Stochastic Time-Variable Rainfall-Runoff Modeling

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SUMMARY A systematic procedure for modelling time-varying and sluggish streamflow responses is reported in this paper. The procedure involves splitting the streamflow into quick and slow response components which are then modelled separately. Adaptive estimation techniques for time-variable parameters are utilized to account for the seasonal variation of streamflow responses.

1 INTRODUCTION

Rainfall undergoes many processes before being transformed to streamflow. However, observations can only be made of the aggregate spatial and temporal effects of these processes, and this invariably leads to uncertainties in determining the dynamic relationships between streamflow and rainfall. Further uncertainties are introduced by insufficient and inaccurate monitoring of variables, due mainly to financial and technological limitations. Such uncertainties must therefore be accounted for in the development of any rainfall-runoff model.

Practical physically-based rainfall-runoff models, typified by the Stanford Watershed Model (Crawford and Linsley, 1966), cannot tolerate uncertainties effectively. In these models, some parameters are supposed to be determinable from catchment characteristics, but often a trial-and-error or automatic optimization method has to be used. This implies that the parameter values are chosen to reflect the average effect of aggregating the various rainfall-runoff processes, as observed in the data. In other words, in practice, one ultimately has to let the data "speak for themselves" as much as possible.

It is for this reason that stochastic transfer function modelling procedures, such as those of Young (1974), are becoming increasingly popular. This alternative technique is quick, relatively simple, and employs efficient parameter estimation techniques that account for uncertainties inherent in the data. However, the widespread application of this technique has been hampered by two shortcomings.

Firstly, the approach is not capable of dealing with catchments which have significant baseflow, because the flow at one instant depends on a long past history of rainfall. The degree of dependence decreases with time, and may be small but none-the-less still significant. Identifying and estimating this dependence, which is mixed with the strongly responding quick flow is difficult, especially when faced with the level of data uncertainties encountered in hydrology.

Secondly, the rainfall-runoff relationship is non-linear and strongly dependent on the wetness, or dryness, of the catchment. Heuristic trial-and-error methods are often used to transform the rainfall so that a linear model can be used to relate the streamflow to this transformed rainfall, (Whitehead and Young, 1975).

Techniques developed for overcoming these problems, so as to make the stochastic transfer function modelling approach a more attractive proposition, are outlined in this paper.

2 QUICK/SLOW RESPONSE SEPARATION

The dynamics which govern the transformation of rainfall to runoff often cover a broad spectrum of response time scales. On one hand, there are quick runoff components that respond within minutes or hours, and, on the other hand, there are baseflow components which take days or weeks to respond. The approach adopted in dealing with this problem was to separate the streamflow into quick and slow response components which were then modelled separately. The purpose of the separation was to enable aggregate data to be used so that the slow flow response dynamics could be identified better. However, separation of this slow flow is difficult because of the wide spectrum of response time-scales, and many conventional baseflow separation procedures are arbitrary and not suited to computer implementation (Argomaniz, 1970). An alternative separation method based on a recursive digital filter, commonly used in signal analysis and processing (Lynn, 1973), was developed. The filter was of the simple form:

$$f_k = a\, f_{k-1} + \frac{(1+a)}{2}\,(y_k - y_{k-1}) \qquad (1)$$

where, f_k is the filtered quick response at the k^{th} sampling instant.
y_k is the original streamflow.
and, a the filter parameter.

The output from this filter was constrained so that the separated slow flow was not negative or greater than the original streamflow. In addition, after the forward pass filtering, a reverse pass filter was applied to the slow flow starting from the end of the data. This was done to nullify any phase distortion of the data due to the forward pass filter (i.e. to prevent any shift in the position of the peaks and troughs; see Kormylo and Jain, 1974). Furthermore, once separated, the slow flow could be passed through the same filtering procedure again for further separation. Figure 1 gives an illustration of the separated flow for the Harvey River, Dingo Road catchment of Western Australia.

