

CS 5522: Artificial Intelligence II

Reinforcement Learning



Instructor: Wei Xu

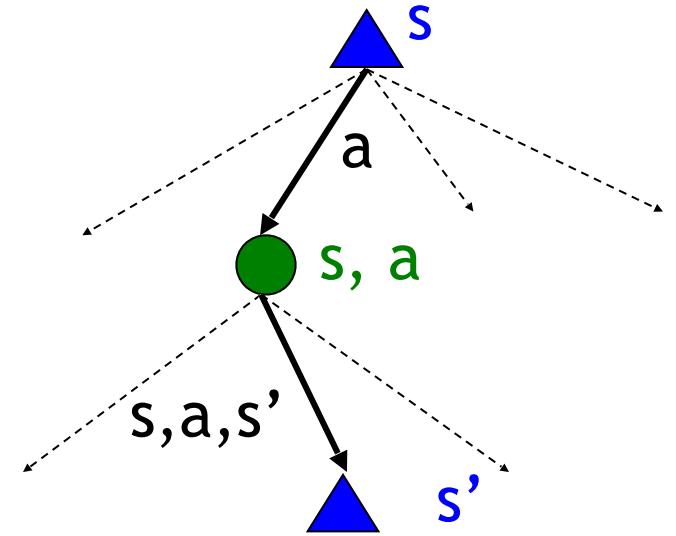
Ohio State University

[These slides were adapted from CS188 Intro to AI at UC Berkeley.]

Recap: MDPs

- **Markov decision processes:**

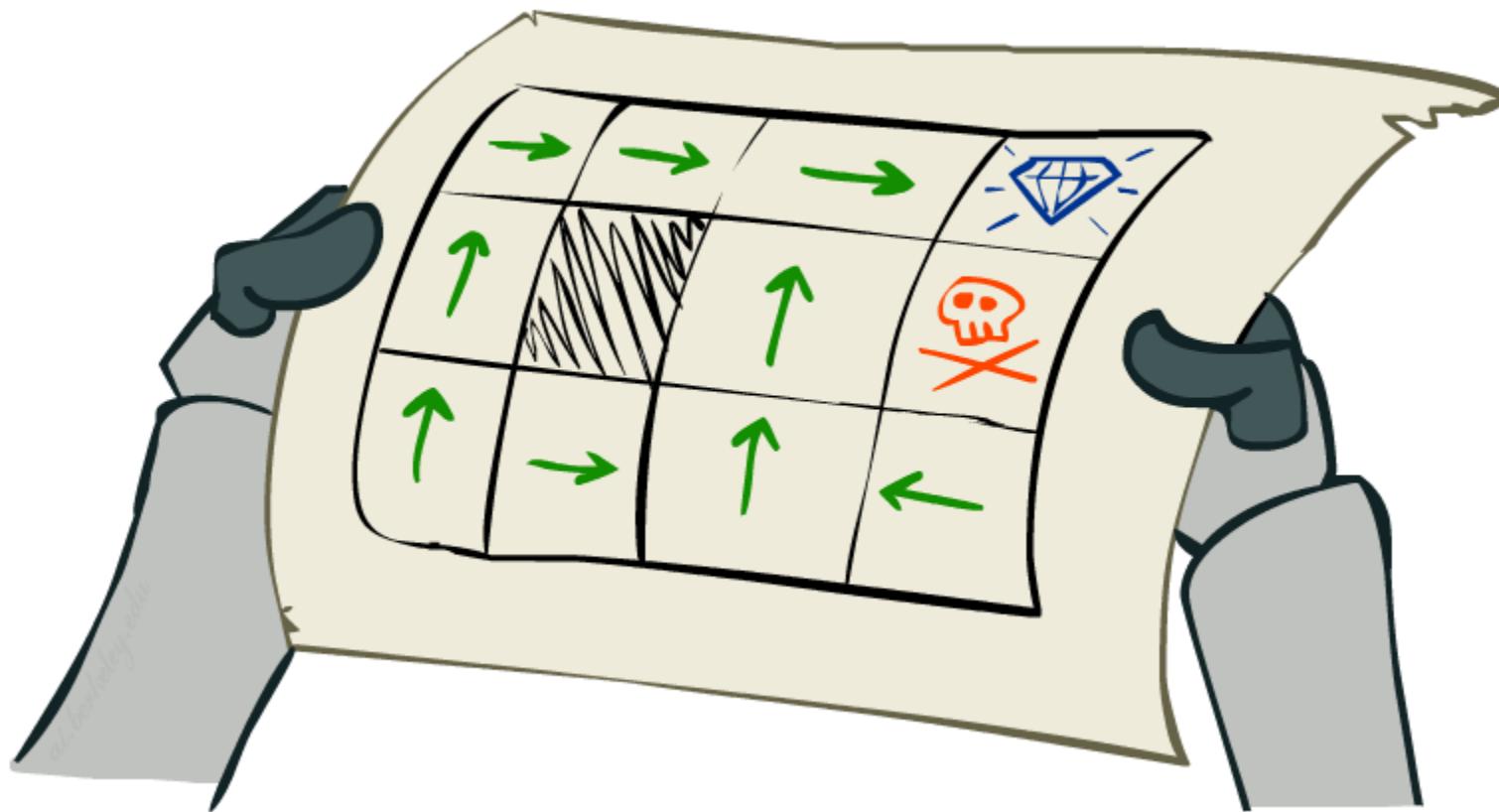
- States S
- Actions A
- Transitions $P(s'|s, a)$ (or $T(s, a, s')$)
- Rewards $R(s, a, s')$ (and discount γ)
- Start state s_0



- **Quantities:**

- Policy = map of states to actions
- Utility = sum of discounted rewards
- Values = expected future utility from a state (max node)
- Q-Values = expected future utility from a q-state (chance node)

Solving MDPs



The Bellman Equations

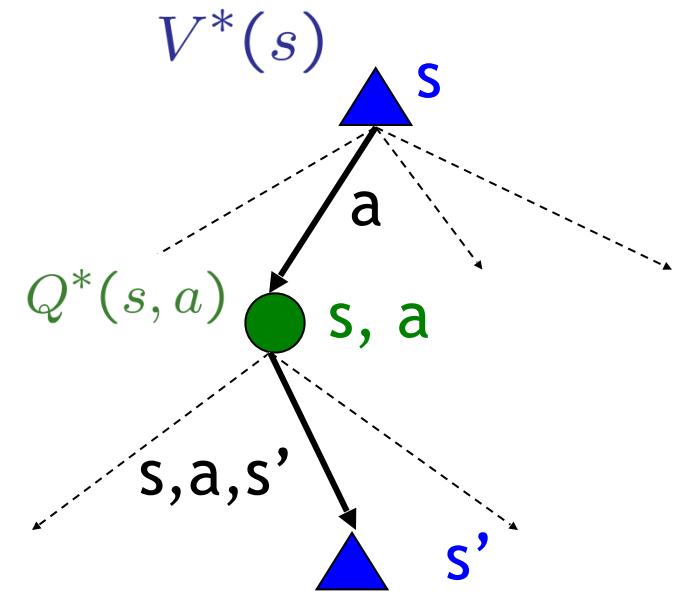
- Definition of “optimal utility” via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s, a)$$

$$Q^*(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

$$V^*(s) = \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

- These are the Bellman equations, and they characterize optimal values in a way we'll use over and over

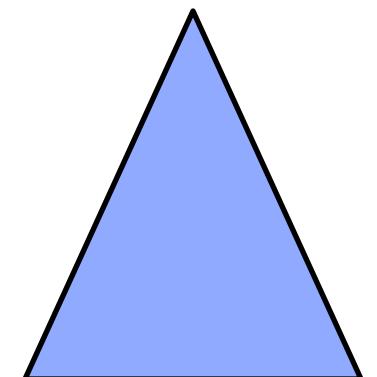


Value Iteration

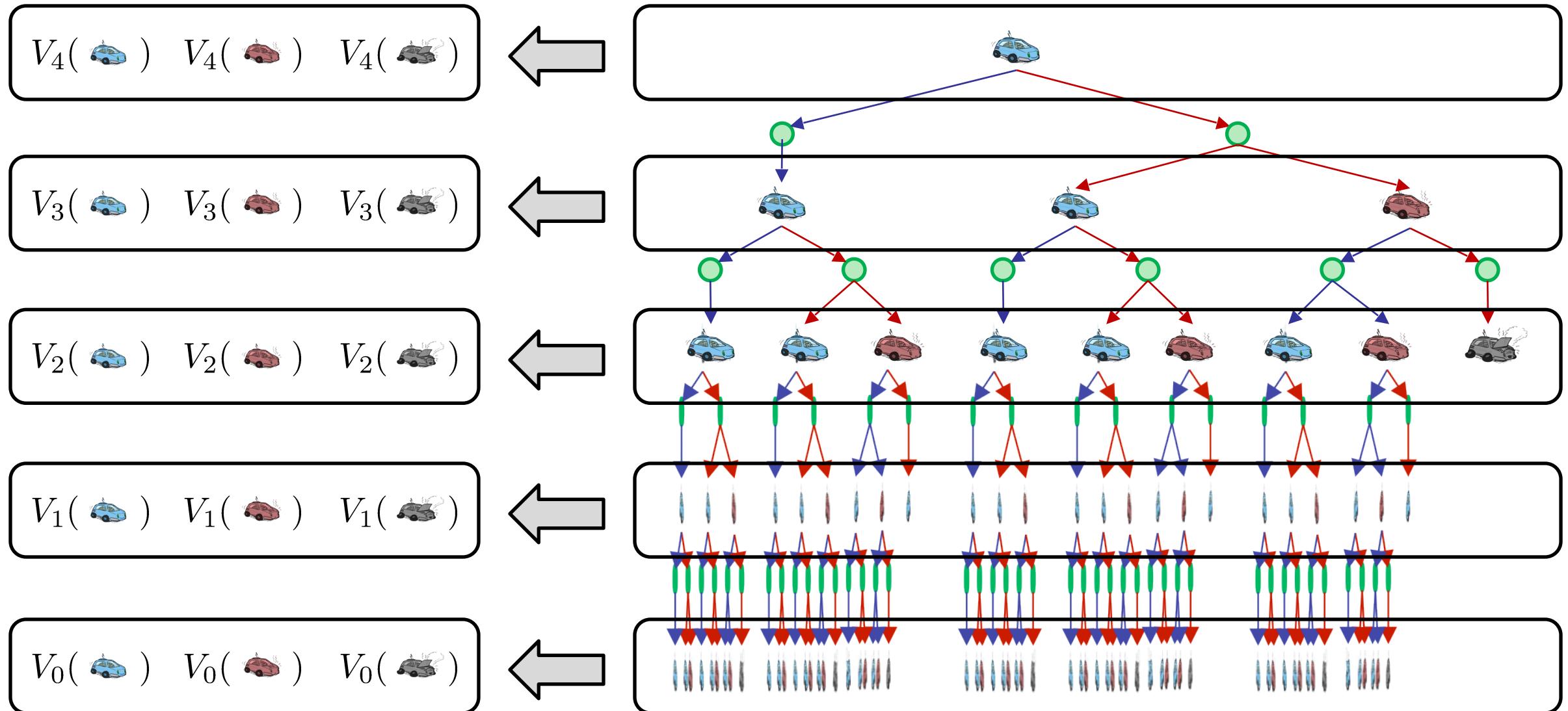
- Start with $V_0(s) = 0$: no time steps left means an expected reward sum of zero
- Given vector of $V_k(s)$ values, do one ply of expectimax from each state:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

