

Neural Network Language Modeling

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CSE 5525

Many slides from Marek Rei, Philipp Koehn and Noah Smith

Course Project

- Sign up your course project
- In-class presentation on next Friday, 5 minute each

Language Modeling (Recap)

- Goal:
 - calculate the probability of a sentence
 - calculate probability of a word in the sentence

N-gram Language Modeling (Recap)

$$P(w_1 \ w_2 \ \dots \ w_N) = \prod_{i=1}^N P(w_i|w_{i-1})$$

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1} \ w_i)}{C(w_{i-1})}$$

Zero Probabilities (Recap)

- When we have sparse statistics:

$P(w \mid \text{denied the})$

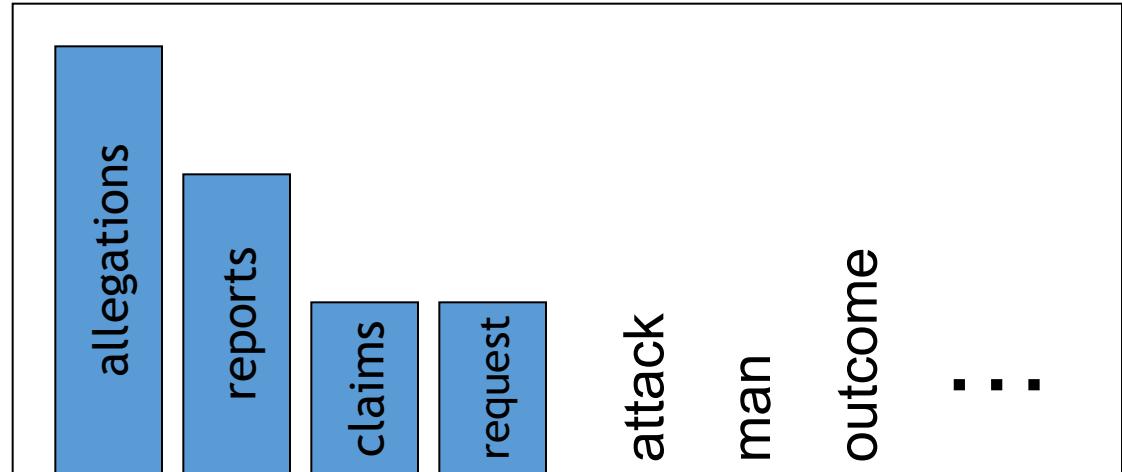
3 allegations

2 reports

1 claims

1 request

7 total



- Steal probability mass to generalize better

$P(w \mid \text{denied the})$

2.5 allegations

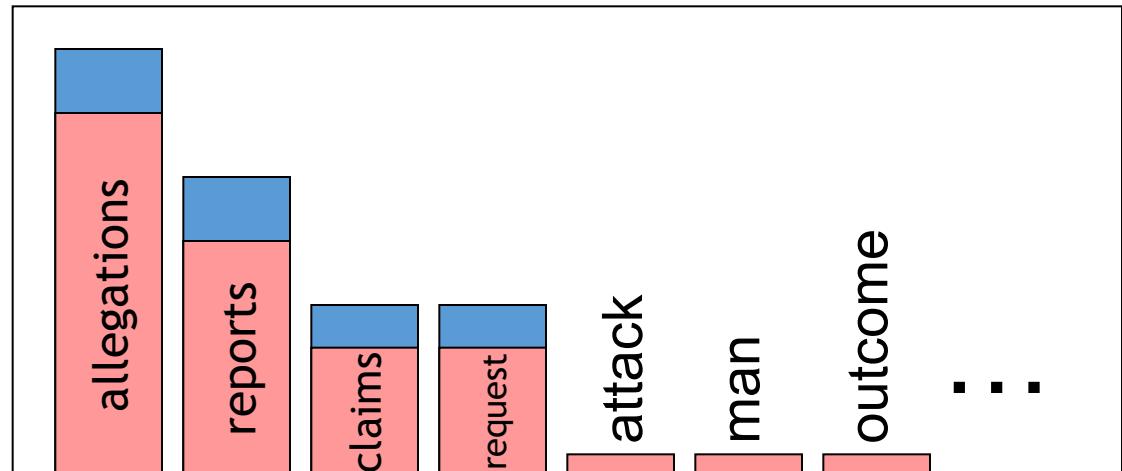
1.5 reports

0.5 claims

0.5 request

2 other

7 total



Smoothing (Recap)

- Backoff
- Interpolation

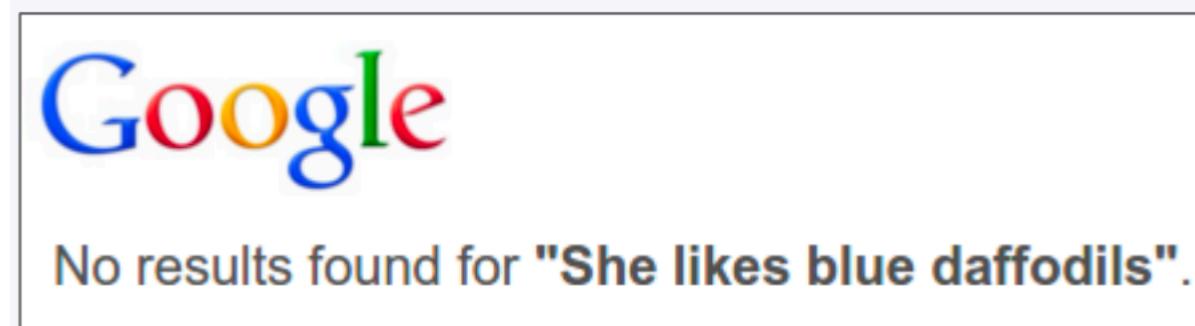
$$P_{\text{interp}}(w_i | w_{i-2} \ w_{i-1}) = \lambda_1 P(w_i | w_{i-2} \ w_{i-1}) + \lambda_2 P(w_i | w_{i-1}) + \lambda_3 P(w_i)$$

- Kneser-Ney Smoothing

$$P_{KN}(w_i | w_{i-1}) = \frac{\max(C(w_{i-1} \ w_i) - D, 0)}{C(w_{i-1})} + \lambda(w_{i-1}) P_{\text{continuation}}(w_i)$$

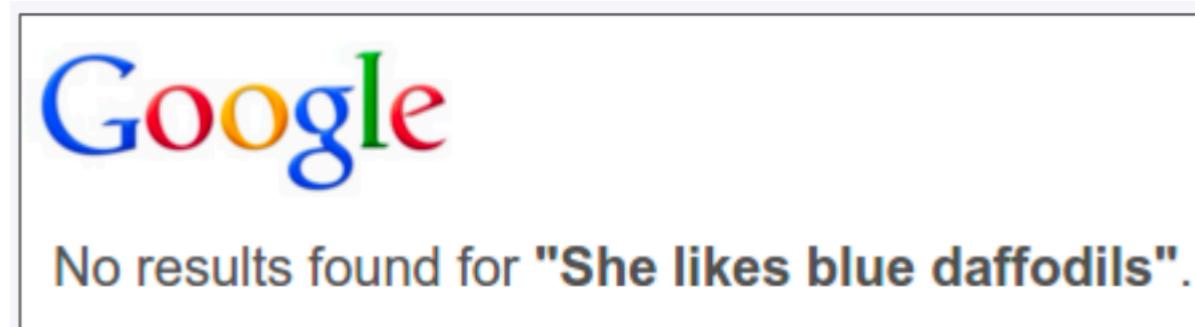
Problems with N-grams

- Problem 1: N-grams are sparse.
 - There are V^4 possible 4-grams. With $V=10,000$, that's 10^{16} 4-grams.
 - We will only see a tiny fraction of them in our training data.



Problems with N-grams

- Problem 2: Words are independent.
 - It only map together identical words, but ignore similar or related words.
 - If we have seen “yellow daffodil” in the data, we could use the intuition that “blue” is related to “yellow” to handle “blue daffodils”.



Vector Representation

- Let's represent words (or any objects) as vectors.
- Let's choose them, so that similar words have similar vectors.

One-hot Word Vectors

- Each element represents the word.

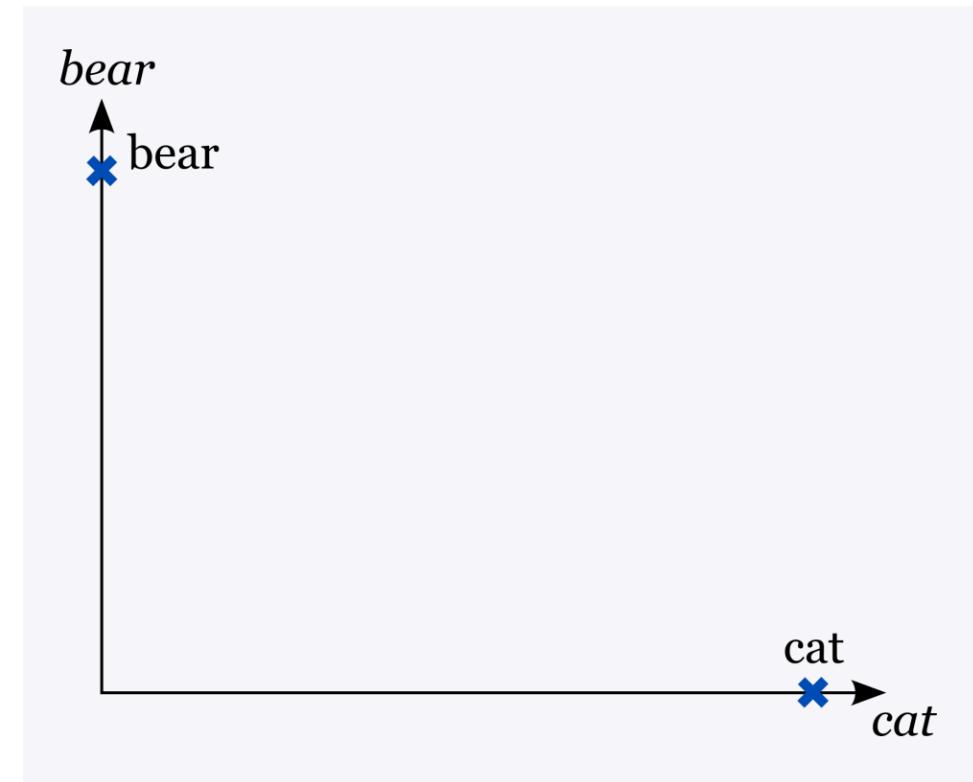
	<i>bear</i>	<i>cat</i>	<i>frog</i>
bear	1	0	0
cat	0	1	0
frog	0	0	1

$\text{bear}=[1.0, 0.0, 0.0]$

$\text{cat}=[0.0, 1.0, 0.0]$

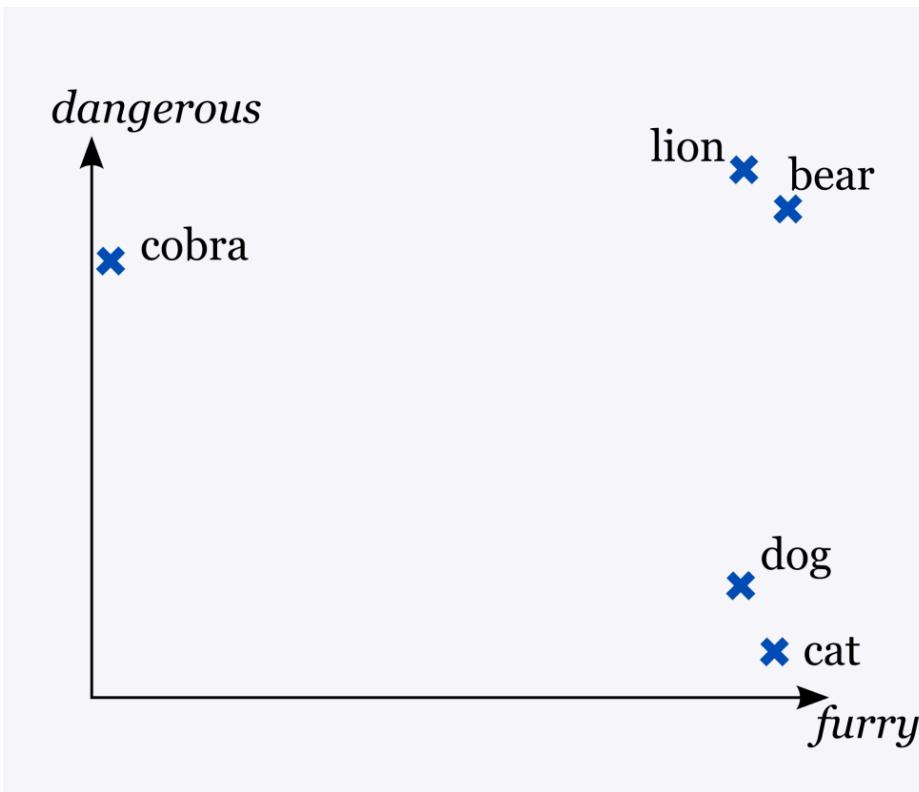
One-hot Word Vectors

- That's a very large vector!
- They tell us very little.



Distributed Vectors (Representations)

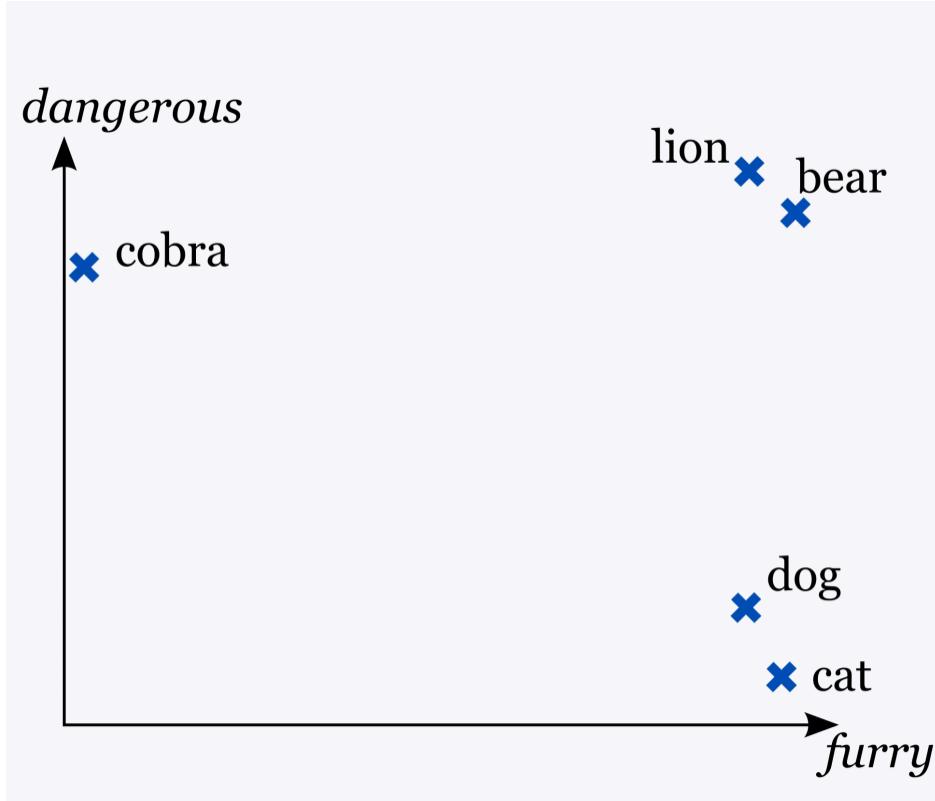
- Each element represents a property, and they are shared between the words.



