Deep Learning Basics

Lecture 4: Convolutional Neural Networks

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WARNING: 본 교육 콘텐츠의 지식재산권은 재단법인 네이버커넥트에 귀속됩니다. 본 <mark>콘텐츠를 어떠한 경로로든 외부로 유출 및 수정하는 행위를 엄격히 금합니다.</mark> 다만, 비영리적 교육 및 연구활동에 한정되어 사용할 수 있으나 재단의 허락을 받아야 합니다. 이를 위반하는 경우, 관련 법률에 따라 책임을 질 수 있습니다.



Continuous convolution

$$(f * g)(t) = \int f(\tau)g(t - \tau)d\tau = \int f(t - \tau)g(t)d\tau$$

Discrete convolution

$$(f * g)(t) = \sum_{i=-\infty}^{\infty} f(i)g(t-i) = \sum_{i=-\infty}^{\infty} f(t-i)g(i)$$

2D image convolution

$$(I * K)(i,j) = \sum_{m} \sum_{n} I(m,n)K(i-m,j-n) = \sum_{m} \sum_{n} I(i-m,i-n)K(m,n)$$

K (3x3 filter)

K ₁₁	K ₁₂	K ₁₃
K ₂₁	K ₂₂	K ₂₃
K ₃₁	K ₃₂	K ₃₃

I (7x7 image)

<i>I</i> ₁₁	I ₁₂	I ₁₃	I ₁₄	I ₁₅	I ₁₆	I ₁₇
I ₂₁	I ₂₂	I ₂₃	I ₂₄	I ₂₅	I ₂₆	I ₂₇
I ₃₁	I ₃₂	I ₃₃	I ₃₄	I ₃₅	I ₃₆	I ₃₇
I ₄₁	I ₄₂	I ₄₃	I_{44}	I_{45}	I ₄₆	I ₄₇
I ₅₁	I ₅₂	I ₅₃	I ₅₄	I ₅₅	I ₅₆	I ₅₇
I ₆₁	I ₆₂	I ₆₃	I ₆₄	I ₆₅	I ₆₆	I ₆₇
I ₇₁	I ₇₂	I ₇₃	I ₇₄	I ₇₅	I ₇₆	I ₇₇

0 ₁₁	0 ₁₂	0 ₁₃	0 ₁₄	0 ₁₅
021	022	023	024	0 ₂₅
031	032	033	034	035
041	042	043	044	045
0 ₅₁	052	0 ₅₃	0 ₅₄	055

K (3x3 filter)

K ₁₁	K ₁₂	K ₁₃
K ₂₁	K ₂₂	K ₂₃
K ₃₁	K ₃₂	K ₃₃

I (7x7 image)									
K_{11}	1	$I_{12}^{K_1}$	2	$I_{13}^{K_1}$	3	I ₁₄	I ₁₅	I ₁₆	I ₁₇
$I_{21}^{K_{21}}$	1	$I_{22}^{K_2}$	2	$I_{23}^{K_2}$	3	I ₂₄	I ₂₅	I ₂₆	I ₂₇
$I_{31}^{K_{31}}$		$I_{32}^{K_3}$	2	$I_{33}^{K_3}$	3	I ₃₄	I ₃₅	I ₃₆	I ₃₇
I ₄₁		I ₄₂		I ₄₃		I ₄₄	I ₄₅	I ₄₆	I ₄₇
I ₅₁		I ₅₂		I ₅₃		I ₅₄	I ₅₅	I ₅₆	I ₅₇
I ₆₁		I ₆₂		I ₆₃		I ₆₄	I ₆₅	I ₆₆	I ₆₇
I ₇₁		I ₇₂		I ₇₃		I ₇₄	I ₇₅	I ₇₆	I ₇₇

Output (5x5)

r					
	011	012	013	014	015
	0 ₂₁	022	023	024	025
	031	032	033	034	035
	041	042	043	044	045
	0 ₅₁	052	0 ₅₃	0 ₅₄	0 ₅₅

 $O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$

K (3x3 filter)

K ₁₁	K ₁₂	K ₁₃
K ₂₁	K ₂₂	K ₂₃
K ₃₁	K ₃₂	К ₃₃

I (7x7 image)

(1111 1113.83)								
I ₁₁	$K_{1/1}$	K_{12}	K ₁₃	I ₁₅	I ₁₆	I ₁₇		
I ₂₁	$K_{2_{l_{22}}^1}$	$K_{\frac{2}{1}_{23}}$	K_{23}	I ₂₅	I ₂₆	I ₂₇		
I ₃₁	$K_{3_{1_{32}}}$	K ₃₂	K ₃₃	I ₃₅	I ₃₆	I ₃₇		
I ₄₁	I ₄₂	I ₄₃	I ₄₄	I ₄₅	I ₄₆	I ₄₇		
I ₅₁	I ₅₂	I ₅₃	I ₅₄	I ₅₅	I ₅₆	I ₅₇		
I ₆₁	I ₆₂	I ₆₃	I ₆₄	I ₆₅	I ₆₆	I ₆₇		
I ₇₁	I ₇₂	I ₇₃	I ₇₄	I ₇₅	I ₇₆	I ₇₇		