- Repeat until convergence
- Complexity of each iteration: $O(S^2A)$
- **Theorem: will converge to unique optimal values**
 - Basic idea: approximations get refined towards optimal values
 - Policy may converge long before values do



Computing Time-Limited Values



Policy Iteration

- Evaluation: For fixed current policy π , find values with policy evaluation:
 - Iterate until values converge:

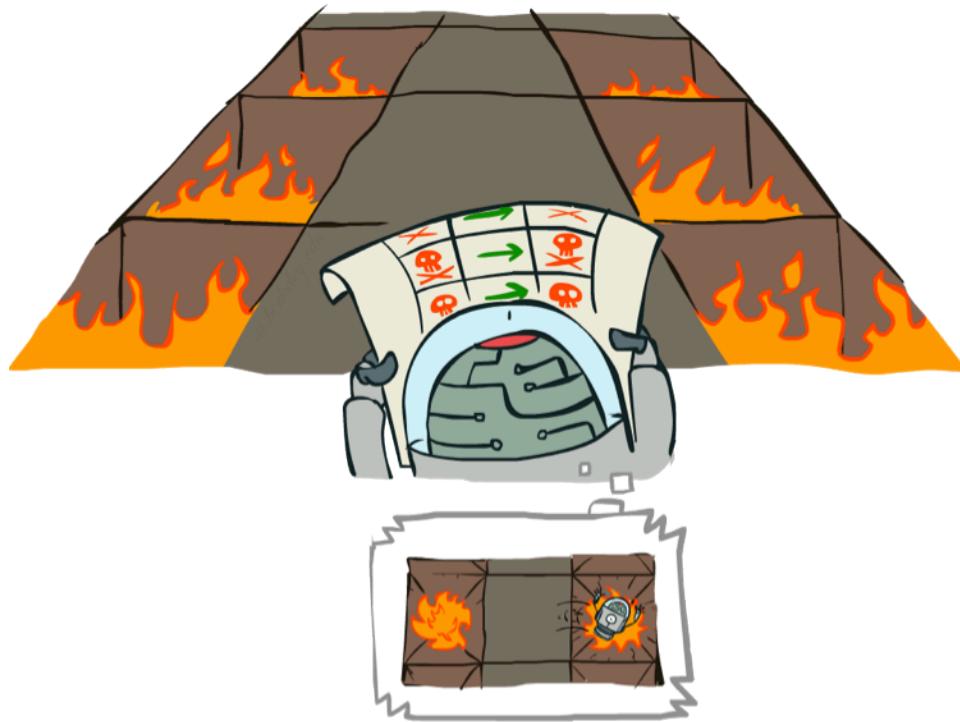
$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') [R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')]$$

- Improvement: For fixed values, get a better policy using policy extraction
 - One-step look-ahead:

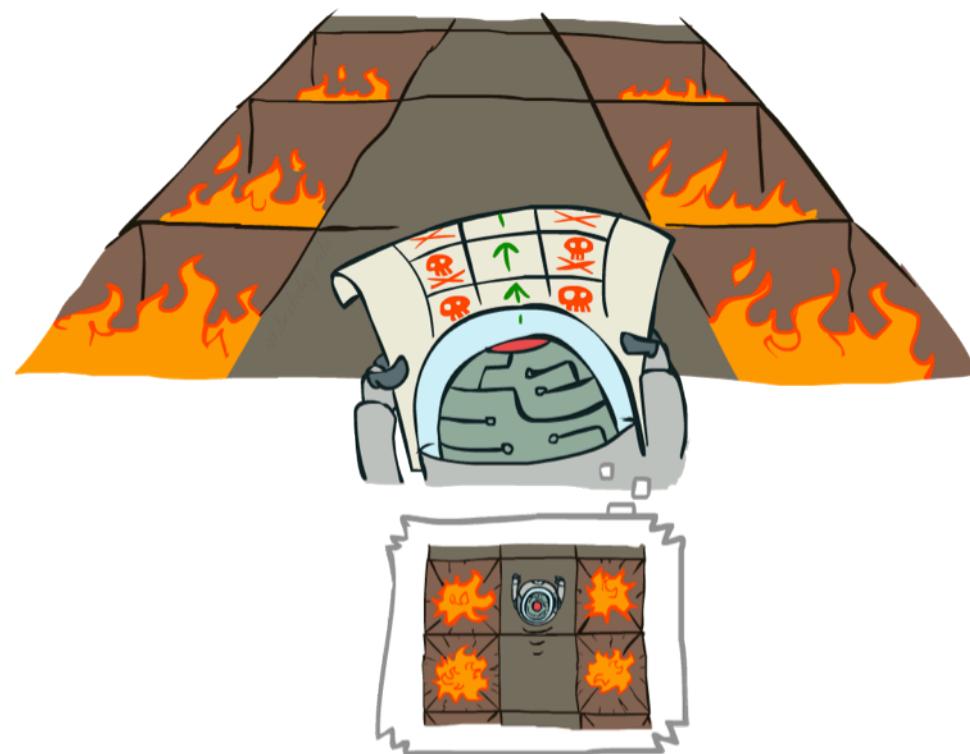
$$\pi_{i+1}(s) = \arg \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^{\pi_i}(s')]$$

Example: Policy Evaluation

Always Go Right



Always Go Forward



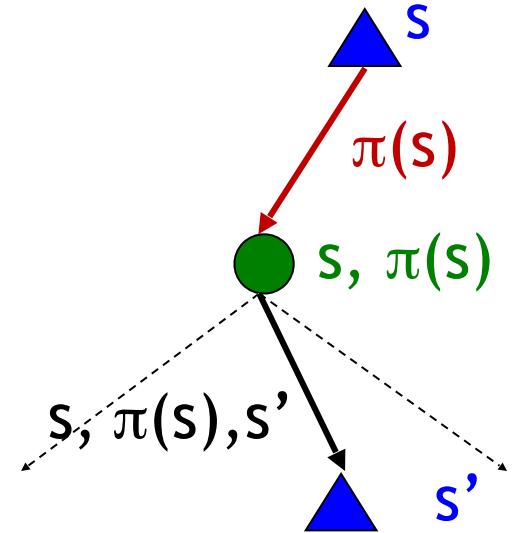
Policy Evaluation

- How do we calculate the V's for a fixed policy π ?
- Idea 1: Turn recursive Bellman equations into updates (like value iteration)

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

- Efficiency: $O(S^2)$ per iteration
- Idea 2: Without the maxes, the Bellman equations are just a linear system
 - Solve with Matlab (or your favorite linear system solver)



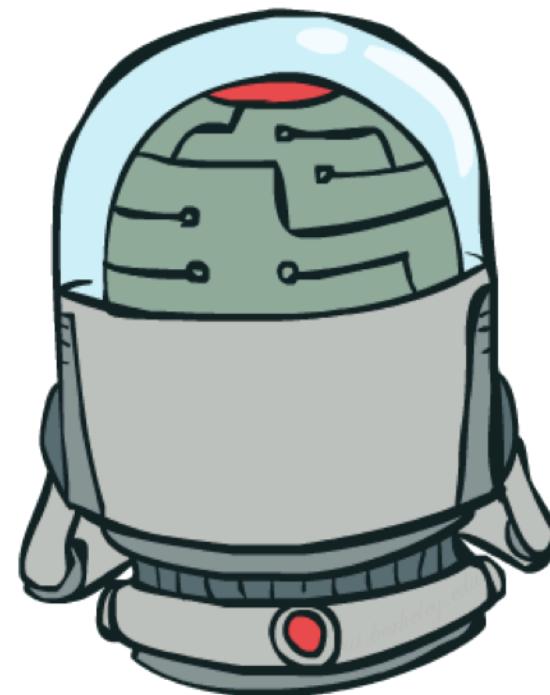
Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
 - Every iteration updates both the values and (implicitly) the policy
 - We don't track the policy, but taking the max over actions implicitly recomputes it
- In policy iteration:
 - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
 - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
 - The new policy will be better (or we're done)
- Both are dynamic programs for solving MDPs

Summary: MDP Algorithms

- So you want to....
 - Compute optimal values: use value iteration or policy iteration
 - Compute values for a particular policy: use policy evaluation
 - Turn your values into a policy: use policy extraction (one-step lookahead)
- These all look the same!
 - They basically are - they are all variations of Bellman updates
 - They all use one-step lookahead expectimax fragments
 - They differ only in whether we plug in a fixed policy or max over actions