The choice of an appropriate set of separated components was not based on a consideration of time scales, but rather on which set of components

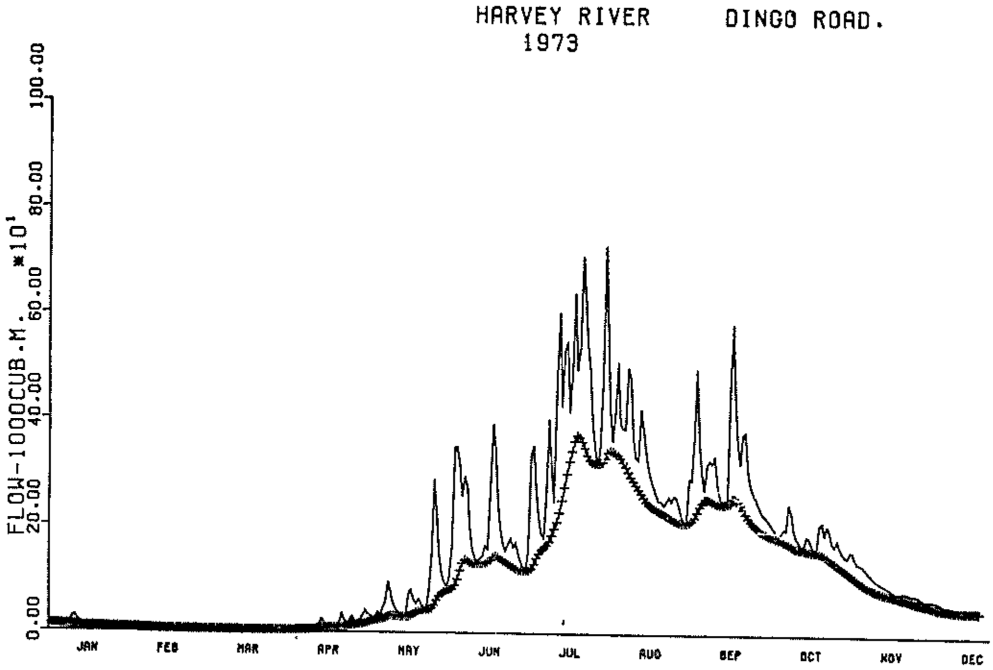

Figure 1. Illustration of the slow flow separation filter performance. Two stage filter with a = 0.8.

could be modelled best. A good range of separated components can be achieved by successive separation with a filter parameter between 0.75 and 0.9. However, Lyne (1979) has shown that, for any reasonable components, the performance of the final models arrived at may be much the same. Lyne also proposes an alternative method based on "impulse response separation" for choosing a set of separated components.

3 RAINFALL TRANSFORMATION

The solution to the problem of rainfall transformation was based on tracking parameter variations by an adaptive estimation algorithm and modelling these variations as a function of rainfall.

Initially, the standard Box and Jenkins (1970) method of correlation analysis for ARMA (Autoregressive Moving Average) models was used to identify a base model for the quick flow. Here, identification refers to the process of determining how many autoregressive and moving average terms would be required in a model of the form :

$$y_k = a_1 y_{k-1} + a_2 y_{k-2} + \dots + a_r y_{k-r} + b_o x_k + b_1 x_{k-1} + \dots + b_s x_{k-s} + e_k \qquad (2)$$

where the autoregressive terms are those involving the flow (y_k) which describe the decay characteristics, or autoregression, of the flow; and the moving average terms (x_k) describe the initial dependence of the flow on the rainfall. The term (e_k) is the stochastic noise. Equation 2 can be expressed in the more convenient vector form :