011	012	0 ₁₃	0 ₁₄	0 ₁₅
021	022	O_{23}	024	025
031	032	033	034	035
041	042	043	044	0 ₄₅
0 ₅₁	052	0 ₅₃	0 ₅₄	055

$$O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$$

$$O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$$

K (3x3 filter)

K ₁₁	K ₁₂	K ₁₃
K ₂₁	K ₂₂	K ₂₃
K ₃₁	K ₃₂	K ₃₃

	I (/x/ image)							
I ₁₁	I ₁₂	$K_{1/1_{1:3}}$	K ₁₂	K_{13}	I ₁₆	I ₁₇		
I ₂₁	I ₂₂	$K_{2_{l_{23}^{1}}}$	K_{22}	$K_{23} = K_{25}$	I ₂₆	I ₂₇		
I ₃₁	I ₃₂	$K_{3_{1_{33}}}$	K ₃₂	K ₃₃	I ₃₆	I ₃₇		
I ₄₁	I ₄₂	I ₄₃	I ₄₄	I_{45}	I ₄₆	I_{47}		
I ₅₁	I ₅₂	I ₅₃	I ₅₄	I ₅₅	I ₅₆	I ₅₇		
I ₆₁	I ₆₂	I ₆₃	I ₆₄	I ₆₅	I ₆₆	I ₆₇		
I ₇₁	I ₇₂	I ₇₃	I ₇₄	I ₇₅	I ₇₆	I ₇₇		

0 ₁₁	0 ₁₂	0 ₁₃	014	0 ₁₅
021	022	0 ₂₃	O_{24}	0 ₂₅
031	032	033	034	035
041	042	043	044	045
0 ₅₁	052	0 ₅₃	0 ₅₄	0 ₅₅

$$O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$$

$$O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$$

$$O_{13} = I_{13}K_{11} + I_{14}K_{12} + I_{15}K_{13} + I_{23}K_{21} + I_{24}K_{22} + I_{25}K_{23} + I_{33}K_{31} + I_{34}K_{32} + I_{35}K_{33} + bias$$

K (3x3 filter)

K ₁₁	K ₁₂	K ₁₃
K ₂₁	K ₂₂	K ₂₃
K ₃₁	K ₃₂	K ₃₃

I (7x7 image)

1 (777) 1114867							
<i>I</i> ₁₁	I ₁₂	I_{13}	$K_{\frac{11}{14}}$	K_{12}	$K_{\frac{13}{16}}$	I ₁₇	
I ₂₁	I ₂₂	I ₂₃	$K_{\frac{21}{I_{24}}}$	$K_{\frac{12}{125}}$	K_{23}	I ₂₇	
I ₃₁	I ₃₂	I ₃₃	$K_{\frac{31}{1_{34}}}$	K ₃₂	K ₃₃	I ₃₇	
I ₄₁	I ₄₂	I ₄₃	I_{44}	I ₄₅	I ₄₆	I ₄₇	
I ₅₁	I ₅₂	I ₅₃	I ₅₄	I ₅₅	I ₅₆	I ₅₇	
I ₆₁	I ₆₂	I ₆₃	I ₆₄	I ₆₅	I ₆₆	I ₆₇	
I ₇₁	I ₇₂	I ₇₃	I ₇₄	I ₇₅	I ₇₆	I ₇₇	

0 ₁₁	0 ₁₂	0 ₁₃	0 ₁₄	O ₁₅
021	022	O ₂₃	024	O ₂₅
031	032	033	034	035
041	042	043	044	045
0 ₅₁	052	0 ₅₃	0 ₅₄	0 ₅₅

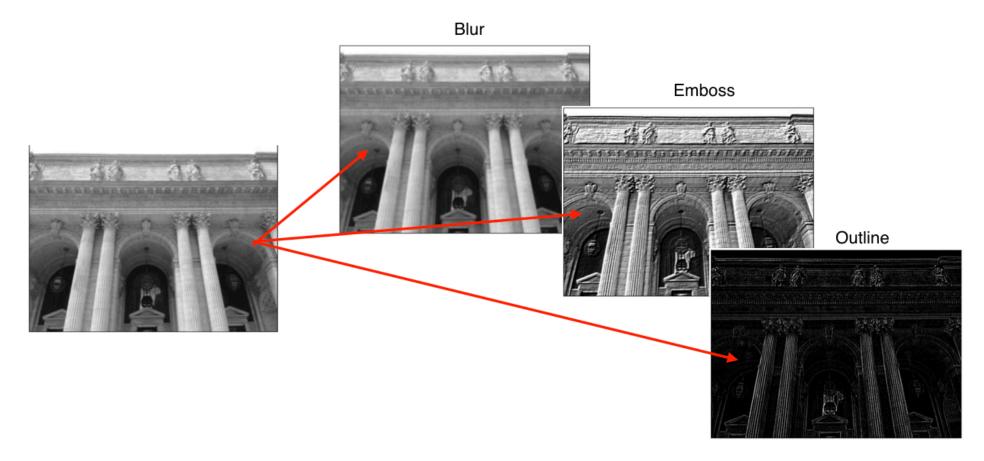
$$O_{11} = I_{11}K_{11} + I_{12}K_{12} + I_{13}K_{13} + I_{21}K_{21} + I_{22}K_{22} + I_{23}K_{23} + I_{31}K_{31} + I_{32}K_{32} + I_{33}K_{33} + bias$$

