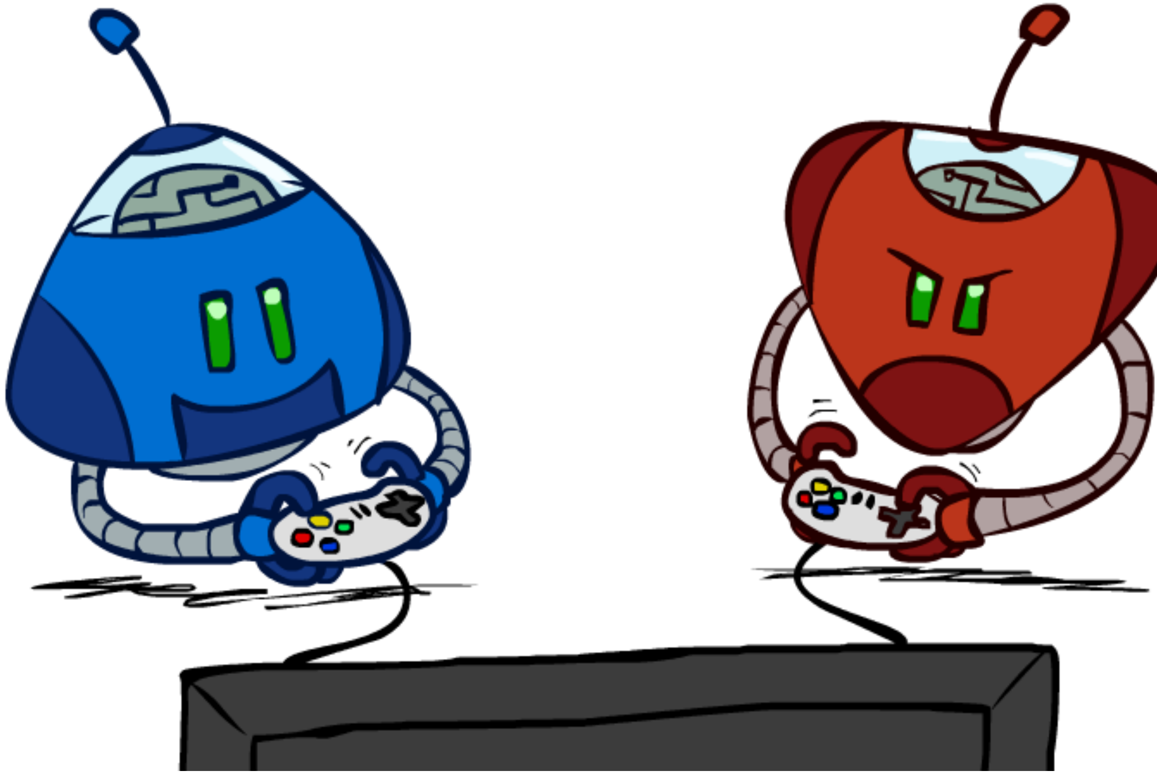
CS 5522: Artificial Intelligence II

Adversarial Search



Instructor: Wei Xu

Ohio State University

[These slides were adapted from CS188 Intro to AI at UC Berkeley.]

Machine Translation

Garcia and associates .

Garcia y asociados .

Carlos Garcia has three associates .

Carlos Garcia tiene tres asociados .

his associates are not strong .

sus asociados no son fuertes .

Garcia has a company also .

Garcia tambien tiene una empresa .

its clients are angry .

sus clientes estan enfadados .

the associates are also angry .

los asociados tambien estan enfadados .

the clients and the associates are enemies .

los clientes y los asociados son enemigos .

the company has three groups .

la empresa tiene tres grupos .

its groups are in Europe .

sus grupos estan en Europa .

the modern groups sell strong pharmaceuticals .

los grupos modernos venden medicinas fuertes .

the groups do not sell zanzanine .

los grupos no venden zanzanina .

the small groups are not modern .

los grupos pequenos no son modernos .

Word aligner

Garcia and associates .
Garcia y asociados .

Carlos Garcia has three associates .
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the small groups are not modern .
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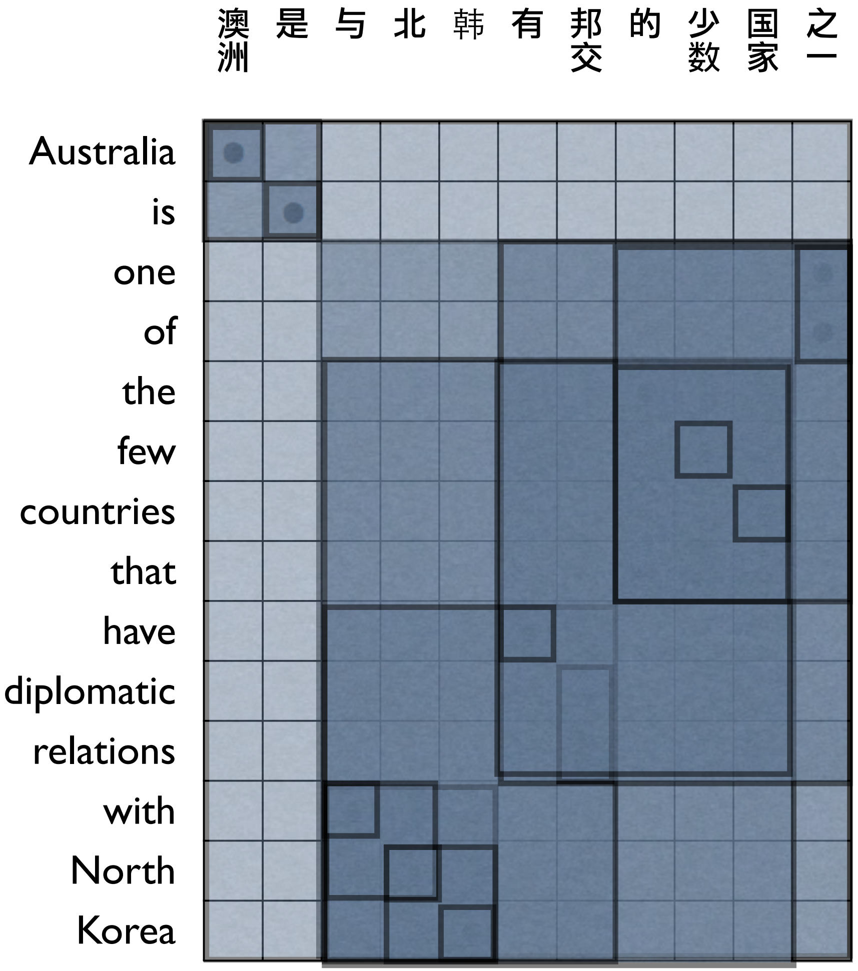
Phrase Extractor

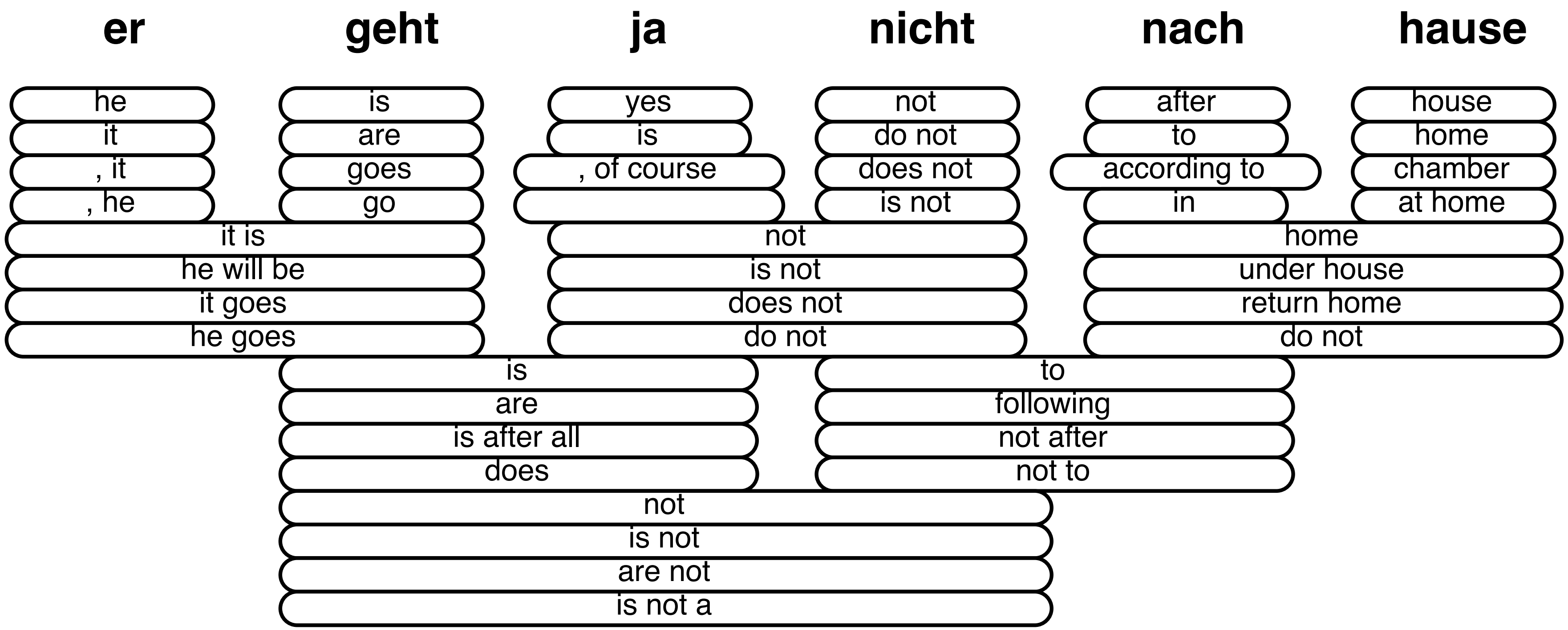


北 韩
North Korea

邦交
diplomatic relations

与 北 韩 有 邦交
have diplomatic relations with North Korea

Phrase-based Decoder



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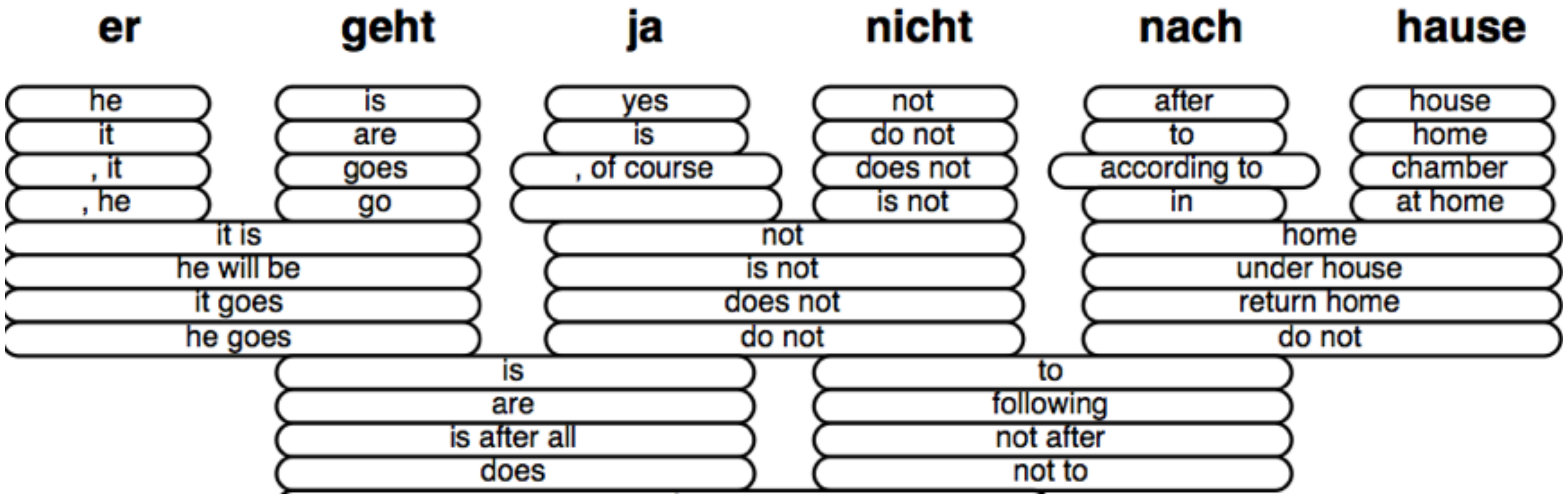

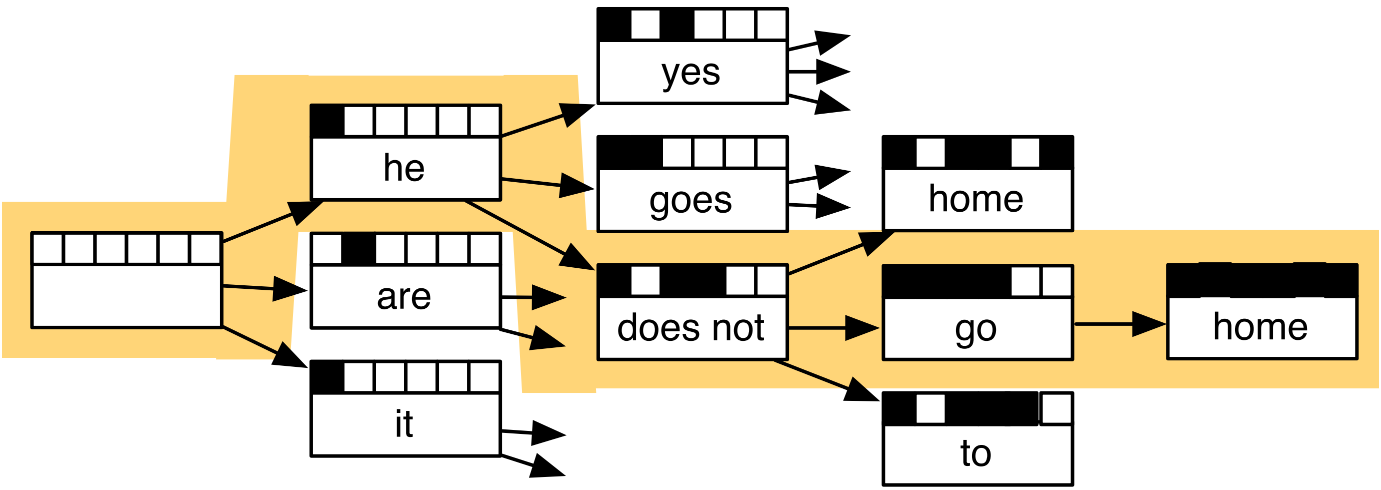

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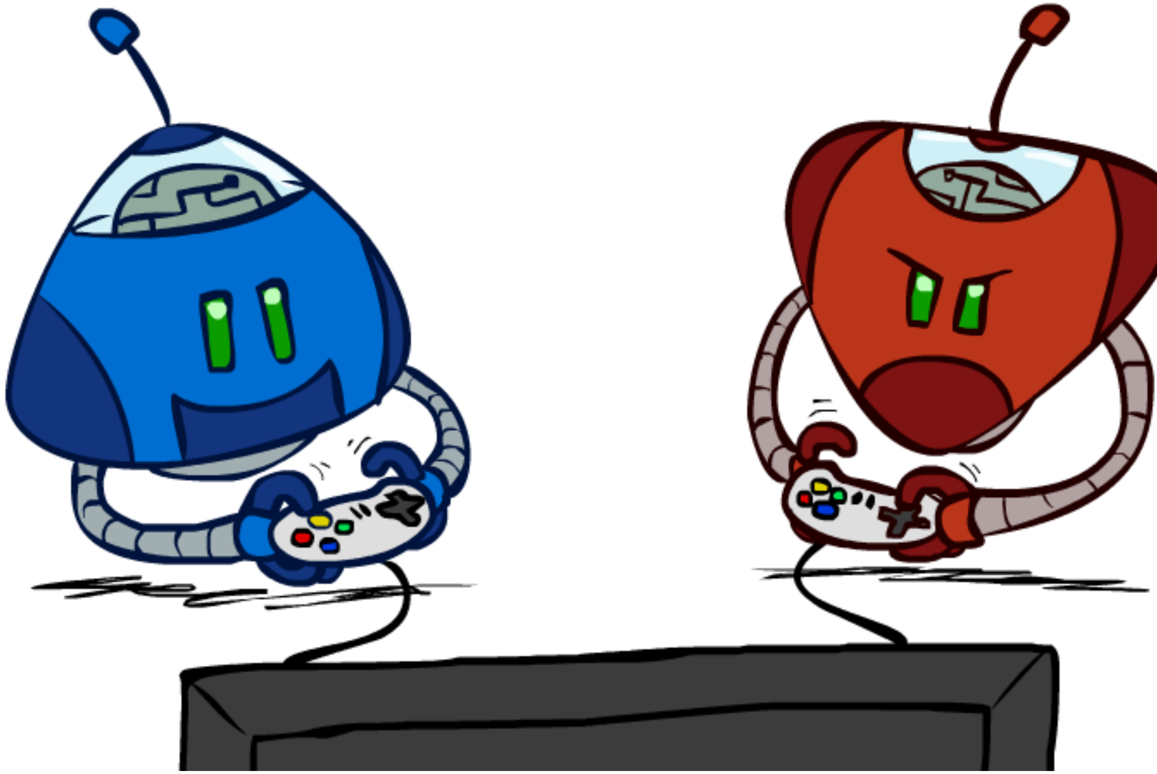

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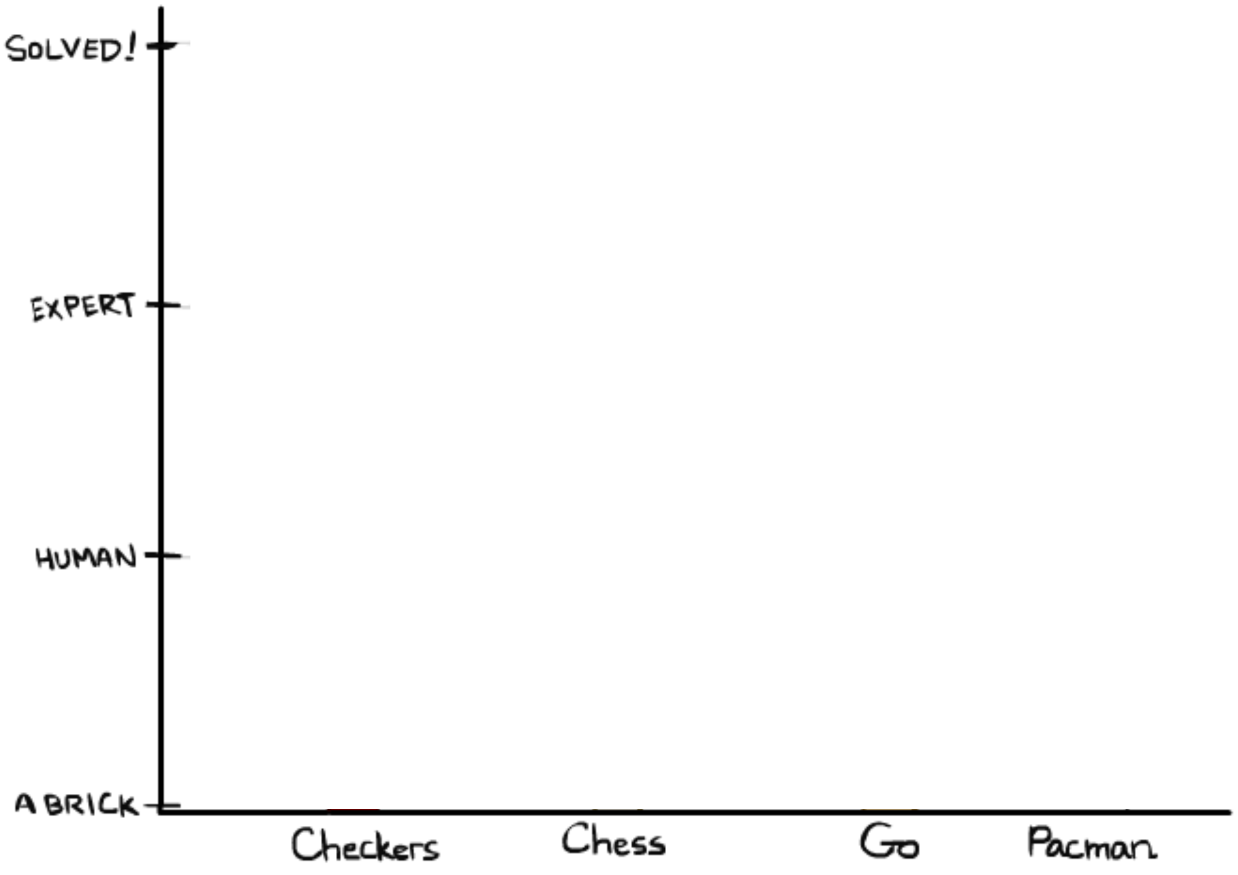
Game Playing State-of-the-Art



Game Playing State-of-the-Art

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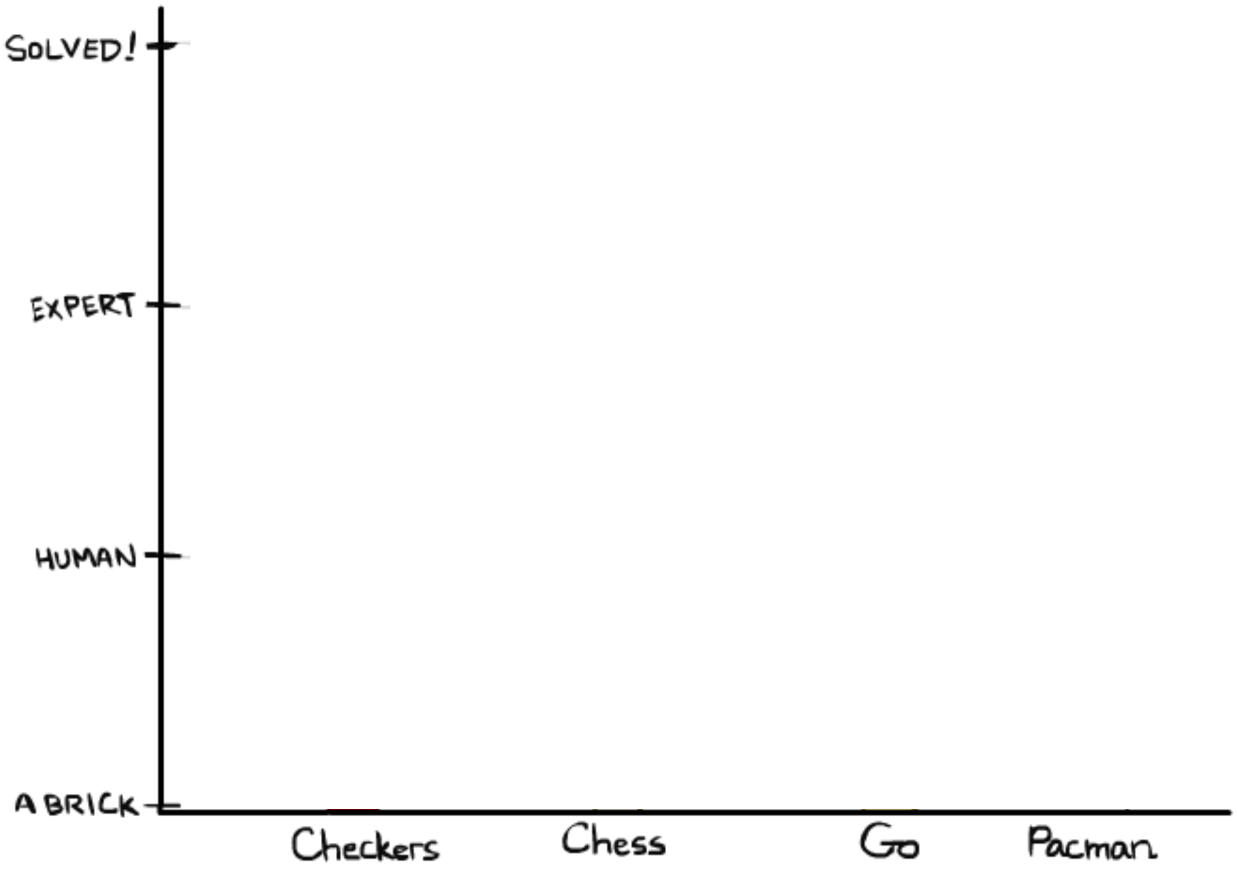