$$y_k = \underline{z}_k^T \underline{a} + e_k \qquad (3)$$

where,

$$\underline{z}_k^T = [y_k, \dots, y_{k-r}, x_k, \dots x_{k-s}] \; ;$$

$$\underline{a}^T = [a_1, \dots, a_r, b_o, \dots, b_s]$$

Once the number of AR and MA terms required in the model had been identified, their values were estimated using the iterative Instrumental Variable (IV) algorithm of Young (1974). The algorithm shown in Figure 2 operates through the agency of an auxiliary model whose function is to provide an output that is highly correlated with the noise-free system output (flow). The parameters of the model are derived from the most recent estimates provided by the algorithm. At each instant, the algorithm estimates the parameters from the difference between the observed flow and the output from the auxiliary model. The correction is of the form :

$$\hat{\underline{a}}_k = \hat{\underline{a}}_{k-1} + \underline{w}_k \left\{ y_k - \hat{y}_k \right\} \qquad (4)$$

where, $\hat{y}_k = \hat{\underline{z}}_k^T \hat{\underline{a}}_{k-1}$ is the output from the auxiliary model.
Estimated variables are hatted.

$\underline{w}_k$ is a vector of weights which is applied to the error in order to update the parameter estimates. The magnitudes of the weights are estimated by a recursive algorithm which takes account of the accuracy of the estimates. For normal recursive least-squares estimation, $\underline{w}_k$ decreases in magnitude as more data come to hand i.e., as the estimates of $\underline{a}_k$ become more accurate. However, for estimating parameter variations, $\underline{w}_k$ must be artificially increased so as to give more emphasis to recent data. A simple way of doing this is to assume that the parameters vary as random walks, i.e.

$$\underline{a}_k = \underline{a}_{k-1} + \underline{v}_k \qquad (5)$$

where, $\underline{v}_k$ is a vector of serially uncorrelated noise. The uncertainty in the parameter estimates resulting from the addition of the noise $\underline{v}_k$ provides the means by which $\underline{w}_k$ can be increased at each instant. Thus, more weight is given to recent data and parameter variations can be detected (see Young (1974) for a full discussion). Better estimates of the parameter variations can be obtained by using a fixed-interval smoothing algorithm (Meditch, 1973; Gelb, 1974). This algorithm has the ability to use all of the available information in contrast to recursive algorithms which can only use past and present infor-

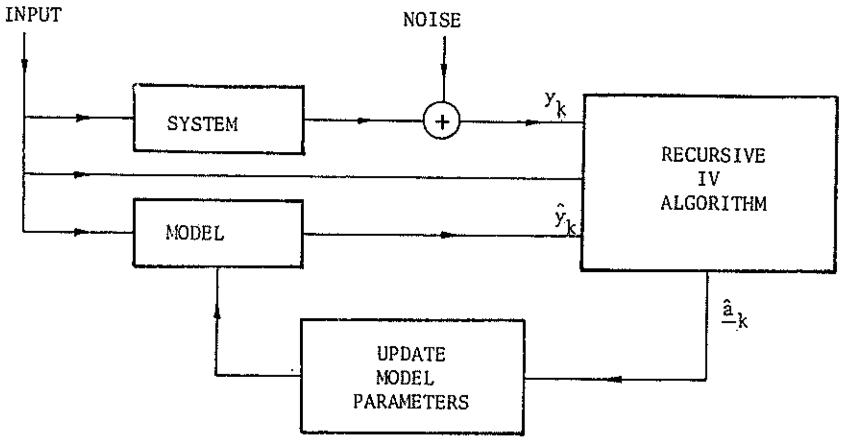

Figure 2. The Instrumental Variable estimation algorithm of Young (1974).

mation. Smoothing consists essentially of a forward and reverse recursive filter and smoothed parameter estimates at each instant are a weighted average of the estimates from the two filter passes (see Norton, 1975 and Lyne, 1979 for applications of smoothing algorithms to rainfall-runoff data).