$$O_{12} = I_{12}K_{11} + I_{13}K_{12} + I_{14}K_{13} + I_{22}K_{21} + I_{23}K_{22} + I_{24}K_{23} + I_{32}K_{31} + I_{33}K_{32} + I_{34}K_{33} + bias$$

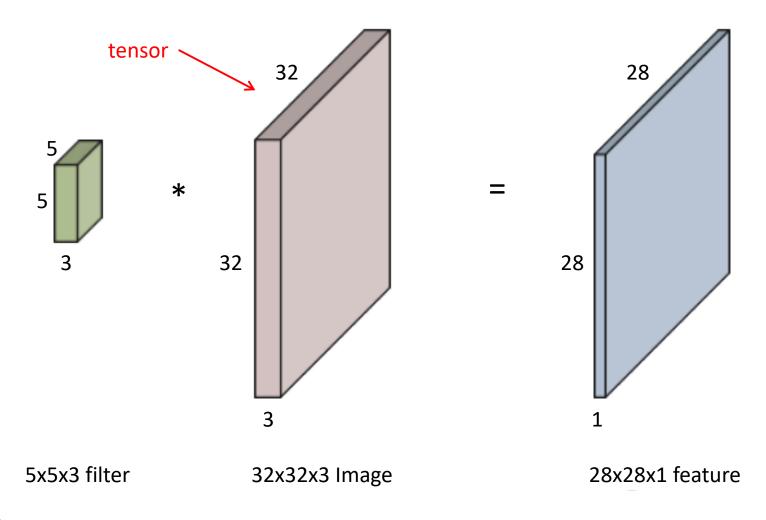
$$O_{13} = I_{13}K_{11} + I_{14}K_{12} + I_{15}K_{13} + I_{23}K_{21} + I_{24}K_{22} + I_{25}K_{23} + I_{33}K_{31} + I_{34}K_{32} + I_{35}K_{33} + bias$$

$$O_{14} = I_{14}K_{11} + I_{15}K_{12} + I_{16}K_{13} + I_{24}K_{21} + I_{25}K_{22} + I_{26}K_{23} + I_{34}K_{31} + I_{35}K_{32} + I_{36}K_{33} + bias$$

