

# Neural Networks

Wei Xu

(many slides from Greg Durrett and Philipp Koehn)

# This Lecture

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- ▶ Neural network history
- ▶ Neural network basics
- ▶ Feedforward neural networks + backpropagation
- ▶ Applications
- ▶ Implementing neural networks (if time)

# A Bit of History

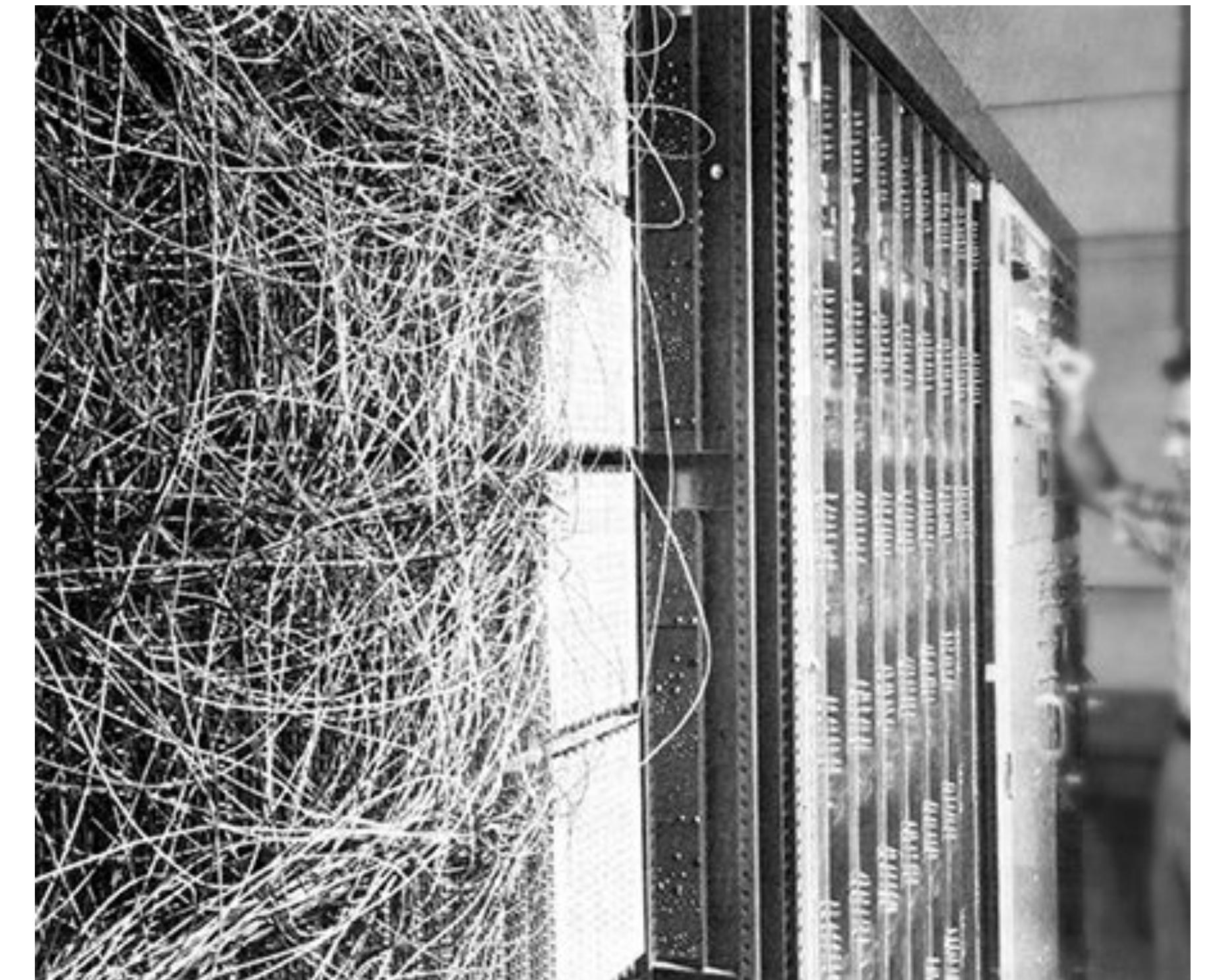
- ▶ The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.
- ▶ Perceptron (Frank Rosenblatt, 1957)
- ▶ Artificial Neuron (McCulloch & Pitts, 1943)

McCulloch Pitts Neuron  
(assuming no inhibitory inputs)

$$y = 1 \quad if \sum_{i=0}^n x_i \geq 0$$
$$= 0 \quad if \sum_{i=0}^n x_i < 0$$

Perceptron

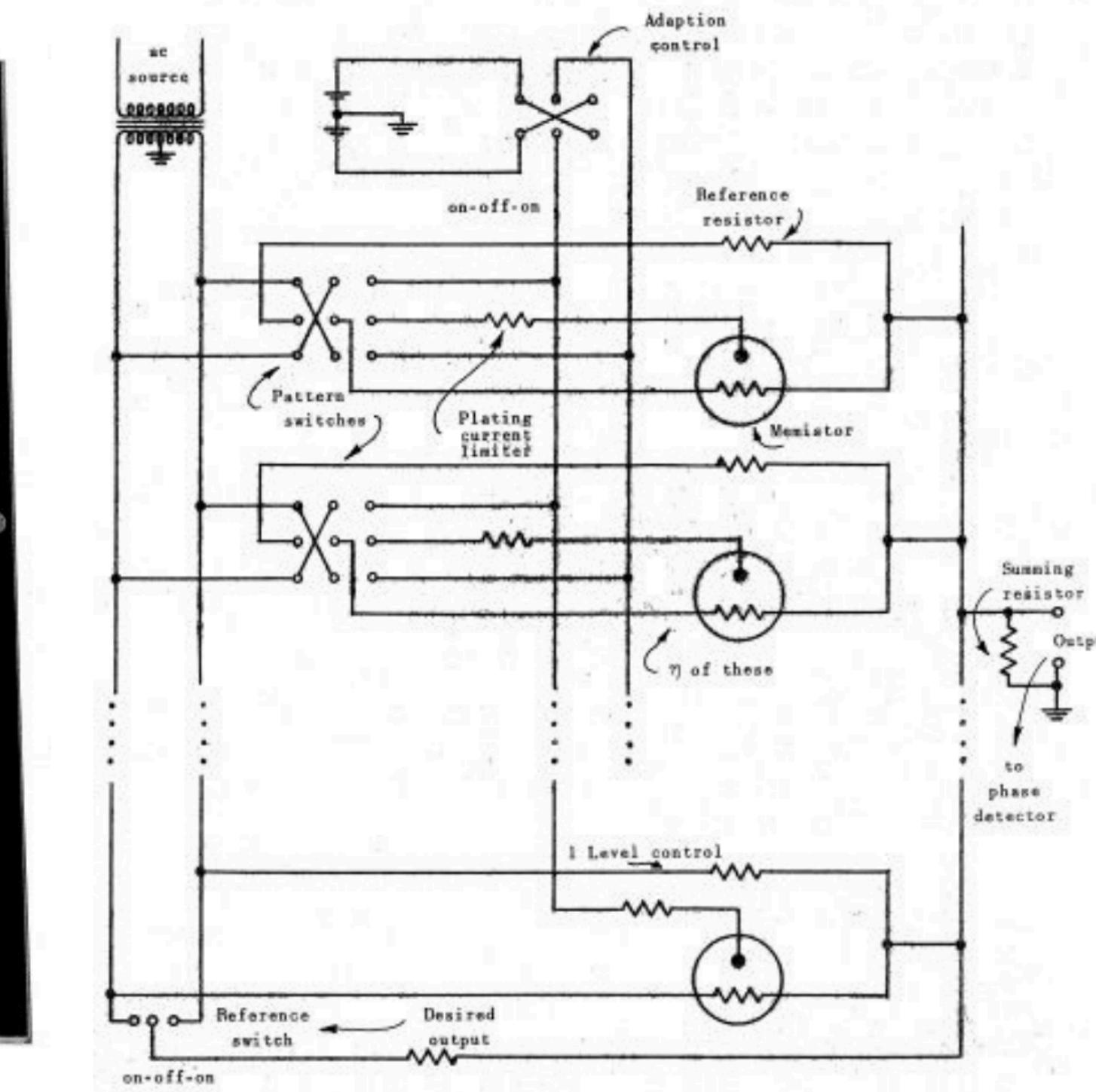
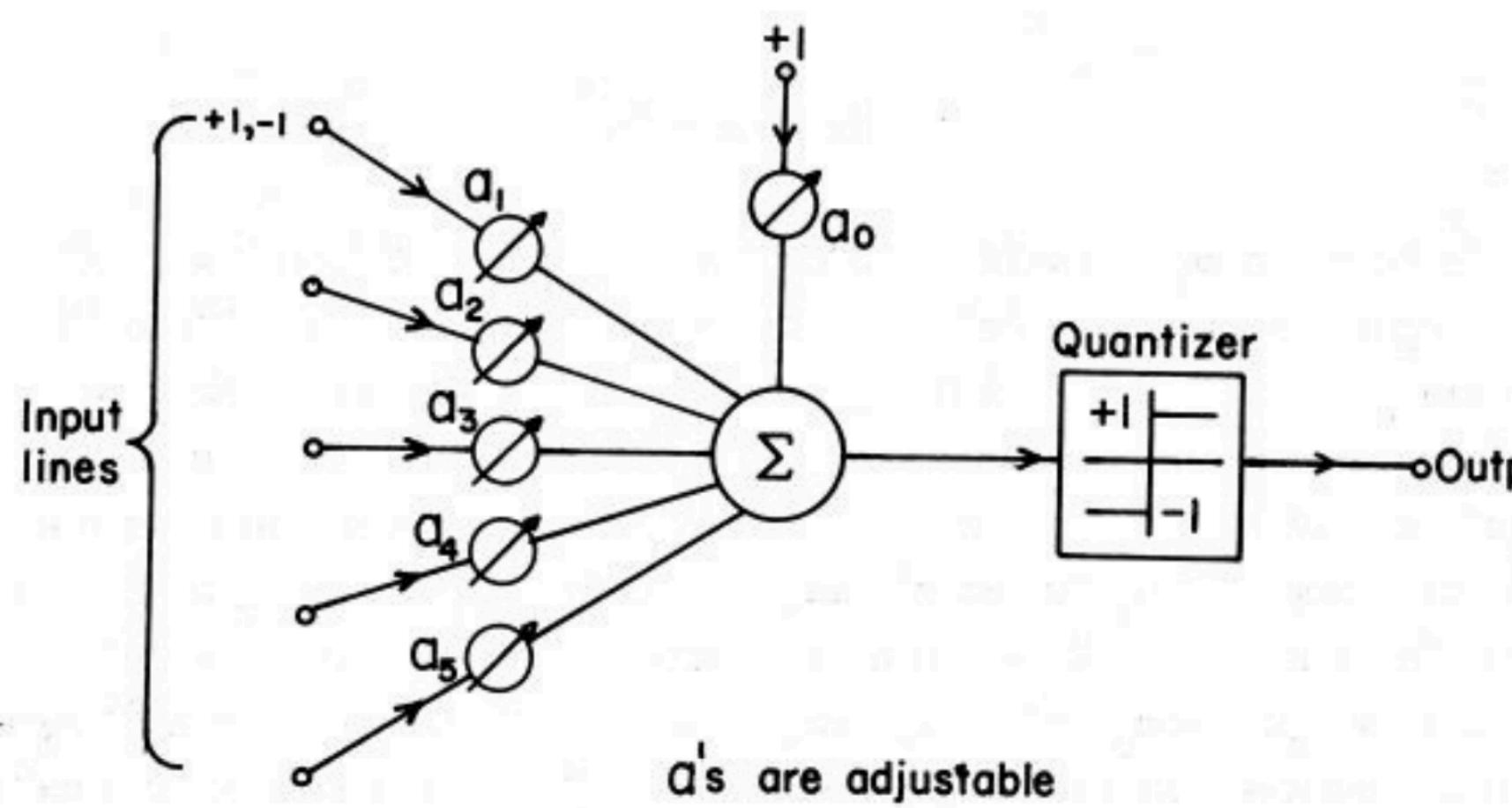
$$y = 1 \quad if \sum_{i=0}^n w_i * x_i \geq 0$$
$$= 0 \quad if \sum_{i=0}^n w_i * x_i < 0$$



The IBM Automatic Sequence Controlled Calculator, called Mark I by Harvard University's staff. It was designed for image recognition: it had an array of 400 photocells, randomly connected to the "neurons". Weights were encoded in potentiometers, and weight updates during learning were performed by electric motors.

# A Bit of History

- ▶ Adaline/Madeline - single and multi-layer “artificial neurons”  
(Widrow and Hoff, 1960)



# A Bit of History

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## ► First time back-propagation became popular (Rumelhart et al, 1986)

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### Learning representations by back-propagating errors

David E. Rumelhart\*, Geoffrey E. Hinton†  
& Ronald J. Williams\*

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San Diego, La Jolla, California 92093, USA

† Department of Computer Science, Carnegie-Mellon University,  
Pittsburgh, Philadelphia 15213, USA

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We describe a new learning procedure, back-propagation, for networks of neurone-like units. The procedure repeatedly adjusts the weights of the connections in the network so as to minimize a measure of the difference between the actual output vector of the net and the desired output vector. As a result of the weight adjustments, internal 'hidden' units which are not part of the input or output come to represent important features of the task domain, and the regularities in the task are captured by the interactions of these units. The ability to create useful new features distinguishes back-propagation from earlier, simpler methods such as the perceptron-convergence procedure<sup>1</sup>.

There have been many attempts to design self-organizing neural networks. The aim is to find a powerful synaptic modification rule that will allow an arbitrarily connected neural network to develop an internal structure that is appropriate for a particular task domain. The task is specified by giving the desired state vector of the output units for each state vector of the input units. If the input units are directly connected to the output units it is relatively easy to find learning rules that iteratively adjust the relative strengths of the connections so as to progressively reduce the difference between the actual and desired output vectors<sup>2</sup>. Learning becomes more interesting but

more difficult when we introduce hidden units whose actual or desired states are not specified by the task. (In perceptrons, there are 'feature analysers' between the input and output that are not true hidden units because their input connections are fixed by hand, so their states are completely determined by the input vector: they do not learn representations.) The learning procedure must decide under what circumstances the hidden units should be active in order to help achieve the desired input-output behaviour. This amounts to deciding what these units should represent. We demonstrate that a general purpose and relatively simple procedure is powerful enough to construct appropriate internal representations.

The simplest form of the learning procedure is for layered networks which have a layer of input units at the bottom; any number of intermediate layers; and a layer of output units at the top. Connections within a layer or from higher to lower layers are forbidden, but connections can skip intermediate layers. An input vector is presented to the network by setting the states of the input units. Then the states of the units in each layer are determined by applying equations (1) and (2) to the connections coming from lower layers. All units within a layer have their states set in parallel, but different layers have their states set sequentially, starting at the bottom and working upwards until the states of the output units are determined.

The total input,  $x_j$ , to unit  $j$  is a linear function of the outputs,  $y_i$ , of the units that are connected to  $j$  and of the weights,  $w_{ji}$ , on these connections

$$x_j = \sum_i y_i w_{ji} \quad (1)$$

Units can be given biases by introducing an extra input to each unit which always has a value of 1. The weight on this extra input is called the bias and is equivalent to a threshold of the opposite sign. It can be treated just like the other weights.

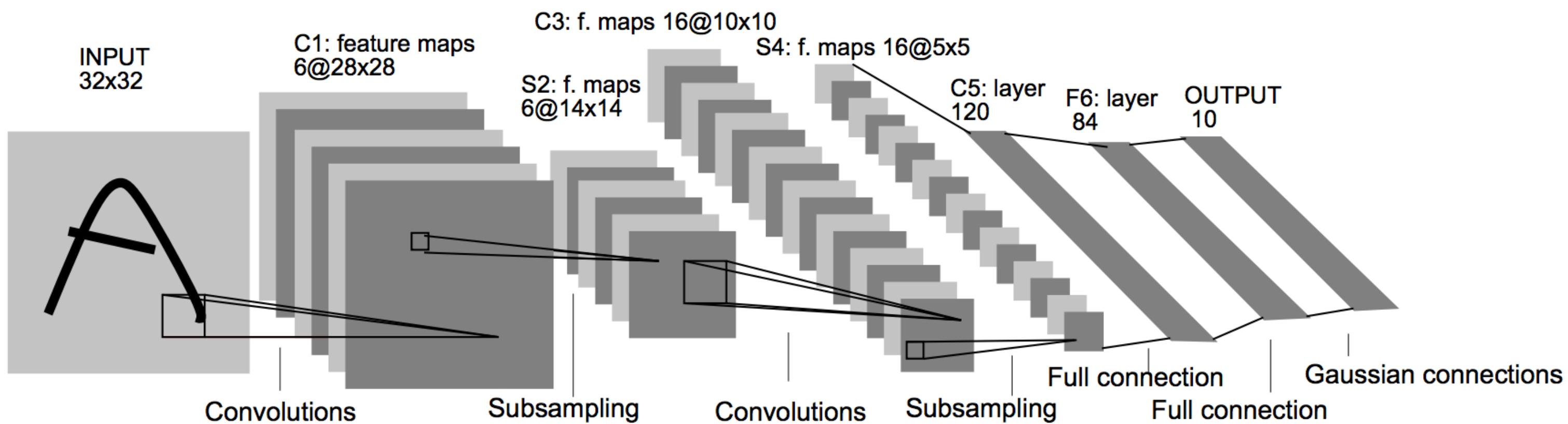
A unit has a real-valued output,  $y_j$ , which is a non-linear function of its total input

$$y_j = \frac{1}{1 + e^{-x_j}} \quad (2)$$

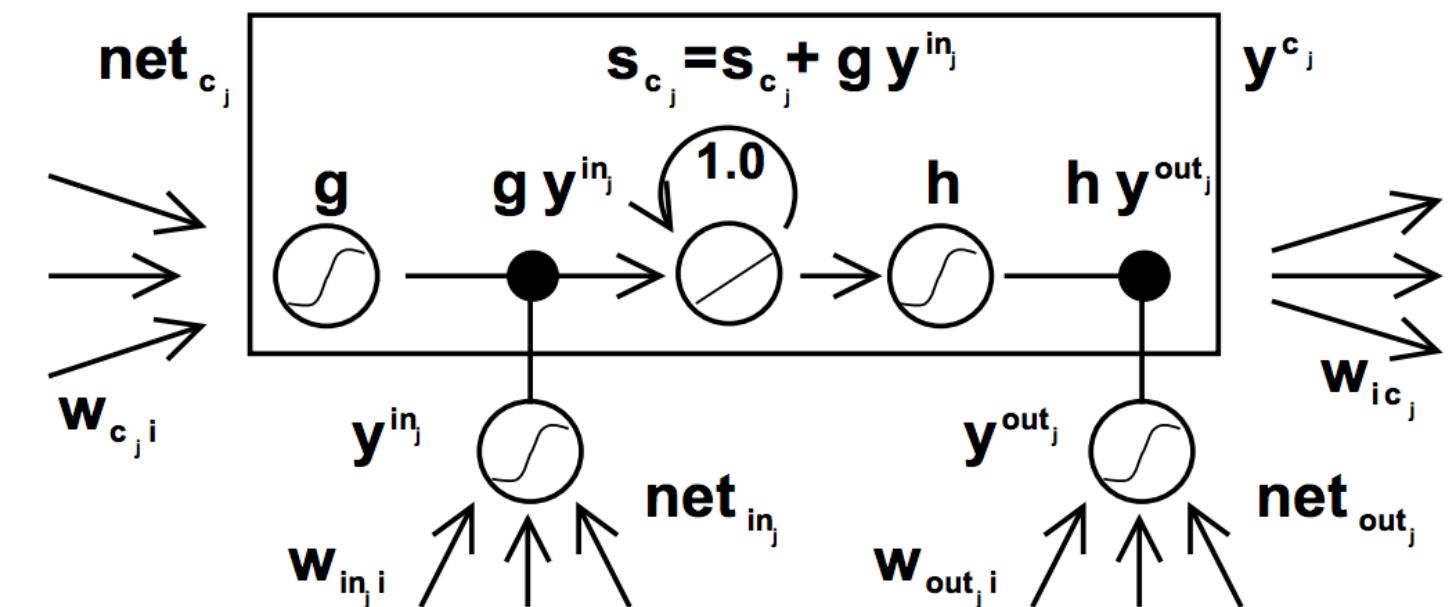
† To whom correspondence should be addressed.

