Predicting Mortgage Yield using Regression Analysis

Group 42

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1 Introduction

The study of A. H. Schaaf, 1966, "Regional Differences in Mortgage Financing Costs", investigates the existence and causes of regional differences in Mortgage financing costs in the United States. While these differences in Mortgage Yields were decreasing in the early 20th century, they suprisingly remained stable after World War II. The paper explores two main explanations for this phenomenon:

1. Differences in investment value due to risk, terms, and liquidity.
2. Market imperfections such as legal barriers and information gaps.

The data used in this study comes from the Federal Home Loan Bank Board, which contains interest rates and fees in 18 SMSAs (Standard Metropolitan Statistical Areas). The findings suggest that distance from major financial centers, risk levels, and local demand for savings significantly affect Mortgage Yields. However, market structure and overall savings levels play a lesser important role.

The aim of this report is to analyze the data and develop a predictive model to predict Mortgage Yield (`mortYld` in %) based on 6 explanatory variables:

- **X1:** Loan-to-Mortgage Ratio, in % → High values indicate low down payments.
- **X2:** Distance from Boston, in miles → Measures regional proximity to financial centers.
- **X3:** Savings per New Unit Built, in $ → Indicator of regional credit demand.
- **X4:** Savings per Capita, in $ → Measures local savings levels (credit supply).
- **X5:** Population Increase, 1950-1960, in % → Proxy for housing demand growth.
- **X6:** Percentage of First Mortgages from Inter-Regional Banks, in % → Indicator of external financing reliance.

2 Exploratory Data Analysis (EDA)

2.1 Load Data and Libraries

Table 1: First few rows of the dataset

smsa	mortYld	X1	X2	X3	X4	X5	X6
Los Angeles-Long Bea	6.17	78.1	3042	91.3	1738.1	45.5	33.1
Denver	6.06	77.0	1997	84.1	1110.4	51.8	21.9
San Francisco-Oaklan	6.04	75.7	3162	129.3	1738.1	24.0	46.0
Dallas-Fort Worth	6.04	77.4	1821	41.2	778.4	45.7	51.3

Miami	6.02	77.4	1542	119.1	1136.7	88.9	18.7
Atlanta	6.02	73.6	1074	32.3	582.9	39.9	26.6

Here is a display of the first few rows of the dataset. Each SMSA is described by its Mortgage Yield (`mortYld`) as the dependent variable and six explanatory variables (X1 to X6). All data consist of numerical values and quick checking confirms that there are no missing values in any region.

2.2 Univariate Analysis

2.2.1 Summary Statistics

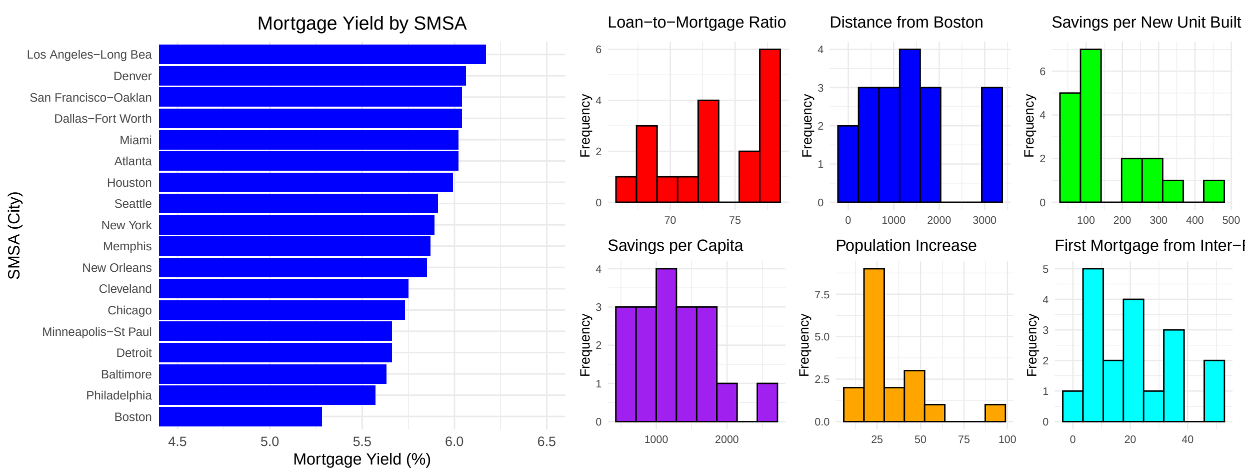
Table 2: Summary Statistics of Variables