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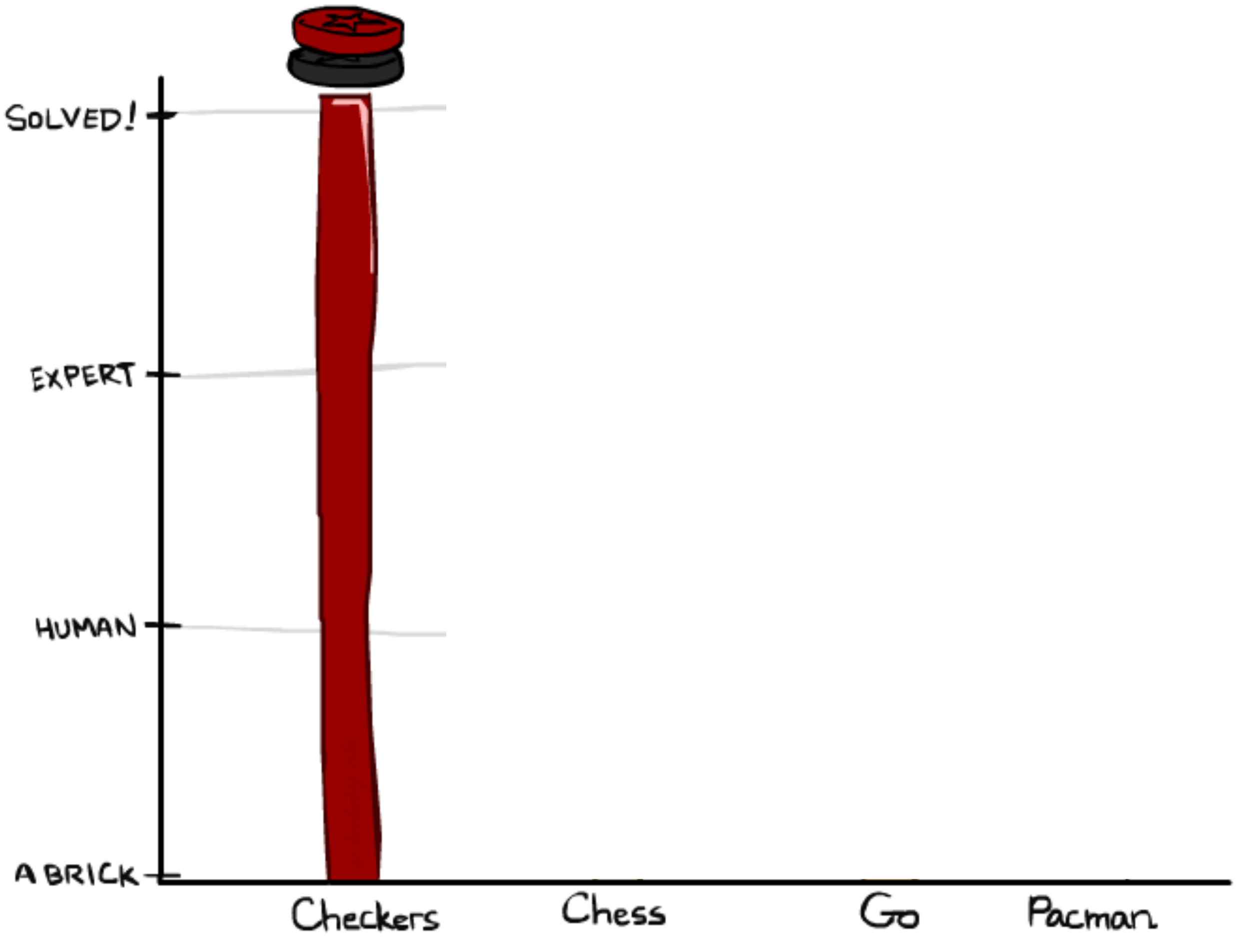

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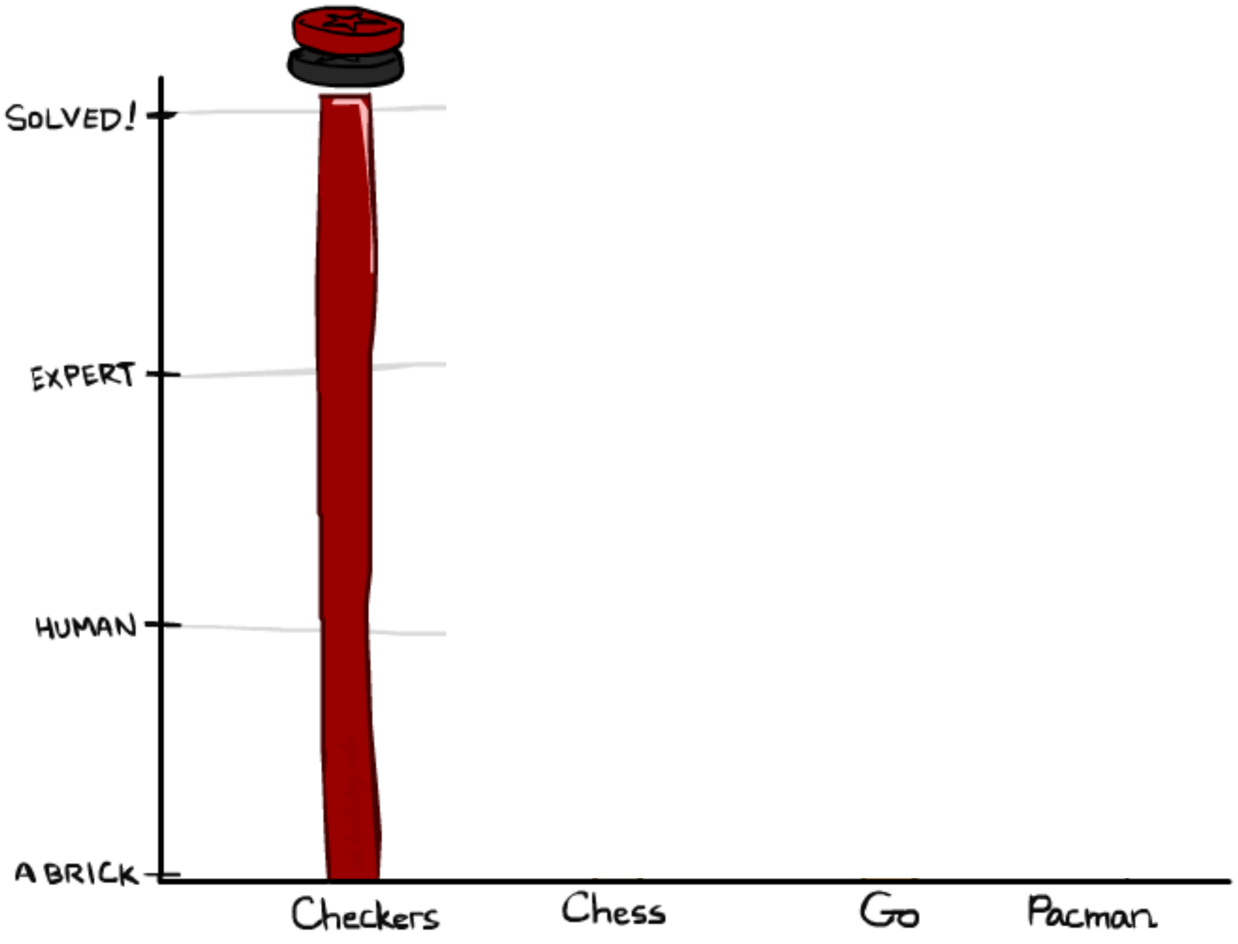

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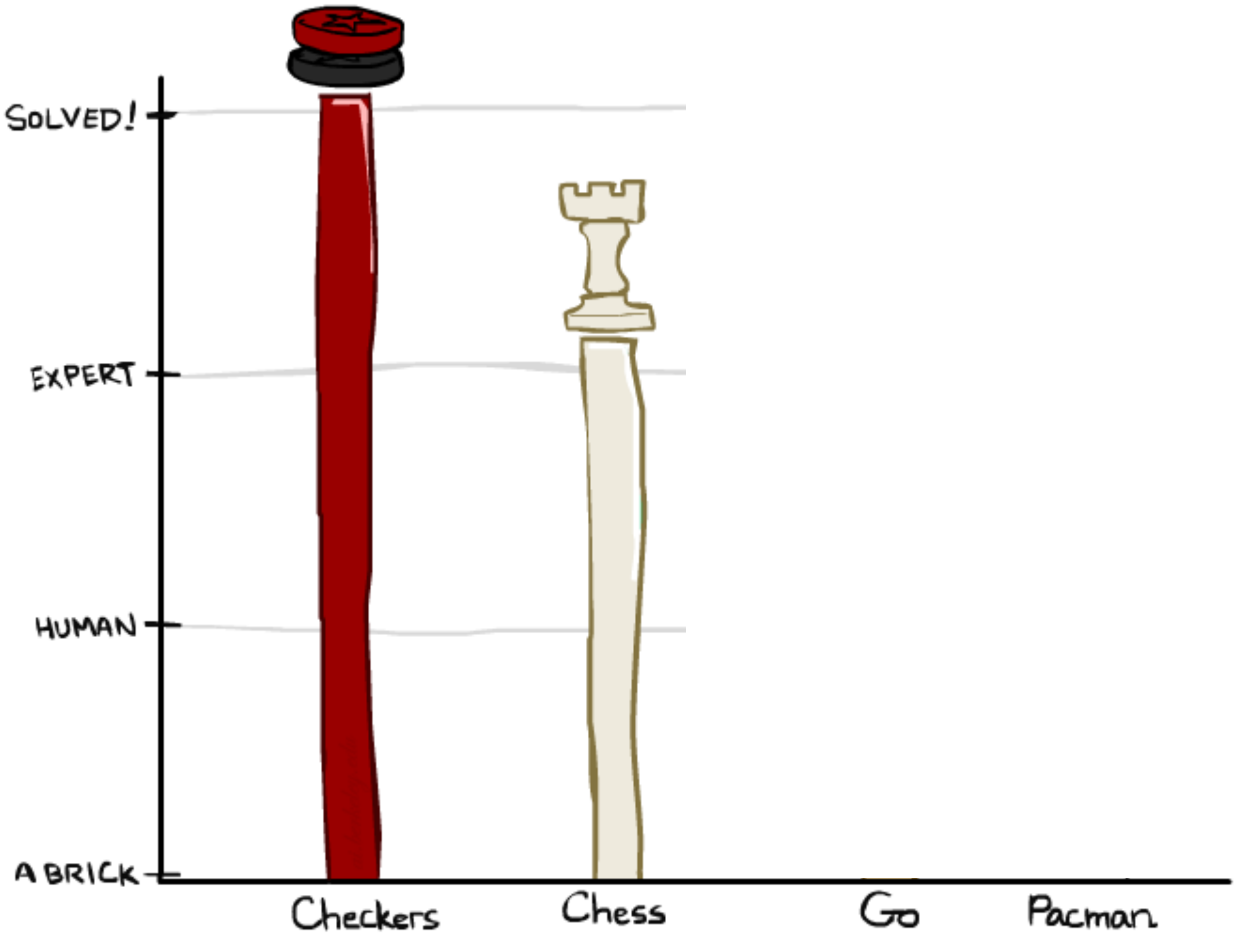

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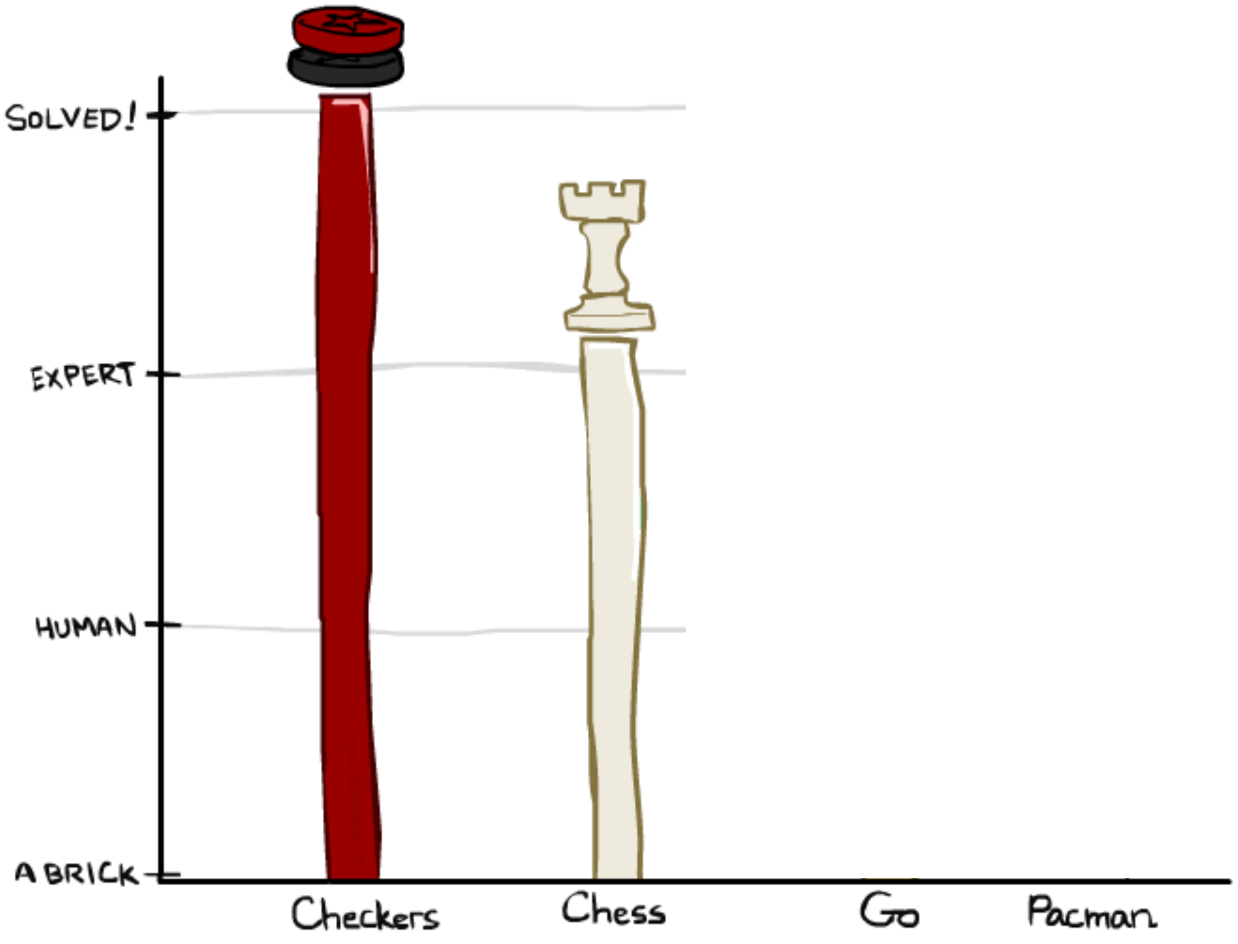

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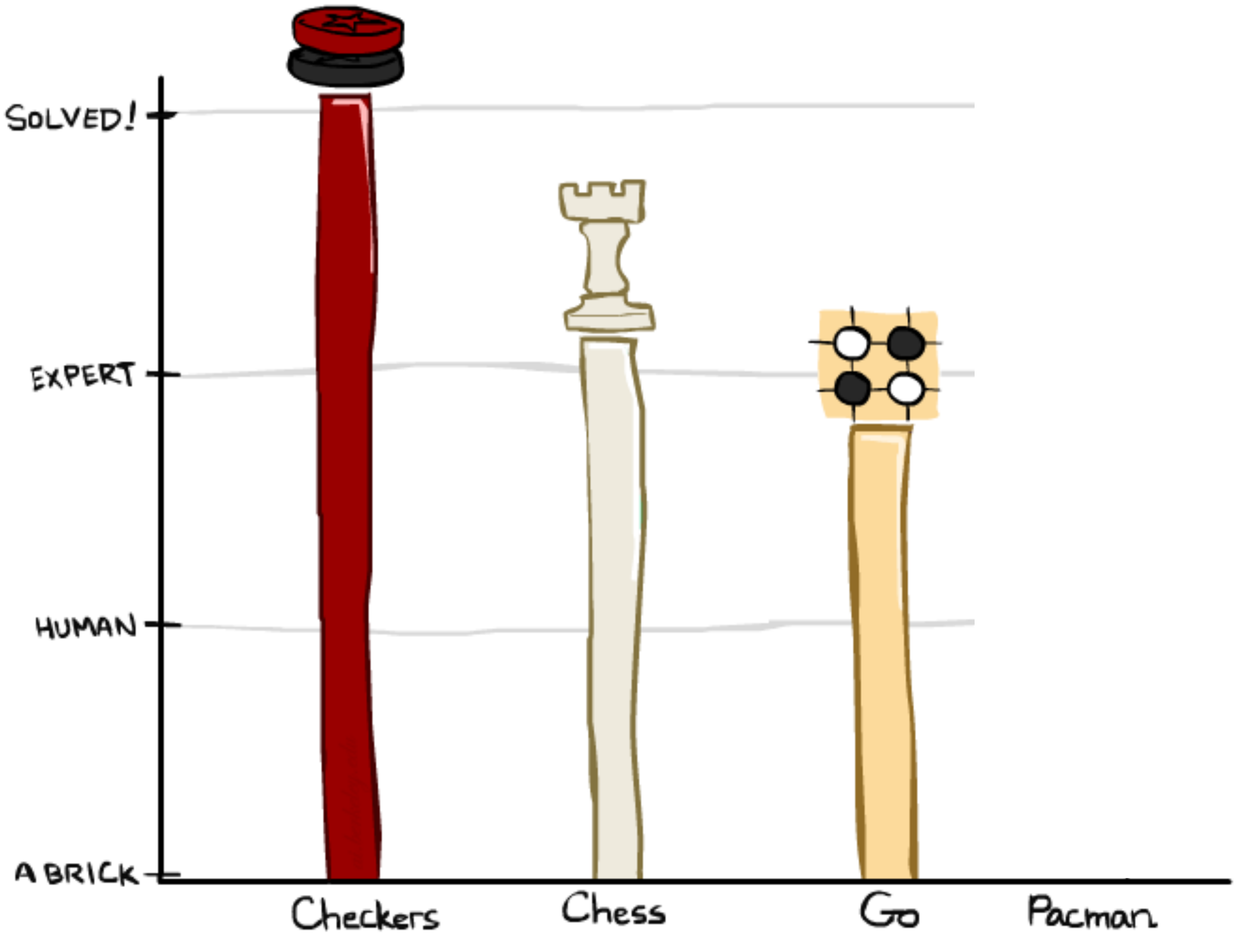

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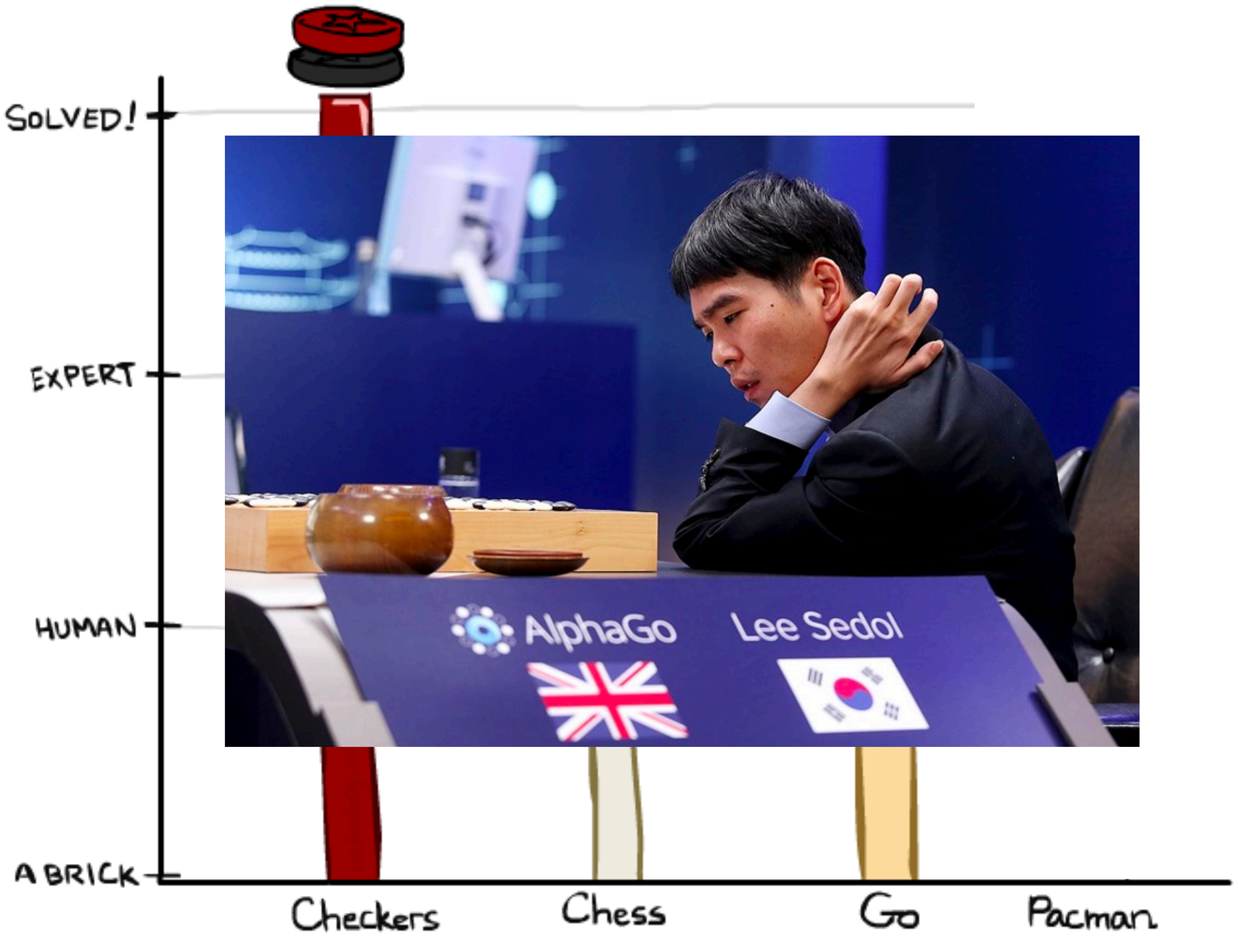

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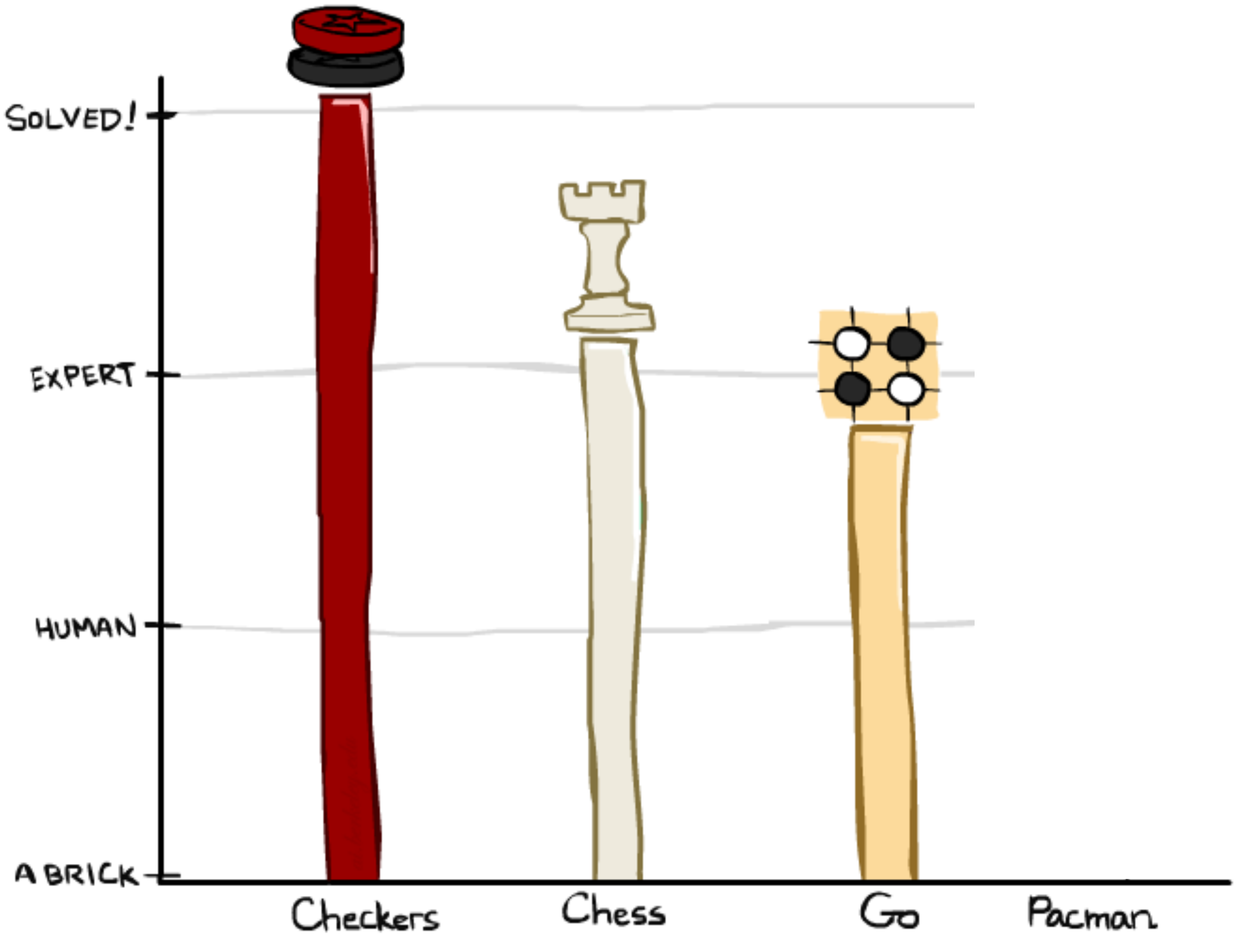
- **Pacman**



