Predicting Mortgage Yield using Regression Analysis

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1 Introduction

The study of A. H. Schaaf, 1966, “Regional Differences in Mortgage Financing Costs” (Schaaf 1966), investigates the existence and causes of regional differences in Mortgage financing costs in the United States. While these differences in Mortgage Yields were decreasing in the early 20th century, they suprisingly remained stable after World War II. The paper explores two main explanations for this phenomenon: differences in investment value due to risk, terms, and liquidity, and market imperfections such as legal barriers and information gaps.

The dataset, provided by the Federal Home Loan Bank Board, includes interest rates and fees in 18 SMSAs (Standard Metropolitan Statistical Areas). Key findings suggest that distance from major financial centers, local risk levels, and savings demand influence Mortgage Yields more than market structure and overall savings levels.

The aim of this report is to analyze the dataset and build a predictive model for Mortgage Yield (in %) using 6 explanatory variables:

- **X1**: Loan-to-Mortgage Ratio, in % → High values indicate low down payments.
- **X2**: Distance from Boston, in miles → Measures regional proximity to financial centers.
- **X3**: Savings per New Unit Built, in $ → Indicator of regional credit demand.
- **X4**: Savings per Capita, in $ → Measures local savings levels (credit supply).
- **X5**: Population Increase, 1950-1960, in % → Proxy for housing demand growth.
- **X6**: Percentage of First Mortgages from Inter-Regional Banks, in % → Indicator of external financing reliance.

2 Exploratory Data Analysis (EDA)

2.1 Univariate analysis

2.1.1 Numerical analysis

We begin with a numerical summary of each variable:

mortYld	X1	X2	X3	X4	X5	X6
Min. :5.280	Min. :67.00	Min. : 0	Min. : 32.3	Min. : 582.9	Min. : 7.50	Min. : 2.00
1st Qu.:5.678	1st Qu.:70.03	1st Qu.: 648	1st Qu.: 85.9	1st Qu.: 792.9	1st Qu.:23.18	1st Qu.: 9.55
Median :5.880	Median :73.25	Median :1364	Median :122.2	Median :1161.3	Median :27.35	Median :18.70
Mean :5.841	Mean :73.38	Mean :1389	Mean :159.8	Mean :1245.9	Mean :33.03	Mean :20.95
3rd Qu.:6.020	3rd Qu.:77.22	3rd Qu.:1847	3rd Qu.:218.2	3rd Qu.:1556.6	3rd Qu.:44.10	3rd Qu.:30.43
Max. :6.170	Max. :78.10	Max. :3162	Max. :428.2	Max. :2582.4	Max. :88.90	Max. :51.30

Table 1: Summary Statistics of all Variables. The minimum, 1st quartile, median, mean, 3rd quartile, and maximum are shown for all six variables and the Mortgage Yield.

Through this summary, we already observe that Mortgage Yields (**mortYld**) don’t vary much across regions. Most values are between 5.2% and 6.2%, suggesting relatively stable Mortgage rates.

Loan-to-Mortgage Ratios (**X1**) are concentrated in between 67% and 78.1%. Distance from Boston (**X2**) has a vast range (0–3162 miles), highlighting geographical diversity and potential financial access disparities. Savings per New Unit Built (**X3**) and Savings per Capita (**X4**) are characterized by means bigger than medians, representing right-skewed distributions. Population Increase (**X5**) from 1950 to 1960 varies widely (7.5–88.9%). Lastly, Percentage of

First Mortgages from Inter-Regional Banks (**X6**) spans from 2.0% to 51.3%, meaning that some areas depend heavily on external financing while others rely more on local institutions.

