Neural Network Language Modeling

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CSE 5525

Many slides from Marek Rei, Philipp Koehn and Noah Smith

Language Modeling (Recap)

- Goal:
  - calculate the probability of a sentence
  - calculate probability of a word in the sentence

N-gram Language Modeling (Recap)

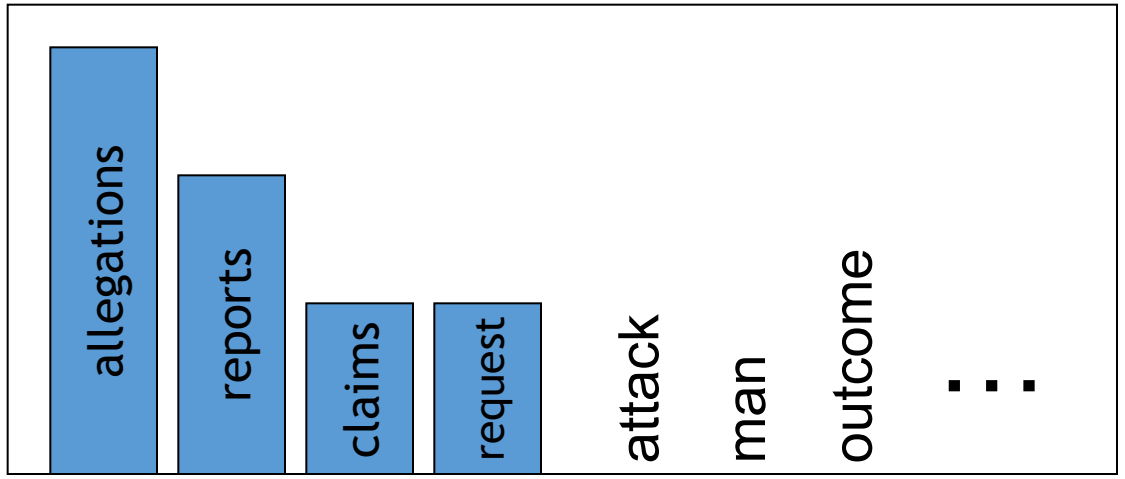
$$P(w_1\ w_2\ \ldots\ w_N) = \prod_{i=1}^{N} P(w_i|w_{i-1})$$

$$P(w_i|w_{i-1}) = \frac{C(w_{i-1}\ w_i)}{C(w_{i-1})}$$

Zero Probabilities (Recap)

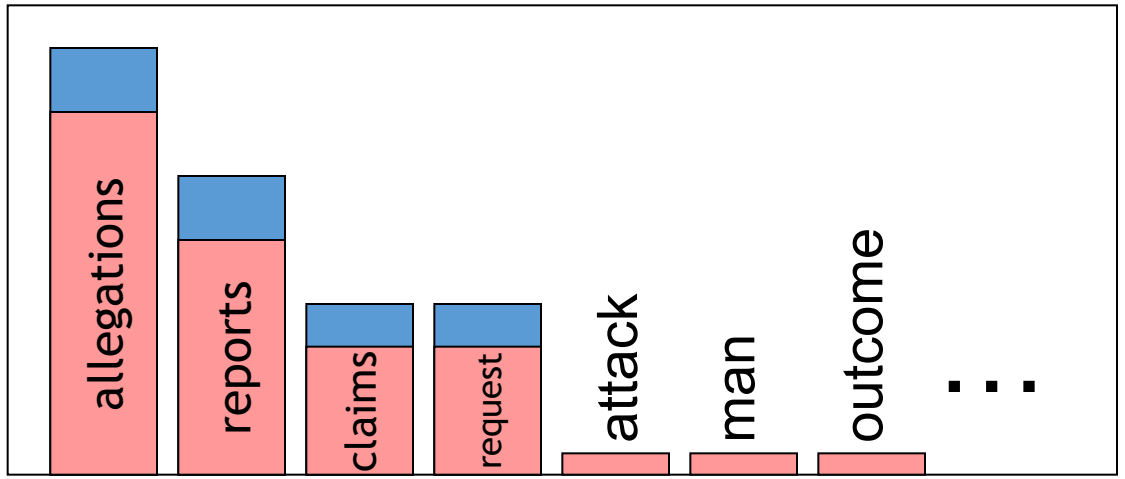
- When we have sparse statistics:

  P(w | denied the)
  3 allegations
  2 reports
  1 claims
  1 request
  7 total



- Steal probability mass to generalize better

  P(w | denied the)
  2.5 allegations
  1.5 reports
  0.5 claims
  0.5 request
  2 other
  7 total



Smoothing (Recap)

- Backoff
- Interpolation

$$P_{interp}(w_i|w_{i-2}\ w_{i-1}) = \lambda_1 P(w_i|w_{i-2}\ w_{i-1}) + \lambda_2 P(w_i|w_{i-1}) + \lambda_3 P(w_i)$$

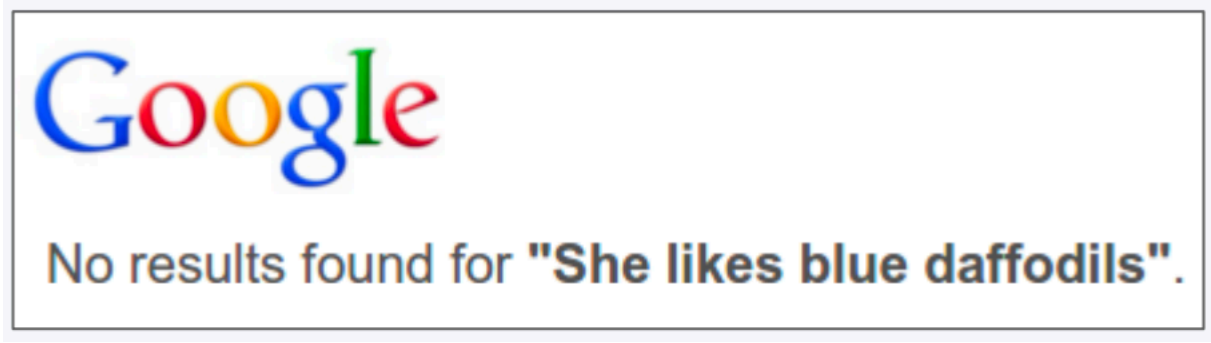
- Kneser-Ney Smoothing

$$P_{KN}(w_i|w_{i-1}) = \frac{max(C(w_{i-1}\ w_i) - D, 0)}{C(w_{i-1})} + \lambda(w_{i-1}) P_{continuation}(w_i)$$

Problems with N-grams

- Problem 1: N-grams are sparse.

  - There are V^4 possible 4-grams. With V=10,000, that's 10^{16} 4-grams.
  - We will only see a tiny fraction of them in our training data.



Problems with N-grams

- Problem 2: Words are independent.

  - It only map together identical words, but ignore similar or related words.
  - If we have seen “yellow daffodil” in the data, we could use the intuition that “blue” is related to “yellow” to handle “blue daffodils”.

Vector Representation

- Let’s represent words (or any objects) as vectors.
- Let’s choose them, so that similar words have similar vectors.

One-hot Word Vectors

- Each element represents the word.

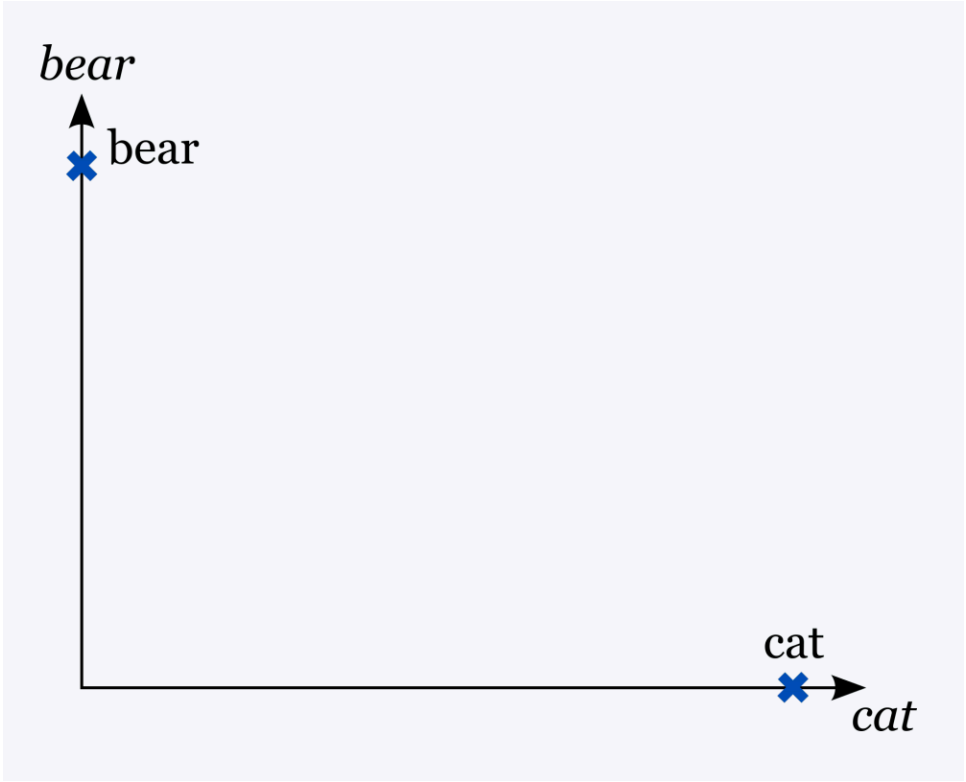
	bear	*cat*	*frog*
bear	1	0	0
cat	0	1	0
frog	0	0	1

bear=[1.0, 0.0, 0.0]

cat=[0.0, 1.0, 0.0]

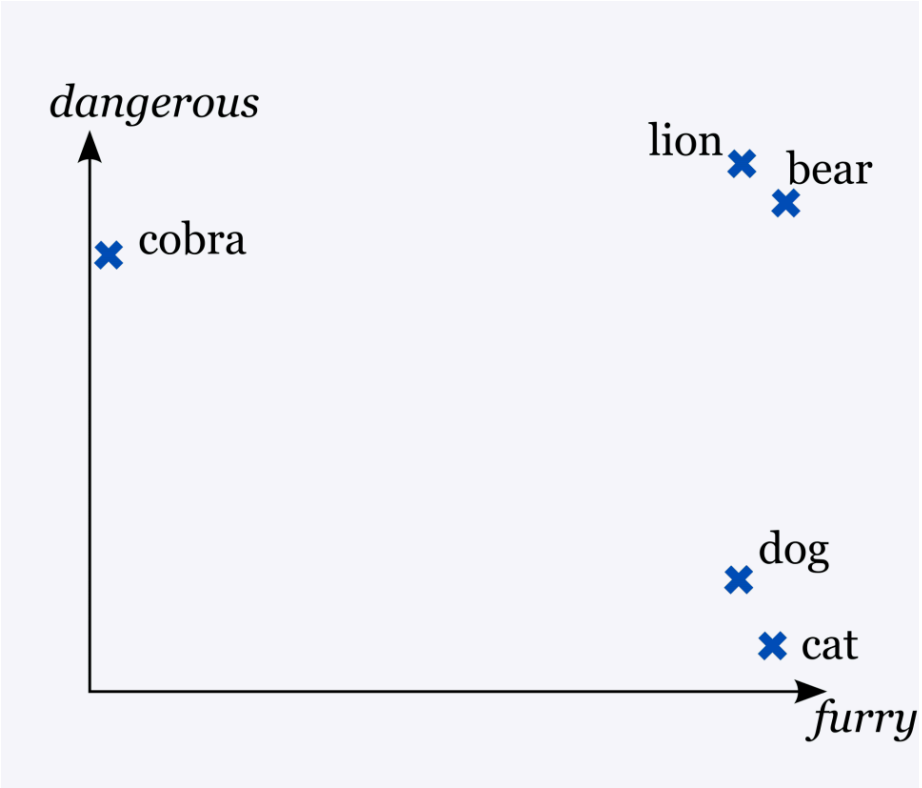
One-hot Word Vectors

- That’s a very large vector!
- They tell us very little.



Distributed Vectors (Representations)

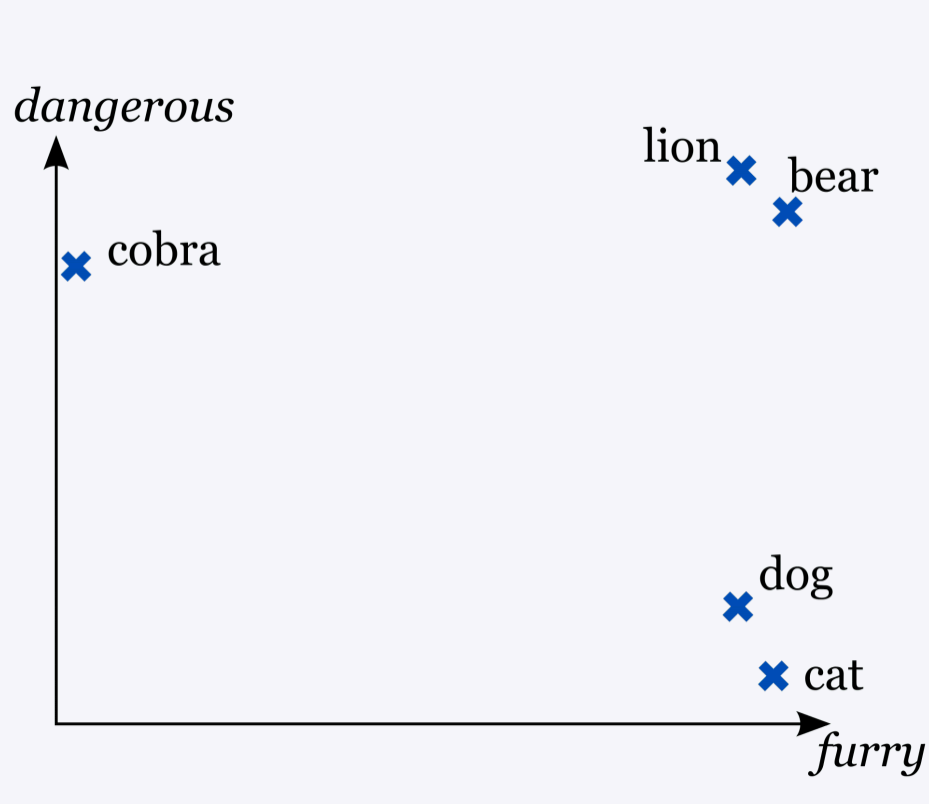
- Each element represents a property, and they are shared between the words.



	furry	*dangerous*
bear	0.9	0.85
cat	0.85	0.15
cobra	0.0	0.8
lion	0.85	0.9
dog	0.8	0.15

Distributed Vectors

- Groups similar words/objects together.