# History: NN “dark ages”

- ▶ ConvNets: applied to MNIST by LeCun in 1998



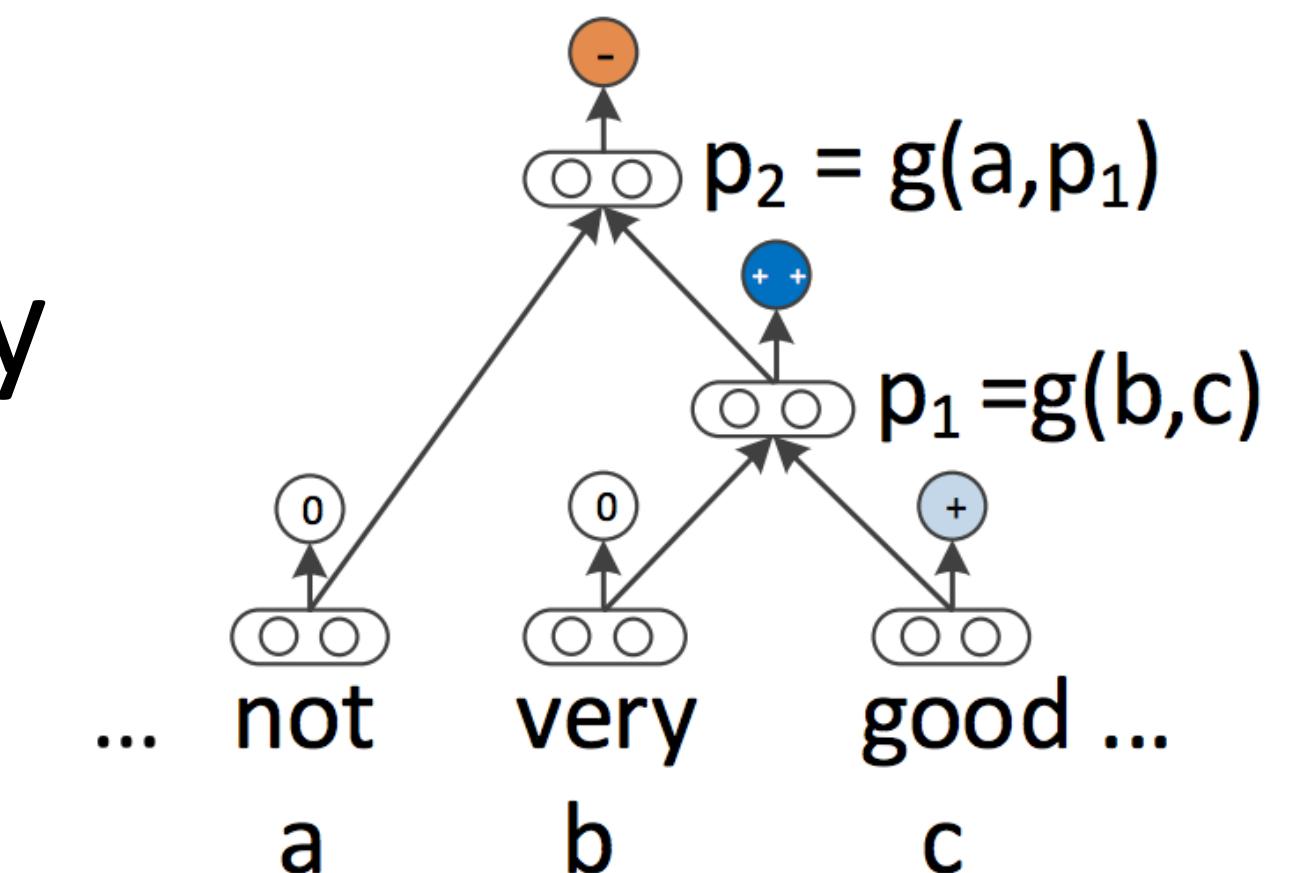
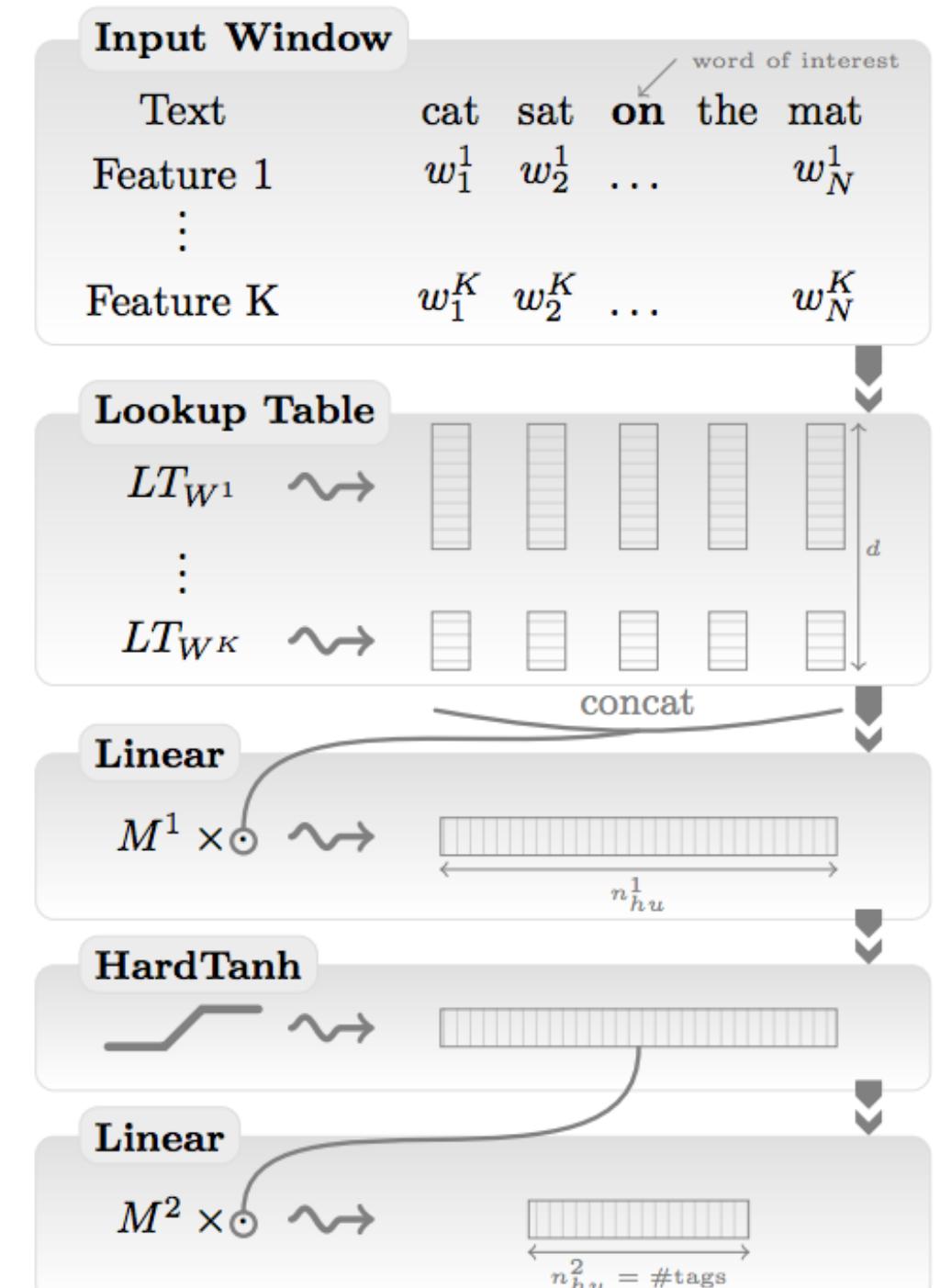
- ▶ LSTMs: Hochreiter and Schmidhuber (1997)



- ▶ Henderson (2003): neural shift-reduce parser, not SOTA

# 2008-2013: A glimmer of light...

- ▶ Collobert and Weston 2011: “NLP (almost) from scratch”
  - ▶ Feedforward neural nets induce features for sequential CRFs (“neural CRF”)
  - ▶ 2008 version was marred by bad experiments, claimed SOTA but wasn’t, 2011 version tied SOTA
- ▶ Krizhevsky et al. (2012): AlexNet for vision
- ▶ Socher 2011-2014: tree-structured RNNs working okay



# 2014: Stuff starts working

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- ▶ Kim (2014) + Kalchbrenner et al. (2014): sentence classification / sentiment (convnets work for NLP?)
- ▶ Sutskever et al. (2014) + Bahdanau et al. (2015) : seq2seq + attention for neural MT (LSTMs work for NLP?)
- ▶ Chen and Manning (2014) transition-based dependency parser (even feedforward networks work well for NLP?)
- ▶ 2015: explosion of neural nets for everything under the sun

# Why didn't they work before?

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- ▶ **Datasets too small:** for MT, not really better until you have 1M+ parallel sentences (and really need a lot more)
- ▶ **Optimization not well understood:** good initialization, per-feature scaling + momentum (AdaGrad / AdaDelta / Adam) work best out-of-the-box
  - ▶ **Regularization:** dropout is pretty helpful
  - ▶ **Computers not big enough:** can't run for enough iterations
- ▶ **Inputs:** need word representations to have the right continuous semantics
- ▶ **Libraries:** TensorFlow (Nov 2015), PyTorch (Sep 2016)

# Neural Net Basics

# Neural Networks: motivation

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- ▶ Linear classification:  $\operatorname{argmax}_y w^\top f(x, y)$
- ▶ How can we do nonlinear classification? Kernels are too slow...
- ▶ Want to learn intermediate conjunctive features of the input

*the movie was **not** all that good*

I[contains *not* & contains *good*]

# Neural Networks: XOR

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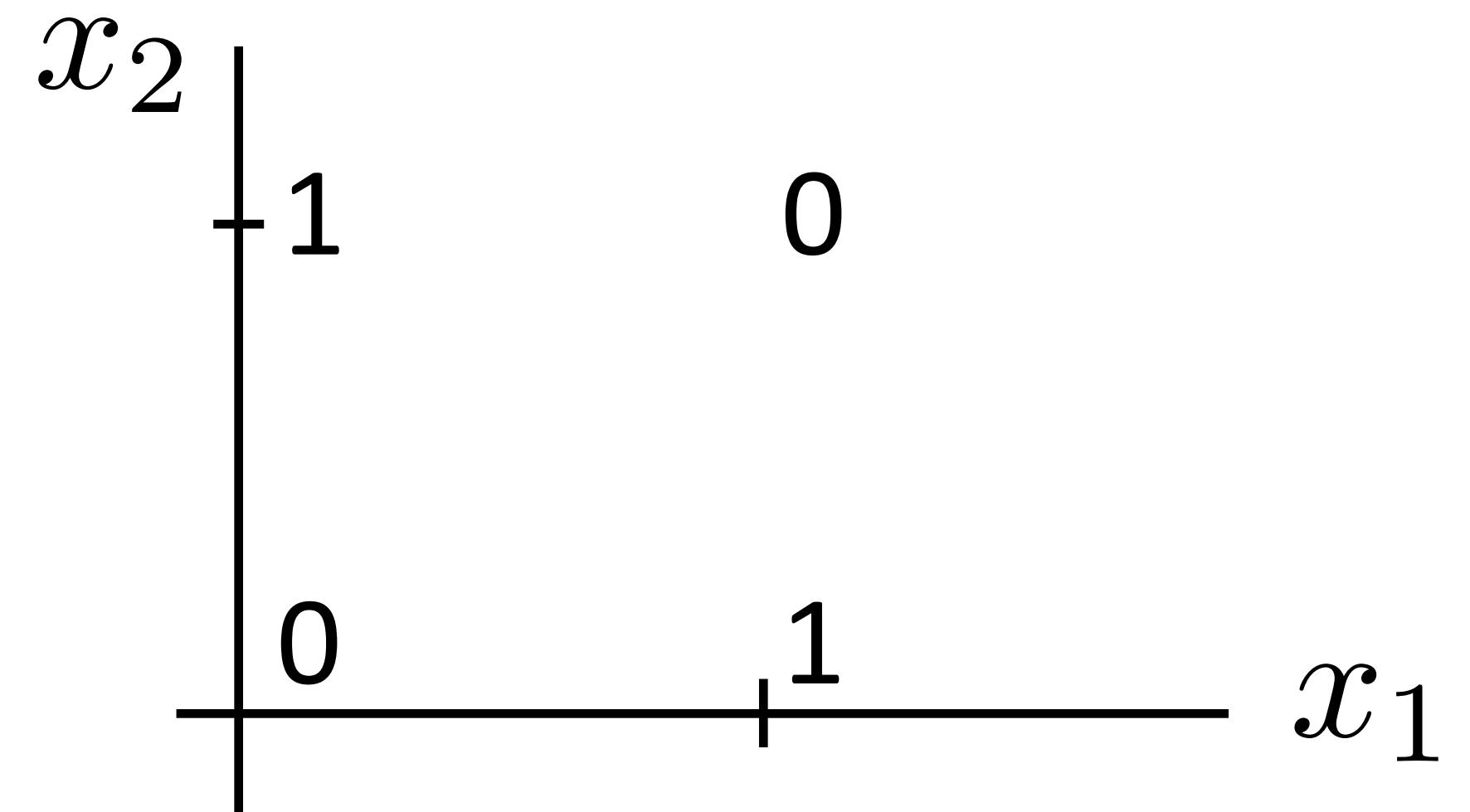
- ▶ Let's see how we can use neural nets to learn a simple nonlinear function

- ▶ Inputs  $x_1, x_2$

(generally  $\mathbf{x} = (x_1, \dots, x_m)$ )

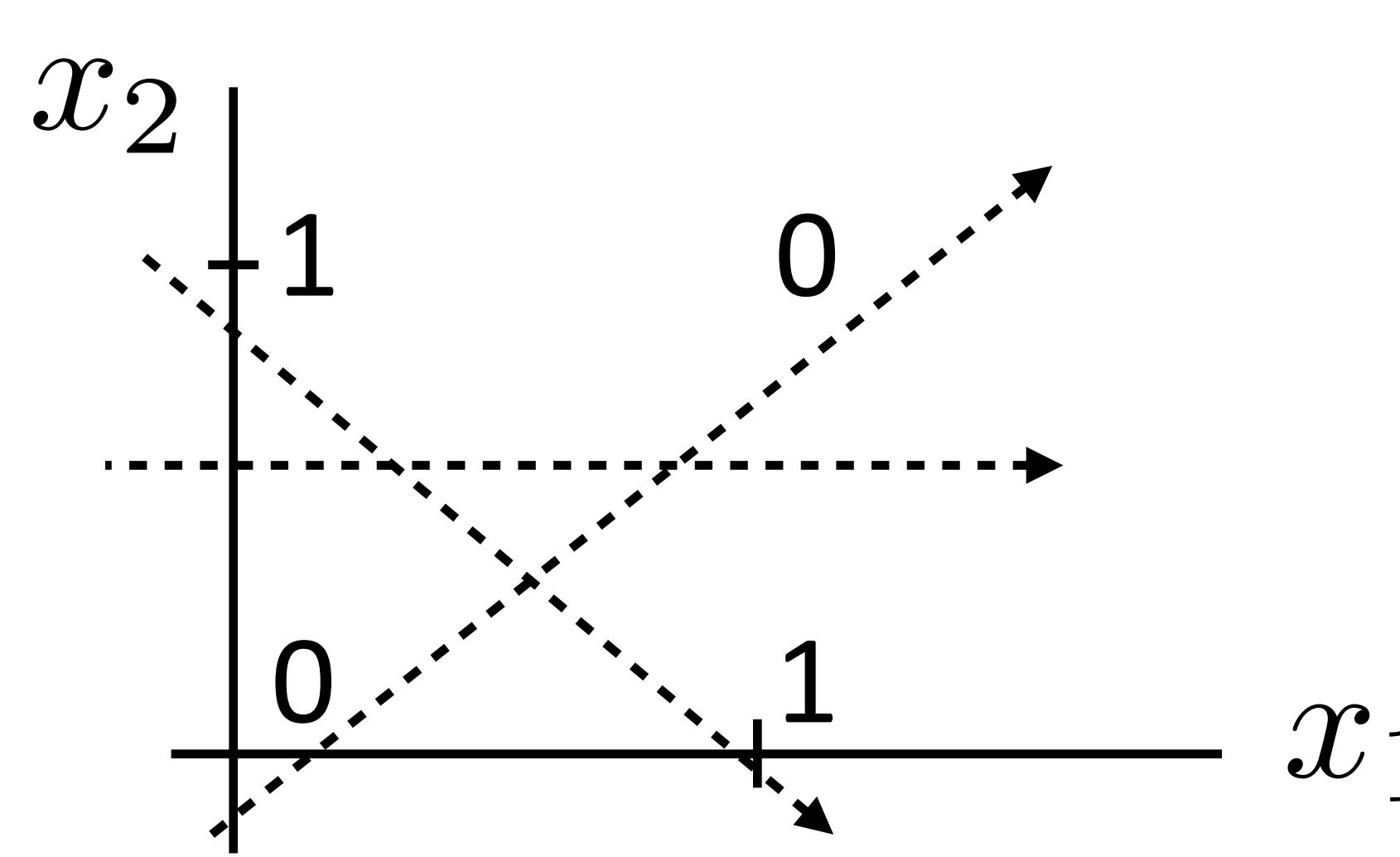
- ▶ Output  $y$

(generally  $\mathbf{y} = (y_1, \dots, y_n)$ )



$x_1$	$x_2$	$y = x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0

# Neural Networks: XOR



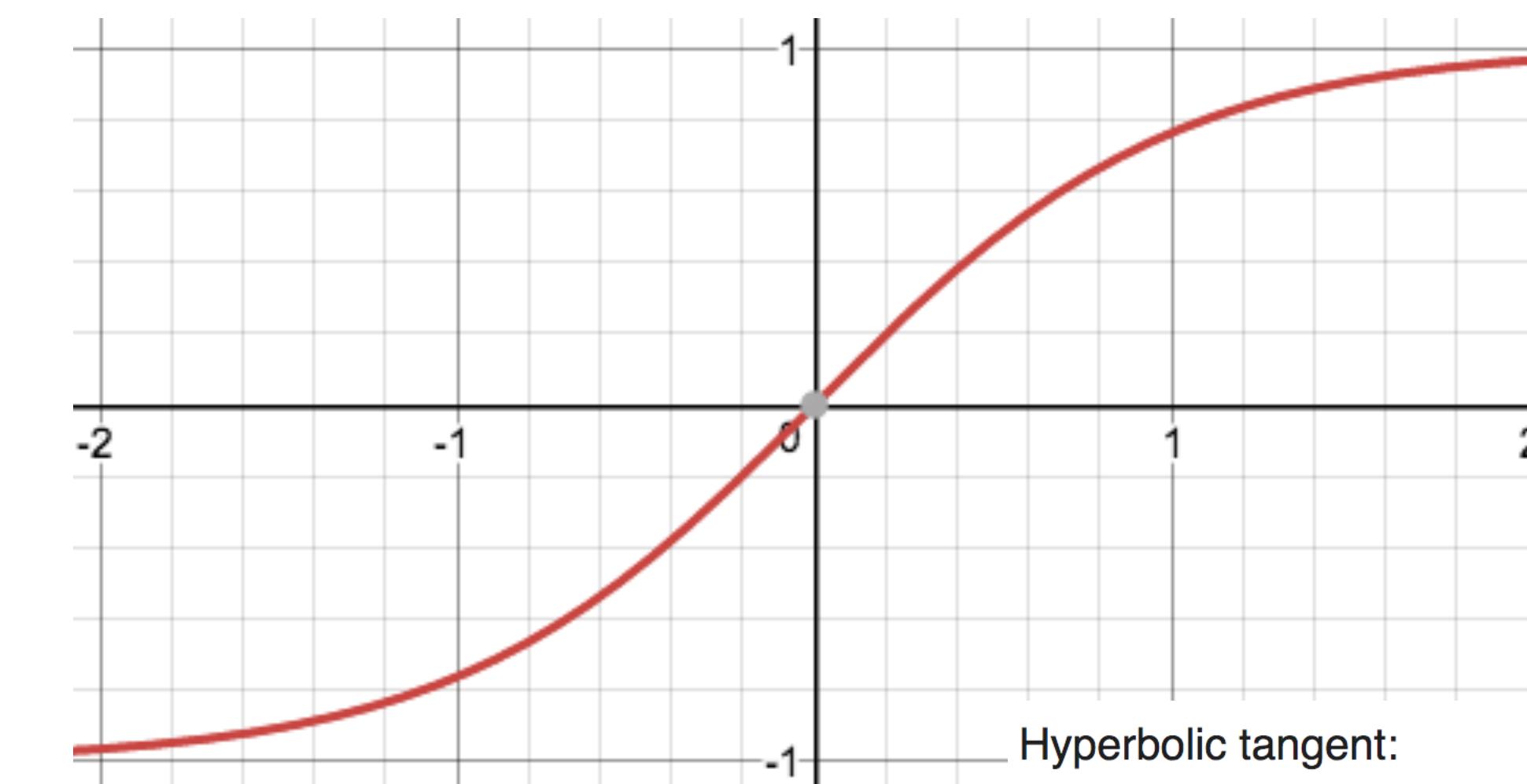
$$y = a_1 x_1 + a_2 x_2$$

$$y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$$

“or”

(looks like action potential in neuron)

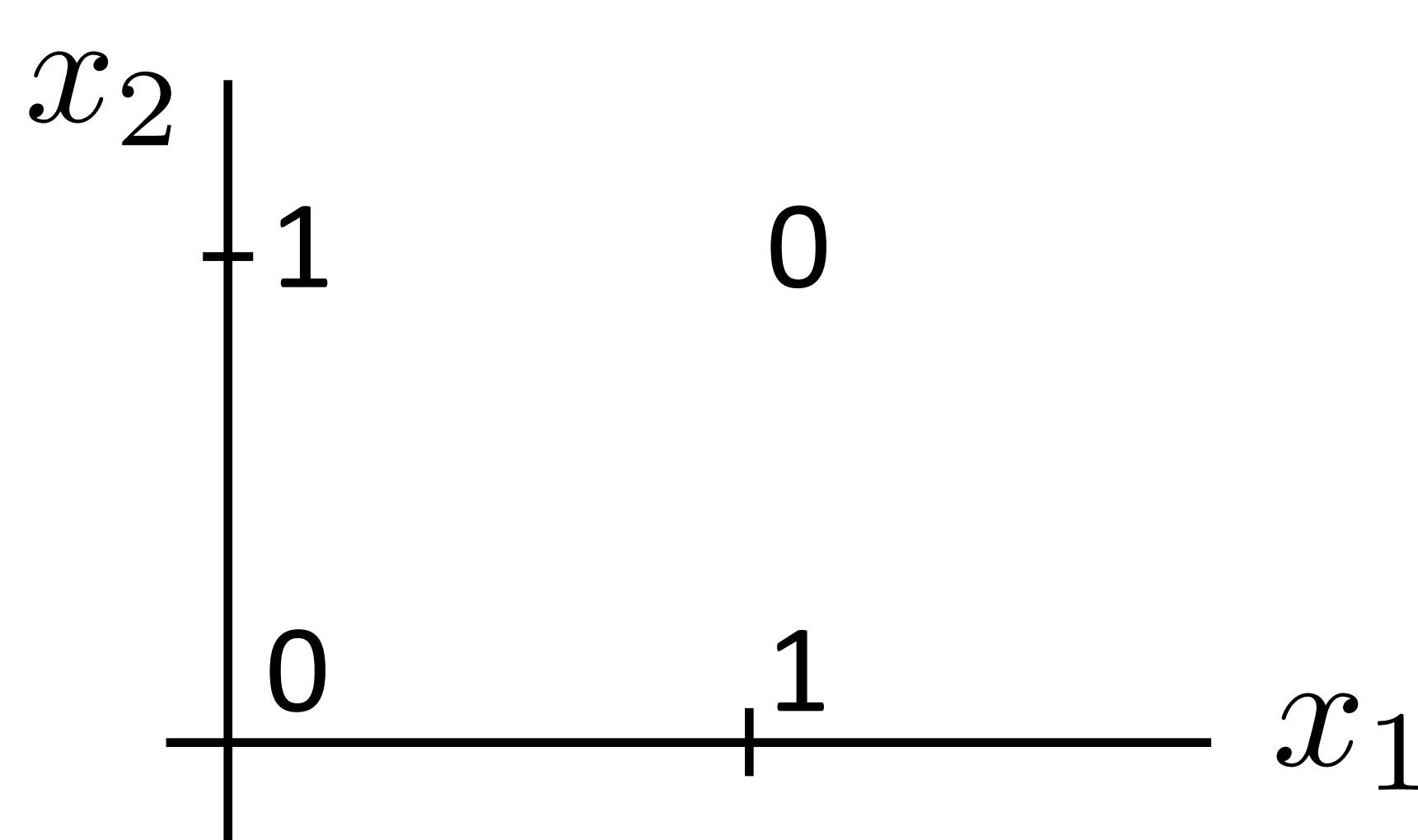
$x_1$	$x_2$	$x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0



Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

# Neural Networks: XOR



$x_1$	$x_2$	$x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0

$$y = a_1 x_1 + a_2 x_2$$

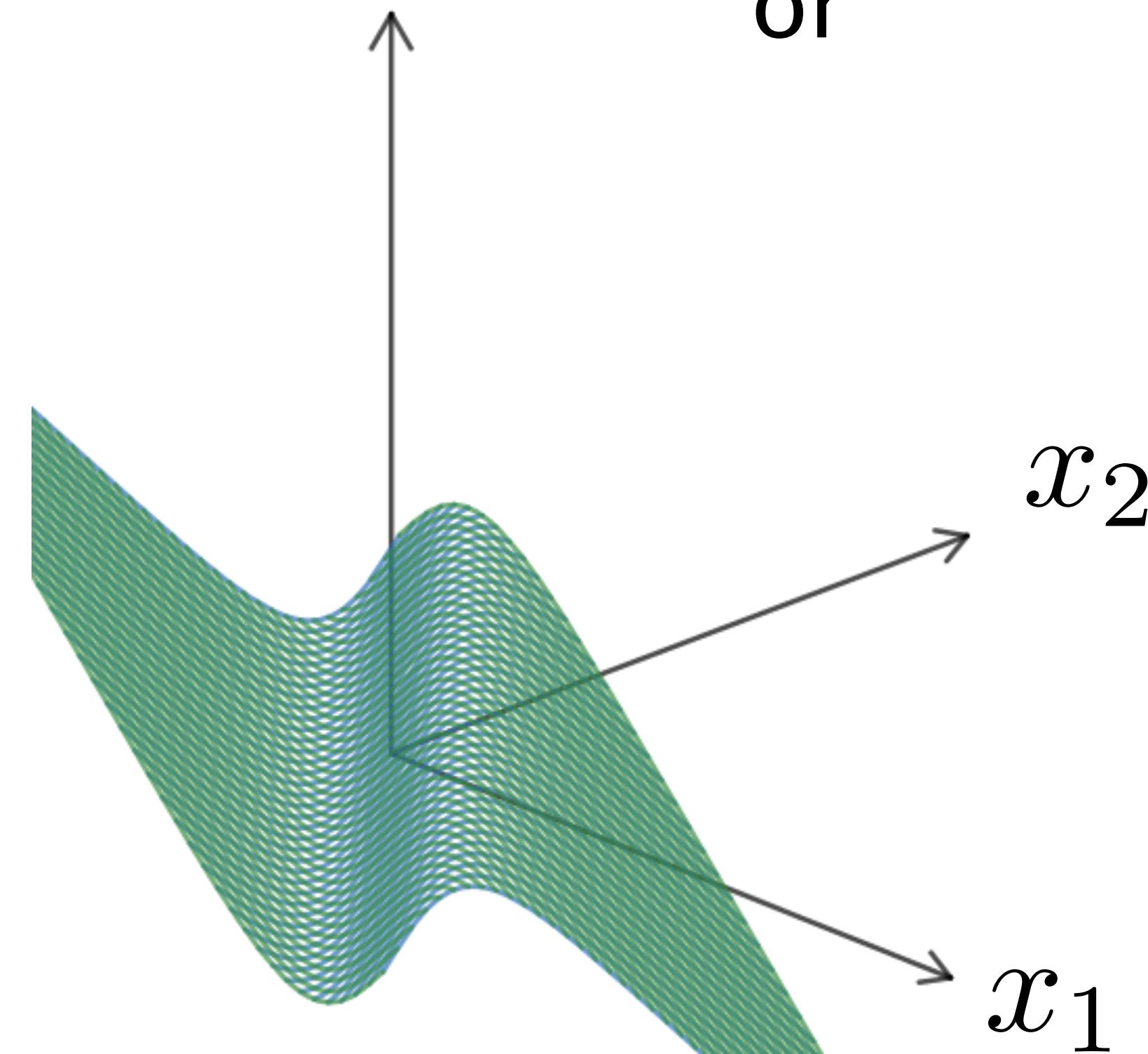
$$y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$$

$$y = -x_1 - x_2 + 2 \tanh(x_1 + x_2)$$

“or”

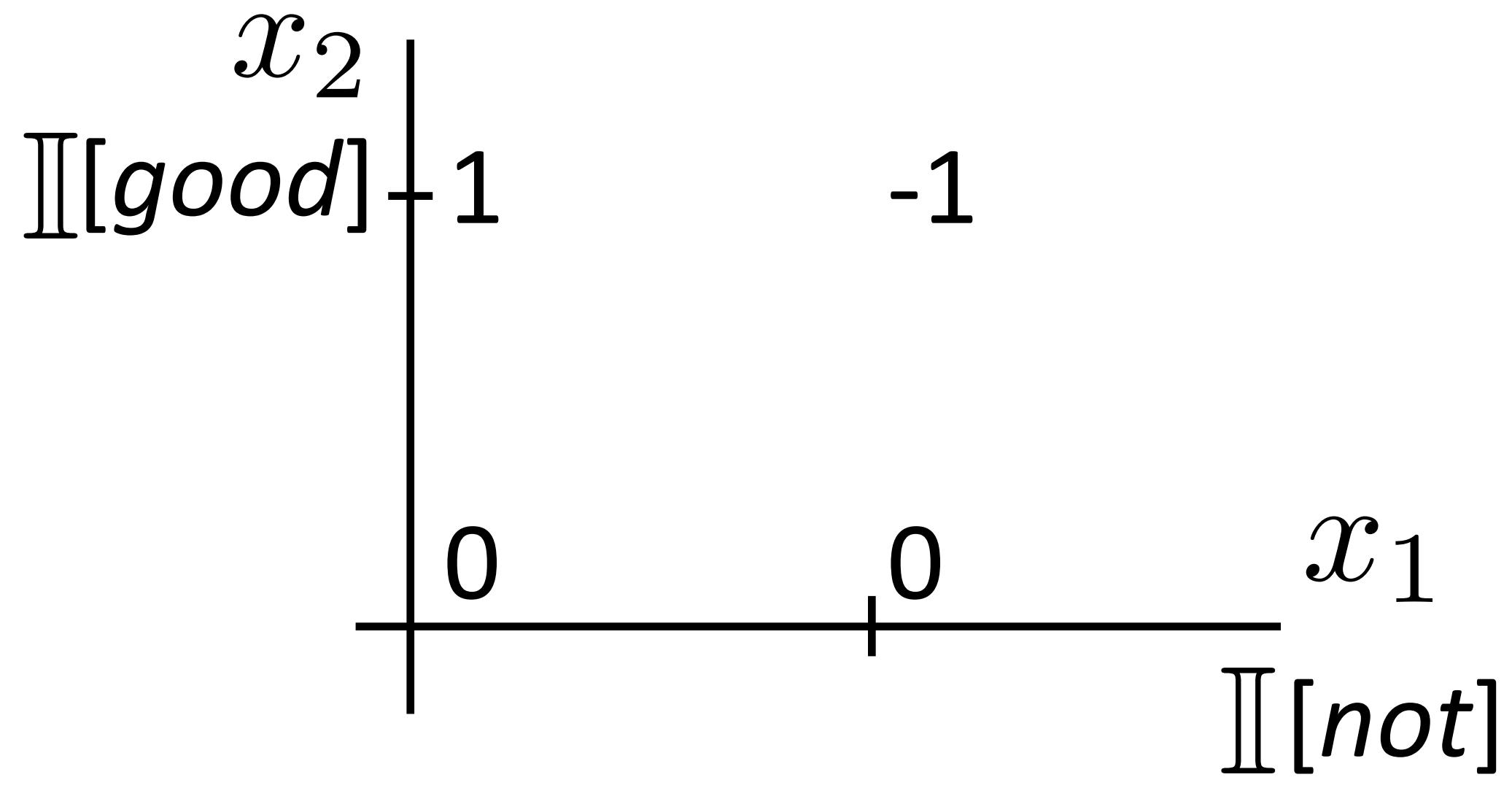
X

✓



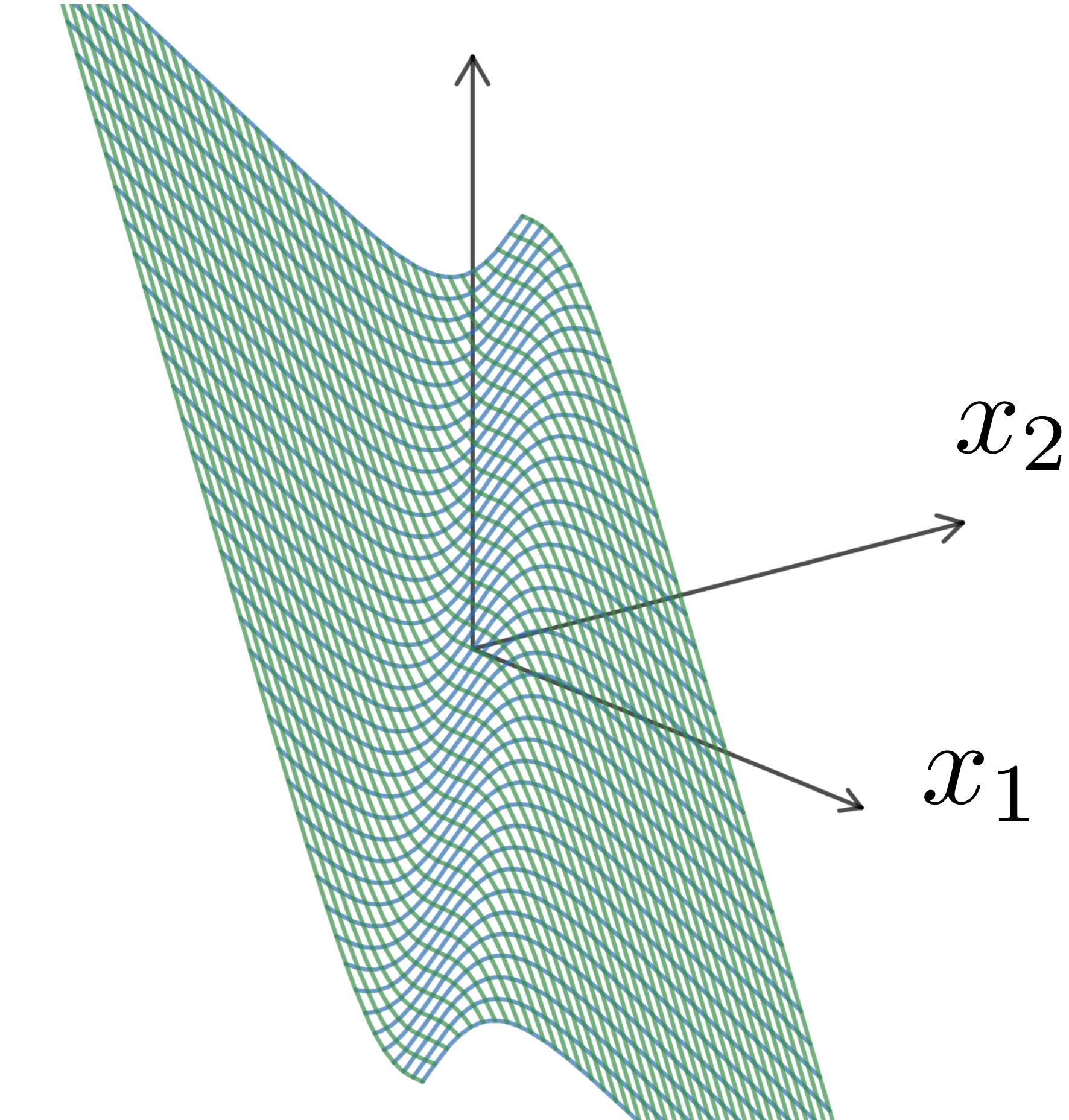
# Neural Networks: XOR

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$$y = -2x_1 - x_2 + 2 \tanh(x_1 + x_2)$$

*the movie was **not** all that good*



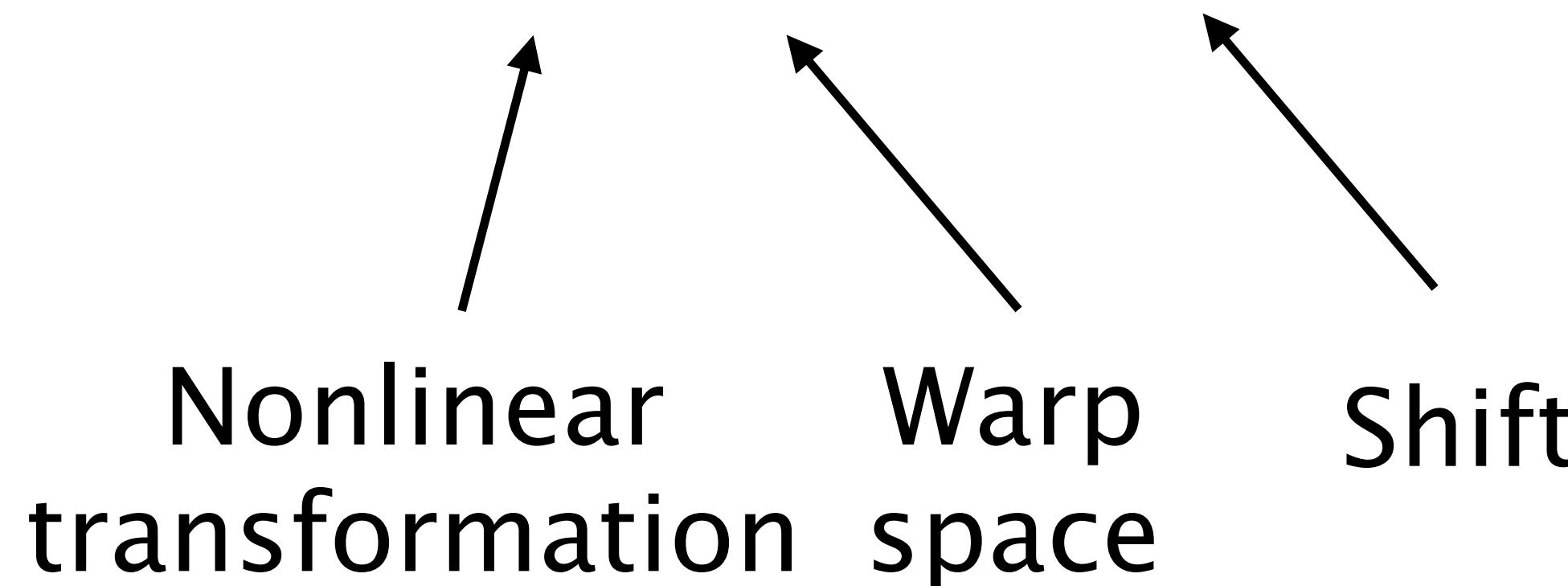
# Neural Networks

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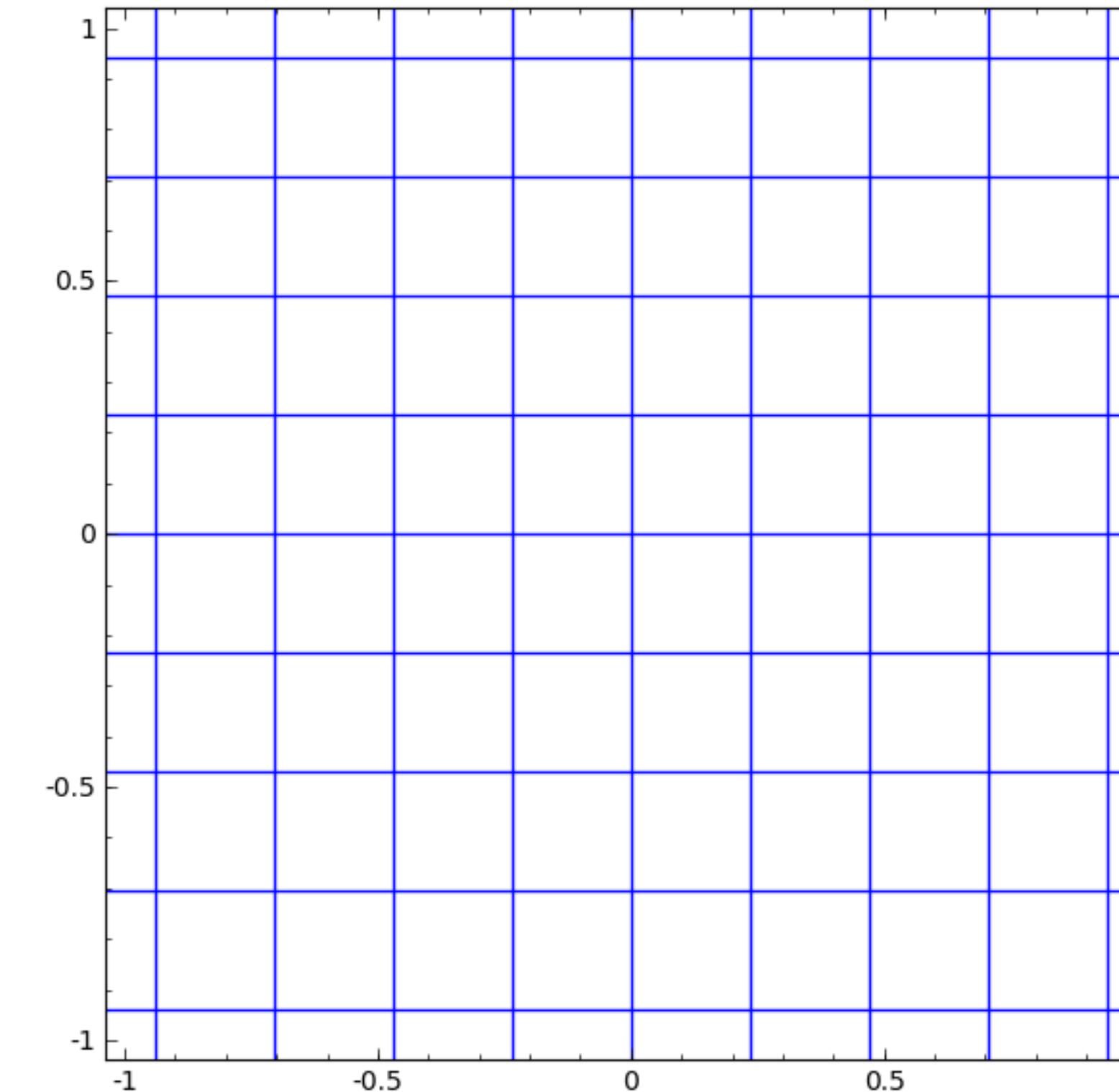
Linear model:  $y = \mathbf{w} \cdot \mathbf{x} + b$