2D convolution in action

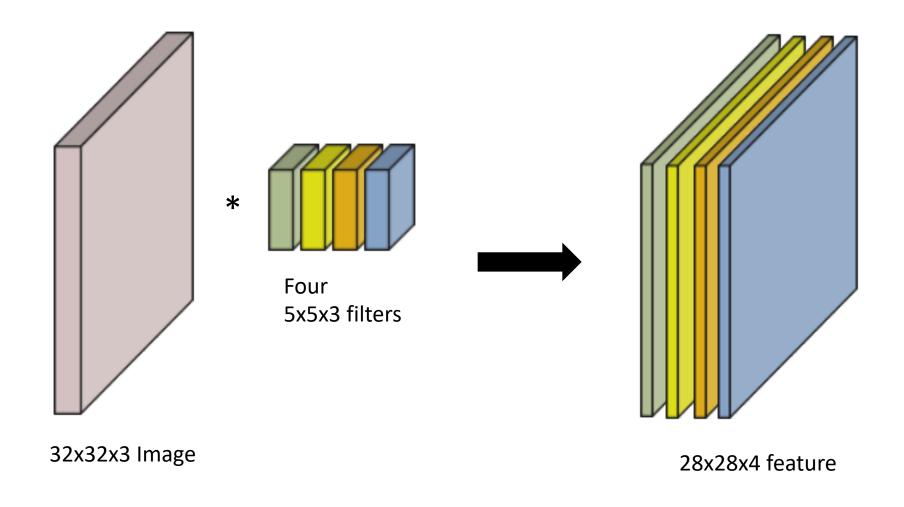


RGB Image Convolution



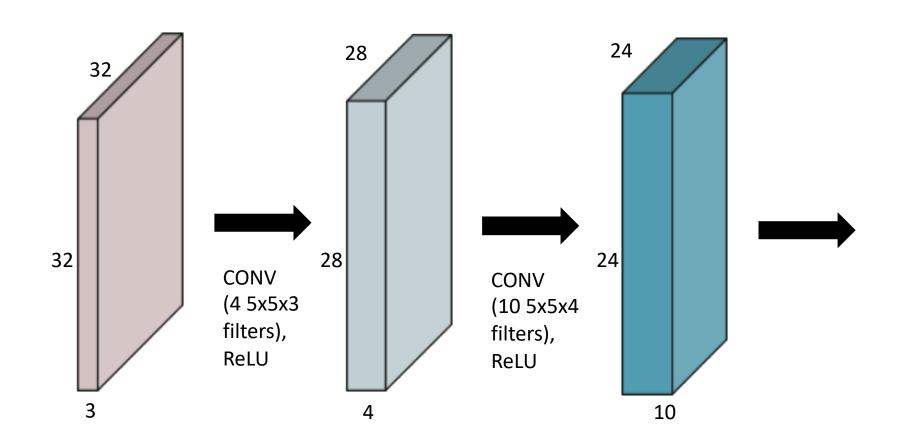


RGB Image Convolution



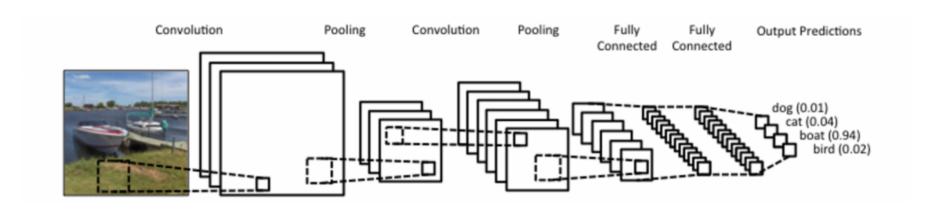


Stack of Convolutions



Convolutional Neural Networks

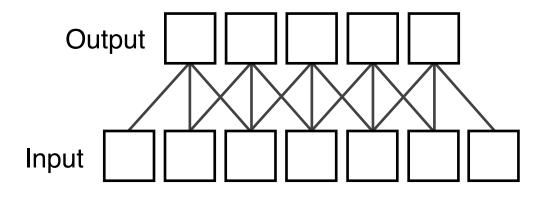
- CNN consists of convolution layer, pooling layer, and fully connected layer.
 - Convolution and pooling layers: feature extraction
 - Fully connected layer: decision making (e.g., classification)



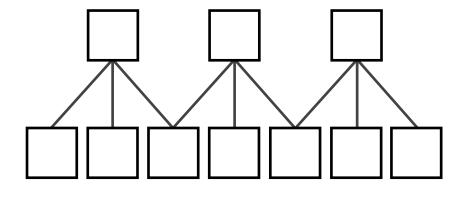
Convolution Arithmetic (of GoogLeNet)

	Stride	Channel	_						#	param	
type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	$7 \times 7/2$	$112{\times}112{\times}64$	1							2.7K	34M
max pool	$3\times3/2$	$56 \times 56 \times 64$	0								
convolution	$3\times3/1$	$56 \times 56 \times 192$	2		64	192				112K	360M
max pool	$3 \times 3/2$	$28 \times 28 \times 192$	0								
inception (3a)		$28 \times 28 \times 256$	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	$3\times3/2$	14×14×480	0								
inception (4a)		$14 \times 14 \times 512$	2	192	96	208	16	48	64	364K	73M
inception (4b)		$14 \times 14 \times 512$	2	160	112	224	24	64	64	437K	88M
inception (4c)		$14 \times 14 \times 512$	2	128	128	256	24	64	64	463K	100M
inception (4d)		$14 \times 14 \times 528$	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	$3 \times 3/2$	$7 \times 7 \times 832$	0								
inception (5a)		$7 \times 7 \times 832$	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	$1\times1\times1024$	0								
dropout (40%)		$1\times1\times1024$	0								
linear		1×1×1000	1							1000K	1M
softmax		$1\times1\times1000$	0								