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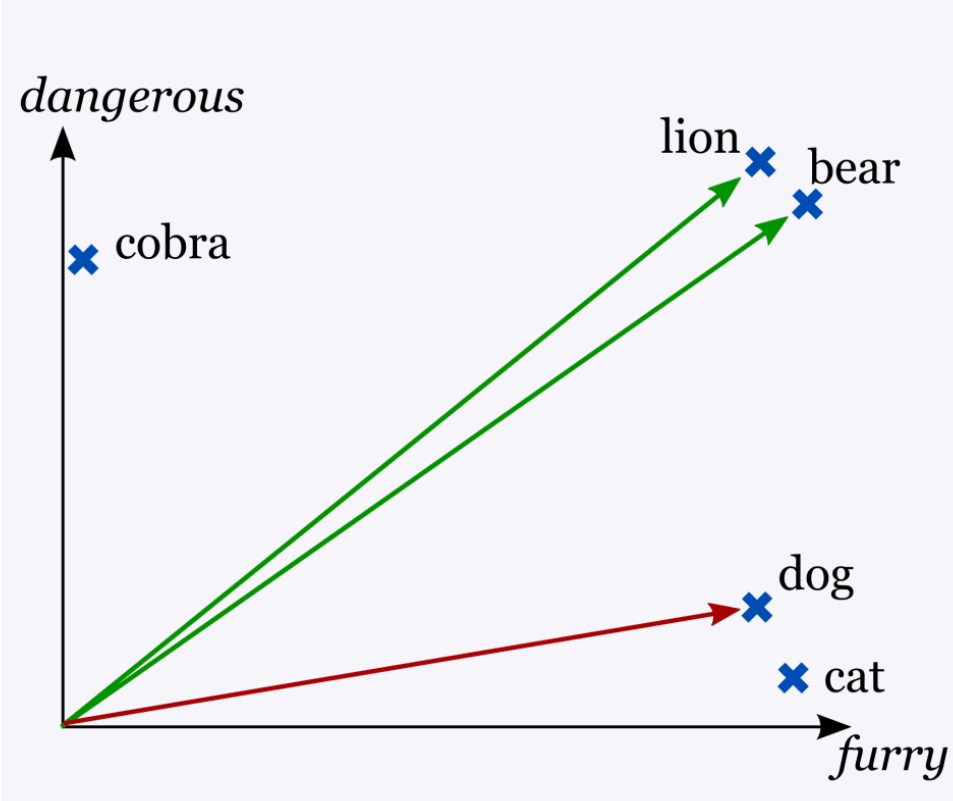
Distributed Vectors

- Use cosine similarity to calculate similarity between two words



$$cos(a,b) = \frac{\sum_i a_i \cdot b_i}{\sqrt{\sum_i a_i^2} \cdot \sqrt{\sum_i b_i^2}}$$

cos(lion, bear) = 0.998
cos(lion, dog) = 0.809
cos(cobra, dog) = 0.727

Distributed Vectors

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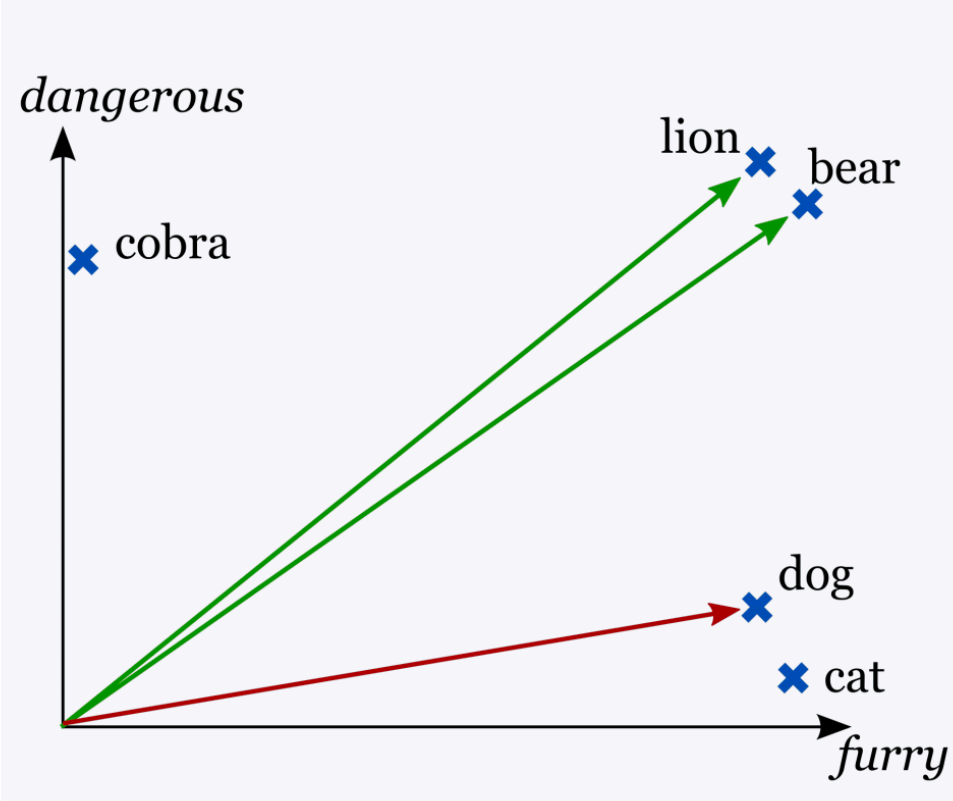

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cos(lion, bear) = 0.998
cos(lion, dog) = 0.809
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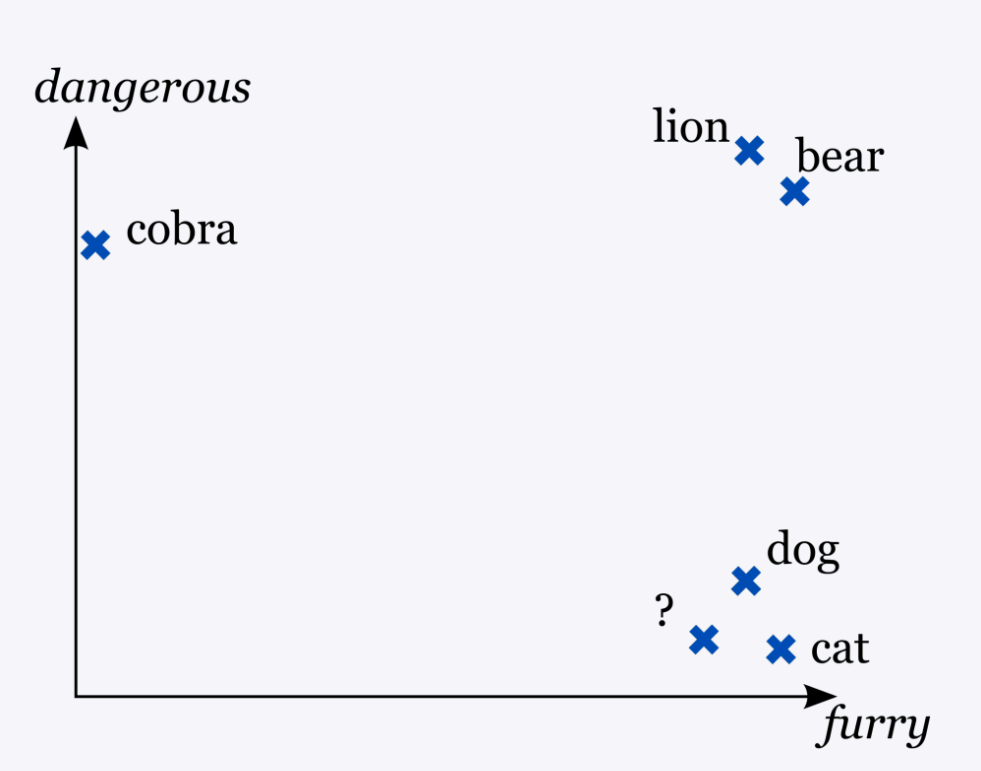
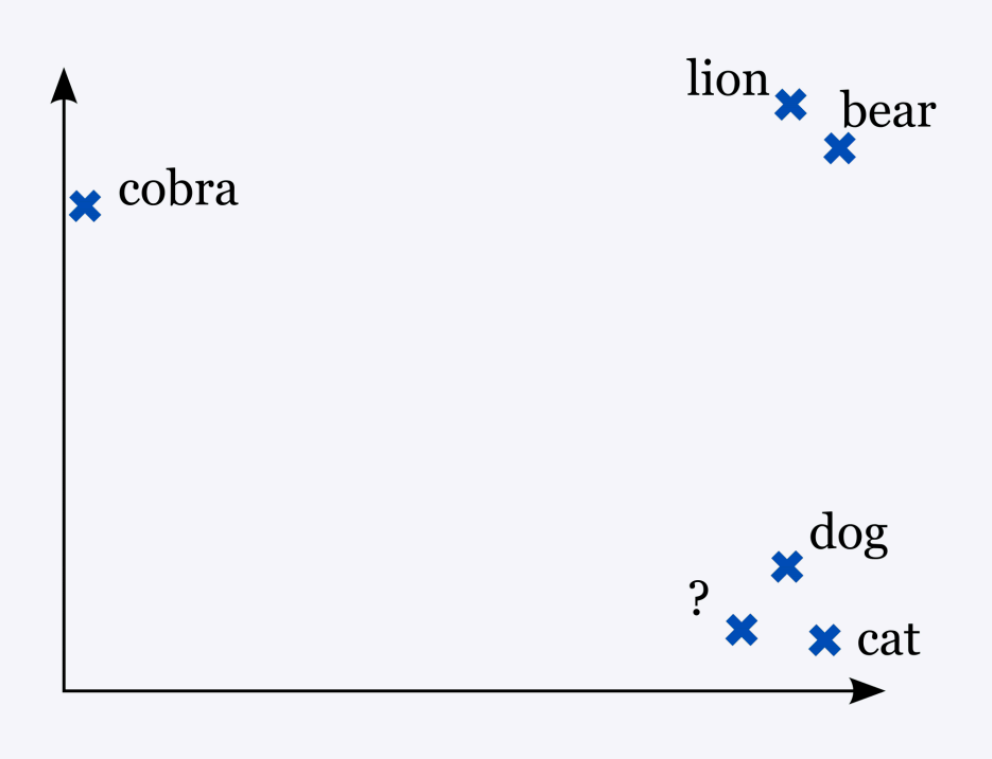
Distributed Vectors

- We can infer some information based only on the vector of the word.



Distributed Vectors

- We don’t even need to know the labels on the vector elements.



Distributed Vectors

- The vectors are usually not 2 or 3-dimensional. More often 100-1000 dimensions.

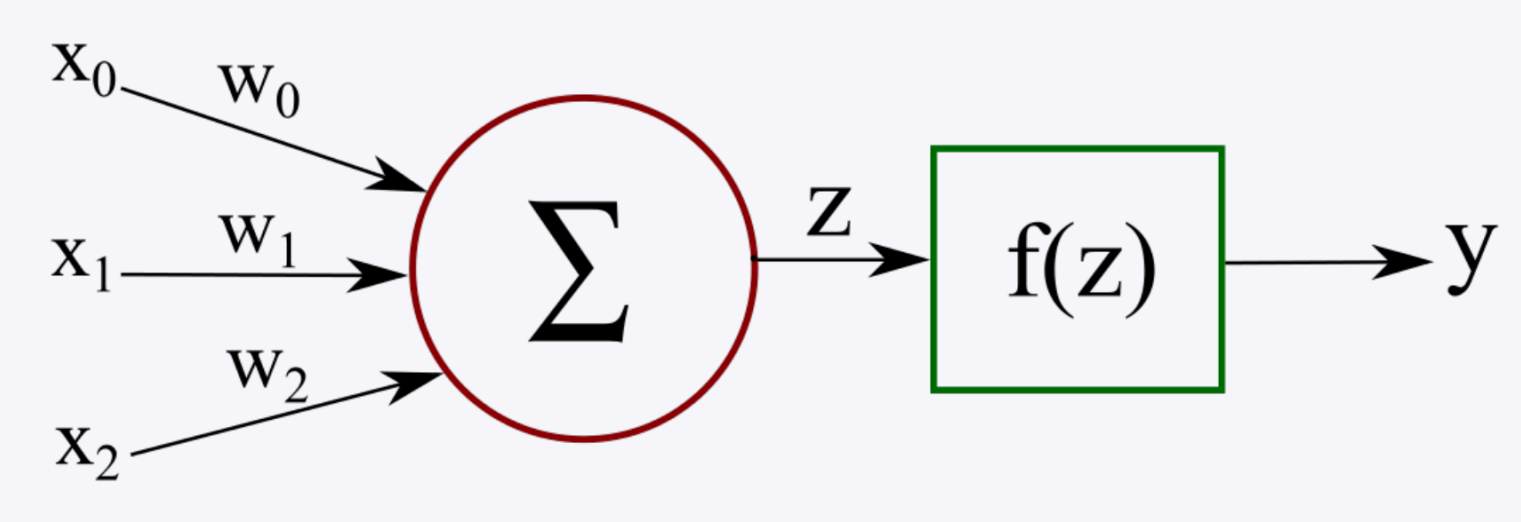
bear -0.089383 -0.375981 -0.337130 0.025117 -0.232542 -0.224786 0.148717 -0.154768 -0.260046
-0.156737 -0.085468 0.180366 -0.076509 0.173228 0.231817 0.314453 -0.253200 0.170015
-0.111660 0.377551 -0.025207 -0.097520 -0.020041 0.117727 0.105745 -0.352382 0.010241
0.114237 -0.315126 0.196771 -0.116824 -0.091064 -0.291241 -0.098721 0.297539 0.213323
-0.158814 -0.157823 0.152232 0.259710 0.335267 0.195840 -0.118898 0.169420 -0.201631
0.157561 0.351295 0.033166 0.003641 -0.046121 0.084251 0.021727 -0.065358 -0.083110
-0.265997 0.027450 0.372135 0.040659 0.202577 -0.109373 0.183473 -0.380250 0.048979
0.071580 0.152277 0.298003 0.017217 0.072242 0.541714 -0.110148 0.266429 0.270824 0.046859
0.150756 -0.137924 -0.099963 -0.097112 -0.110336 -0.018136 -0.032682 0.182723 0.260882
-0.146807 0.502611 0.034849 -0.092219 -0.103714 -0.034353 0.112178 0.065348 0.161681
0.006538 0.364870 0.153239 -0.366863 -0.149125 0.413624 -0.229378 -0.396910 -0.023116

Neural Network Language Models

- Represent each word as a vector, and similar words with similar vectors.