$$y = g(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$



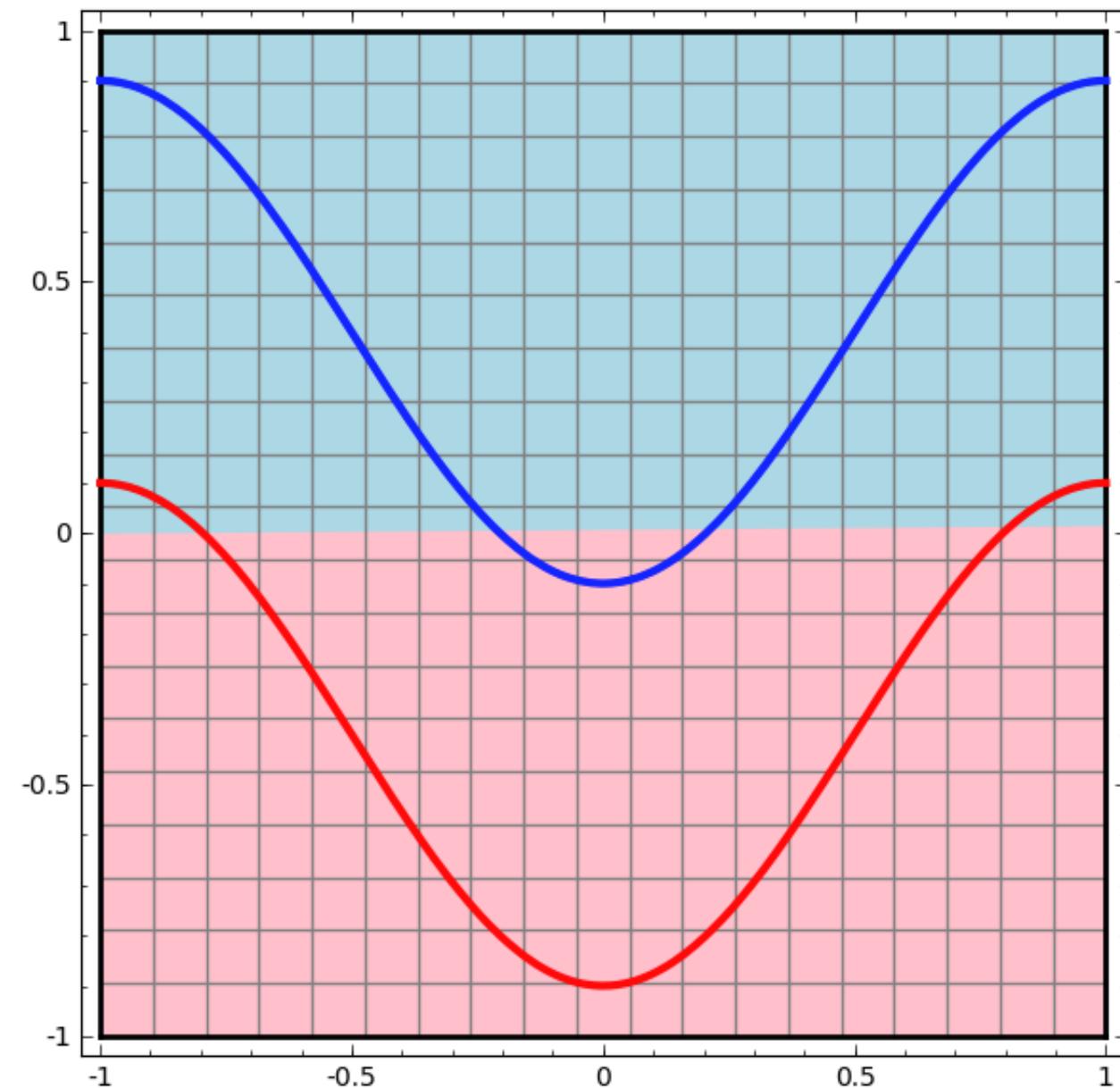
tanh



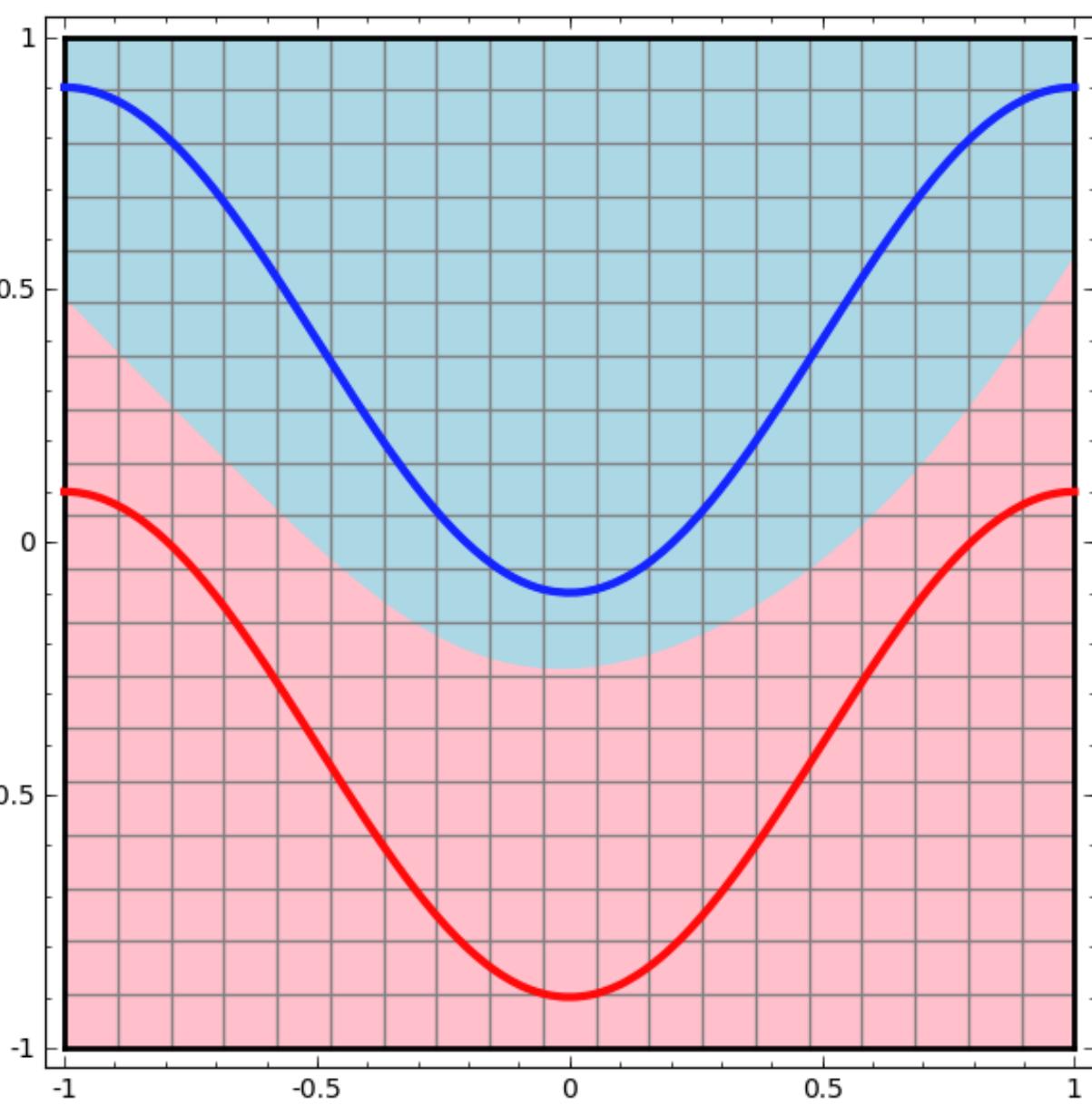
# Neural Networks

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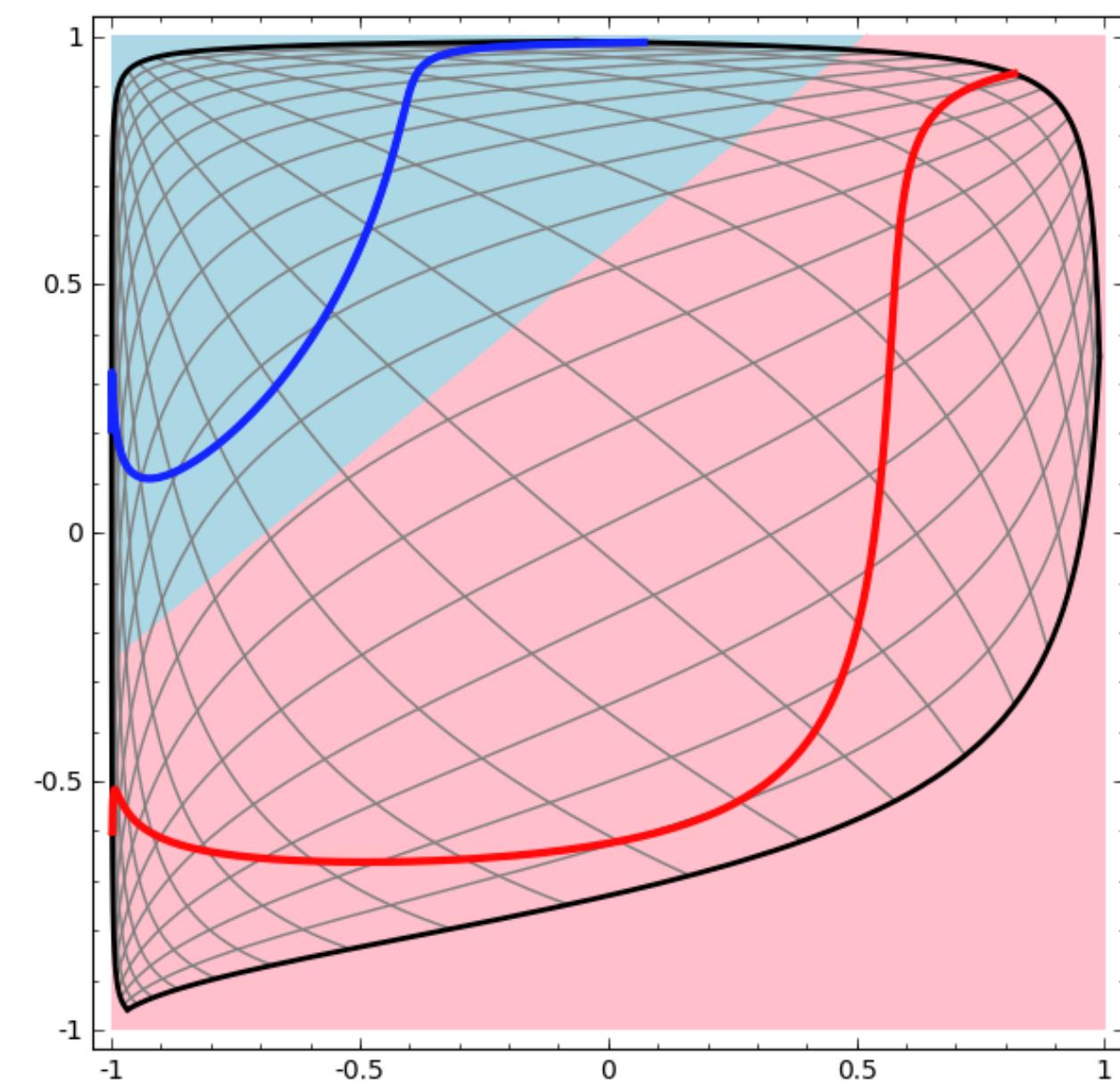
Linear classifier



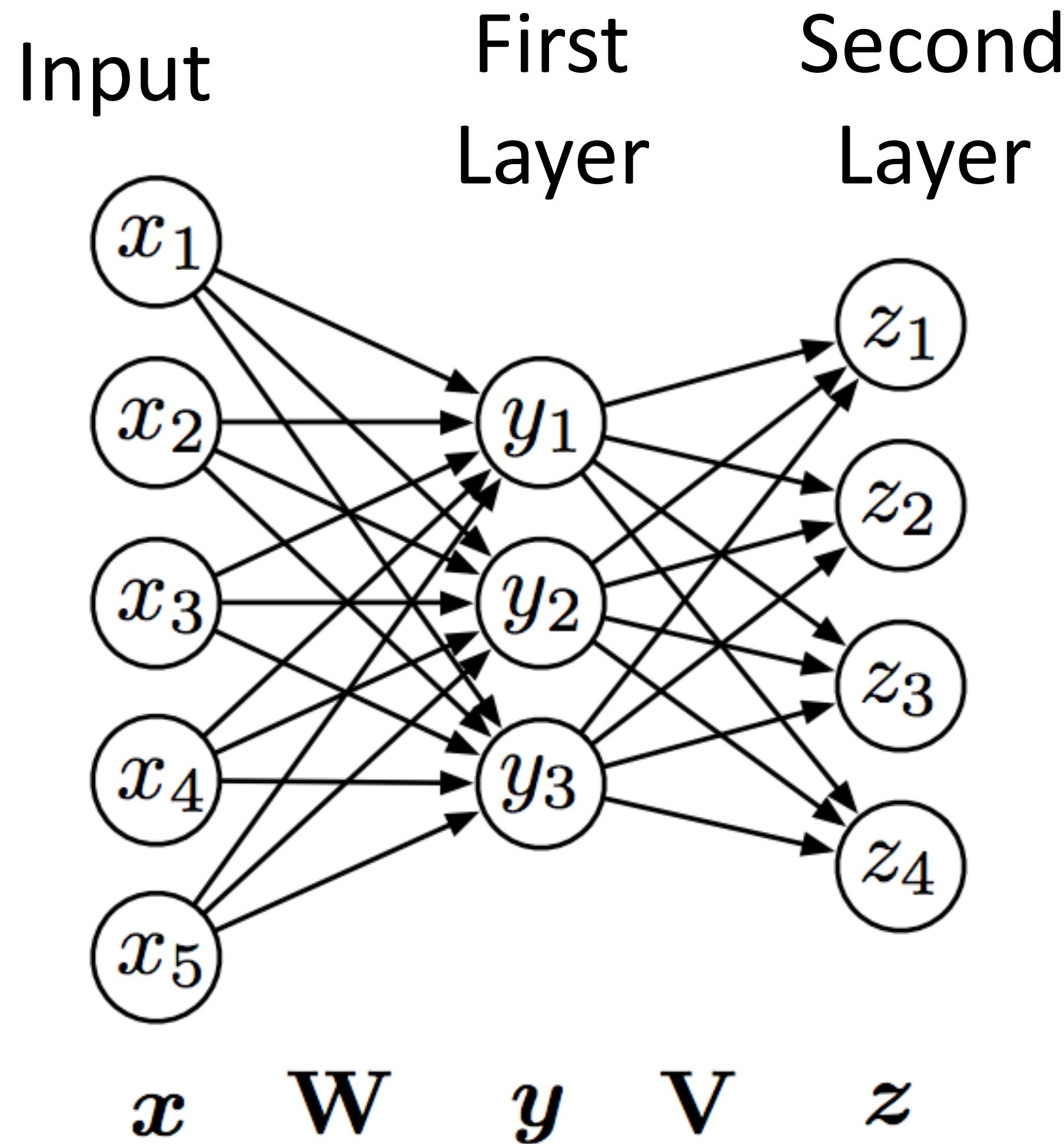
Neural network



...possible because  
we transformed the  
space!



# Deep Neural Networks



$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

$$\mathbf{z} = g(\mathbf{V}\mathbf{y} + \mathbf{c})$$

$$\mathbf{z} = g(\mathbf{V}g(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c})$$

output of first layer

“Feedforward” computation (not recurrent)

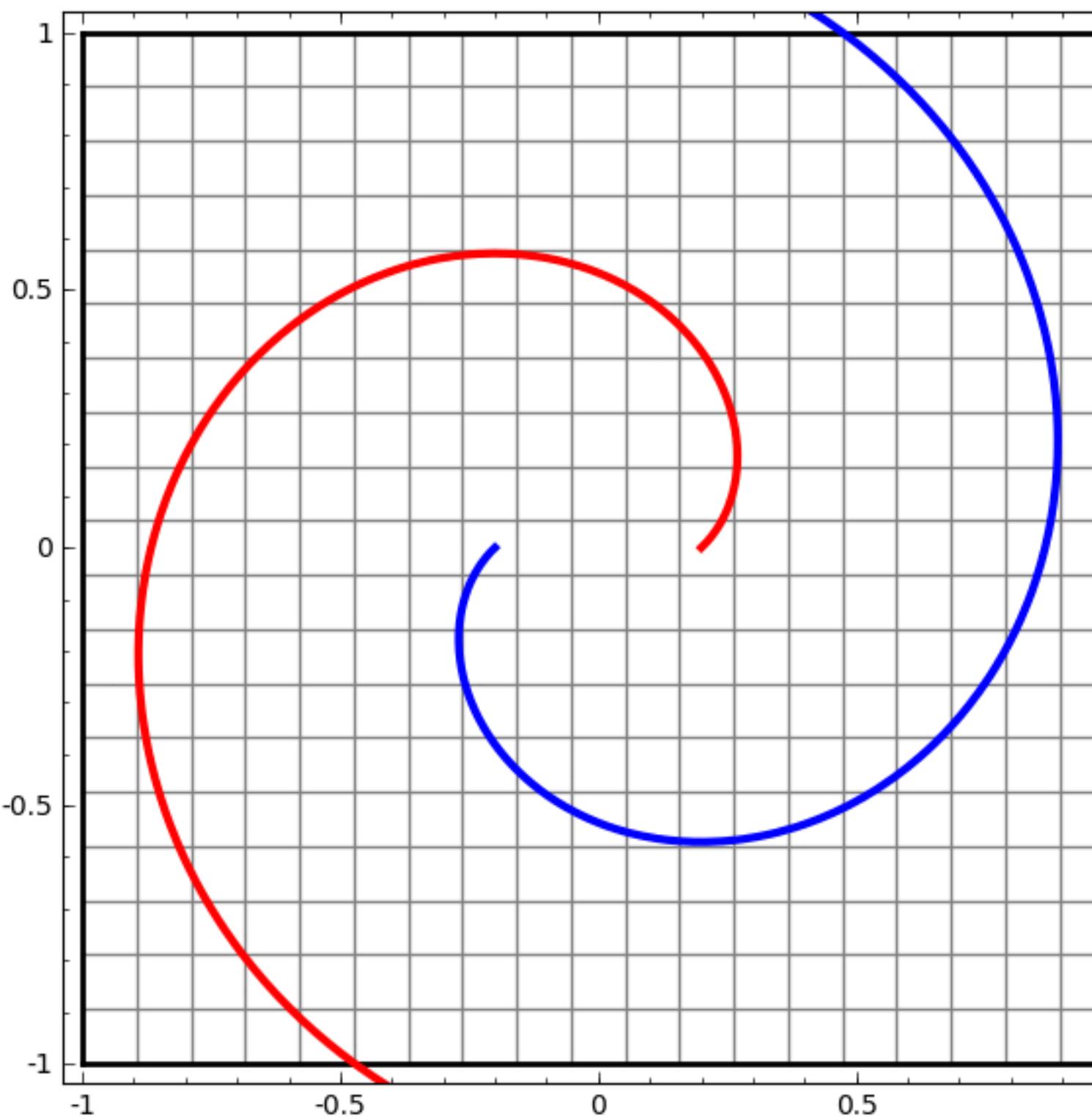
Check: what happens if no nonlinearity?  
More powerful than basic linear models?

$$\mathbf{z} = \mathbf{V}(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c}$$

Adopted from Chris Dyer

# Deep Neural Networks

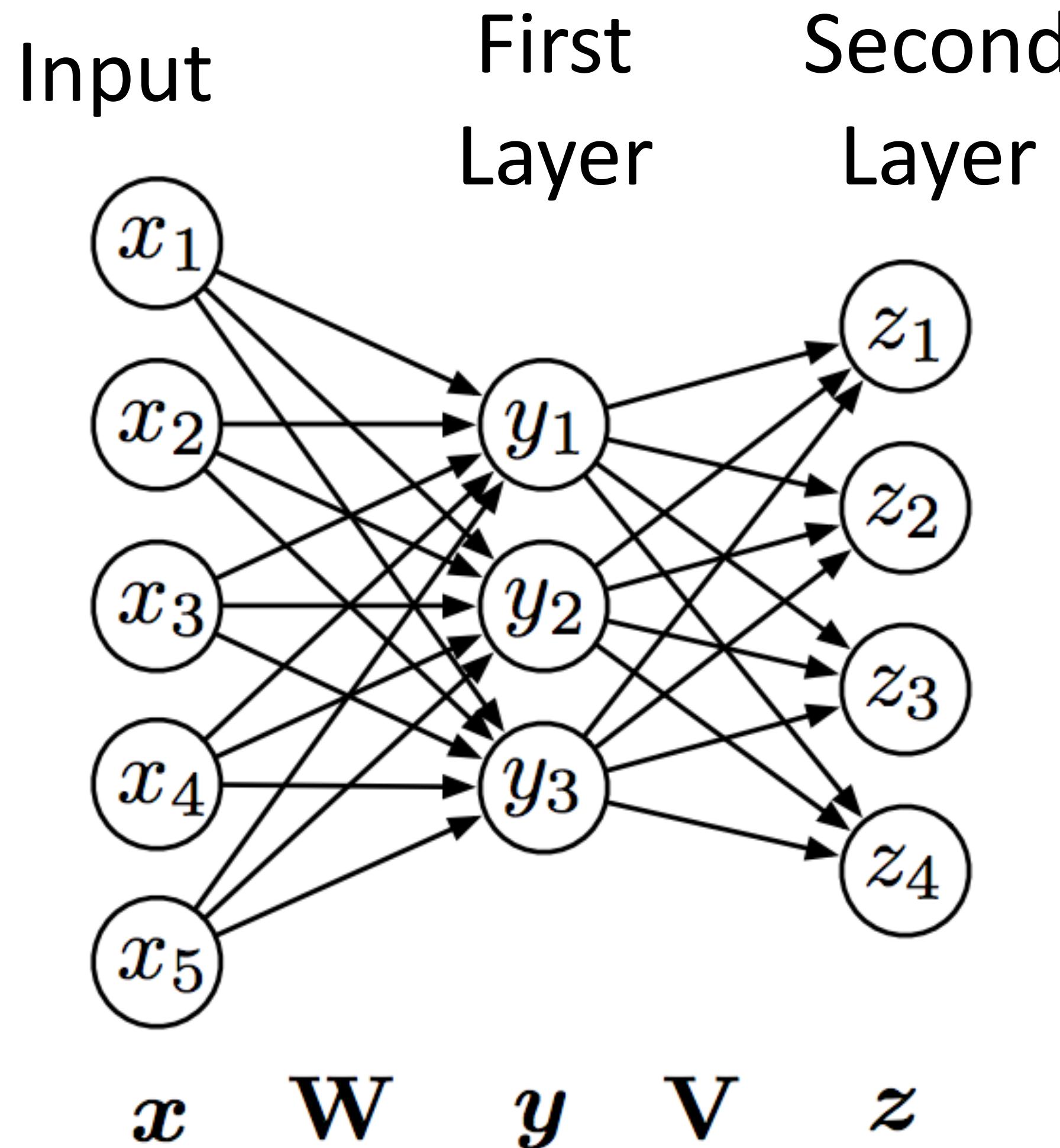
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Taken from <http://colah.github.io/posts/2014-03-NN-Manifolds-Topology/>

