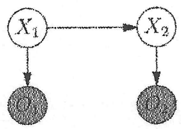
1. **(7 points) There will be a few questions that are based on your programming project #3 (Reinforcement Learning) and #4 (Ghostbusters). Similar to the midterm, it may include identifying errors in a given block of code or writing out a small piece of pseudocode.**

2. **(6 points)** Consider the following Hidden Markov Model.



X_1	$P(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$P(X_{t+1}\|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$P(O_t\|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that $O_1 = A$ and $O_2 = B$ is observed.

(a) Use the Forward algorithm to compute the probability distribution $P(X_2, O_1 = A, O_2 = B)$. Show your work. You do not need to evaluate arithmetic expressions involving only numbers.

$P(X_1)P(O_1=A|X_1)$

X_1	$P(X_1, O_1=A)$
0	0.3×0.9
1	0.7×0.5

X_2	$P(X_2, O_1=A, O_2=B)$
0	$0.1\times[(0.3\times 0.9)\times 0.4 + (0.7\times 0.5)\times 0.8] = 0.0388$
1	$0.5\times[(0.3\times 0.9)\times 0.6 + (0.7\times 0.5)\times 0.2] = 0.1160$

$$P(X_2, O_1=A, O_2=B) = \sum_{X_1} P(X_1, X_2, O_1=A, O_2=B) = \sum_{X_1} P(X_1, O_1=A)P(X_2|X_1)P(O_2=B|X_2)$$
$$= P(O_2=B|X_2)\sum_{X_1} P(X_1, O_1=A)\,P(X_2|X_1)$$

(b) Compute the probability $P(X_1 = 1|O_1 = A, O_2 = B)$. Show your work.

X_1	X_2	$P(X_1, X_2, O_1=A, O_2=B)$
0	0	$0.1\times[(0.3\times 0.9)\times 0.4] = 0.0108$
0	1	$0.5\times[(0.3\times 0.9)\times 0.6] = 0.081$
1	0	$0.1\times[(0.7\times 0.5)\times 0.8] = 0.028$
1	1	$0.5\times[(0.7\times 0.5)\times 0.2] = 0.035$

$$P(X_1=1|O_1=A, O_2=B) = \frac{0.028+0.035}{0.0108+0.081+0.028+0.035}$$

(c) *True* or *False*? Variable elimination is generally more accurate than the Forward algorithm. Explain your answer.

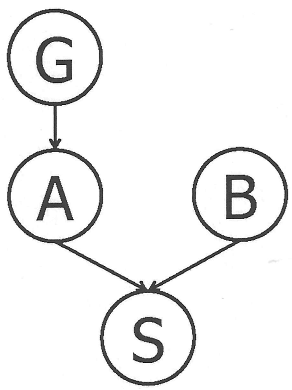
False. Both variable elimination and the Forward algorithm are exact inference.

Cont.

3. **(6 points)** Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown below. For each part, you may leave your answer as an arithmetic expression.

$P(G)$	
$+g$	0.1
$-g$	0.9

$P(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$P(B)$	
$+b$	0.4
$-b$	0.6

$P(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

(a) Compute the following entry from the joint distribution:

$P(+g, +a, +b, +s) = P(+g)P(+a|+g)P(+b)P(+s|+a,+b)$

$= 0.1 \times 1.0 \times 0.4 \times 1.0$

$= 0.04$

(b) What is the probability that a patient has disease A?

$P(+a) = P(+a|+g)P(+g) + P(+a|-g)P(-g)$

$= 1.0 \times 0.1 + 0.1 \times 0.9$

$= 0.19$

(c) What is the probability that a patient has disease A given that they have disease B?

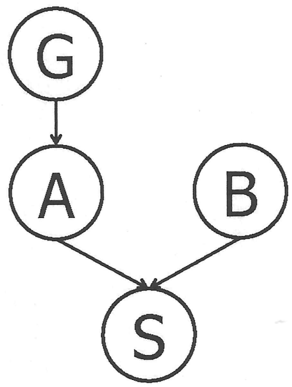
$P(+a| +b) = P(+a) = 0.19$ because $A \perp\!\!\!\perp B$

Cont.

The figures and table below are identical to the ones on the previous page and are repeated here for your convenience.