- Idea:
  - similar contexts have similar words
  - so we define a model that aims to predict between a word w_t and context words: $P(w_t|context)$ or $P(context|w_t)$
  - *Optimize the vectors together with the model, so we end up with vectors that perform well for language modeling (aka representation learning)*

Artificial Neuron

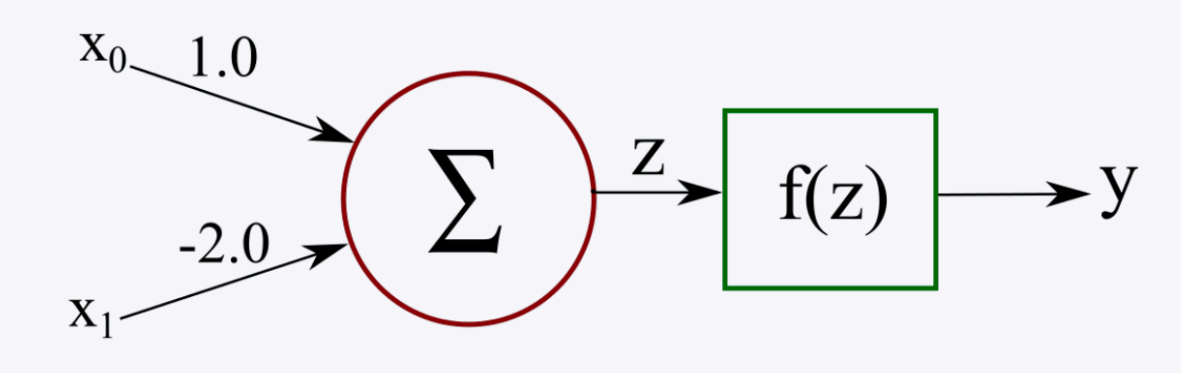


Input: $[x_0, x_1, x_2]$

Output: y

Perceptrons = Single-Layer Neural Networks !

Artificial Neuron



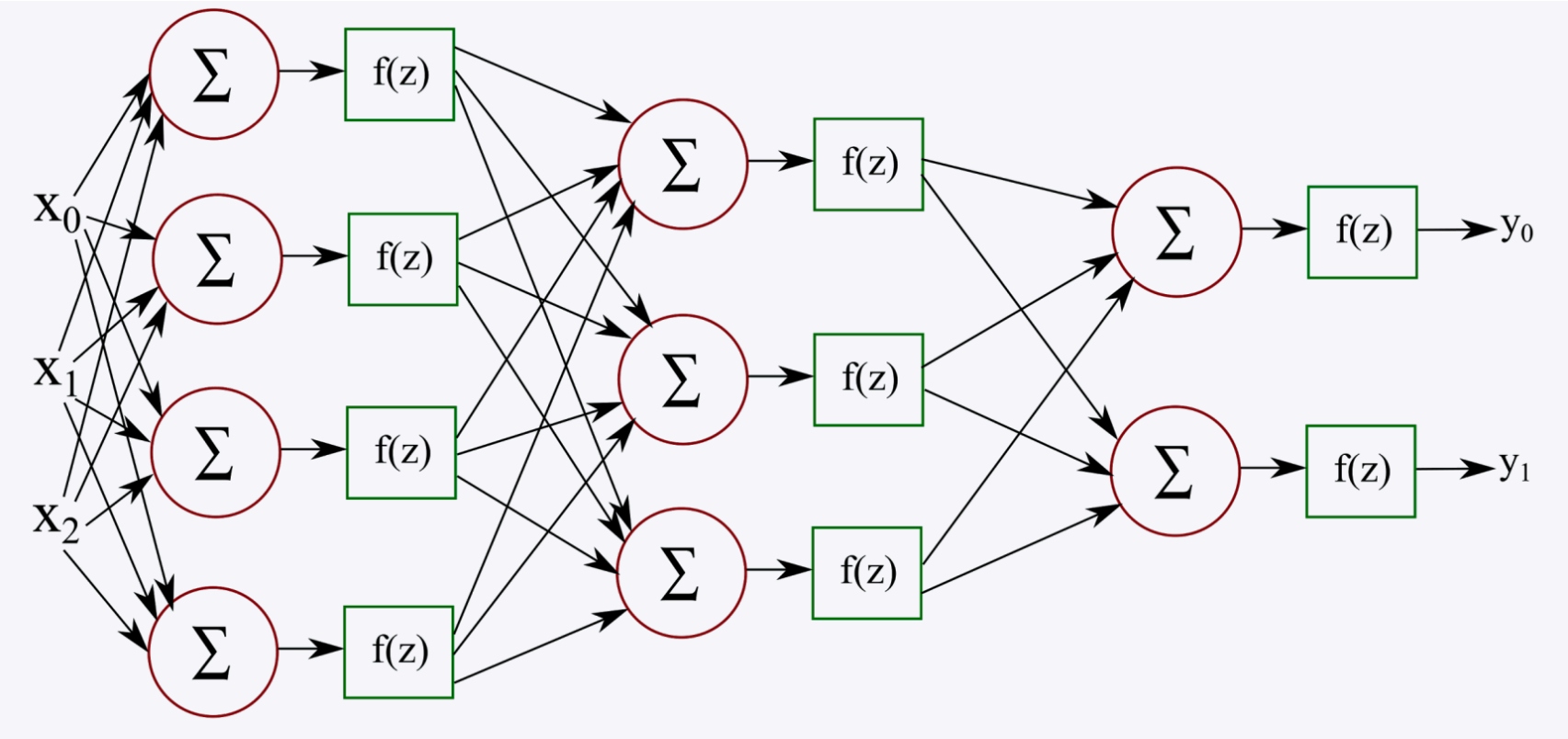
$$z = \sum_i x_i w_i$$

$$y = \frac{1}{1 + exp(-z)}$$

	x0	*x1*	*z*	*y*
bear	0.9	0.85	-0.8	0.31
cat	0.85	0.15	0.55	0.63
cobra	0.0	0.8	-1.6	0.17
lion	0.85	0.9	-0.95	0.28
dog	0.8	0.15	0.5	0.62

Neural Network

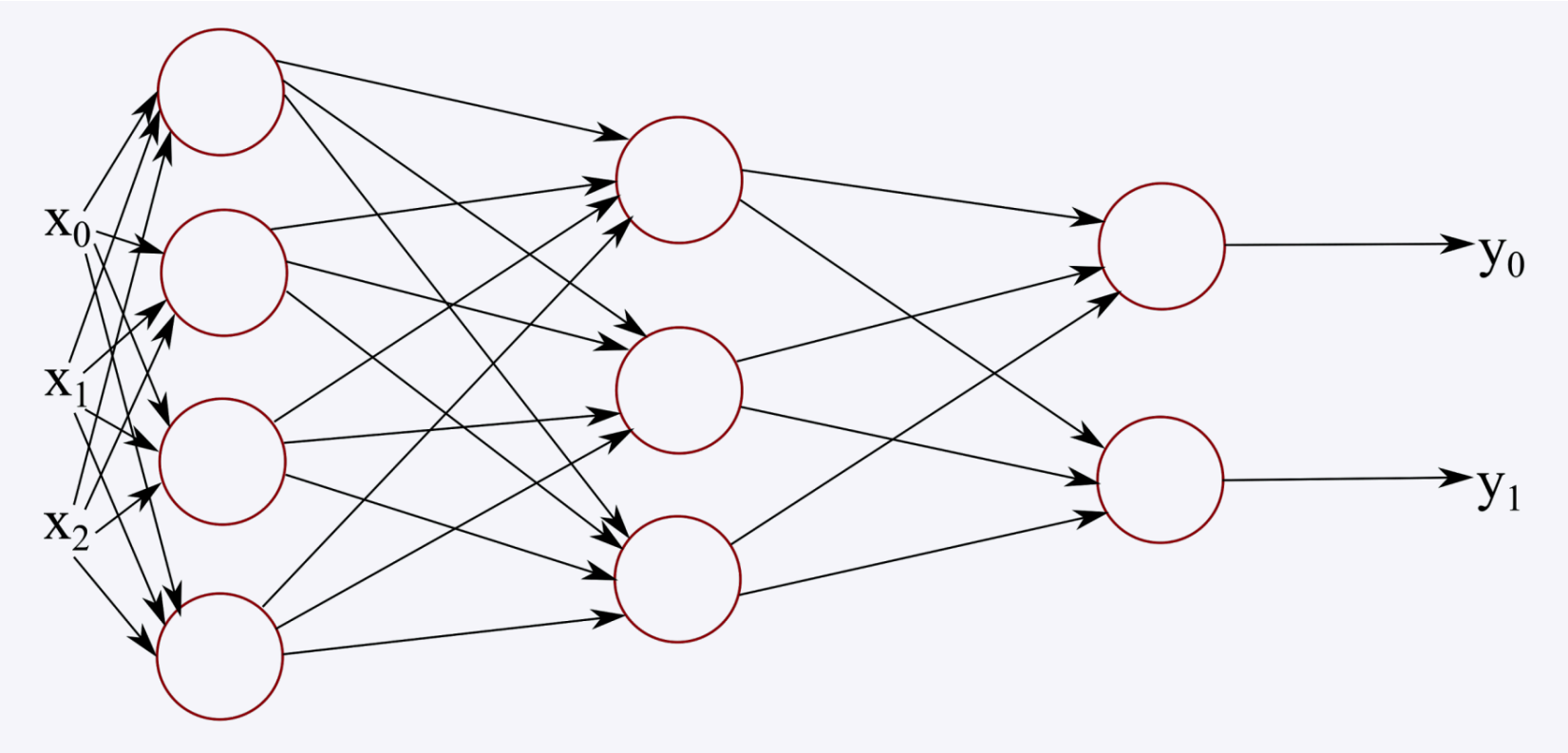
- Many neurons connected together



Multi-Layer Feed-Forward Networks!