	<i>furry</i>	<i>dangerous</i>
bear	0.9	0.85
cat	0.85	0.15
cobra	0.0	0.8
lion	0.85	0.9
dog	0.8	0.15

Distributed Vectors

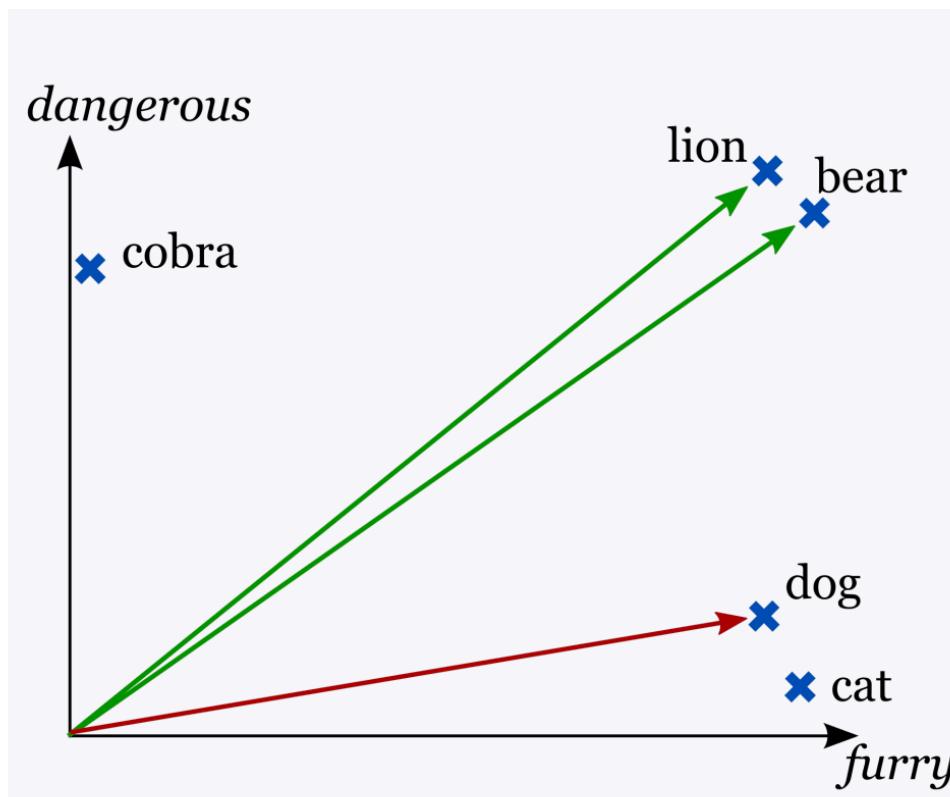
- Groups similar words/objects together.



	furry	dangerous
bear	0.9	0.85
cat	0.85	0.15
cobra	0.0	0.8
lion	0.85	0.9
dog	0.8	0.15

Distributed Vectors

- Use cosine similarity to calculate similarity between two words

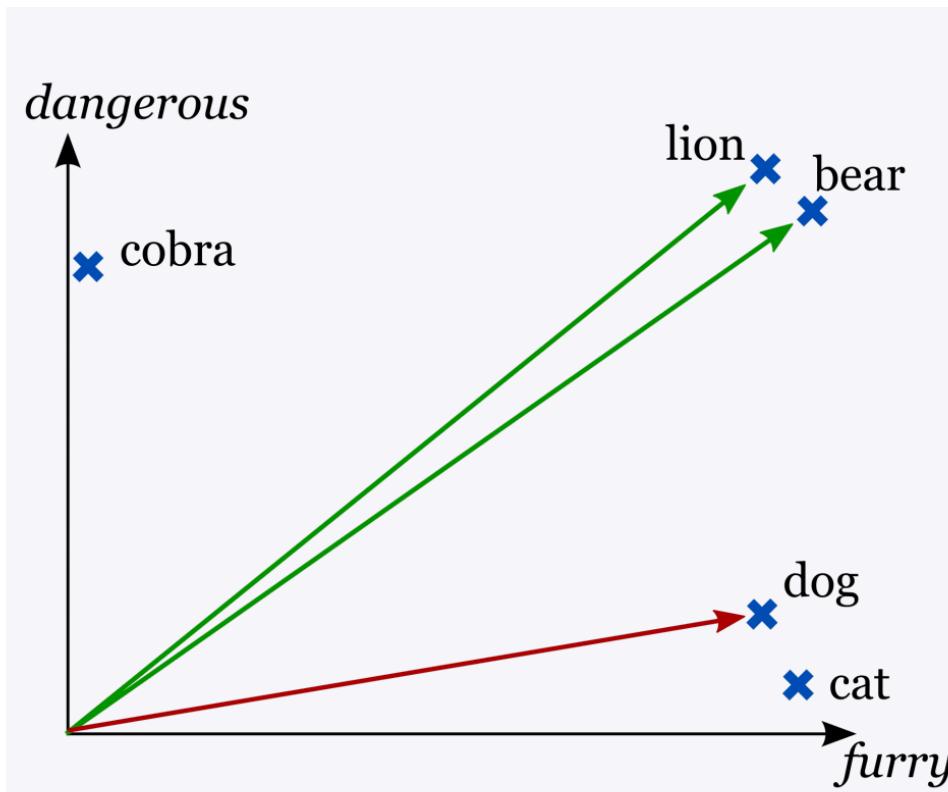


$$\cos(a, b) = \frac{\sum_i a_i \cdot b_i}{\sqrt{\sum_i a_i^2} \cdot \sqrt{\sum_i b_i^2}}$$

$$\begin{aligned}\cos(\text{lion}, \text{bear}) &= 0.998 \\ \cos(\text{lion}, \text{dog}) &= 0.809 \\ \cos(\text{cobra}, \text{dog}) &= 0.727\end{aligned}$$

Distributed Vectors

- Use cosine similarity to calculate similarity between two words

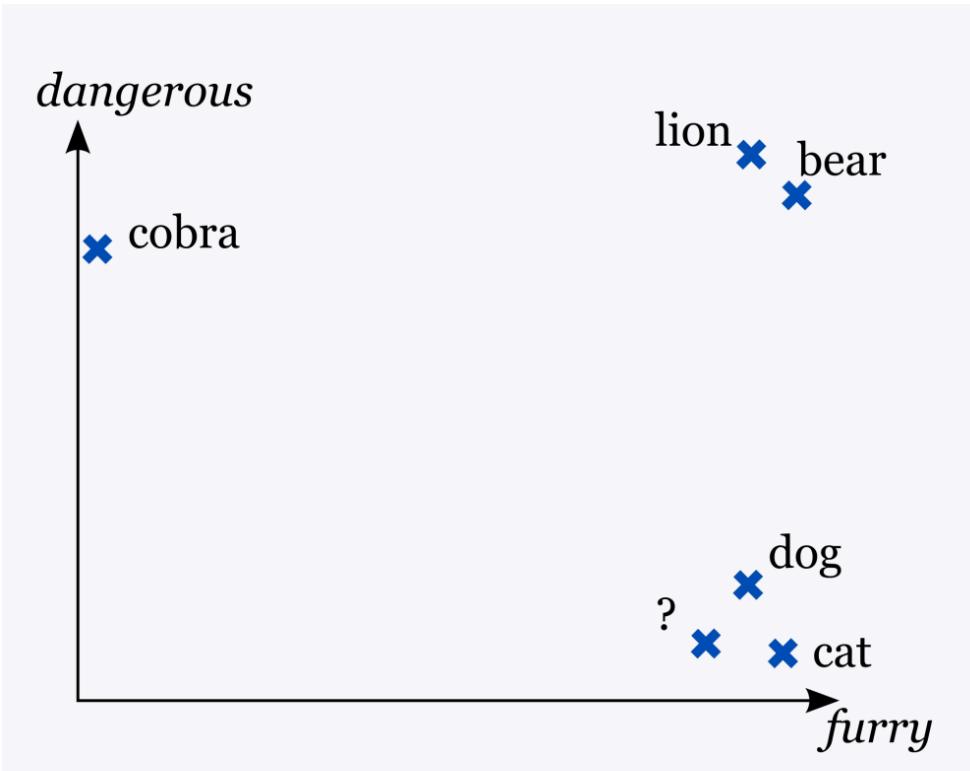


$$\cos(a, b) = \frac{\sum_i a_i \cdot b_i}{\sqrt{\sum_i a_i^2} \cdot \sqrt{\sum_i b_i^2}}$$

$$\begin{aligned}\cos(\text{lion}, \text{bear}) &= 0.998 \\ \cos(\text{lion}, \text{dog}) &= 0.809 \\ \cos(\text{cobra}, \text{dog}) &= 0.727\end{aligned}$$

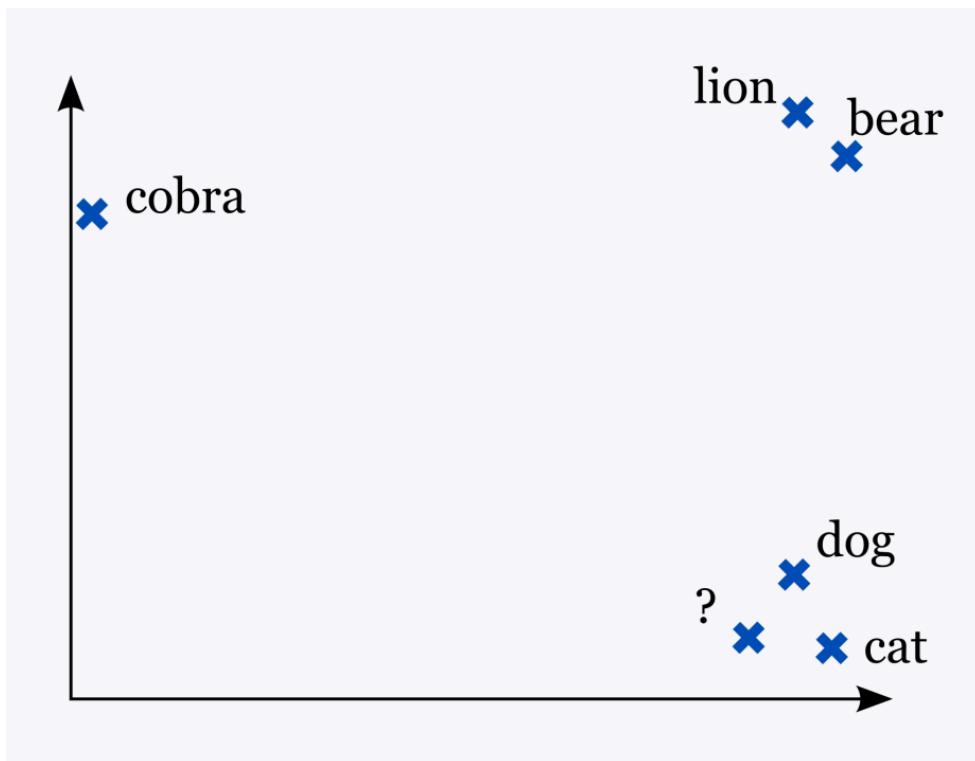
Distributed Vectors

- We can infer some information based only on the vector of the word.



Distributed Vectors

- We don't even need to know the labels on the vector elements.



Distributed Vectors

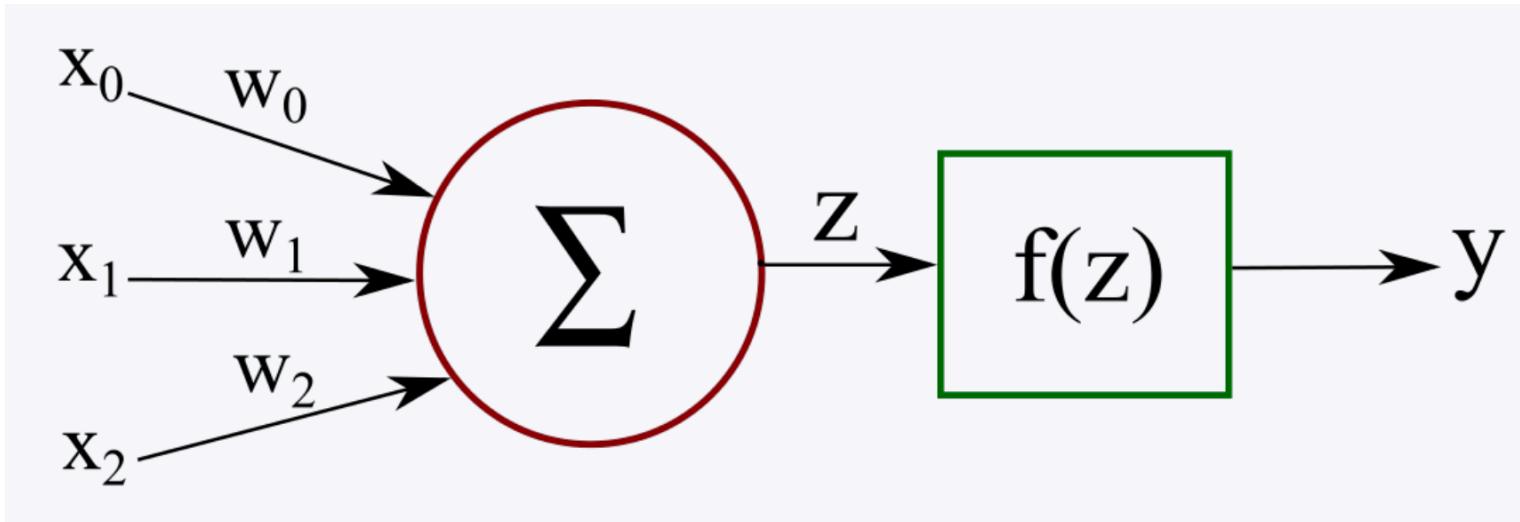
- The vectors are usually not 2 or 3-dimensional. More often 100-1000 dimensions.

```
bear -0.089383 -0.375981 -0.337130 0.025117 -0.232542 -0.224786 0.148717 -0.154768 -0.260046  
-0.156737 -0.085468 0.180366 -0.076509 0.173228 0.231817 0.314453 -0.253200 0.170015  
-0.111660 0.377551 -0.025207 -0.097520 -0.020041 0.117727 0.105745 -0.352382 0.010241  
0.114237 -0.315126 0.196771 -0.116824 -0.091064 -0.291241 -0.098721 0.297539 0.213323  
-0.158814 -0.157823 0.152232 0.259710 0.335267 0.195840 -0.118898 0.169420 -0.201631  
0.157561 0.351295 0.033166 0.003641 -0.046121 0.084251 0.021727 -0.065358 -0.083110  
-0.265997 0.027450 0.372135 0.040659 0.202577 -0.109373 0.183473 -0.380250 0.048979  
0.071580 0.152277 0.298003 0.017217 0.072242 0.541714 -0.110148 0.266429 0.270824 0.046859  
0.150756 -0.137924 -0.099963 -0.097112 -0.110336 -0.018136 -0.032682 0.182723 0.260882  
-0.146807 0.502611 0.034849 -0.092219 -0.103714 -0.034353 0.112178 0.065348 0.161681  
0.006538 0.364870 0.153239 -0.366863 -0.149125 0.413624 -0.229378 -0.396910 -0.023116
```

Neural Network Language Models

- Represent each word as a vector, and similar words with similar vectors.
- Idea:
 - similar contexts have similar words
 - so we define a model that aims to predict between a word w_t and context words: $P(w_t | \text{context})$ or $P(\text{context} | w_t)$
 - *Optimize the vectors together with the model, so we end up with vectors that perform well for language modeling (aka representation learning)*

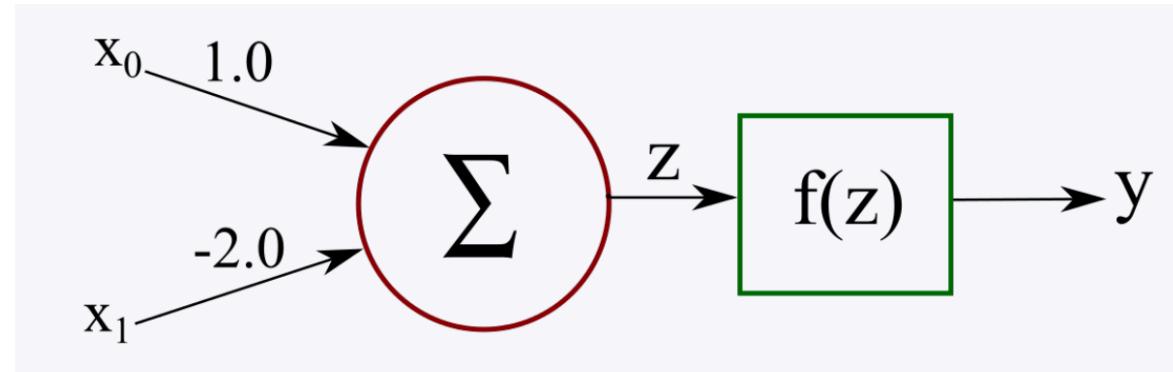
Artificial Neuron



Input: $[x_0, x_1, x_2]$
Output: y

Perceptrons = Single-Layer Neural Networks !