Once the parameter variations have been determined, there are many ways in which they can be modelled as a function of rainfall. One scheme involves modelling the peak response of the impulse response (unit hydrograph) as a function of rainfall. (The impulse response can be determined from the recursive equation (2) with $x_o = 1$ and $x_k = 0$ for $k>o$). The transformed rainfall is then obtained by multiplying the modelled peak variation by the original rainfall. Another scheme may be obtained by noting that the volume of runoff per unit rainfall (V) is:

$$V = \frac{\text{Sum of MA Parameters}}{1 - \text{Sum of AR Parameters}} \qquad (6)$$

The modelled variation of V can then be used to transform the rainfall in the same manner as for the peak variation. The choice of a transformation scheme depends on the intended use of the model, i.e., whether it is for forecasting peak floods, monthly or annual flow volumes and so on.

Conceptually, the procedure of rainfall transformation obtained by using, for example, the peak variation, is similar to the Antecedent Precipitation Index method commonly used in hydrology. However, the procedure described here has the great advantage that the transformation model can be tailor-made to suit a particular catchment. Furthermore, systematic model identification techniques and efficient recursive parameter estimation algorithms can be used to obtain the transformation model.

4 APPLICATION

Once the transformed rainfall had been obtained, it was used to model the quick flow using correlation analysis for identifying the model, and the IV algorithm for estimating the parameters. While the same rainfall transformation procedure could be repeated for the slow flow, it was found that the transformed rainfall for the quick flow could also be used for the slow flow. The only difference in modelling the slow flow was that the data had to be aggregated so as to amplify the weak dependence between the rainfall and slow flow. The scale of aggregation was chosen as a compromise between the resulting loss of detail and the enhancement of the dominant slow flow response characteristics (Lyne, 1979). Typical scales of aggregation used ranged from 5 to 30 days. It is also shown in Lyne (1979) that it is possible to disaggregate the slow flow model back to its basic time unit, so that predictions of the total flow can be obtained by combining the output from the quick and disaggregated slow flow models.

The modelling procedure outlined above has been applied to several catchments of the South West Coastal Drainage Division of Western Australia, and the preliminary results have been encouraging. Figure 3, for example shows a comparison of the observed and predicted flows for the sluggish Dwellingerup Brook catchment (1.35 km^2). The data for this catchment were of poor quality with many days of missing record. Weeks (1977) tested the performance of several conceptual models, and one simple time series model, on Dwellingerup Brook, and the results obtained by using the modelling procedure reported here compare very favourably with the best of those obtained by Weeks (Lyne, 1979).

In general, large catchments were found to be easier to model than some small catchments which seemed to have a fast wetting up behaviour, due to large storms, superimposed on the slower seasonal wetting behaviour. Here again, the problem is the range of response time scales, and it may be possible to separately model these effects by separating the quick and slow parameter variations. The modelling could be done in the same way as for the quick and slow flow.

Another problem is the effect of evaporation. The inclusion of evaporation in the transformation model can give improved results (Lyne, 1979), but identifying and estimating the effect is difficult. This is because evaporation is a slowly varying variable and it has its greatest effects in summer when the catchment is relatively passive. The problem needs further investigation involving the use of calibration data with some rainfall in summer.

5 CONCLUSIONS

An outline of a new procedure for stochastic transfer function modelling of the rainfall-runoff process has been presented in this paper. The procedure allows for the separate modelling of stream-