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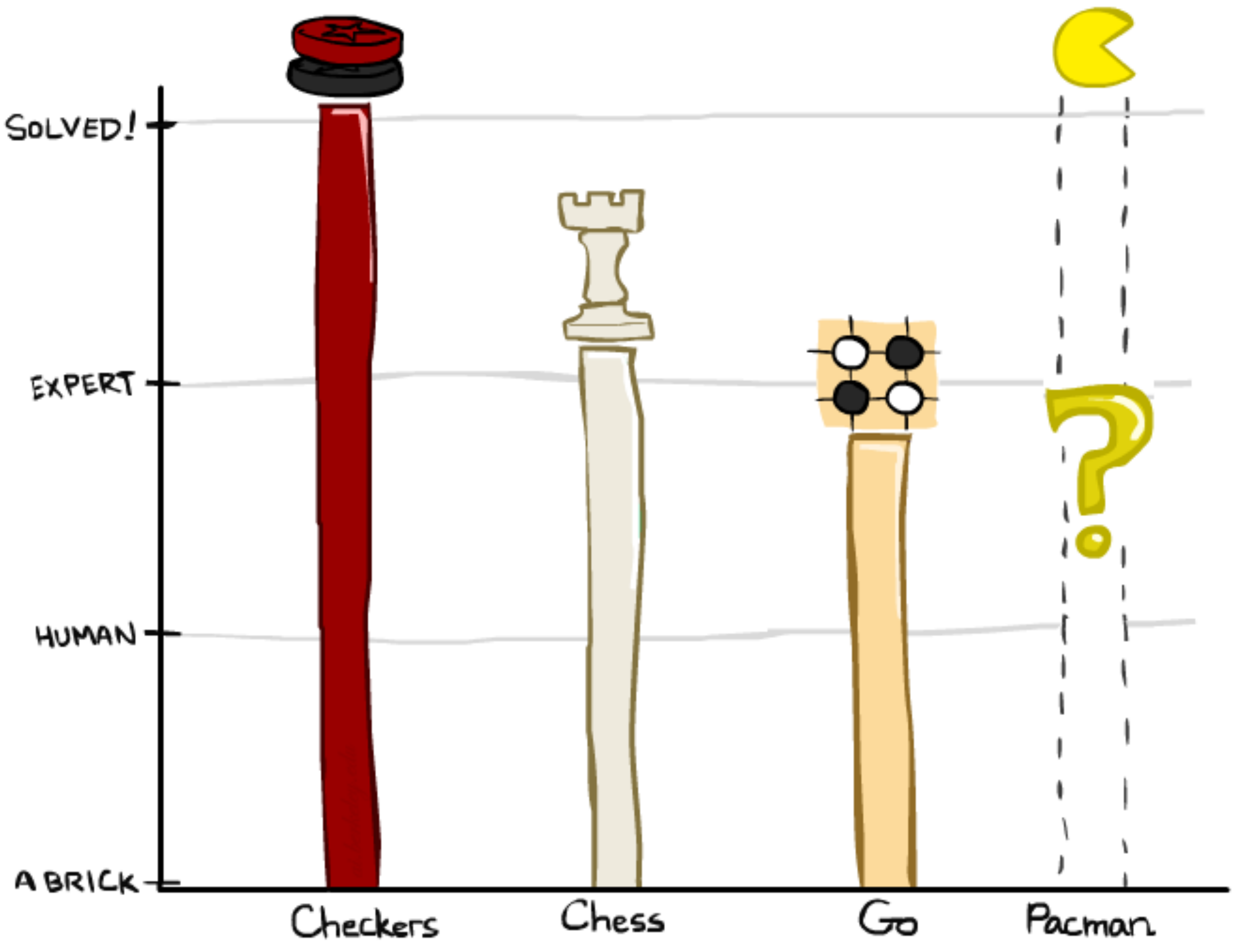
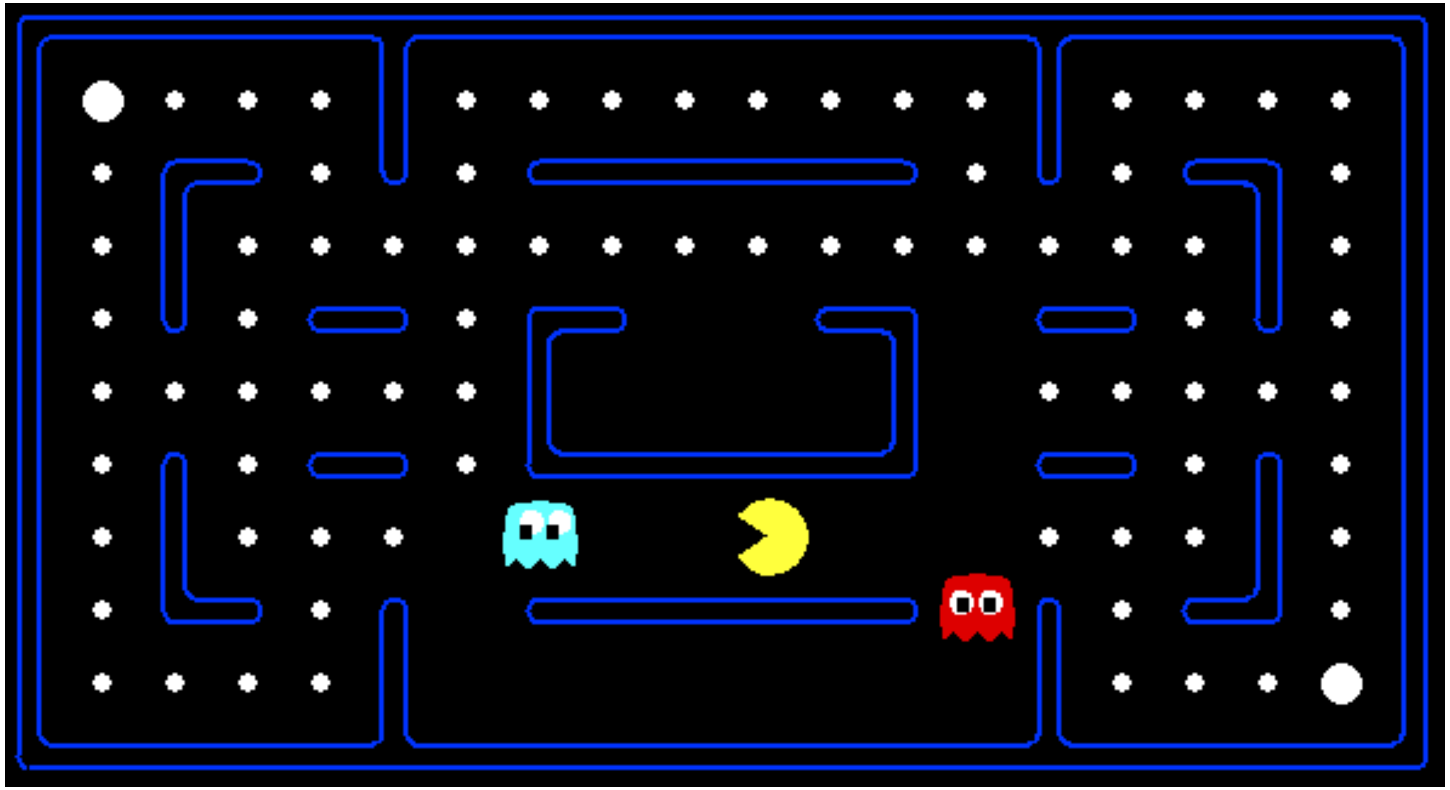
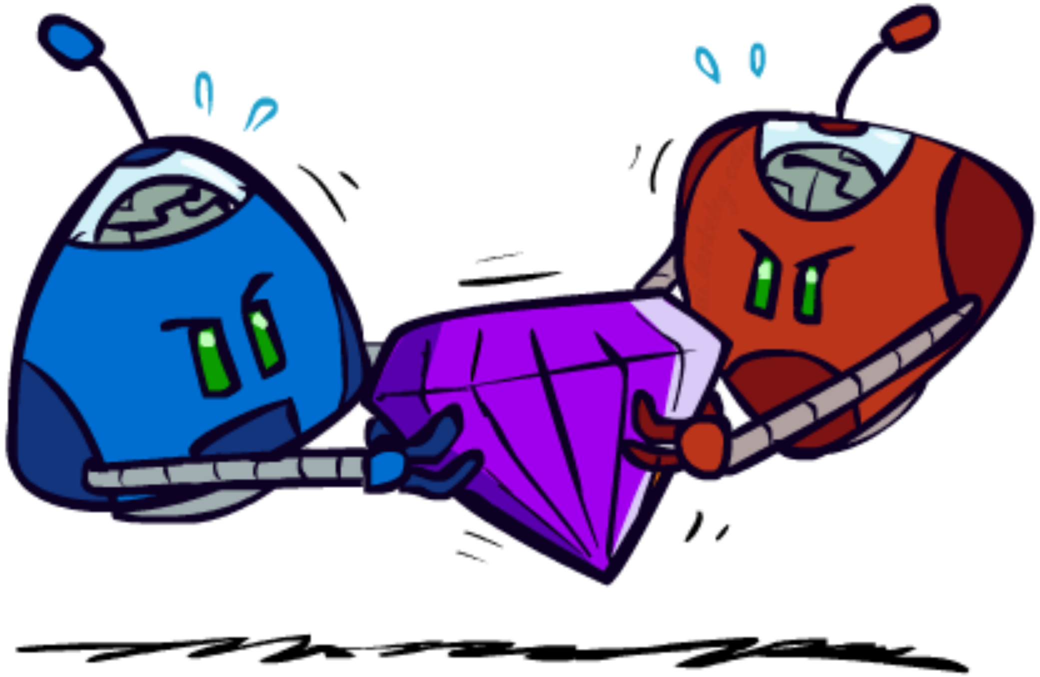


Behavior from Computation



[Demo: mystery pacman (L6D1)]

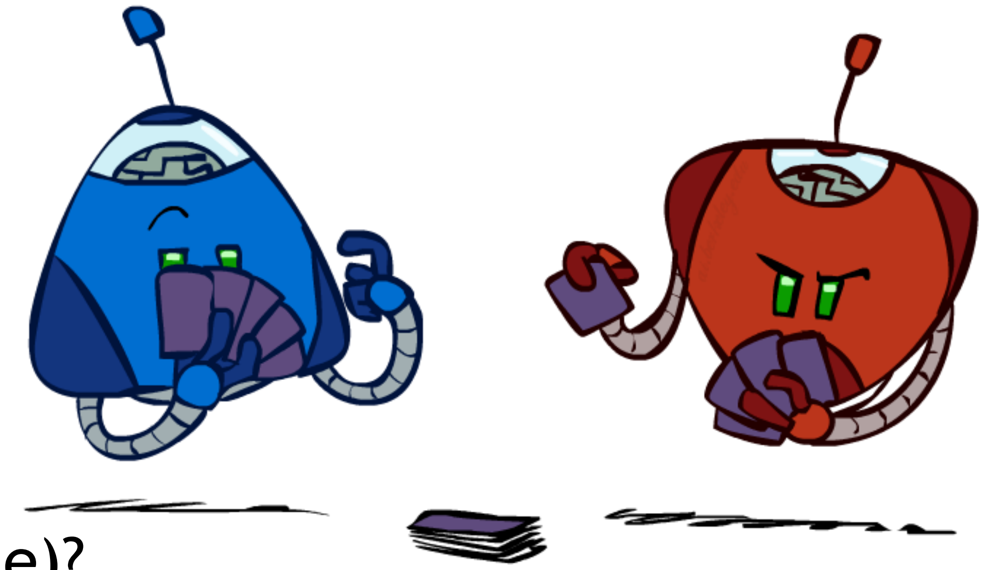
Adversarial Games



Types of Games

- Many different kinds of games!

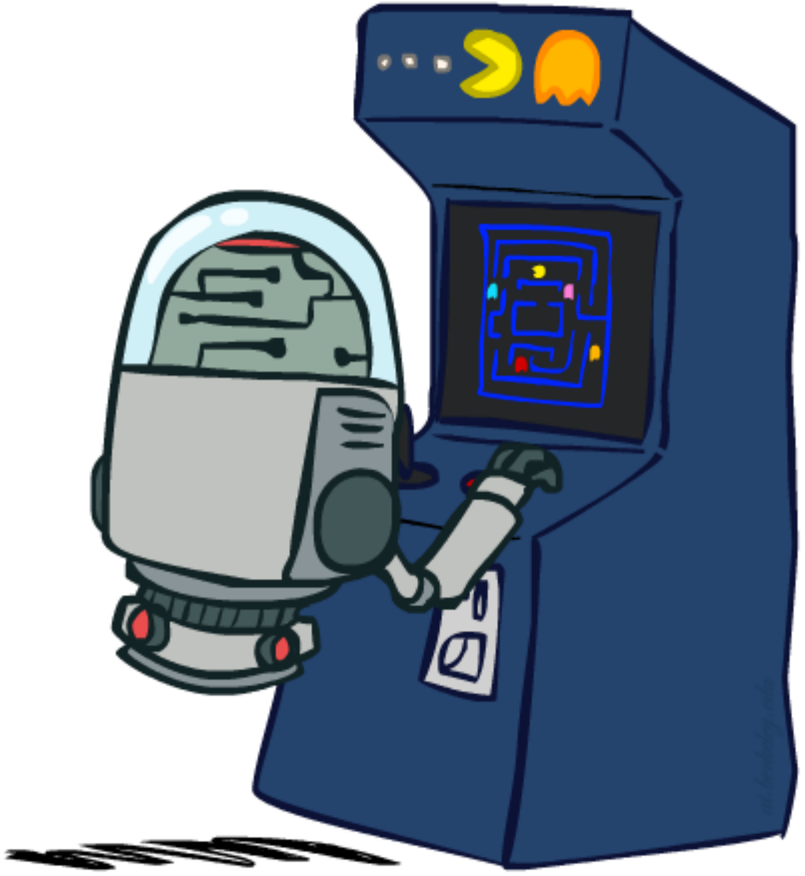
- Axes:
  - Deterministic or stochastic?
  - One, two, or more players?
  - Zero sum?
  - Perfect information (can you see the state)?



- Want algorithms for calculating a strategy (policy) which recommends a move from each state

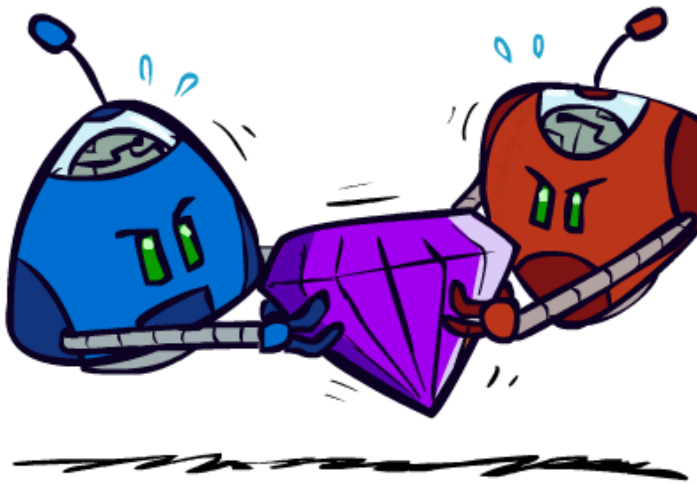
Deterministic Games

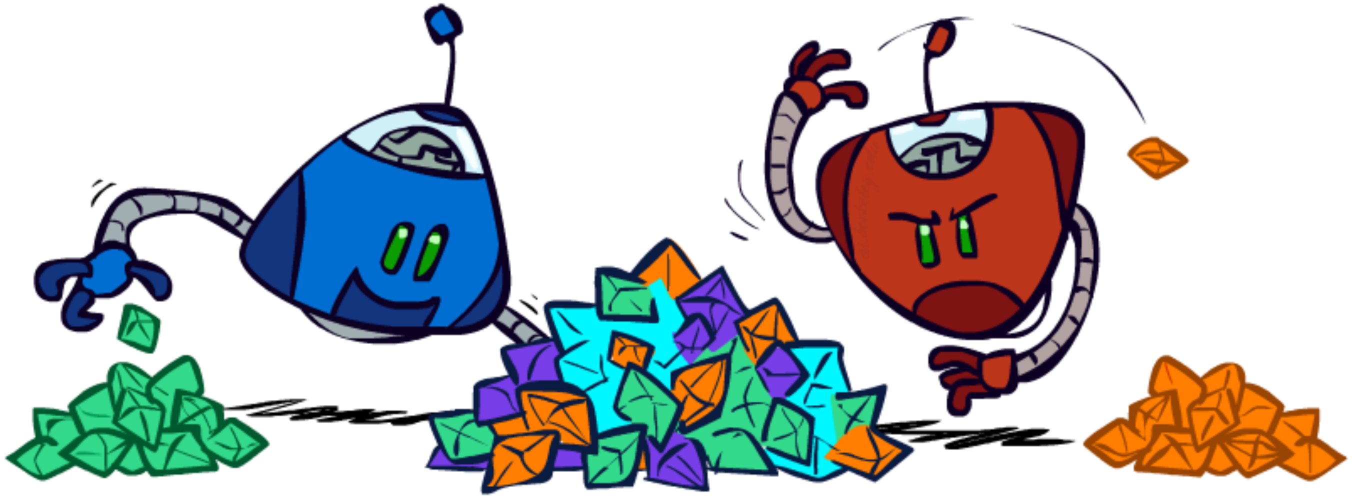
- Many possible formalizations, one is:
  - States: S (start at s_0)
  - Players: P={1...N} (usually take turns)
  - Actions: A (may depend on player / state)
  - Transition Function: SxA $\rightarrow$ S
  - Terminal Test: S $\rightarrow$ {t,f}
  - Terminal Utilities: SxP $\rightarrow$ R



- Solution for a player is a **policy**: S $\rightarrow$ A

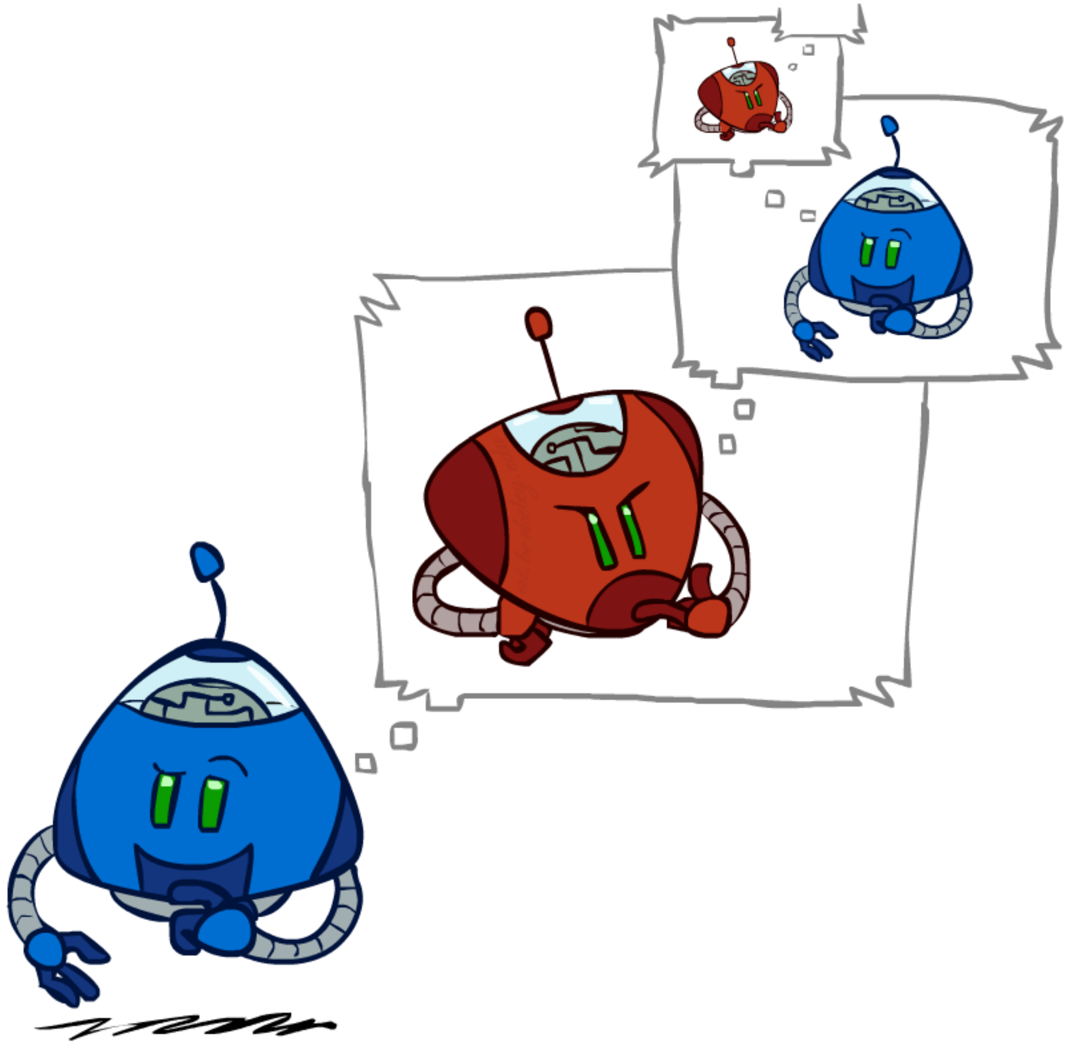
Zero-Sum Games





- Zero-Sum Games
  - Agents have opposite utilities (values on outcomes)
  - Lets us think of a single value that one maximizes and the other minimizes
  - Adversarial, pure competition
- General Games
  - Agents have independent utilities (values on outcomes)
  - Cooperation, indifference, competition, and more are all possible
  - More later on non-zero-sum games

Adversarial Search



Single-Agent Trees

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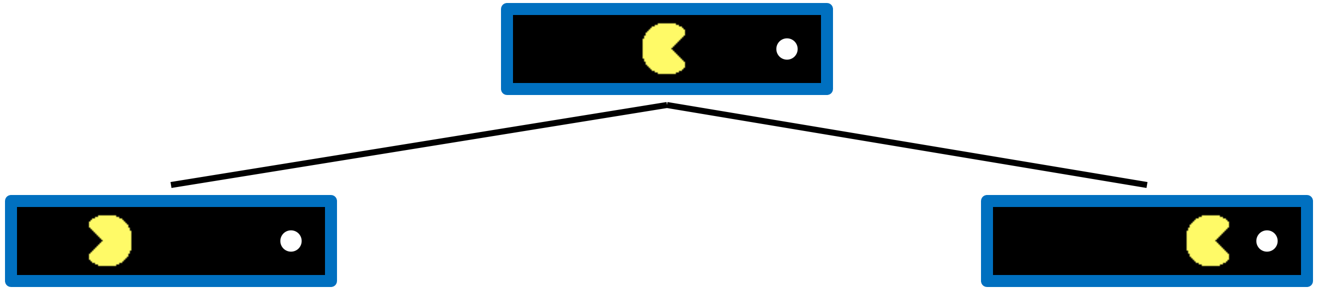

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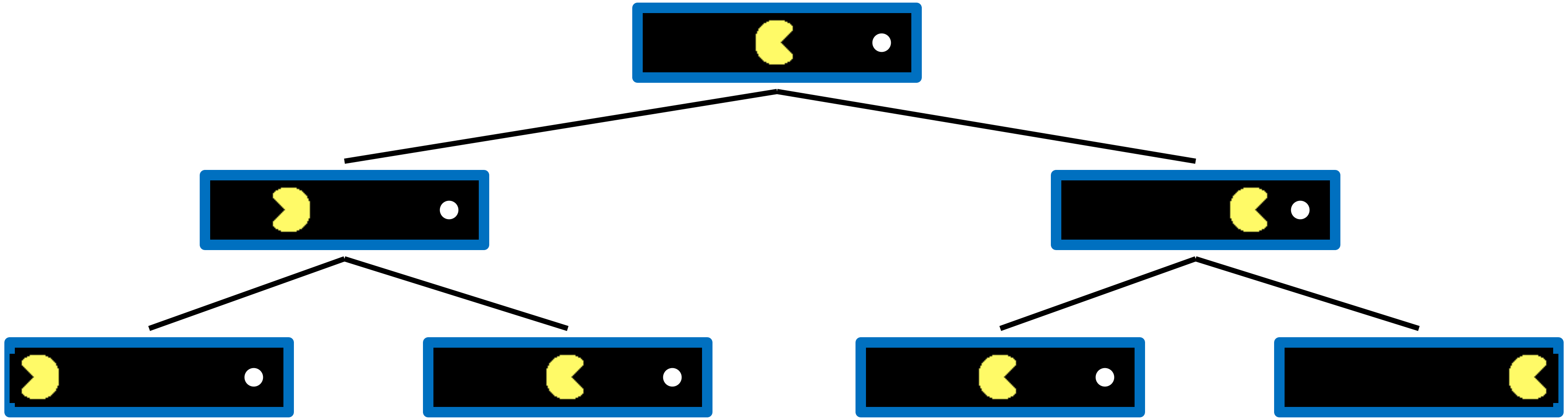

