# CS7650 Problem Sets (Fall 2021)

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Please write your answers and any work on a separate sheet of paper.

## 1 Joint and Marginal Probabilities

Assume the following joint distribution for $P(A, B)$:

$$\begin{aligned} P(A=0, B=0) &= 0.2 \\ P(A=0, B=1) &= 0.5 \\ P(A=1, B=0) &= 0.1 \\ P(A=1, B=1) &= 0.2 \end{aligned}$$

Q1.1) What is the marginal probability of $P(A = 0)$?

Q1.2) What is $P(B = 0|A = 1)$?

Q1.3) What is $P(A = B)$?

## 2 Independence

Q2) Assume $X$ is conditionally independent of $Y$ given $Z$. Which of the following statements are always true? Note that there may be more than 1 correct answer.

(a) $P(X, Y) = \sum_{c \in \mathcal{X}_Z} P(X, Y, Z = c)$

(b) $P(X, Y, Z) = P(X) + P(Y) + P(Z = c), c \in \mathcal{X}_Z$

(c) $P(X, Y|Z) = P(X|Z)P(Y|Z)$

(d) $P(X, Y, Z) = P(X) + P(Y) - P(Z)$

(e) $P(X, Y) = P(X)P(Y)$

# 3 Bayes Rule

Q3) There is a 10% chance that a thunderstorm is approaching at any given moment. You own a dog that has a 75% chance of barking when a thunderstorm is approaching and only a 25% chance of barking when there is no thunderstorm approaching. If your dog is currently barking, how likely is it that a thunderstorm is approaching?

# 4 Entropy

The entropy of a random variable $x$ with a probability distribution $p(x)$ is given by:

$$\mathrm{H}[x] = -\sum_x p(x) \log_2 p(x)$$

Consider two binary random variables $x$ and $y$ having the joint distribution:

| | | y | |
|---|---|---|---|
| | | 0 | 1 |
| x | 0 | 0 | 1/5 |
| | 1 | 2/5 | 2/5 |

Q4.1) Evaluate $\mathrm{H}[x]$

Q4.2) Evaluate $\mathrm{H}[y]$

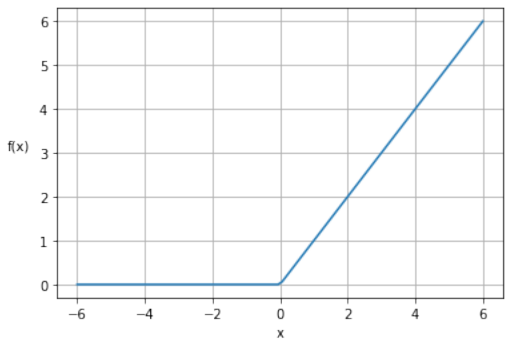
Q4.3) Evaluate $\mathrm{H}[x, y]$

# 5 Calculus Review

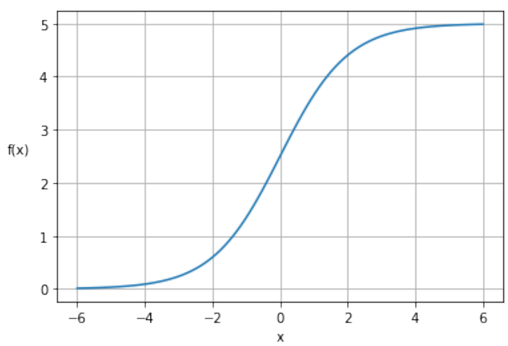
Consider the following function (often referred to as the *logistic* or *sigmoid* function):

$$f(x) = \frac{1}{1 + e^{-x}}$$

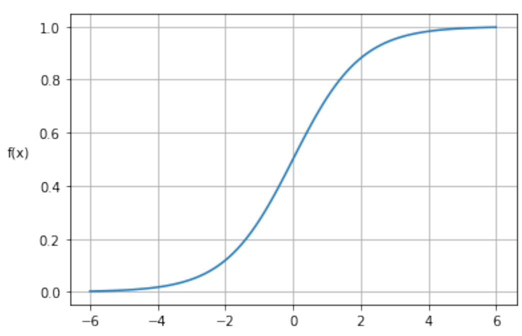
Q5.1) Select the plot that matches the given equation.



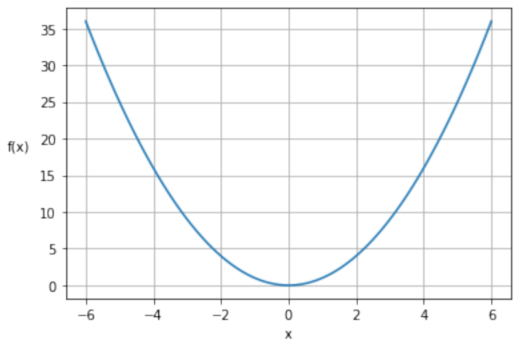

(a)




(b)




(c)




(d)

Q5.2) What are the maximum and minimum values of this function? Why might this function be useful when considering probability? (1-2 sentences)

Q5.3) Show that the derivative of $f(x)$ can be written simply in terms of the function's value like so:

$$\frac{df(x)}{dx} = f(x)(1 - f(x))$$

Q5.4) Consider the function $g : \mathbb{R}^n \to \mathbb{R}, \quad g(\mathbf{x}) = \mathbf{x}^T\mathbf{x} + b^T\mathbf{x}$ where $b$ is a constant vector $b \in \mathbb{R}^n$. For a constant matrix $A \in \mathbb{R}^{n \times n}$, calculate the following in terms of $A$, $b$, and $\mathbf{x}$:

$$\frac{d(g(A\mathbf{x}))}{d\mathbf{x}}$$