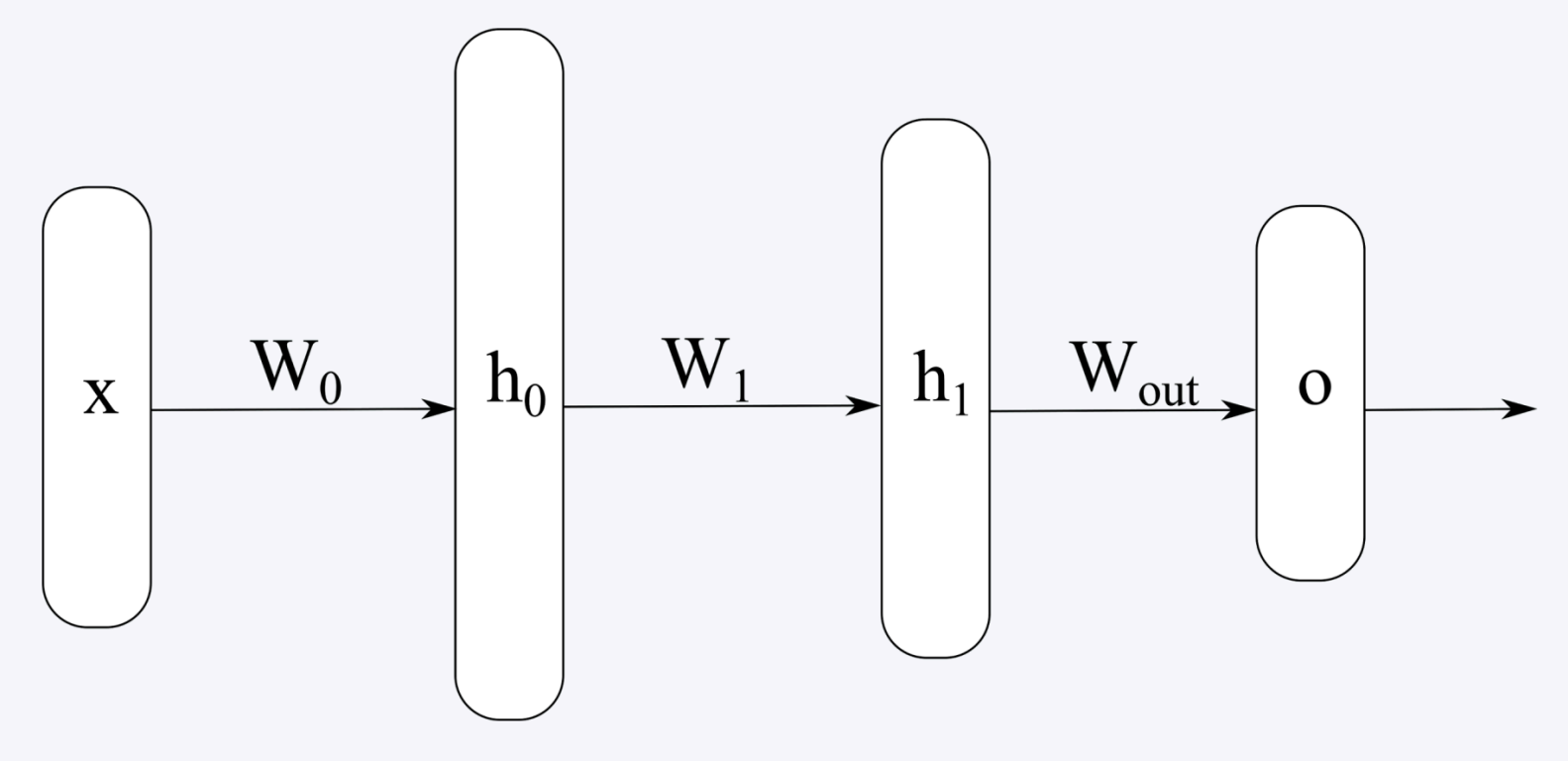
Neural Network

- Usually, a neuron is shown as a single unit

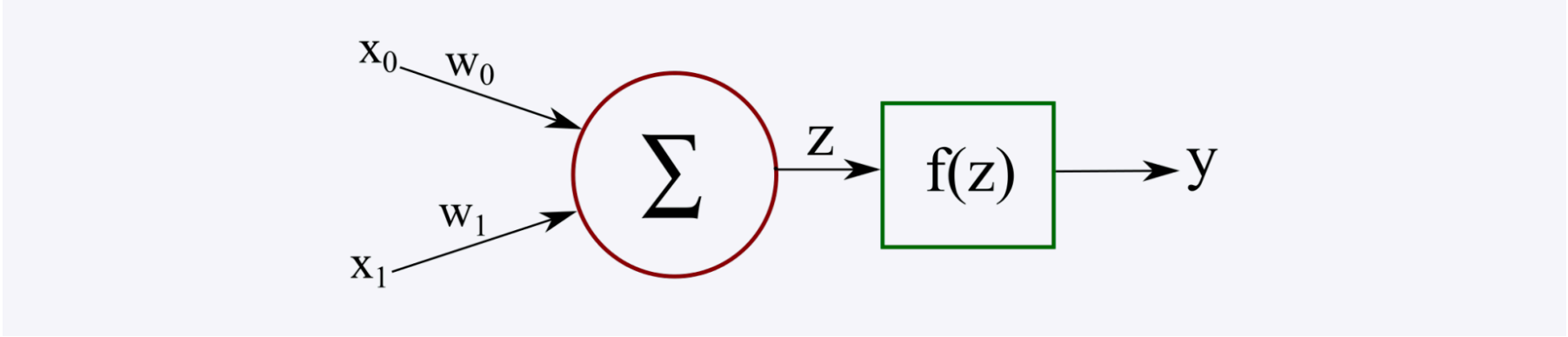


Neural Network

- Or a whole layer of neurons is represented as a block



Neuron Activation w/ Vectors



$$z = \sum_i x_i w_i$$

$$y = \frac{1}{1 + exp(-z)}$$

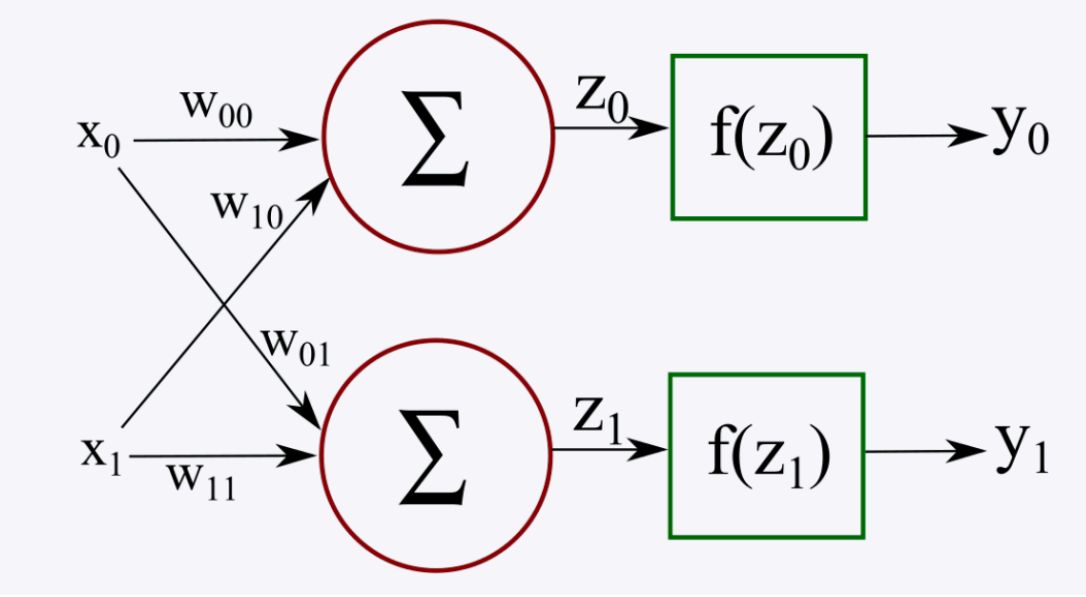
$$x = \begin{vmatrix} x_0 \\ x_1 \end{vmatrix} \qquad W = \begin{vmatrix} w_0 & w_1 \end{vmatrix}$$

$$z = W \cdot x \qquad f(z) = \frac{1}{1 + e^{-z}}$$

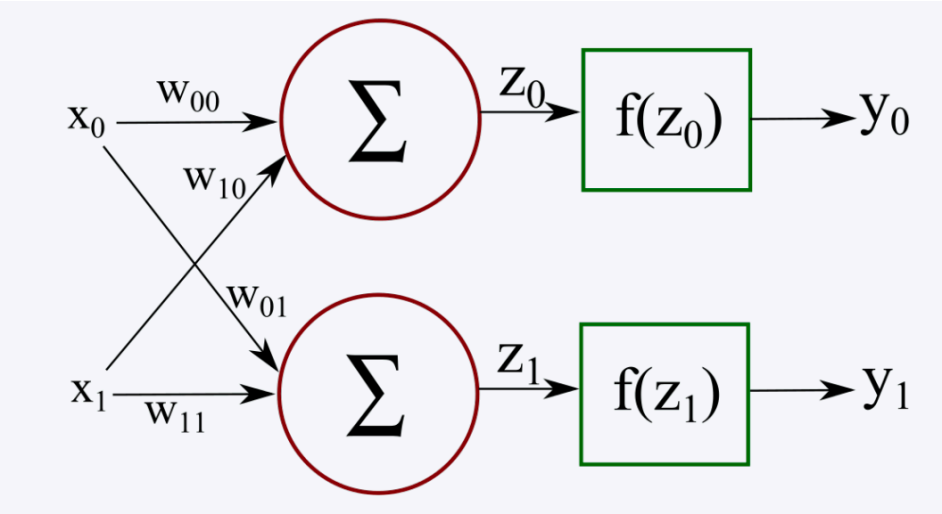
$$y = f(z) = f(W \cdot x)$$

Feedforward Activation w/ Matrices

- The same process applies when activating multiple neurons
- Now the weights are in a matrix as opposed to a vector
- Activation $f(z)$ is applied to each neuron separately



Feedforward Activation w/ Matrices



$$x = \begin{vmatrix} x_0 \\ x_1 \end{vmatrix}$$

$$W = \begin{vmatrix} w_{00} & w_{10} \\ w_{01} & w_{11} \end{vmatrix}$$

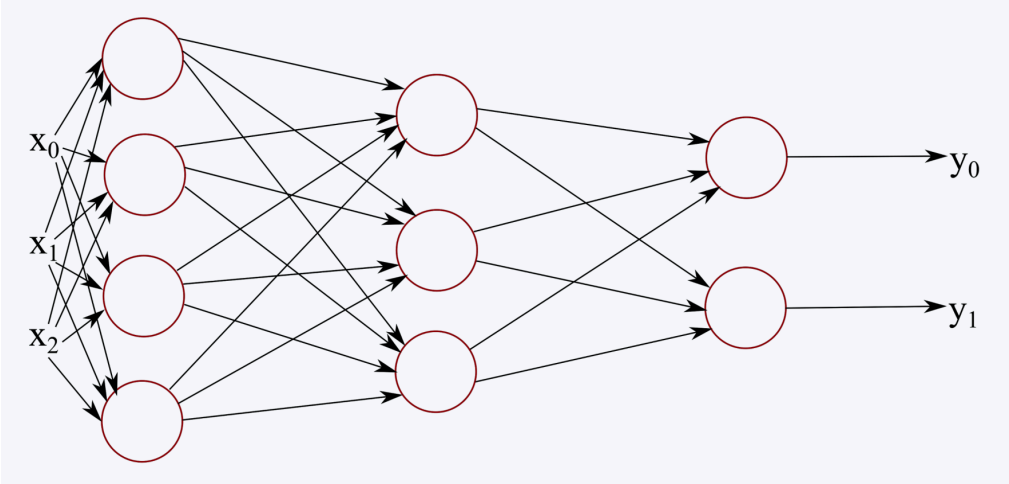
$$z = W \cdot x = \begin{vmatrix} w_{00} & w_{10} \\ w_{01} & w_{11} \end{vmatrix} \cdot \begin{vmatrix} x_0 \\ x_1 \end{vmatrix}$$

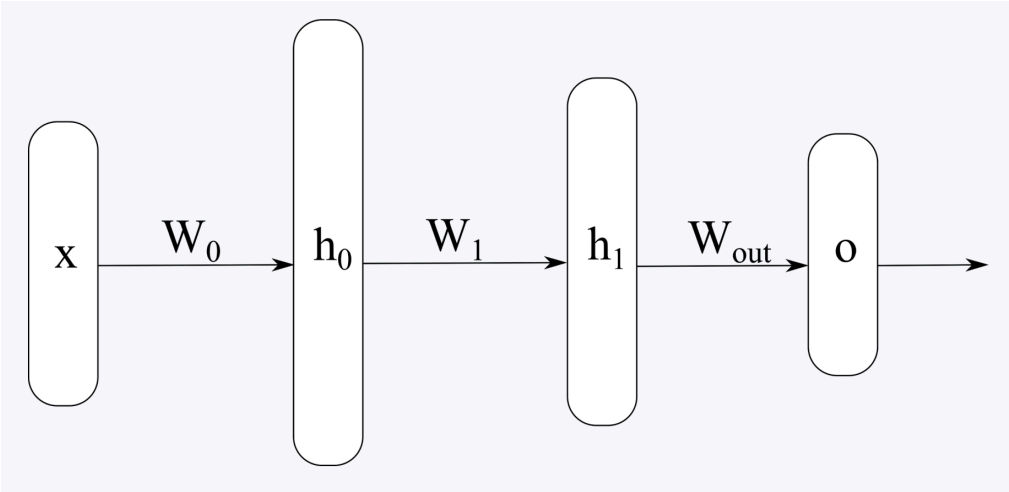
$$= \begin{vmatrix} w_{00} \cdot x_0 + w_{10} \cdot x_1 \\ w_{01} \cdot x_0 + w_{11} \cdot x_1 \end{vmatrix} = \begin{vmatrix} z_0 \\ z_1 \end{vmatrix}$$

$$f(z) = \frac{1}{1 + e^{-z}}$$

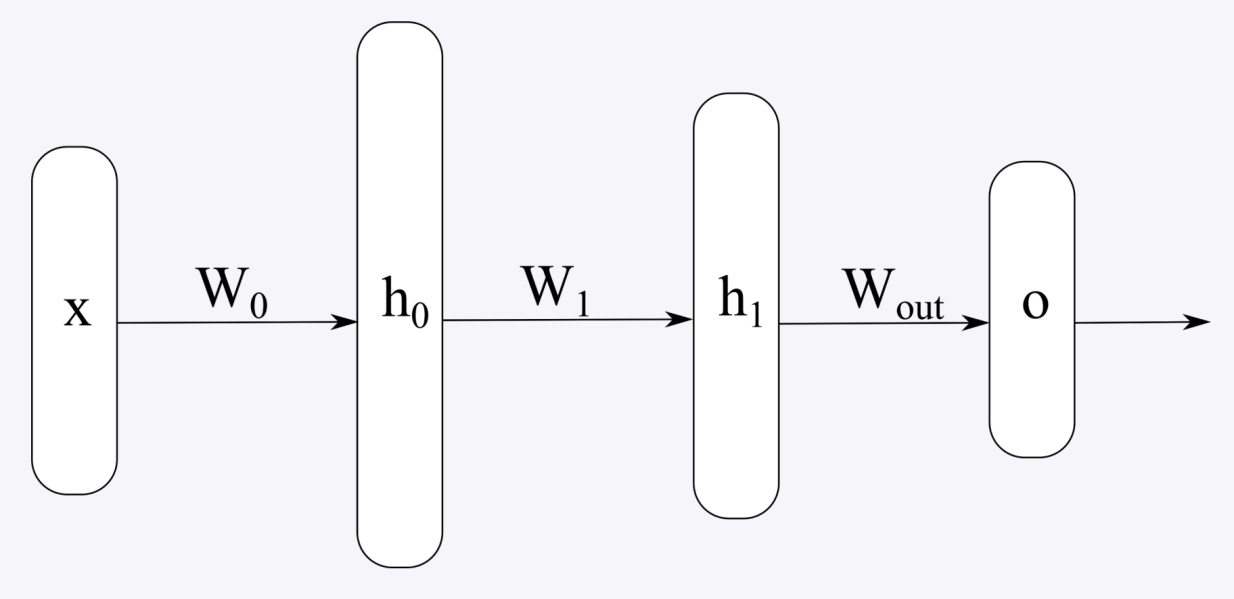
$$y = f(z) = f(W \cdot x) = \begin{vmatrix} f(z_0) \\ f(z_1) \end{vmatrix} = \begin{vmatrix} y_0 \\ y_1 \end{vmatrix}$$

Neural Network



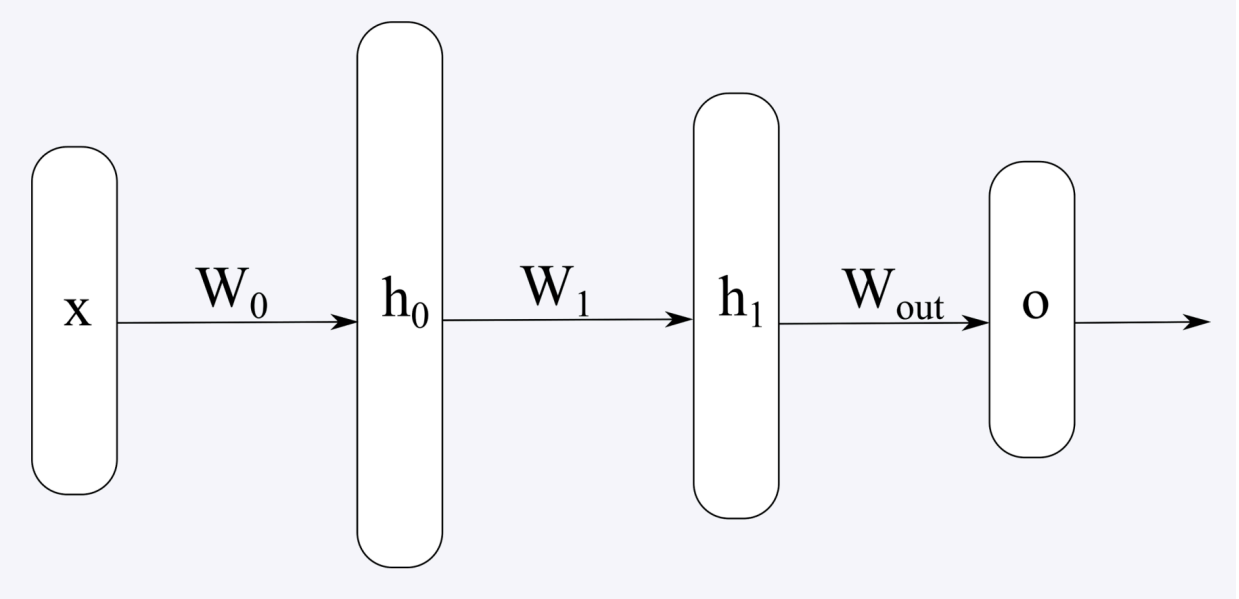


Feedforward Activation



1) Take vector from the previous layer
2) Multiply it with the weight matrix
3) Apply the activation function
4) Repeat