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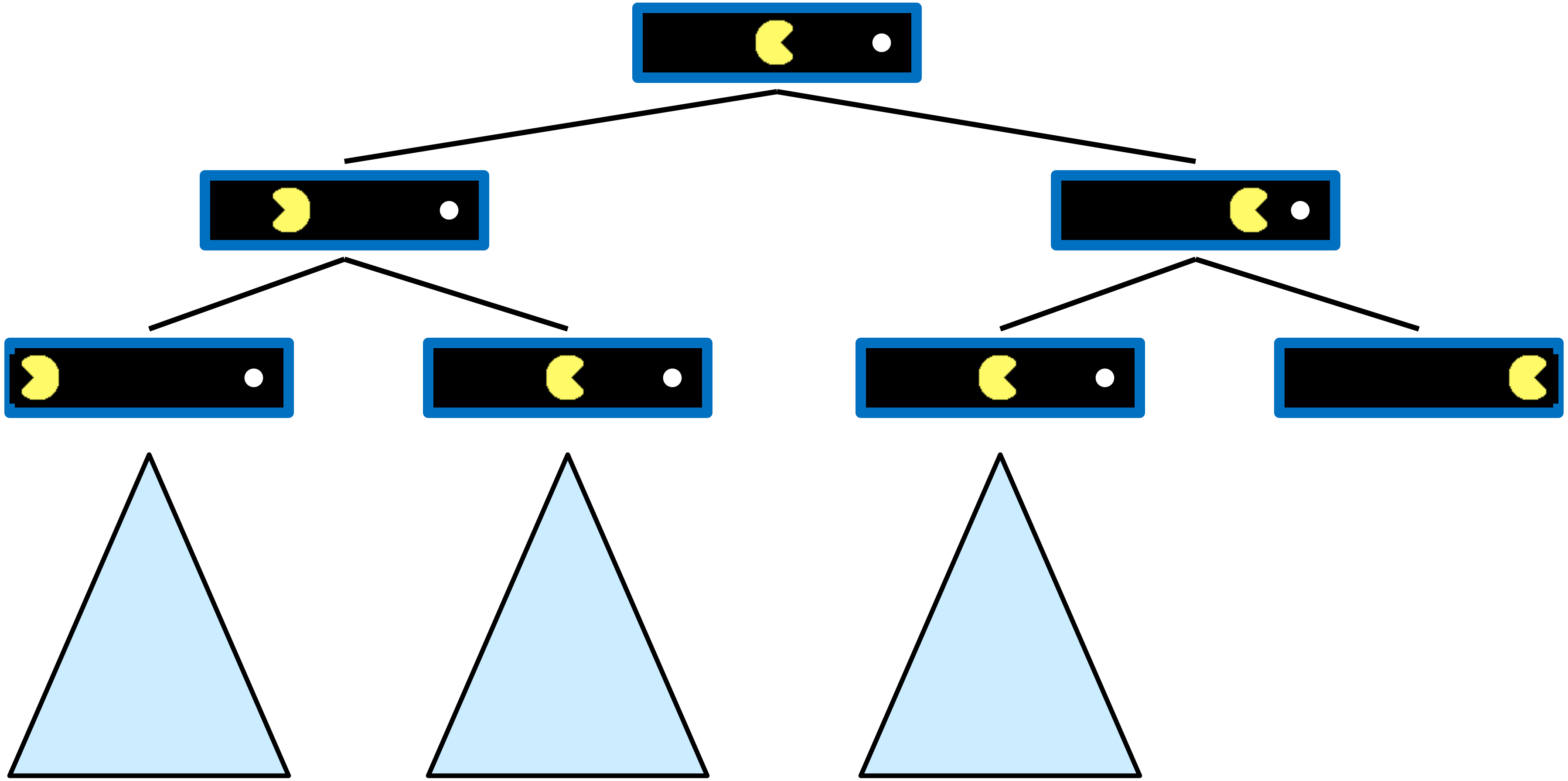

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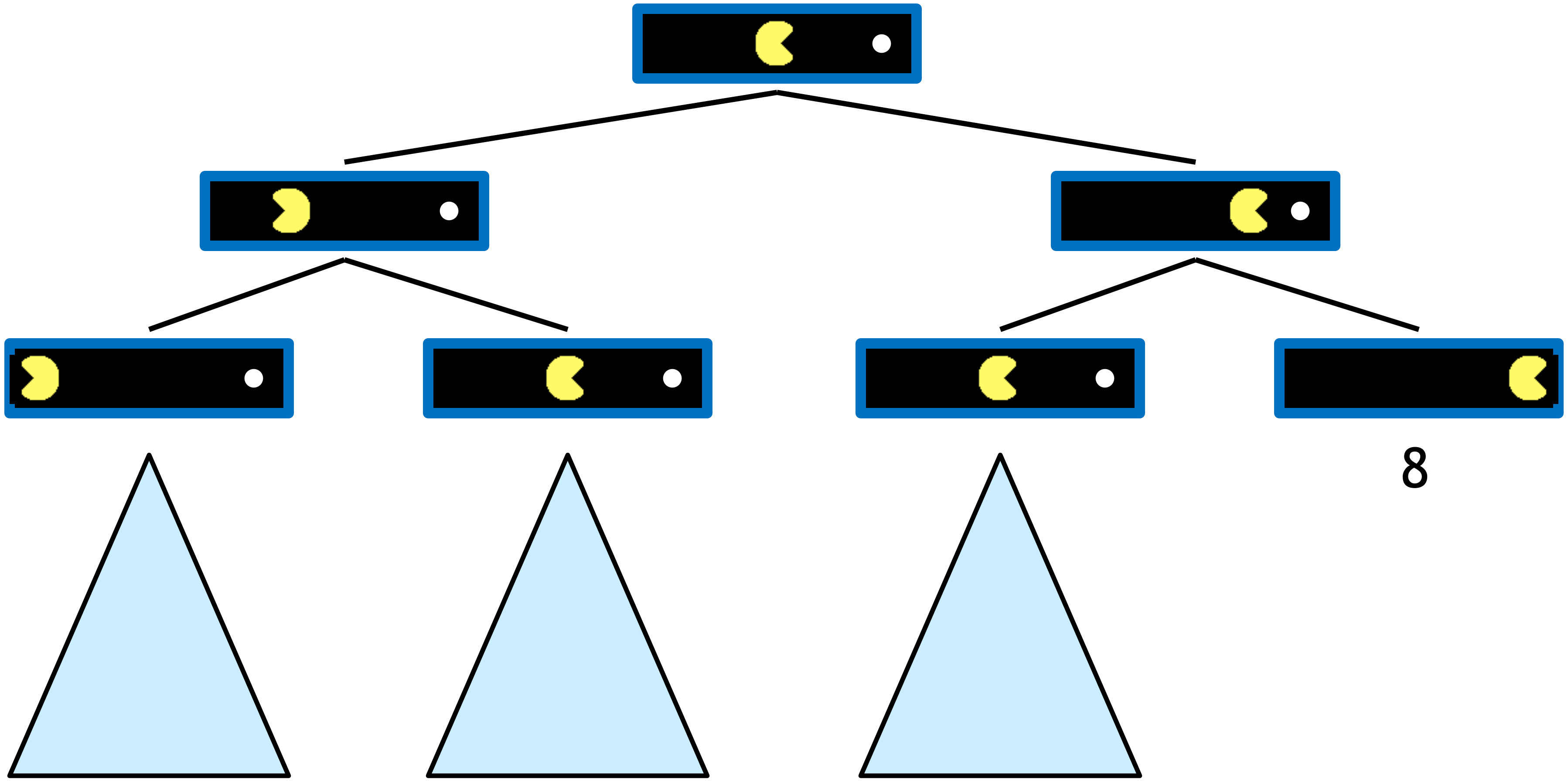

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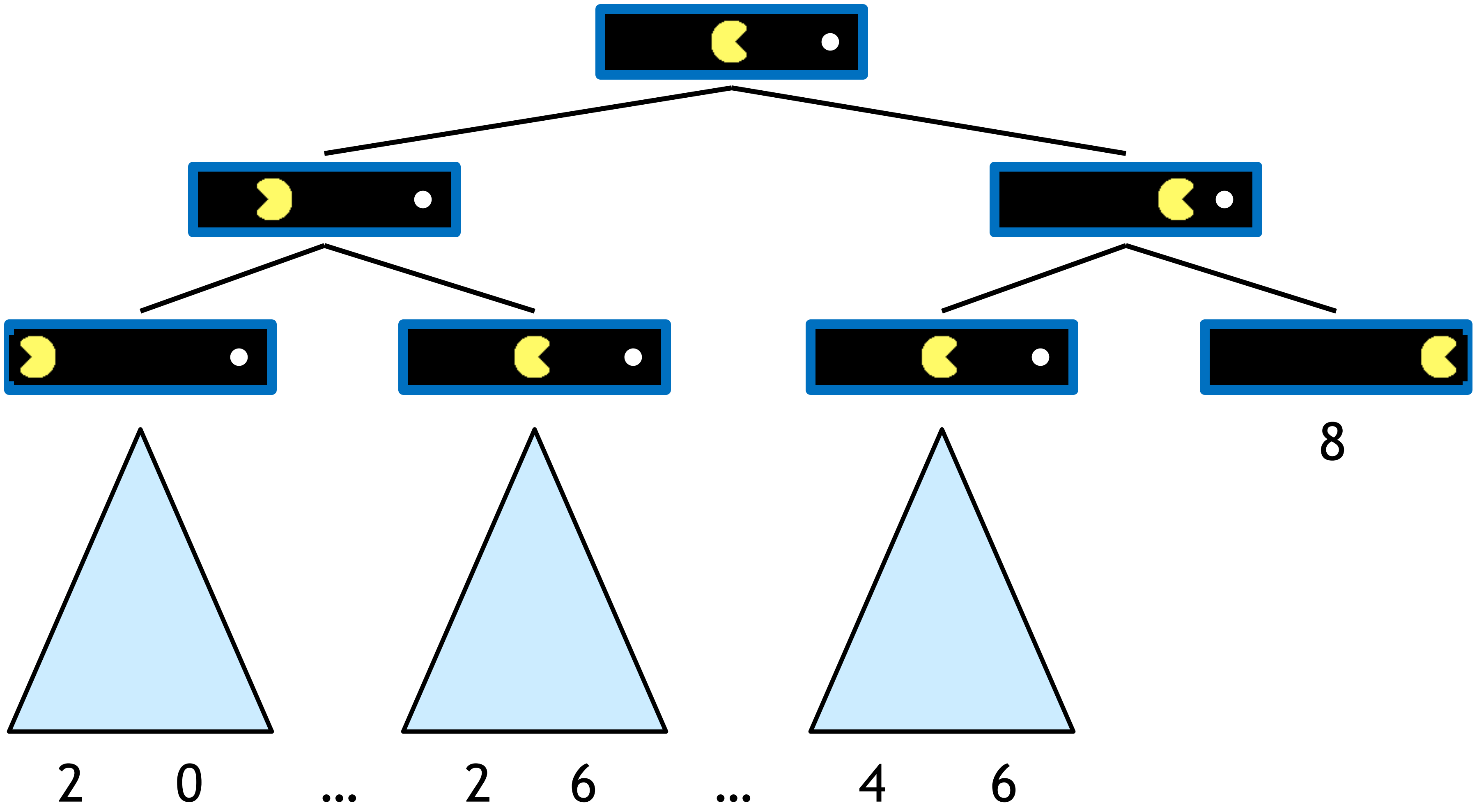
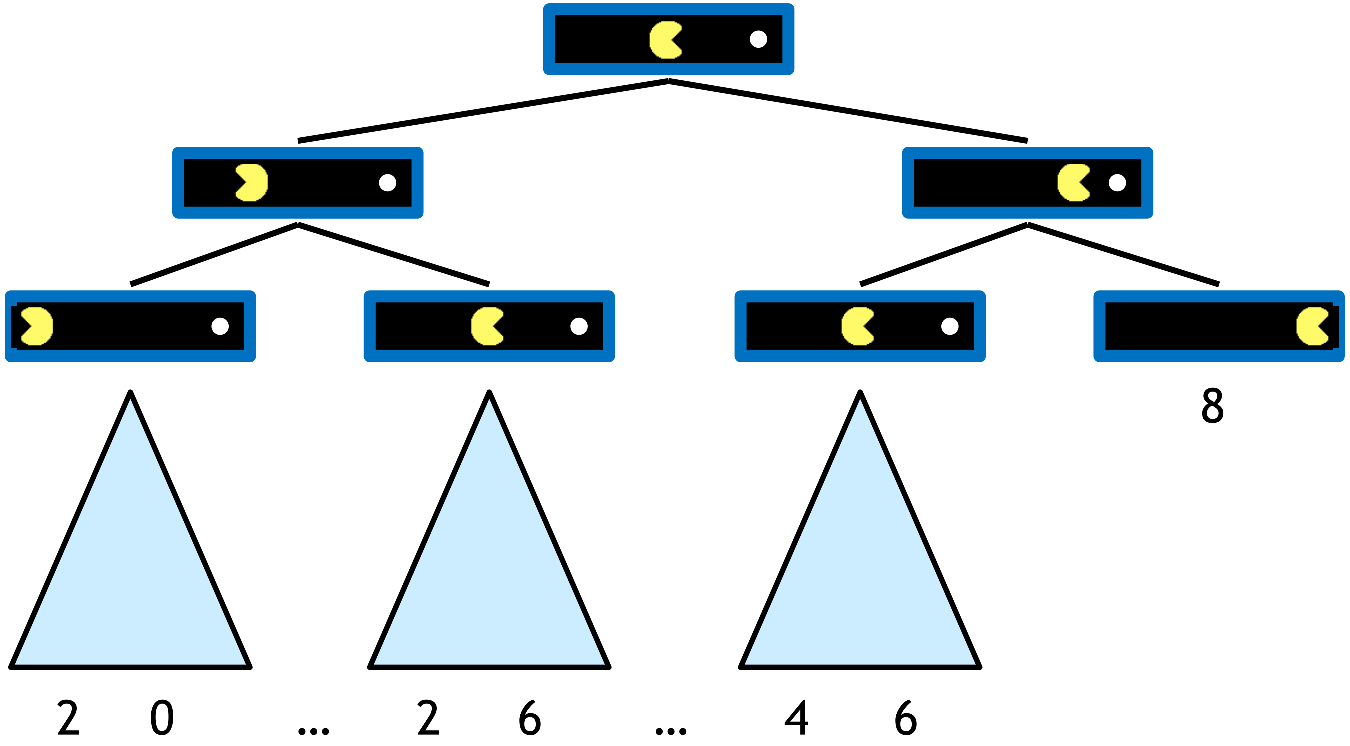
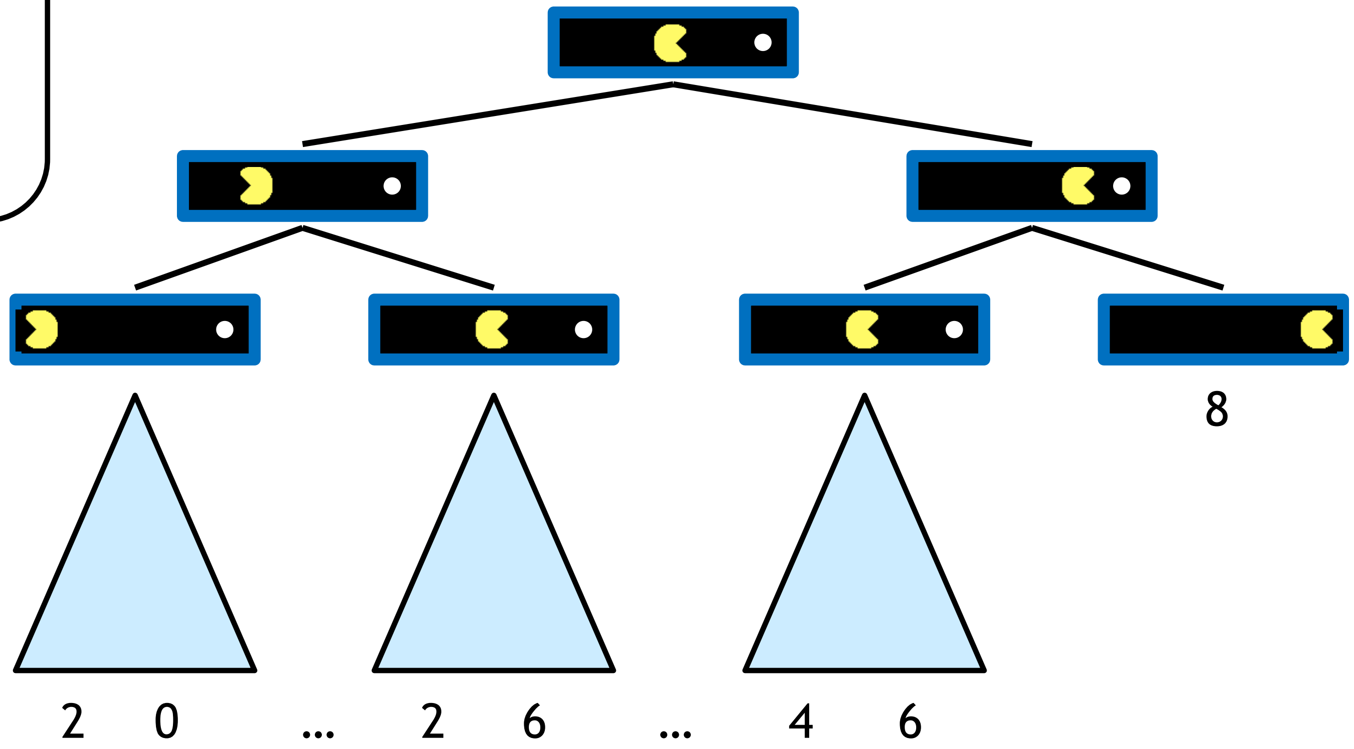


Value of a State



Value of a State

Value of a state:
The best achievable outcome (utility) from that state



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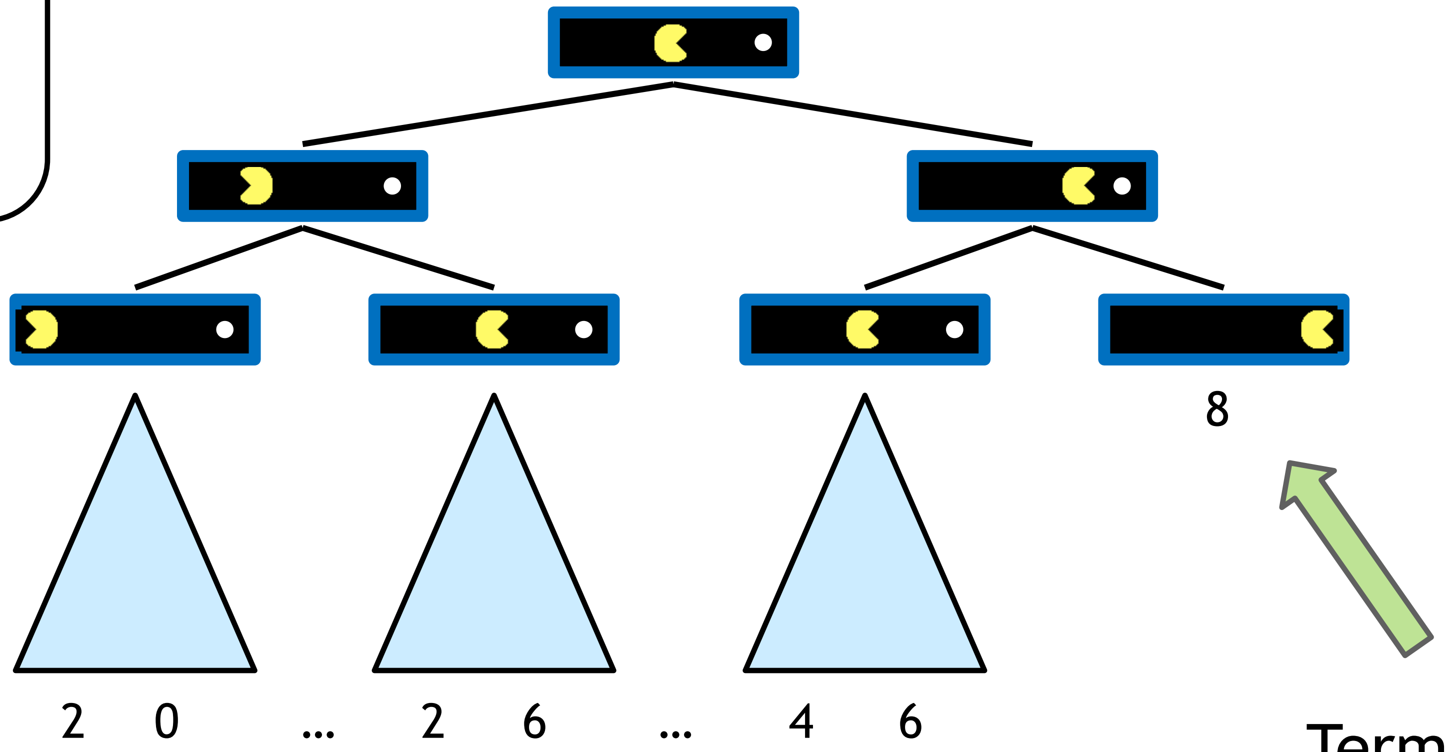

Terminal States:

$V(s) = \text{known}$

Value of a State

Value of a state:
The best
achievable
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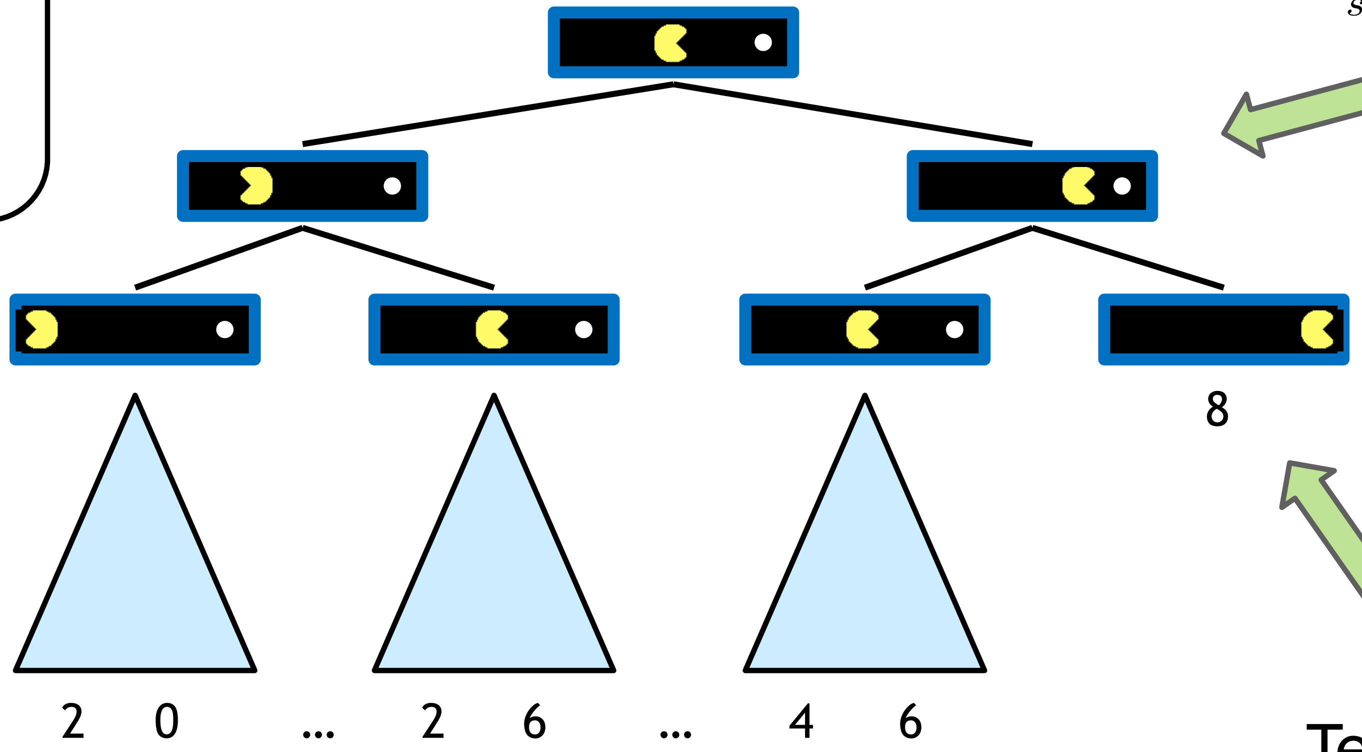
Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$



Terminal States:

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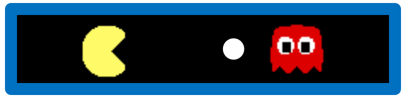
Adversarial Game Trees



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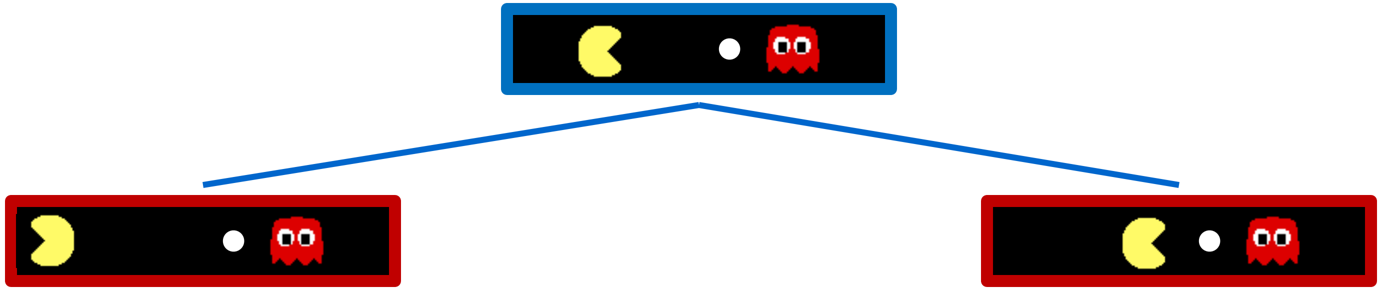

