Log-Sum-Exp Trick

$a_1 = 3.96 \times 10^{-107}$ $\quad k_1 = \log(a_1) = -245$

$a_2 = 1.80 \times 10^{-111}$ $\quad k_2 = \log(a_2) = -255$

compute

$a_1 + a_2 = ?$ $\qquad M = \max(k_1, k_2) = -245$

$$\log(a_1 + a_2)$$

$$= \log(e^{k_1} + e^{k_2})$$

$$= \log\left(e^{M} \cdot (e^{k_1 - M} + e^{k_2 - M})\right)$$

$$= \log e^{M} + \log\left(e^{k_1 - M} + e^{k_2 - M}\right)$$

$$= M + \log(e^{0} + e^{-10})$$

$$= -245 + \log(e^{0} + e^{-10})$$