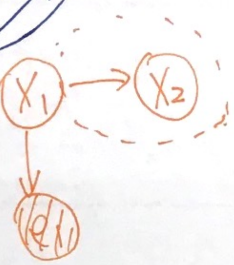
HMM



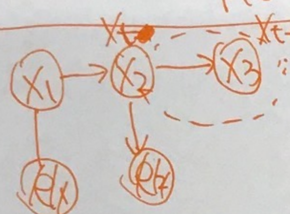
Given $P(X_t | e_{1:t})$

also $P(X_0)$. $P(X_{t+1} | X_t)$. $P(e_t | X_t)$

How to get $P(X_{t+1} | e_{1:t})$?

Base

$$P(X_1|e_1) = \frac{P(X_1,e_1)}{P(e_1)} \propto P(X_1,e_1) = P(X_1)P(e_1|X_1) \qquad P(X_2) = \sum_{x_1} P(X_1,X_2) = \sum_{x_1} P(X_1)P(X_2|X_1)$$

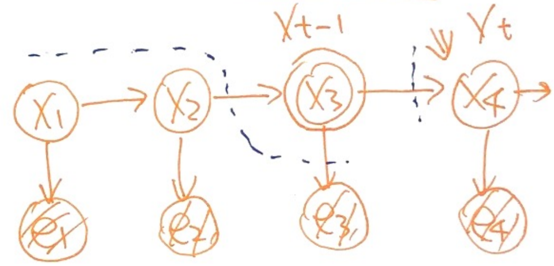


$$\begin{aligned} P(X_{t+1}|e_{1:t}) &= \sum_{x_t} P(X_{t+1}, x_t | e_{1:t}) \\ &= \sum_{x_t} \underbrace{P(X_{t+1} | x_t, e_{1:t})}_{\text{HMM independence assumption}} P(x_t | e_{1:t}) \\ &= \sum_{x_t} P(X_{t+1} | x_t) P(x_t | e_{1:t}) \end{aligned}$$

Independence

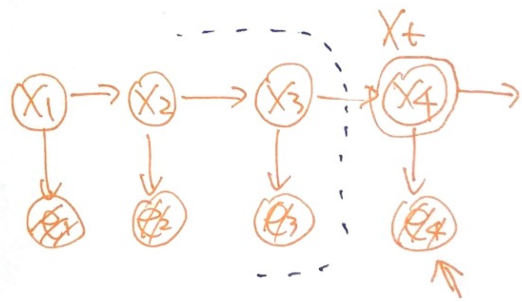
$X_t \perp\!\!\!\perp X_1, \cdots X_{t-2} \mid X_{t-1}$

$X_t \perp\!\!\!\perp E_1, \cdots E_{t-1} \mid X_{t-1}$



$E_t \perp\!\!\!\perp X_1, \cdots, X_{t-1} \mid X_t$

$E_t \perp\!\!\!\perp E_1, \cdots, E_{t-1} \mid X_t$



$P(X_t | X_{0:t-1}) = P(X_t | X_{t-1})$

$P(E_t | X_{0:t}; E_{0:t-1}) = P(E_t | X_t)$

Weather HMM

$B_0(+r, -r) = P(R_0) = \langle 0.5, 0.5 \rangle$ initialization

↓ time pass

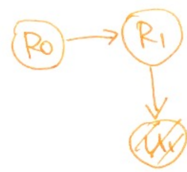
$B_0'(+r, -r) = P(R_1) = \sum_{r_0} P(R_1 | r_0) P(r_0)$

$= \langle 0.7, 0.3 \rangle \times 0.5 + \langle 0.3, 0.7 \rangle \times 0.5$

$= \langle 0.5, 0.5 \rangle$

↓ observe an evidence U_1 = true (umbrella appears)

$B_1(+r, -r) = P(R_1 | u_1) = \dfrac{P(R_1, u_1)}{P(u_1)}$



$\propto_{R_1} P(R_1, u_1)$

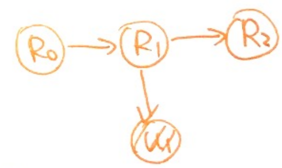
$= P(u_1 | R_1) P(R_1)$

$= \langle 0.9, 0.2 \rangle \langle 0.5, 0.5 \rangle$

$= \langle 0.45, 0.1 \rangle$

normalize ↓ $\approx \langle 0.818, 0.182 \rangle$ so. sum to 1

↓ time pass

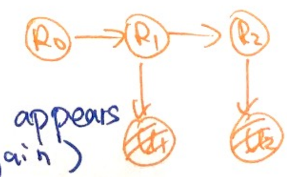


$B_1'(+r, -r) = P(R_2 | u_1) = \sum_{r_1} P(R_2 | r_1) P(r_1 | u_1)$

$= \langle 0.7, 0.3 \rangle \times 0.818 + \langle 0.3, 0.7 \rangle \times 0.182$

$\approx \langle 0.627, 0.373 \rangle$

↓ observe another evidence



U_2 = true (umbrella appears again)

$B_2(+r, -r) = P(R_2 | u_1, u_2) = \ldots$