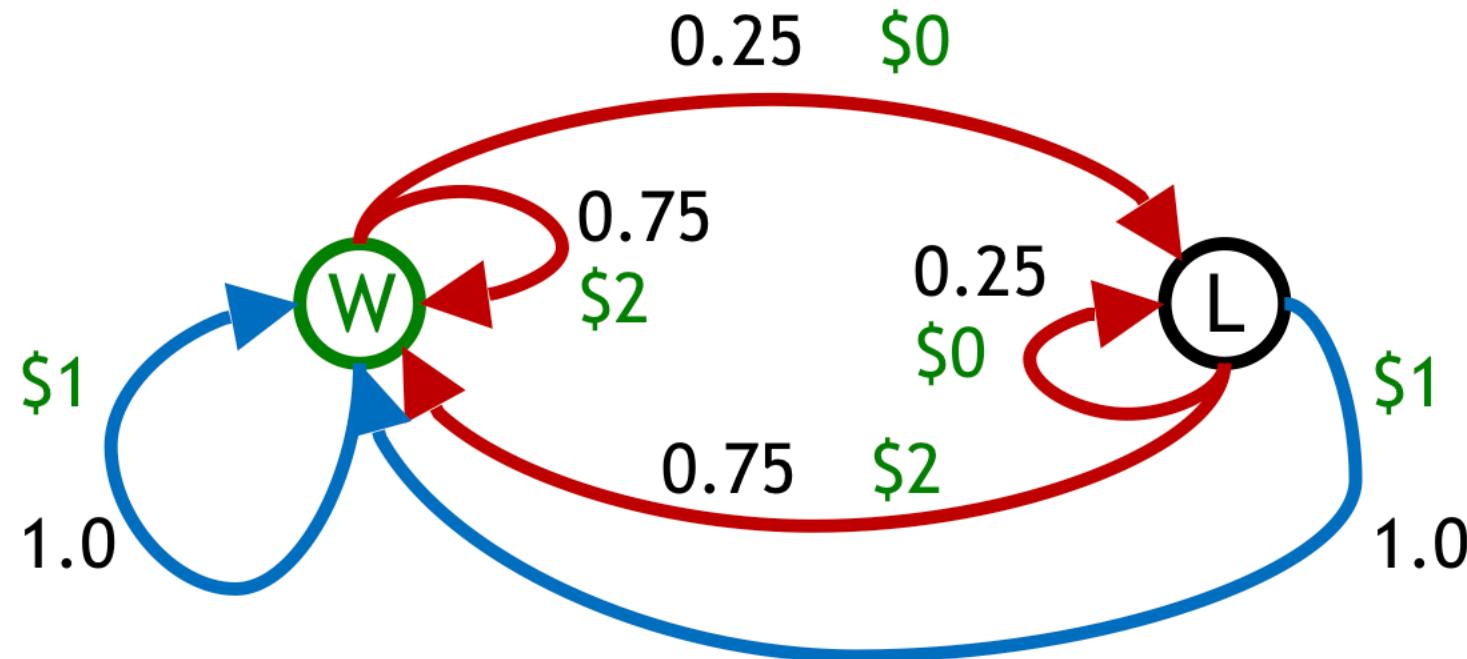
Double Bandits



Double-Bandit MDP

- Actions: *Blue, Red*
- States: *Win, Lose*

*No discount
100 time steps
Both states have
the same value*



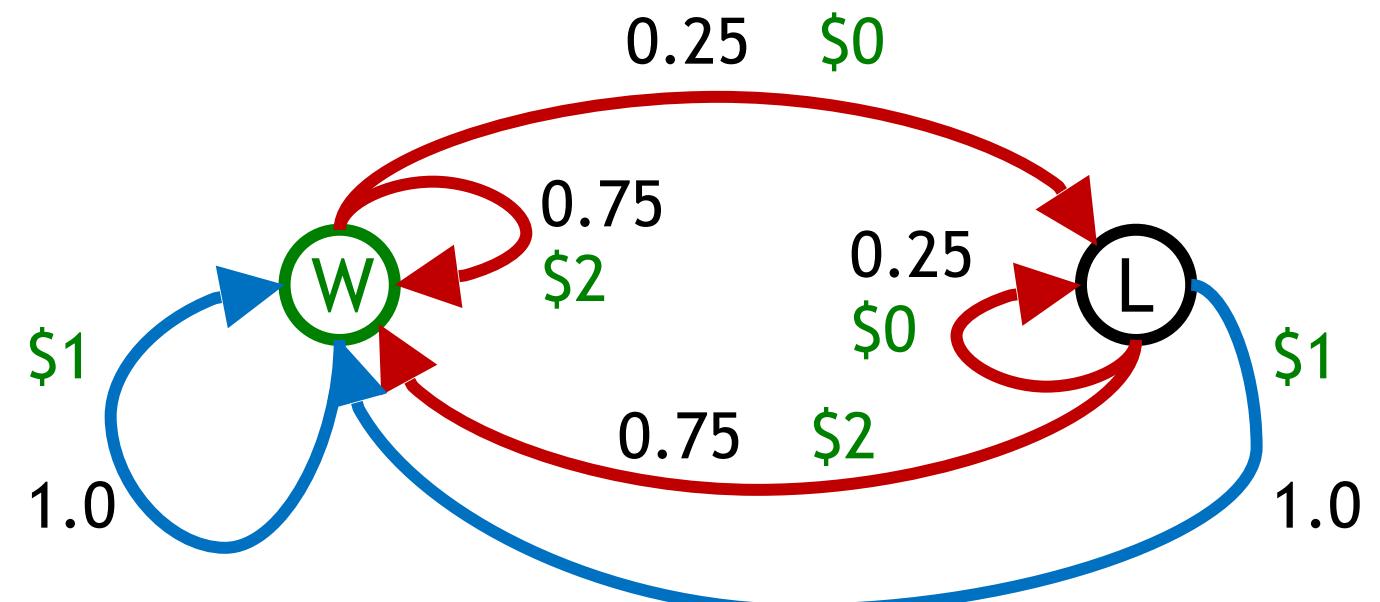
Offline Planning

- Solving MDPs is offline planning

- You determine all quantities through computation
- You need to know the details of the MDP
- You do not actually play the game!

*No discount
100 time steps
Both states have
the same value*

	Value
Play Red	150
Play Blue	100



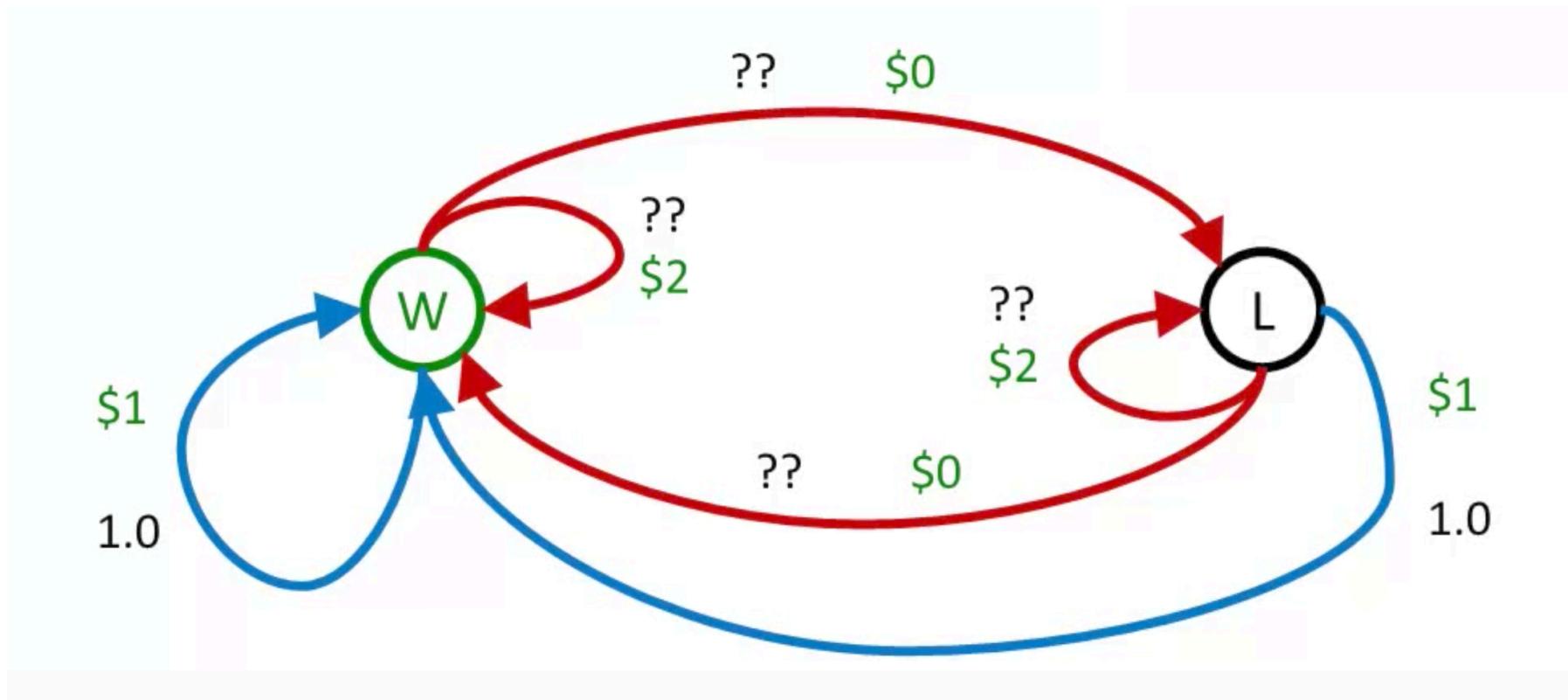
Let's Play!



\$2	\$2	\$0	\$2	\$2
\$2	\$2	\$0	\$0	\$0

Online Planning

- Rules changed! Red's win chance is different.



Let's Play!



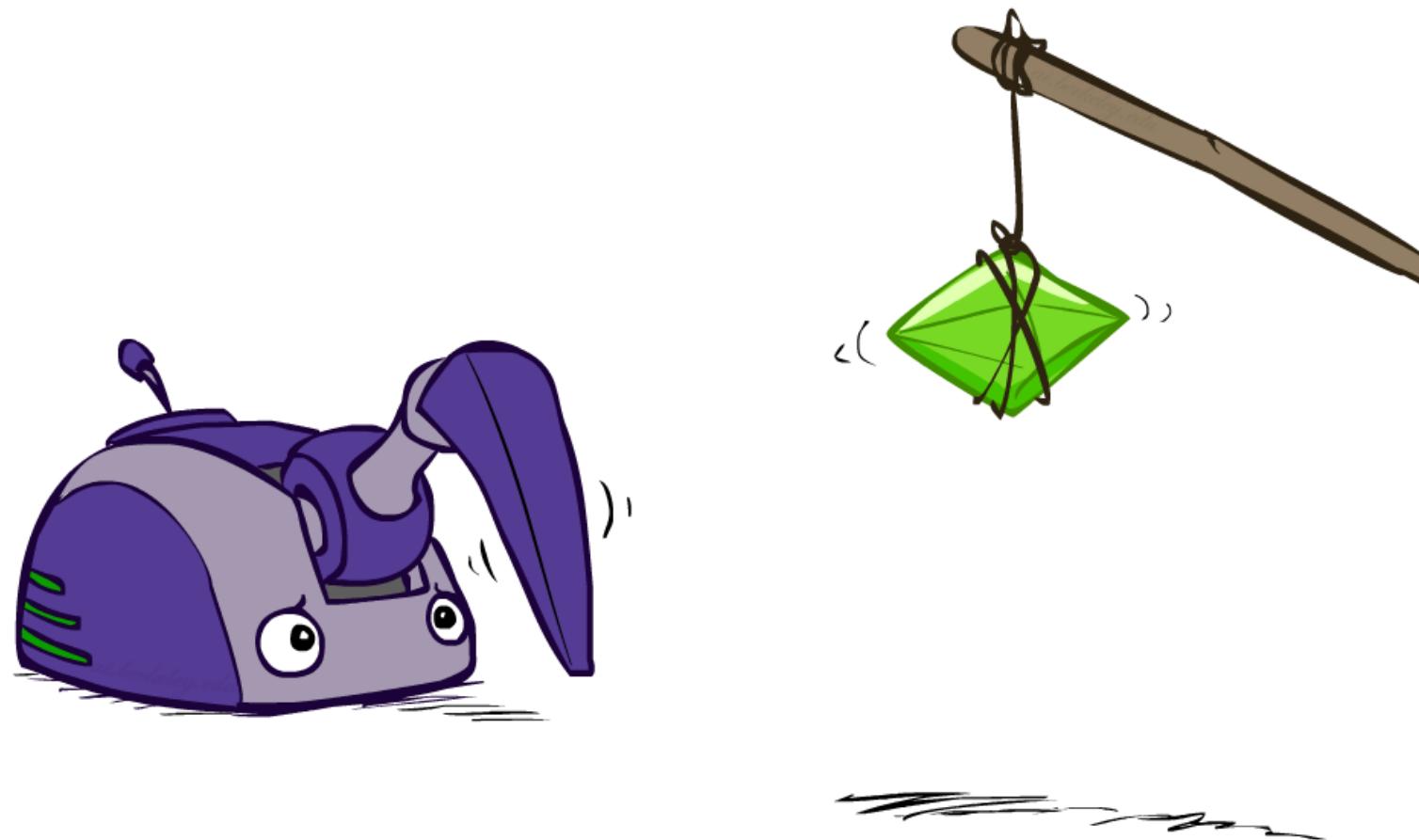
\$0	\$0	\$0	\$2	\$0
\$2	\$0	\$0	\$0	\$0

What Just Happened?