2.1.2 Graphical analysis

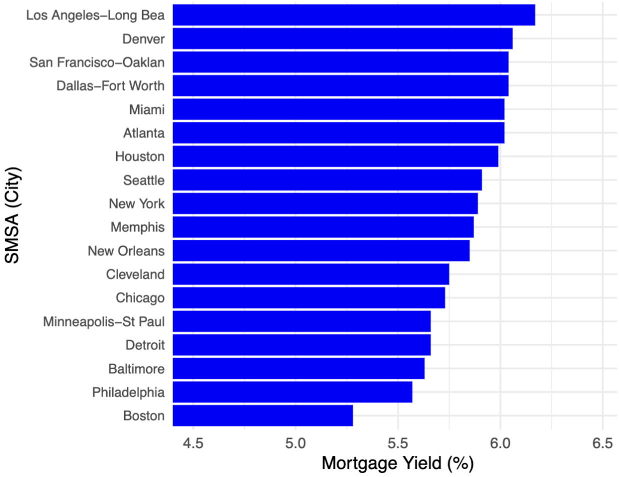


Figure 1: Histogram of Mortgage Yield across SMSAs (Standard Metropolitan Statistical Area)

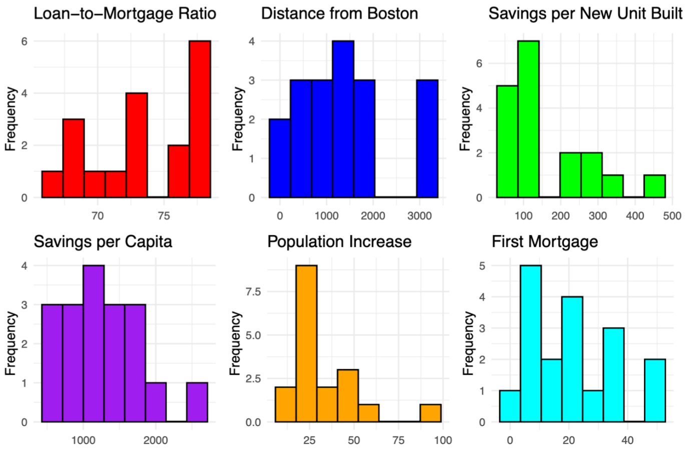


Figure 2: Histograms of Mortgage Yield across Predictor Variables

Although the variation in Mortgage Yields across SMSAs is limited, regional differences are still observable, likely due to factors such as savings rates, loan characteristics, and local banking practices:

X1 shows low variance, indicating that the distance from Boston does not vary much across SMSAs. **X2** has a wide and uniform distribution, reflecting the broad spatial distribution of the regions. **X3** and **X4** are right-skewed, suggesting that a small number of SMSAs have significantly higher savings levels. **X5** is also right-skewed, with a potential outlier, indicating uneven population growth. **X6** shows that most SMSAs have low reliance on external financing, with a few depending heavily on it.

These patterns suggest that regional heterogeneity in economic conditions and credit access contributes to the observed differences in Mortgage Yields.

2.2 Bivariate analysis

2.2.1 Graphical analysis

The Association Matrix offers a visual overview of pairwise relationships, including form (linear, curved, weak), direction (positive or negative), and outlier presence. It complements the Correlation Matrix.

Most plots show random dispersion: some indicate linear or curved trends. **X3** is positively related to **X4**, and negatively to **X5** and **X6**. **X2** and **X3** show a negative exponential relationship.

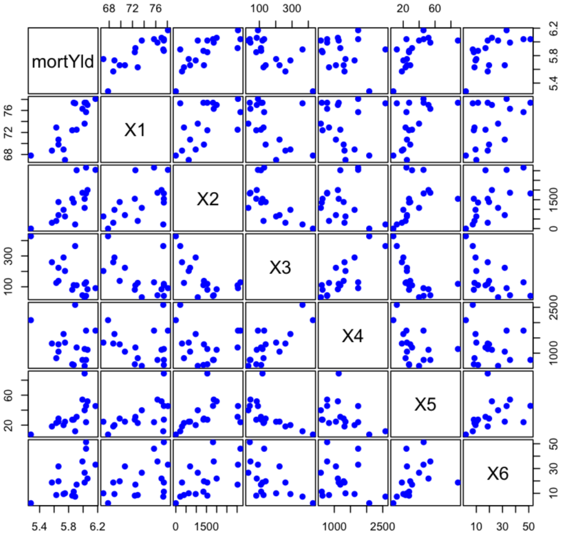


Figure 3: Association Matrix Between Mortgage Yield and Predictors. Displays pairwise correlations, highlighting relationship strength and direction between Mortgage Yield and the 6 variables.

Now let's examine the relationship between Mortgage Yield (%) and each explanatory variable (x-axis), corresponding to the first row of the Association Matrix.