# Feedforward Networks, Backpropagation

# Recap: Feedforward Neural Networks



$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

$$\mathbf{z} = g(\mathbf{V}\mathbf{y} + \mathbf{c})$$

$$\mathbf{z} = g(\mathbf{V}g(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c})$$

output of first layer

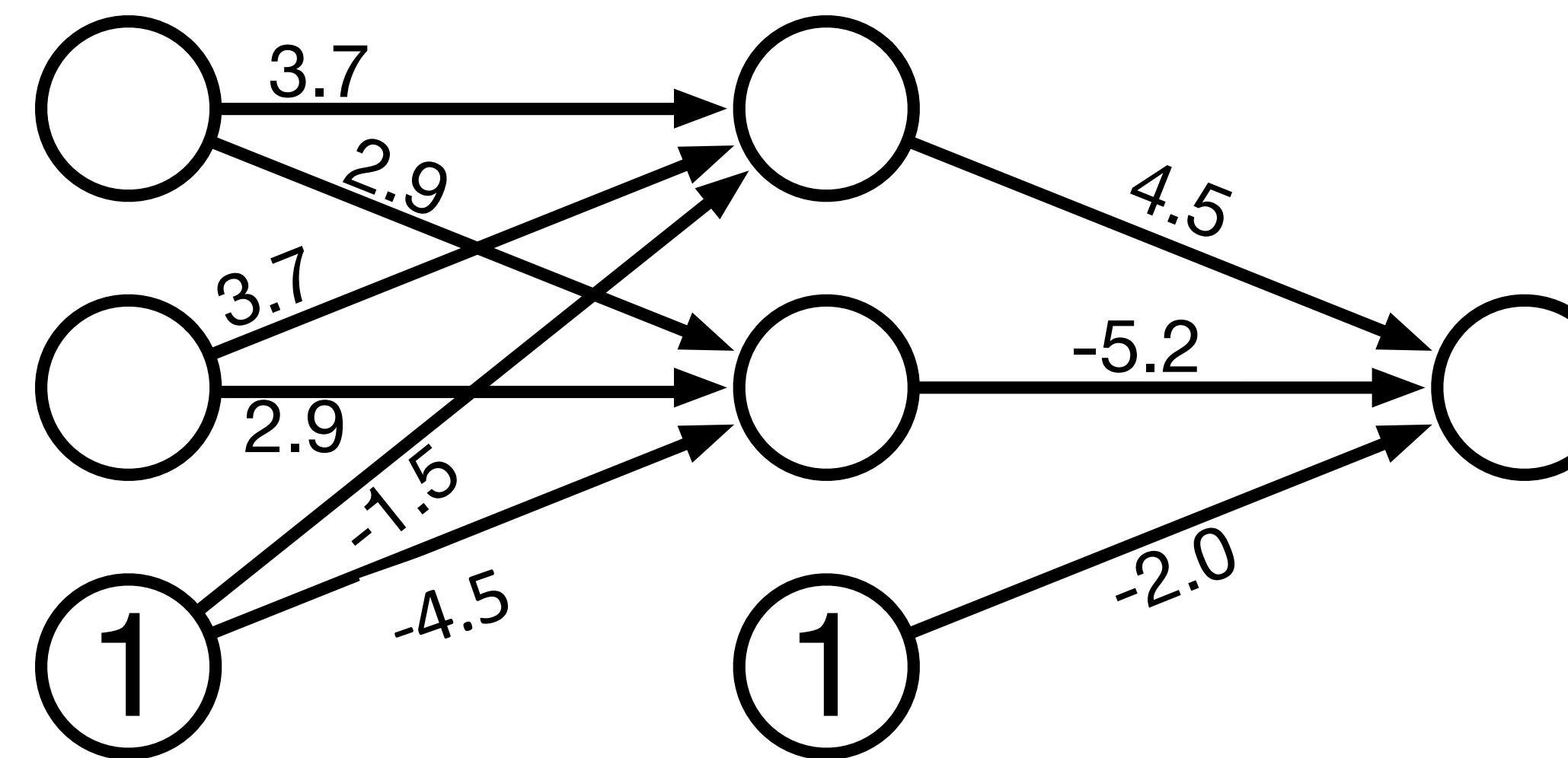
“Feedforward” computation (not recurrent)

Check: what happens if no nonlinearity?  
More powerful than basic linear models?

$$\mathbf{z} = \mathbf{V}(\mathbf{W}\mathbf{x} + \mathbf{b}) + \mathbf{c}$$

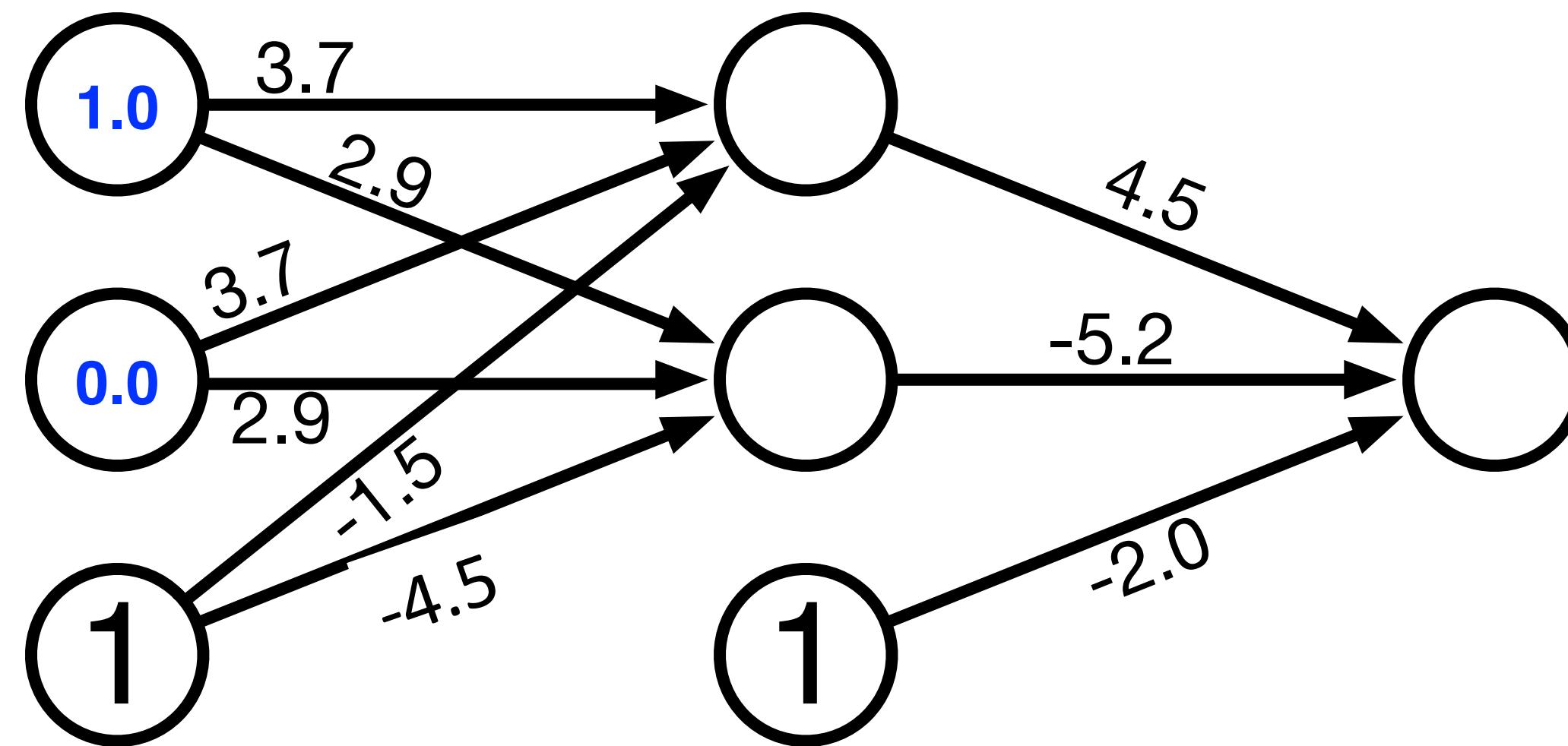
# Simple Neural Network

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# Simple Neural Network

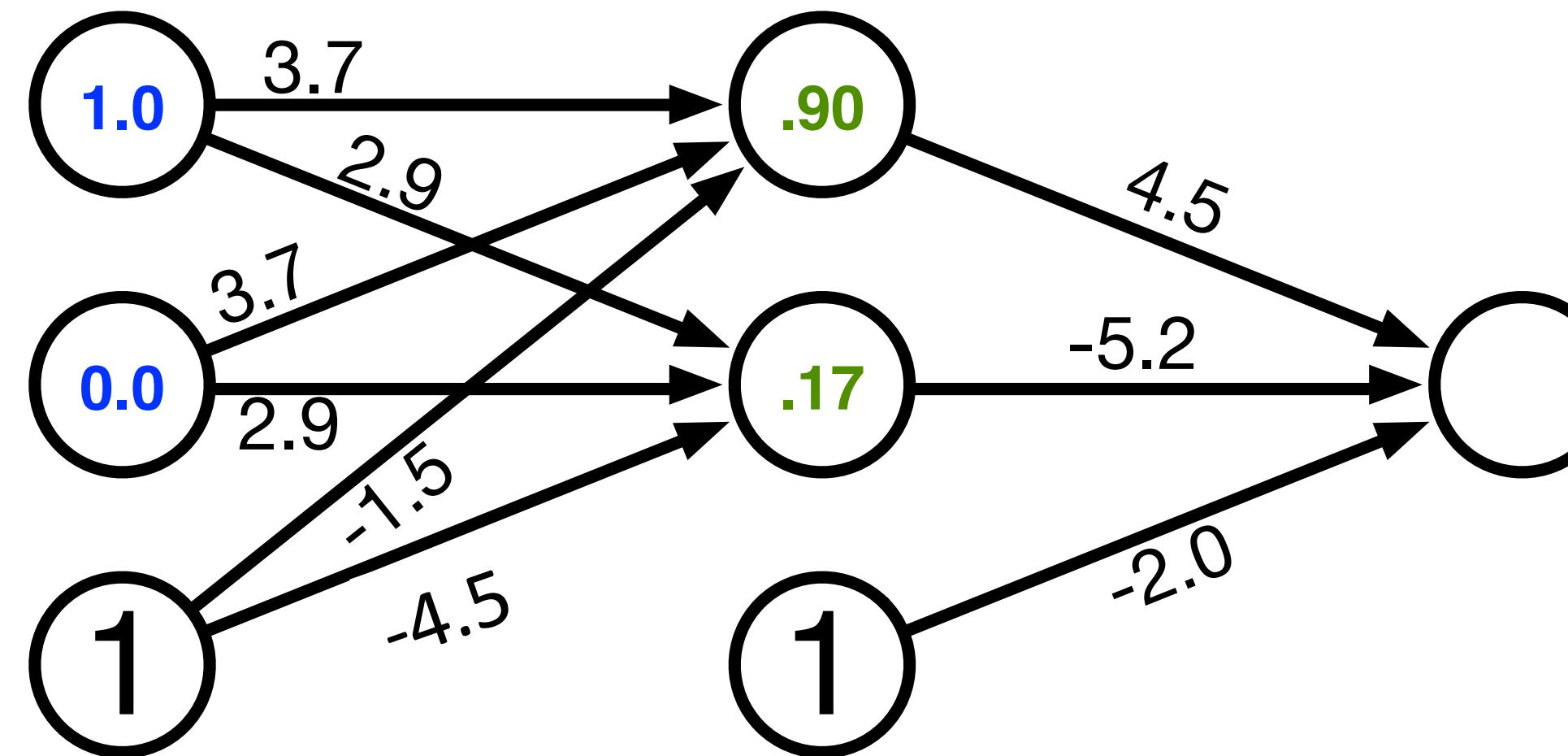
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► Try out two input values

# Simple Neural Network

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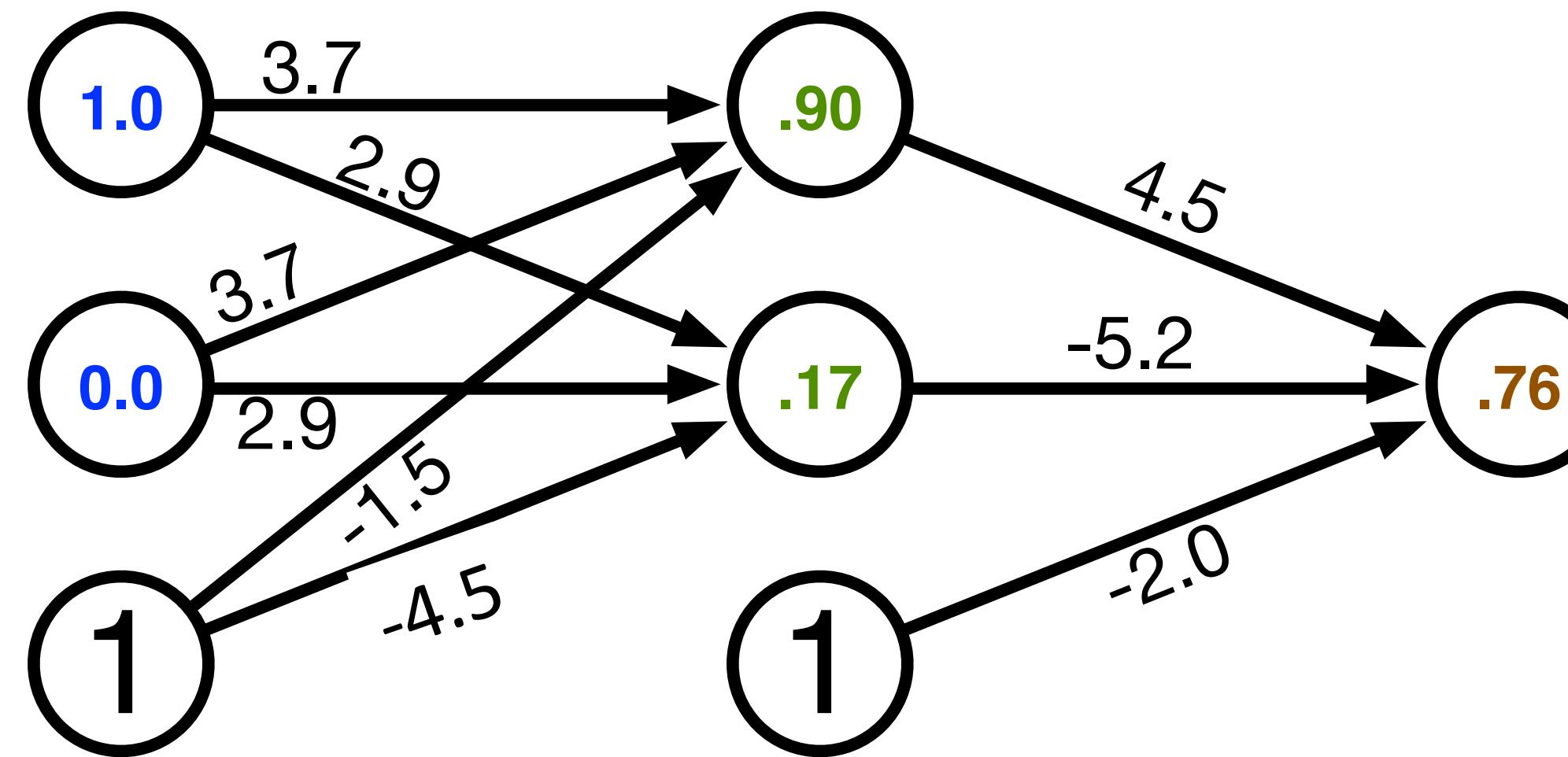
- ▶ Try out two input values
- ▶ Hidden unit computation

$$\text{sigmoid}(1.0 \times 3.7 + 0.0 \times 2.9 + 1 \times -1.5) = \text{sigmoid}(2.2) = \frac{1}{1 + e^{-2.2}} = 0.90$$

$$\text{sigmoid}(1.0 \times 2.9 + 0.0 \times 3.7 + 1 \times -4.5) = \text{sigmoid}(-1.6) = \frac{1}{1 + e^{1.6}} = 0.17$$

# Simple Neural Network

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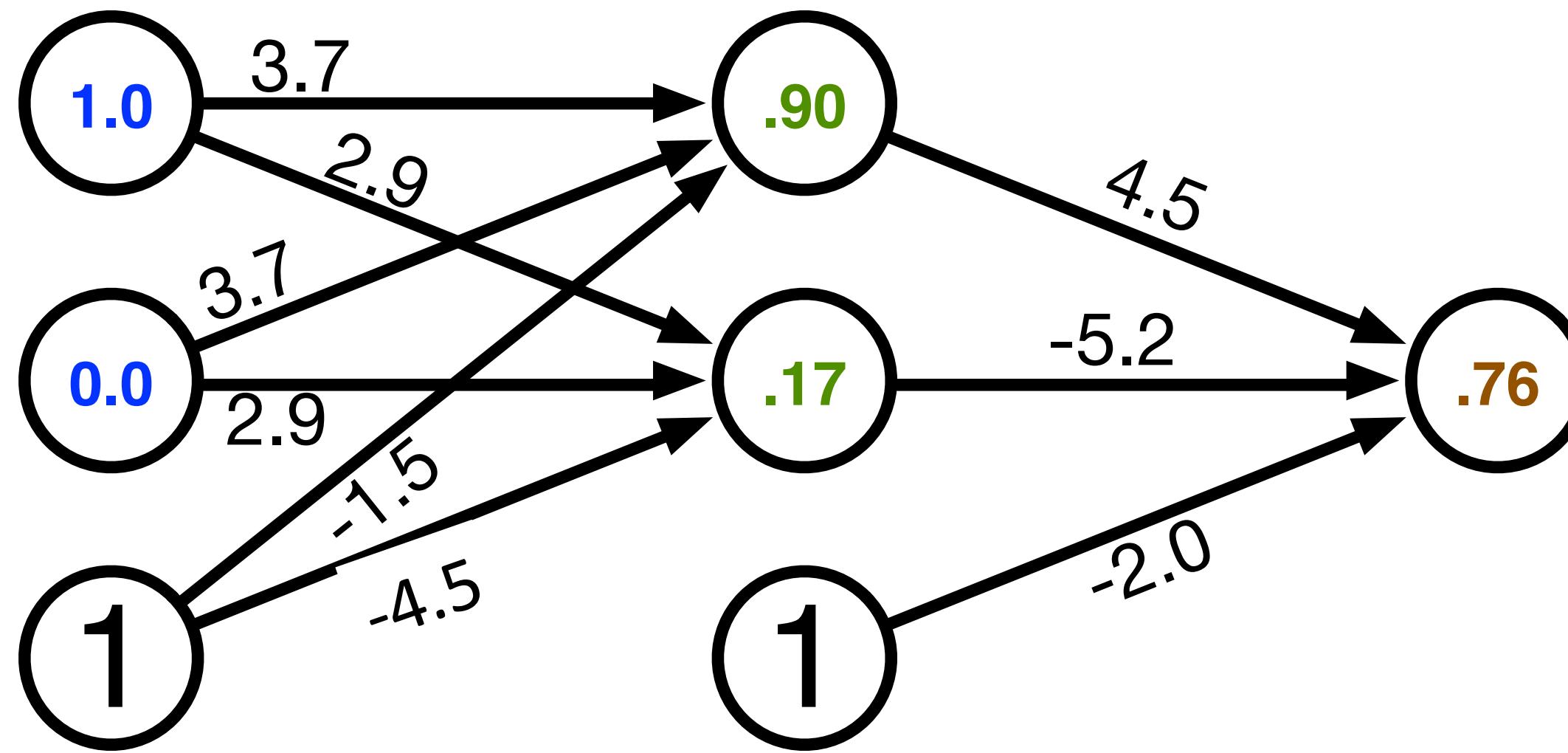


## ► Output unit computation

$$\text{sigmoid}(.90 \times 4.5 + .17 \times -5.2 + 1 \times -2.0) = \text{sigmoid}(1.17) = \frac{1}{1 + e^{-1.17}} = 0.76$$

# Error

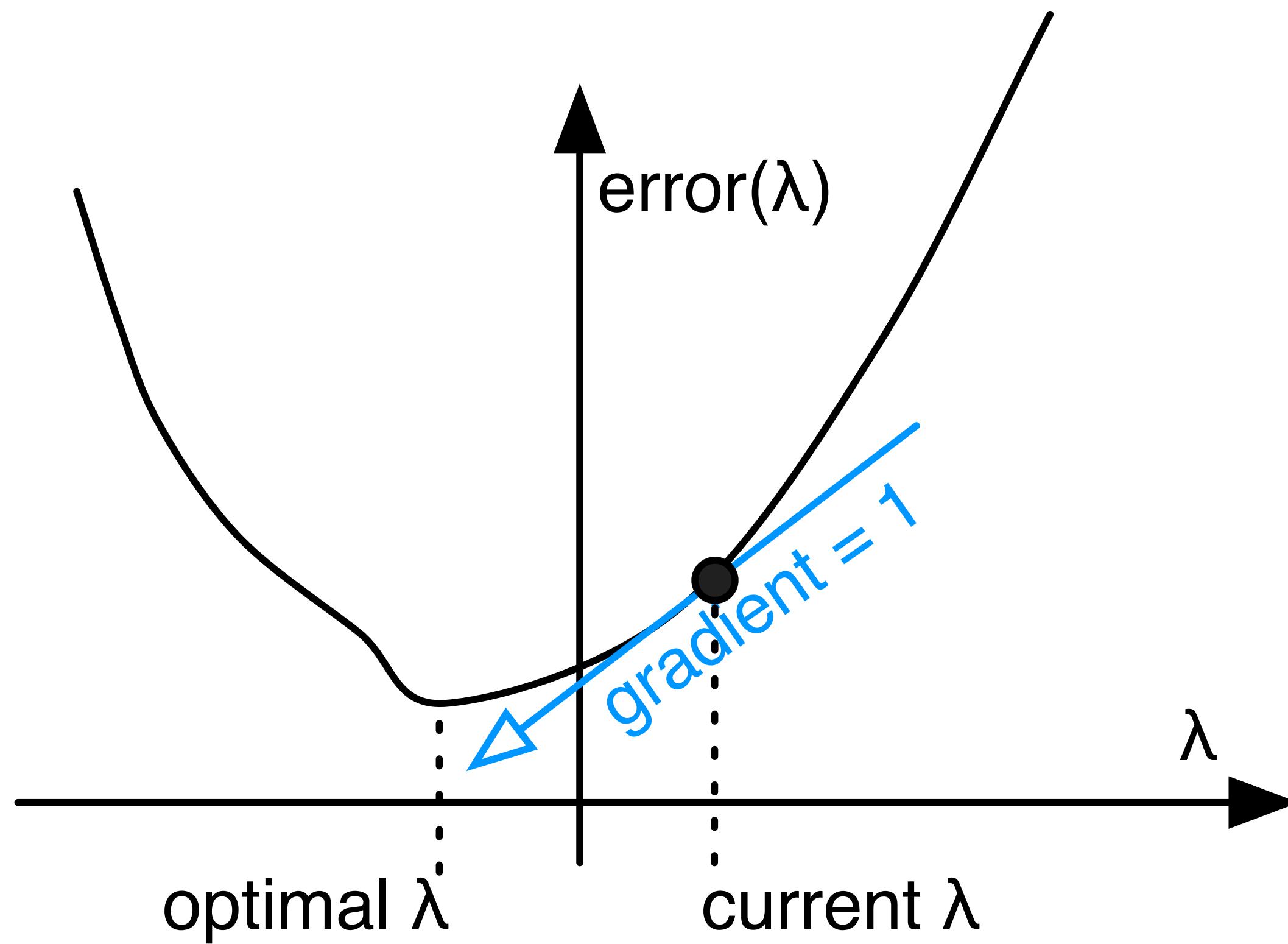
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- ▶ Computed output:  $y = .76$
- ▶ Correct output:  $t = 1.0$
- ▶ Q: how do we adjust the weights?