mortYld	X1	X2	X3	X4	X5	X6
Min. :5.280	Min. :67.00	Min. : 0	Min. : 32.3	Min. : 582.9	Min. : 7.50	Min. : 2.00
1st Qu.:5.678	1st Qu.:70.03	1st Qu.: 648	1st Qu.: 85.9	1st Qu.: 792.9	1st Qu.:23.18	1st Qu.: 9.55
Median :5.880	Median :73.25	Median :1364	Median :122.2	Median :1161.3	Median :27.35	Median :18.70
Mean :5.841	Mean :73.38	Mean :1389	Mean :159.8	Mean :1245.9	Mean :33.03	Mean :20.95
3rd Qu.:6.020	3rd Qu.:77.22	3rd Qu.:1847	3rd Qu.:218.2	3rd Qu.:1556.6	3rd Qu.:44.10	3rd Qu.:30.43
Max. :6.170	Max. :78.10	Max. :3162	Max. :428.2	Max. :2582.4	Max. :88.90	Max. :51.30

Through this summary, we already observe that Mortgage Yields don't vary much across regions. Most values are between 5.2% and 6.2%, suggesting relatively stable Mortgage rates.

Loan-to-Mortgage Ratios (X1) are concentrated in between 67% and 78.1%, indicating relatively consistent lending practices across regions. Distance from Boston (X2) has a vast range (0–3162 miles), highlighting geographical diversity and potential financial access disparities. Savings per New Unit Built (X3) and Savings per Capita (X4) are characterized by means bigger than medians, representing right-skewed distributions, thus suggesting regional imbalances in credit demand and supply/in housing affordability across regions. Population Increase (X5) from 1950 to 1960 varies widely (7.5–88.9%), reflecting differing housing market pressures. Lastly, Percentage of First Mortgages from Inter-Regional Banks (X6) spans from 2.0% to 51.3%, meaning that some areas depend heavily on external financing while others rely more on local institutions.

2.2.2 Graphical Representation



With deeper analysis, although the variation across SMSAs is small, we see that regional differences still exist in Mortgage Yields, possibly due to economic factors like savings, loan terms, and regional banking practices.

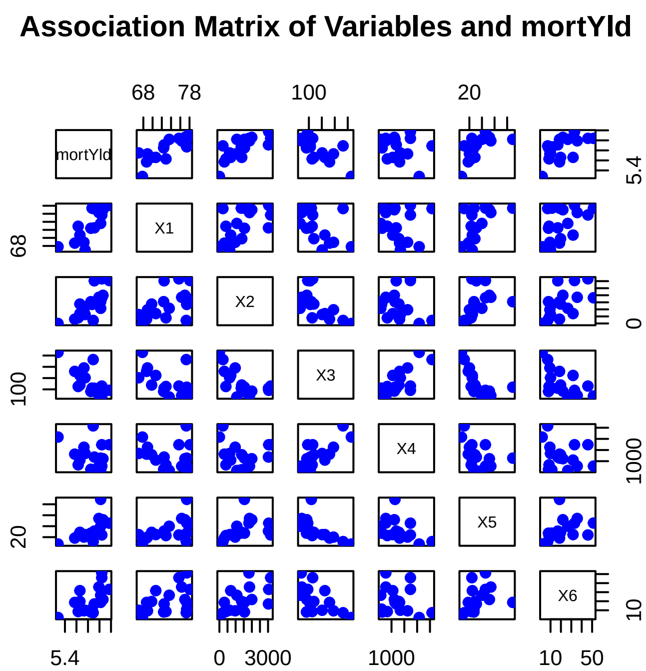
The histograms confirm the distribution of the explanatory variables:

The Loan-to-Mortgage Ratio (X1) shows low variance with most values concentrated between 67% and 80%, possibly indicating limited variability across regions. Distance from Boston (X2) displays a wide and almost homogeneous distribution, reflecting substantial geographic spread among SMSAs. Savings per New Unit Built (X3) and Savings per Capita (X4) both exhibit right-skewed distributions, suggesting that a few cities have notably higher savings levels. Population Increase (X5) is also highly right-skewed with one major outlier (increase of ~25%), indicating that most regions had moderate growth, while a few experienced rapid expansion. Finally, the percentage of First Mortgages from Inter-Regional Banks (X6) is also right-skewed, with most cities relying minimally on external financing and a few showing heavy dependence.

Overall, the data suggests regional variation in housing finance conditions, credit accessibility, and Mortgage market dynamics.

2.3 Bivariate Numerical Analysis

2.3.1 Association Analysis



The Association Matrix provides a quick visual assessment of bivariate relationships (how each variable relates to the others and `mortYld`), of types of associations among predictors (if a relationship looks linear, curved or weak, as well as positive or negative), and of outlier presence. It complements numerical analyses like the correlation matrix and VIF.

We can see that most of the plots are random dispersion, while some are linear, and some are curved. X3 seems to be positively associated with X4 and negatively with X5. X2 and X3 seem negatively exponentially associated. X6 seems to be negatively associated with X3.

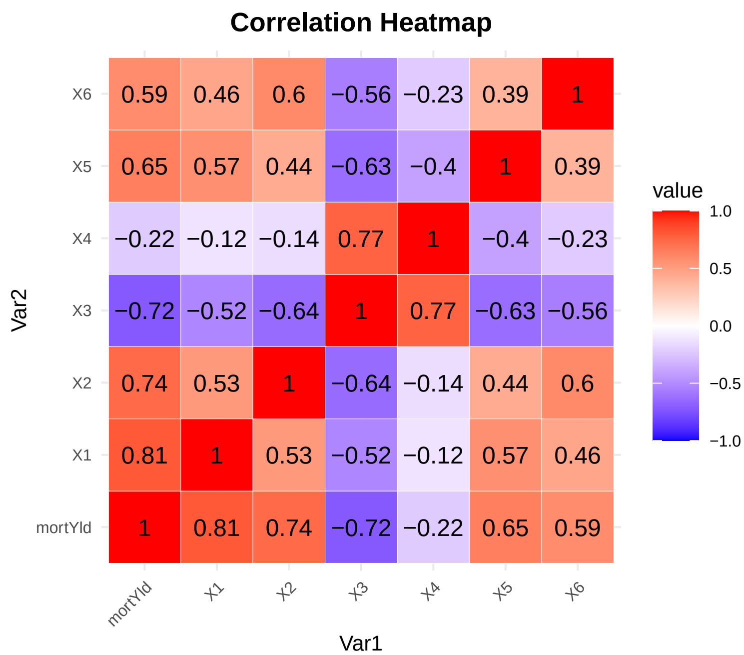
Let's take a closer look into the Association Matrix, regarding the relationship between Mortgage Yield (%) and the explanatory variables (x-axis), representing the first row in the precedent figure.

As the Loan-to-Mortgage Ratio (X1) increases, the Mortgage Yield increases. This suggests a positive correlation, and that higher Loan-to-Mortgage Ratios (more borrowed money relative to the property value) are associated with higher Mortgage Yields. Distance from Boston (X2) reveals a positive correlation with `mortYld`. Boston represents a major financial center with surplus capital. Regions further from Boston might have higher Yields. Savings per New Unit Built (X3) seems to be negatively correlated with `mortYld`. This indicates that areas with more savings dedicated to new construction have better access to local financing, resulting in lower Mortgage Yields. Savings per Capita (X4)'s influence is less distinguishable but appears to be a weak negative correlation or a random dispersion. Population Increase (X5) shows a positive association which can be seen as a square-root relationship. High population growth may imply higher demand for housing, increasing Mortgage Yields due to heightened competition for available funds. We can observe a potential outlier at the right side of the plot. The Percentage of First Mortgages from Inter-Regional Banks (X6)'s variation shows no clear trend. It seems like the reliance on external financing does not significantly influence Mortgage Yields.