X1 shows a positive correlation: higher Loan-to-Mortgage Ratios are linked to higher yields. **X2** also correlates positively; yields rise with distance from Boston, likely due to reduced capital access. **X3** is negatively correlated - regions investing more construction savings tend to have lower yields. **X4** shows little to no clear association. **X5** has a mildly positive, possibly non-linear relationship, with higher population growth linked to higher yields. **X6** shows no clear trend, suggesting limited impact of external financing.

These observations support the findings of Schaaf (1966) stating that distance from financial centers, risk factors, and local demand for savings contribute to Mortgage Yield variations.

2.2.2 Numerical analysis

The Correlation Matrix confirms earlier patterns. **X3** is strongly correlated with **X4** (0.77) and negatively with **X2** (−0.64), **X5** (−0.63), and **X6** (−0.56). **X1** and **X2** are strongly positively correlated with **mortYld**, **X5** and **X6** moderately, and **X3** strongly negatively. **X4** is weakly correlated.

This suggests potential multicollinearity. Removing one of the highly correlated predictors may improve model stability. Still, low correlation doesn't rule out non-linear relationships.

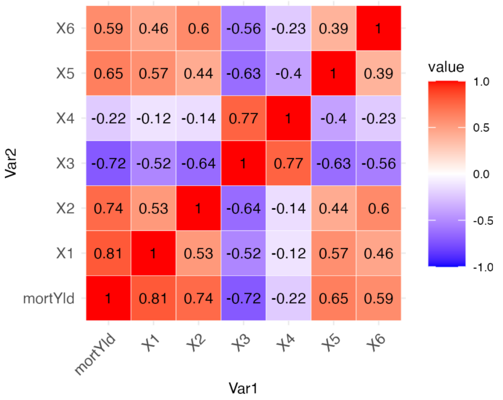


Figure 5: Correlation Heatmap of Mortgage Yield and Predictor Variables. red: strong positive, blue: strong negative, white: no correlation.

3 Model Fitting

All predictors are continuous, and each observation corresponds to a unique SMSA. With no grouping or unequal group sizes, this is a standard multiple linear regression with one observation per row. The order of predictors does not affect the results.

The multiple linear regression model is defined mathematically as:

$$\text{mortYld}_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \beta_6 X_{6i} + \varepsilon_i$$

where: mortYld_i is the mortgage yield for the i-th SMSA, X_{1i} to X_{6i} are the predictors, β_0 are coefficients, β_1 to β_6 are the regression coefficients, and ε_i is the error term.

We assume the classical linear regression assumptions:
1. Linearity: The relationship between each predictor and the outcome is linear.
2. Independence: The errors ε_i are independent across observations.
3. Homoscedasticity: The errors have constant variance: $\mathrm{Var}(\varepsilon_i) = \sigma^2$.
4. Normality: The errors follow a normal distribution: $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$.
5. No multicollinearity: The predictors are not perfectly linearly correlated.

The model is fitted using Ordinary Least Squares (OLS), which minimizes the sum of squared residuals: $\min_\beta \sum_{i=1}^{18} \left(\mathrm{mortYld}_i - \beta_0 - \sum_{j=1}^{6} \beta_j X_{ji}\right)^2$.

3.1 Null Model vs Full Model Comparison

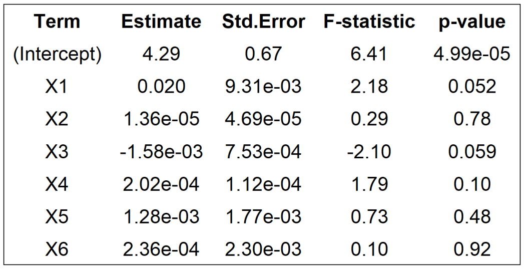
To test whether the explanatory variables significantly improve the model fit compared to the intercept-only model, we conduct an ANOVA comparing the null model and the full (alternative) model. In order to do so, we test the following hypotheses:

- **Null Hypothesis** (H_0): $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$. i.e. the explanatory variables do not improve the model.
- **Alternative Hypothesis** (H_1): At least one $\beta_j \neq 0 \quad \text{for } j = 1, \ldots, 6$. i.e. at least one explanatory variable significantly contributes to predicting mortgage yield.