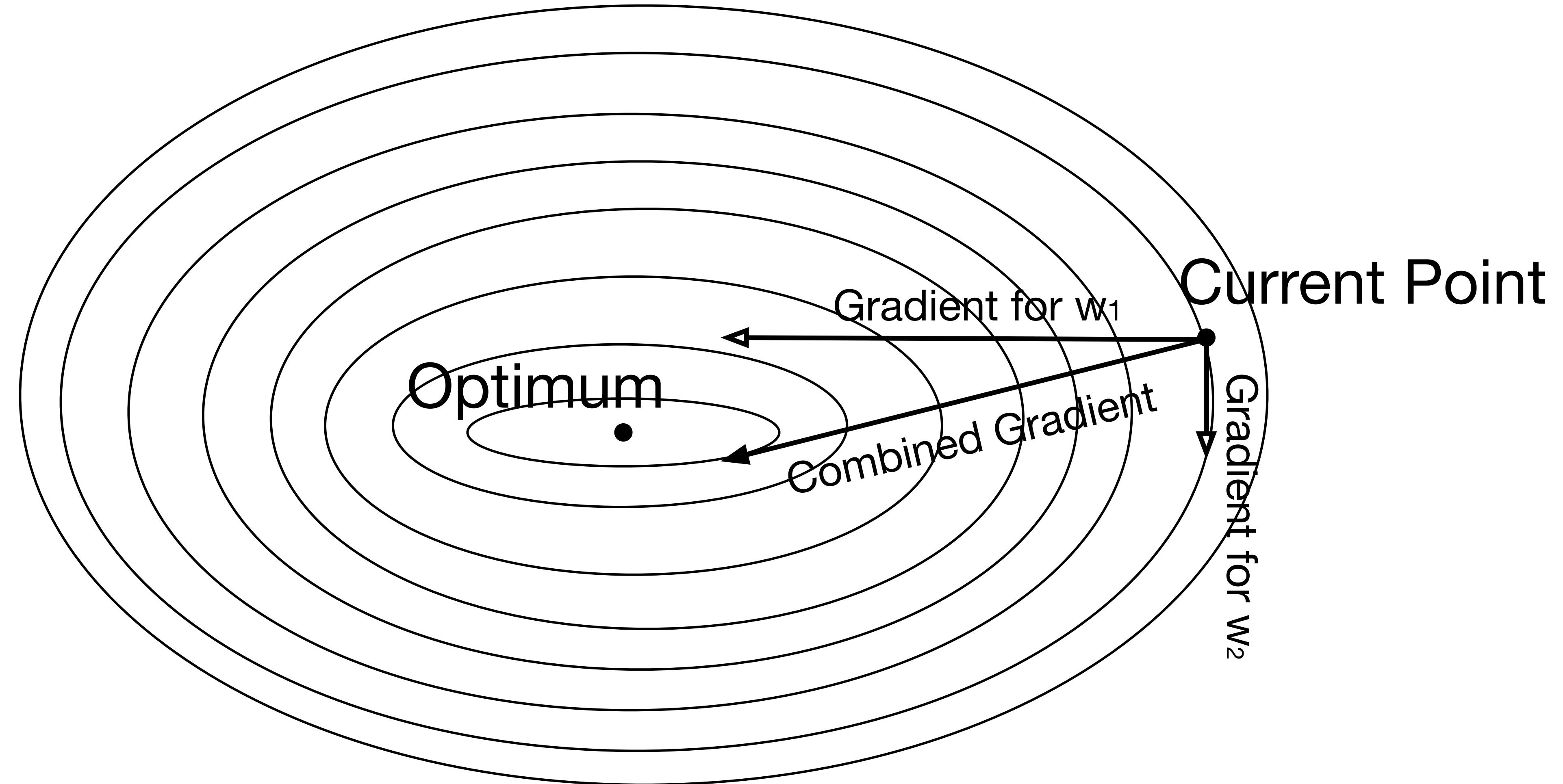
# Gradient Descent

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# Gradient Descent

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# Derivative of Sigmoid

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- ▶ Sigmoid function:

$$\text{sigmoid}(x) = \frac{1}{1 + e^{-x}}$$

- ▶ Derivative:

$$\begin{aligned}\frac{d \text{ sigmoid}(x)}{dx} &= \frac{d}{dx} \frac{1}{1 + e^{-x}} \\ &= \frac{0 \times (1 - e^{-x}) - (-e^{-x})}{(1 + e^{-x})^2} \\ &= \frac{1}{1 + e^{-x}} \left( \frac{e^{-x}}{1 + e^{-x}} \right) \\ &= \frac{1}{1 + e^{-x}} \left( 1 - \frac{1}{1 + e^{-x}} \right) \\ &= \text{sigmoid}(x)(1 - \text{sigmoid}(x))\end{aligned}$$

# Final Layer Update

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- ▶ Linear combination of weights:  $s = \sum_k w_k h_k$
- ▶ Activation function:  $y = \text{sigmoid}(s)$
- ▶ Error (L2 norm):  $E = \frac{1}{2}(t - y)^2$
- ▶ Derivative of error with regard to one weight  $w_k$  :

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

# Final Layer Update (1)

---

- ▶ Linear combination of weights:  $s = \sum_k w_k h_k$
- ▶ Activation function:  $y = \text{sigmoid}(s)$
- ▶ Error (L2 norm):  $E = \frac{1}{2}(t - y)^2$
- ▶ Derivative of error with regard to one weight  $w_k$  :

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- ▶ Error  $E$  is defined with respect to  $y$  :

$$\frac{dE}{dy} = \frac{d}{dy} \frac{1}{2}(t - y)^2 = -(t - y)$$

# Final Layer Update (2)

---

- ▶ Linear combination of weights:  $s = \sum_k w_k h_k$
- ▶ Activation function:  $y = \text{sigmoid}(s)$
- ▶ Error (L2 norm):  $E = \frac{1}{2}(t - y)^2$
- ▶ Derivative of error with regard to one weight  $w_k$  :

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- ▶  $y$  with respect to  $s$  is  $\text{sigmoid}(s)$  :

$$\frac{dy}{ds} = \frac{d \text{sigmoid}(s)}{ds} = \text{sigmoid}(s)(1 - \text{sigmoid}(s)) = y(1 - y)$$

# Final Layer Update (3)

---

- ▶ Linear combination of weights:  $s = \sum_k w_k h_k$
- ▶ Activation function:  $y = \text{sigmoid}(s)$
- ▶ Error (L2 norm):  $E = \frac{1}{2}(t - y)^2$
- ▶ Derivative of error with regard to one weight  $w_k$  :

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- ▶  $s$  is weighted linear combination of hidden node values  $h_k$  :

$$\frac{ds}{dw_k} = \frac{d}{dw_k} \sum_k w_k h_k = h_k$$

# Putting it All Together

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- ▶ Derivative of error with regard to one weight  $w_k$  :

$$\begin{aligned}\frac{dE}{dw_k} &= \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k} \\ &= -(t - y) \quad y(1 - y) \quad h_k\end{aligned}$$

error      derivative of sigmoid:  $y'$

- ▶ Weighted adjustment will be scaled by a fixe learning rate  $\mu$  :

$$\Delta w_k = \mu (t - y) y' h_k$$

# Multiple Output Nodes

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- ▶ Previous slides discussed the situation with only one output node:

$$E = \frac{1}{2}(t - y)^2$$

$$\Delta w_k = \mu(t - y) y' h_k$$

- ▶ But, typically neural networks have multiple output nodes

- ▶ Error is computed over all j output nodes:

$$E = \sum_j \frac{1}{2}(t_j - y_j)^2$$

- ▶ Weights are adjusted according to the node they point to:

$$\Delta w_{j \leftarrow k} = \mu(t_j - y_j) y'_j h_k$$

# Hidden Layer Update

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- ▶ In a hidden layer, we do not have a target output value
- ▶ But, we can compute how much each node contributed to downstream error
- ▶ Definition of error term of each node:

$$\delta_j = (t_j - y_j) y'_j$$

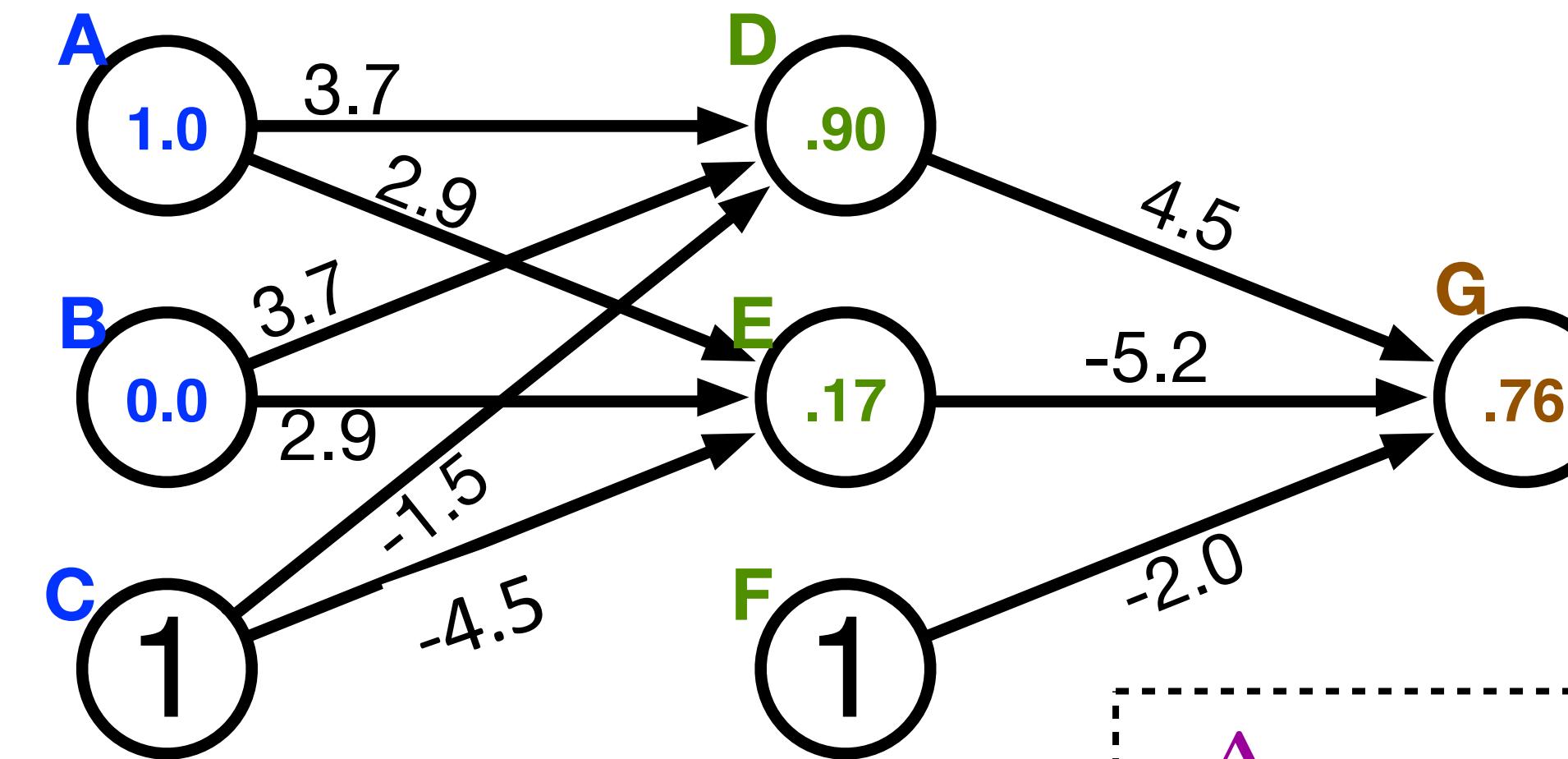
- ▶ Back-propagate the error term:

$$\delta_i = \left( \sum_j w_{j \leftarrow i} \delta_j \right) y'_i$$

- ▶ Universal update formula:

$$\Delta w_{j \leftarrow k} = \mu \delta_j h_k$$

# Our Example



- ▶ Computed output:  $y = .76$
- ▶ Correct output:  $t = 1.0$
- ▶ Q: how do we adjust the weights?

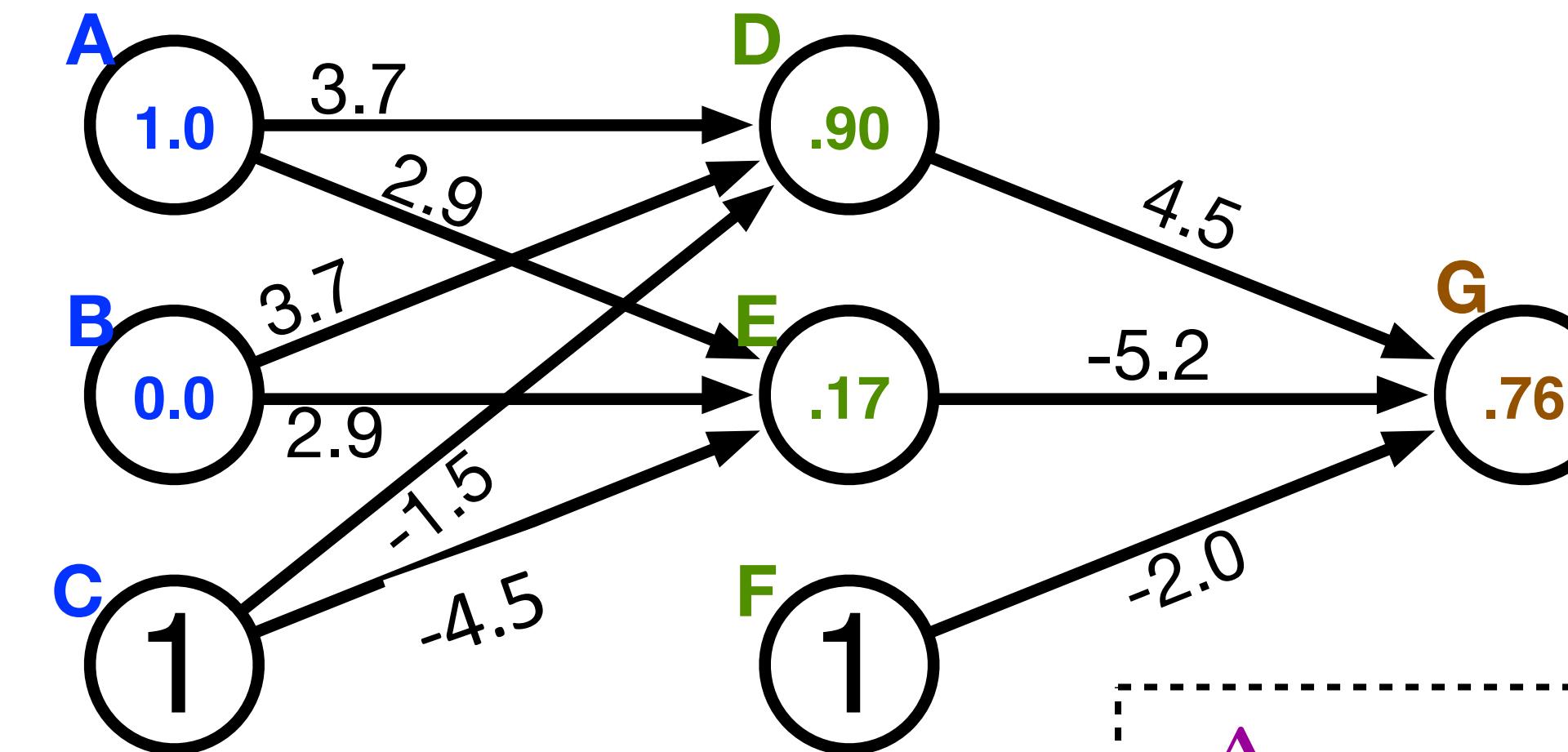
$$\Delta w_{j \leftarrow k} = \mu(t_j - y_j) \frac{y'_j}{\text{error term}} h_k$$

learning rate

error term

hidden node value

# Our Example



- ▶ Computed output:  $y = .76$
  - ▶ Correct output:  $t = 1.0$
  - ▶ Final layer weight updates (learning rate  $\mu = 10$ ):

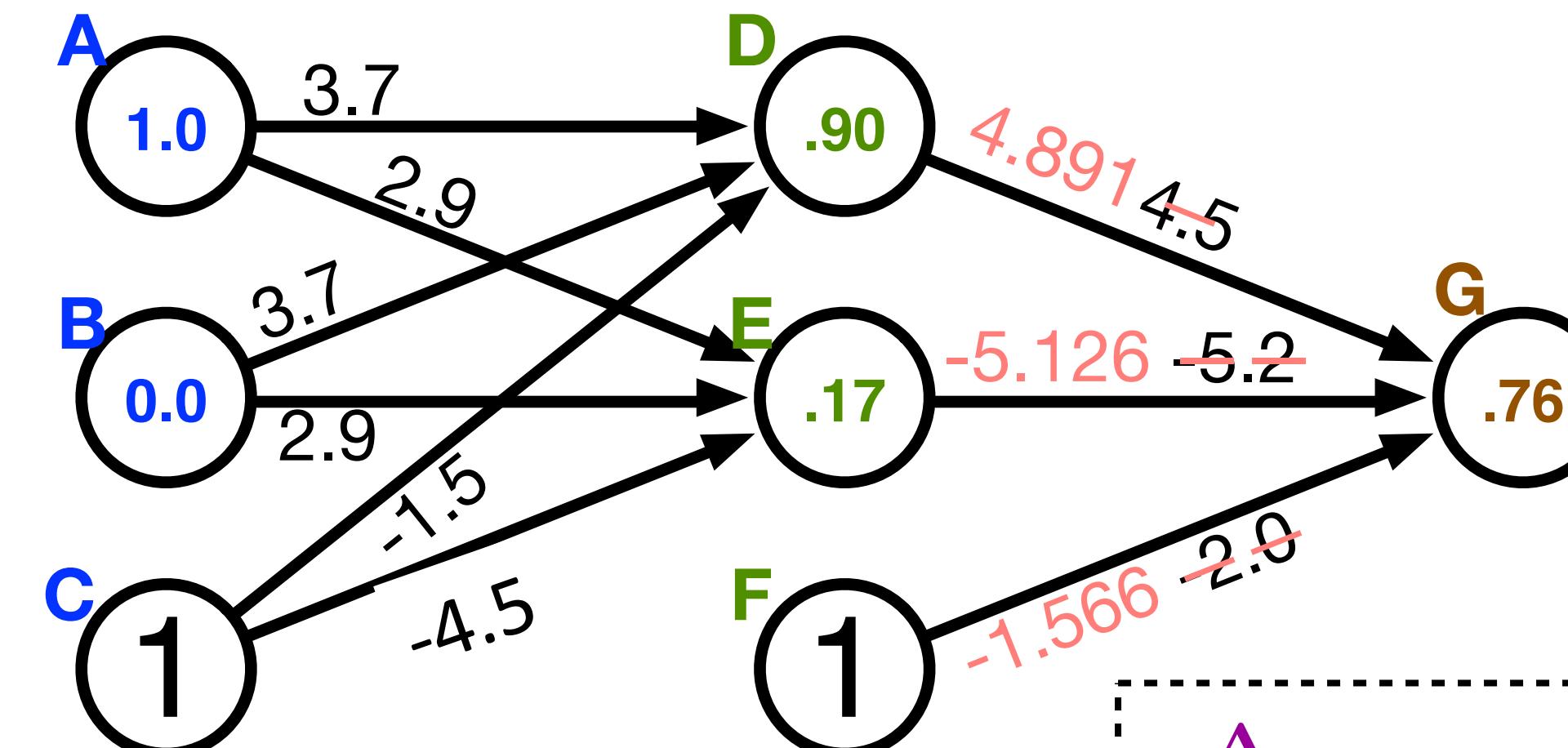
$$\delta_G = (t - y) y' = (1 - .76) 0.181 = .0434$$

$$\Delta w_{GD} = \mu \delta_G h_D = 10 \times .0434 \times .90 = .391$$

$$\Delta w_{GE} = \mu \delta_G h_E = 10 \times .0434 \times .17 = .074$$

$$\Delta w_{GF} = \mu \delta_G h_F = 10 \times .0434 \times 1 = .434$$

# Our Example



- Computed output:  $y = .76$
- Correct output:  $t = 1.0$
- Final layer weight updates (learning rate  $\mu = 10$  ):