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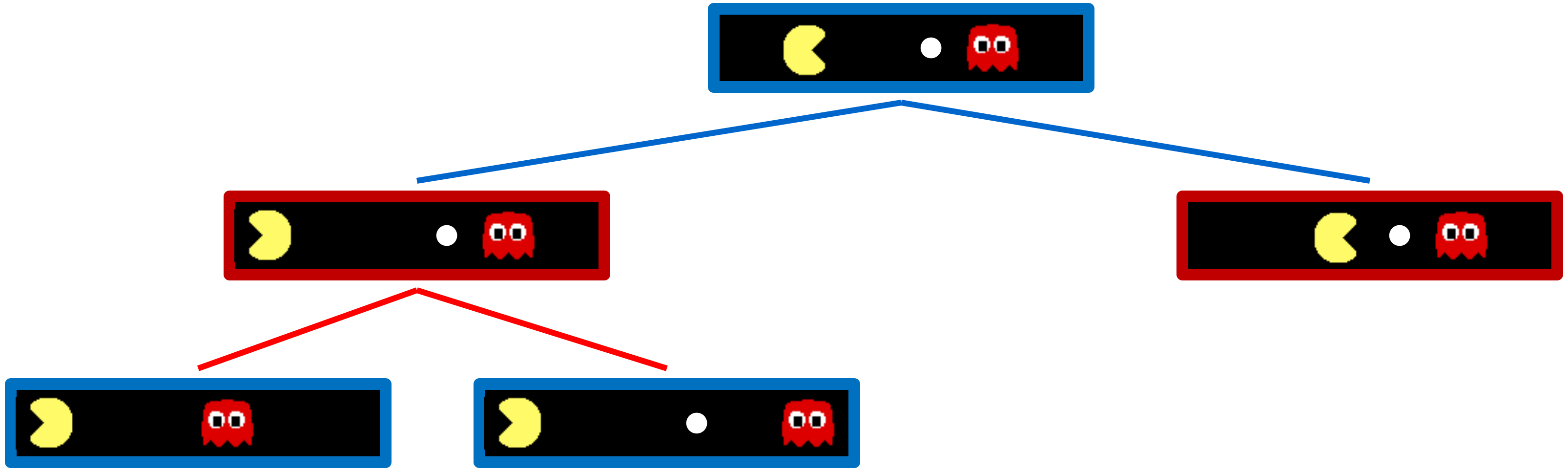

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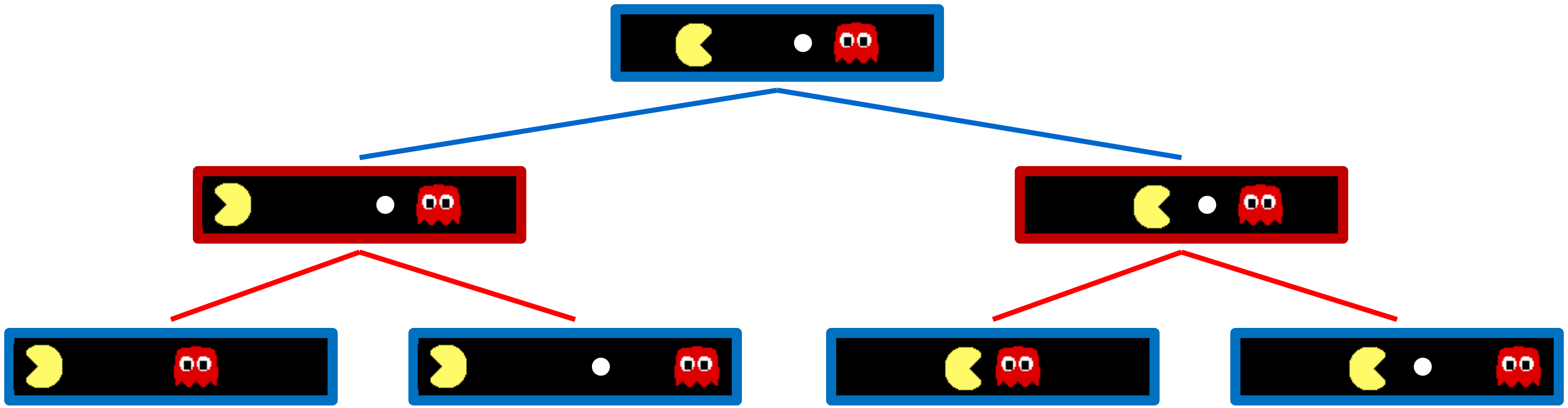

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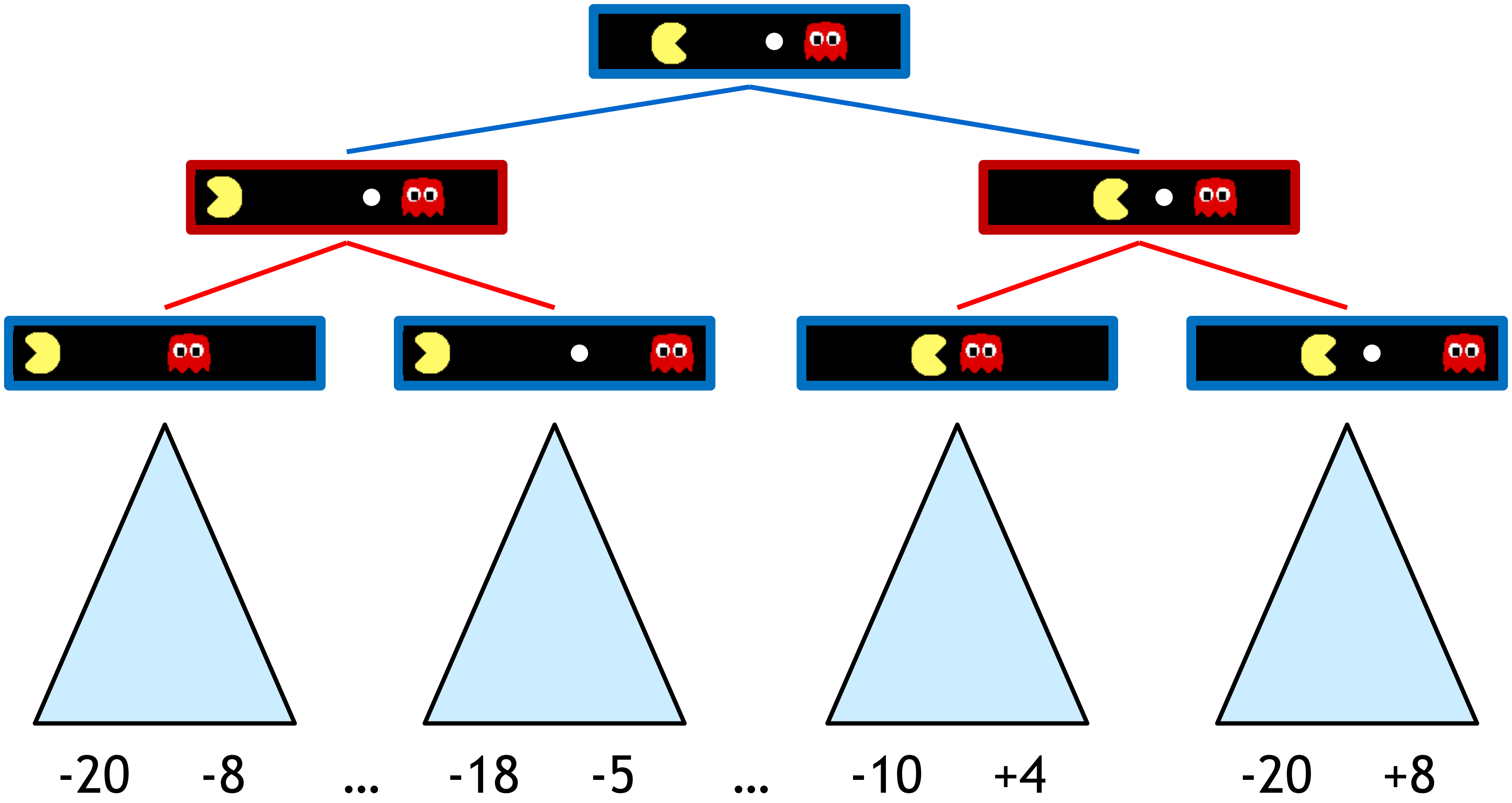
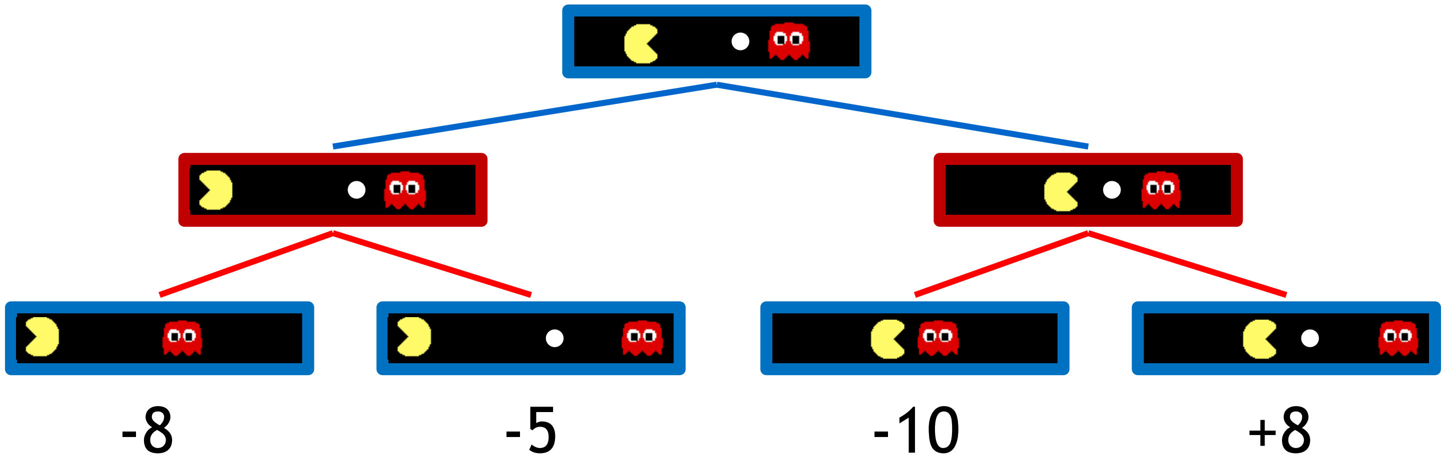


Minimax Values



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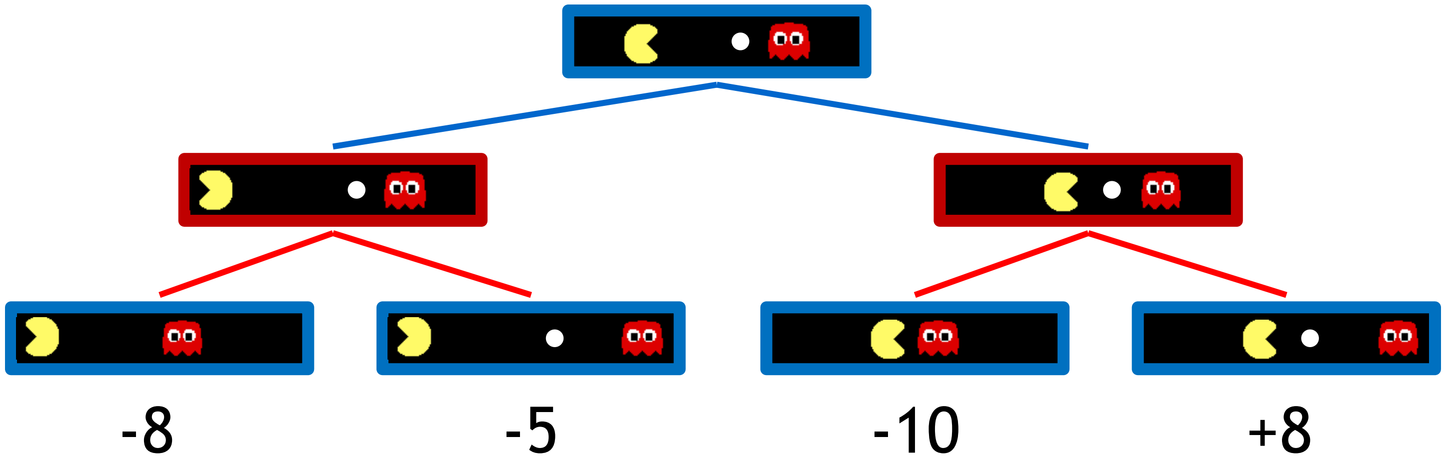

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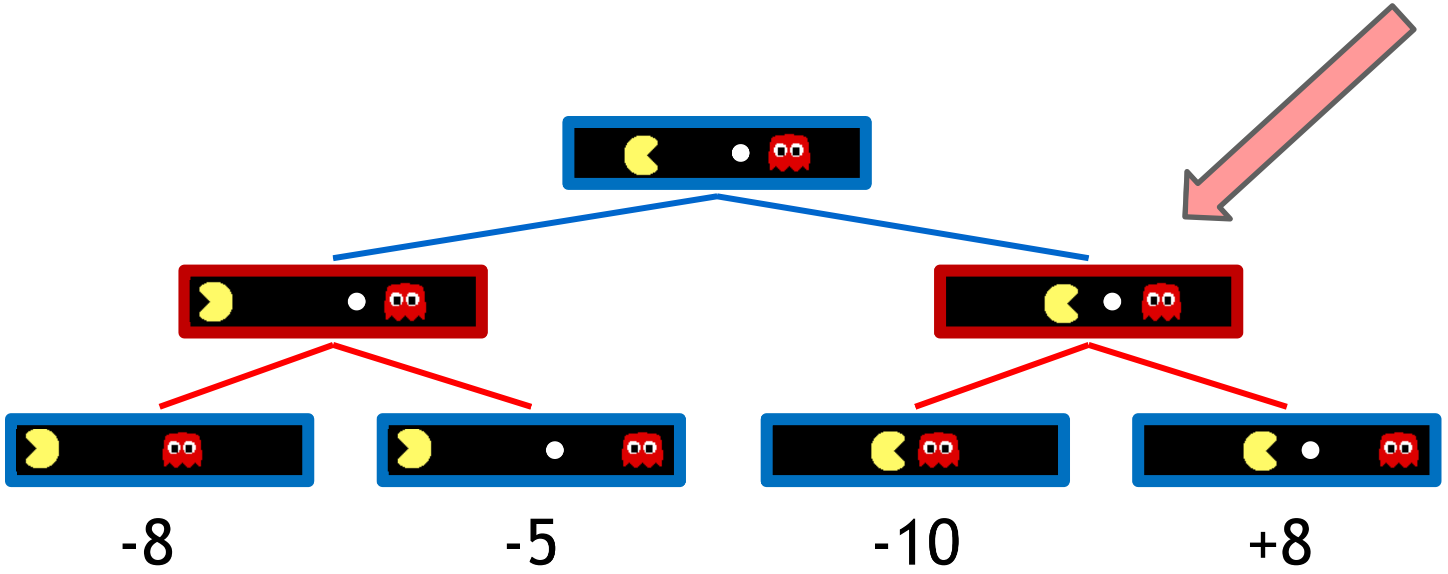
Minimax Values

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



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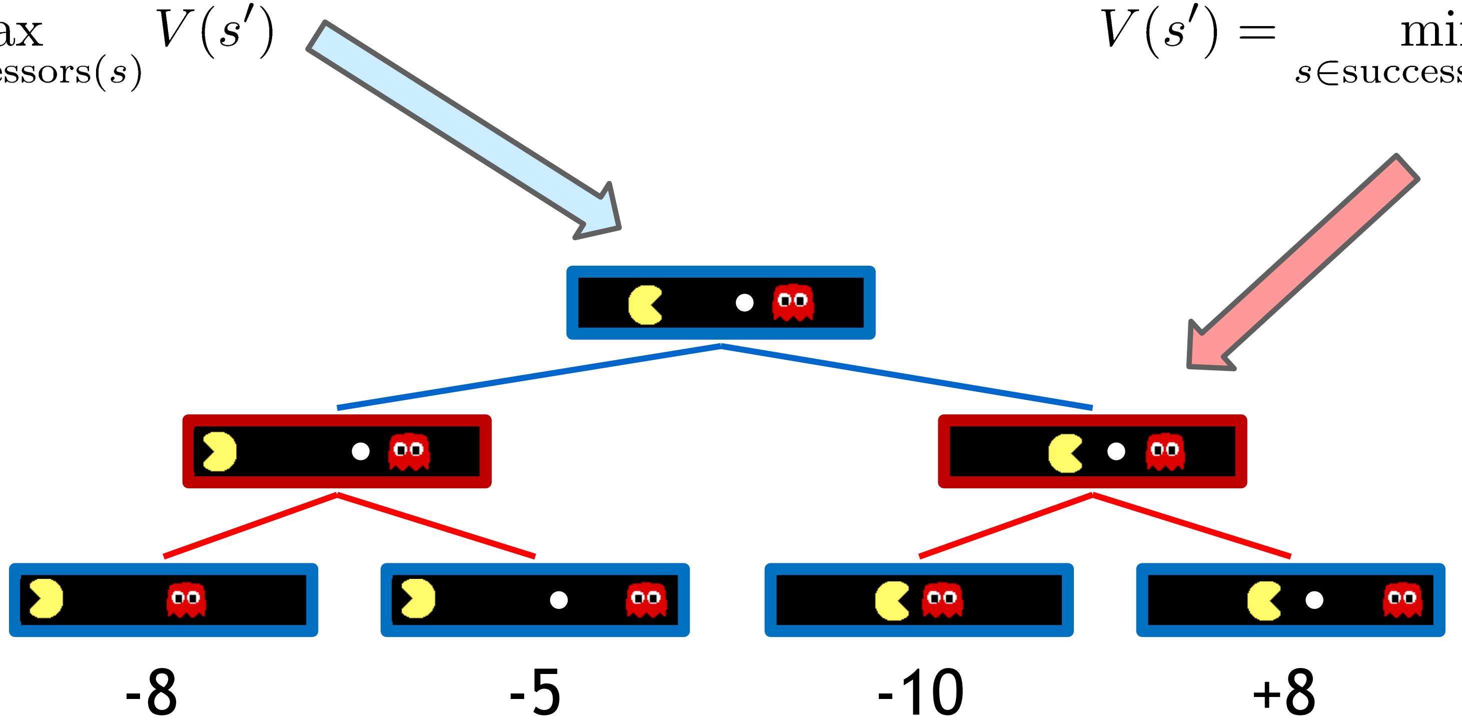

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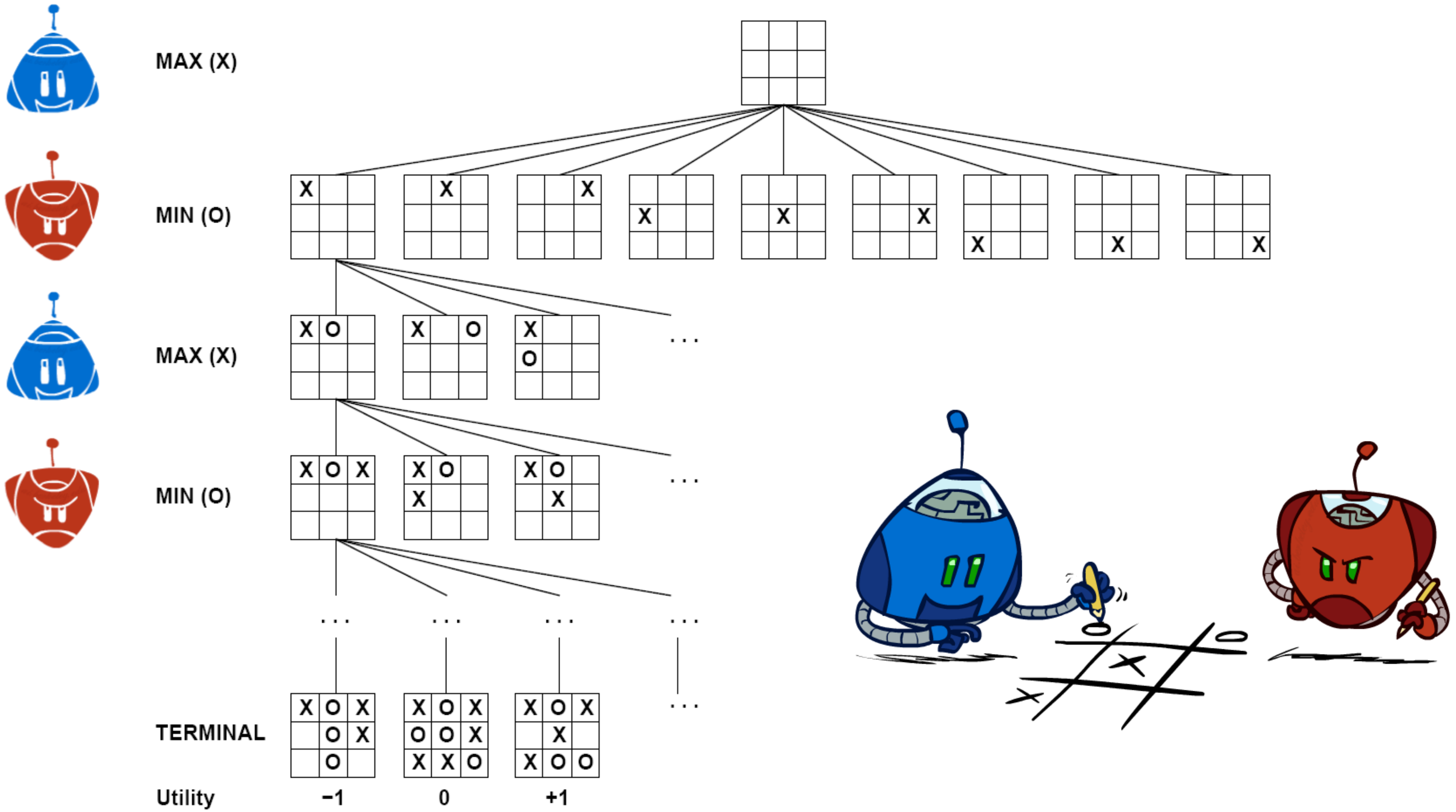
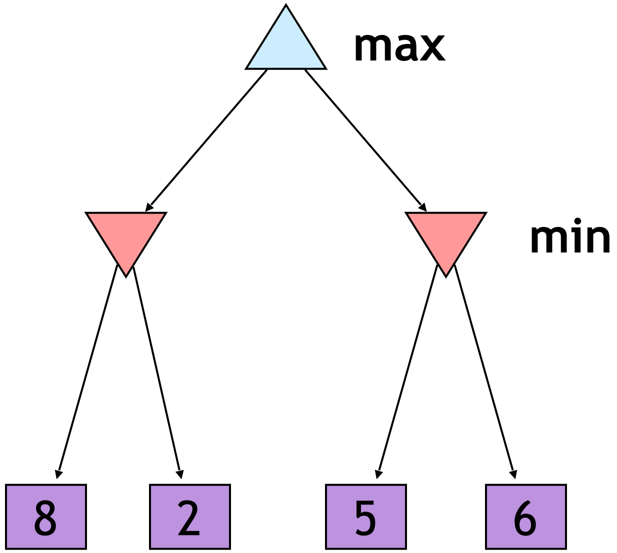
Tic-Tac-Toe Game Tree



Adversarial Search (Minimax)

- Deterministic, zero-sum games:
  - Tic-tac-toe, chess, checkers
  - One player maximizes result
  - The other minimizes result
- Minimax search:
  - A state-space search tree
  - Players alternate turns
  - Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



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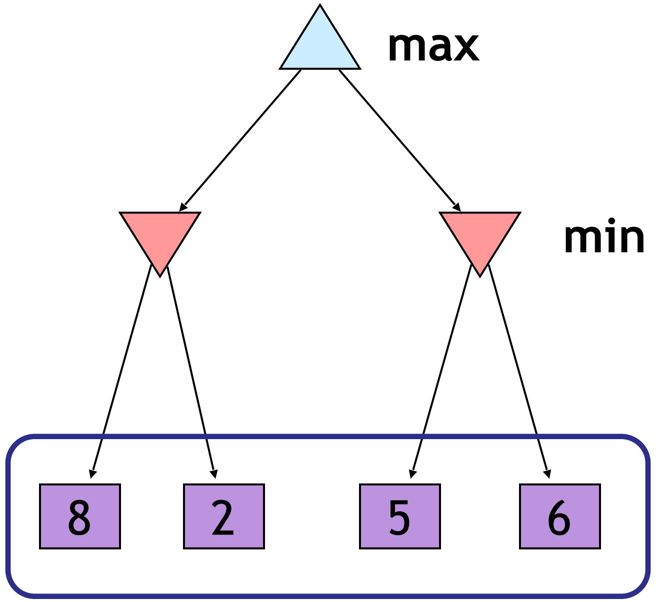

