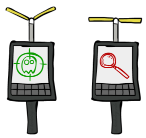
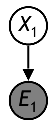
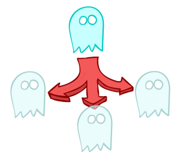
Inference: Base Cases

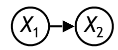




$$P(X_1|e_1)$$

$$P(x_1|e_1) = P(x_1,e_1)/P(e_1)$$
$$\propto_{X_1} P(x_1,e_1)$$
$$= P(x_1)P(e_1|x_1)$$

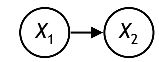




$$P(X_2)$$

$$P(x_2) = \sum_{x_1} P(x_1,x_2)$$
$$= \sum_{x_1} P(x_1)P(x_2|x_1)$$

Passage of Time



- Assume we have current belief P(X | evidence to date)

  $$B(X_t) = P(X_t|e_{1:t})$$

- Then, after one time step passes:

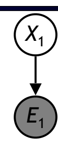
$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1},x_t|e_{1:t})$$
$$= \sum_{x_t} P(X_{t+1}|x_t,e_{1:t})P(x_t|e_{1:t})$$
$$= \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})$$

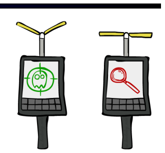
- Or compactly:

  $$B'(X_{t+1}) = \sum_{x_t} P(X'|x_t)B(x_t)$$

- Basic idea: beliefs get "pushed" through the transitions
  - With the "B" notation, we have to be careful about what time step t the belief is about, and what evidence it includes

Observation





- Assume we have current belief P(X | previous evidence):

  $$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$

- Then, after evidence comes in:

$$P(X_{t+1}|e_{1:t+1}) = P(X_{t+1},e_{t+1}|e_{1:t})/P(e_{t+1}|e_{1:t})$$
$$\propto_{X_{t+1}} P(X_{t+1},e_{t+1}|e_{1:t})$$
$$= P(e_{t+1}|e_{1:t},X_{t+1})P(X_{t+1}|e_{1:t})$$
$$= P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$$

- Or, compactly:

  $$B(X_{t+1}) \propto_{X_{t+1}} P(e_{t+1}|X_{t+1})B'(X_{t+1})$$

- Basic idea: beliefs "reweighted" by likelihood of evidence
- Unlike passage of time, we have to renormalize

The Forward Algorithm

- We are given evidence at each time and want to know

  $$B_t(X) = P(X_t|e_{1:t})$$

- We can derive the following updates

$$P(x_t|e_{1:t}) \propto_X P(x_t,e_{1:t})$$
$$= \sum_{x_{t-1}} P(x_{t-1},x_t,e_{1:t})$$
$$= \sum_{x_{t-1}} P(x_{t-1},e_{1:t-1})P(x_t|x_{t-1})P(e_t|x_t)$$
$$= P(e_t|x_t)\sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1},e_{1:t-1})$$

We can normalize as we go if we want to have P(x|e) at each time step, or just once at the end…

Active / Inactive Paths

- Question: Are X and Y conditionally independent given evidence variables {Z}?
  - Yes, if X and Y "d-separated" by Z
  - Consider all (undirected) paths from X to Y
  - No active paths = independence!
- A path is active if each triple is active:
  - Causal chain A → B → C where B is unobserved (either direction)
  - Common cause A ← B → C where B is unobserved
  - Common effect (aka v-structure)
    A → B ← C where B *or one of its descendents* is observed
- All it takes to block a path is a single inactive segment