$P(G)$	
$+g$	0.1
$-g$	0.9

$P(A\|G)$		
$+g$	$+a$	1.0
$+g$	$-a$	0.0
$-g$	$+a$	0.1
$-g$	$-a$	0.9



$P(B)$	
$+b$	0.4
$-b$	0.6

$P(S\|A,B)$			
$+a$	$+b$	$+s$	1.0
$+a$	$+b$	$-s$	0.0
$+a$	$-b$	$+s$	0.9
$+a$	$-b$	$-s$	0.1
$-a$	$+b$	$+s$	0.8
$-a$	$+b$	$-s$	0.2
$-a$	$-b$	$+s$	0.1
$-a$	$-b$	$-s$	0.9

(d) What is the probability that a patient has disease A given that they have symptom S and disease B?

$$P(+a|+s,+b) = \frac{P(+a,+b,+s)}{P(+a,+b,+s)+P(-a,+b,+s)} = \frac{P(+a)P(+b)P(+s|+a,+b)}{P(+a)P(+b)P(+s|+a,+b)+P(-a)P(+b)P(+s|-a,+b)}$$

$$= \frac{0.19\times 0.4\times 1.0}{0.19\times 0.4\times 1.0 + 0.81\times 0.4\times 0.8}$$

(e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

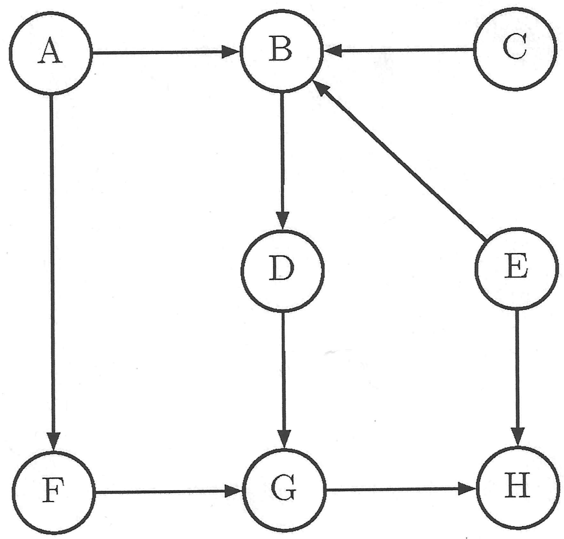
$$P(+g|+a) = \frac{P(+g)P(+a|+g)}{P(+g)P(+a|+g)+P(-g)P(+a|-g)} = \frac{0.1\times 1.0}{0.1\times 1.0+0.9\times 0.1}$$

(f) What is the probability that a patient has the disease carrying gene variation G given that they have disease B?

$P(+g|+b) =$ $P(+g) = 0.1$ because $G \perp\!\!\!\perp B$

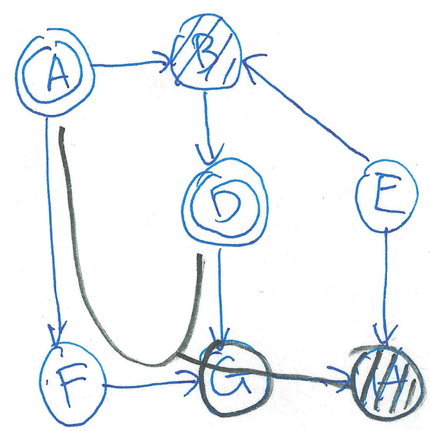
Cont.

4. **(4 points)** Consider the Bayes' net given below.



Remember that $X \perp\!\!\!\perp Y$ reads as "X is independent of Y given nothing", and $X \perp\!\!\!\perp Y|\{Z, W\}$ reads as "X is independent of Y given Z and W."

For each expression, fill in the corresponding circle to indicate whether it is True or False.



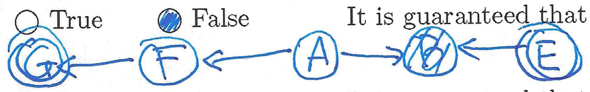
(i) ○ True ● False It is guaranteed that $A \perp\!\!\!\perp B$

(ii) ● True ○ False It is guaranteed that $A \perp\!\!\!\perp C$

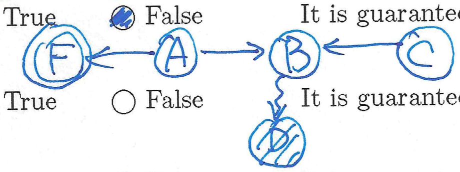
(iii) ○ True ● False It is guaranteed that $A \perp\!\!\!\perp D \mid \{B, H\}$

(iv) ● True ○ False It is guaranteed that $A \perp\!\!\!\perp E|F$

(v) ○ True ● False It is guaranteed that $G \perp\!\!\!\perp E|B$

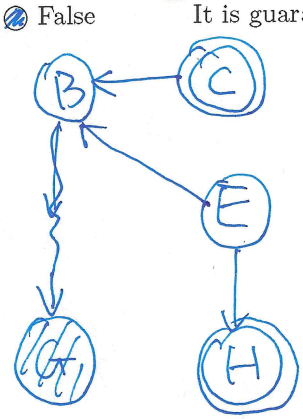


(vi) ○ True ● False It is guaranteed that $F \perp\!\!\!\perp C|D$



(vii) ● True ○ False It is guaranteed that $E \perp\!\!\!\perp D|B$

(viii) ○ True ● False It is guaranteed that $C \perp\!\!\!\perp H|G$



The End.