Model	Residual_DF	RSS	DF (num,den)	SS	F-statistic	p-value
mortYld ~ 1	17	0.85	NA	NA	NA	NA
mortYld ~ X1 + X2 + X3 + X4 + X5 + X6	11	0.11	6,11	0.74	12.33	2.52e-04

Table 2: ANOVA Comparison between Null and Full Models. It shows Residual degrees of freedom (Residual_DF), residual sum of squares (RSS), the numerator and denominator degrees of freedom (DF (num, den)), the sum of squares (SS), F-statistic, and p-value to assess whether the full model improves fit. A p-value < 0.05 indicates that at least one predictor is relevant.

The ANOVA test confirms that the full model (with predictors) explains Mortgage Yield significantly better than the null model, as shown by the low p-value (p < 0.001). This means at least one variable contributes meaningfully to the prediction.



Term	Estimate	Std.Error	F-statistic	p-value
(Intercept)	4.29	0.67	6.41	4.99e-05
X1	0.020	9.31e-03	2.18	0.052
X2	1.36e-05	4.69e-05	0.29	0.78
X3	-1.58e-03	7.53e-04	-2.10	0.059
X4	2.02e-04	1.12e-04	1.79	0.10
X5	1.28e-03	1.77e-03	0.73	0.48
X6	2.36e-04	2.30e-03	0.10	0.92

Table 3: Summary of Full Linear Model

The Full model explains ∼87% of the variance in Mortgage Yield, and 80% after adjusting for the number of predictors, which highlights a strong fit. The Residual Standard Error is low, and the overall model is statistically significant, with a very low p-value ($p < 0.001$). Once again, it means that at least one term contributes significantly to explaining the variation in **mortYld**.

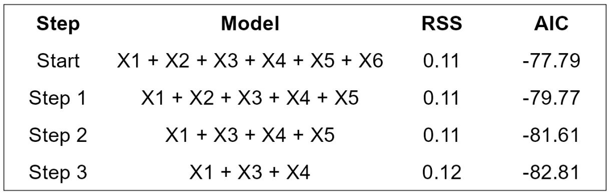
Adjusted_R^2	R^2	Std.Error	DF	F-statistic	p-value
0.80	0.87	0.10	6	12.33	2.52e-04

Table 4: Fit Statistics of Full Linear Model. R^2 shows the proportion of variance explained, Adjusted R^2 accounts for the number of predictors, and Standard Error measures the average distance of observed values from the fitted values.

The intercept is highly significant ($p < 0.001$). Among the predictors, only **X1** and **X3** show weak significance ($p \approx 0.05$), while **X2**, **X5**, and **X6** are not individually significant. We begin by fitting the full model to assess both combined and individual effects. Given the limited contribution of several variables, a reduced model may be more appropriate. We proceed with stepwise regression, using statistical significance and AIC as selection criteria. We end up with : $\widehat{mortYld} = 4.29 + 0.020\cdot\mathbf{X1} + 1.36\cdot10^{-5}\cdot\mathbf{X2} - 1.58\cdot10^{-3}\cdot\mathbf{X3} + 2.02\cdot10^{-4}\cdot\mathbf{X4} + 1.28\cdot10^{-3}\cdot\mathbf{X5} + 2.36\cdot10^{-4}\cdot\mathbf{X6}$

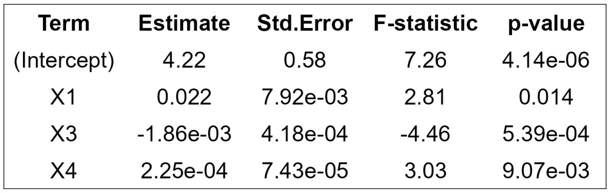
3.2 Make stepwise regression to select the best model

AIC is the Akaike Information Criterion. A lower AIC indicates a better trade-off between model fit and complexity.



Step	Model	RSS	AIC
Start	X1 + X2 + X3 + X4 + X5 + X6	0.11	-77.79
Step 1	X1 + X2 + X3 + X4 + X5	0.11	-79.77
Step 2	X1 + X3 + X4 + X5	0.11	-81.61
Step 3	X1 + X3 + X4	0.12	-82.81

Table 5: Stepwise AIC Process.