- That wasn't planning, it was learning!
 - Specifically, reinforcement learning
 - There was an MDP, but you couldn't solve it with just computation
 - You needed to actually act to figure it out
- Important ideas in reinforcement learning that came up
 - Exploration: you have to try unknown actions to get information
 - Exploitation: eventually, you have to use what you know
 - Regret: even if you learn intelligently, you make mistakes
 - Sampling: because of chance, you have to try things repeatedly
 - Difficulty: learning can be much harder than solving a known MDP

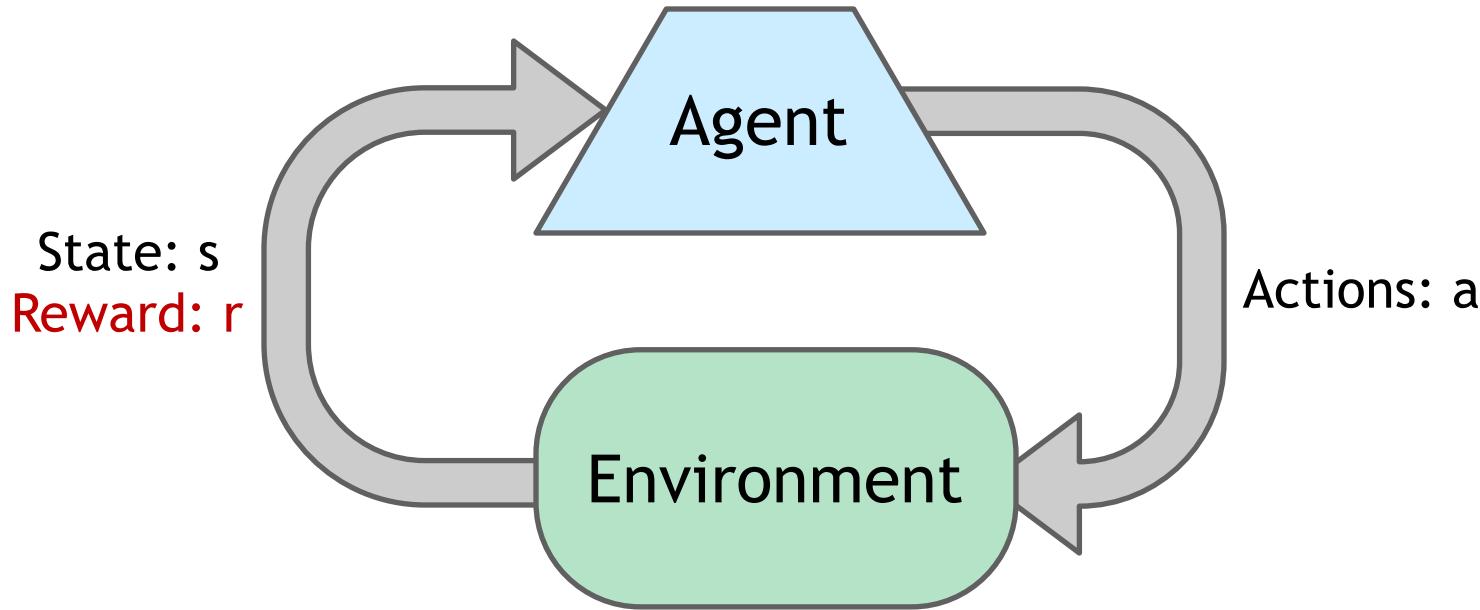


Reinforcement Learning



<https://www.youtube.com/watch?v=7a7MqMg2JPY&t=8s>

Reinforcement Learning



- Basic idea:
 - Receive feedback in the form of **rewards**
 - Agent's utility is defined by the reward function
 - Must (learn to) act so as to **maximize expected rewards**
 - All learning is based on observed samples of outcomes!

Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

Example: Learning to Walk



Initial

Example: Learning to Walk



Training

[Kohl and Stone, ICRA 2004]

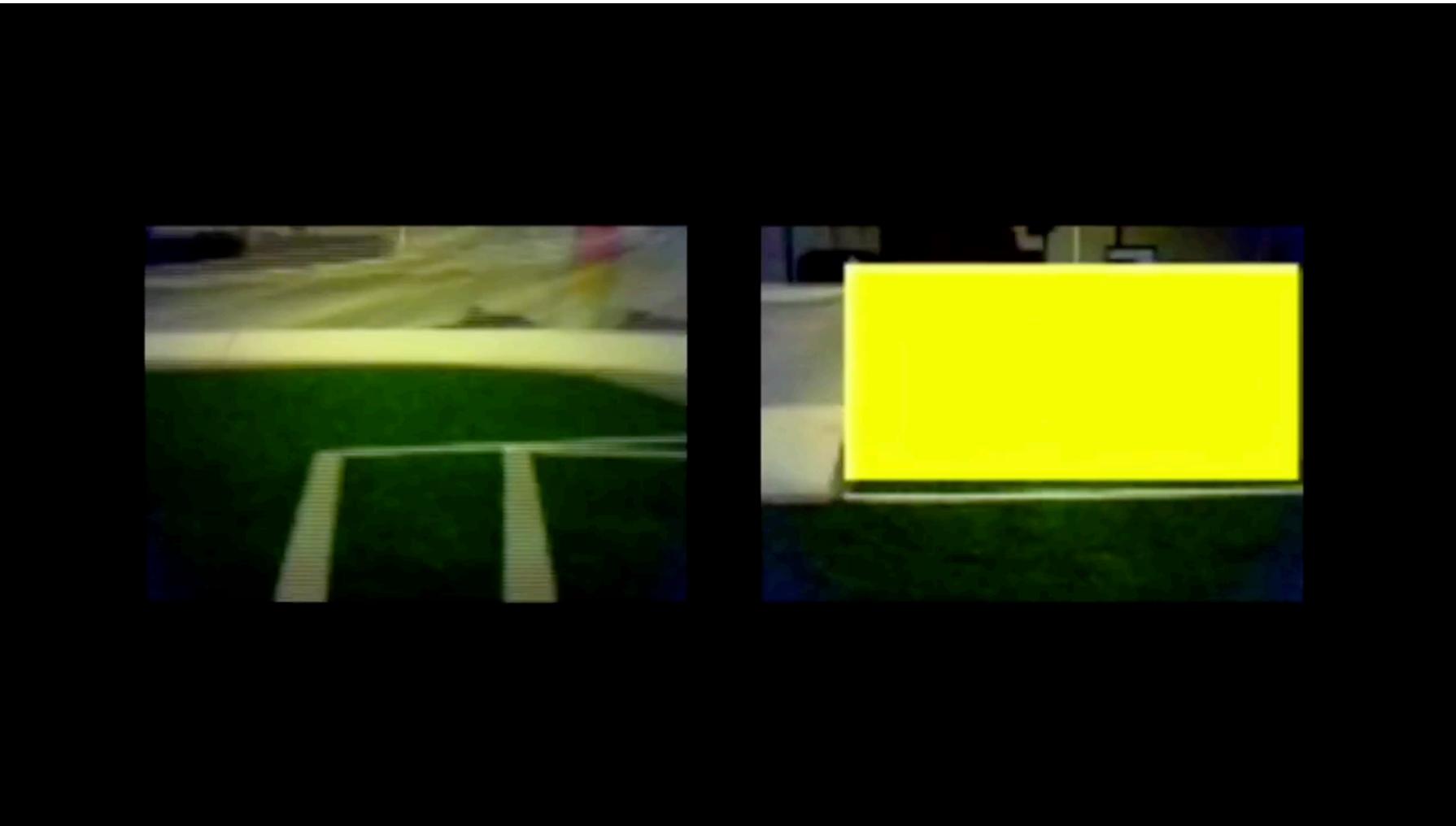
[Video: AIBO WALK - training]

Example: Learning to Walk



Finished

Example: Learning to Walk



Robot View

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK - robot view]

The Crawler!



[You, in Homework 3]

Reinforcement Learning

- Still assume a Markov decision process (MDP):

- A set of states $s \in S$
 - A set of actions (per state) A
 - A model $T(s,a,s')$
 - A reward function $R(s,a,s')$

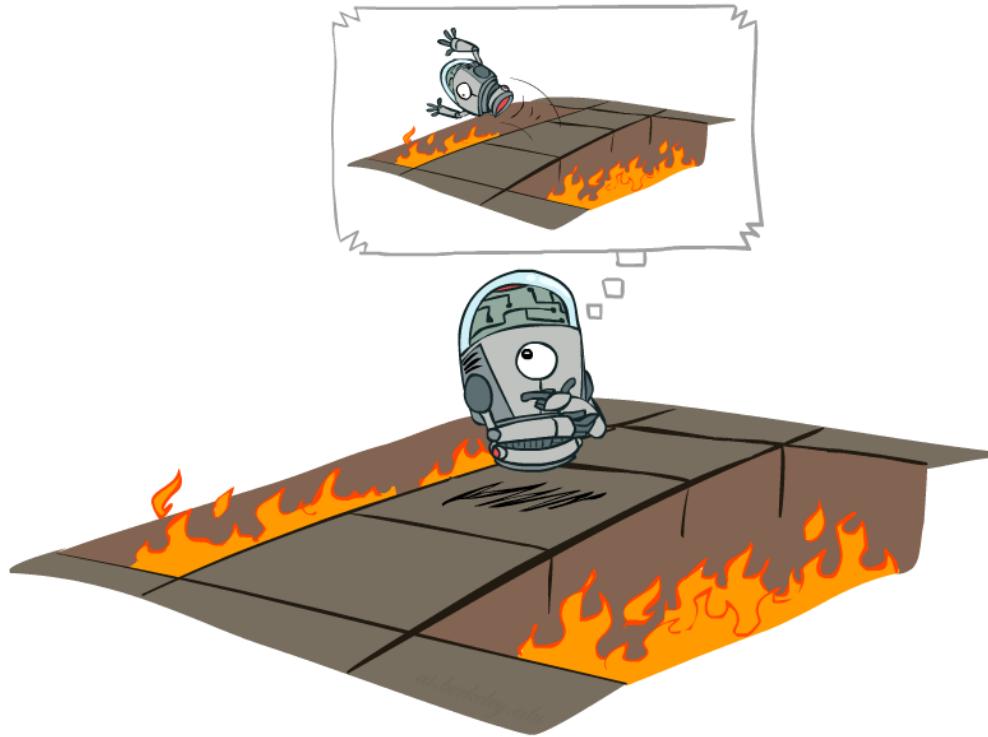
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
 - Must actually try actions and states out to learn