Artificial Neuron



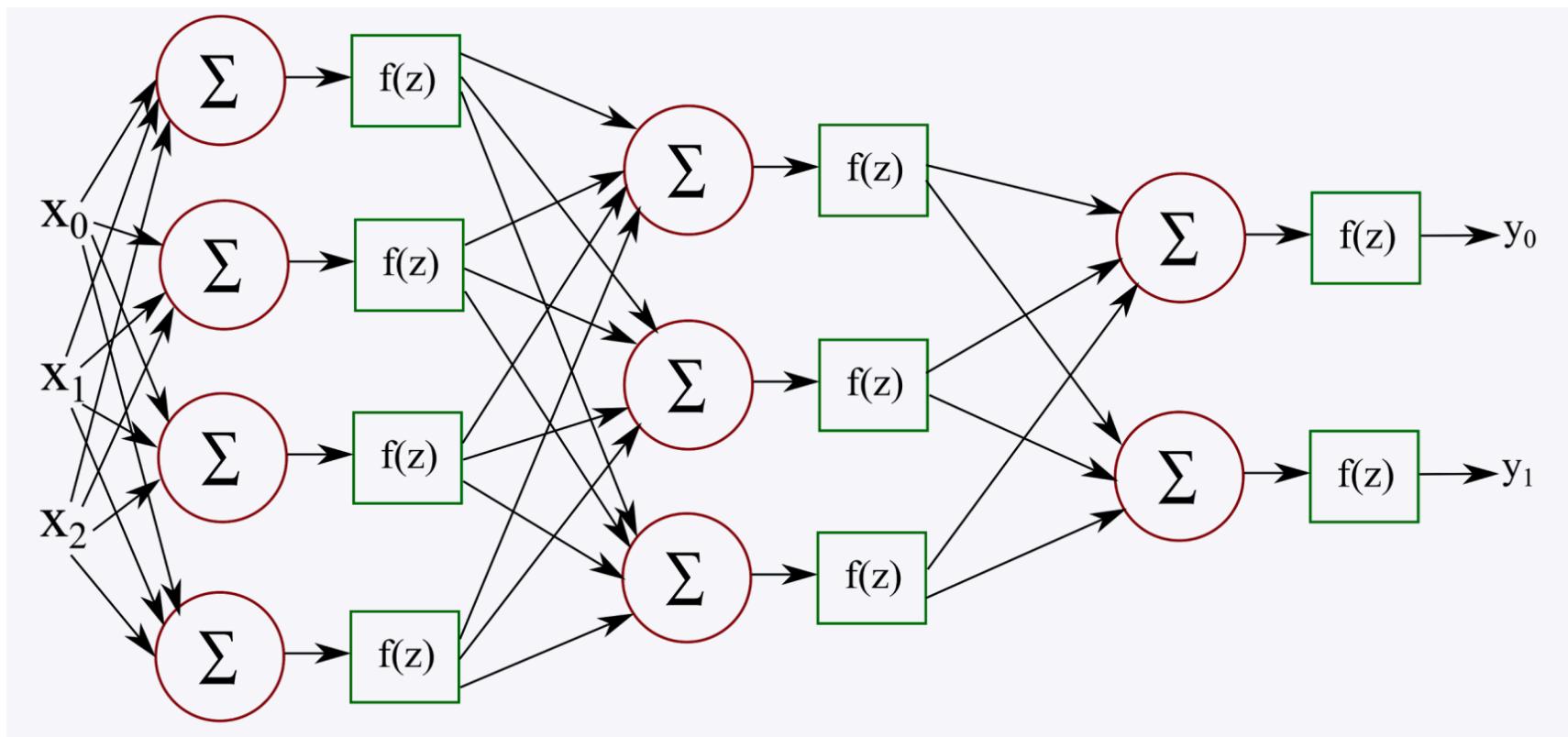
$$z = \sum_i x_i w_i$$

$$y = \frac{1}{1 + \exp(-z)}$$

	x_0	x_1	z	y
bear	0.9	0.85	-0.8	0.31
cat	0.85	0.15	0.55	0.63
cobra	0.0	0.8	-1.6	0.17
lion	0.85	0.9	-0.95	0.28
dog	0.8	0.15	0.5	0.62

Neural Network

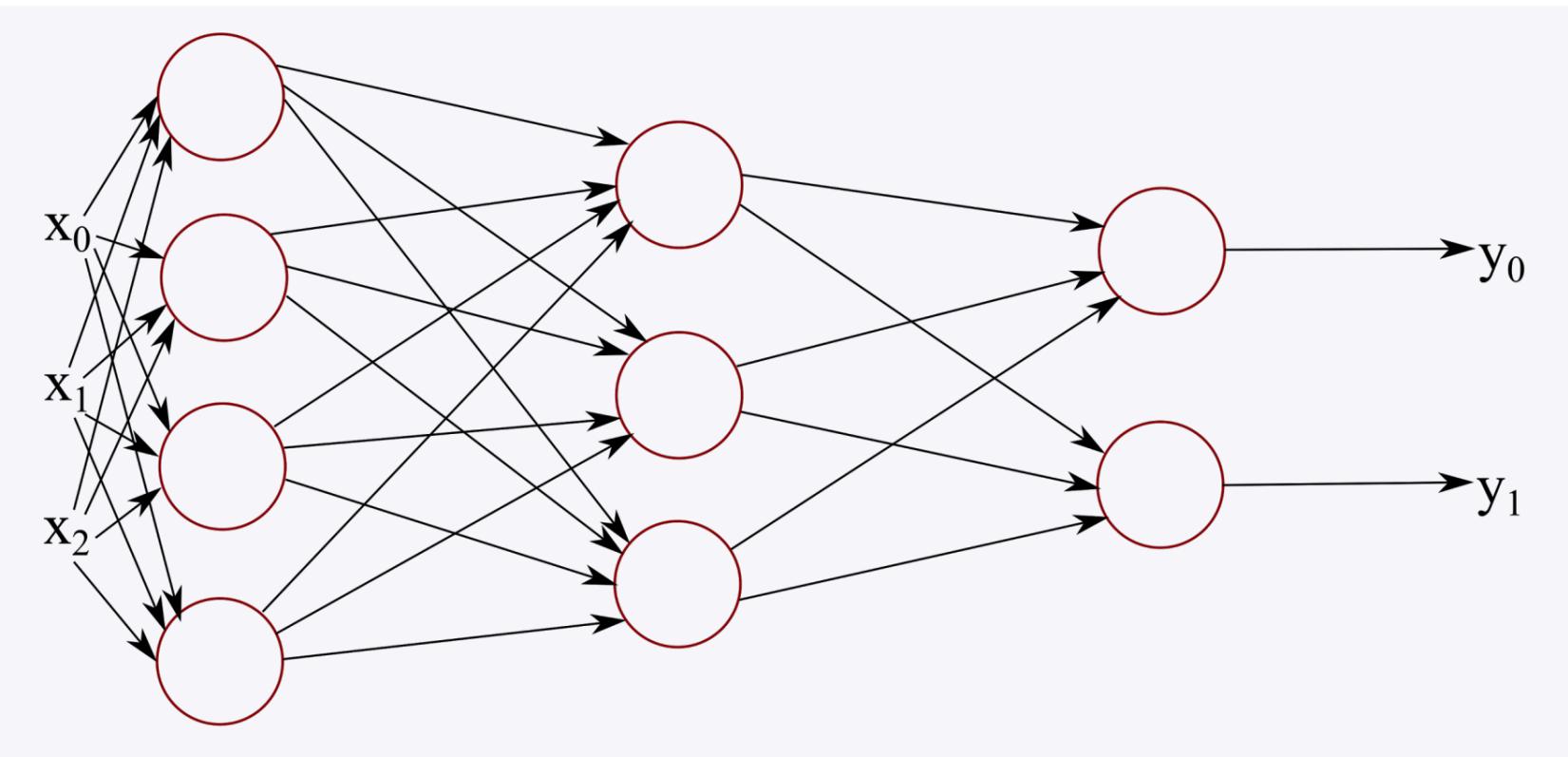
- Many neurons connected together



Multi-Layer Feed-Forward Networks!

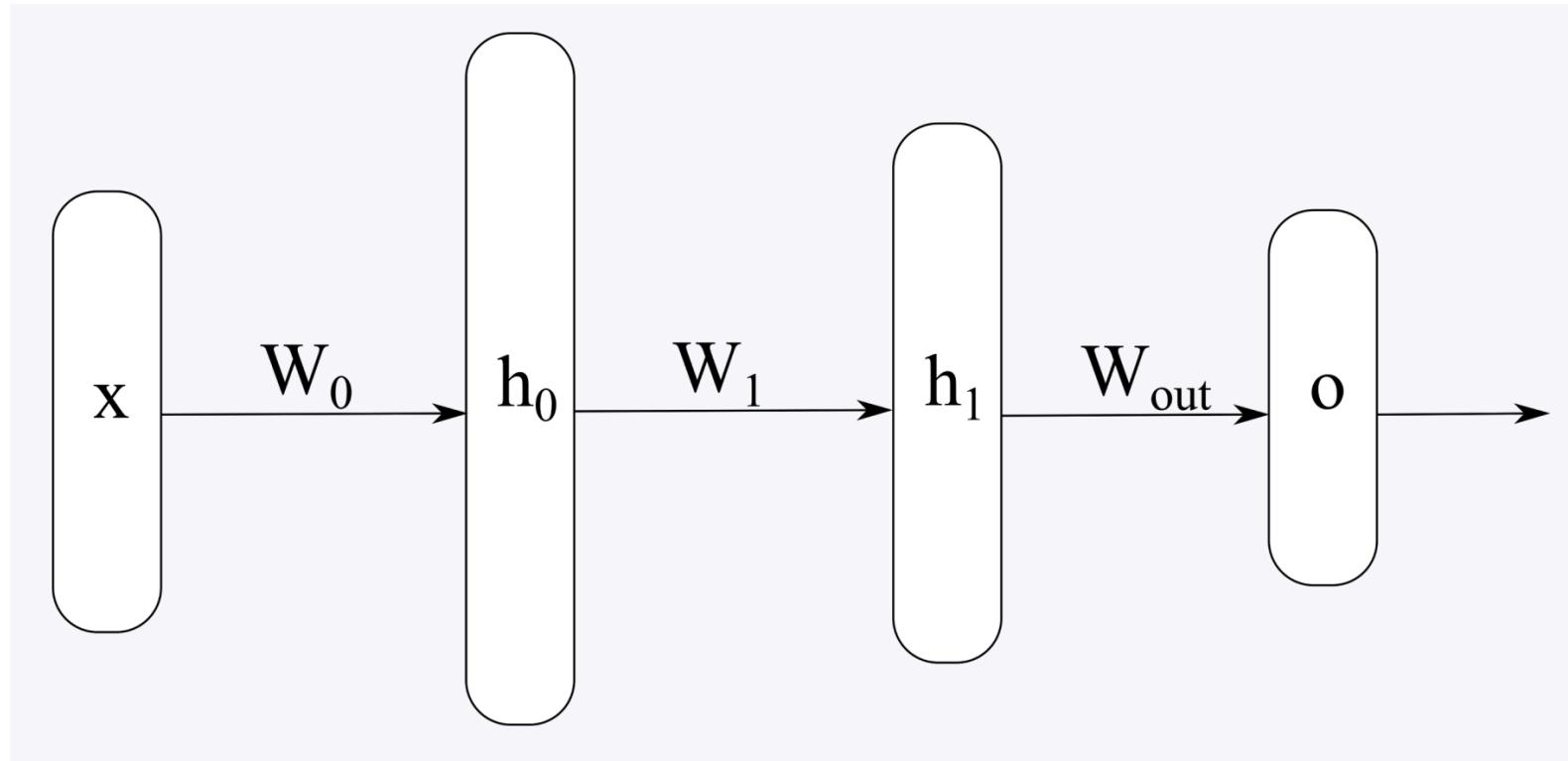
Neural Network

- Usually, a neuron is shown as a single unit

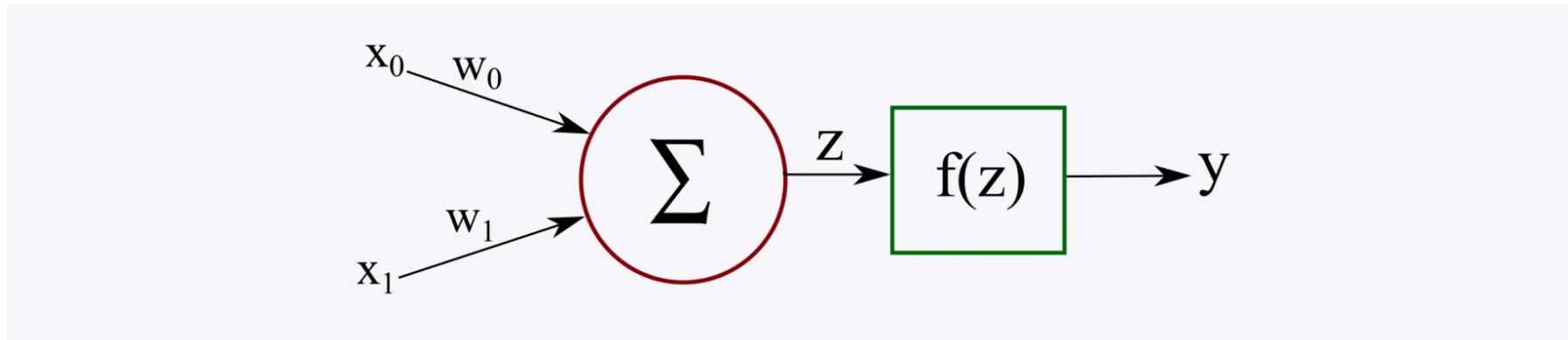


Neural Network

- Or a whole layer of neurons is represented as a block



Neuron Activation w/ Vectors



$$z = \sum_i x_i w_i$$

$$y = \frac{1}{1 + \exp(-z)}$$

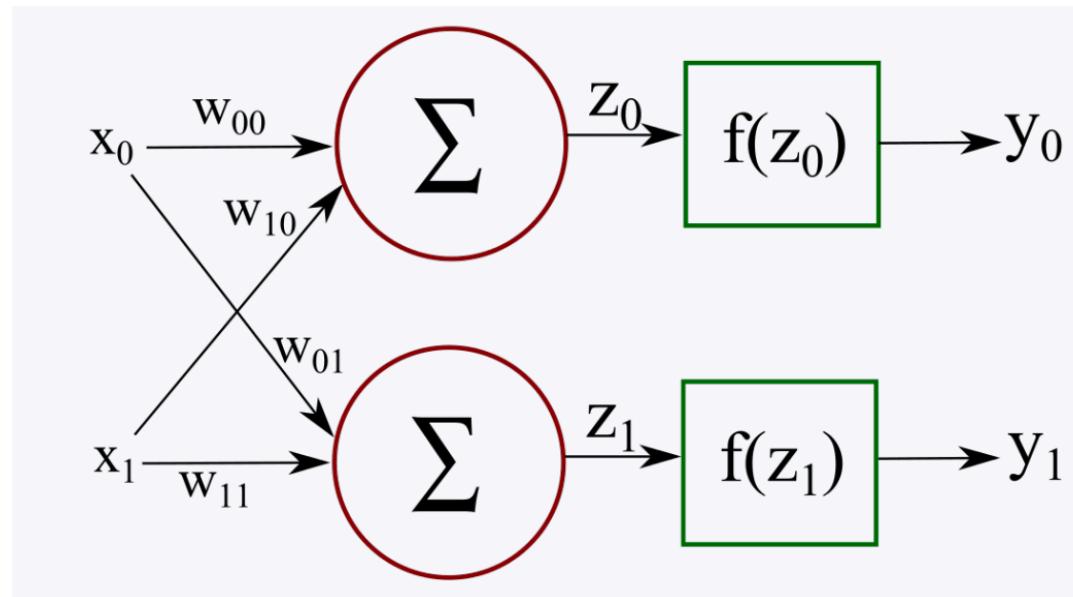
$$x = \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} \quad W = \begin{vmatrix} w_0 & w_1 \end{vmatrix}$$

$$z = W \cdot x \quad f(z) = \frac{1}{1 + e^{-z}}$$

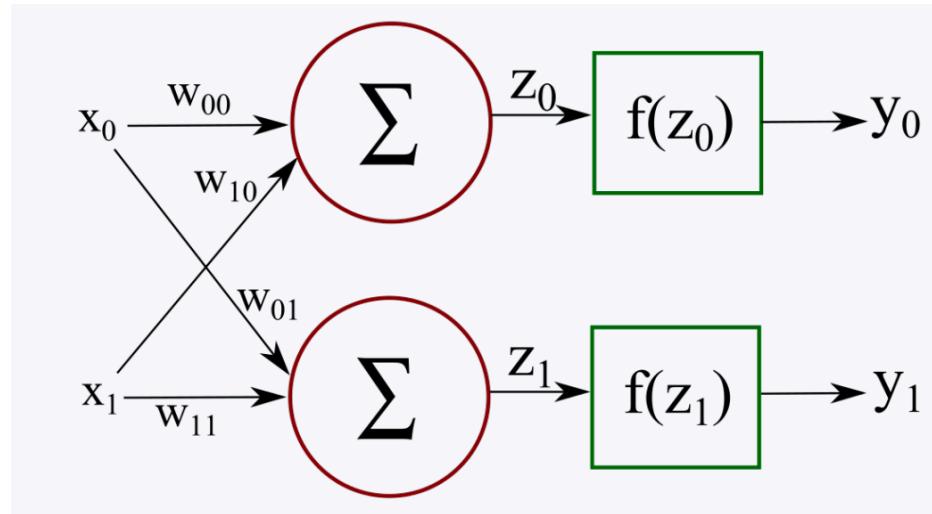
$$y = f(z) = f(W \cdot x)$$

Feedforward Activation w/ Matrices

- The same process applies when activating multiple neurons
- Now the weights are in a matrix as opposed to a vector
- Activation $f(z)$ is applied to each neuron separately