Stride

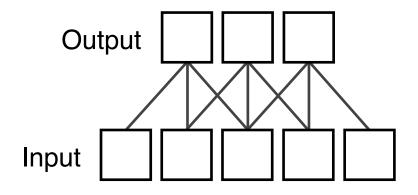




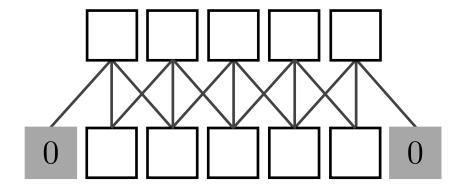


Stride = 2

Padding

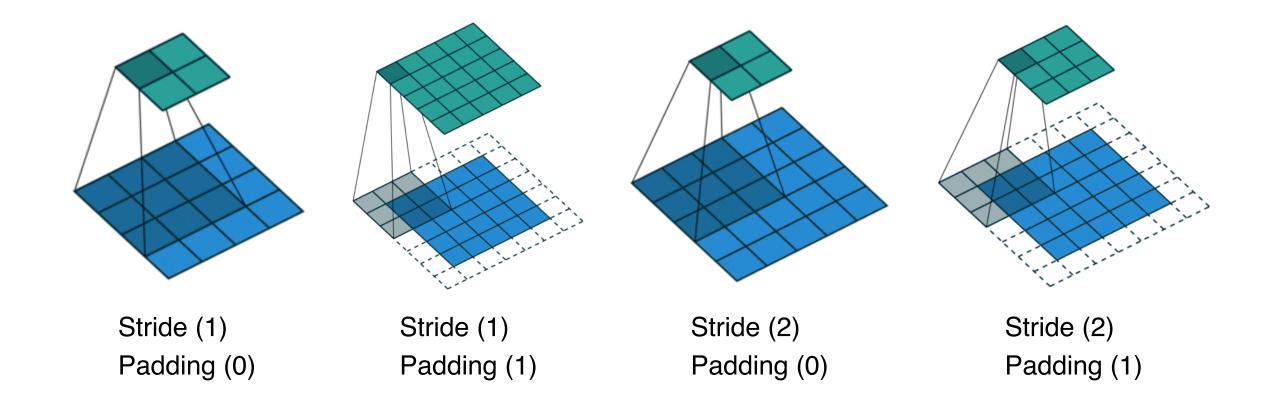


No padding (stride=1)



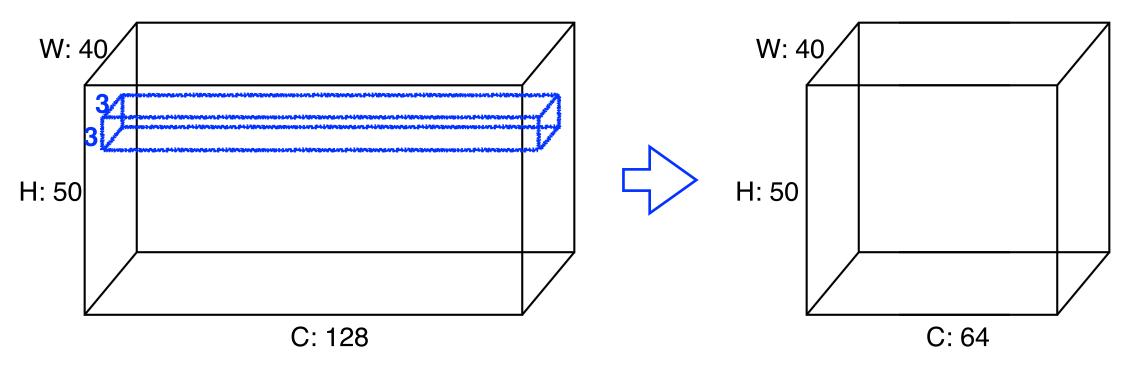
Zero padding (stride=1)

Stride? Padding?