Offline (MDPs) vs. Online (RL)

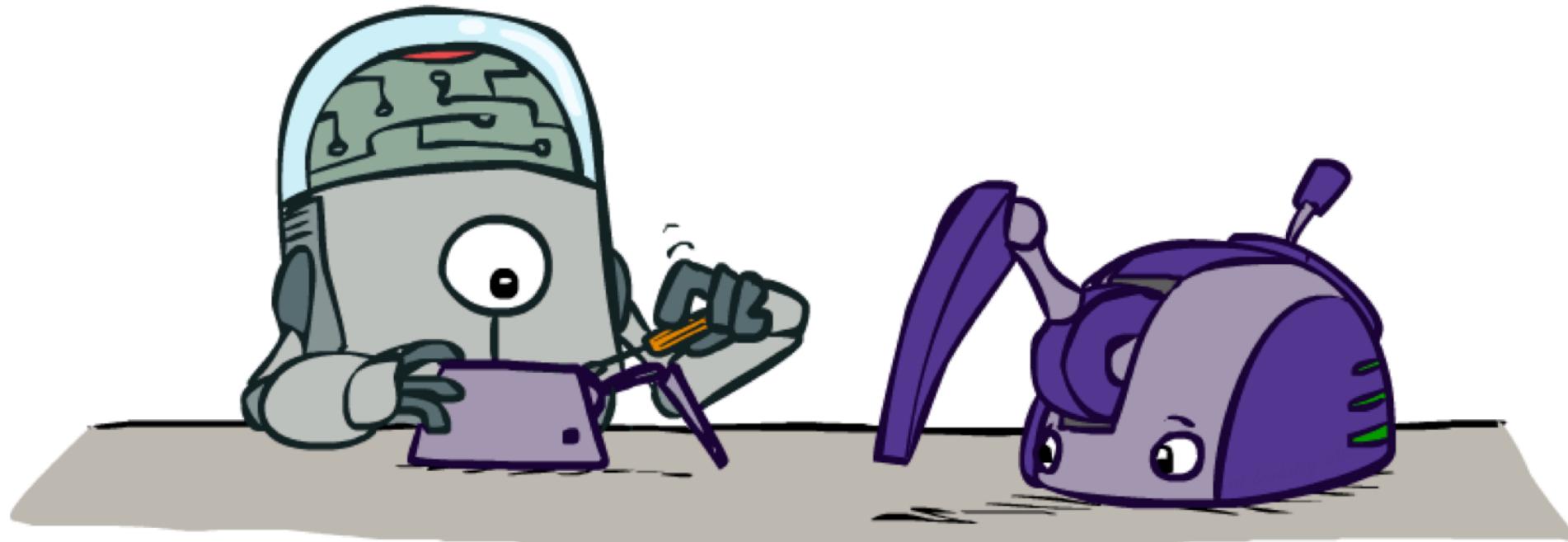


Offline Solution



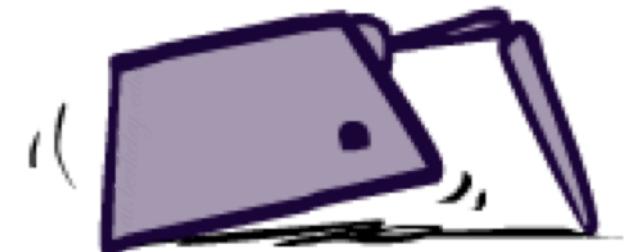
Online Learning

Model-Based Learning



Model-Based Learning

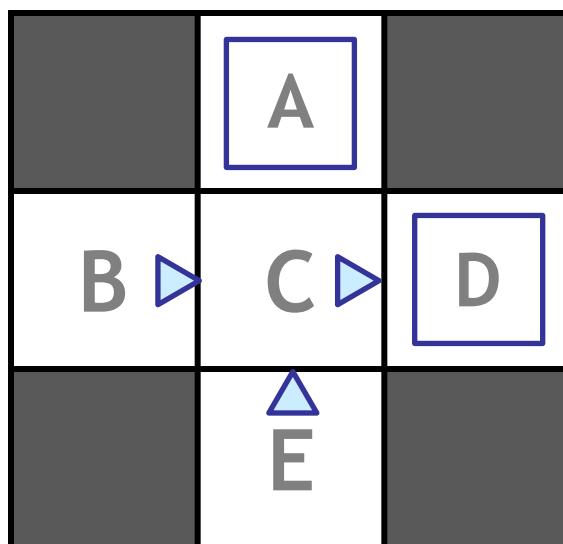
- Model-Based Idea:
 - Learn an approximate model based on experiences
 - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
 - Count outcomes s' for each s, a
 - Normalize to give an estimate of $\hat{T}(s, a, s')$
 - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
- Step 2: Solve the learned MDP
 - For example, use value iteration, as before



Example: Model-Based Learning

Input Policy

π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{T}(s, a, s')$

$T(B, \text{east}, C) = 1.00$
 $T(C, \text{east}, D) = 0.75$
 $T(C, \text{east}, A) = 0.25$
...

$\hat{R}(s, a, s')$

$R(B, \text{east}, C) = -1$
 $R(C, \text{east}, D) = -1$
 $R(D, \text{exit}, x) = +10$
...

Example: Expected Age

Goal: Compute expected age of cse5522 students

Known $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without $P(A)$, instead collect samples $[a_1, a_2, \dots a_N]$

Unknown $P(A)$: “Model Based”

Why does this work? Because eventually you learn the right model.

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

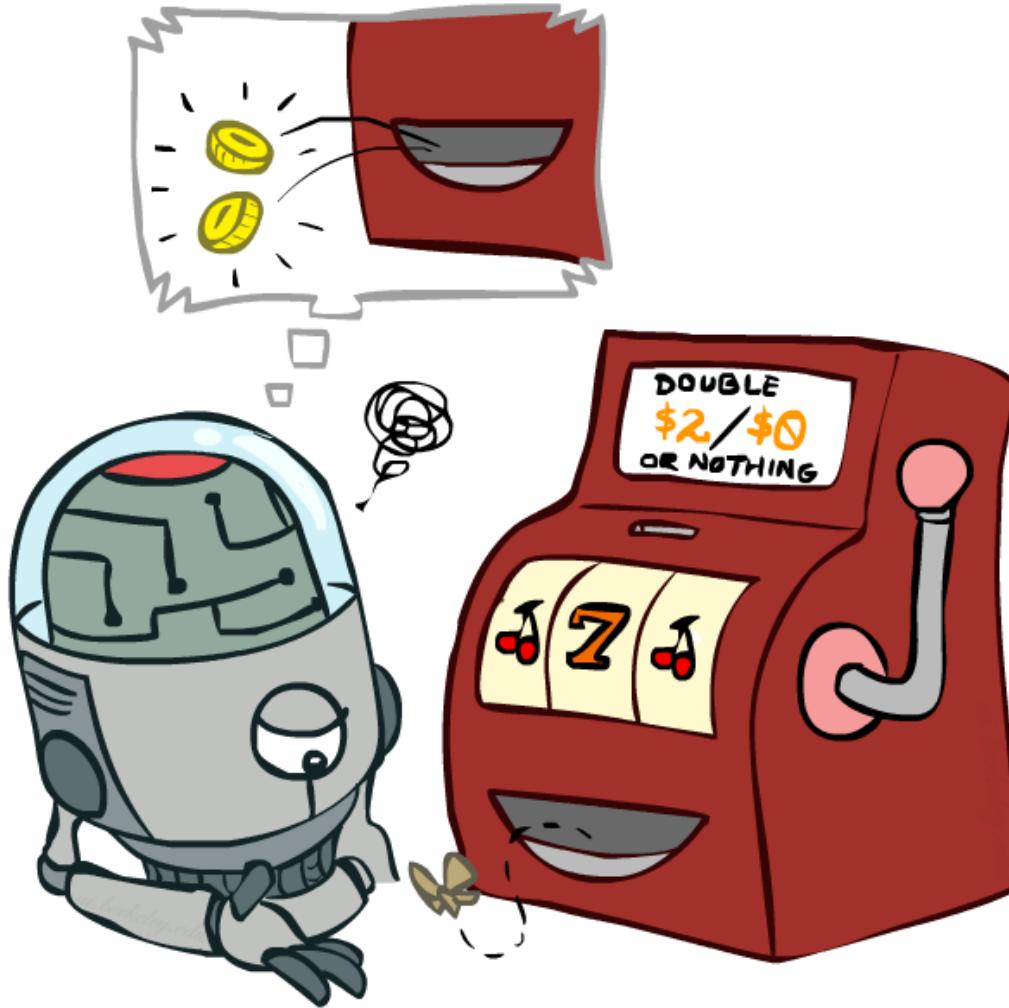
$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown $P(A)$: “Model Free”