$$\delta_G = (t - y) y' = (1 - .76) 0.181 = .0434$$

$$\Delta w_{GD} = \mu \delta_G h_D = 10 \times .0434 \times .90 = .391$$

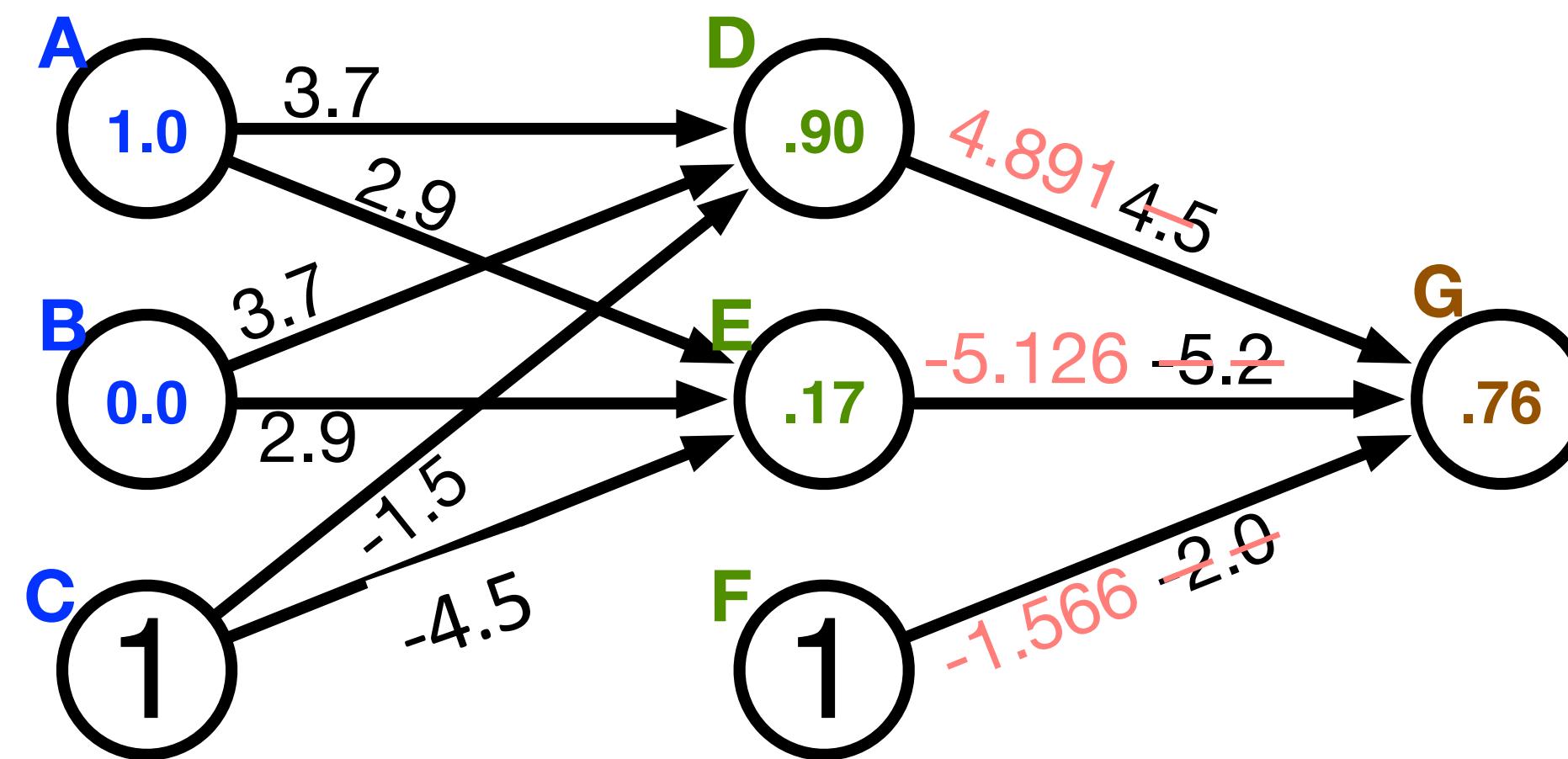
$$\Delta w_{GE} = \mu \delta_G h_E = 10 \times .0434 \times .17 = .074$$

$$\Delta w_{GF} = \mu \delta_G h_F = 10 \times .0434 \times 1 = .434$$

$$\Delta w_{j \leftarrow k} = \frac{\mu(t_j - y_j) y'_j}{\text{learning rate}} h_k$$

error term      hidden node value

# Hidden Layer Updates



## ▶ Hidden node D:

$$\delta_D = \left( \sum_j w_{j \leftarrow i} \delta_j \right) y'_D = w_{GD} \delta_G y'_D = 4.5 \times .0434 \times .0898 = .0175$$

$$\Delta w_{DA} = \mu \delta_D h_A = 10 \times .0175 \times 1.0 = .175$$

$$\Delta w_{DB} = \mu \delta_D h_B = 10 \times .0175 \times 0.0 = 0$$

$$\Delta w_{DC} = \mu \delta_D h_C = 10 \times .0175 \times 1 = .175$$

## ▶ Hidden node E:

$$\delta_E = \left( \sum_j w_{j \leftarrow i} \delta_j \right) y'_E = w_{GE} \delta_G y'_E = -5.2 \times .0434 \times 0.2055 = -.0464$$

$$\Delta w_{EA} = \mu \delta_E h_A = 10 \times -.0464 \times 1.0 = -.464$$

etc.

# Logistic Regression with NNs

---

$$P(y|\mathbf{x}) = \frac{\exp(w^\top f(\mathbf{x}, y))}{\sum_{y'} \exp(w^\top f(\mathbf{x}, y'))}$$

$$P(y|\mathbf{x}) = \text{softmax}_y(w^\top f(\mathbf{x}, y))$$

$$P(y|\mathbf{x}) = \text{softmax}_y(\underbrace{w_y^\top g(Vf(\mathbf{x}))}_{\text{Hidden representation } \mathbf{z}})$$

Hidden representation  $\mathbf{z}$ , can see this as “induced features”

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$

- ▶ Assumes that the labels  $y$  are indexed and associated with coordinates in a vector space

- ▶ Single scalar probability
- ▶  $\text{softmax}_y$ : score vector -> prob of  $y$
- ▶ Feature function no longer looks at label — same shared processing for each label.
- ▶ softmax: score vector -> probability vector

# Logistic Regression with NNs

---

$$P(y|\mathbf{x}) = \frac{\exp(w^\top f(\mathbf{x}, y))}{\sum_{y'} \exp(w^\top f(\mathbf{x}, y'))}$$

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}\left([w^\top f(\mathbf{x}, y)]_{y \in \mathcal{Y}}\right)$$

$$\text{softmax}(p)_i = \frac{\exp(p_i)}{\sum_{i'} \exp(p_{i'})}$$

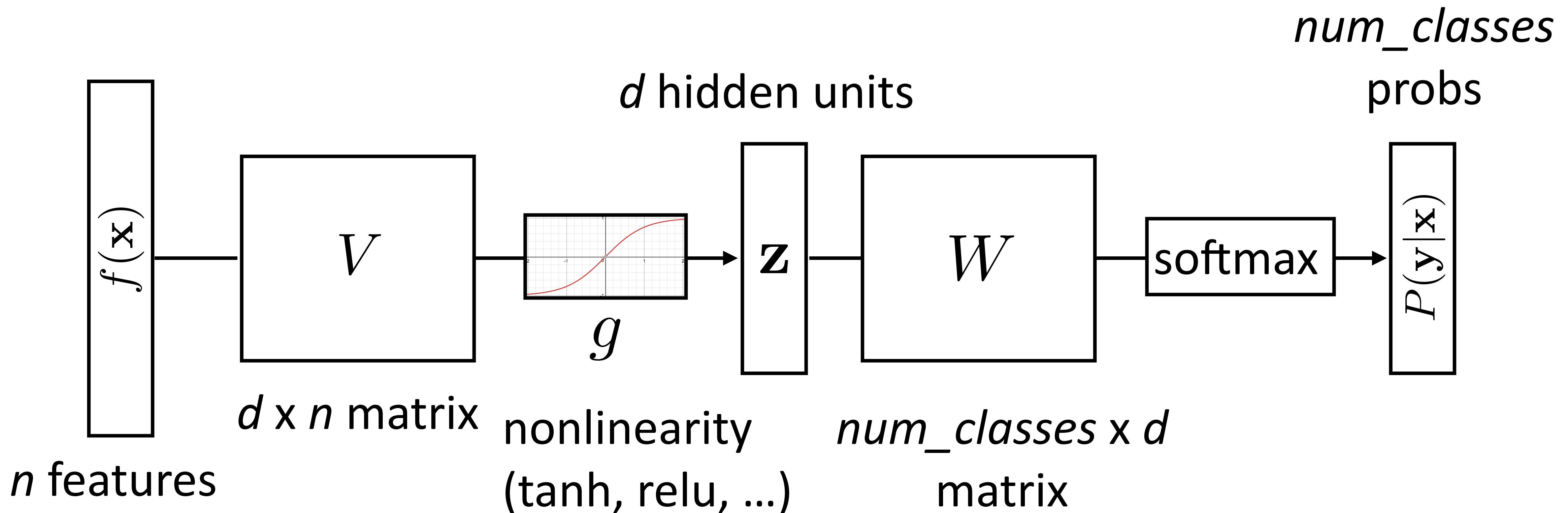
$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wf(\mathbf{x}))$$

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$

- ▶ Single scalar probability
- ▶ Compute scores for all possible labels at once (returns vector)
- ▶ softmax: exps and normalizes a given vector
- ▶ Weight vector per class; W is [num classes x num feats]
- ▶ Now one hidden layer

# Neural Networks for Classification

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$



We can think of a neural network classifier with one hidden layer as building a vector  $\mathbf{z}$  which is a hidden layer representation (i.e. latent features) of the input, and then running standard logistic regression on the features that the network develops in  $\mathbf{z}$ .

# Training Neural Networks

---

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(W\mathbf{z}) \quad \mathbf{z} = g(Vf(\mathbf{x}))$$

- ▶ Maximize log likelihood of training data

$$\mathcal{L}(\mathbf{x}, i^*) = \log P(y = i^* | \mathbf{x}) = \log (\text{softmax}(W\mathbf{z}) \cdot e_{i^*})$$

- ▶  $i^*$ : index of the gold label
- ▶  $e_i$ : 1 in the  $i$ th row, zero elsewhere. Dot by this = select  $i$ th index

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_j \exp(W\mathbf{z}) \cdot e_j$$

# Computing Gradients

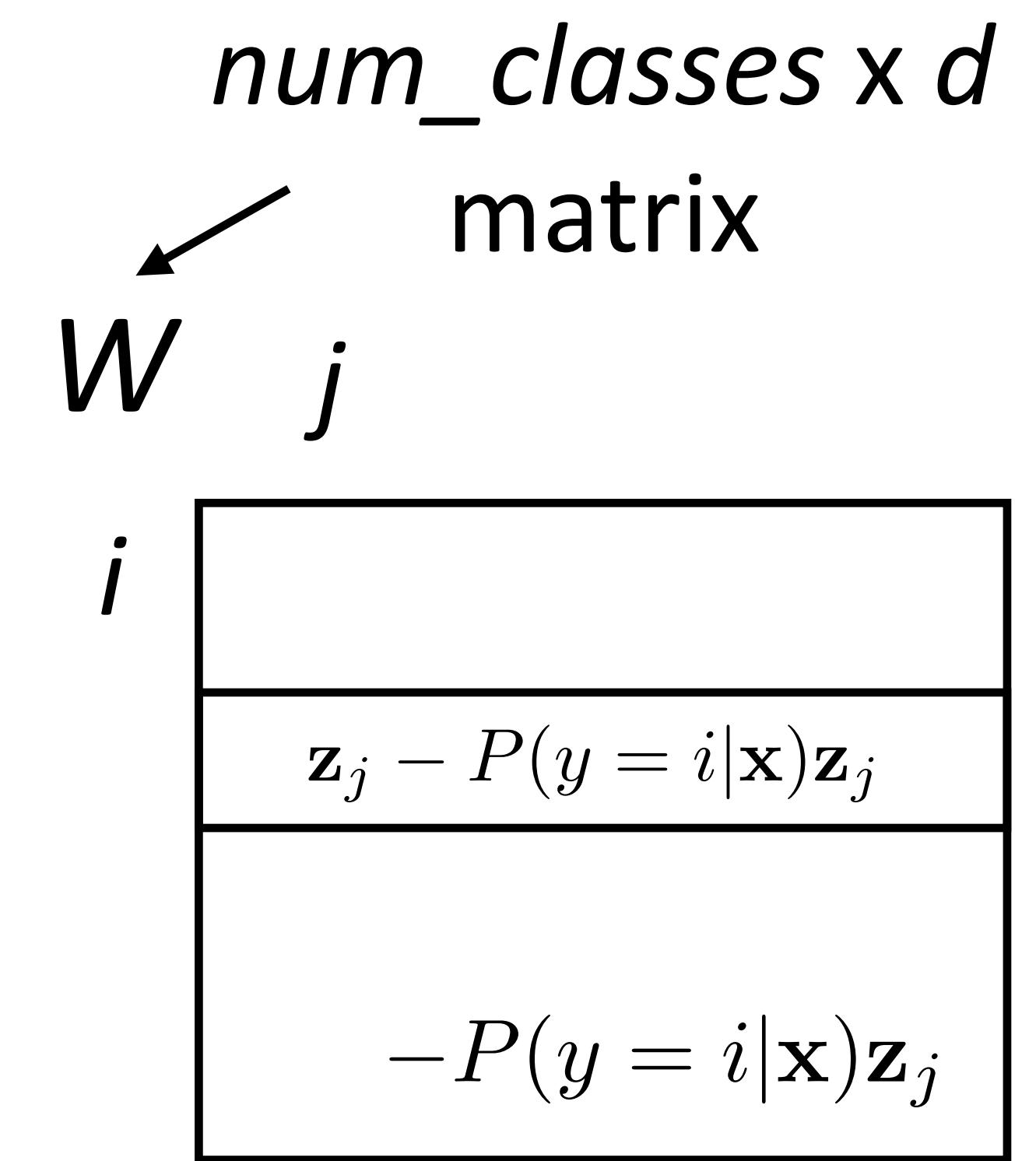
$$\mathcal{L}(\mathbf{x}, i^*) = \mathbf{Wz} \cdot e_{i^*} - \log \sum_j \exp(\mathbf{Wz}) \cdot e_j$$

- Gradient with respect to  $\mathbf{W}$

$$\frac{\partial}{\partial W_{ij}} \mathcal{L}(\mathbf{x}, i^*) = \begin{cases} \mathbf{z}_j - P(y = i|\mathbf{x})\mathbf{z}_j & \text{if } i = i^* \\ -P(y = i|\mathbf{x})\mathbf{z}_j & \text{otherwise} \end{cases}$$

index of  
output space  $\gamma$

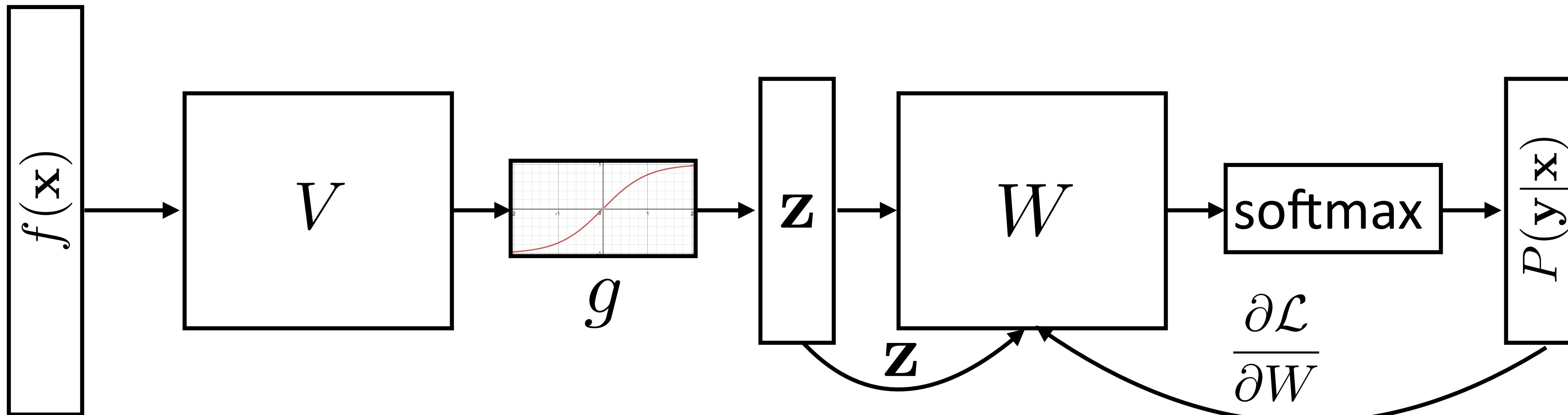
index of vector  $\mathbf{z}$



- Looks like logistic regression with  $\mathbf{z}$  as the features!

# Neural Networks for Classification

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$



$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{W}} = \mathbf{z}(e_{i^*} - P(\mathbf{y}|\mathbf{x})) = \mathbf{z} \cdot err(\text{root})$$

# Computing Gradients

---

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_j \exp(W\mathbf{z}) \cdot e_j$$

- Gradient with respect to  $\mathbf{z}$  [some math...]

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = W_{i^*} - \sum_j P(y = j | \mathbf{x}) W_j$$

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^\top err(\text{root})$$

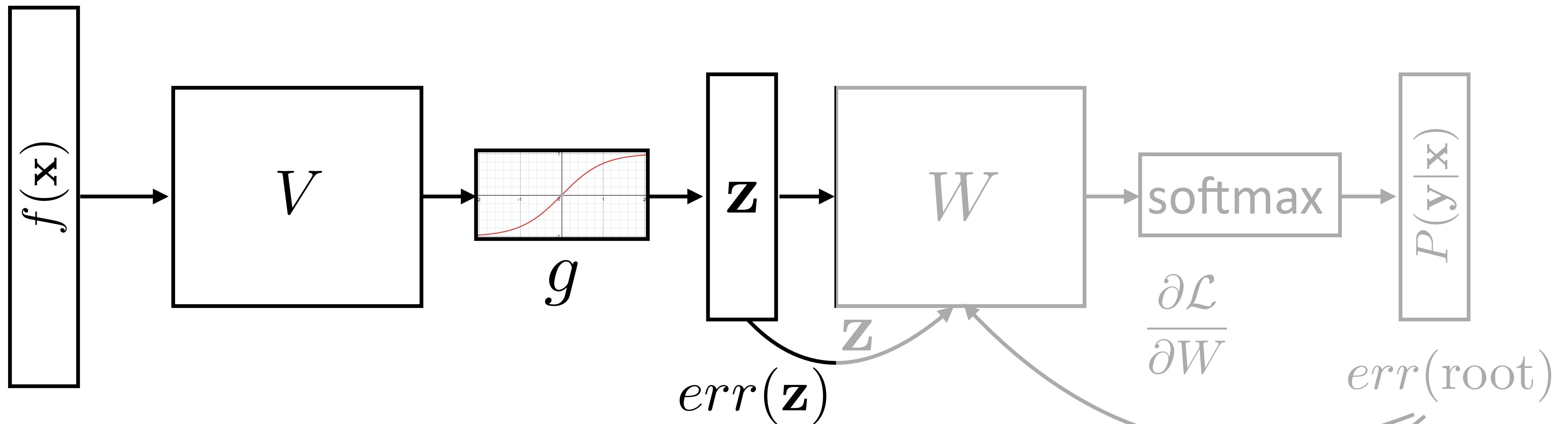
$\dim = d$

$$err(\text{root}) = e_{i^*} - P(\mathbf{y} | \mathbf{x})$$

$\dim = \text{num\_classes}$

# Backpropagation: Picture

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$



- ▶ Can forget everything after  $\mathbf{z}$ , treat it as the output and keep backpropping

$$\boxed{\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^\top err(\text{root})}$$

# Computing Gradients: Backpropagation

$$\mathcal{L}(\mathbf{x}, i^*) = W\mathbf{z} \cdot e_{i^*} - \log \sum_j \exp(W\mathbf{z}) \cdot e_j \quad \mathbf{z} = g(Vf(\mathbf{x}))$$

Activations at  
hidden layer

- Gradient with respect to  $V$ : apply the chain rule

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial V_{ij}}$$

[some math...]

$$err(\text{root}) = e_{i^*} - P(y|\mathbf{x})$$

dim = num\_classes

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^\top err(\text{root})$$

dim = d

# Computing Gradients: Backpropagation

$$\mathcal{L}(\mathbf{x}, i^*) = \mathbf{W}\mathbf{z} \cdot e_{i^*} - \log \sum_{j=1}^m \exp(\mathbf{W}\mathbf{z} \cdot e_j)$$