Feedforward Activation w/ Matrices



$$x = \begin{vmatrix} x_0 \\ x_1 \end{vmatrix}$$

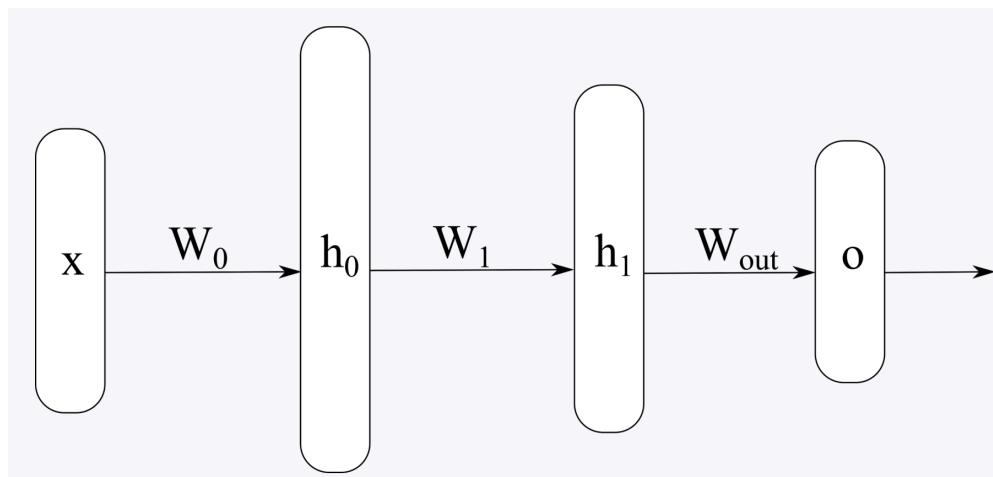
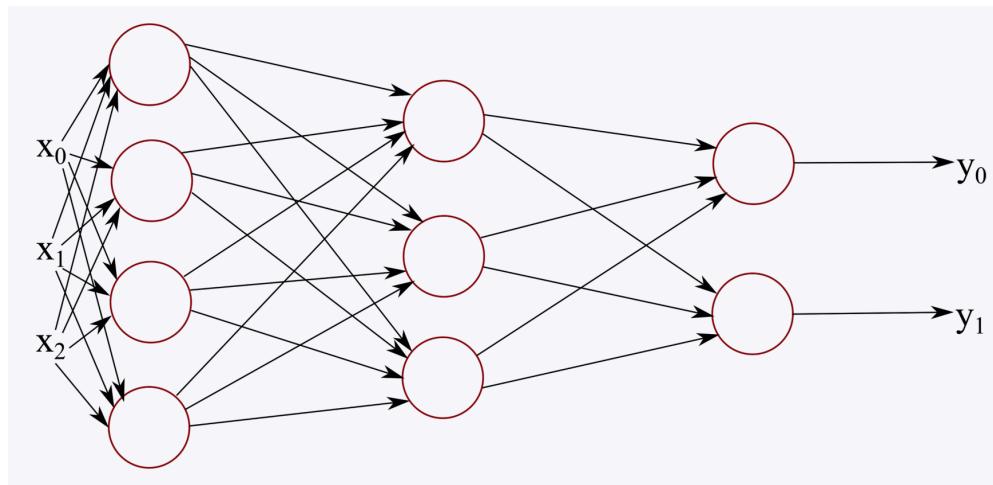
$$W = \begin{vmatrix} w_{00} & w_{10} \\ w_{01} & w_{11} \end{vmatrix}$$

$$\begin{aligned} z &= W \cdot x = \begin{vmatrix} w_{00} & w_{10} \\ w_{01} & w_{11} \end{vmatrix} \cdot \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} \\ &= \begin{vmatrix} w_{00} \cdot x_0 + w_{10} \cdot x_1 \\ w_{01} \cdot x_0 + w_{11} \cdot x_1 \end{vmatrix} = \begin{vmatrix} z_0 \\ z_1 \end{vmatrix} \end{aligned}$$

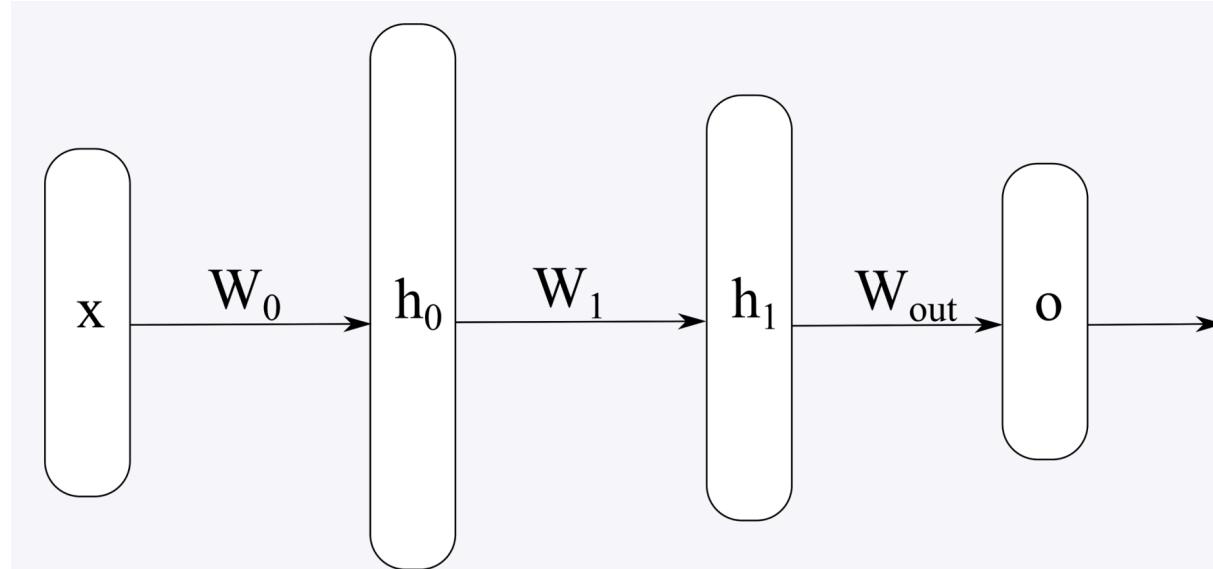
$$f(z) = \frac{1}{1 + e^{-z}}$$

$$y = f(z) = f(W \cdot x) = \begin{vmatrix} f(z_0) \\ f(z_1) \end{vmatrix} = \begin{vmatrix} y_0 \\ y_1 \end{vmatrix}$$

Neural Network

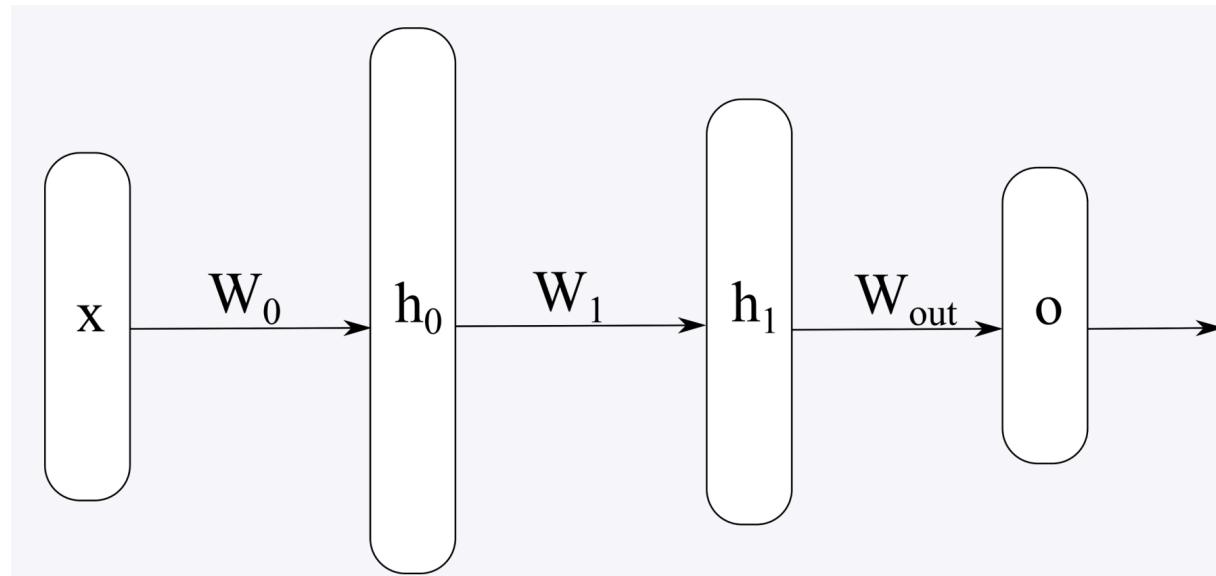


Feedforward Activation



- 1) Take vector from the previous layer
- 2) Multiply it with the weight matrix
- 3) Apply the activation function
- 4) Repeat

Feedforward Activation



$$h_0 = f(W_0 \cdot x)$$

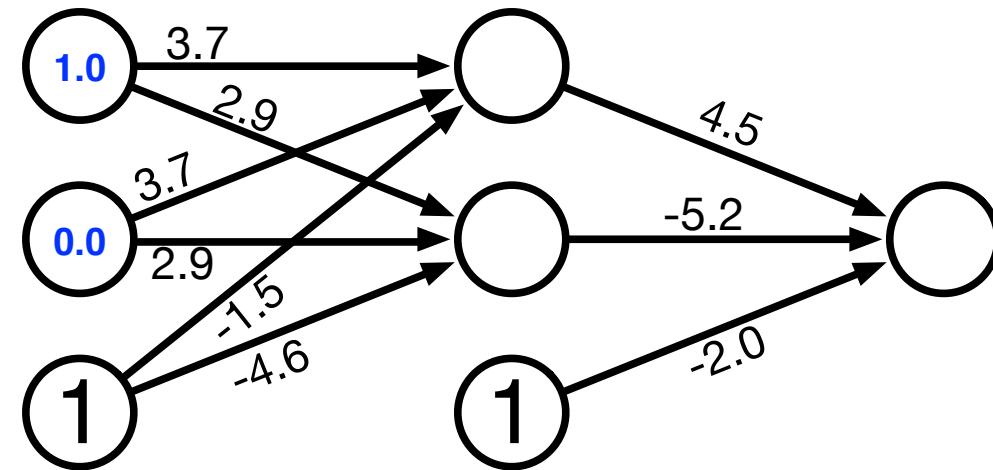
$$h_1 = f(W_1 \cdot h_0)$$

$$o = f(W_{out} \cdot h_1)$$

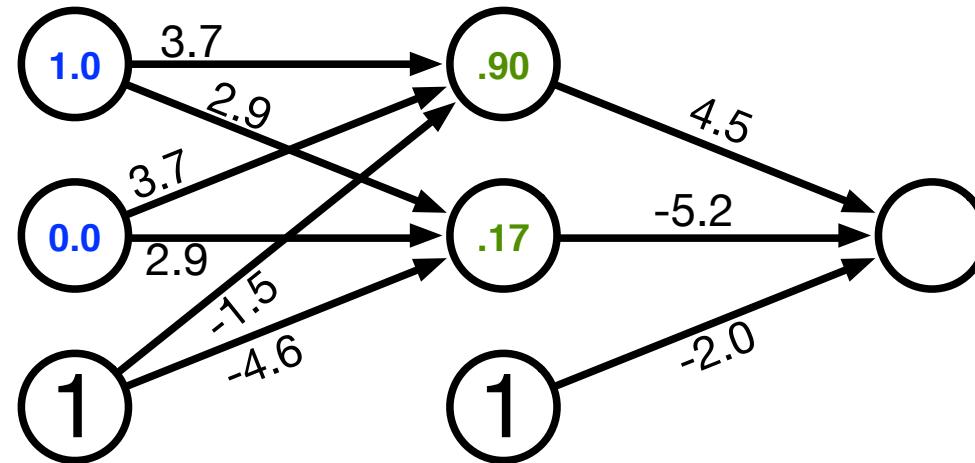
$$o = f(W_{out} \cdot f(W_1 \cdot f(W_0 \cdot x)))$$

Exercise

Input Layer



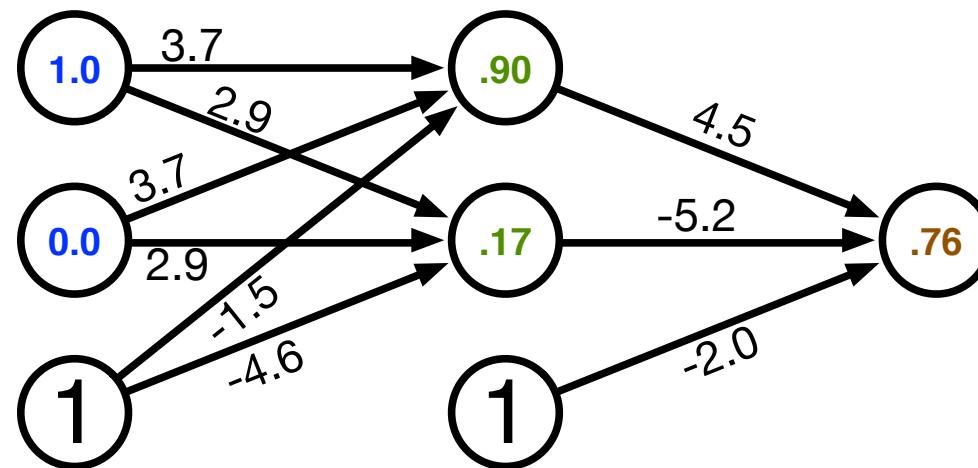
Hidden Layer Computation



$$\text{sigmoid}(1.0 \times 3.7 + 0.0 \times 3.7 + 1 \times -1.5) = \text{sigmoid}(2.2) = \frac{1}{1 + e^{-2.2}} = 0.90$$

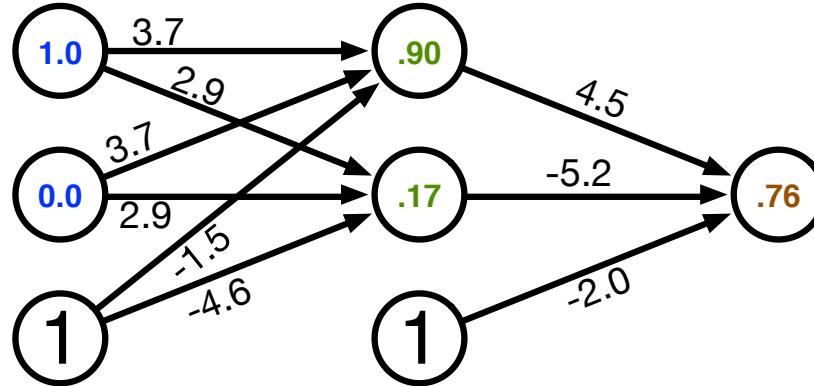
$$\text{sigmoid}(1.0 \times 2.9 + 0.0 \times 2.9 + 1 \times -4.5) = \text{sigmoid}(-1.6) = \frac{1}{1 + e^{1.6}} = 0.17$$

Output Layer Computation



$$\text{sigmoid}(.90 \times 4.5 + .17 \times -5.2 + 1 \times -2.0) = \text{sigmoid}(1.17) = \frac{1}{1 + e^{-1.17}} = 0.76$$

Error



- Computed output: $y = .76$
- Correct output: $t = 1.0$

⇒ How do we adjust the weights?