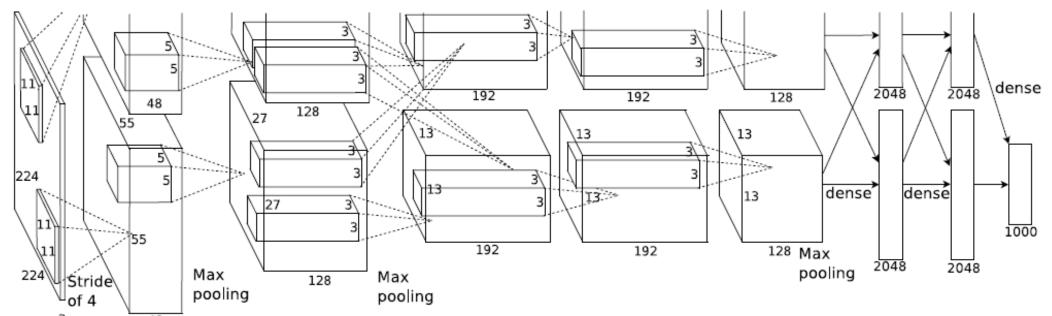
Convolution Arithmetic

 \bullet Padding (1), Stride (1), 3×3 Kernel



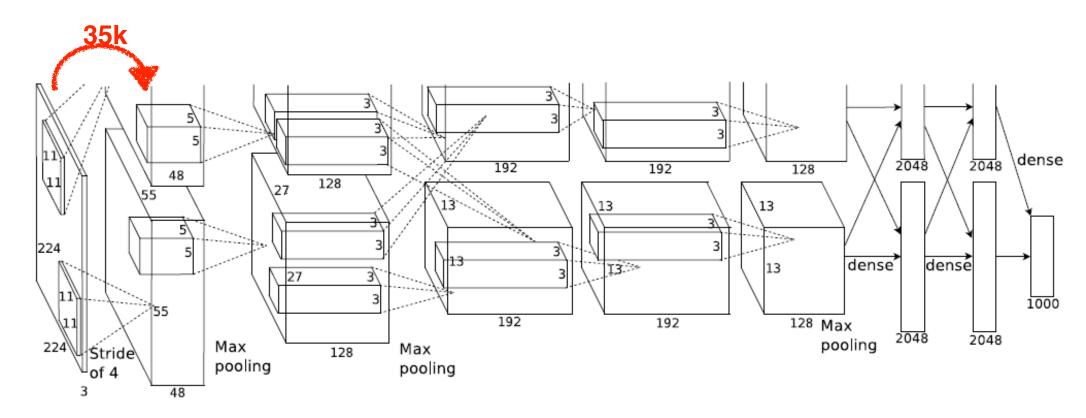
What is the **number of parameters** of this model?

The answer is $3 \times 3 \times 128 \times 64 = 73,728$



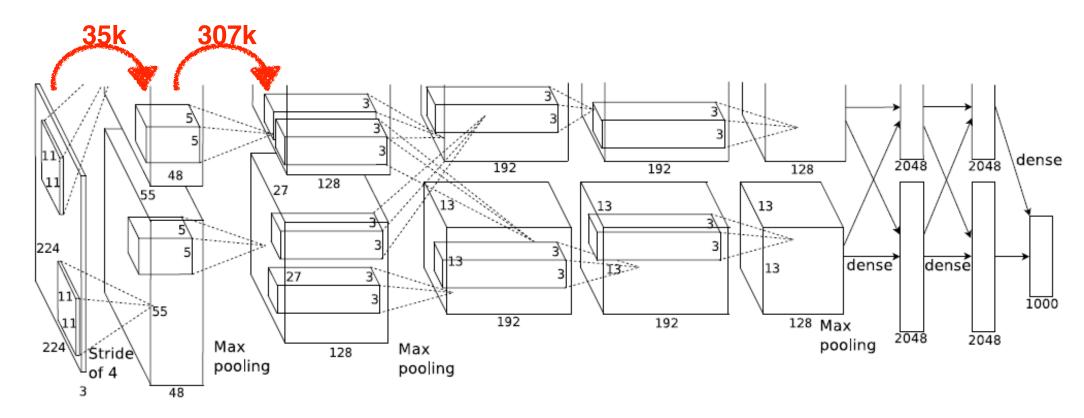
ImageNet Classification with Deep Convolutional Neural Networks, 2012





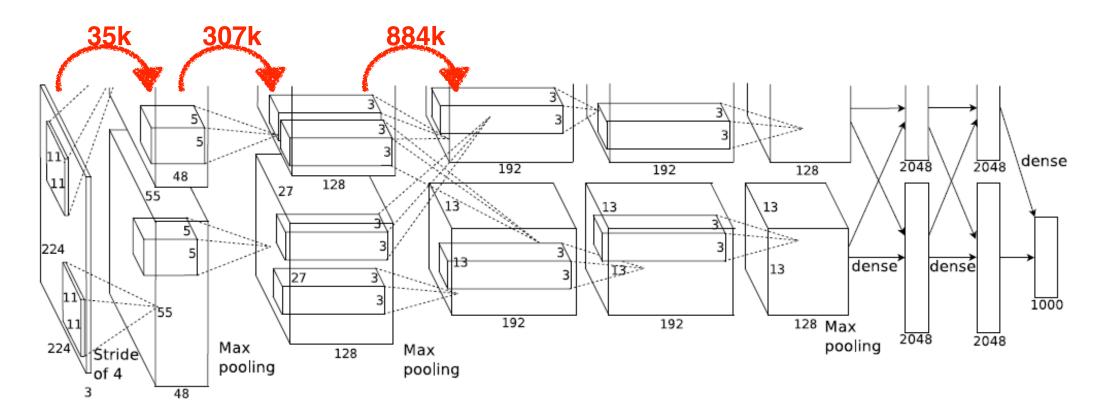
$$11 \times 11 \times 3 \times 48 * 2 \approx 35k$$