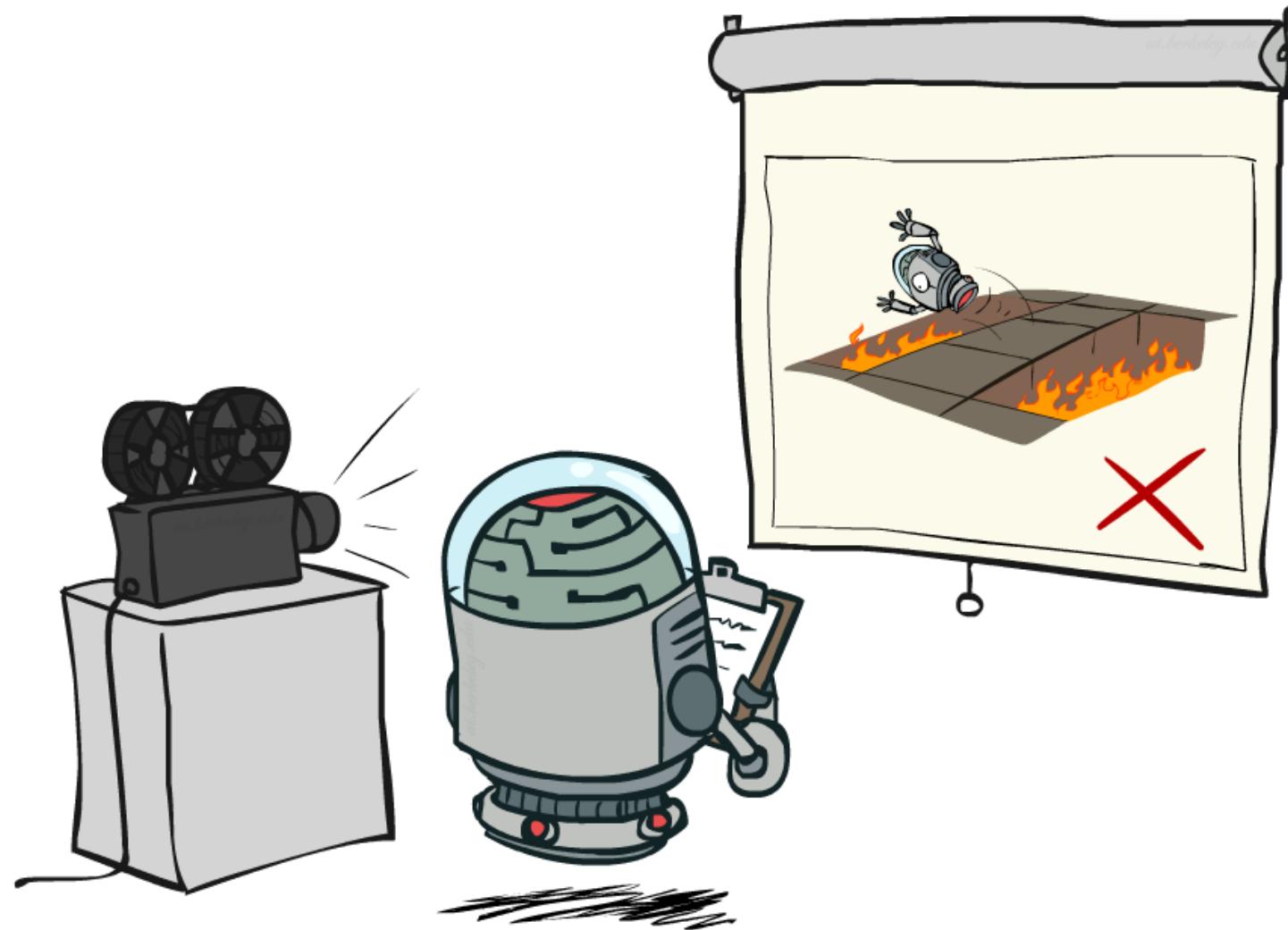
$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Model-Free Learning



Passive Reinforcement Learning



Passive vs. Active Learning

- **Passive Learning:**
 - The agent has a fixed policy and tried to learn the utilities of states by observing the world go by
 - Analogous to policy evaluation
 - Often serves as a component of active learning
 - Often inspires active learning algorithm
- **Active Learning:**
 - The agent attempts to find an optimal (or at least good) policy by acting in the world
 - Analogous to solving the underlying MDP, but without first being given the MDP model

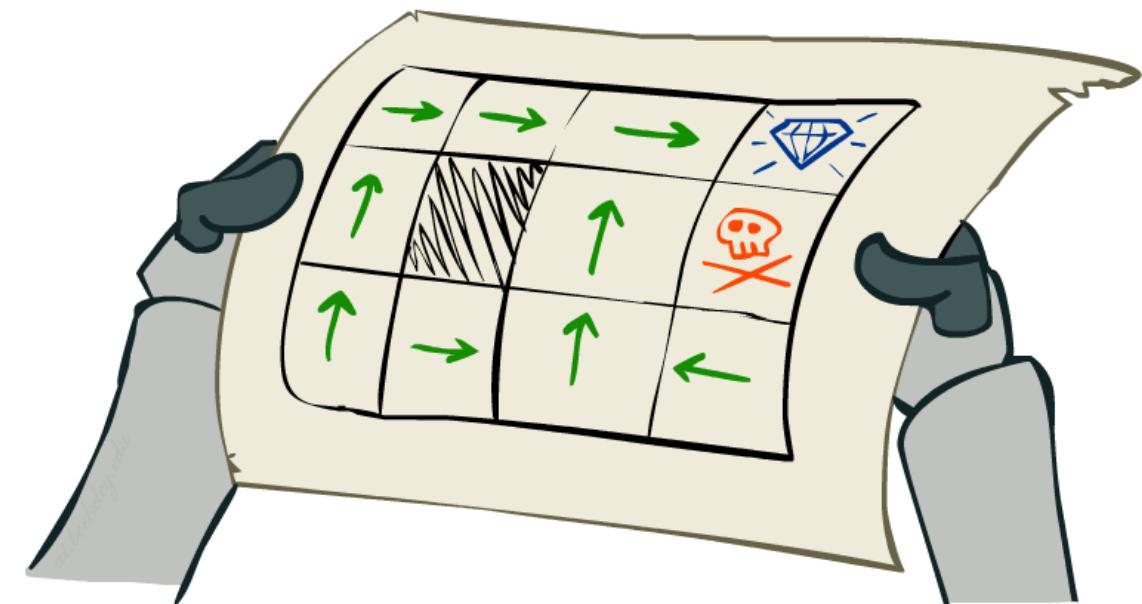
Passive Reinforcement Learning

- Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don't know the transitions $T(s,a,s')$
- You don't know the rewards $R(s,a,s')$
- Goal: learn the state values

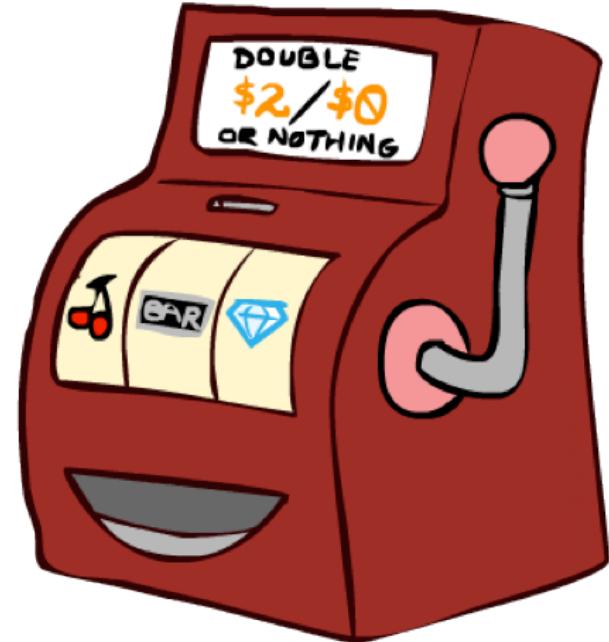
- In this case:

- Learner is “along for the ride”
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.



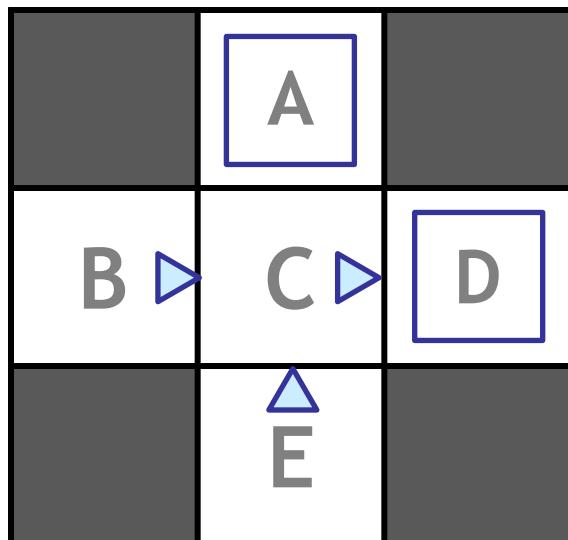
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
 - Act according to π
 - Every time you visit a state, write down what the sum of discounted rewards turned out to be
 - Average those samples
- This is called direct evaluation



Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

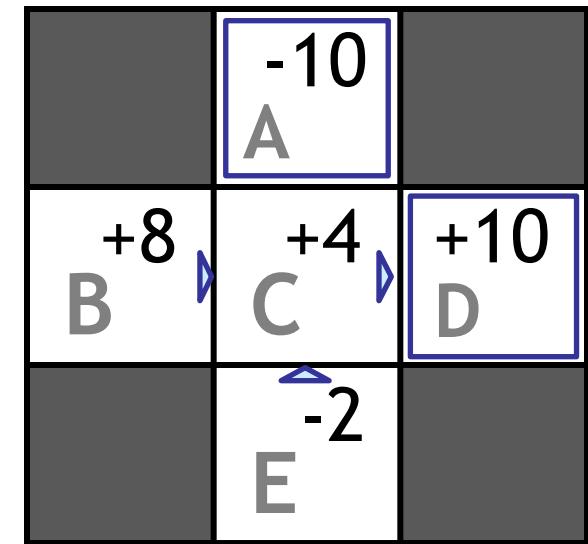
Output Values

	-10	
A	+4	+10
B	+8	
C		
D		
E	-2	

Problems with Direct Evaluation

- What's good about direct evaluation?
 - It's easy to understand
 - It doesn't require any knowledge of T, R
 - It eventually computes the correct average values, using just sample transitions
- What bad about it?
 - It wastes information about state connections
 - Each state must be learned separately
 - So, it takes a long time to learn

Output Values



If B and E both go to C under this policy, how can their values be different?

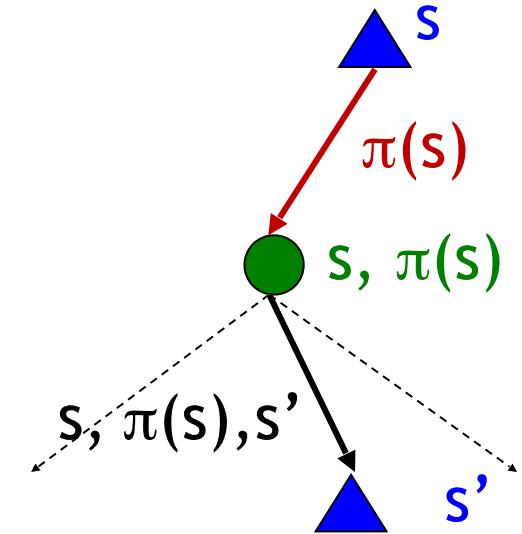
Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:

- Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
 - Unfortunately, we need T and R to do it!