Term	Estimate	Std.Error	F-statistic	p-value
(Intercept)	4.22	0.58	7.26	4.14e-06
X1	0.022	7.92e-03	2.81	0.014
X3	-1.86e-03	4.18e-04	-4.46	5.39e-04
X4	2.25e-04	7.43e-05	3.03	9.07e-03

Table 6: Summary of Final Stepwise Model

Adjusted_R^2	R^2	Std.Error	DF	F-statistic	p-value
0.83	0.86	0.091	3	29.49	2.62e-06

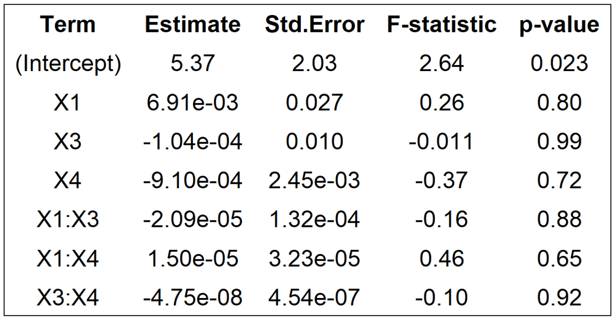
Table 7: Fit Statistics of Stepwise Model

The Stepwise regression process identifies **X1**, **X3**, and **X4** as the most significant predictors of Mortgage Yield, constituting the final model. It is interesting to note that **X4** appears among the 3 most significant predictors although it shows very weak correlation in the Correlation Matrix. Multiple regression measures the effect of each variable while holding all others constant. As **X4** has very strong correlation with **X3** (0.77), holding **X3** can make the unique contribution of **X4** clearer.

The final Stepwise model explains approximately 83.4% of the variance in Mortgage Yield using only these three predictors. The AIC doesn't increase a lot when keeping more predictors, meaning that even if these predictors can still be statistically valid to keep, they are not so useful to the model. Though the final model is simpler, it explains the data just as well or better than more complex models. The RSE (0.09) is low, and the overall model is highly significant ($p < 0.001$), indicating a good fit.

We end up with : $\widehat{mortYld} = 4.22 + 0.022\cdot\mathbf{X1} - 1.86\cdot10^{-3}\cdot\mathbf{X3} + 2.25\cdot10^{-4}\cdot\mathbf{X4}$

Let's now try a model with 2-way interactions. The 2-way Interaction model, which is more complex than the Stepwise model, explains approximately 79.9% of the variance in Mortgage Yield. The Residual Standard Error (0.10) is low, and the overall model is highly significant ($p < 0.001$), indicating that at least one of the terms has a significant influence on Mortgage Yield.



Term	Estimate	Std.Error	F-statistic	p-value
(Intercept)	5.37	2.03	2.64	0.023
X1	6.91e-03	0.027	0.26	0.80
X3	-1.04e-04	0.010	-0.011	0.99
X4	-9.10e-04	2.45e-03	-0.37	0.72
X1:X3	-2.09e-05	1.32e-04	-0.16	0.88
X1:X4	1.50e-05	3.23e-05	0.46	0.65
X3:X4	-4.75e-08	4.54e-07	-0.10	0.92

Table 8: Summary of 2-way Interaction Model

Adjusted_R^2	R^2	Std.Error	DF	F-statistic	p-value
0.80	0.87	0.10	6	12.25	2.60e-04

Table 9: Fit Statistics of 2-way Interaction Model

None of the variables show statistically significant individual contributions: only the intercept appears to be moderately significant to fit the model (p < 0.05). This suggests that a reduced model may be more appropriate.

We end up with : $\widehat{mortYld} = 5.37 + 6.91\cdot10^{-3}\cdot\mathbf{X1} - 1.04\cdot10^{-4}\cdot\mathbf{X3} - 9.10\cdot10^{-4}\cdot\mathbf{X4} - 2.09\cdot10^{-5}\cdot\mathbf{X1}:\mathbf{X3} + 1.50\cdot10^{-5}\cdot\mathbf{X1}:\mathbf{X4} - 4.75\cdot10^{-8}\cdot\mathbf{X3}:\mathbf{X4}$

We decided not to include a 3-way Interaction model in our analysis. Given the small sample size (18 observations), adding high-order interactions would significantly reduce degrees of freedom and increase the risk of overfitting. Moreover, 3-way interactions are often difficult to interpret meaningfully.