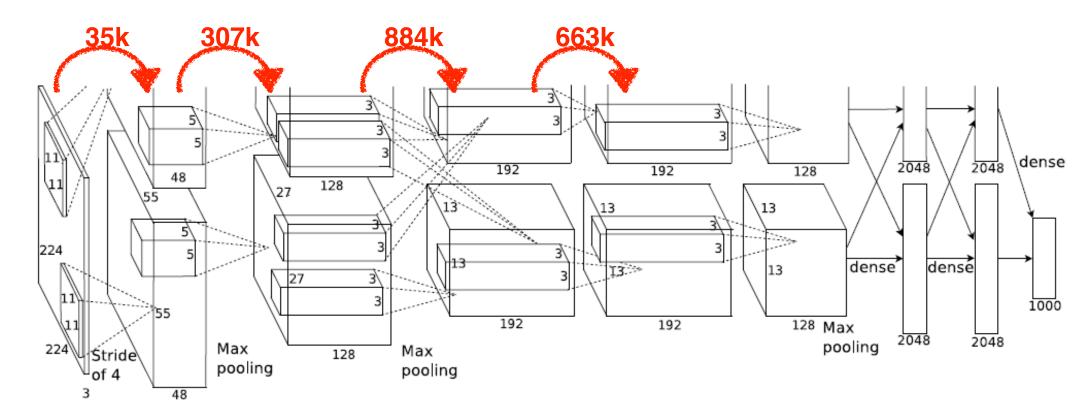
$$5 \times 5 \times 48 \times 128 * 2 \approx 307k$$





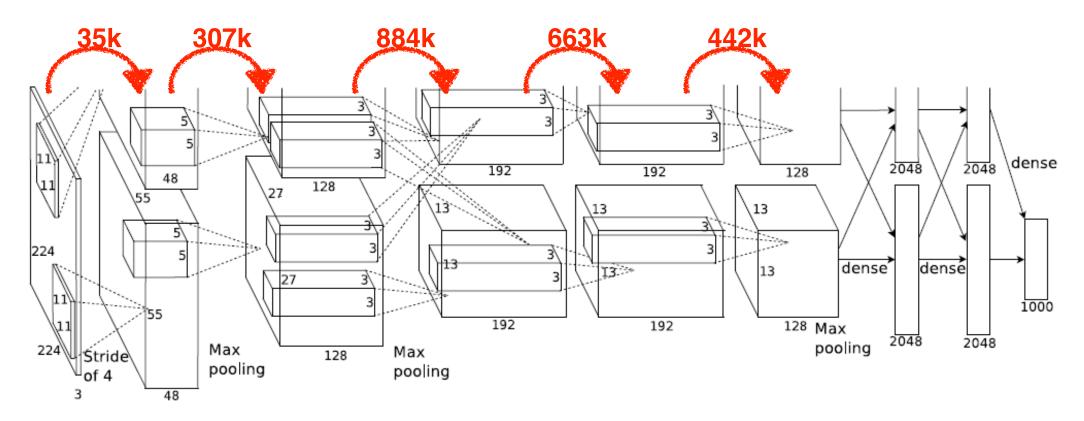
$$3 \times 3 \times 128 * 2 \times 192 * 2 \approx 884k$$