- Key question: how can we do this update to V without knowing T and R ?
- In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

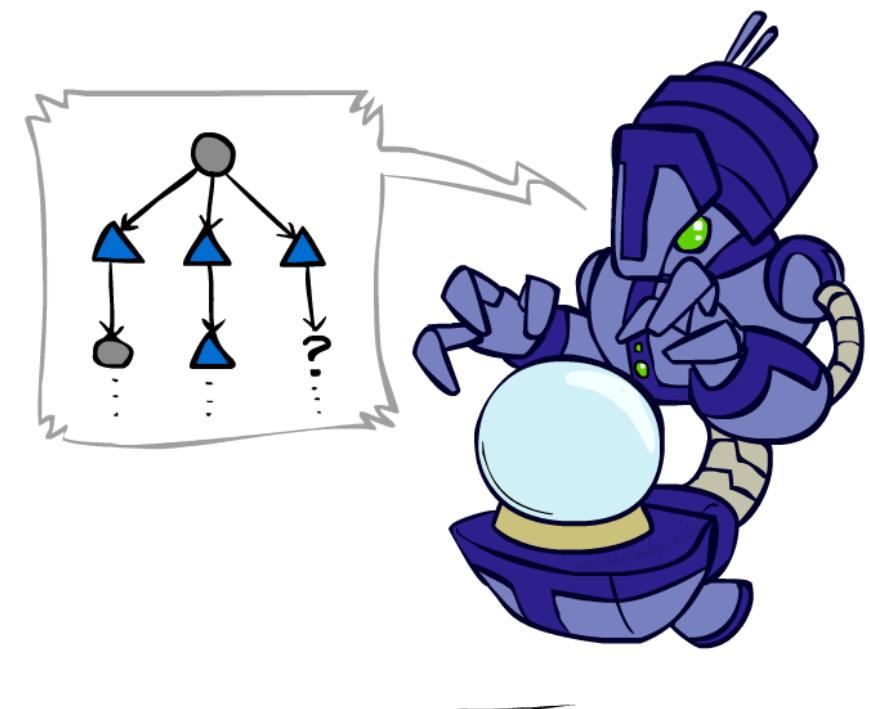
$$\text{sample}_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$\text{sample}_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

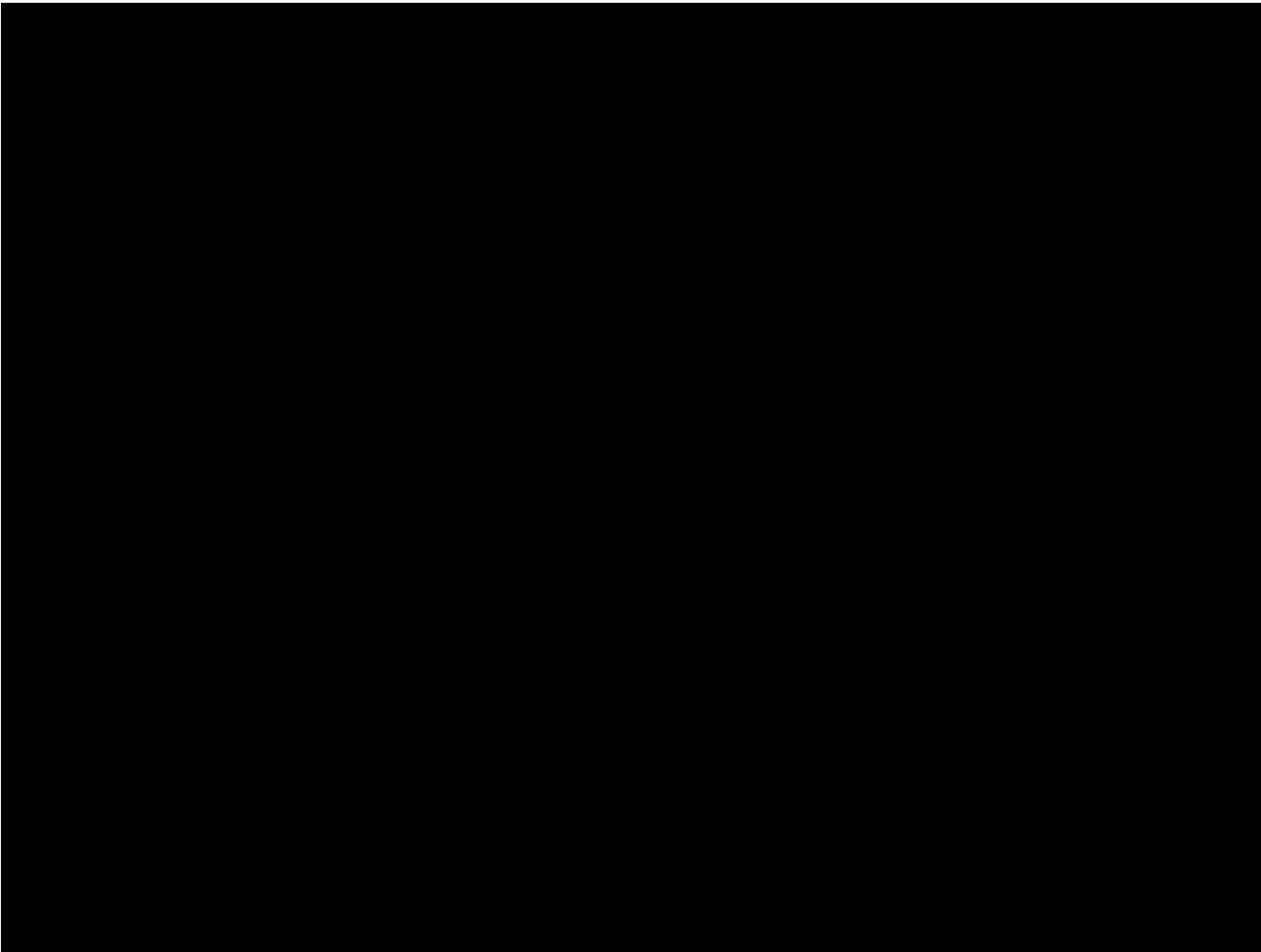
...

$$\text{sample}_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i \text{sample}_i$$



Crawler in Real-world



[Francis Waffels 2010] An example of Q-learning using 16 states (4 positions for each servo's) and 4 possible actions (moving 1 position up or down for on of the servos). The robot is build with lego components, two servo's, a wheel encoder and a dwengo-board with PIC18F4550 microcontroller. The robot learns using the Q-learning algorithm and its reward is related with the moved distance.

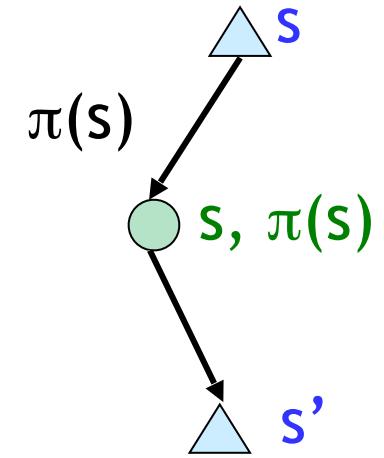
Temporal Difference Learning

- Big idea: learn from every experience!
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
 - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
 - Policy still fixed, still doing evaluation!
 - Move values toward value of whatever successor occurs: running average

Sample of $V(s)$: $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$



Exponential Moving Average

- Exponential moving average

- The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$

- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

	A	
B	C	D
	E	

Observed Transitions

$R(s, \pi(s), s')$

B, east, C, -2

C, east, D, -2

$V^\pi(s)$	0	0
0	0	8
	0	

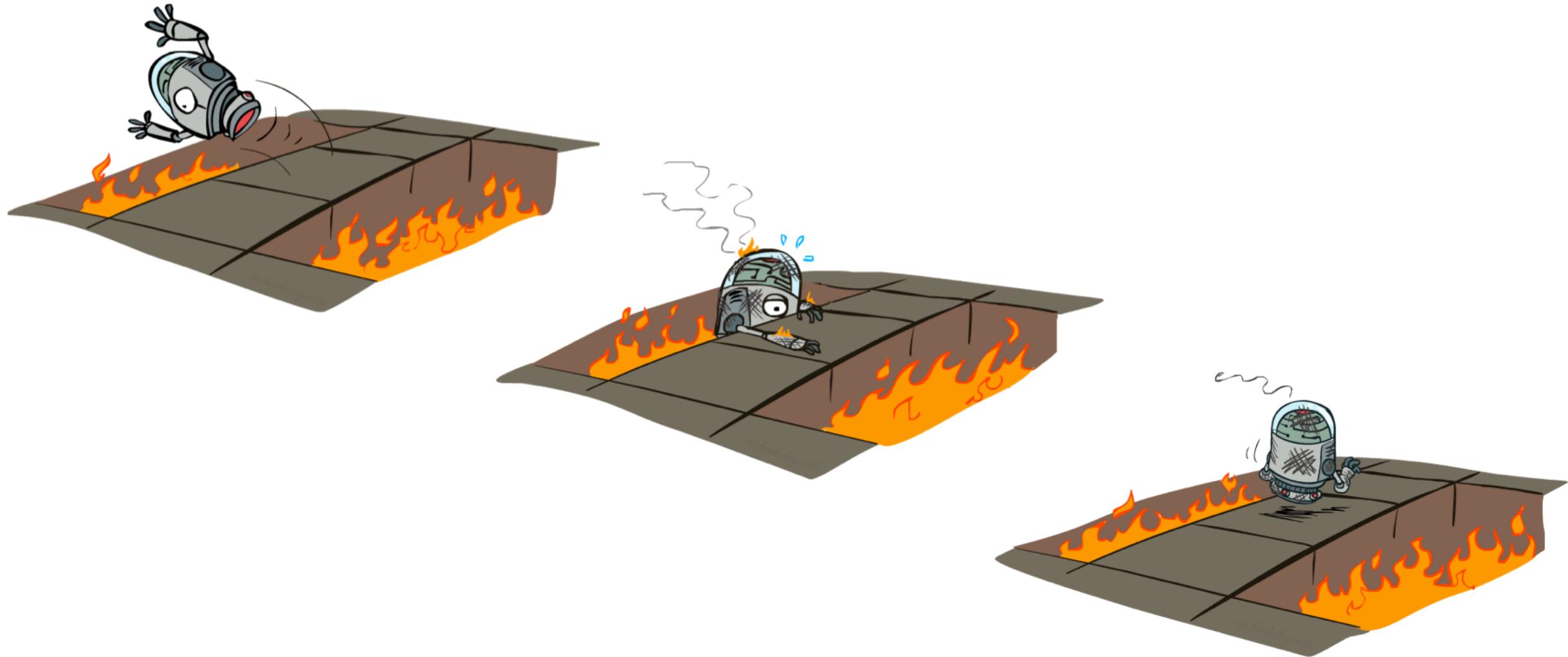
$V^\pi(s) \leftarrow$	-1	0
-1	0	8
	0	

-1	3	8
-1	3	8
0	0	

Assume: $\gamma = 1$,
 $\alpha = 1/2$

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

Active Reinforcement Learning



Passive vs. Active Learning

- **Passive Learning:**
 - The agent has a fixed policy and tried to learn the utilities of states by observing the world go by
 - Analogous to policy evaluation
 - Often serves as a component of active learning
 - Often inspires active learning algorithm
- **Active Learning:**
 - The agent attempts to find an optimal (or at least good) policy by acting in the world
 - Analogous to solving the underlying MDP, but without first being given the MDP model