To resume:
- X1, X2 and X5 seem to be the most influential variables positively correlated with Mortgage Yield.
- X3 is the most influential variable negatively correlated with Mortgage Yield.
- X6 shows moderate positive influence on Mortgage Yield.
- X4 variable shows a weak relationship with Mortgage Yields.

These observations support the findings of Schaaf (1966) stating that distance from financial centers, risk factors, and local demand for savings contribute to Mortgage Yield variations.

2.3.2 Correlation Analysis



Now, let's take a look at the correlations between each variable and confirm our previous observations.

- X3 is are strongly positively correlated to X4 (0.77) and moderately negatively correlated to X2 (-0.64), X5 (-0.63) and X6 (-0.56). This confirms our observations with the Association Matrix.
- X1 and X2 exhibit strong positive correlation with `mortYld`, while X5 shows moderate positive correlation, and X3 a strong negative one. X6 shows moderate positive correlation with `mortYld` as well. X4 shows only weak correlation with `mortYld`. In consequence, we can confirm our previous statements about the scatter-plots.

We can then think about removing one of the highly correlated predictors, to see if multicollinearity affects the regression model. However, these correlations only indicate if two variables are linearly associated. Thus, a low value doesn't necessarily mean that the variables are not correlated in another way.

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3 Model Fitting

In this analysis, all predictors are continuous variables and each observation corresponds to a unique SMSA. Since the dataset contains no grouping or categorical factors with unequal group sizes, this is a standard multiple regression model with one observation per row. Therefore, the design is not factorial and does not involve unbalanced group structures. As a result, the order in which predictors are entered into the lm() function does not influence the coefficient estimates, F-tests, or model interpretation.

3.1 Pairwise Simple Regressions

Table 3: Simple Linear Regressions: R^2 and p-values

Predictor	R_squared	p_value
X1	0.654	0.0000
X2	0.546	0.0005
X3	0.517	0.0008
X4	0.049	0.3763
X5	0.419	0.0037
X6	0.346	0.0103

The table summarizes the individual linear relationships between each predictor (X1–X6) and Mortgage Yield, assuming all other variables remain constant, based on simple linear regression.
- X1 is the strongest and most significant predictor of Mortgage Yield, explaining 65.4% of its variance ($p < 0.0001$).
- X2 and X3 also show strong and significant linear associations ($R^2 = 0.546$ and 0.517, respectively with $p < 0.001$ and $p < 0.001$).
- X5 and X6 show moderate yet significant associations ($R^2 = 0.419$ and 0.346, respectively with $p < 0.005$ and $p < 0.05$).
- X4 doesn't exhibit a significant linear relationship with Mortgage Yield ($R^2 = 0.049$, $p = 0.3763$), suggesting it may not be a strong individual linear predictor.

This preliminary analysis indicates that variables X1, X2, and X3 may be the most promising candidates for predicting Mortgage Yield in a multivariate linear model.

3.2 Null Model vs Full Model Comparison

Table 4: Comparison of Null and Full Model (ANOVA)

term	df.residual	rss	df	sumsq	statistic	p.value
mortYld ~ 1	17	0.8485778	NA	NA	NA	NA
mortYld ~ X1 + X2 + X3 + X4 + X5 + X6	11	0.1098038	6	0.738774	12.3349	0.0002523

The ANOVA comparison between the null model (intercept-only) and the full model (including all predictors), reveals that the full model better explains the Mortgage Yield, as shown by the significant F-statistic and p-value ($p < 0.001$). This indicates that at least one of the predictors is significantly related to Mortgage Yield, and is useful for improving the model.
The Full model explains ~87% of the variance in Mortgage Yield, and 80% after adjusting for the number of predictors, which highlights a strong fit. The Residual Standard Error (?) is low, and the overall model is statistically significant, with a very low p-value ($p < 0.001$). Once again, it means that at least one term contributes significantly to explaining the variation in `mortYld`.

