.9069
```

An ANOVA was conducted to assess whether including 2-way interaction terms significantly improved the model fit. The test yielded an F-statistic of 0.18 and a p-value of 0.91, indicating that the additional interaction terms did not meaningfully reduce the residual variance.

As a result, we retained the simpler model with only main effects (X1, X3, and X4), which offers comparable explanatory power and better interpretability.

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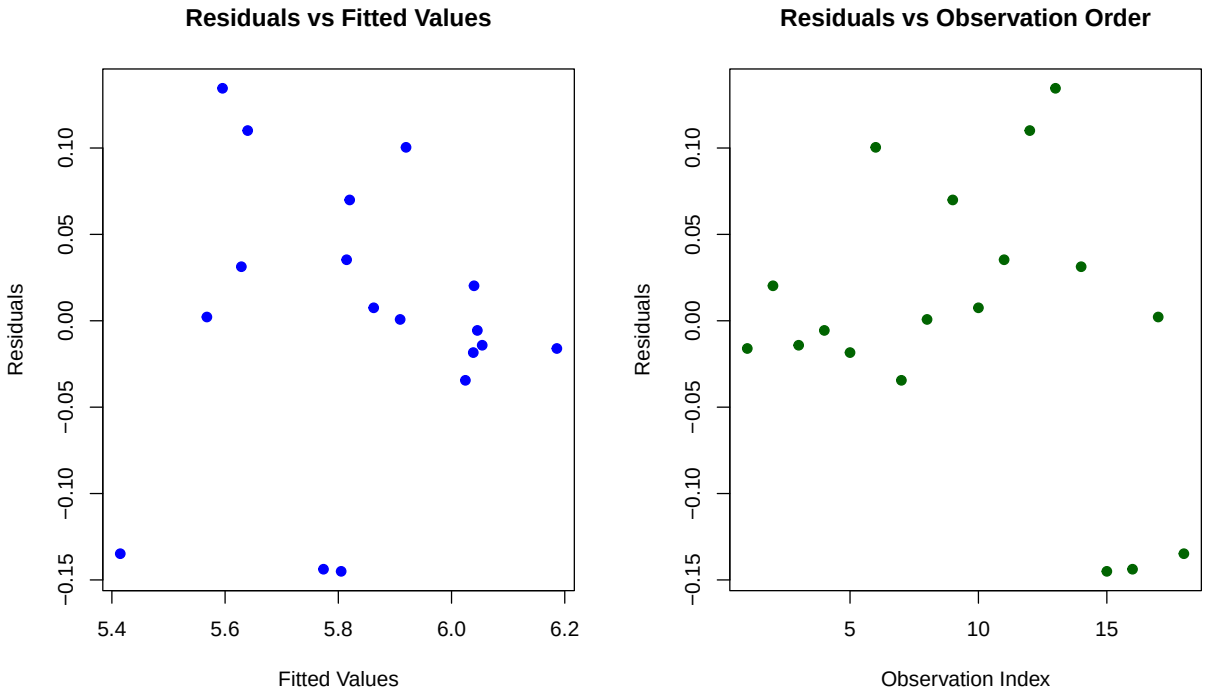
# 4 Model assumptions and Diagnostics

## 4.1 Independence evaluation




The 1st graph shows that the residuals appear randomly scattered around 0. There's no clear pattern. This suggests the assumptions of linearity and constant error variance (homoscedasticity) are reasonably met.

The 2nd graph can help us conclude that there is no consistent trend so the independance is verified.

## 4.2 Multicolinearity diagnostic

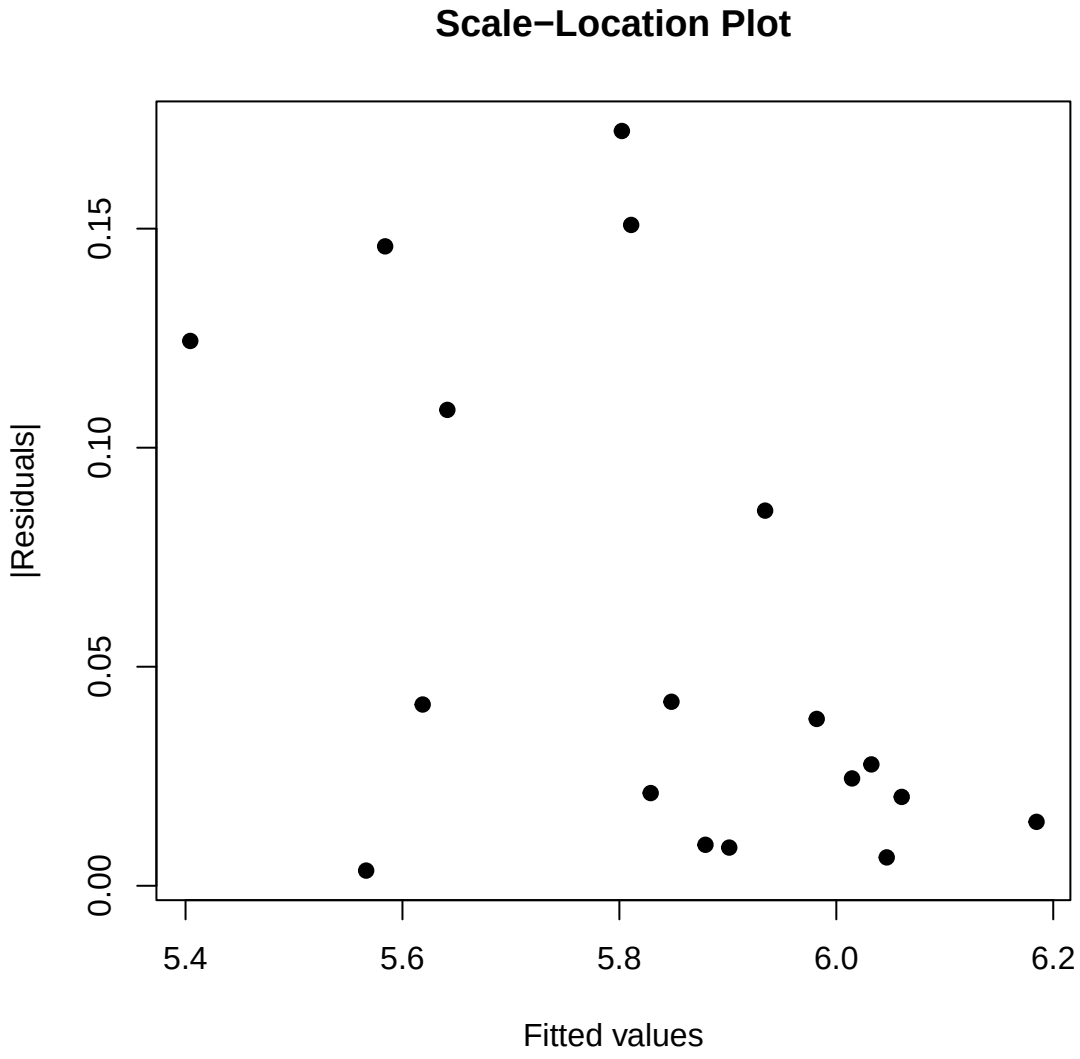
Table 5: Variance Inflation Factors (VIF)

| | vif_values |
|---|---|
| X1 | 1.886802 |
| X3 | 4.550125 |
| X4 | 3.348330 |