Feedforward Activation



$$h_0 = f(W_0 \cdot x) \qquad h_1 = f(W_1 \cdot x)$$

$$o = f(W_{out} \cdot h_1)$$

$$o = f(W_{out} \cdot f(W_1 \cdot f(W_0 \cdot x)))$$

Initialization of Weights

- Weights are initialized randomly
  e.g., uniformly from interval $[-0.01, 0.01]$

- Glorot and Bengio (2010) suggest
  - for shallow neural networks

$$\left[-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right]$$

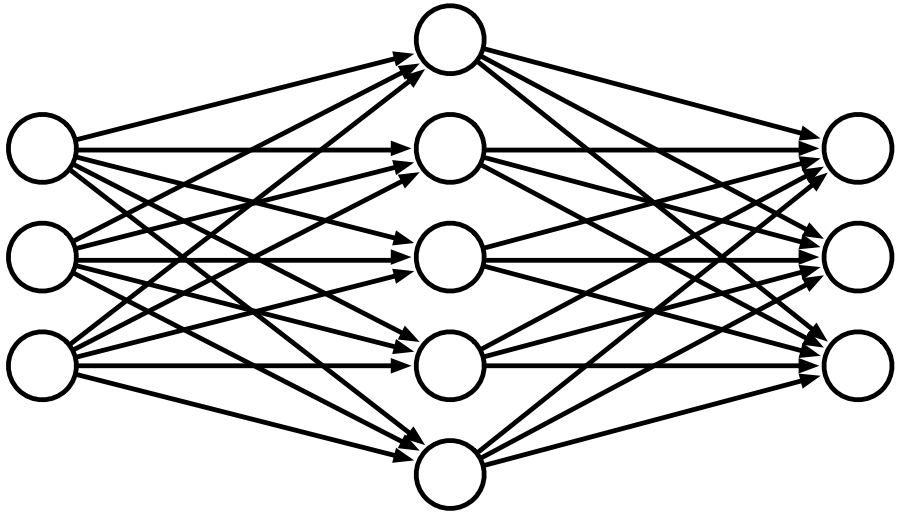
    n is the size of the previous layer

  - for deep neural networks

$$\left[-\frac{\sqrt{6}}{\sqrt{n_j + n_{j+1}}}, \frac{\sqrt{6}}{\sqrt{n_j + n_{j+1}}}\right]$$

    n_j is the size of the previous layer, n_j size of next layer

Neural Networks for Classification

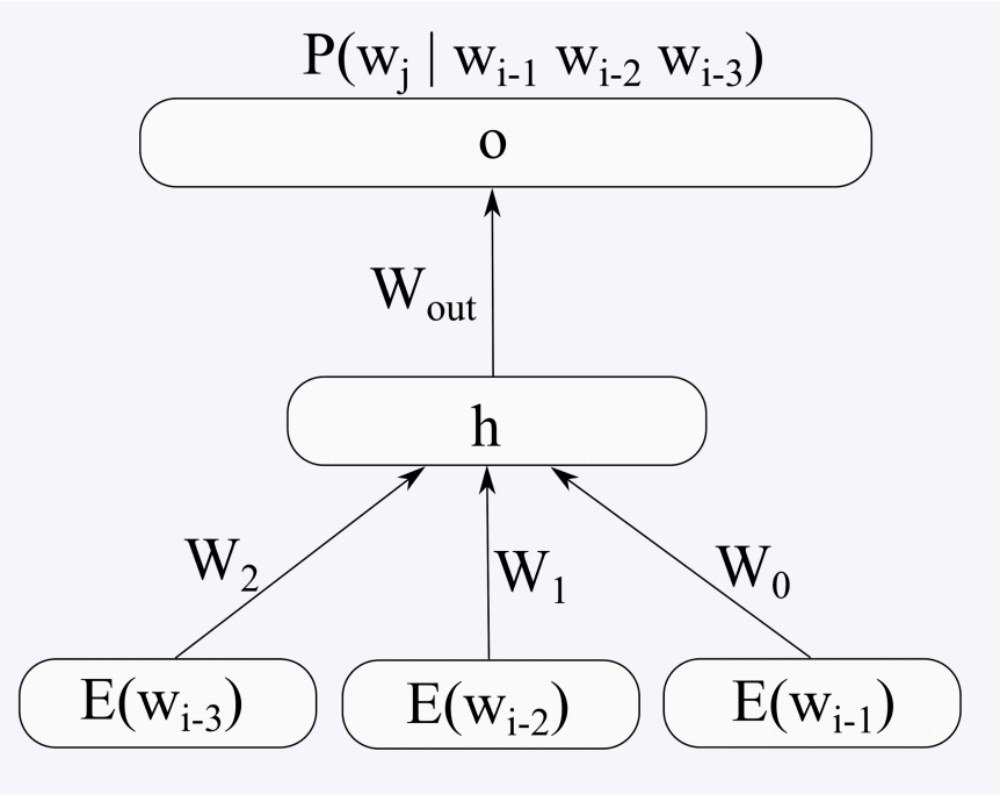


- Predict class: one output node per class
- Training data output: "One-hot vector", e.g., $\vec{y} = (0, 0, 1)^T$
- Prediction
  - predicted class is output node y_i with highest value
  - obtain posterior probability distribution by soft-max

$$\text{softmax}(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

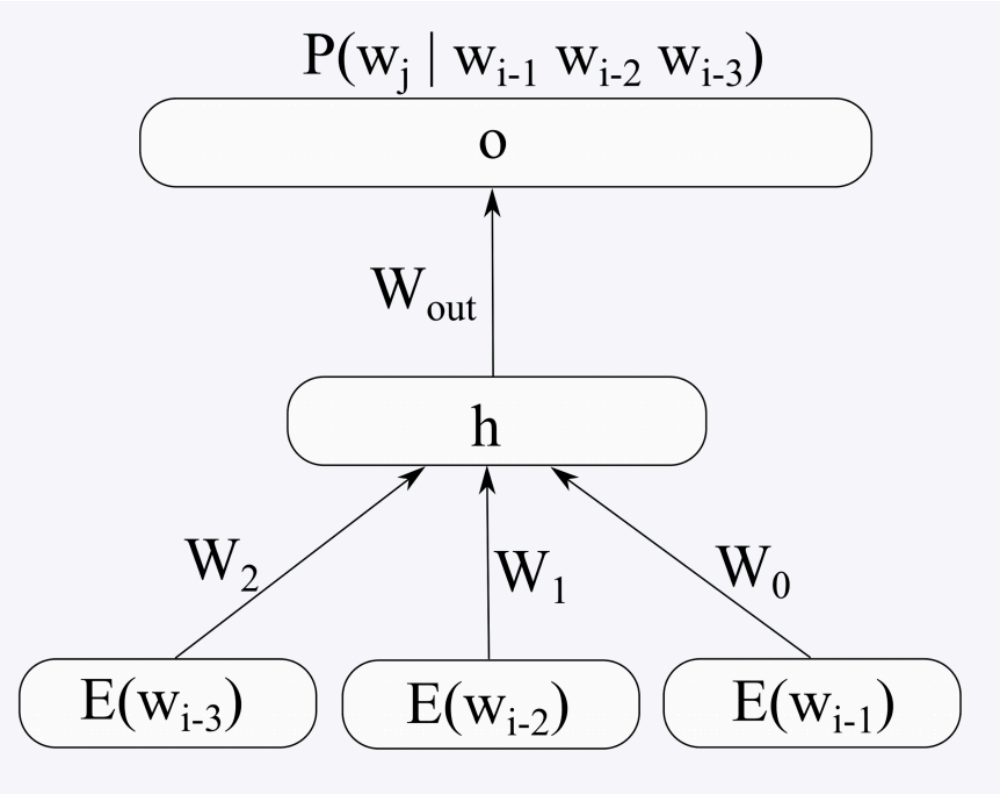
Feedforward Neural Network Language Model

- Input: vector representations of previous words $E(w_{i-3})$ $E(w_{i-2})$ $E(w_{i-1})$
- Output: the conditional probability of w_j being the next word



Feedforward Neural Network Language Model

- We can also think of the input as a concatenation of the context vectors
- The hidden layer *h* is calculated as in previous examples

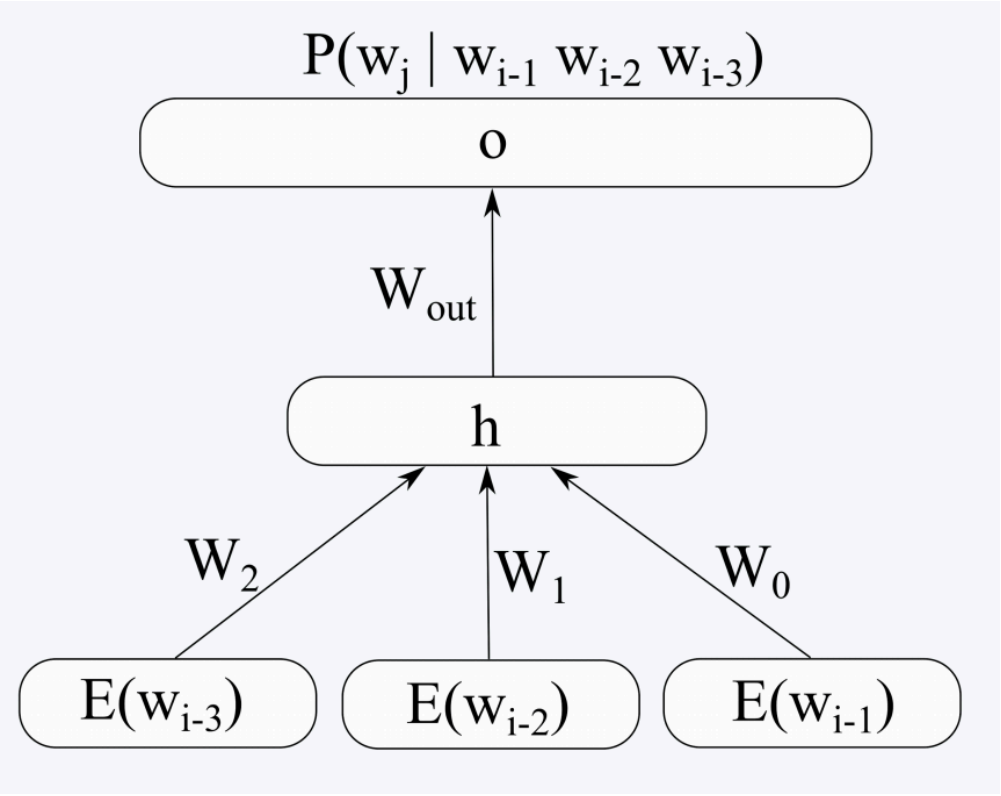


How do we calculate

$P(w_j \mid w_{i-1}\ w_{i-2}\ w_{i-3})$?

Feedforward Neural Network Language Model

- Our output vector o has an element for each possible word w_j
- We take a softmax over that vector