Table 5: Summary of Full Linear Model

term	estimate	std.error	statistic	p.value
(Intercept)	4.2852	0.6682	6.4127	0.0000
X1	0.0203	0.0093	2.1835	0.0515
X2	0.0000	0.0000	0.2896	0.7775
X3	-0.0016	0.0008	-2.1029	0.0593
X4	0.0002	0.0001	1.7944	0.1002
X5	0.0013	0.0018	0.7267	0.4826
X6	0.0002	0.0023	0.1024	0.9203

Table 6: Fit Statistics of Full Linear Model

R^2	Adjusted R^2	Residual Std. Error	F-statistic	DF	p-value
0.8706	0.8	0.0999	12.3349	6	3e-04

The intercept appears to be strongly significant to fit the model ($p < 0.001$). On the other hand, most of the variables do not show statistically significant individual contributions: only X1 and X3 show weak significance ($p \approx 0.05$), while the other variables, X2, X5 and X6, do not show significant individual effects. This suggests that a reduced model may be more appropriate.

We end up with : `mortYld` = 4.2852 + 0.0203*X1 + 0.0*X2 - 0.0016*X3 + 0.0002*X4 + 0.0013*X5 + 0.0002*X6

3.3 Make stepwise regression to select the best model

Table 7: Stepwise AIC Steps

Step	Model	RSS	AIC
Start	X1 + X2 + X3 + X4 + X5 + X6	0.1098	-77.79
Step 1	X1 + X2 + X3 + X4 + X5	0.1099	-79.77
Step 2	X1 + X3 + X4 + X5	0.1109	-81.61
Step 3	X1 + X3 + X4	0.1159	-82.81

Table 8: Coefficients of Final Stepwise Model

term	estimate	std.error	statistic	p.value
(Intercept)	4.2226	0.5814	7.2629	0.0000
X1	0.0223	0.0079	2.8143	0.0138
X3	-0.0019	0.0004	-4.4599	0.0005
X4	0.0002	0.0001	3.0261	0.0091

Table 9: Fit Statistics of Stepwise Model

R^2	Adjusted R^2	Residual Std. Error	F-statistic	DF	p-value
0.8634	0.8341	0.091	29.4933	3	2.62e-06

The Stepwise regression process identifies X1, X3, and X4 as the most significant predictors of Mortgage Yield, constituting the final model.

It is interesting to note that X4 appears among the 3 most significant predictors although it shows very weak correlation in the Correlation Matrix. Multiple regression measures the effect of each variable while holding all others constant. As X4 has very strong correlation with X3 (0.77), holding X3 can make the unique contribution of X4 clearer.

The final Stepwise model explains approximately 83.4% of the variance in Mortgage Yield using only these three predictors. The AIC doesn't increases a lot when keeping more predictors, meaning that even if these predictors can still be statistically valid to keep, they are not so useful to the model. Though the final model is simpler, it explains the data just as well or better than more complex models. The Residual Standard Error (0.091) is low, and the overall model is highly significant ($p < 0.001$), indicating a good fit.

We end up with : `mortYld` = 4.223 + 0.02229*X1 - 0.001863*X3 + 0.0002249*X4

Let's now try a model with 2-way interactions.