Back-propagation Training

- Gradient descent
 - error is a function of the weights
 - we want to reduce the error
 - gradient descent: move towards the error minimum
 - compute gradient → get direction to the error minimum
 - adjust weights towards direction of lower error
- Back-propagation
 - first adjust last set of weights
 - propagate error back to each previous layer
 - adjust their weights

Final Layer Update

- Linear combination of weights $s = \sum_k w_k h_k$
- Activation function $y = \text{sigmoid}(s)$
- Error (L2 norm) $E = \frac{1}{2}(t - y)^2$
- Derivative of error with regard to one weight w_k

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

Final Layer Update (1)

- Linear combination of weights $s = \sum_k w_k h_k$
- Activation function $y = \text{sigmoid}(s)$
- Error (L2 norm) $E = \frac{1}{2}(t - y)^2$
- Derivative of error with regard to one weight w_k

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- Error E is defined with respect to y

$$\frac{dE}{dy} = \frac{d}{dy} \frac{1}{2}(t - y)^2 = -(t - y)$$

Final Layer Update (2)

- Linear combination of weights $s = \sum_k w_k h_k$
- Activation function $y = \text{sigmoid}(s)$
- Error (L2 norm) $E = \frac{1}{2}(t - y)^2$
- Derivative of error with regard to one weight w_k

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- y with respect to x is $\text{sigmoid}(s)$

$$\frac{dy}{ds} = \frac{d \text{ sigmoid}(s)}{ds} = \text{sigmoid}(s)(1 - \text{sigmoid}(s)) = y(1 - y)$$

Final Layer Update (3)

- Linear combination of weights $s = \sum_k w_k h_k$
- Activation function $y = \text{sigmoid}(s)$
- Error (L2 norm) $E = \frac{1}{2}(t - y)^2$
- Derivative of error with regard to one weight w_k

$$\frac{dE}{dw_k} = \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k}$$

- x is weighted linear combination of hidden node values h_k

$$\frac{ds}{dw_k} = \frac{d}{dw_k} \sum_k w_k h_k = h_k$$

Putting it Together

- Derivative of error with regard to one weight w_k

$$\begin{aligned}\frac{dE}{dw_k} &= \frac{dE}{dy} \frac{dy}{ds} \frac{ds}{dw_k} \\ &= -(t - y) \quad y(1 - y) \quad h_k\end{aligned}$$

- error
- derivative of sigmoid: y'
- Weight adjustment will be scaled by a fixed learning rate μ

$$\Delta w_k = \mu (t - y) y' h_k$$

Multiple Output Nodes

- Our example only had one output node
- Typically neural networks have multiple output nodes
- Error is computed over all j output nodes

$$E = \sum_j \frac{1}{2} (t_j - y_j)^2$$

- Weights $k \rightarrow j$ are adjusted according to the node they point to

$$\Delta w_{j \leftarrow k} = \mu(t_j - y_j) y'_j h_k$$

Hidden Layer Update

- In a hidden layer, we do not have a target output value
- But we can compute how much each node contributed to downstream error
- Definition of error term of each node

$$\delta_j = (t_j - y_j) \ y'_j$$

- Back-propagate the error term

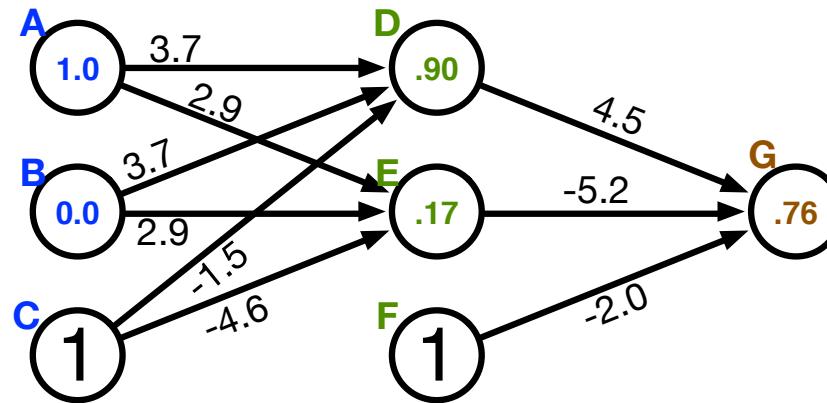
(why this way? there is math to back it up...)

$$\delta_i = \left(\sum_j w_{j \leftarrow i} \delta_j \right) y'_i$$

- Universal update formula

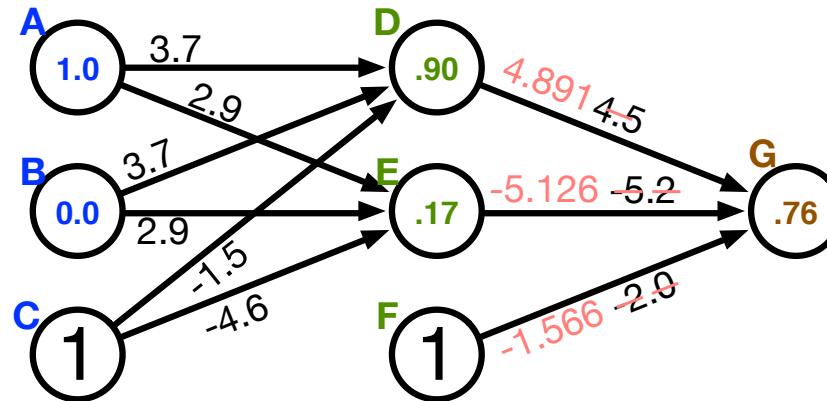
$$\Delta w_{j \leftarrow k} = \mu \ \delta_j \ h_k$$

Final Layer Update (example)



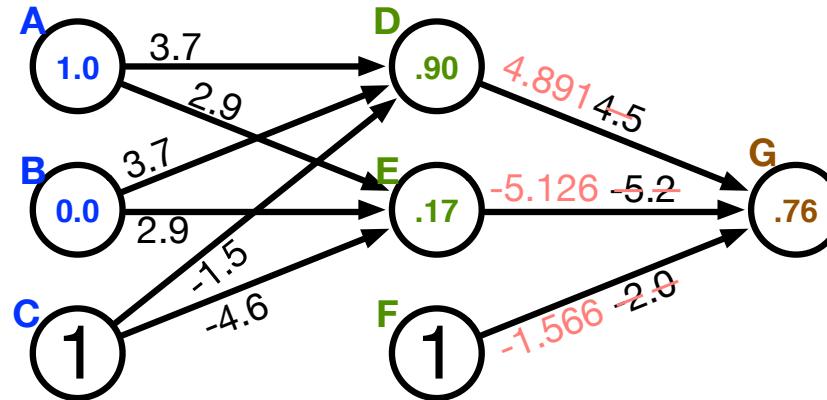
- Computed output: $y = .76$
- Correct output: $t = 1.0$
- Final layer weight updates (learning rate $\mu = 10$)
 - $\delta_G = (t - y) y' = (1 - .76) 0.181 = .0434$
 - $\Delta w_{GD} = \mu \delta_G h_D = 10 \times .0434 \times .90 = .391$
 - $\Delta w_{GE} = \mu \delta_G h_E = 10 \times .0434 \times .17 = .074$
 - $\Delta w_{GF} = \mu \delta_G h_F = 10 \times .0434 \times 1 = .434$

Final Layer Update (example)



- Computed output: $y = .76$
- Correct output: $t = 1.0$
- Final layer weight updates (learning rate $\mu = 10$)
 - $\delta_G = (t - y) y' = (1 - .76) 0.181 = .0434$
 - $\Delta w_{GD} = \mu \delta_G h_D = 10 \times .0434 \times .90 = .391$
 - $\Delta w_{GE} = \mu \delta_G h_E = 10 \times .0434 \times .17 = .074$
 - $\Delta w_{GF} = \mu \delta_G h_F = 10 \times .0434 \times 1 = .434$

Hidden Layer Update (example)



- Hidden node **D**

- $\delta_D = \left(\sum_j w_{j \leftarrow i} \delta_j \right) y'_D = w_{GD} \delta_G y'_D = 4.5 \times .0434 \times .0898 = .0175$
- $\Delta w_{DA} = \mu \delta_D h_A = 10 \times .0175 \times 1.0 = .175$
- $\Delta w_{DB} = \mu \delta_D h_B = 10 \times .0175 \times 0.0 = 0$
- $\Delta w_{DC} = \mu \delta_D h_C = 10 \times .0175 \times 1 = .175$

- Hidden node **E**

- $\delta_E = \left(\sum_j w_{j \leftarrow i} \delta_j \right) y'_E = w_{GE} \delta_G y'_E = -5.2 \times .0434 \times 0.2055 = -.0464$
- $\Delta w_{EA} = \mu \delta_E h_A = 10 \times -.0464 \times 1.0 = -.464$
- etc.

Initialization of Weights

- Weights are initialized randomly
e.g., uniformly from interval $[-0.01, 0.01]$
- Glorot and Bengio (2010) suggest
 - for shallow neural networks

$$\left[-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}} \right]$$

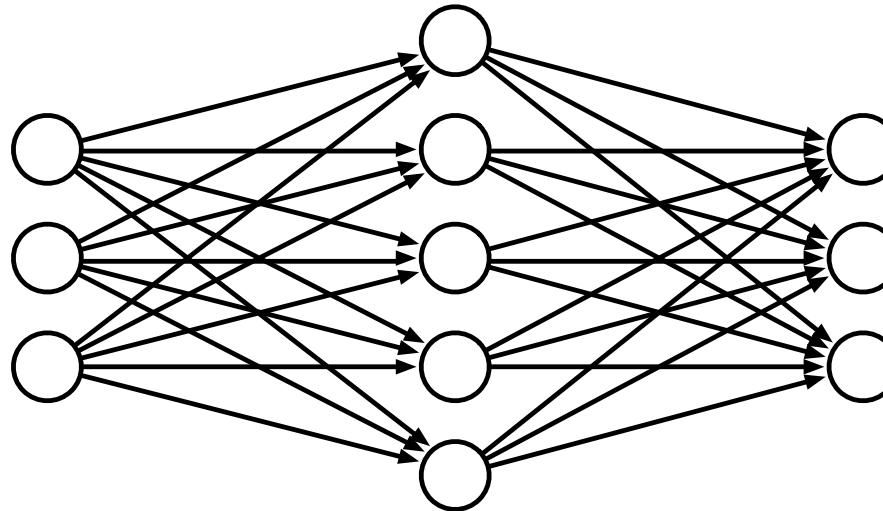
n is the size of the previous layer

- for deep neural networks

$$\left[-\frac{\sqrt{6}}{\sqrt{n_j + n_{j+1}}}, \frac{\sqrt{6}}{\sqrt{n_j + n_{j+1}}} \right]$$

n_j is the size of the previous layer, n_j size of next layer

Neural Networks for Classification

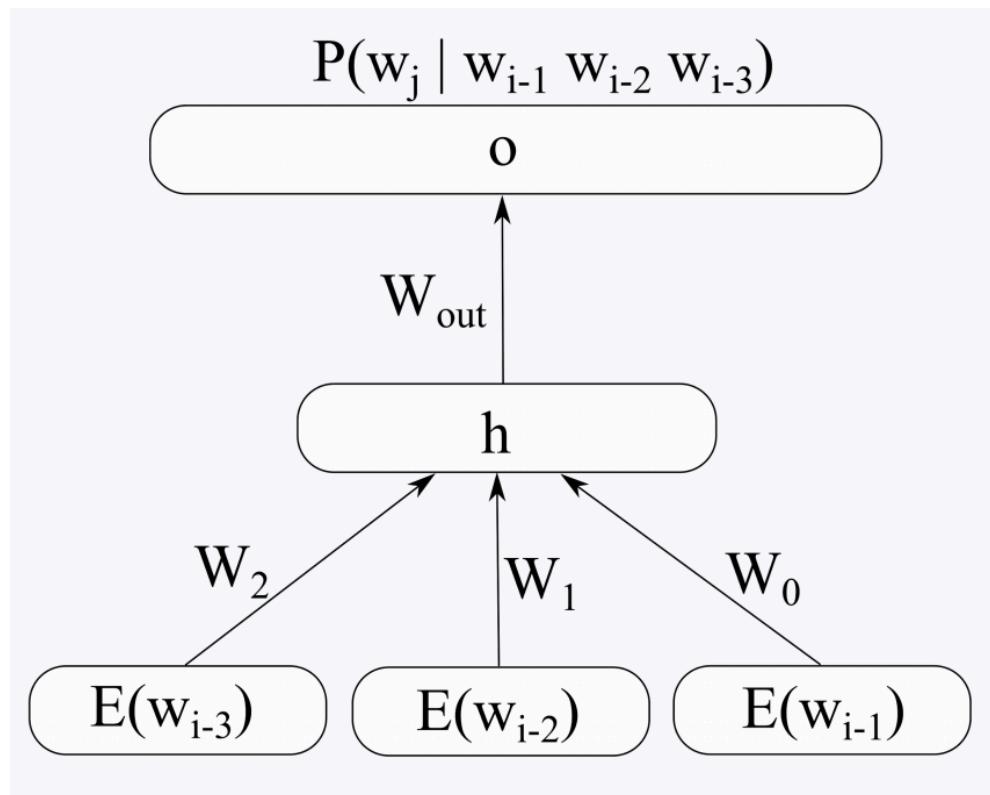


- Predict class: one output node per class
- Training data output: "One-hot vector", e.g., $\vec{y} = (0, 0, 1)^T$
- Prediction
 - predicted class is output node y_i with highest value
 - obtain posterior probability distribution by soft-max

$$\text{softmax}(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

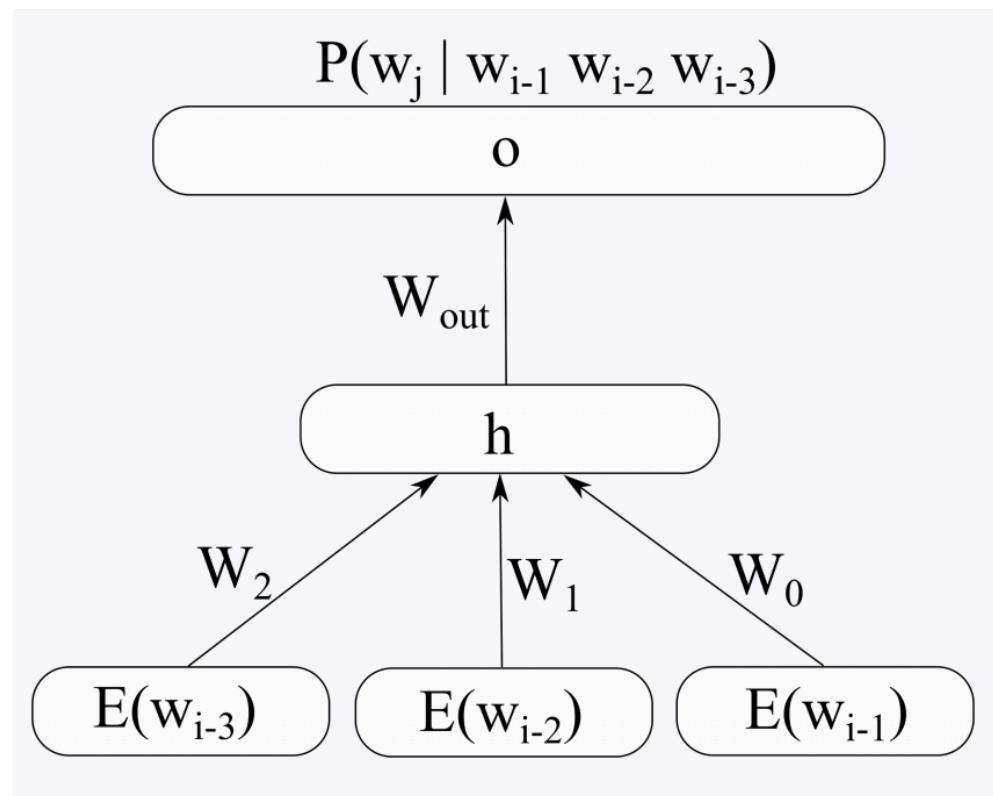
Feedforward Neural Network Language Model