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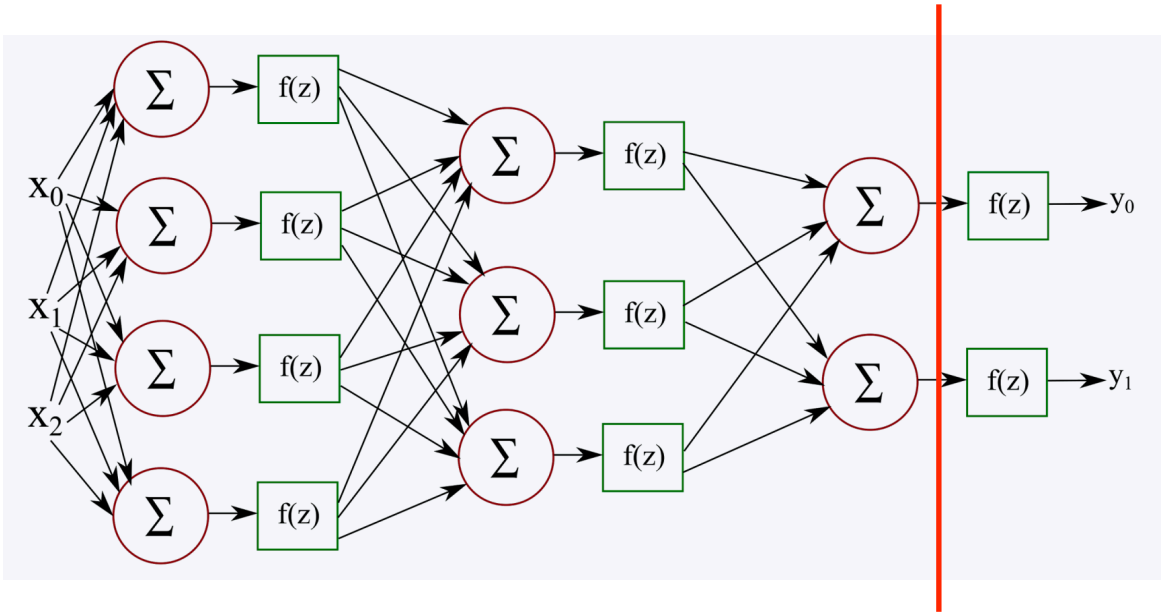
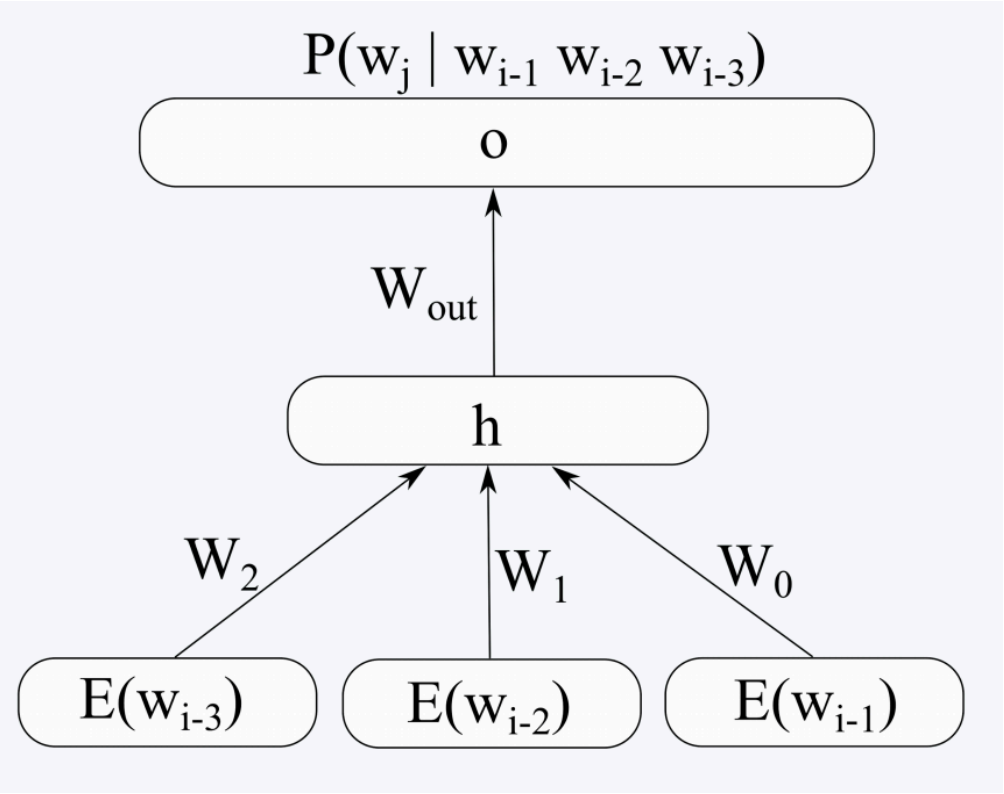


$$softmax(z)_j = \frac{e^{z_j}}{\sum_k e^{z_k}}$$

	0	1	2	3	SUM
z	-5.0	-4.5	-4.0	-6.0	-19.5
exp(z)	0.007	0.011	0.018	0.002	0.038
softmax(z)	0.184	0.289	0.474	0.053	1.0

Feedforward Neural Network Language Model



1) Multiple input vectors with weights

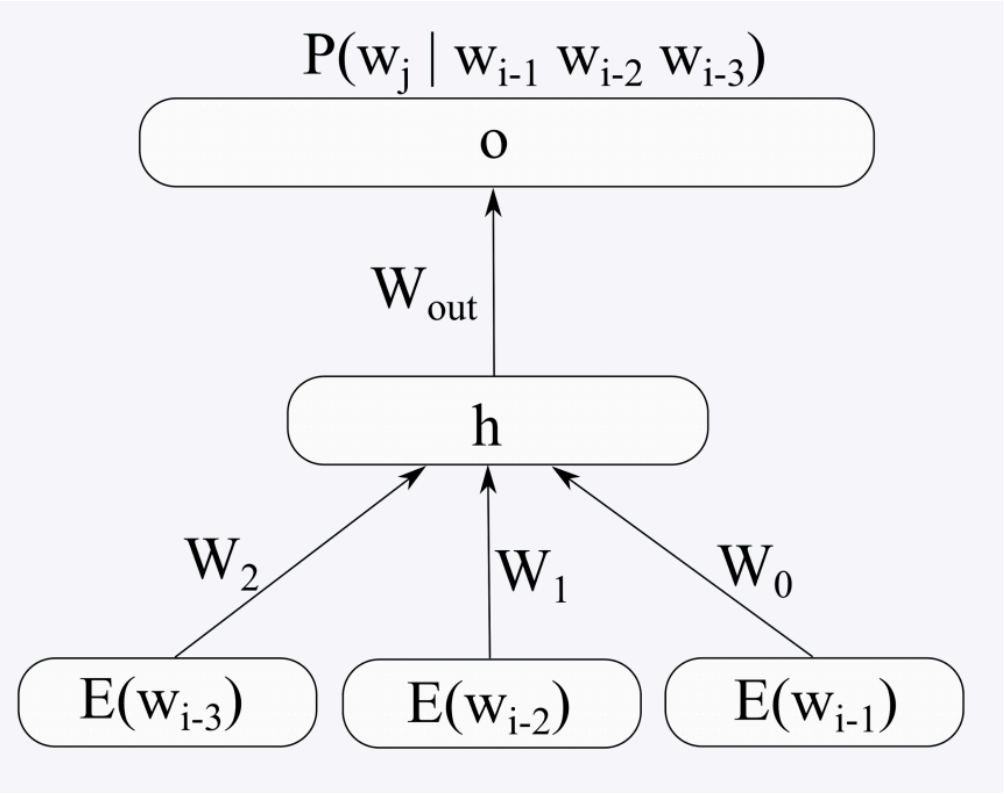
$$z = W_2 \cdot E(w_{i-3}) + W_1 \cdot E(w_{i-2}) + W_0 \cdot E(w_{i-1})$$

2) Apply the activation function

$$h = f(z)$$

Bengio et al. (2003)

Feedforward Neural Network Language Model



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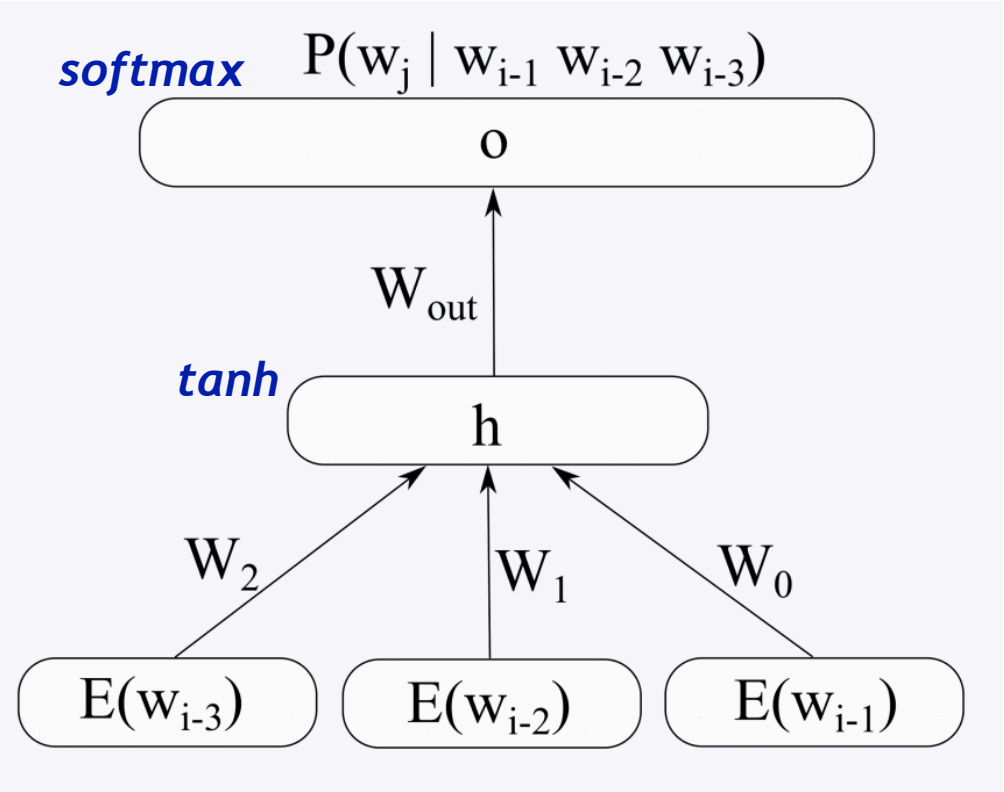


3) Multiply hidden vector with output weights

$$s = W_{out} \cdot h$$

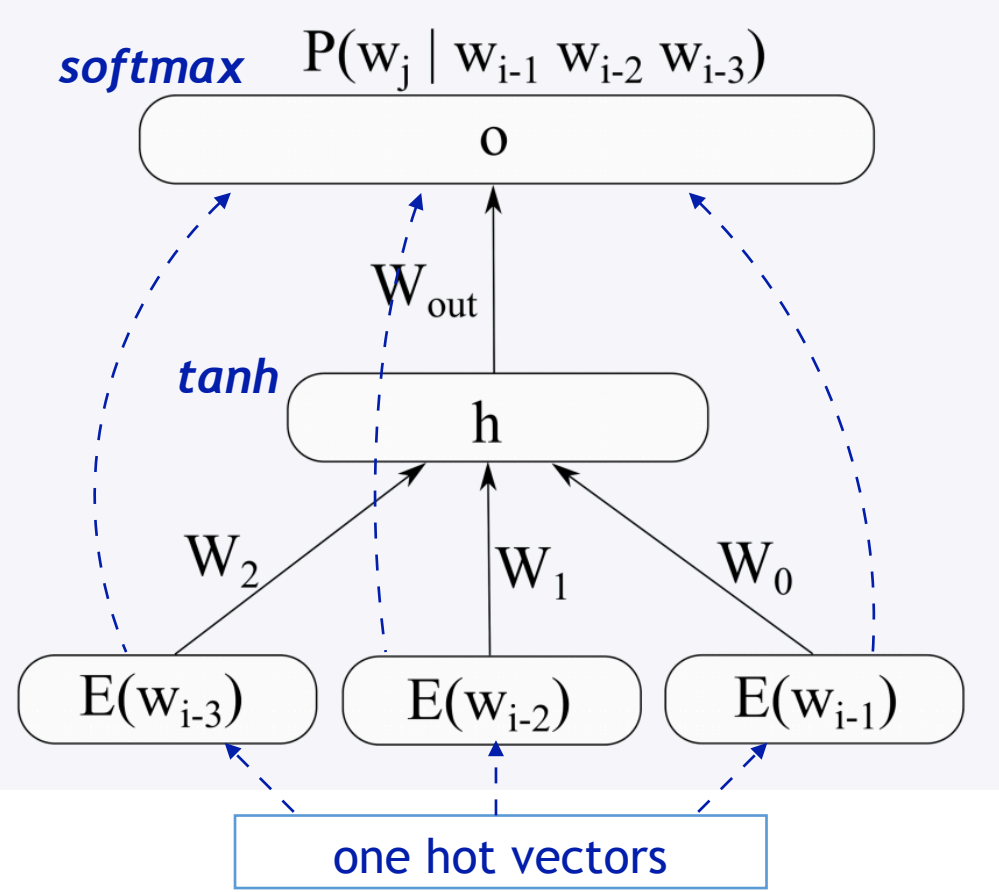
4) Apply softmax to the output vector

$$o = softmax(s)$$

Now the *j*-th element in the output vector o_j contains the probability of w_j being the next word. $0 \leq j < V$

Bengio et al. (2003)

Feedforward Neural Network Language Model



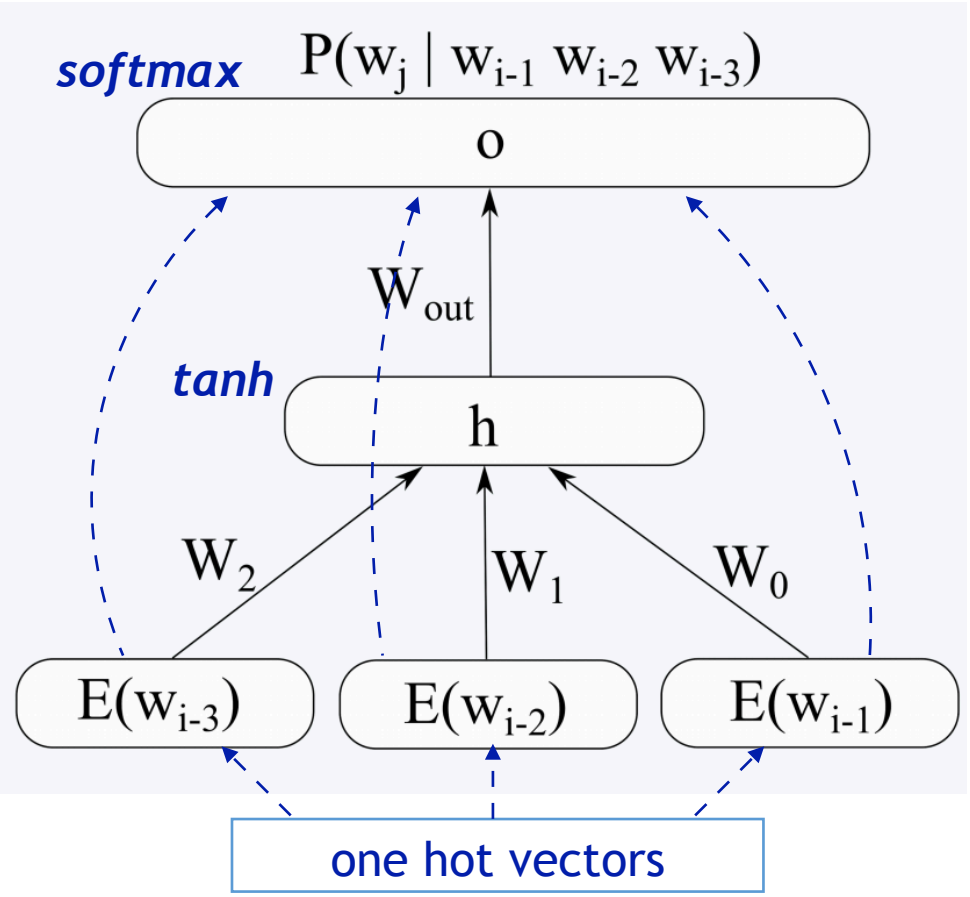
Bengio et al. (2003)