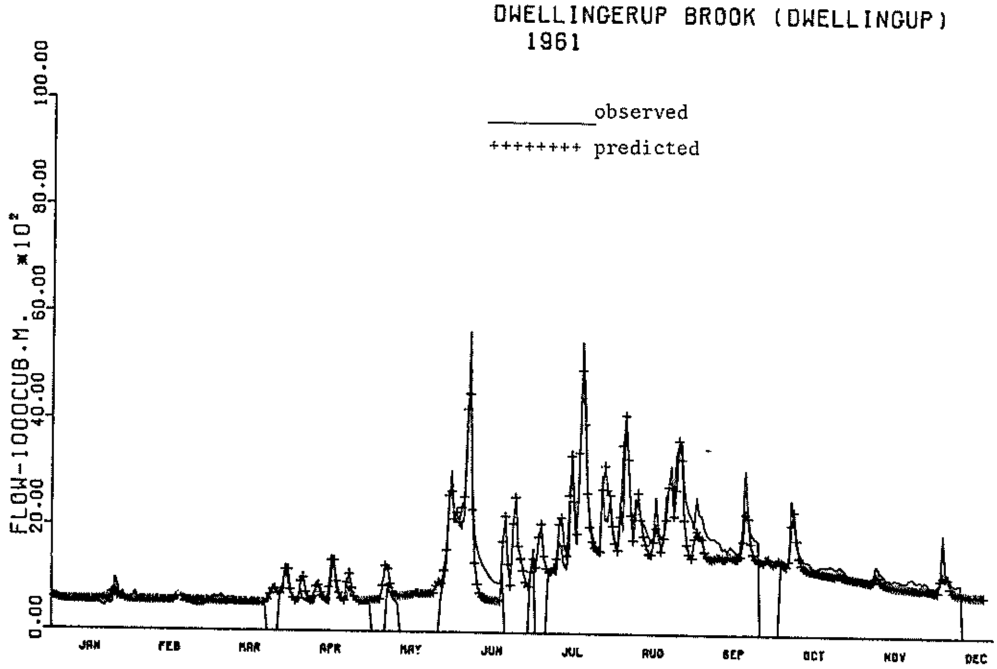

Figure 3. Comparison of observed and predicted flow for the Dwellingerup Brook catchment. Gaps in the observed flow correspond to missing flow data.

flow components exhibiting significantly different response time scales. In addition, non-linear streamflow responses can be modelled by adaptively estimating time-varying parameters, and modelling these as a function of rainfall. Preliminary applications of the procedure have provided encouraging results and it promises to offer the rainfall-runoff modeller a systematic and quick method of constructing a model. Furthermore, the procedure may also be applicable to other non-linear hydrologic processes which exhibit differing response time scales.

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7 REFERENCES

ARGOMANIZ, R. (1970). Separation of quick flow on small watersheds. M.Sc.Thesis, Penn. State Univ., Grad. Sch., Dept. of Civil Eng.

BOX, G.E.P. and JENKINS, G.M. (1970). Time series analysis: forecasting and control. San Francisco Calif., Holden-Day.

CRAWFORD, N.H. and LINSLEY, R.K. (1966). Digital simulation in Hydrology. Stanford Watershed Model IV. Tech Report No.39, Stanford Univ., Dept. of Civil Eng.

GELB, A. (Ed) (1974). Applied optimal estimation Analytic Sciences Corporation, Cambridge Mass., MIT Press.

KORMYLO, J.J. and JAIN, V.K. (1974). 2-pass recursive digital-filter with zero phase-shift. IEEE Acoust., Vol. AS22 (5), pp.384-385.

LYNE, V.D. (1979). Recursive modelling of sluggish and time-varying streamflow responses. M. Eng.Sc. Thesis, Univ. of Western Aust., Dept.of Civil Eng.

LYNN, P.A. (1973). An introduction to the analysis and processing of signals. London, Macmillan.

MEDITCH, J.S. (1973). A survey of data smoothing for linear and non-linear dynamic systems. Automatica Vol.9, pp 151-162.

NORTON, J.P. (1975). Optimal smoothing in the identification of linear time-varying systems. Proc. IEEE., Vol.122, No.6., pp 663-338 .

WEEKS, W.D. (1977). A comparison of rainfall-runoff models for the south-west of Western Australia. M. Eng. Sc. Thesis, Univ. of Western Aust., Dept. of Civil Eng.

WHITEHEAD, P.G. and YOUNG, P.C. (1975). A dynamic stochastic model for water quality in part of the Bedford-Ouse river system. In. G.C.Vansteenkiste (Ed.), Computer Simulation of Water Resources Systems. North Holland/American Elsevier, Amsterdam/ New York.

YOUNG, P.C. (1974). Recursive approaches to time-series analysis. Bulletin of Inst. Maths. and its Applications, Vol. 10, pp 209-224.