Table 10: Coefficients of Interaction Model

term	estimate	std.error	statistic	p.value
(Intercept)	5.3710	2.0329	2.6421	0.0229
X1	0.0069	0.0266	0.2601	0.7996
X3	-0.0001	0.0096	-0.0108	0.9916
X4	-0.0009	0.0025	-0.3706	0.7180
X1:X3	0.0000	0.0001	-0.1582	0.8772
X1:X4	0.0000	0.0000	0.4641	0.6516
X3:X4	0.0000	0.0000	-0.1046	0.9185

Table 11: Fit Statistics of Interaction Model

R^2	Adjusted R^2	Residual Std. Error	F-statistic	DF	p-value
0.8698	0.7988	0.1002	12.25	6	3e-04

The 2-way Interactions model, which is more complex than the Stepwise model, explains approximately 79.9% of the variance in Mortgage Yield. The Residual Standard Error (0.1002) is low, and the overall model is highly significant ($p < 0.001$), indicating that at least one of the terms has a significant influence on Mortgage Yield.

None of the variables show statistically significant individual contributions: only the intercept appears to be moderately significant to fit the model ($p < 0.05$). This suggests that a reduced model may be more appropriate.

We end up with : `mortYld` = 5.3710 + 0.0069*X1 - 0.0001*X3 - 0.0009*X4 + 0.0*X1:X3 + 0.0*X1:X4 + 0.0*X3:X4

We decided not to include a 3-way Interactions model in our analysis. Given the small sample size (18 observations), adding high-order interactions would significantly reduce degrees of freedom and increase the risk of overfitting. Moreover, 3-way interactions are often difficult to interpret meaningfully.

3.4 Model Comparison

Table 12: Comparison of Model Performance Metrics

Model	R2	Adj_R2	AIC	Residual_SE	F_statistic
Full Model	0.871	0.800	-24.708	0.100	12.335
Stepwise Model	0.863	0.834	-29.731	0.091	29.493
2-Way Interaction Model	0.870	0.799	-24.600	0.100	12.250

The **Stepwise model** offers the best trade-off between simplicity and performance: it has the lowest AIC (~29.7), demonstrating the best model fit among the three. Despite having a slightly lower R^2 than the Full and 2-ways Interactions model, it achieves the highest Adjusted R^2. It also has the lowest Residual Standard Error (0.091) and the highest F-statistic (~29.5). This confirms the overall model significance and parsimony.

Table 13: ANOVA Comparison: Stepwise vs Interactions Model

Model	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
Stepwise model	14	0.1159	NA	NA	NA	NA
Interaction model	11	0.1105	3	0.0055	0.1813	0.9069

An ANOVA is then conducted to confirm that including 2-way Interactions terms do not significantly improve the model fit. The test yields an F-statistic of 0.18 and a p-value of 0.91, indicating that the additional interaction terms do not meaningfully reduce the residual variance.

As a result, the simpler model with only main effects (X1, X3, and X4) truly provides the best fit, as it also offers comparable explanatory power and better interpretability.

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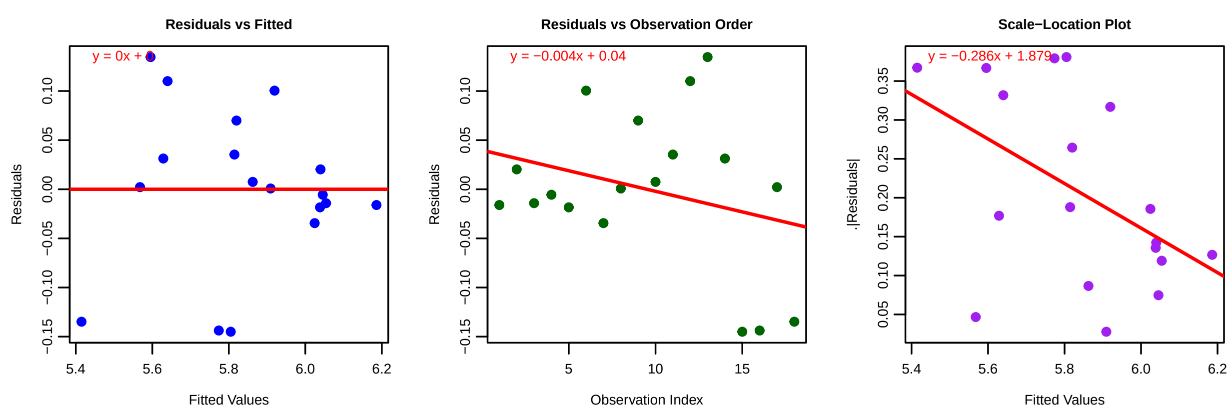
4 Model assumptions and Diagnostics