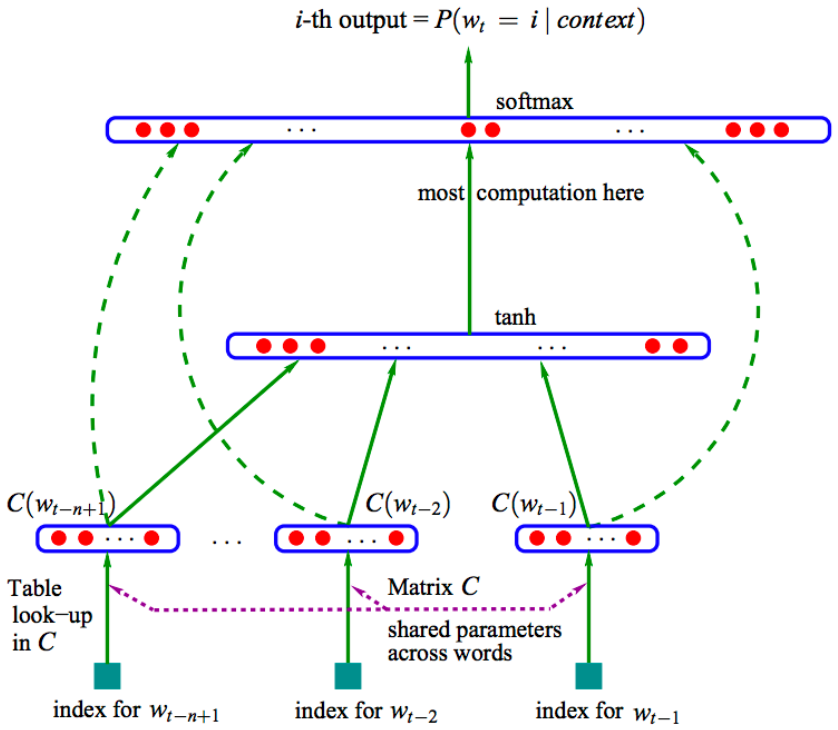
Like a log-linear language model with two kinds of features:

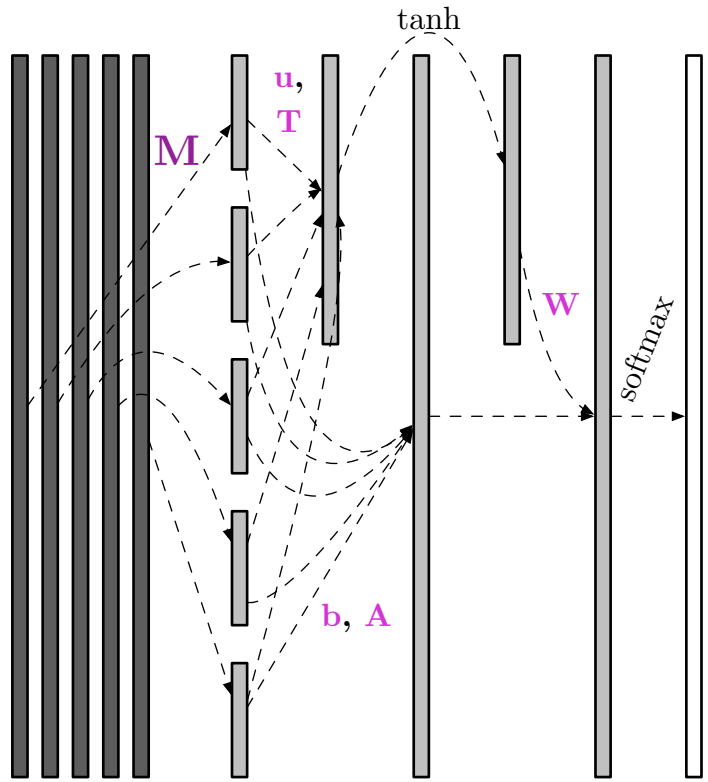
1) concatenation of context-word embedding vectors

2) *tanh*-affine transformation of the above

Feedforward Neural Network Language Model







Bengio et al. (2003)

Neural Network: Definitions

Warning: there is no widely accepted standard notation!

A feedforward neural network $n_{\boldsymbol{\nu}}$ is defined by:

- A function family that maps parameter values to functions of the form $n : \mathbb{R}^{d_{in}} \to \mathbb{R}^{d_{out}}$; typically:
  - Non-linear
  - Differentiable with respect to its inputs
  - "Assembled" through a series of affine transformations and non-linearities, composed together
  - Symbolic/discrete inputs handled through lookups.
- Parameter values $\boldsymbol{\nu}$
  - Typically a collection of scalars, vectors, and matrices
  - We often assume they are linearized into $\mathbb{R}^D$

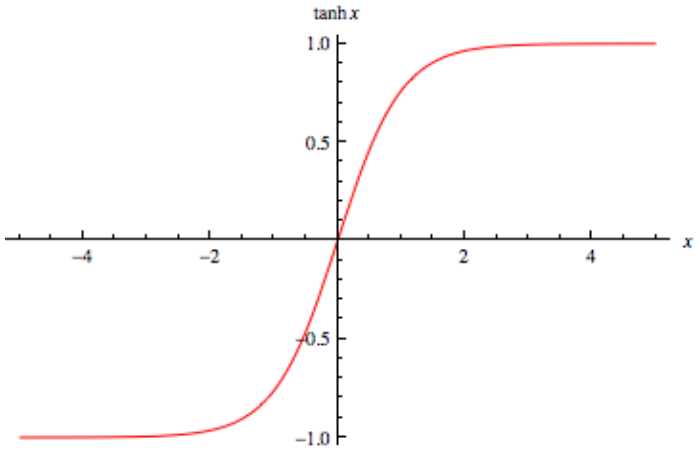
A Couple of Useful Functions

- $\mathrm{softmax} : \mathbb{R}^k \to \mathbb{R}^k$

$$\langle x_1, x_2, \ldots, x_k \rangle \mapsto \left\langle \frac{e^{x_1}}{\sum_{j=1}^{k} e^{x_j}}, \frac{e^{x_2}}{\sum_{j=1}^{k} e^{x_j}}, \ldots, \frac{e^{x_k}}{\sum_{j=1}^{k} e^{x_j}} \right\rangle$$

- $\tanh : \mathbb{R} \to [-1, 1]$

$$x \mapsto \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

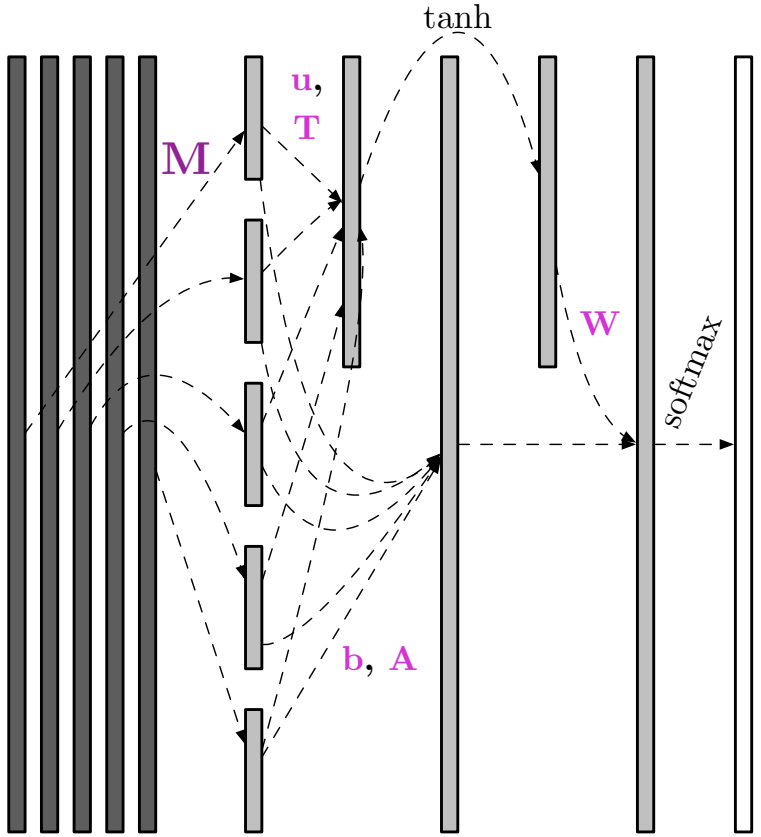


Generalized to be *elementwise*, so that it maps $\mathbb{R}^k \to [-1, 1]^k$.

- Others include: ReLUs, logistic sigmoids, PReLUs, ...

Feedforward Neural Network Language Model

(Bengio et al., 2003)



Define the n-gram probability as follows:

$$p(\cdot \mid \langle h_1, \ldots, h_{\mathsf{n}-1} \rangle) = n_{\boldsymbol{\nu}} \left(\langle \mathbf{e}_{h_1}, \ldots, \mathbf{e}_{h_{\mathsf{n}-1}} \rangle \right) =$$

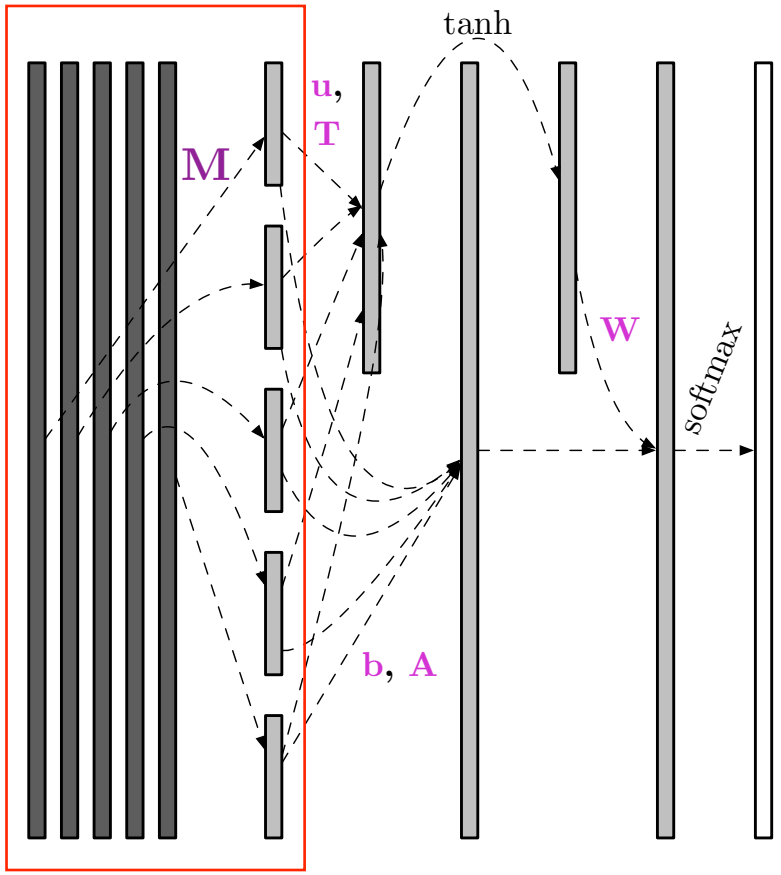
$$\operatorname{softmax} \left(\underset{V}{\mathbf{b}} + \sum_{j=1}^{\mathsf{n}-1} \underset{V}{\mathbf{e}_{h_j}}^{\top} \underset{V \times d}{\mathbf{M}} \underset{d \times V}{\mathbf{A}_j} + \underset{V \times H}{\mathbf{W}} \tanh \left(\underset{H}{\mathbf{u}} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{e}_{h_j}^{\top} \mathbf{M} \underset{d \times H}{\mathbf{T}_j} \right) \right)$$

where each $\mathbf{e}_{h_j} \in \mathbb{R}^V$ is a one-hot vector and H is the number of "hidden units" in the neural network (a "hyperparameter").

Parameters $\boldsymbol{\nu}$ include:

- $\mathbf{M} \in \mathbb{R}^{V \times d}$, which are called "embeddings" (row vectors), one for every word in $\mathcal{V}$
- Feedforward NN parameters $\mathbf{b} \in \mathbb{R}^V$, $\mathbf{A} \in \mathbb{R}^{(\mathsf{n}-1) \times d \times V}$, $\mathbf{W} \in \mathbb{R}^{V \times H}$, $\mathbf{u} \in \mathbb{R}^H$, $\mathbf{T} \in \mathbb{R}^{(\mathsf{n}-1) \times d \times H}$

Breaking It Down

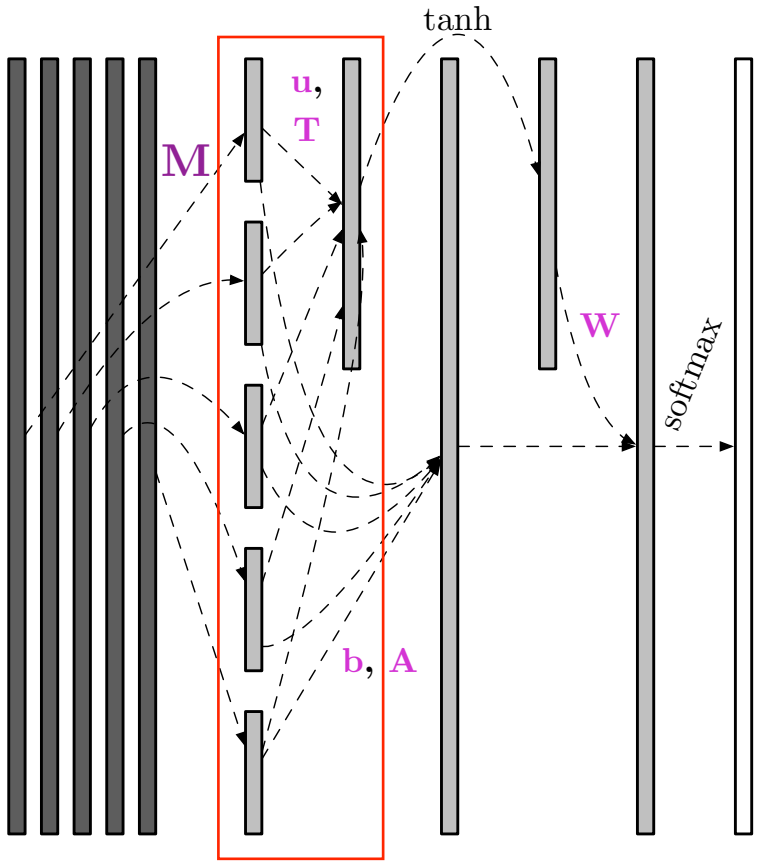


Look up each of the history words $h_j, \forall j \in \{1, \ldots, \mathsf{n}-1\}$ in $\mathbf{M}$; keep two copies. Rename the embedding for h_j as $\mathbf{m}_{h_j}$.