Terminal values: part of the game

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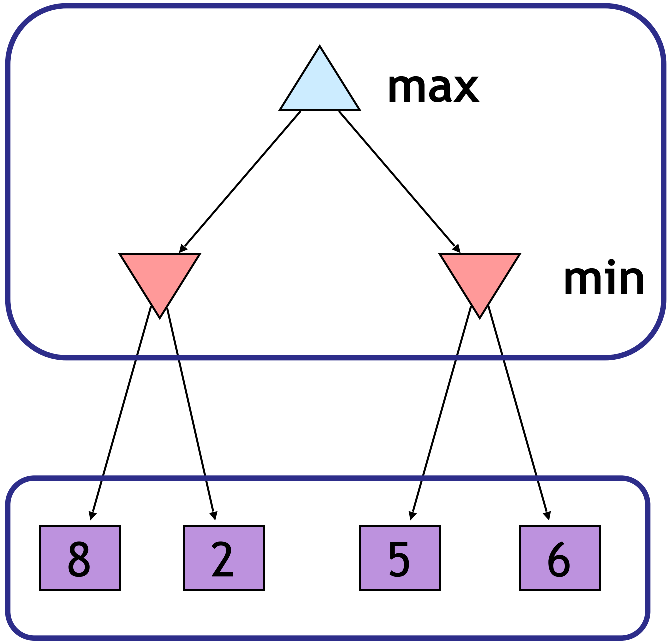
Minimax values: computed recursively



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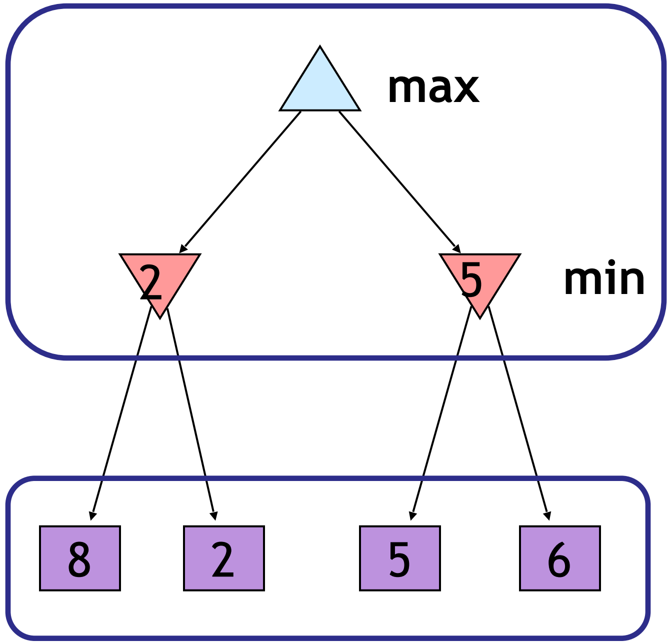

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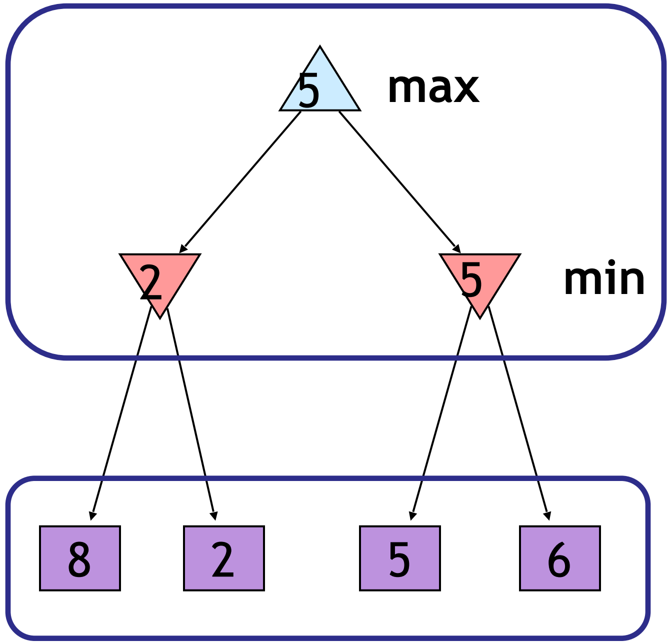
Minimax values: computed recursively



Terminal values: part of the game

Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

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$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

Minimax Implementation (Dispatch)