3.3 Model Comparison

The Stepwise model offers the best trade-off between simplicity and performance: it has the lowest AIC (~29.7), demonstrating the best model fit among the three. Despite having a slightly lower R^2 than the Full and 2-ways Interactions model, it achieves the highest Adjusted R^2. It also has the lowest Residual Standard Error (0.09) and the highest F-statistic (~29.5). This confirms the overall model significance and parsimony.

Model	Adjusted_R^2	R^2	RSE	AIC	F-statistic
Full Model	0.80	0.87	0.10	-24.71	12.33
Stepwise Model	0.83	0.86	0.091	-29.73	29.49
2-Way Interaction Model	0.80	0.87	0.10	-24.60	12.25

Table 10: Comparison of Model Performance Metrics. RSE (residual standard error) reflects the typical size of prediction errors; lower values indicate better fit.

Model	Residual_DF	RSS	DF	SS	F-statistic	p-value
Stepwise model	14	0.12	NA	NA	NA	NA
Interaction model	11	0.11	3	5.46e-03	0.18	0.91

Table 11: ANOVA Comparison Between Stepwise and Interaction Models

An ANOVA is then conducted to compare the Stepwise Model and the Interaction Model, which are nested — the Interaction Model extends the Stepwise Model by including additional two-way interaction terms. The test yields an F-statistic of 0.18 and a p-value of 0.91, indicating that the additional interaction terms do not significantly reduce the residual variance. As a result, the simpler model with only main effects (**X1**, **X3**, and **X4**) truly provides the best fit, as it also offers comparable explanatory power and better interpretability.

4 Model assumptions and Diagnostics

In order to trust the results of our regression model, we must ensure that the residuals satisfy the following assumptions, evaluated using residual diagnostic plots:

1. The residuals have an expected value (mean) of 0: $\mathbb{E}[\varepsilon_i] = 0$
2. The residuals are homoscedastic (have constant variance): $\mathrm{Var}(\varepsilon_i) = \sigma^2$
3. The residuals are uncorrelated: $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$
4. The residuals are normally distributed: $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$

4.1 Independence evaluation

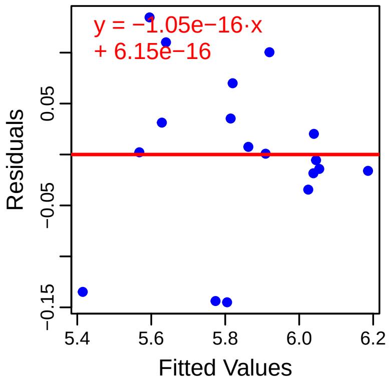


Figure 6: Residuals vs Fitted Plot. It displays residual spread to assess homoscedasticity.

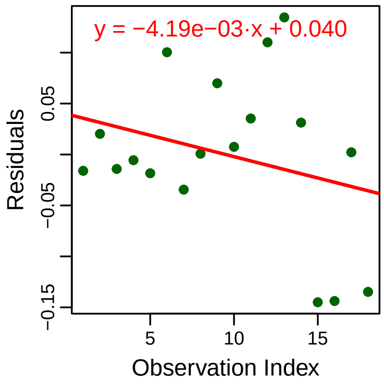


Figure 7: Residuals vs Observation Order Plot. It shows residuals over SMSAs to detect trends or autocorrelation.

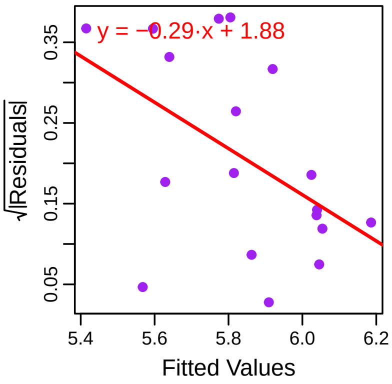


Figure 8: Scale-Location Plot. It shows the variance of residuals versus fitted values to check for homoscedasticity.