$$\mathbf{e}_{h_j}{}^\top \mathbf{M} = \mathbf{m}_{h_j}$$

$$\mathbf{e}_{h_j}{}^\top \mathbf{M} = \mathbf{m}_{h_j}$$

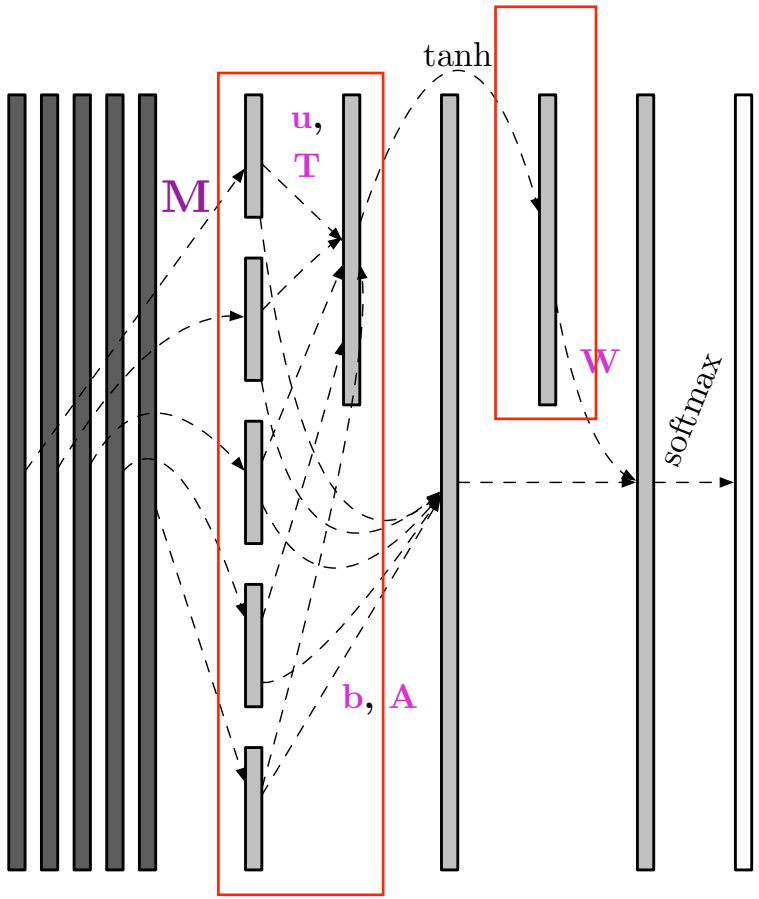
Breaking It Down



Apply an affine transformation to the second copy of the history-word embeddings ($\mathbf{u}$, $\mathbf{T}$)

$$\underset{H}{\mathbf{m}_{h_j}} \quad \underset{H}{\mathbf{u}} + \sum_{j=1}^{n-1} \mathbf{m}_{h_j} \underset{d \times H}{\mathbf{T}_j}$$

Breaking It Down

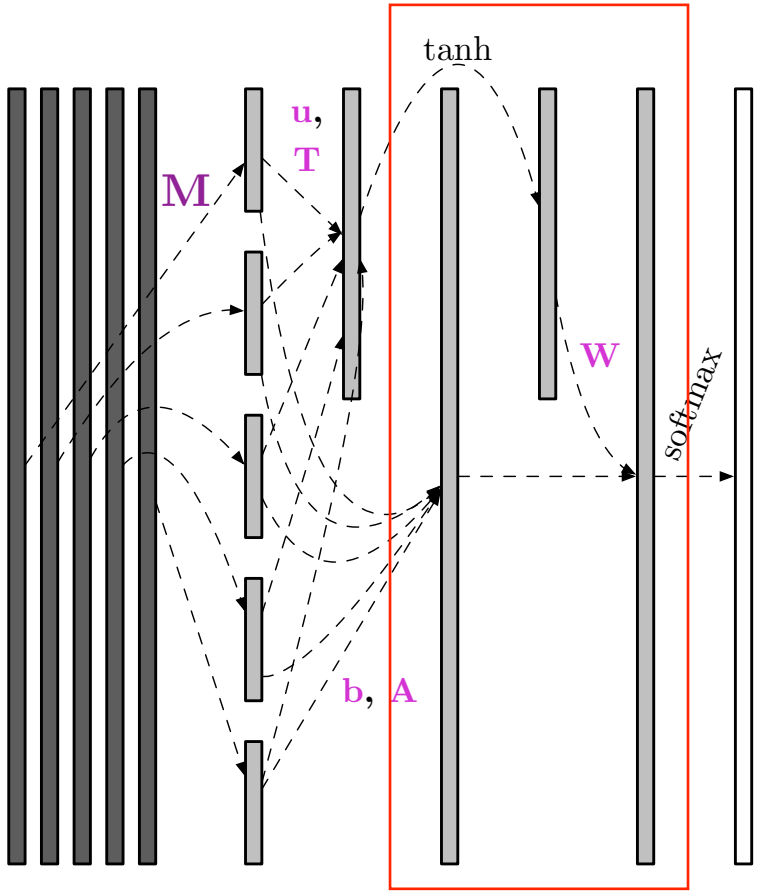


Apply an affine transformation to the second copy of the history-word embeddings ($\mathbf{u}$, $\mathbf{T}$) and a $\tanh$ nonlinearity.

$$\mathbf{m}_{h_j}$$

$$\tanh\left(\mathbf{u} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}\ \mathbf{T}_j\right)$$

Breaking It Down

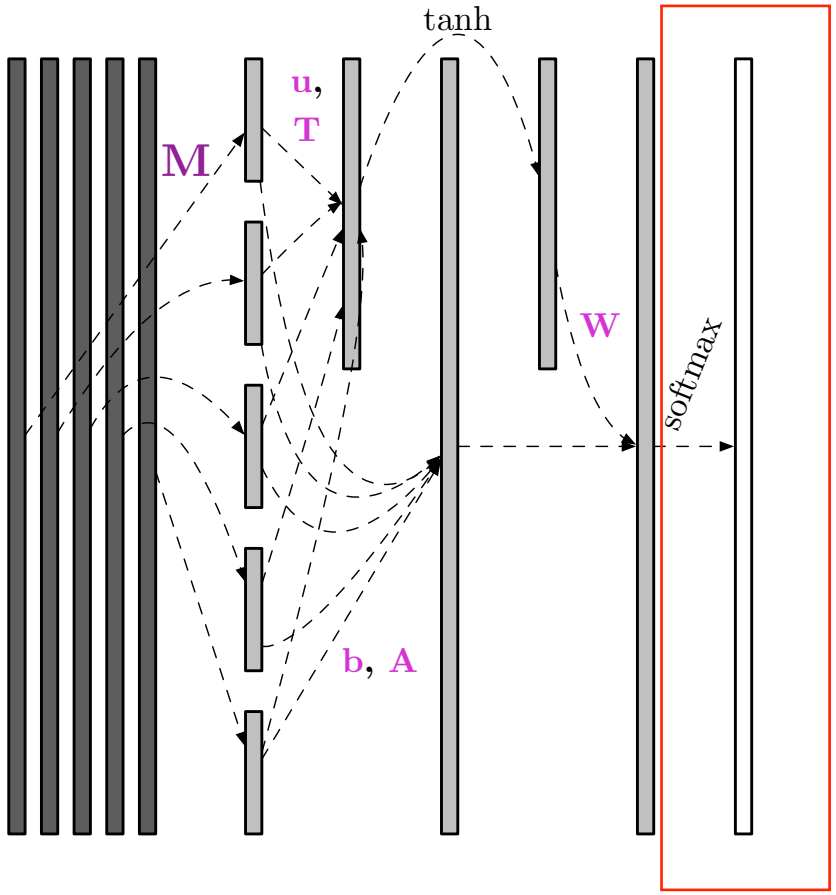


Apply an affine transformation to everything ($\mathbf{b}$, $\mathbf{A}$, $\mathbf{W}$).

$$\underset{V}{\mathbf{b}} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j} \underset{d \times V}{\mathbf{A}_j}$$

$$+ \underset{V \times H}{\mathbf{W}} \tanh \left(\mathbf{u} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j} \mathbf{T}_j \right)$$

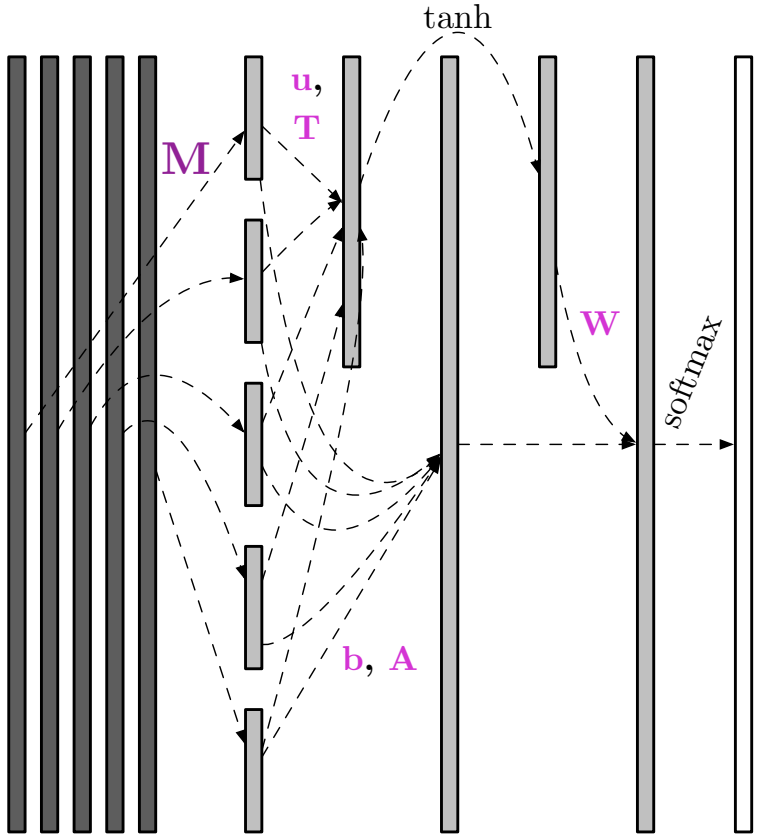
Breaking It Down



Apply a softmax transformation to make the vector sum to one.

$$\text{softmax}\left(\mathbf{b} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}\ \mathbf{A}_j + \mathbf{W}\ \tanh\left(\mathbf{u} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}\ \mathbf{T}_j\right)\right)$$

Breaking It Down



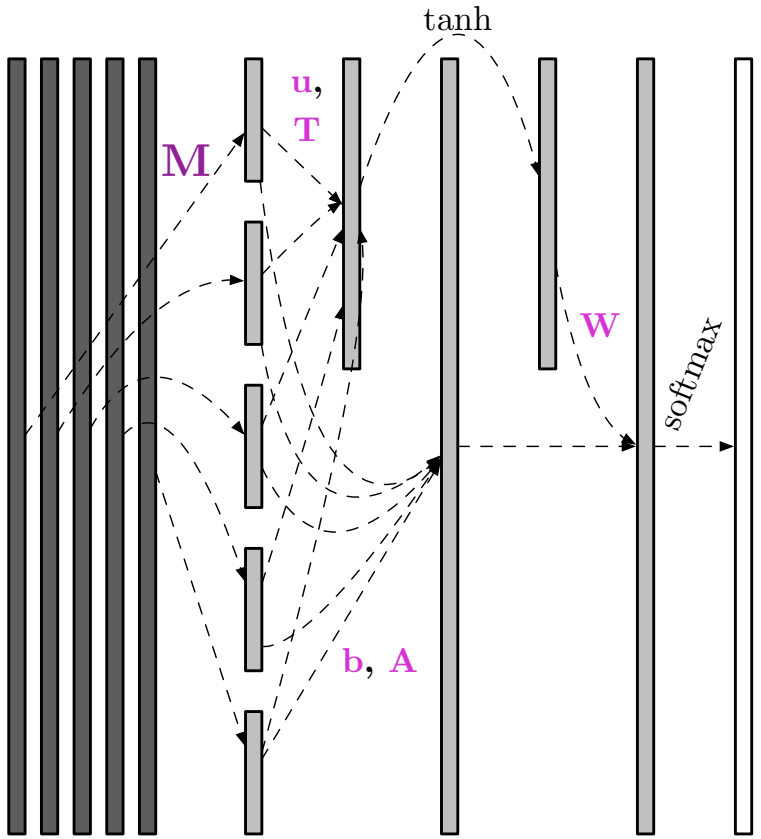
$$\text{softmax}\left(\mathbf{b} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}\ \mathbf{A}_j + \mathbf{W}\ \tanh\left(\mathbf{u} + \sum_{j=1}^{\mathsf{n}-1} \mathbf{m}_{h_j}\ \mathbf{T}_j\right)\right)$$

Like a log-linear language model with two kinds of features:

- Concatenation of context-word embeddings vectors $\mathbf{m}_{h_j}$
- $\tanh$-affine transformation of the above

New parameters arise from (i) embeddings and (ii) affine transformation "inside" the nonlinearity.

Number of Parameters



$$D = \underbrace{Vd}_{\mathbf{M}} + \underbrace{V}_{\mathbf{b}} + \underbrace{(\mathsf{n}-1)dV}_{\mathbf{A}} + \underbrace{VH}_{\mathbf{W}} + \underbrace{H}_{\mathbf{u}} + \underbrace{(\mathsf{n}-1)dH}_{\mathbf{T}}$$

- $V \approx 18000$ (after OOV processing)
- $d \in \{30, 60\}$
- $H \in \{50, 100\}$
- $\mathsf{n} - 1 = 5$

So $D = 461V + 30100$ parameters, compared to $O(V^{\mathsf{n}})$ for classical n-gram models.

- Forcing $\mathbf{A} = \mathbf{0}$ eliminated $300V$ parameters and performed a bit better, but was slower to converge.
- If we averaged $\mathbf{m}_{h_j}$ instead of concatenating, we'd get to $221V + 6100$ (this is a variant of "continuous bag of words," Mikolov et al., 2013).

Why does it work?