- Input: vector representations of previous words $E(w_{i-3}) E(w_{i-2}) E(w_{i-1})$
- Output: the conditional probability of w_j being the next word



Feedforward Neural Network Language Model

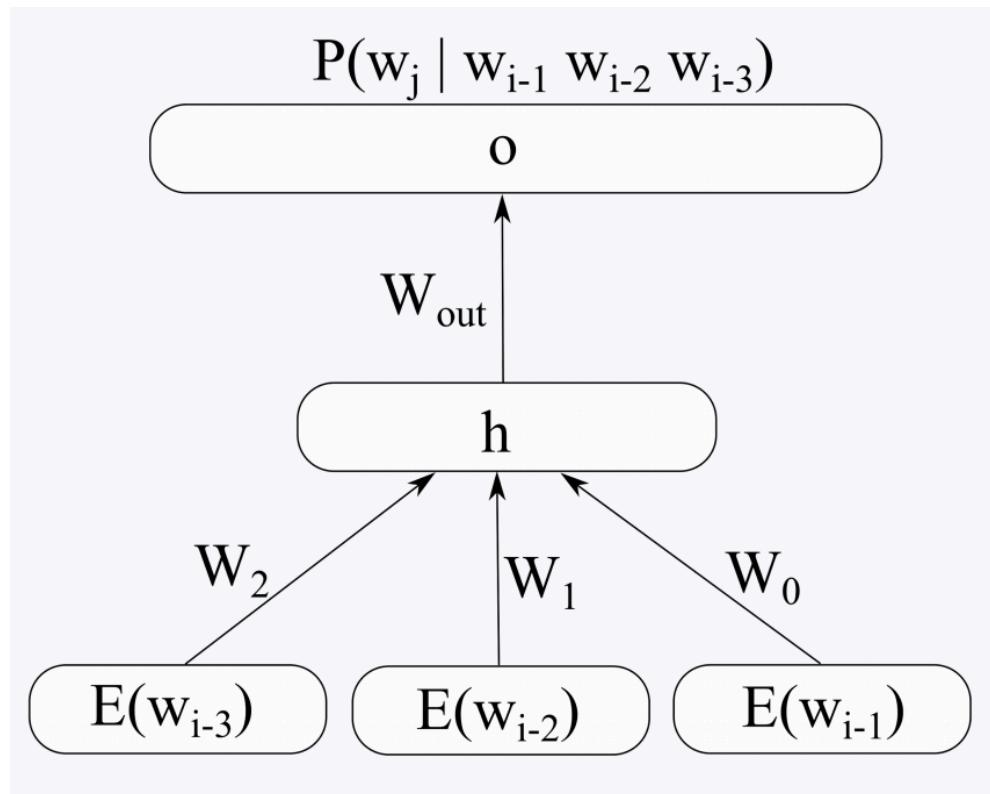
- We can also think of the input as a concatenation of the context vectors
- The hidden layer h is calculated as in previous examples



How do we calculate
 $P(w_j | w_{i-1} w_{i-2} w_{i-3})$?

Feedforward Neural Network Language Model

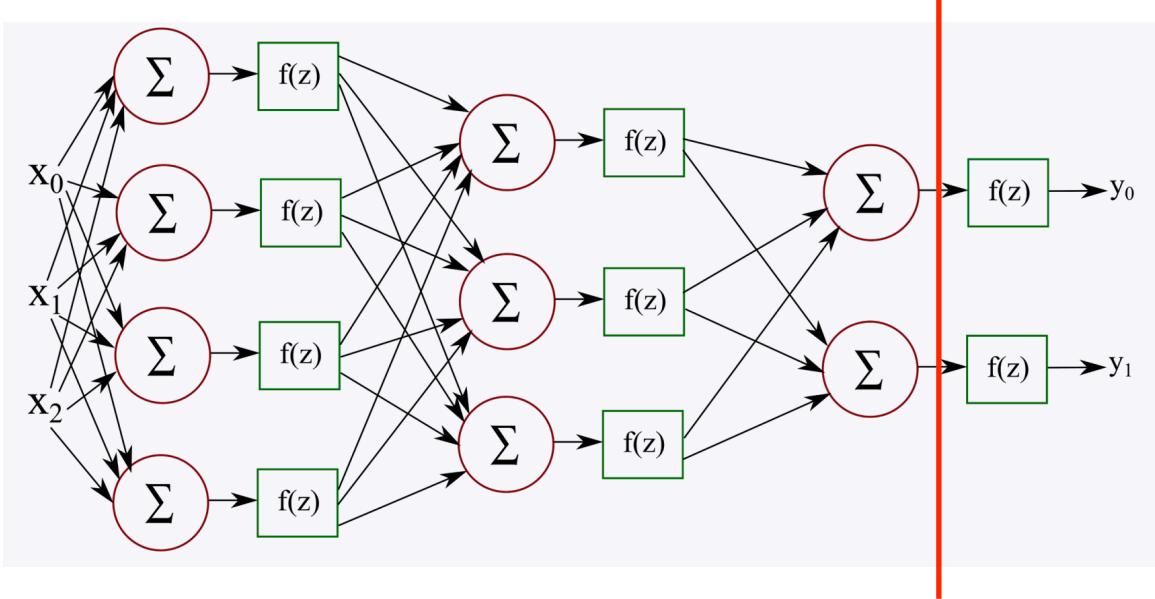
- Our output vector \mathbf{o} has an element for each possible word w_j
- We take a softmax over that vector



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 $P(w_j | w_{i-1} w_{i-2} w_{i-3})$?

Feedforward Neural Network Language Model

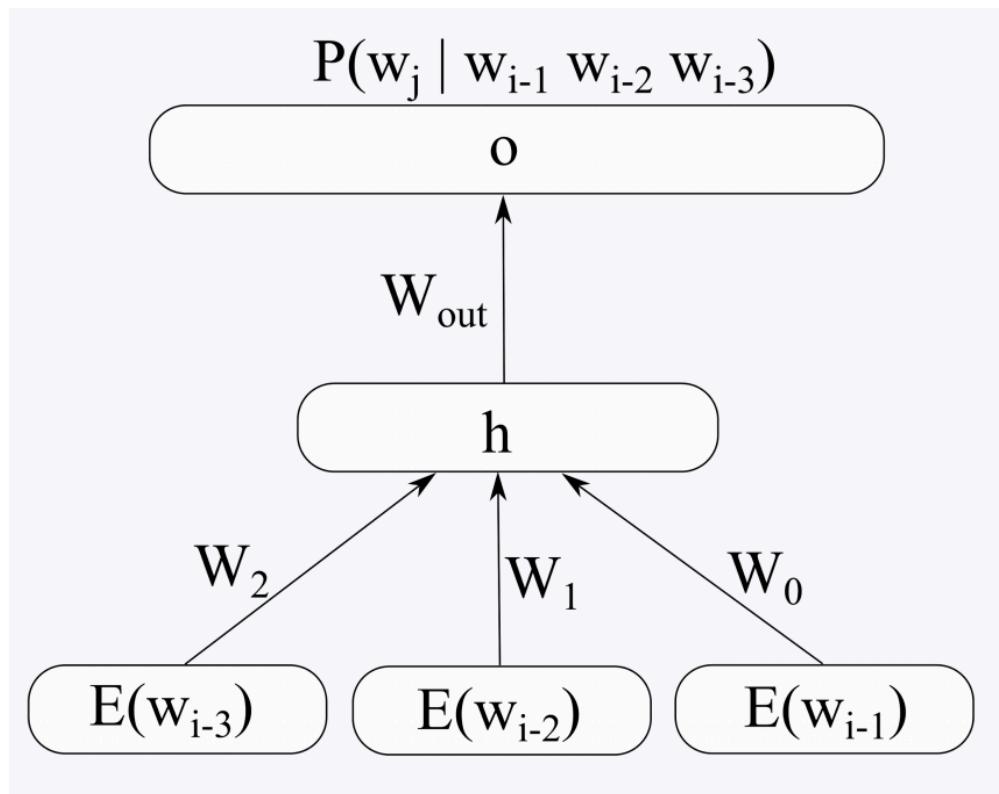
- Our output vector \mathbf{o} has an element for each possible word w_j
- We take a softmax over that vector



$$\text{softmax}(z)_j = \frac{e^{z_j}}{\sum_k e^{z_k}}$$

	0	1	2	3	SUM
z	-5.0	-4.5	-4.0	-6.0	-19.5
exp(z)	0.007	0.011	0.018	0.002	0.038
softmax(z)	0.184	0.289	0.474	0.053	1.0

Feedforward Neural Network Language Model



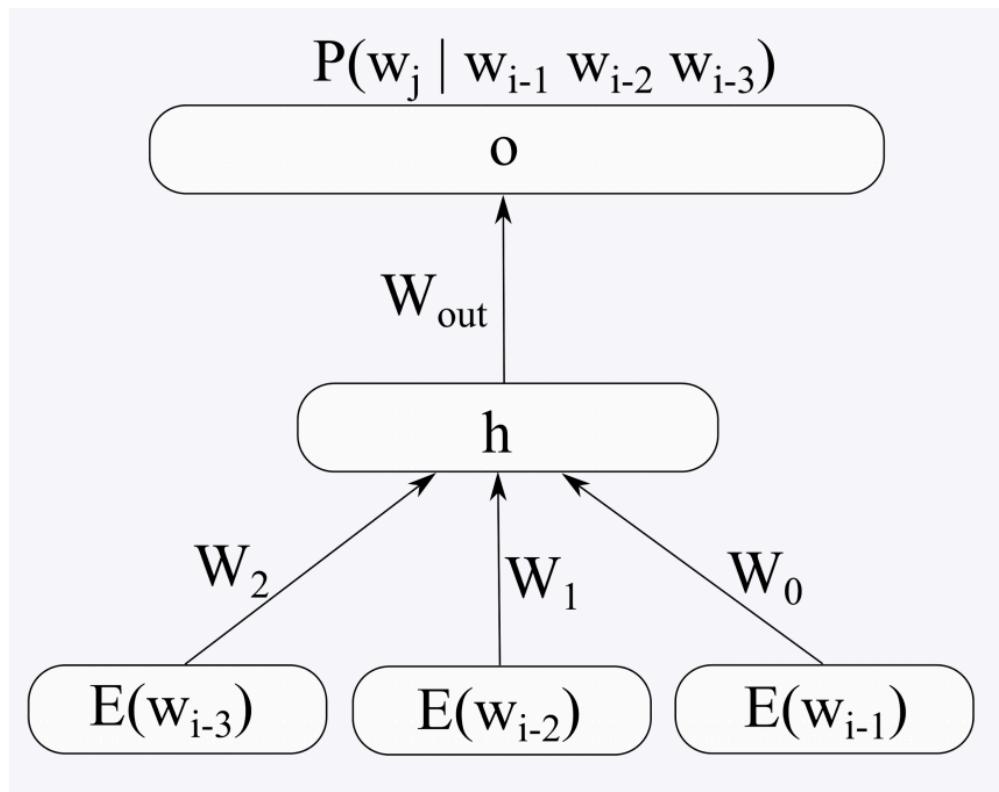
1) Multiple input vectors with weights

$$\begin{aligned} z = & W_2 \cdot E(w_{i-3}) \\ & + W_1 \cdot E(w_{i-2}) \\ & + W_0 \cdot E(w_{i-1}) \end{aligned}$$

2) Apply the activation function

$$h = f(z)$$

Feedforward Neural Network Language Model



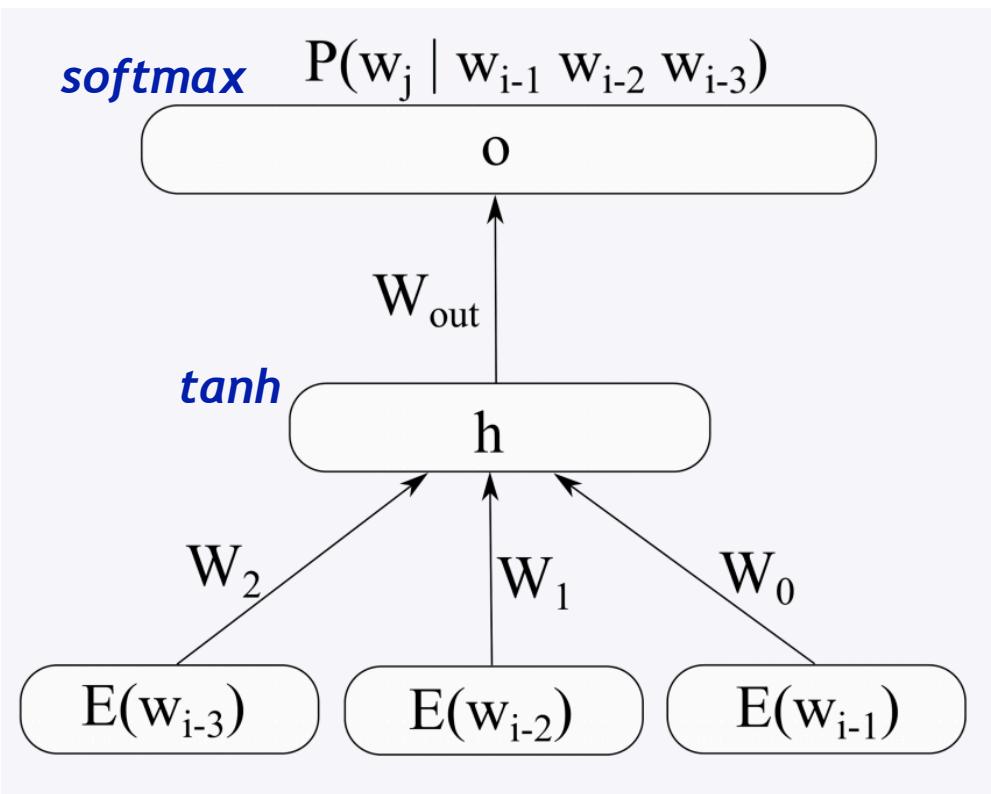
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2) Apply the activation function

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Feedforward Neural Network Language Model