All variables have a VIF value under 5 meaning that variables are not too highly related and that no variable should be eliminated. This confirms our choice of keeping X3 and X4 even if they showed a high correlation coefficient.

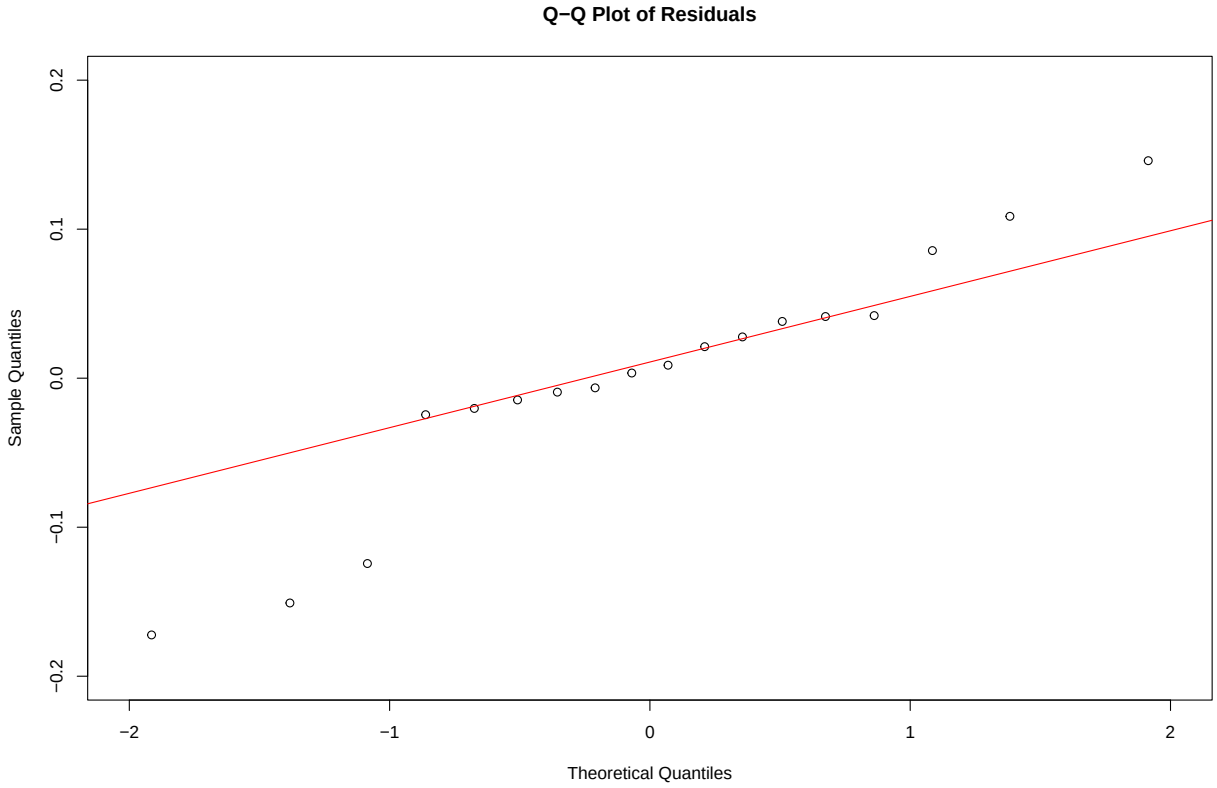
### 4.3 Homoscedasticity

Random scatter indicates good assumption of homeoscedasticity. If we can distinguish a clear pattern, then we have potential heteroscedasticity issue.



### 4.4 Normality Check

If points lie on 45 degrees line, it means the residuals are normally distributed. If we can see a curved pattern, then the normality assumption is violated

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## 5 Final estimated Model

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## 6 Conclusions

- The analysis showed that [mention significant predictors] have a strong relationship with m
- The assumptions of linear regression were [state if met or violated].
- The model provides [good/poor] predictive accuracy based on [$R^2$ and residual analysis].
- Future improvements could involve [mention possible improvements like transformations, addi