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def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
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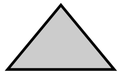
```

Minimax Example



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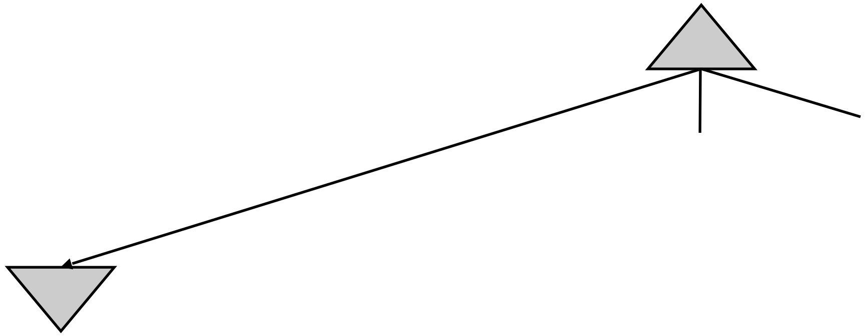

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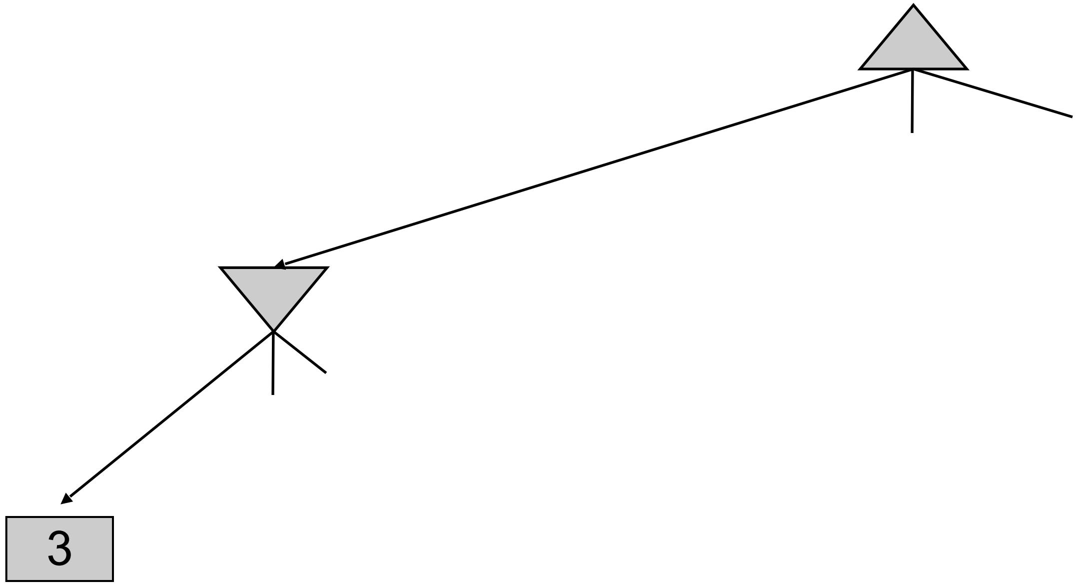

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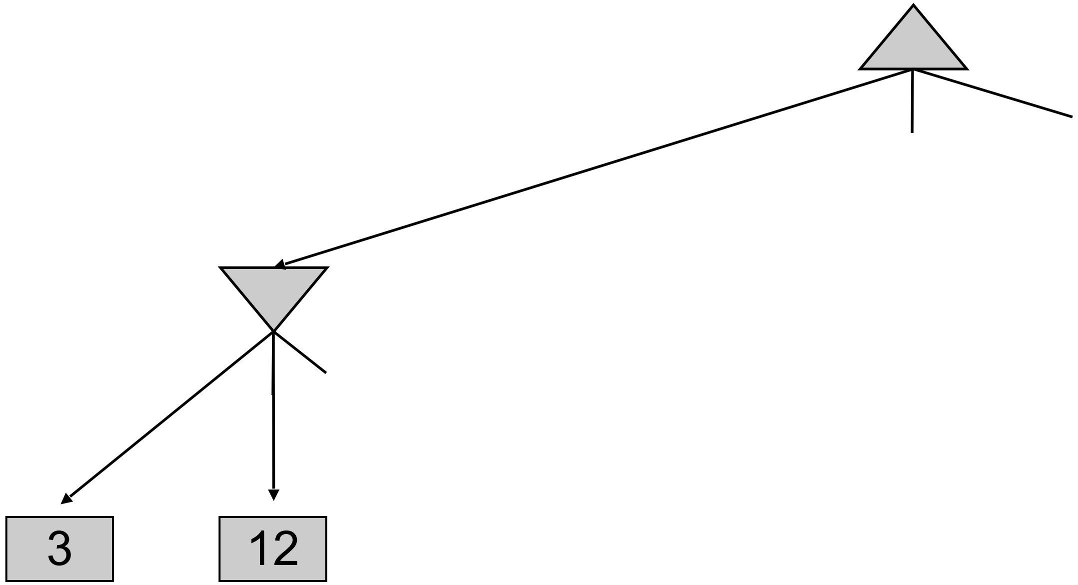

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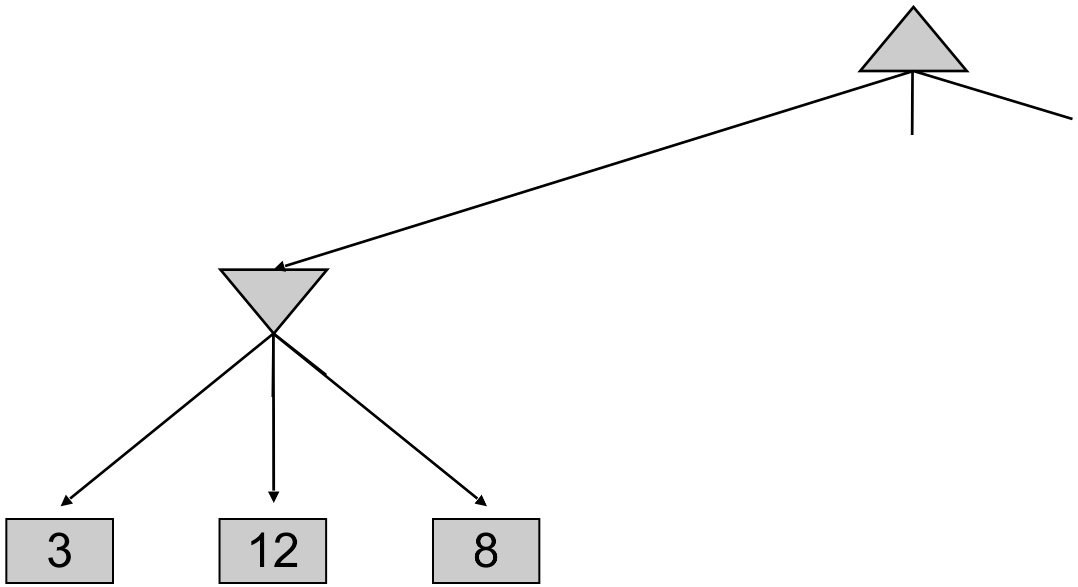

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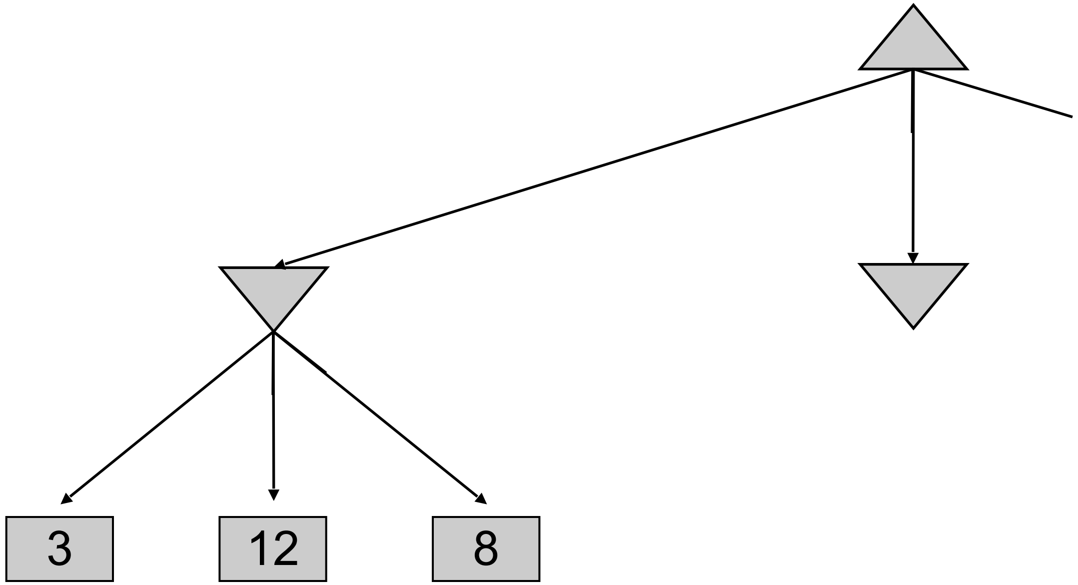

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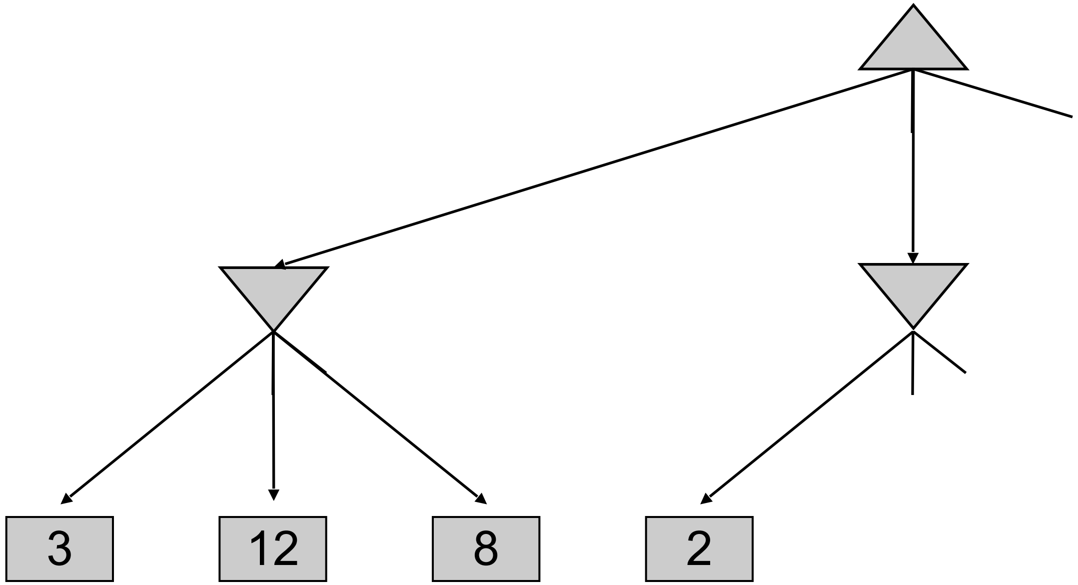

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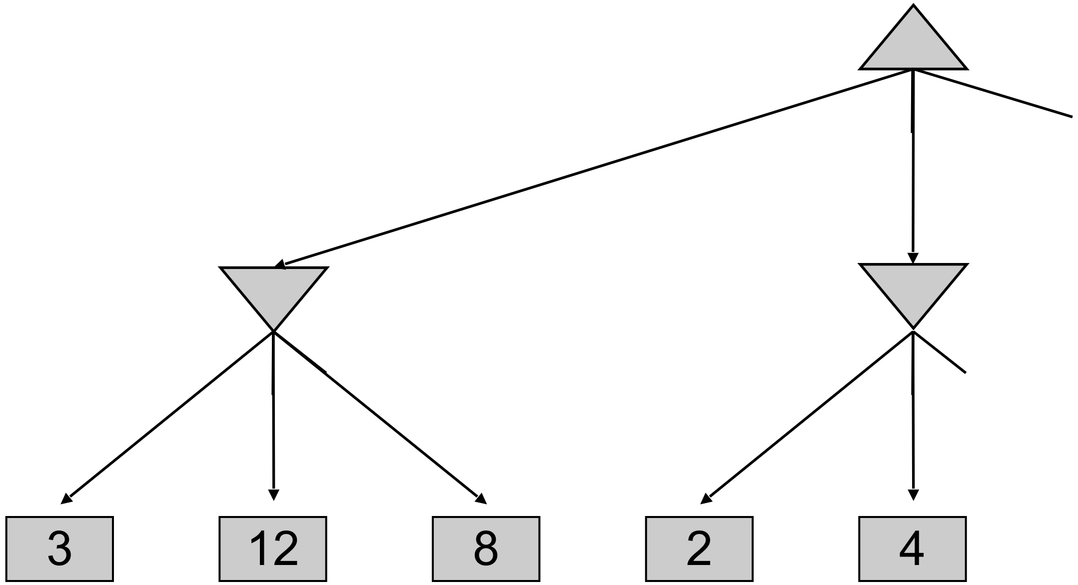

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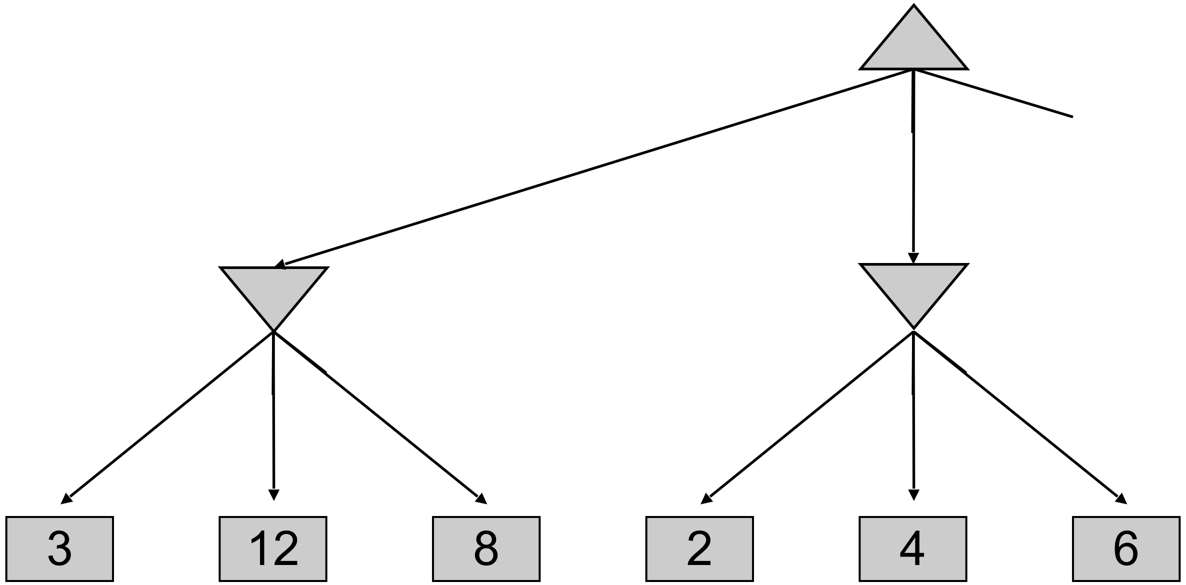

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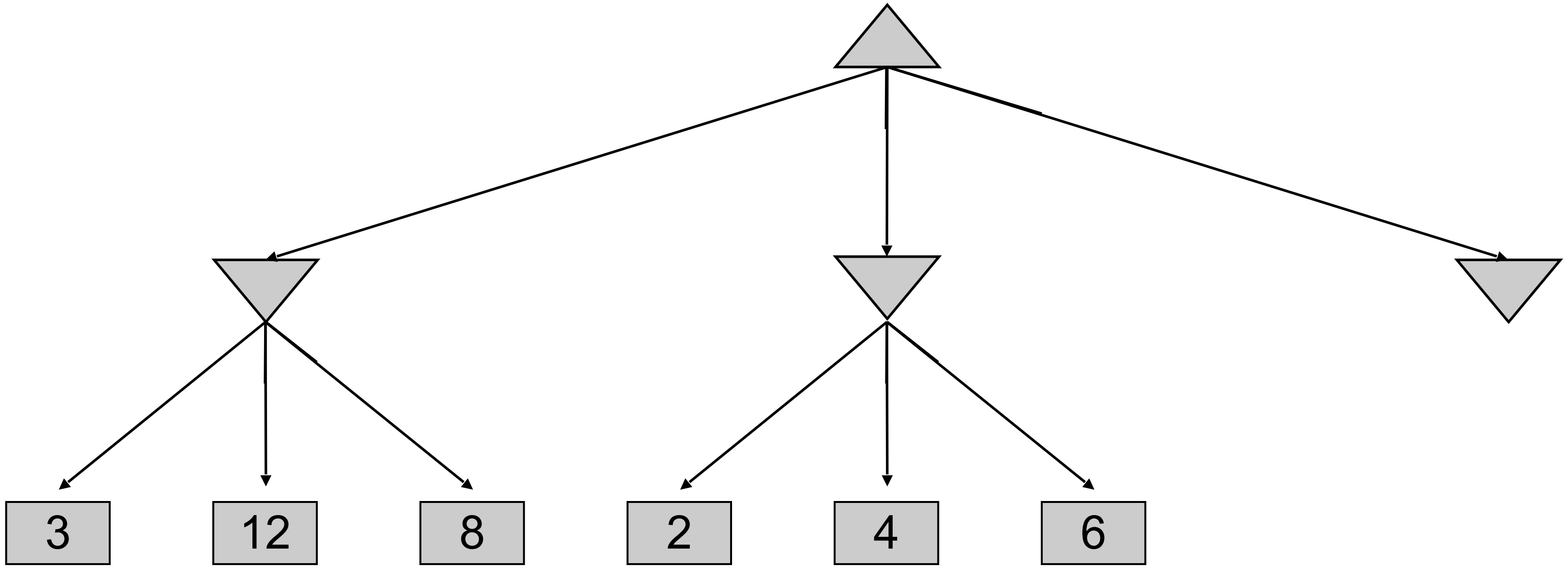

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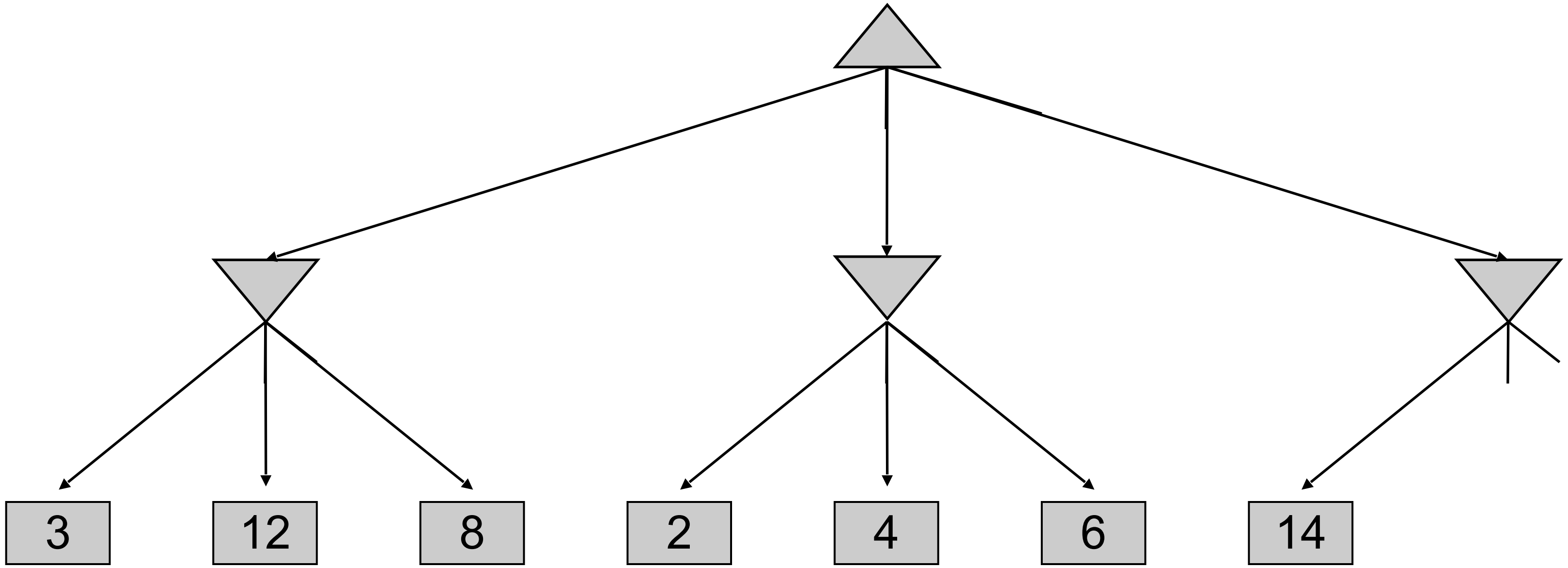

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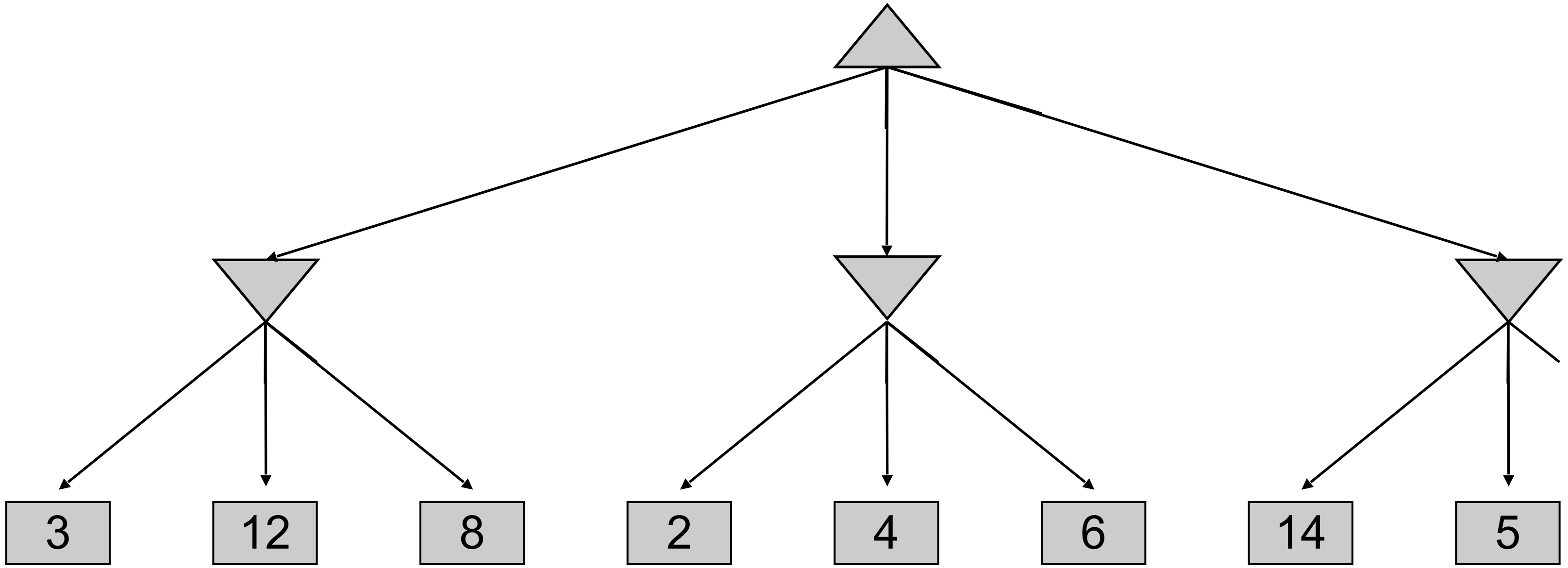

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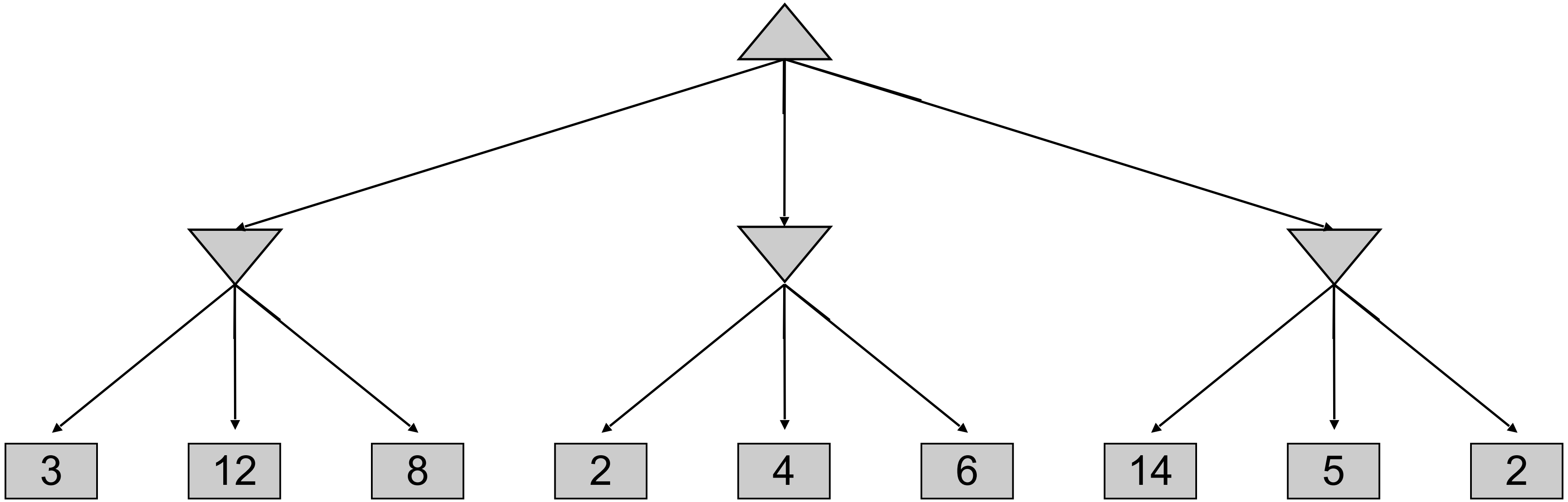
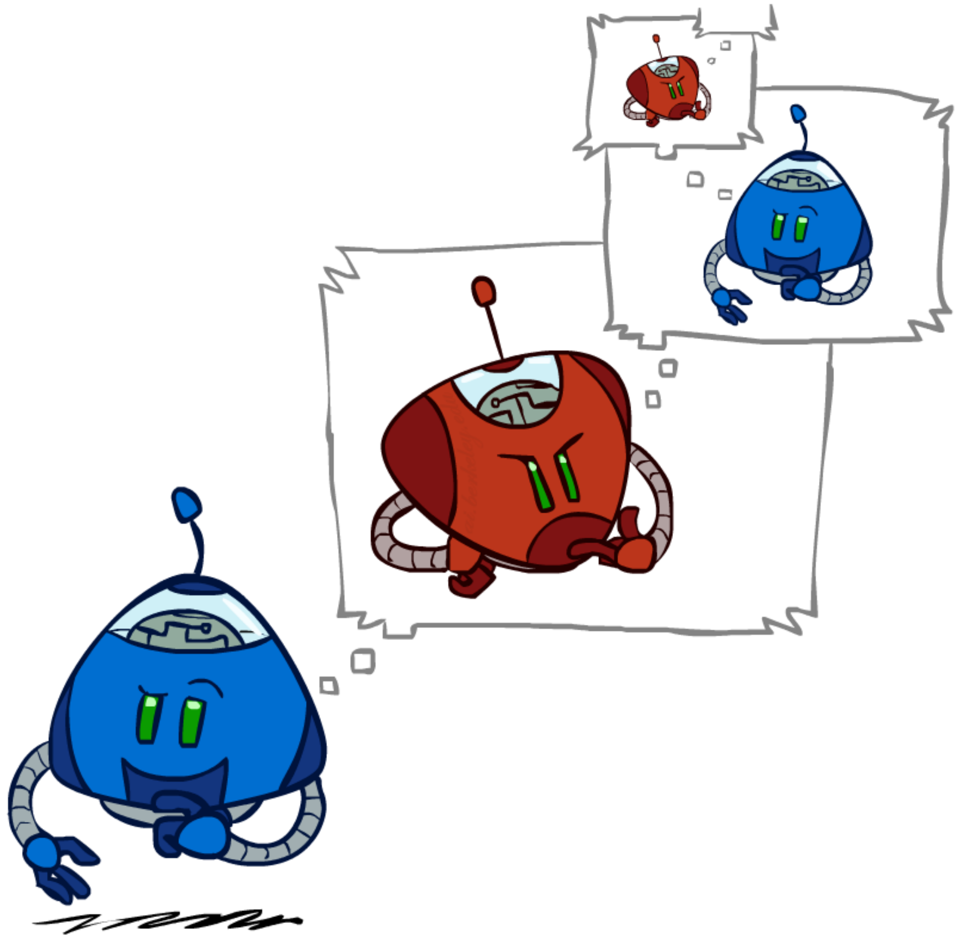


Minimax Efficiency

- How efficient is minimax?
  - Just like (exhaustive) DFS
  - Time: $O(b^m)$
  - Space: $O(bm)$

- Example: For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?



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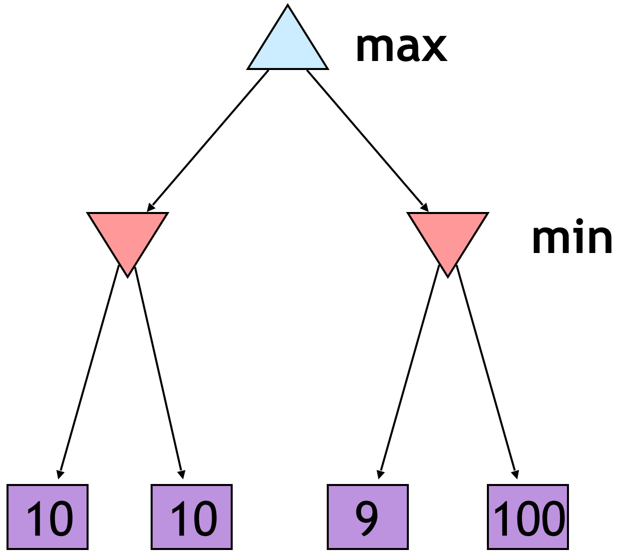

Optimal against a perfect player. Otherwise?