3) Multiply hidden vector with output weights

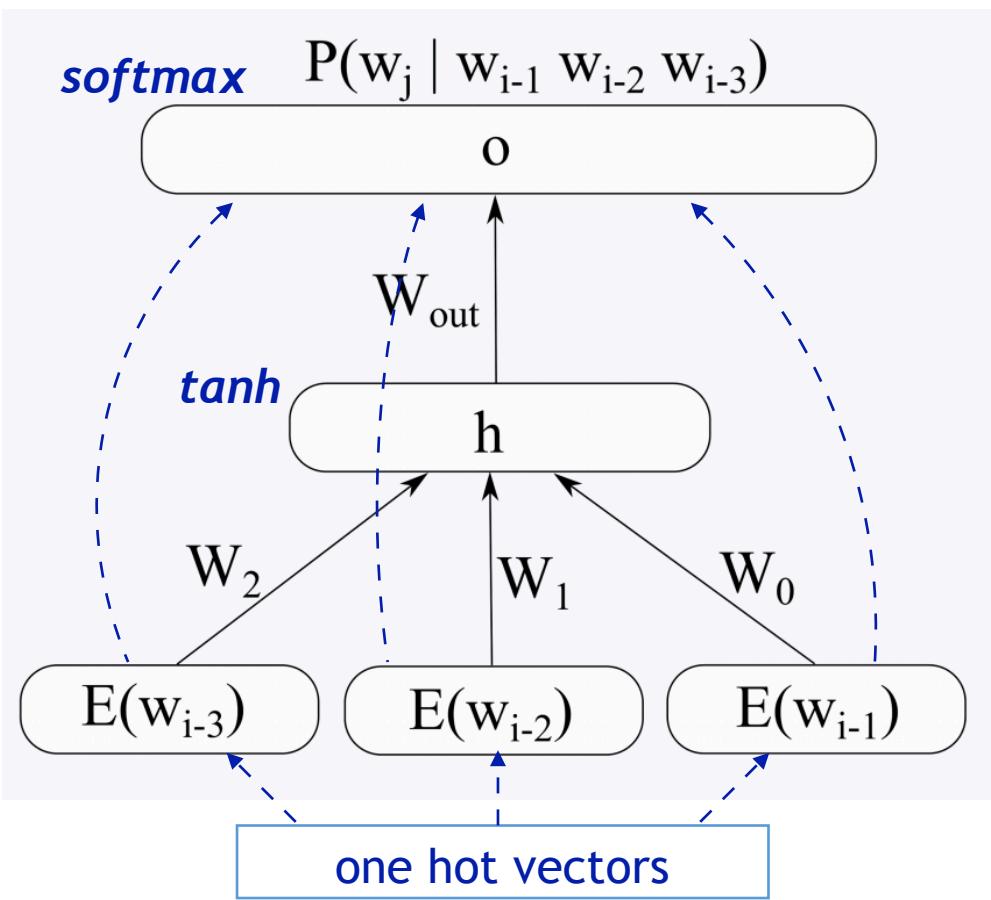
$$s = W_{out} \cdot h$$

4) Apply softmax to the output vector

$$o = softmax(s)$$

Now the j -th element in the output vector o_j contains the probability of w_j being the next word. $0 \leq j < V$

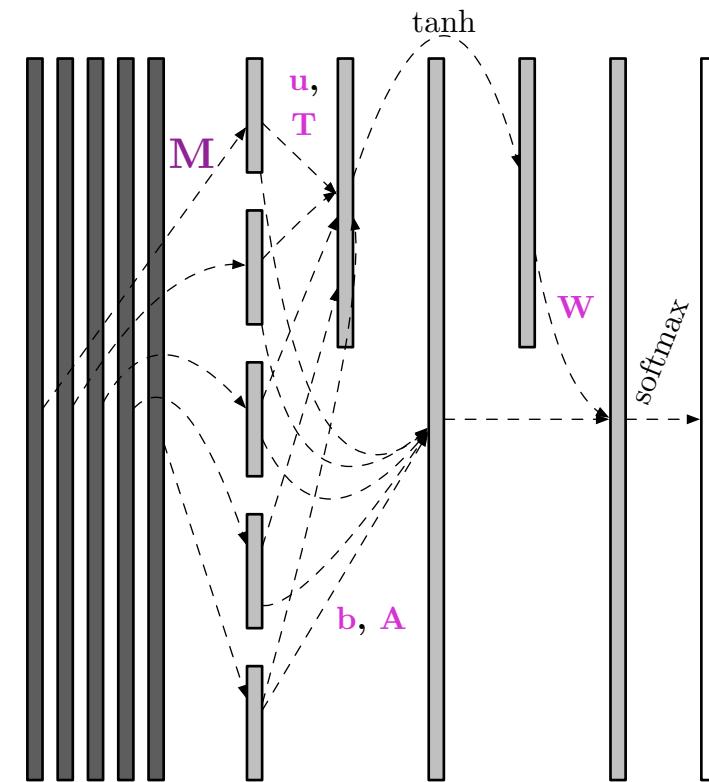
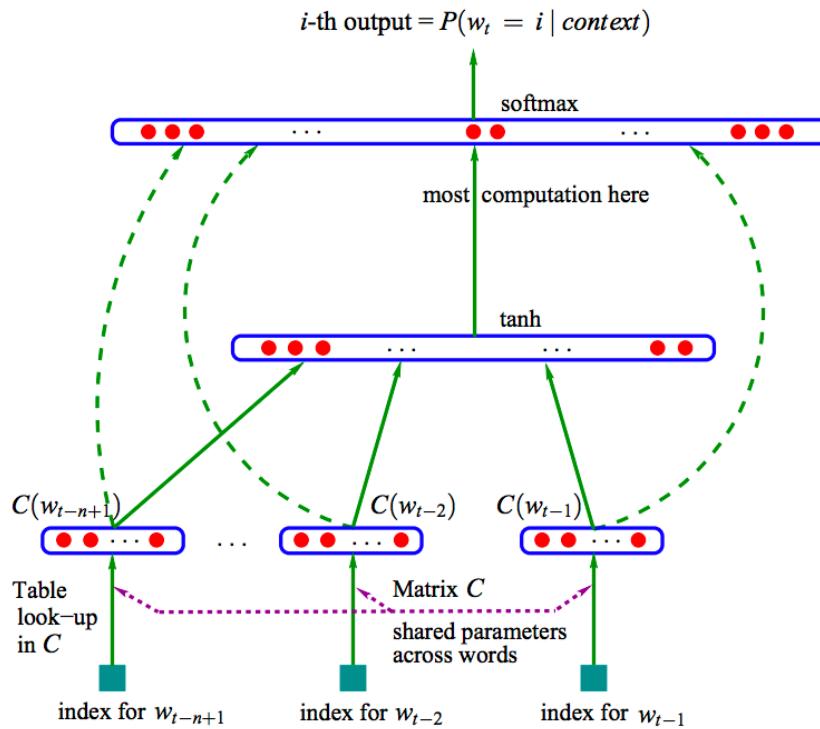
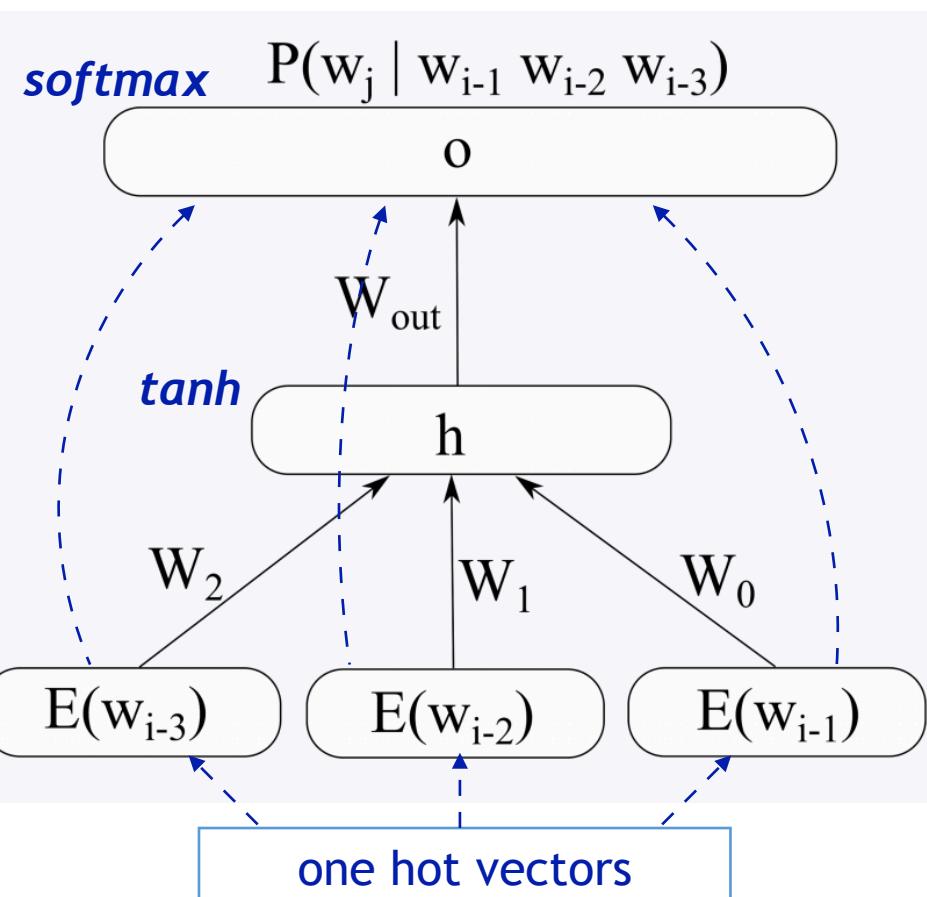
Feedforward Neural Network Language Model



Like a log-linear language model with two kinds of features:

- 1) concatenation of context-word embedding vectors
- 2) *tanh*-affine transformation of the above

Feedforward Neural Network Language Model



Neural Network: Definitions

Warning: there is no widely accepted standard notation!

A feedforward neural network $n_{\textcolor{magenta}{v}}$ is defined by:

- ▶ A function family that maps parameter values to functions of the form $n : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R}^{d_{out}}$; typically:
 - ▶ Non-linear
 - ▶ Differentiable with respect to its inputs
 - ▶ “Assembled” through a series of affine transformations and non-linearities, composed together
 - ▶ Symbolic/discrete inputs handled through lookups.
- ▶ Parameter values $\textcolor{magenta}{v}$
 - ▶ Typically a collection of scalars, vectors, and matrices
 - ▶ We often assume they are linearized into \mathbb{R}^D

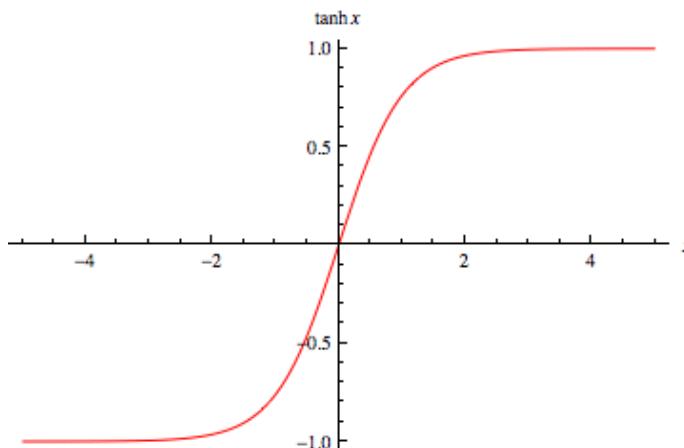
A Couple of Useful Functions

- ▶ softmax : $\mathbb{R}^k \rightarrow \mathbb{R}^k$

$$\langle x_1, x_2, \dots, x_k \rangle \mapsto \left\langle \frac{e^{x_1}}{\sum_{j=1}^k e^{x_j}}, \frac{e^{x_2}}{\sum_{j=1}^k e^{x_j}}, \dots, \frac{e^{x_k}}{\sum_{j=1}^k e^{x_j}} \right\rangle$$

- ▶ tanh : $\mathbb{R} \rightarrow [-1, 1]$

$$x \mapsto \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



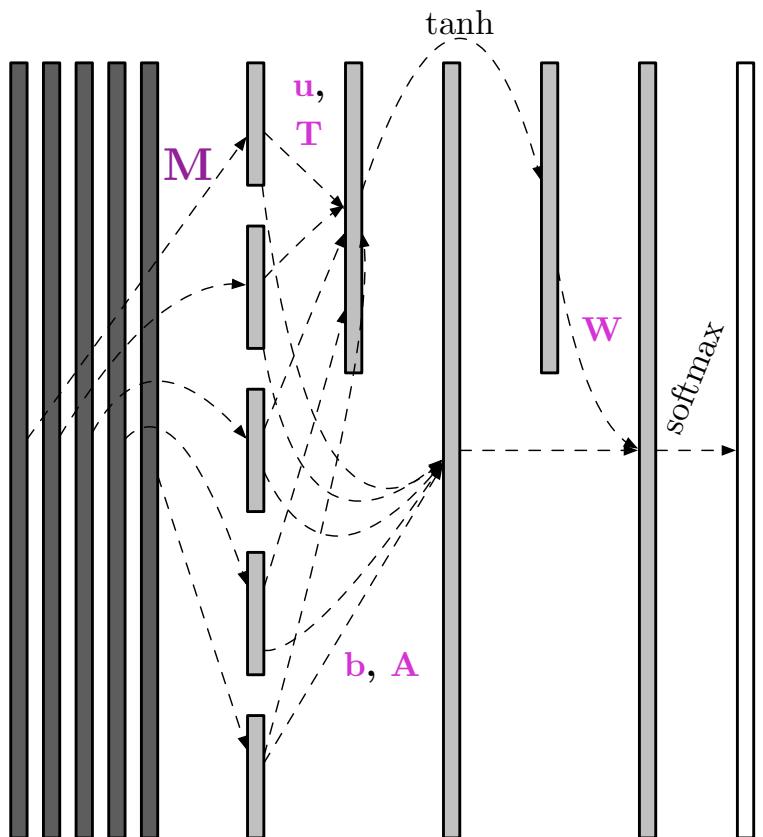
Generalized to be *elementwise*, so that it maps $\mathbb{R}^k \rightarrow [-1, 1]^k$.

- ▶ Others include: ReLUs, logistic sigmoids, PReLU, ...

Feedforward Neural Network Language Model

(Bengio et al., 2003)

Define the n-gram probability as follows:



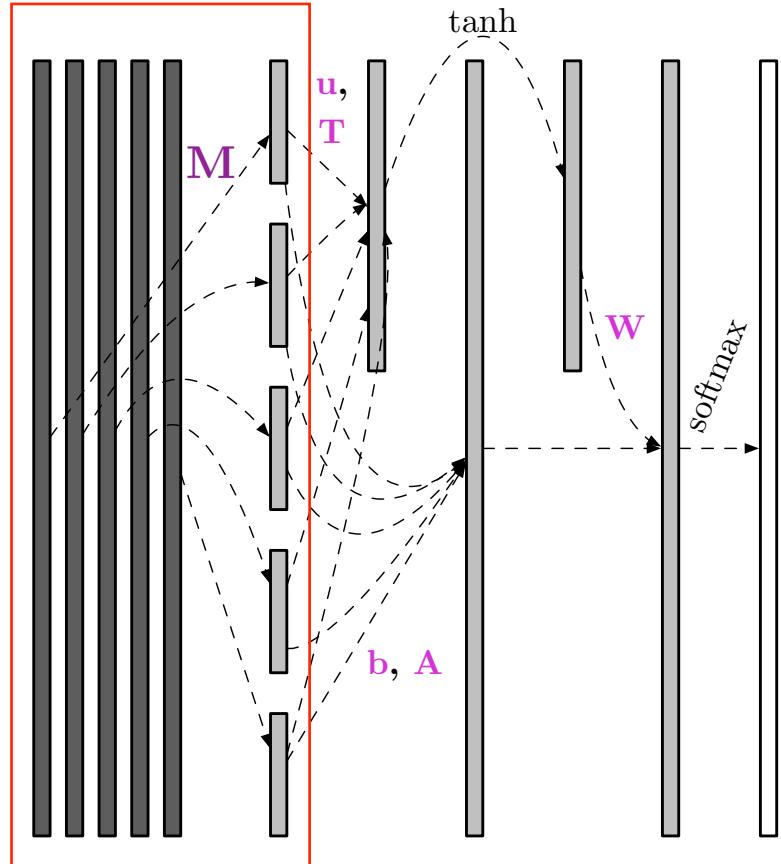
$$p(\cdot | \langle h_1, \dots, h_{n-1} \rangle) = n_{\nu} (\langle \mathbf{e}_{h_1}, \dots, \mathbf{e}_{h_{n-1}} \rangle) = \text{softmax} \left(\mathbf{b} + \sum_{j=1}^{n-1} \mathbf{e}_{h_j}^\top \mathbf{M} \mathbf{A}_j + \mathbf{W} \tanh \left(\mathbf{u} + \sum_{j=1}^{n-1} \mathbf{e}_{h_j}^\top \mathbf{M} \mathbf{T}_j \right) \right)$$

where each $\mathbf{e}_{h_j} \in \mathbb{R}^V$ is a one-hot vector and H is the number of “hidden units” in the neural network (a “hyperparameter”).

