CSE 5525 Artificial Intelligence II

Quiz #5: Probability

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1 Joint and Conditional Distributions

Consider three random variables Toothache, Cavity and Catch. The joint probabilities that each random variable takes on the respective values is given below:

	+toothache		−toothache	
	+catch	−catch	+catch	−catch
+cavity	0.108	0.012	0.072	0.008
−cavity	0.016	0.064	0.144	0.576

Questions:

1) What is $P(+\text{toothache})$? $= 0.108 + 0.016 + 0.012 + 0.064 = 0.2$

2) What is $P(\text{Catch})$? $P(+\text{catch}) = 0.108 + 0.016 + 0.072 + 0.144 = 0.34$

$P(-\text{catch}) = 1 - P(+\text{catch}) = 0.66$

3) What is $P(+\text{cavity}|+\text{catch})$? $= \frac{P(+\text{cavity} \overset{\text{and}}{,} +\text{catch})}{P(+\text{catch})} = \frac{0.108+0.072}{0.34} \approx 0.53$

4) What is $P(+\text{cavity}|+\text{toothache } or + \text{catch})$?

$= \frac{P(+\text{cavity}, +\text{toothache or} + \text{catch})}{P(+\text{toothache or } +\text{catch})} = \frac{0.108+0.012+0.072}{0.108+0.012+0.072+0.016+0.064+0.144} = \frac{0.192}{0.416} \approx 0.46$

5) Is the random variable Catch *conditionally independent* of Toothache, given Cavity?
(Hint: $P(+\text{cavity}|+\text{toothache}, +\text{cavity}) = P(+\text{catch}|+\text{cavity})$?)

Yes. Because

$P(+\text{catch}|+\text{toothache}, +\text{cavity}) = \frac{P(+\text{catch}, +\text{toothache}, +\text{cavity})}{P(+\text{toothache}, +\text{cavity})} = \frac{0.108}{0.108+0.012} = 0.9$

$P(+\text{catch}|+\text{cavity}) = \frac{P(+\text{catch}, +\text{cavity})}{P(+\text{cavity})} = \frac{0.108+0.072}{0.108+0.072+0.012+0.008} = \frac{0.18}{0.20} = 0.9$

2 Conditional Independence

For random variables X, Y, Z, show that the following three statements are equivalent:

(i) $P(X|Y,Z) = P(X|Z)$
(ii) $P(Y|X,Z) = P(Y|Z)$
(iii) $P(X,Y|Z) = P(X|Z)P(Y|Z)$

Equivalence of the first two statements show that conditional independence is symmetric (X and Y are conditionally independent given Z, and the order of X and Y doesn't matter). The third statement is analogous to the definition of *unconditional* independence: $P(X,Y) = P(X)P(Y)$).

Showing equivalence of (i) and (iii)

$$P(X,Y|Z) = P(X|Z)\,P(Y|Z)$$

$$\Downarrow$$

$$P(X|Y,Z) = P(X|Z)$$

plug-in

$$P(X|Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)} = \frac{P(X,Y|Z)\,P(Z)}{P(Y,Z)}$$

$$= \frac{P(X|Z)\,\cancel{P(Y|Z)\,P(Z)}}{\cancel{P(Y,Z)}}$$