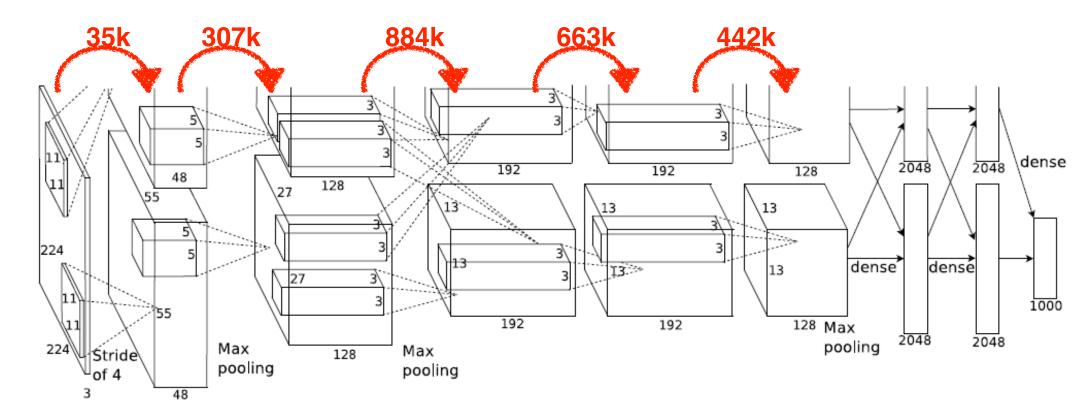


$$3 \times 3 \times 192 \times 192 * 2 \approx 663k$$



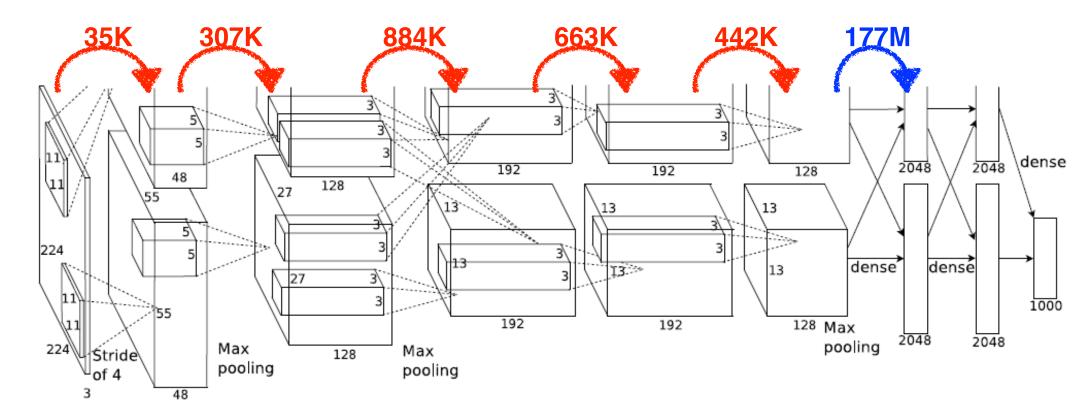


$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$



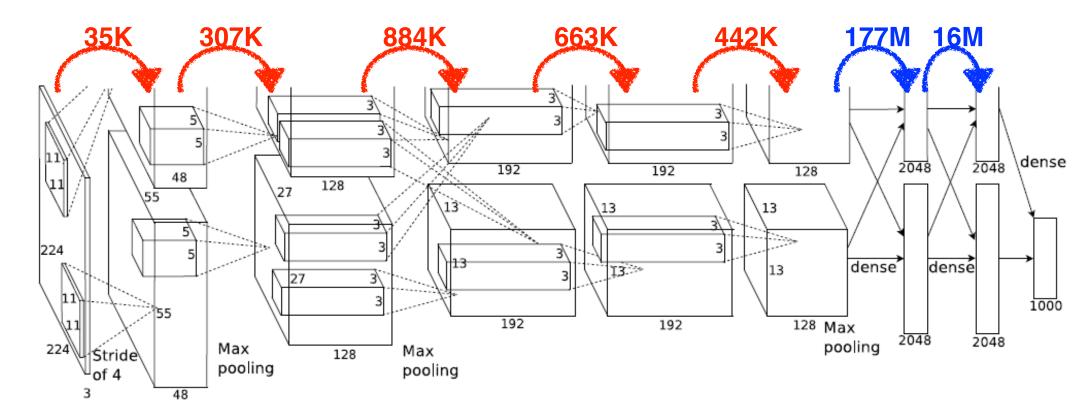
$$3 \times 3 \times 192 \times 128 * 2 \approx 442k$$





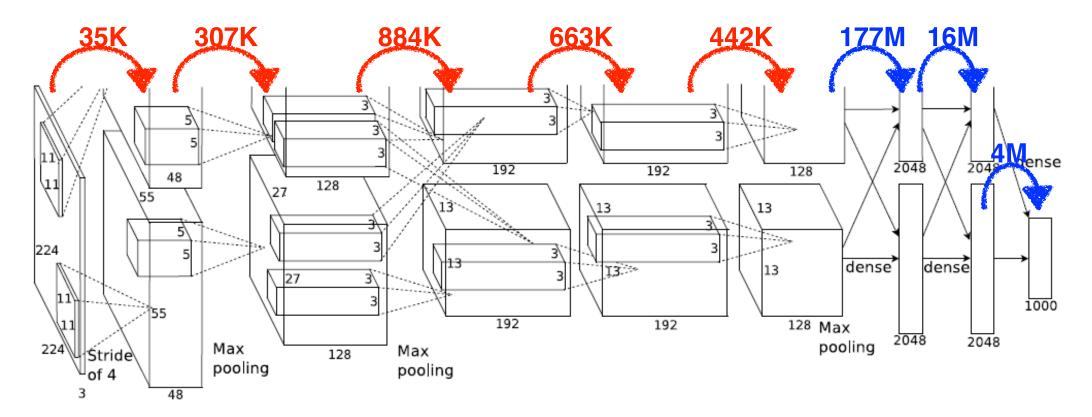
$$13 * 13 * 128 * 2 \times 2048 * 2 \approx 177M$$





$$2048 * 2 \times 2048 * 2 \approx 16M$$

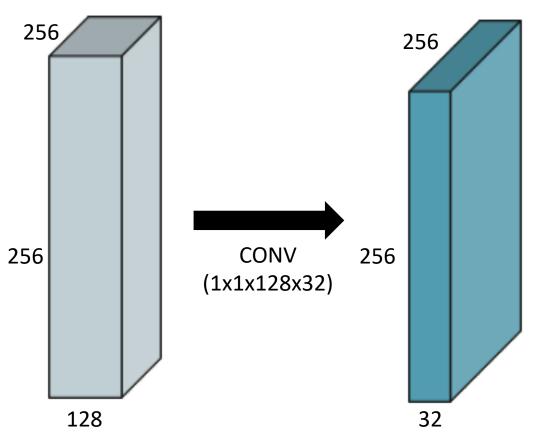




$$2048 * 2 \times 1000 \approx 4M$$



1x1 Convolution



Why?

- Dimension reduction
- To reduce the number of parameters while increasing the depth
- e.g., bottleneck architecture

Thank you for listening