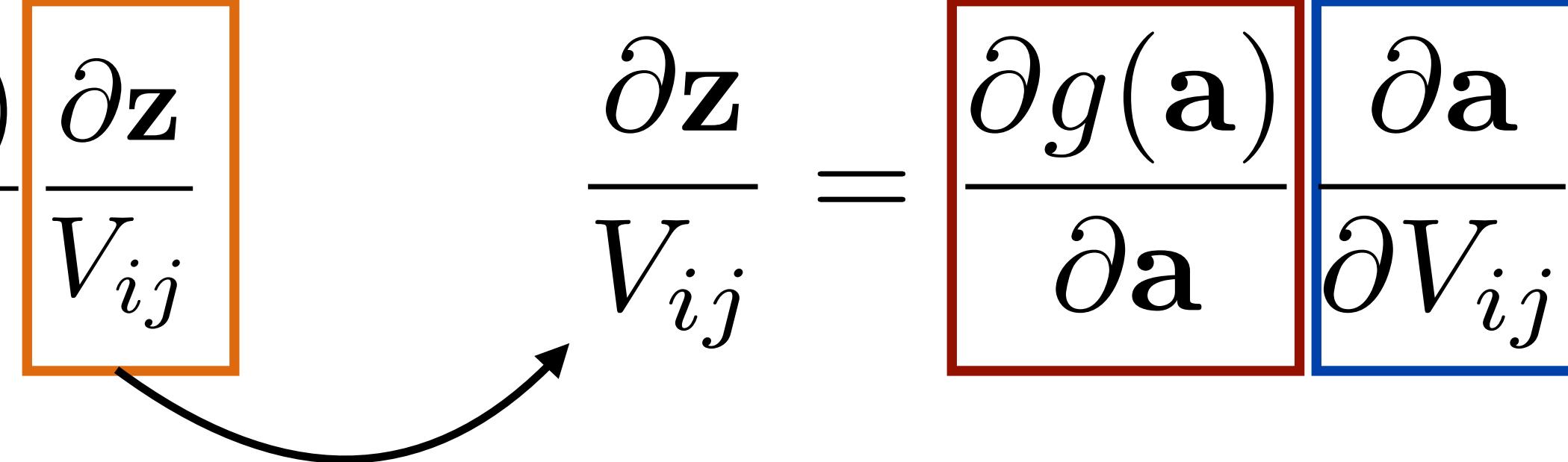
$$\mathbf{z} = g(Vf(\mathbf{x}))$$

Activations at  
hidden layer

- Gradient with respect to  $V$ : apply the chain rule

$$\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial V_{ij}} = \frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} \frac{\partial \mathbf{z}}{\partial V_{ij}}$$
$$\frac{\partial \mathbf{z}}{\partial V_{ij}} = \frac{\partial g(\mathbf{a})}{\partial \mathbf{a}} \frac{\partial \mathbf{a}}{\partial V_{ij}}$$

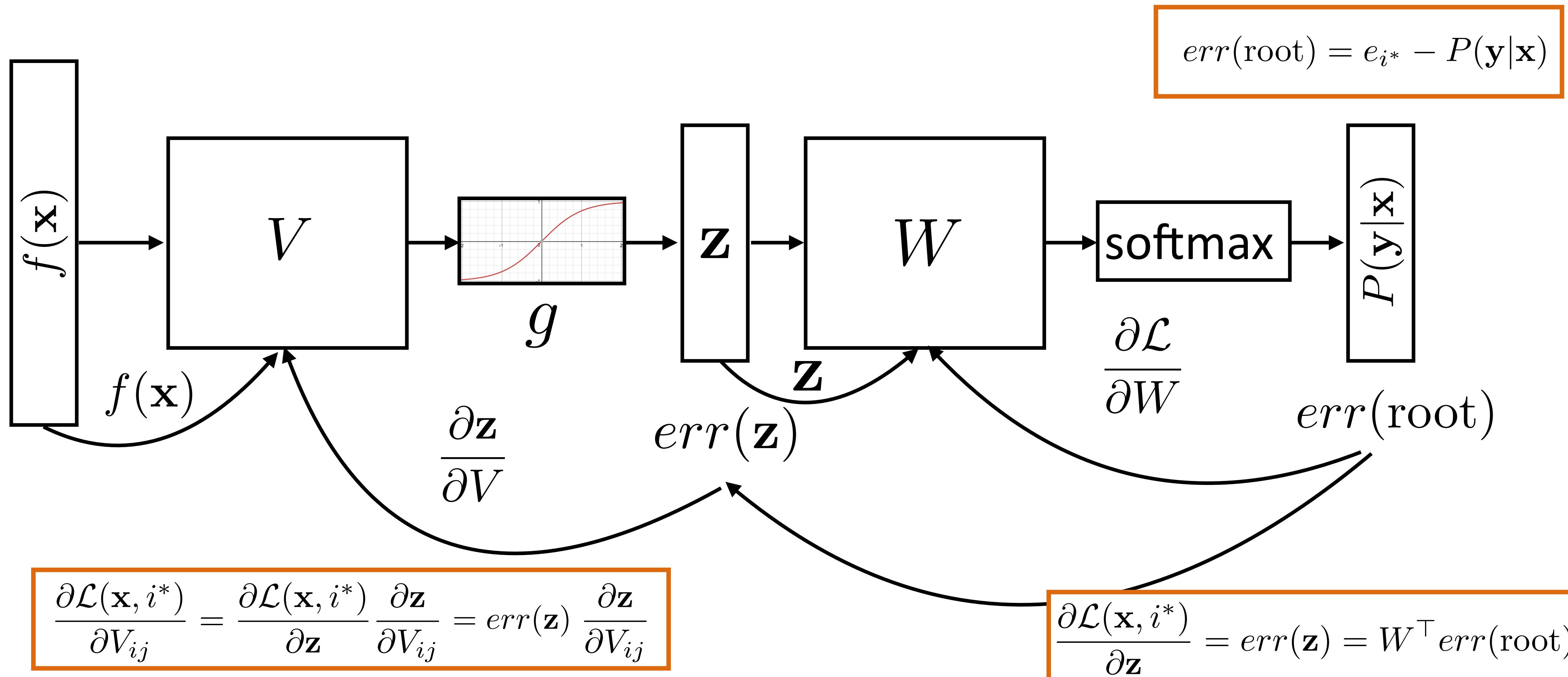
$\mathbf{a} = Vf(\mathbf{x})$



- First term: gradient of nonlinear activation function at  $\mathbf{a}$  (depends on current value)
- Second term: gradient of linear function
- Straightforward computation once we have  $err(\mathbf{z})$

# Backpropagation: Picture

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$



# Backpropagation

---

$$P(\mathbf{y}|\mathbf{x}) = \text{softmax}(Wg(Vf(\mathbf{x})))$$

- ▶ Step 1: compute  $err(\text{root}) = e_{i^*} - P(\mathbf{y}|\mathbf{x})$  (vector)
- ▶ Step 2: compute derivatives of  $W$  using  $err(\text{root})$  (matrix)
- ▶ Step 3: compute  $\frac{\partial \mathcal{L}(\mathbf{x}, i^*)}{\partial \mathbf{z}} = err(\mathbf{z}) = W^\top err(\text{root})$  (vector)
- ▶ Step 4: compute derivatives of  $V$  using  $err(\mathbf{z})$  (matrix)
- ▶ Step 5+: continue backpropagation (compute  $err(f(\mathbf{x}))$  if necessary...)

# Backpropagation: Takeaways

---

- ▶ Gradients of output weights  $W$  are easy to compute — looks like logistic regression with hidden layer  $z$  as feature vector
- ▶ Can compute derivative of loss with respect to  $z$  to form an “error signal” for backpropagation
- ▶ Easy to update parameters based on “error signal” from next layer, keep pushing error signal back as backpropagation
- ▶ Need to remember the values from the forward computation

# Applications

# NLP with Feedforward Networks

- ▶ Part-of-speech tagging with FFNNs

??

*Fed raises interest rates in order to ...*

previous word

- ▶ Word embeddings for each word form input

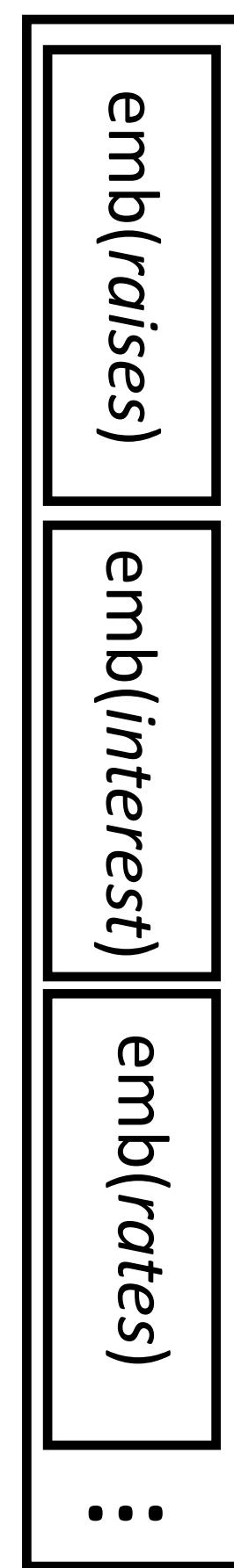
~1000 features here – smaller feature vector  
than in sparse models, but every feature fires on  
every example

curr word

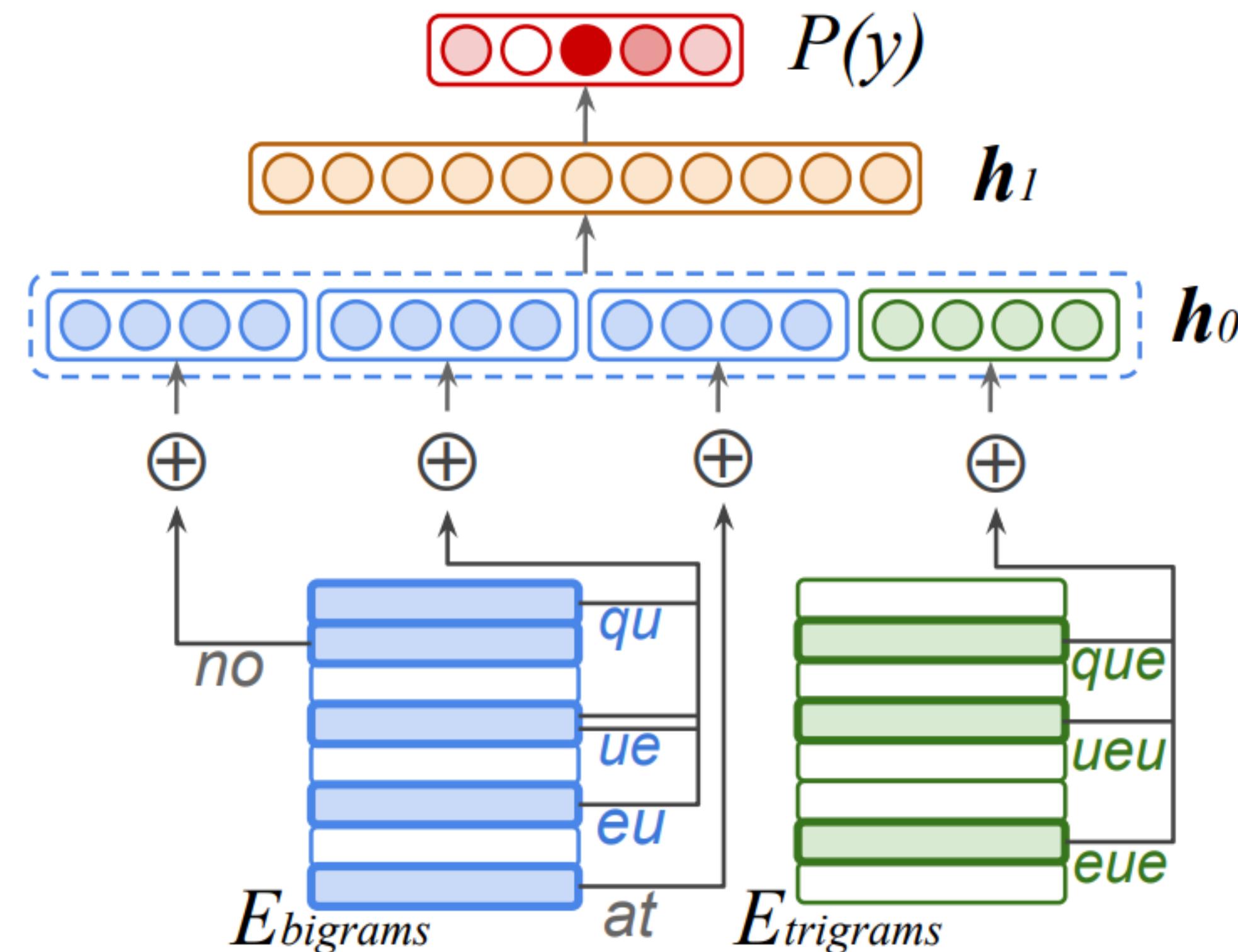
- ▶ Weight matrix learns position-dependent  
processing of the words

other words, feats, etc.

$$f(x)$$



# NLP with Feedforward Networks



There was no queue at the ...

- ▶ Hidden layer mixes these different signals and learns feature conjunctions

# NLP with Feedforward Networks

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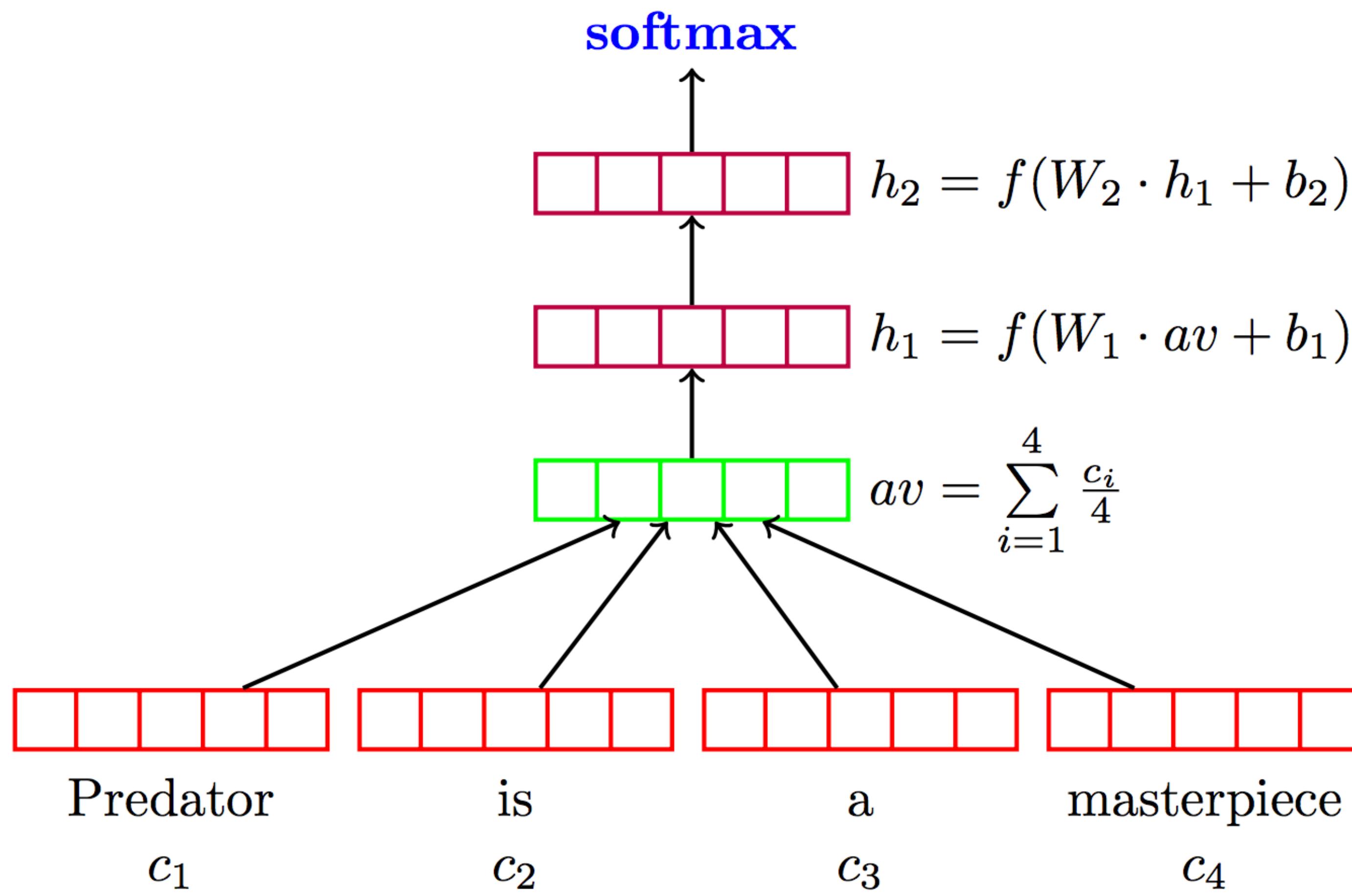
- ▶ Multilingual tagging results:

<b>Model</b>	<b>Acc.</b>	<b>Wts.</b>	<b>MB</b>	<b>Ops.</b>
Gillick et al. (2016)	95.06	900k	-	6.63m
Small FF	94.76	241k	0.6	0.27m
+Clusters	95.56	261k	1.0	0.31m
$\frac{1}{2}$ Dim.	95.39	143k	0.7	0.18m

- ▶ Gillick used LSTMs; this is smaller, faster, and better

# Sentiment Analysis

- ▶ Deep Averaging Networks: feedforward neural network on average of word embeddings from input



Iyyer et al. (2015)

# Sentiment Analysis

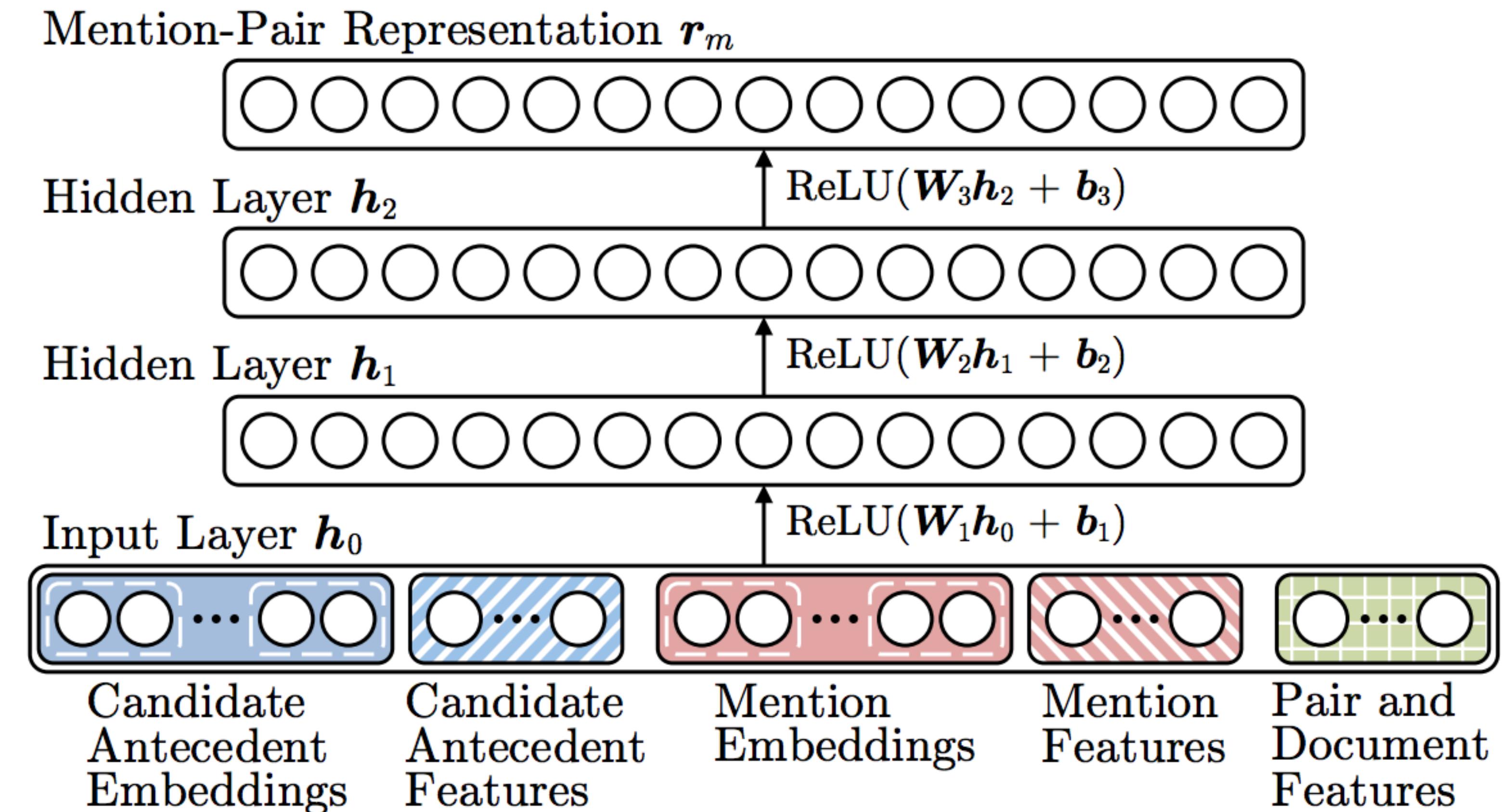
	Model	RT	SST fine	SST bin	IMDB	Time (s)	
Bag-of-words	DAN-ROOT	—	46.9	85.7	—	31	Iyyer et al. (2015)
	DAN-RAND	77.3	45.4	83.2	88.8	136	
	DAN	80.3	47.7	86.3	89.4	136	
	NBOW-RAND	76.2	42.3	81.4	88.9	91	
Tree RNNs / CNNs / LSTMs	NBOW	79.0	43.6	83.6	89.0	91	Wang and Manning (2012)
	BiNB	—	41.9	83.1	—	—	
	NBSVM-bi	79.4	—	—	91.2	—	
	RecNN*	77.7	43.2	82.4	—	—	
Tree RNNs / CNNs / LSTMs	RecNTN*	—	45.7	85.4	—	—	Kim (2014)
	DRecNN	—	49.8	86.6	—	431	
	TreeLSTM	—	<b>50.6</b>	86.9	—	—	
	DCNN*	—	48.5	86.9	89.4	—	
	PVEC*	—	48.7	87.8	<b>92.6</b>	—	
	CNN-MC	<b>81.1</b>	47.4	<b>88.1</b>	—	2,452	
	WRRBMs*	—	—	—	89.2	—	

# Coreference Resolution

- Feedforward networks identify coreference arcs

*President Obama signed...*  
?

*He later gave a speech...*



# Next Class

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- ▶ Training tips for neural networks
- ▶ Word representations
- ▶ word2vec/GloVe
- ▶ Evaluating word embeddings