Example: Sidewinding



Example: Ducklings



[Video: Ducklings vs. Stairs]

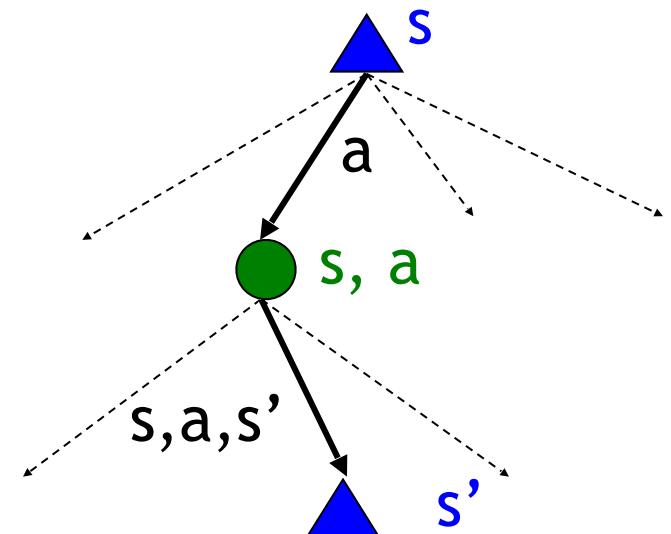
Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

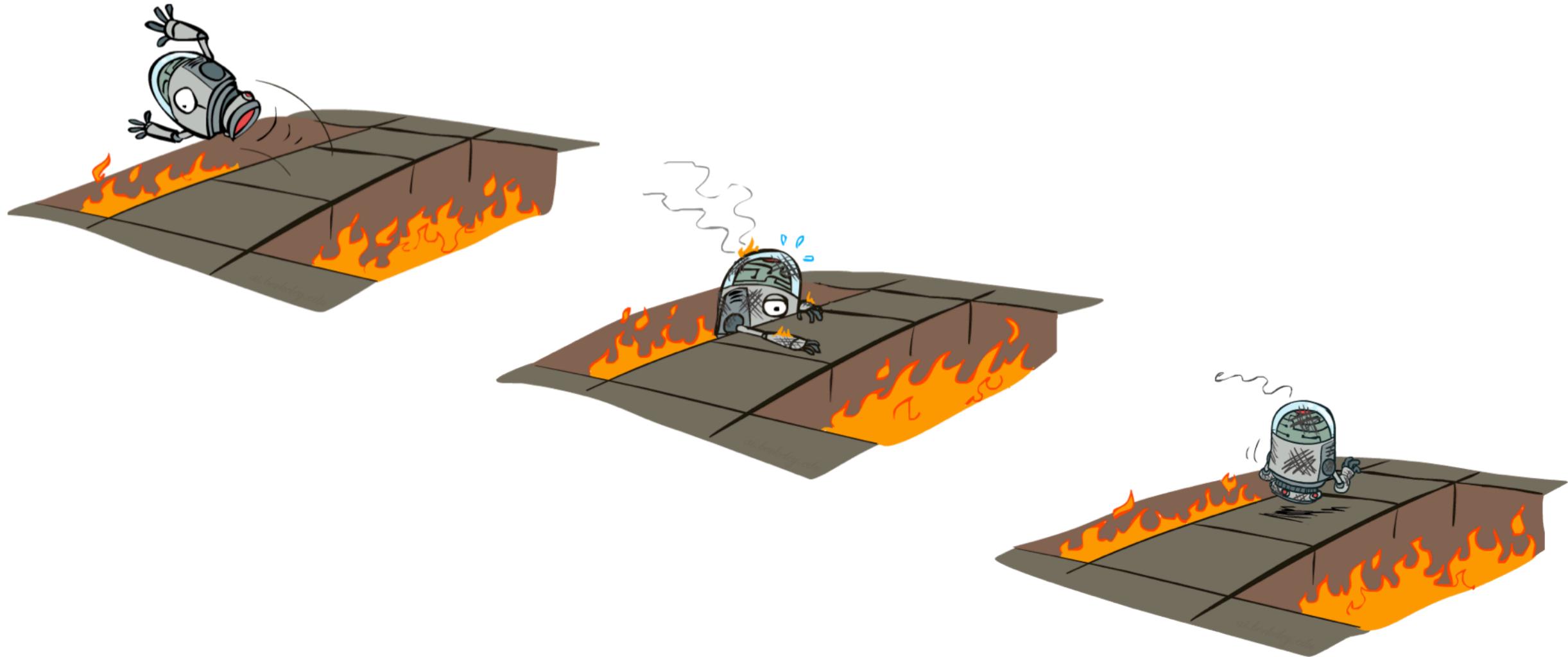
$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!

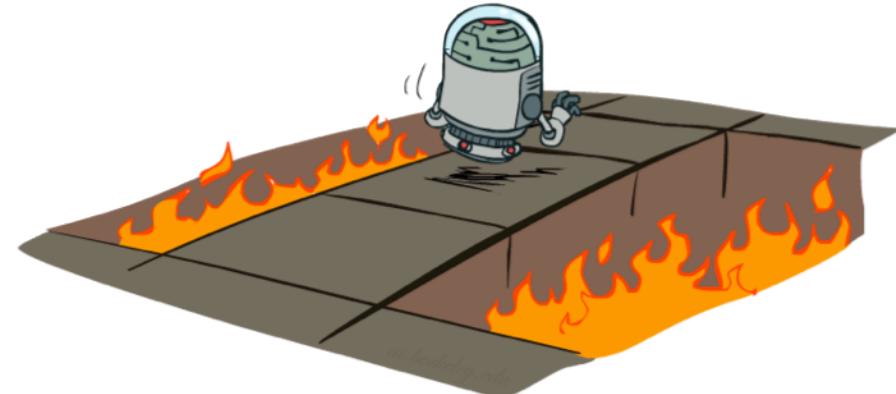


Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - You choose the actions now
 - Goal: learn the optimal policy / values
- In this case:
 - Learner makes choices!
 - Fundamental tradeoff: exploration vs. exploitation
 - This is NOT offline planning! You actually take actions in the world and find out what happens...



Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values

- Start with $V_0(s) = 0$, which we know is right

- Given V_k , calculate the depth $k+1$ values for all states:

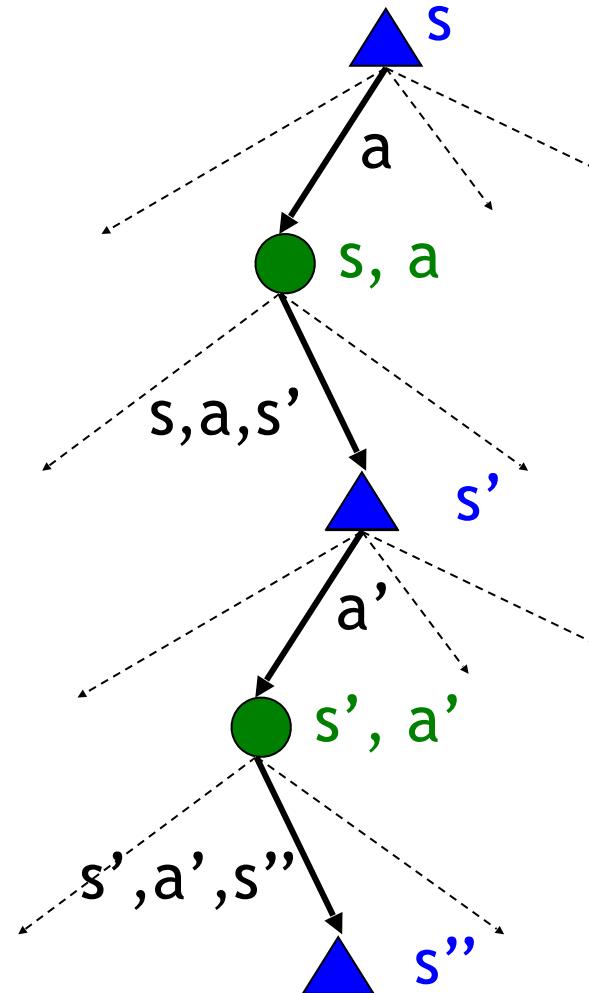
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

- But Q-values are more useful, so compute them instead

- Start with $Q_0(s, a) = 0$, which we know is right

- Given Q_k , calculate the depth $k+1$ q-values for all q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma \max_{a'} Q_k(s', a')]$$



Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

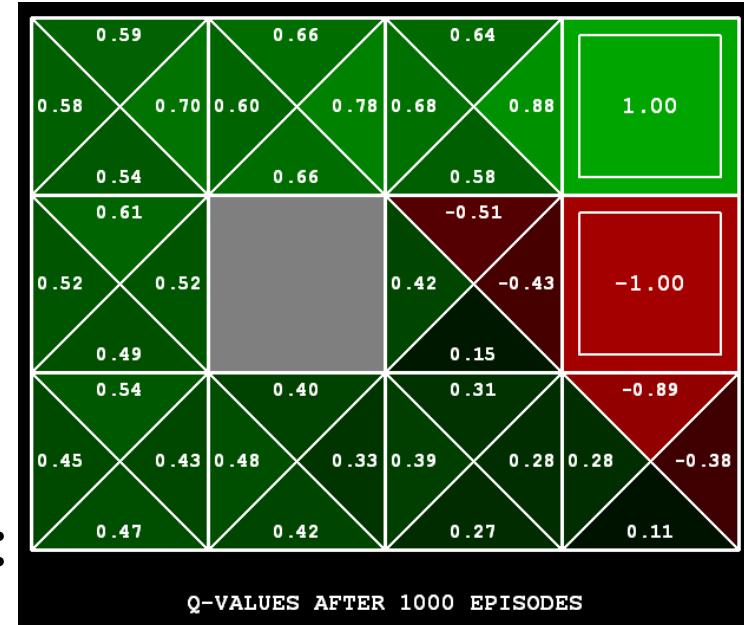
- Learn $Q(s, a)$ values as you go

- Receive a sample (s, a, s', r)
- Consider your old estimate: $Q(s, a)$
- Consider your new sample estimate:

$$\text{sample} = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [\text{sample}]$$



[Demo: Q-learning - gridworld (L10D2)]

Video of Demo Q-Learning -- Gridworld