The `Residuals VS Fitted Values` plot checks for linearity and constant variance. Ideally, residuals should be symmetrically scattered around zero with no clear pattern or funnel shape (indicates constant variance). In our case, the residuals are randomly dispersed with no visible trend (randomness suggests independence), and the red regression line is nearly flat (slope $= -1.05 \times 10^{-16}$), suggesting that the residuals have approximately zero mean and constant variance. Therefore, assumptions 1 and 2 are satisfied.

The `Residuals VS Observation Order` is used to detect autocorrelation in the residuals (assumption 3). A patternless distribution across observations indicates independence. The red regression line has a slope of -0.0042, which is close to zero, and residuals appear randomly scattered, suggesting that residuals are **not autocorrelated**. Hence, assumption 3 appears is verified.

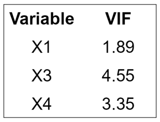
Lastly, the `Scale-Location` plot, shows the square root of standardized residuals vs fitted values. A line with constant spread indicates constant variance : although the slope is somewhat negative (-0.29), the spread remains relatively even. There is no clear increasing or decreasing funnel shape. This is a sign that our model doesn't suffer from heteroscedasticity and is likely a good fit: this supports the homoscedasticity assumption.

In conclusion, based on Figures 6–8, we find that the residuals have a mean close to zero, appear homoscedastic, and show no sign of autocorrelation. Therefore, assumptions 1, 2, and 3 are reasonably satisfied.

4.2 Multicolinearity diagnostic

Variance Inflation Factors (VIF) for all final model variables are below 5 (see Table 12), suggesting no problematic multicollinearity. Though **X3** and **X4** have a correlation of 0.77, their VIFs (4.55 and 3.35) are within acceptable limits. Thus, they provide distinct and valuable information.

In conclusion, there is no evidence of problematic multicollinearity, and all explanatory variables contribute distinct information to the model.



Variable	VIF
X1	1.89
X3	4.55
X4	3.35

Table 12: Variance Inflation Factors (VIF).

Values 5: potential multicollinearity. Values 10: strong multicollinearity.

4.3 Normality Check

The `Q-Q plot` checks if residuals are normally distributed (assumption 4). Most points align with the 45° line, especially in the center, indicating approximate normality. Some deviation at the tails suggests potential outliers or heavy tails, but with only 18 observations, this is not a strong concern. Overall, normality is reasonably satisfied. With only 18 observations, such deviations can be expected and are not strong evidence against normality.

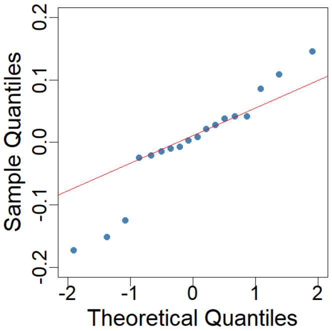


Figure 9: Q-Q Plot of Residuals.

In conclusion, the residuals appear to be approximately normally distributed, and assumption 4 is reasonably met.

5 Conclusion

First, EDA showed regional variation in mortgage yields linked to housing finance, savings, and credit conditions. The final model is:
$mor\hat{t}Yld = 4.22 + 0.022 \cdot \mathbf{X1} - 1.86 \cdot 10^{-3} \cdot \mathbf{X3} + 2.25 \cdot 10^{-4} \cdot \mathbf{X4}$
With **X1** (Loan-to-Mortgage Ratio), **X3** (Savings per New Unit Built), and **X4** (Savings per Capita).
Overall, our analysis shows that higher **X1** increases yield, suggesting greater returns from higher borrowing. Higher **X3** lowers yield, reflecting reduced mortgage reliance. **X4** has a mild positive effect, possibly tied to local financial stability.
The final stepwise regression model explains approximately 83.4% of the variance in Mortgage Yield, indicating a strong fit. Residual diagnostics suggest that linearity, independence, and homoscedasticity assumptions are generally satisfied, though some minor deviations remain. Mortgage Yield is mainly influenced by financial leverage and local savings levels—consistent with economic theory. Future work could explore more predictors, non-linear trends, or robust regression to address residual issues (**wilcox2004robust?**).
In conclusion, the model provides valuable understanding of the factors influencing Mortgage Yield across regions and offers a solid foundation for further predictive or policy analysis.

References

Schaaf, A. H. 1966. "Regional Differences in Mortgage Financing Costs." *The Journal of Finance* 21 (1): 85–94. https://www.jstor.org/stable/2977600.