Parameters ν include:

- ▶ $\mathbf{M} \in \mathbb{R}^{V \times d}$, which are called “embeddings” (row vectors), one for every word in \mathcal{V}
- ▶ Feedforward NN parameters $\mathbf{b} \in \mathbb{R}^V$, $\mathbf{A} \in \mathbb{R}^{(n-1) \times d \times V}$, $\mathbf{W} \in \mathbb{R}^{V \times H}$, $\mathbf{u} \in \mathbb{R}^H$, $\mathbf{T} \in \mathbb{R}^{(n-1) \times d \times H}$

Breaking It Down

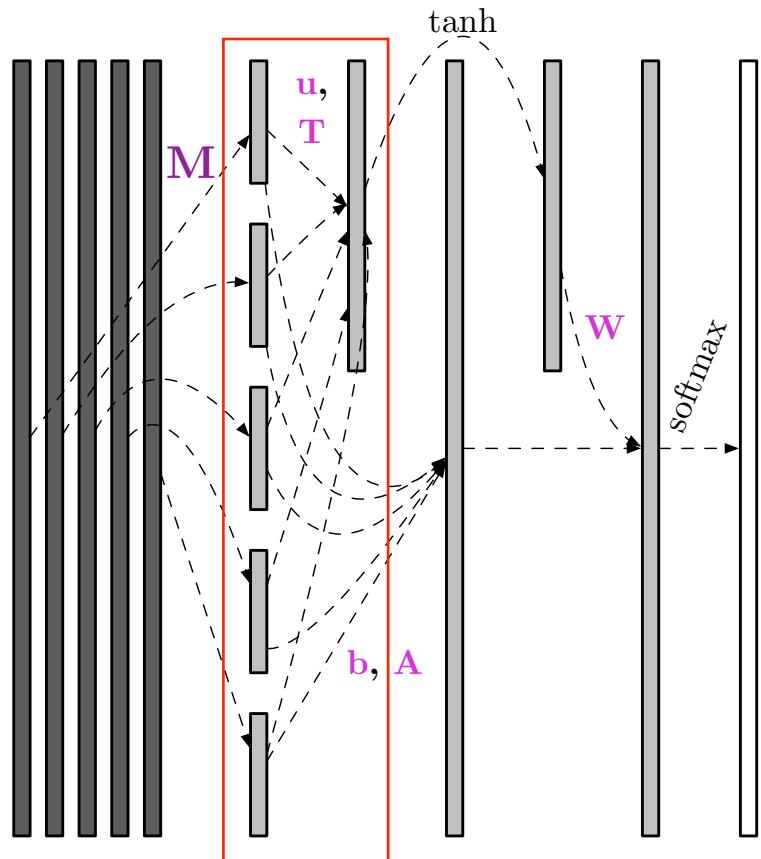


Look up each of the history words $h_j, \forall j \in \{1, \dots, n - 1\}$ in \mathbf{M} ; keep two copies. Rename the embedding for h_j as \mathbf{m}_{h_j} .

$$\mathbf{e}_{h_j}^\top \mathbf{M} = \mathbf{m}_{h_j}$$

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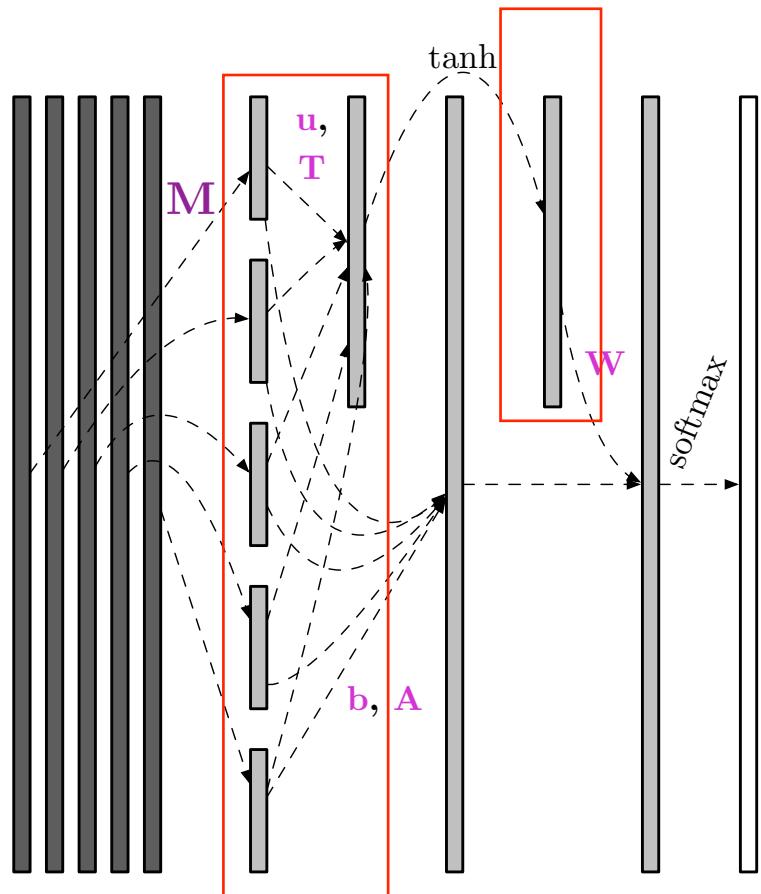
Breaking It Down



Apply an affine transformation to the second copy of the history-word embeddings (\mathbf{u} , \mathbf{T})

$$\mathbf{m}_{h_j} = \mathbf{u}_H + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{T}_j$$

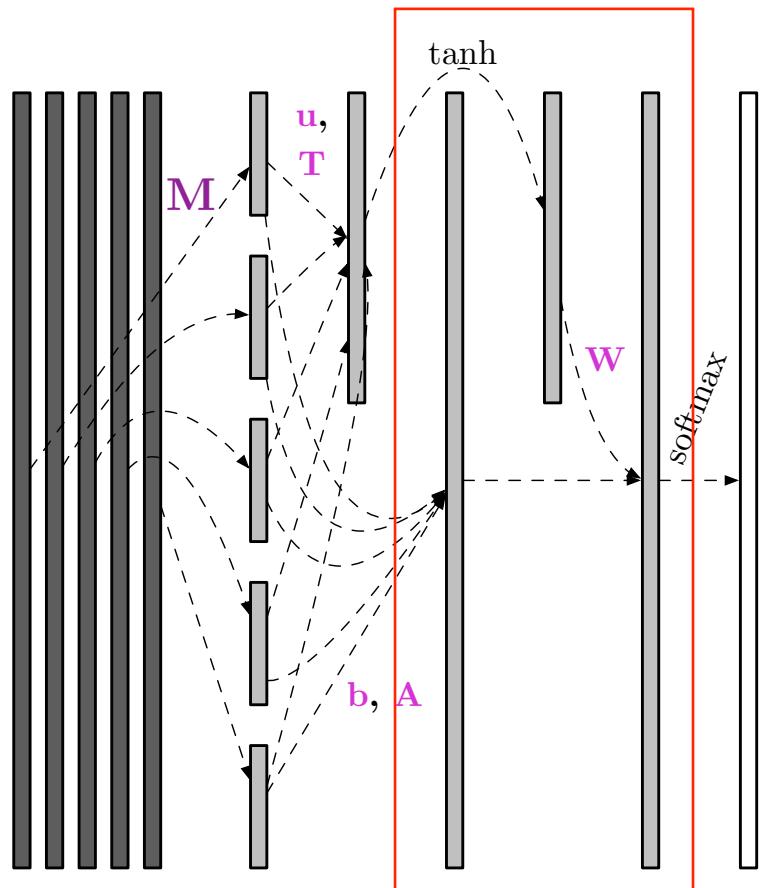
Breaking It Down



Apply an affine transformation to the second copy of the history-word embeddings (\mathbf{u} , \mathbf{T}) and a tanh nonlinearity.

$$\mathbf{m}_{h_j} = \tanh \left(\mathbf{u} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{T}_j \right)$$

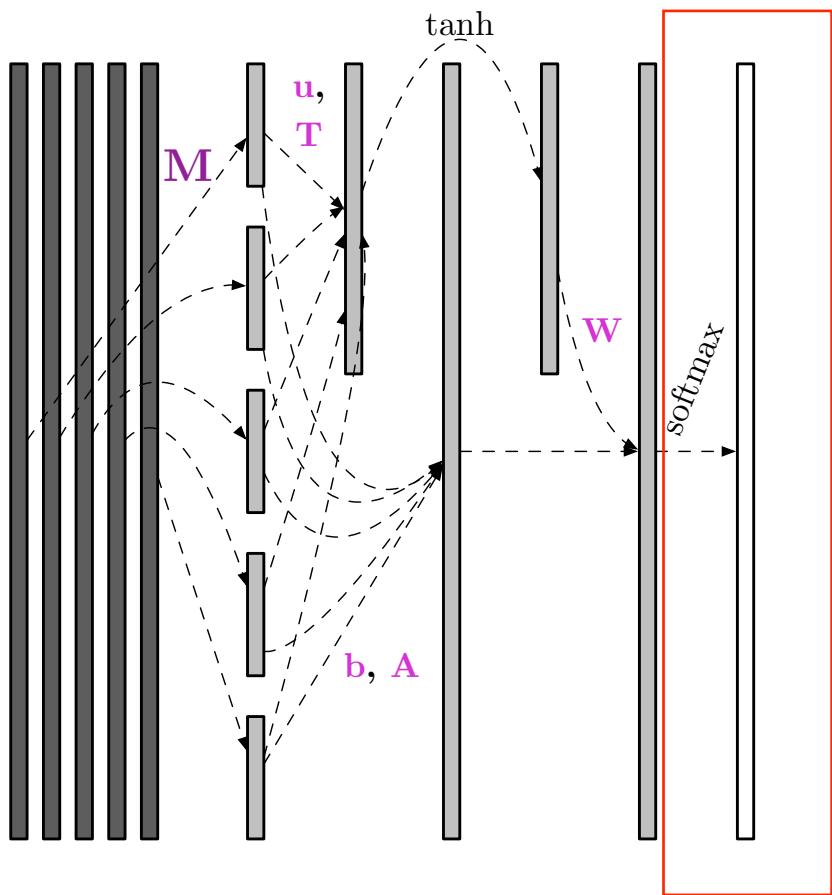
Breaking It Down



Apply an affine transformation to everything (\mathbf{b} , \mathbf{A} , \mathbf{W}).

$$\mathbf{b} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{A}_j \\ + \mathbf{W} \tanh \left(\mathbf{u} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{T}_j \right)$$

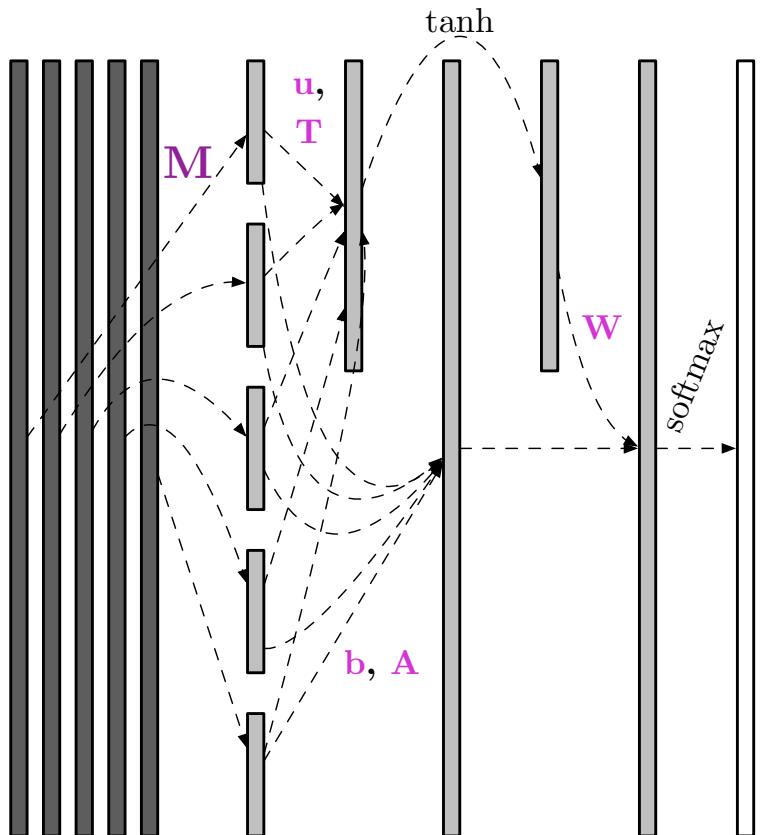
Breaking It Down



Apply a softmax transformation to make the vector sum to one.

$$\text{softmax} \left(\mathbf{b} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{A}_j + \mathbf{W} \tanh \left(\mathbf{u} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{T}_j \right) \right)$$

Breaking It Down



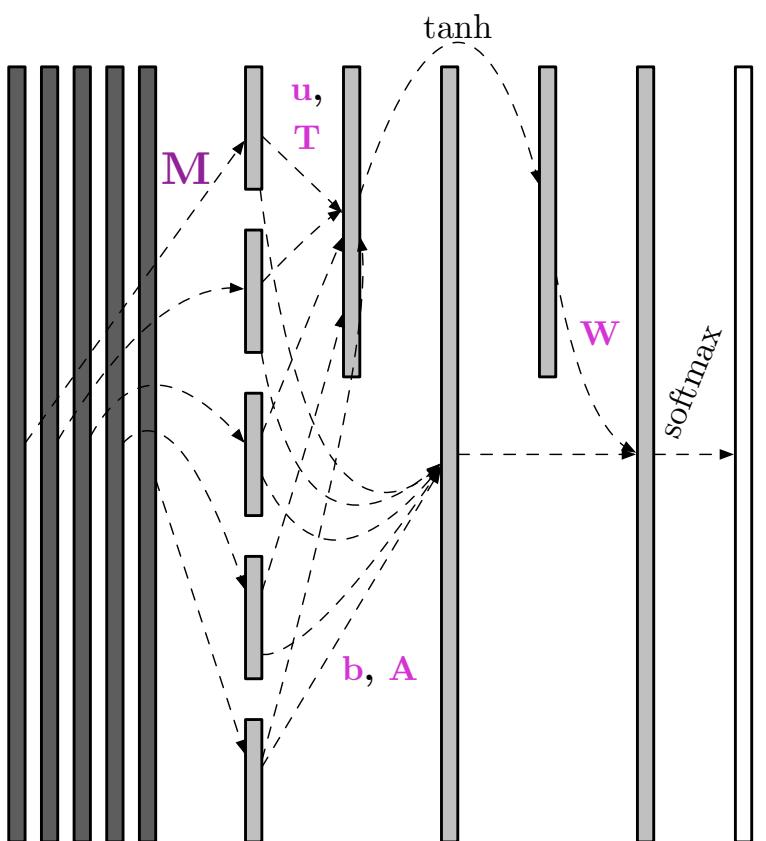
$$\begin{aligned} \text{softmax} & \left(\mathbf{b} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{A}_j \right. \\ & \left. + \mathbf{W} \tanh \left(\mathbf{u} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \mathbf{T}_j \right) \right) \end{aligned}$$

Like a log-linear language model with two kinds of features:

- ▶ Concatenation of context-word embeddings vectors \mathbf{m}_{h_j}
- ▶ tanh-affine transformation of the above

New parameters arise from (i) embeddings and (ii) affine transformation “inside” the nonlinearity.

Number of Parameters



$$D = \underbrace{Vd}_{\mathbf{M}} + \underbrace{V}_{\mathbf{b}} + \underbrace{(n-1)dV}_{\mathbf{A}} + \underbrace{VH}_{\mathbf{W}} + \underbrace{H}_{\mathbf{u}} + \underbrace{(n-1)dH}_{\mathbf{T}}$$

- ▶ $V \approx 18000$ (after OOV processing)
- ▶ $d \in \{30, 60\}$
- ▶ $H \in \{50, 100\}$
- ▶ $n - 1 = 5$

So $D = 461V + 30100$ parameters, compared to $O(V^n)$ for classical n-gram models.

- ▶ Forcing $\mathbf{A} = \mathbf{0}$ eliminated $300V$ parameters and performed a bit better, but was slower to converge.
- ▶ If we averaged \mathbf{m}_{h_j} instead of concatenating, we'd get to $221V + 6100$ (this is a variant of “continuous bag of words,” Mikolov et al., 2013).

Why does it work?