4.1 Independence evaluation



The `Residuals VS Fitted Values` plot shows that the residuals are randomly scattered around 0, with no clear pattern. This suggests that the assumptions of linearity and constant error variance are reasonably met. Secondly, the slightly negative but close to zero slope (-0.286) in the

`Scale-Location Plot` indicates that the spread of residuals is almost constant across fitted values. This is a sign that our model doesn't suffer from heteroscedasticity and is likely a good fit: homoscedasticity seems therefore satisfied.

The `Residuals VS Observation Order` plot helps us conclude that there is no consistent trend: the independence of residuals is verified, as they are not correlated with the order of observations.

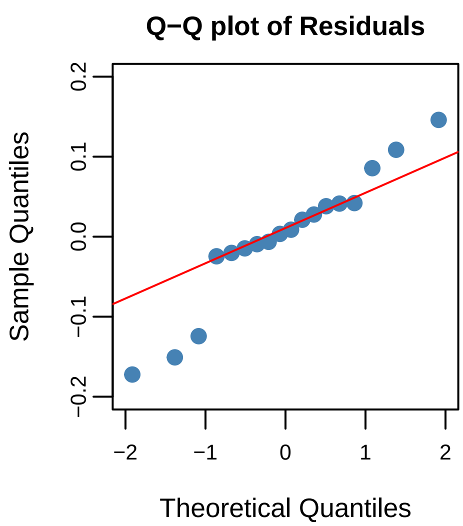
4.2 Multicolinearity diagnostic

Table 14: Variance Inflation Factors (VIF)

	vif_values
X1	1.886802
X3	4.550125
X4	3.348330

As all variables have a VIF value under 5, it means that they don't cause problematic multicollinearity in the final model and that none of them should be eliminated. This confirms our choice of keeping X3 and X4: even if they showed a high correlation coefficient (0.77), these variables still provide enough unique, non-redundant information to justify keeping them in the model.

4.3 Normality Check



The `Q-Q plot of Residuals` indicates that most of the points align closely with the 45-degree line, suggesting that the residuals are approximately normally distributed. However, six points deviate at the extremes of the theoretical quantiles, which indicates the presence of potential outliers or heavy tails in the distribution. In a dataset with just 18 observations, small deviations in the Q-Q plot are usual. Outliers or deviations are common in such a small sample size and do not automatically suggest a violation of normality.

5 Conclusions

The final estimated model is : `mortYld` = 4.223 + 0.02229*X1 - 0.001863*X3 + 0.0002249*X4 where X1 is the Loan-to-Mortgage Ratio, X3 is the Savings per New Unit Built, and X4 is the Savings per Capita.

The analysis thus shows that Loan-to-Mortgage Ratio, Savings per New Unit Built, and Savings per Capita collectively have a significant impact on Mortgage Yield. Mortgage Yield is positively influenced by the Loan-to-Mortgage Ratio (X1), suggesting that when the loan is higher compared to the mortgage, lenders may see better returns on their investment. On the other hand, Mortgage Yield is negatively impacted by Savings per New Unit Built (X3). When there is more capital saved for new construction, there may be less reliance on mortgages, potentially leading to lower returns on those mortgages. Lastly, Savings per Capita (X4) has a positive, though small, effect on the Mortgage Yield. As individuals save more money, it may lead to a slight increase in the return on mortgages, possibly because higher savings per capita can signal a more financially stable environment for lenders, leading to better mortgage performance.

While the assumptions of linear regression are generally satisfied, there are some minor deviations.

The model shows strong predictive performance, accounting for 83.4% of the variance in Mortgage Yield, with homoscedasticity nearly achieved.

Future improvements could include exploring additional predictors, testing for non-linear relationships, or refining the model to better capture any residual heteroscedasticity.