Číslo cvičení: 2
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1. (a)
$$P = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$
   (b) $C = \{0000, 1022, 0121, 1110, 2102, 1201, 2011, 0212, 2220\}$

0000	1022	0121	1110	2102	1201	2011	0212	2220	00
1000	2022	1121	2110	0102	2201	0011	1212	0220	11
0100	1122	0221	1210	2202	1001	2111	0012	2020	12
0010	1002	0101	1120	2112	1211	2021	0222	2200	10
0001	1020	0122	1111	2100	1202	2012	0210	2221	01
2000	0022	2121	0110	1102	0201	1011	2212	1220	22
0200	1222	0021	1010	2002	1101	2211	0112	2120	21
0020	1012	0111	1100	2122	1221	2001	0202	2210	20
0002	1021	0120	1112	2101	1200	2010	0211	2222	02

   (c) 0201→1201, 1111→1110.

2. (a) Basis of such code can be in the form (AA), $(0A)$ or $(A0)$, the last two being equivalent. The respective codes are therefore: $C_1 = (00, 11, 22, 33)$, $C_2 = (00, 01, 02, 03)$. Hence possible values for d are 1, 2.
   (b) Basis of such code is in the form $G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The only possible q to generate exactly 4 codewords is 2. $C = \{00, 01, 10, 11\}$.

3. (a) Because all operations are applied separately on columns "concateration" of two codes results in $n = n_1 + n_2$, $k = k_1 = k_2$, $d = d_1 + d_2$.
   (b) $n = n_1 + n_2$.
   All rows in G' are linearly independent, so $k = k_1 + k_2$. Codewords generated in the form $x_1 \dots x_{n_1} 0 \dots 0$ will exist in the resulting code. Similarly $0 \dots 0 x_1 \dots x_{n_2}$. Therefore $d = min\{d_1, d_2\}$.

4. (a)
$$G \sim \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$
   (b) C has $q^k = 3^3 = 27$ codewords.
   (c) $w(C) = 2 = h(C) < 3 \Rightarrow C$ cannot correct any errors.

5. $R(1, m)$ is obtained by "adding" repetition codes to $R(1, 1)$ like this: $R(1, 1) + R(0, 1) = R(1, 2)$, $R(1, 2) + R(0, 2) = R(1, 3), \dots$ Nuber of cosets generaly equals to q^{n-k}. $q = 2, n = 2^m, k = 1 + \binom{m}{1} = 1 + m \Rightarrow$ number of cosets $= 2^{2^m - m - 1}$.

6. (a) Both conditions of the linear code definition are made using bit-by-bit operations. Removing one bit therefore cannot break any of them.
   (b) $n^i = n - 1$.
   If the removed coordinate has a "twin" bit in the basis (i.e. all bits in a repetition code are "twins"), $k^i = k$. If the basis is like G from 2. (a),$k^i = k - 1$. No other options are possible, because removing one coordinate cannot decrease dimension by 2 or more.
   For similar reasons $d^i = d$ or $d^i = d - 1$.

7. Such codes can be generated by this general $\frac{n}{2} \times n$ basis matrix:
$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 1 & \cdots & 0 \\ \vdots & & & \ddots & & \\ 0 & \cdots & 0 & 0 & 1 & 1 \end{pmatrix}$$