[Demo: min vs exp (L6D2, L6D3)]

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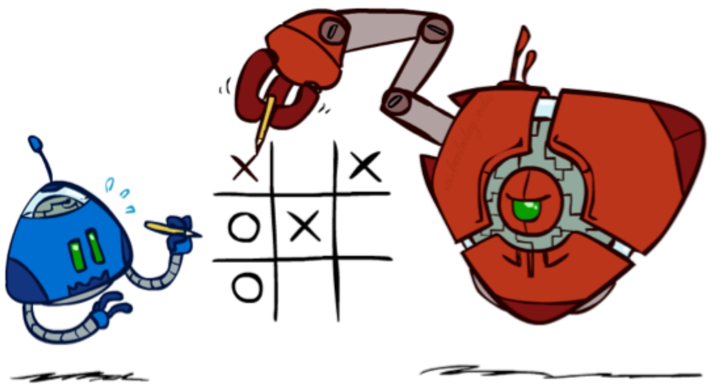

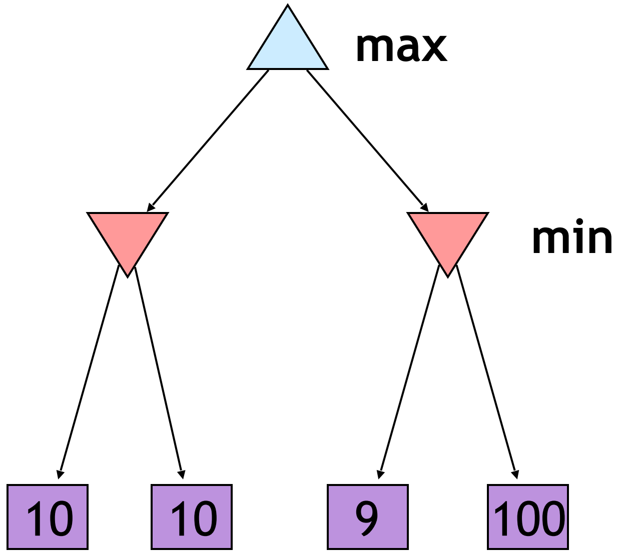

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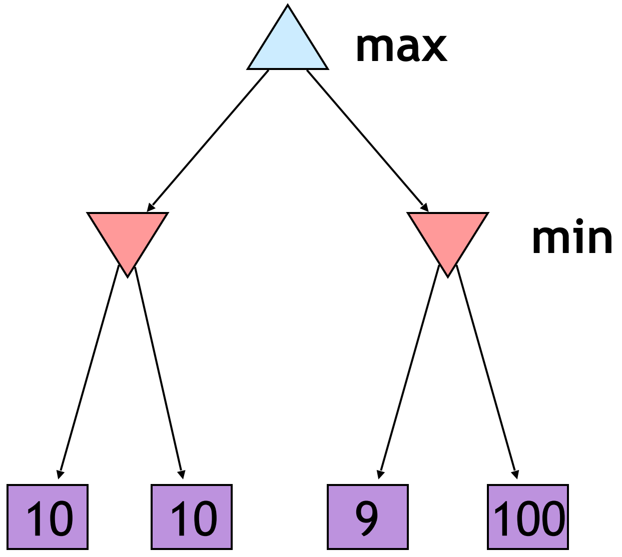

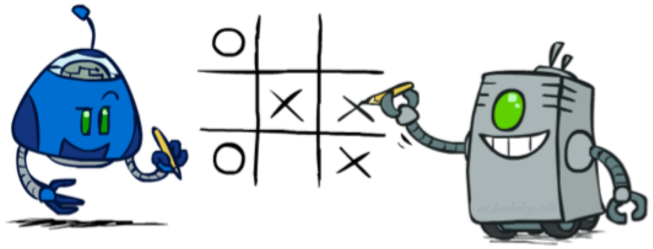

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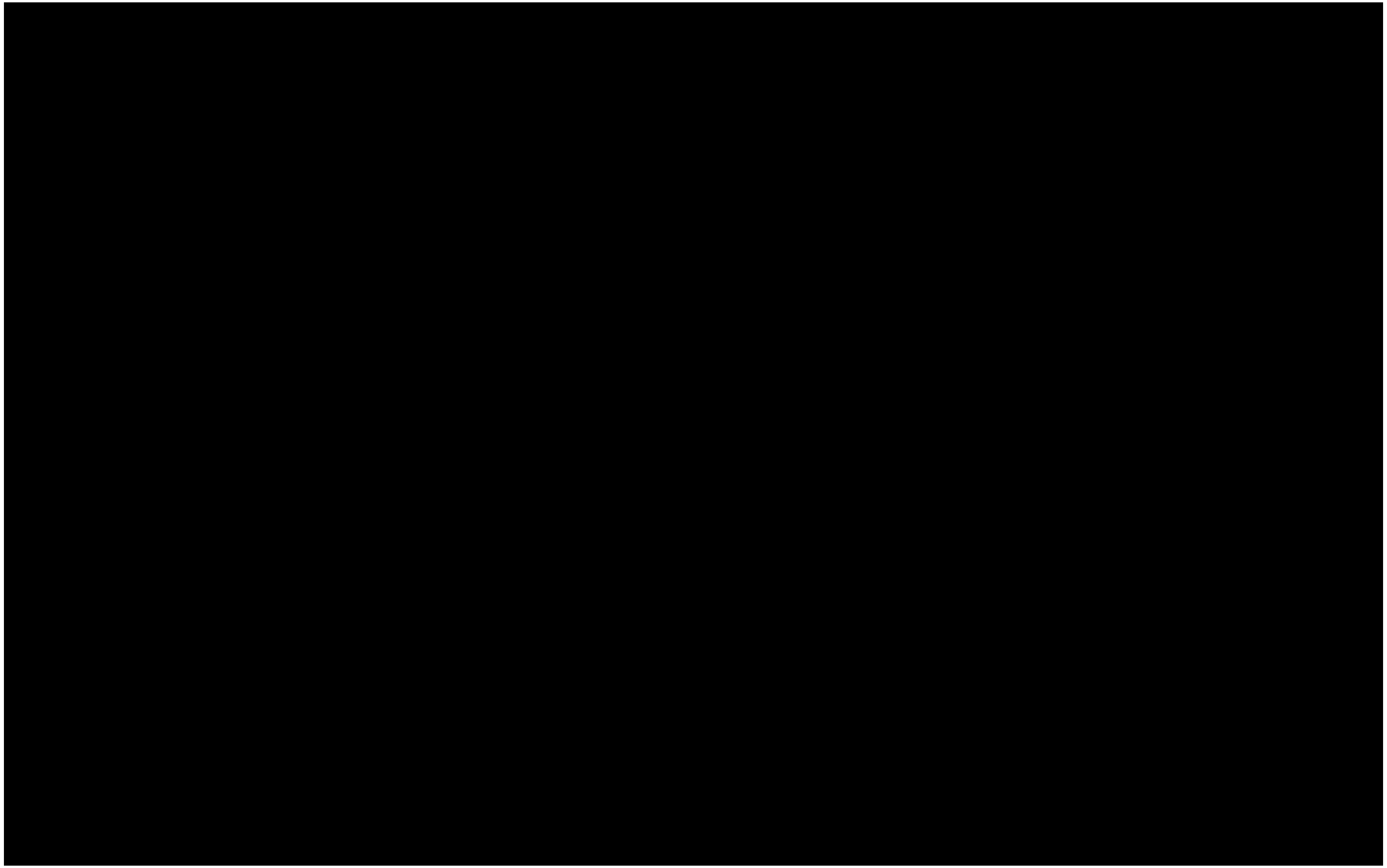
Video of Demo Min vs. Exp (Min)



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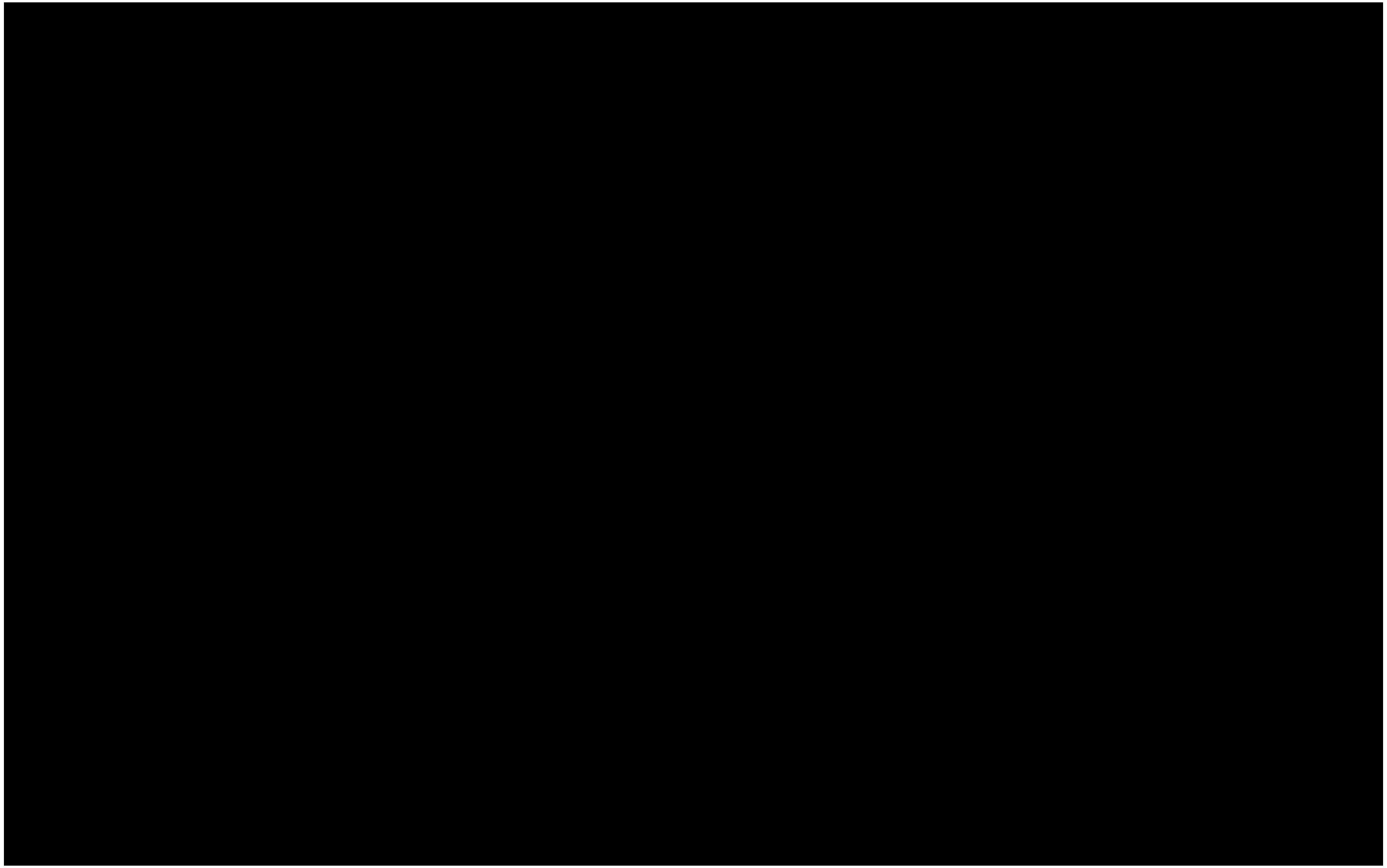

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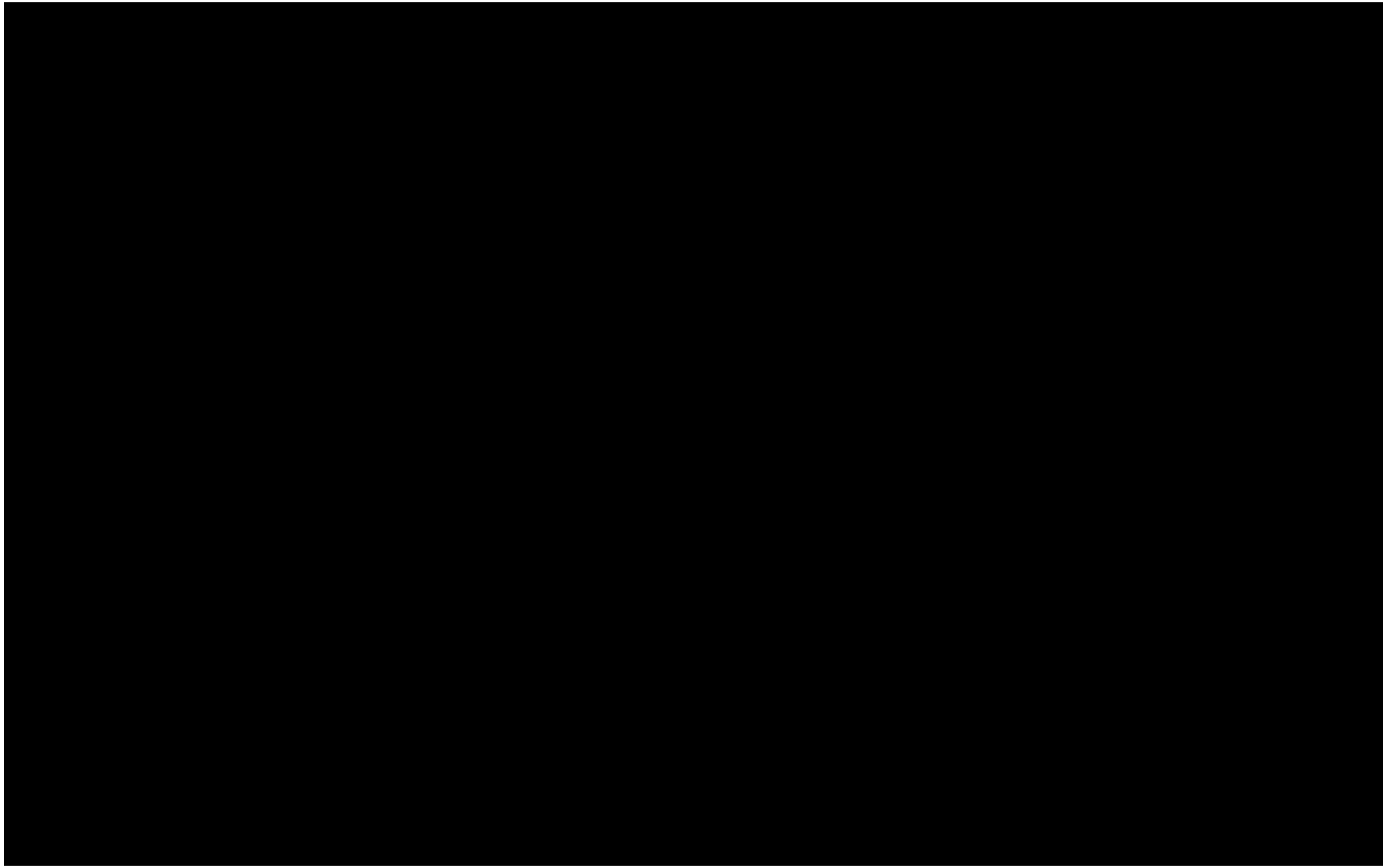

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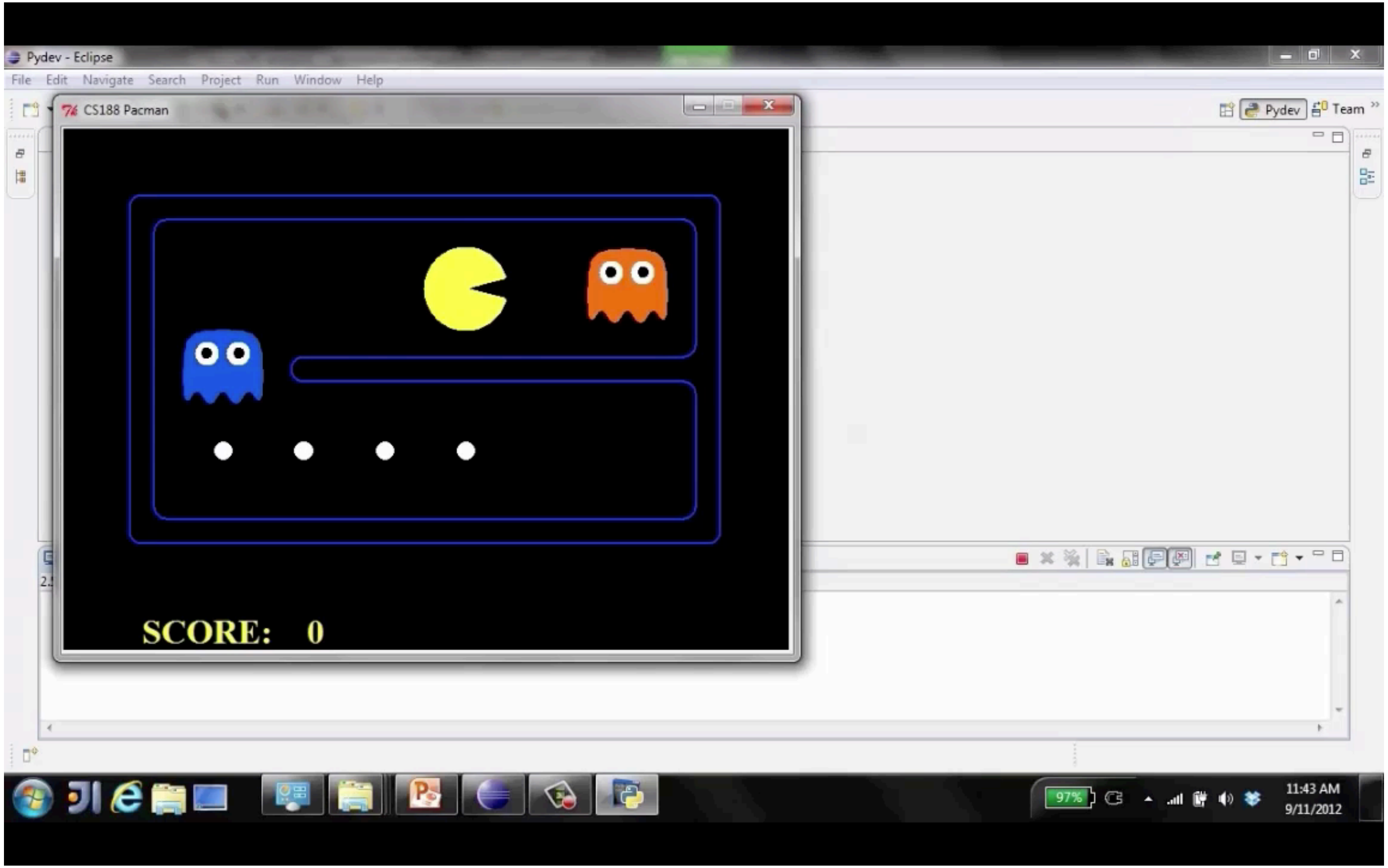

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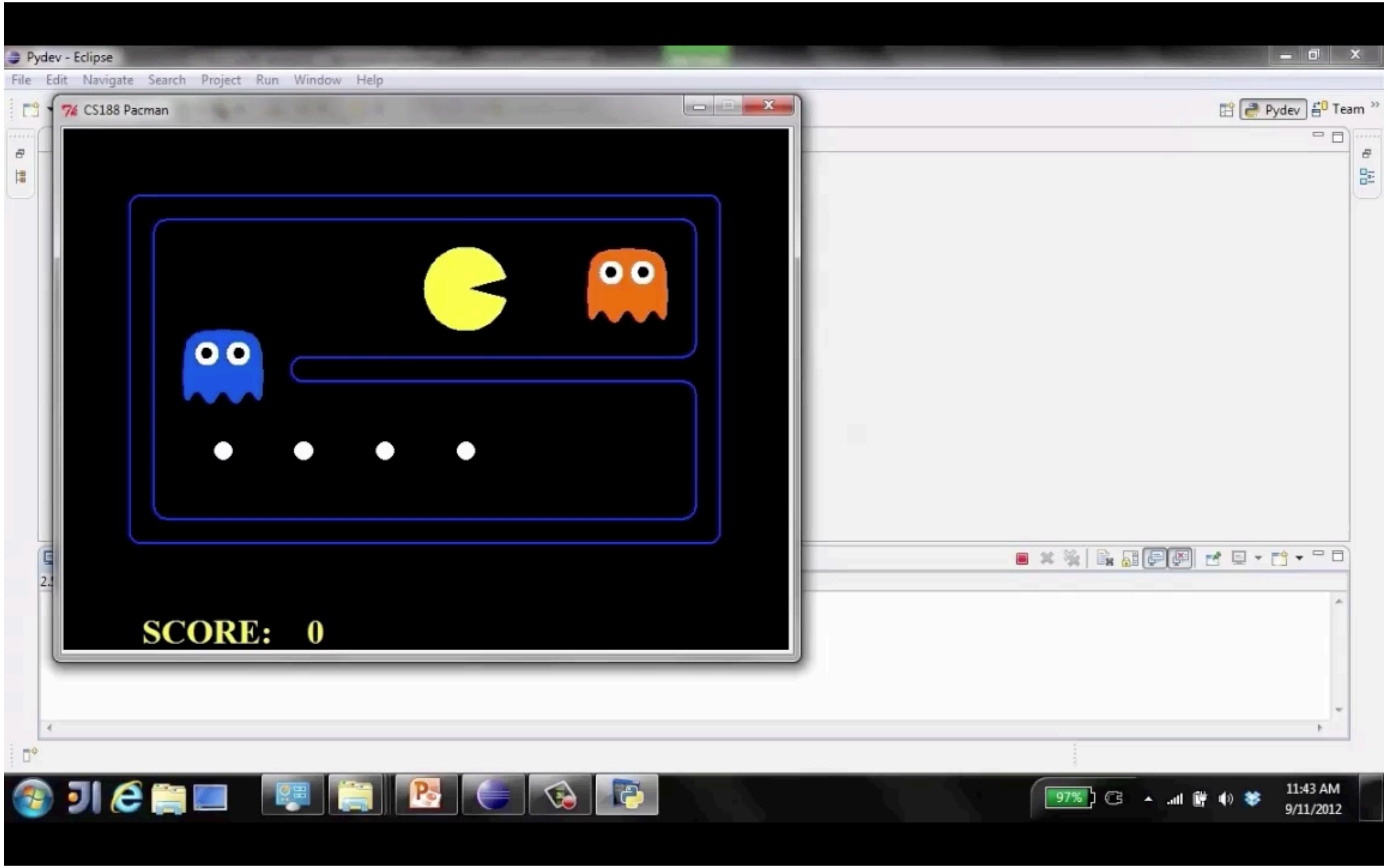

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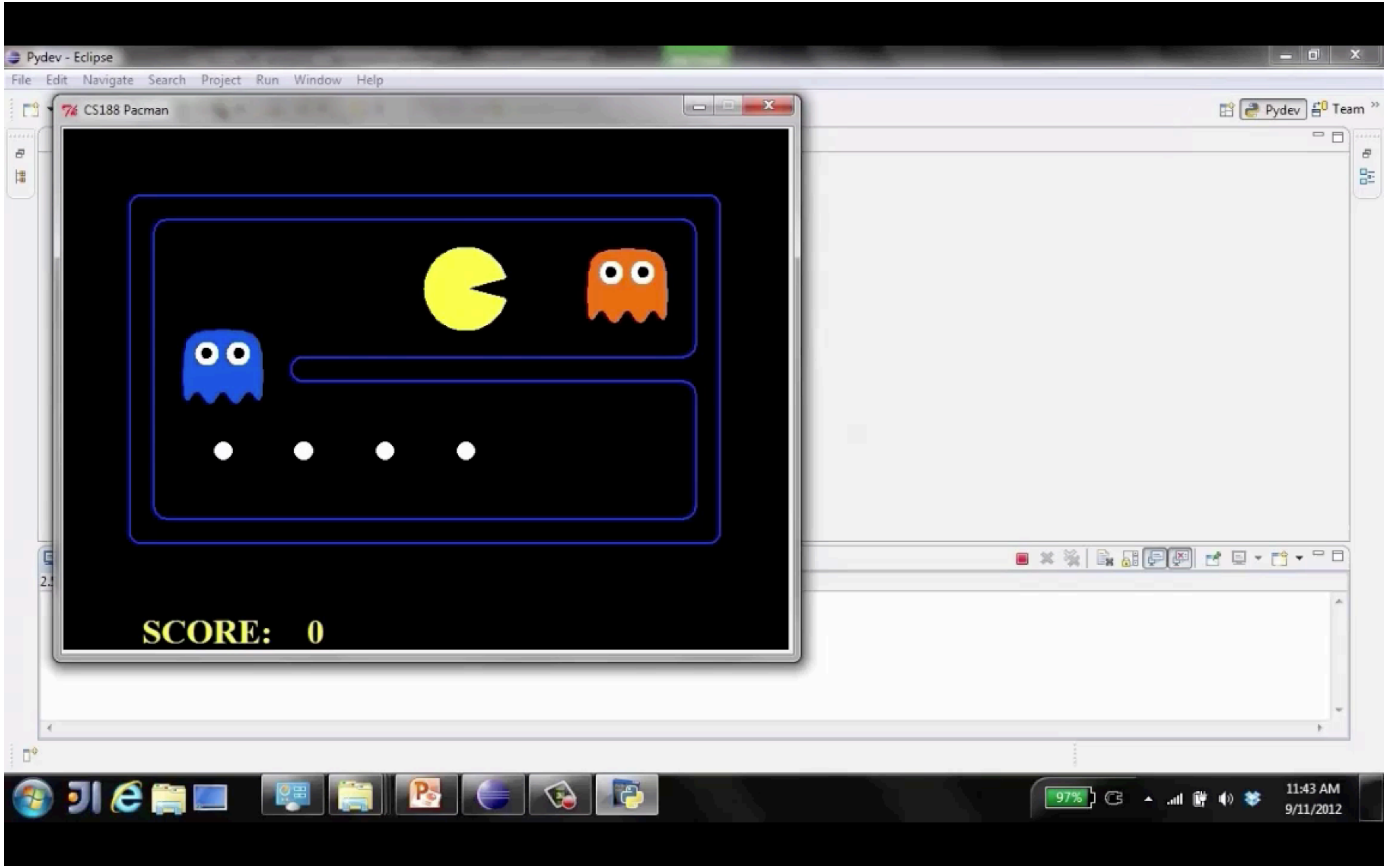

Resource Limits

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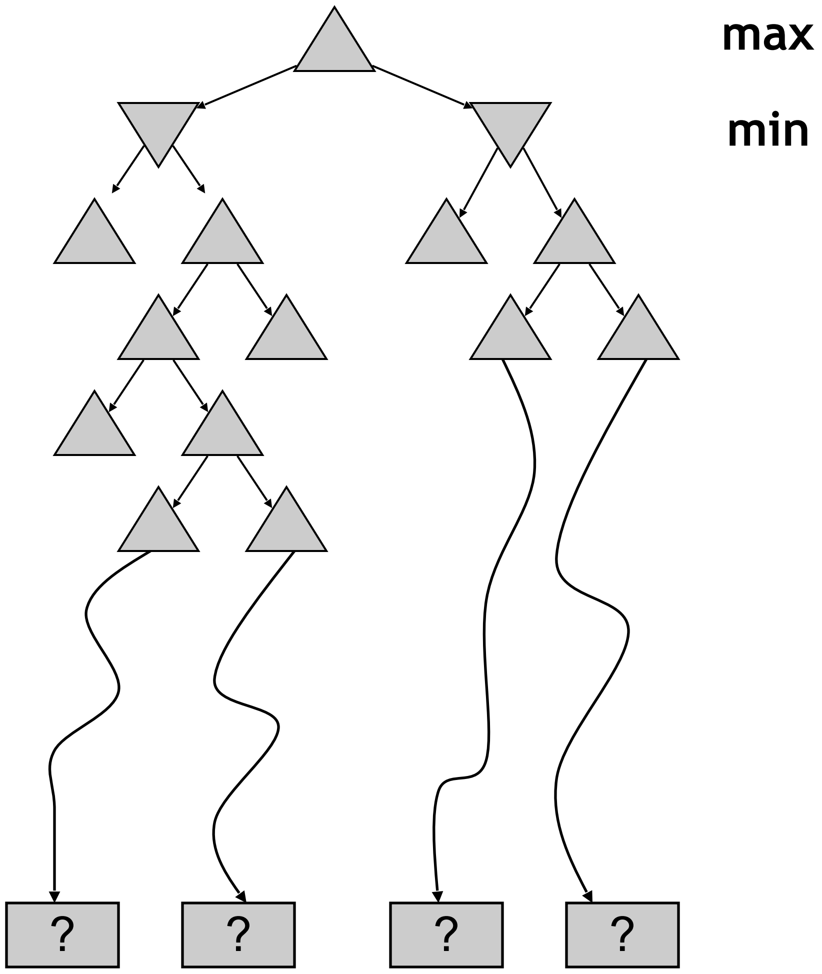

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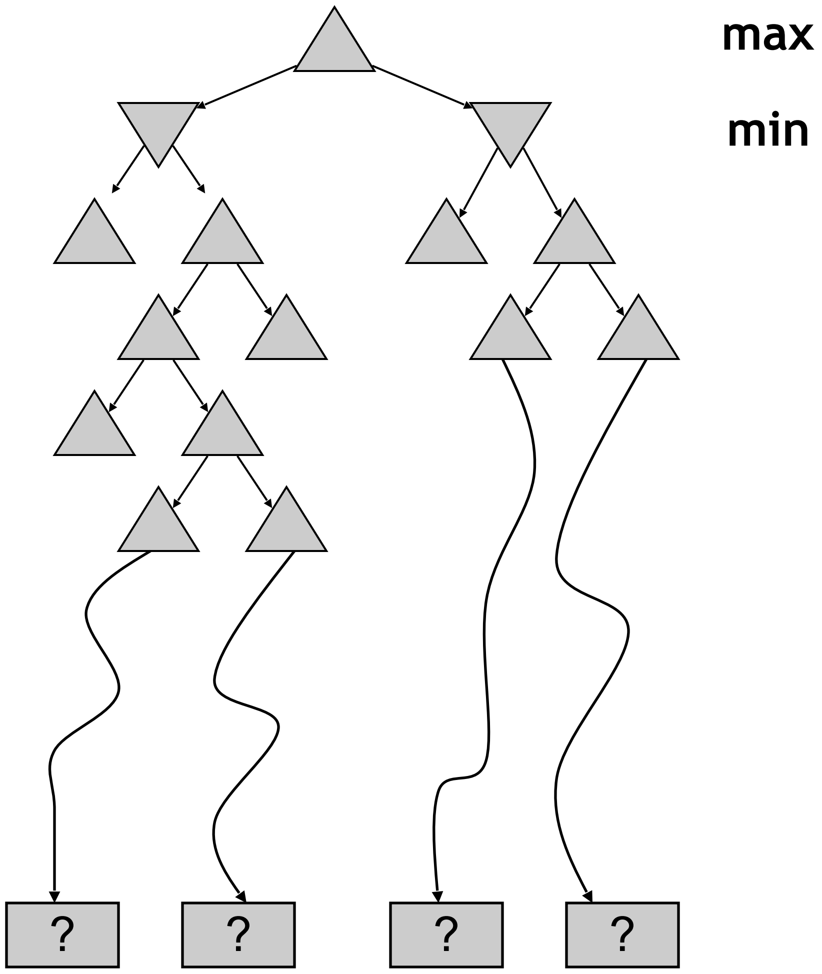
  - Instead, search only to a limited depth in the tree
  - Replace terminal utilities with an evaluation function for non-terminal positions



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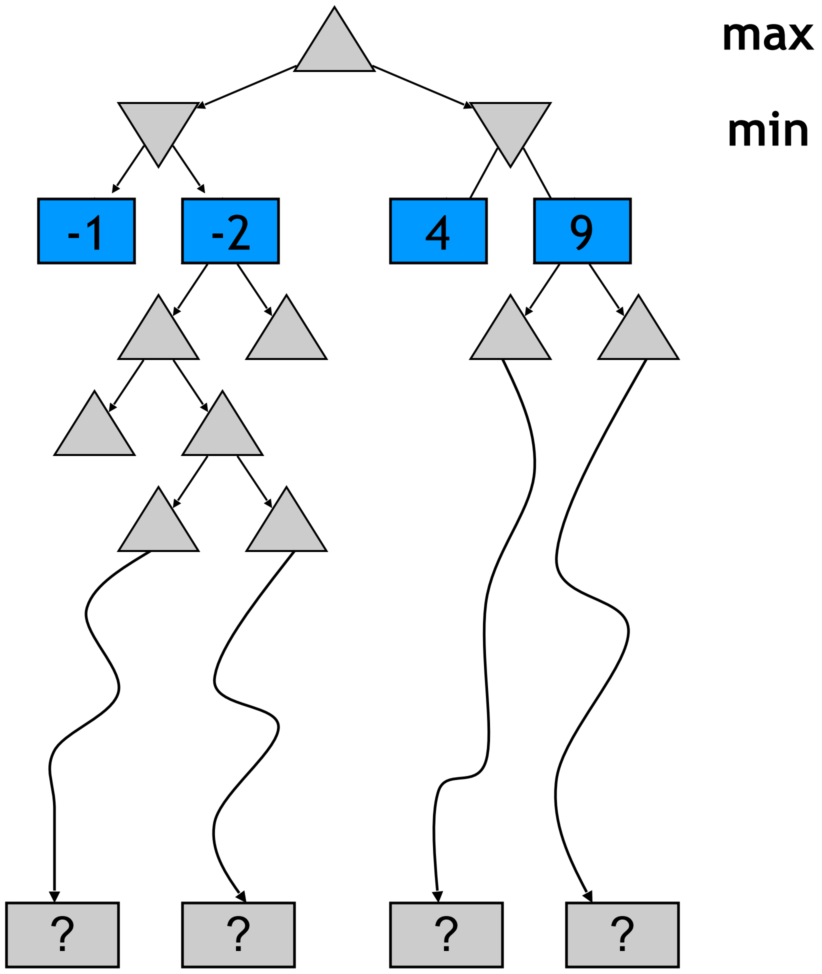

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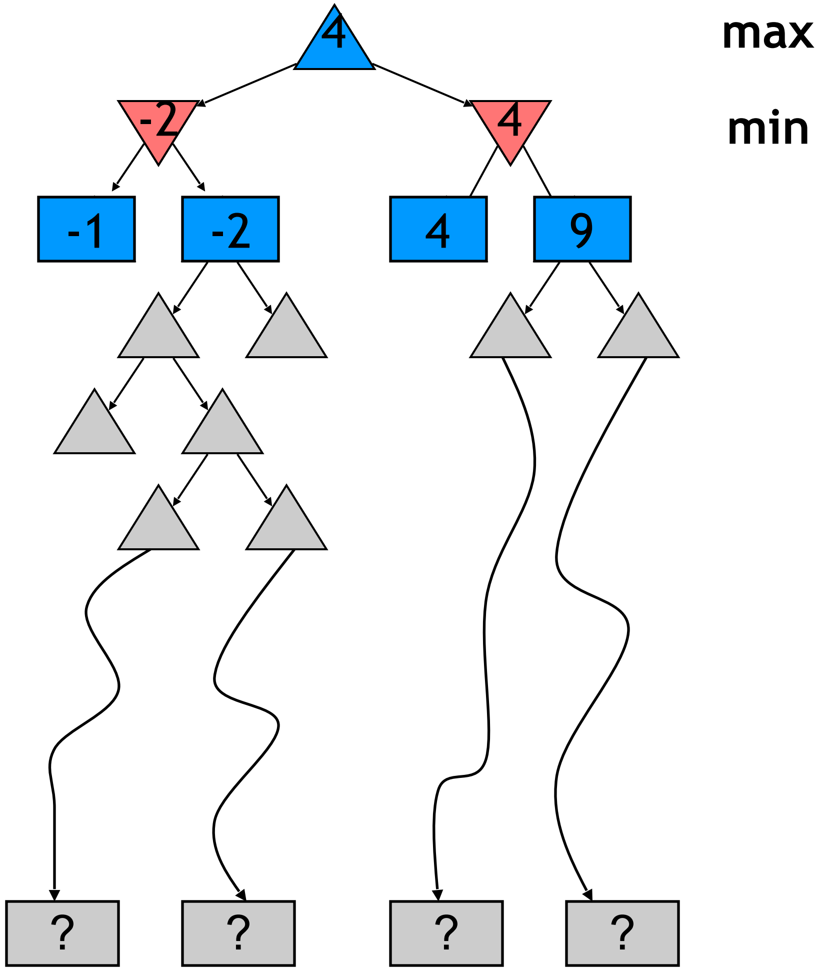

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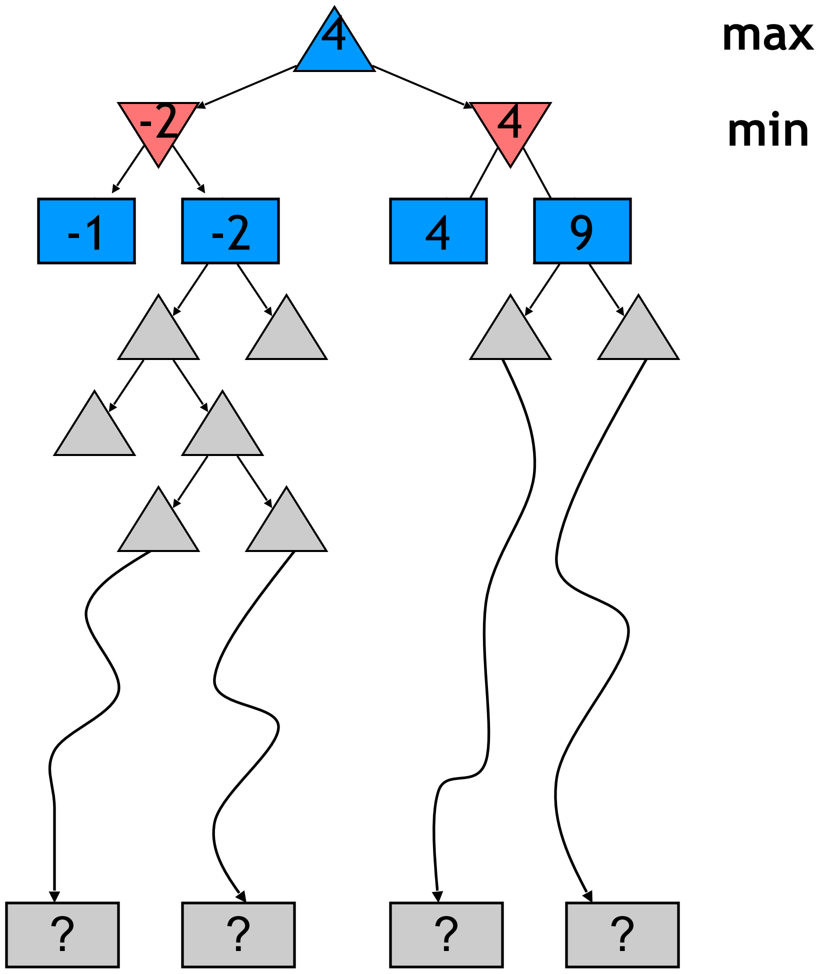
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program



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- Example:
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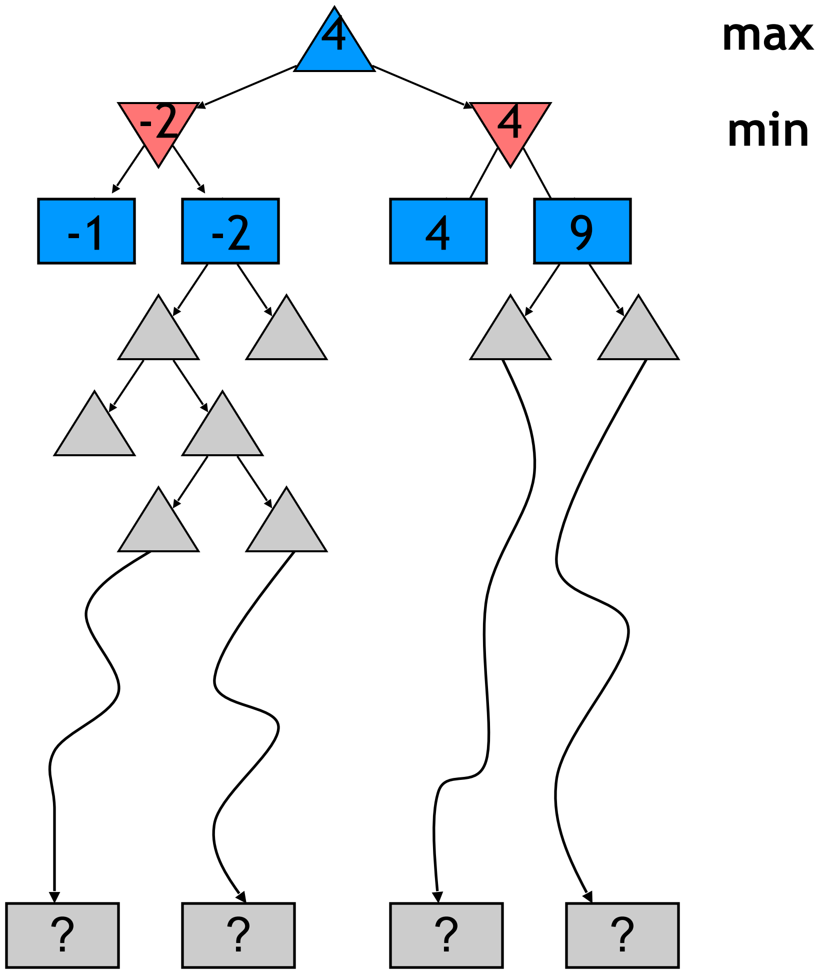
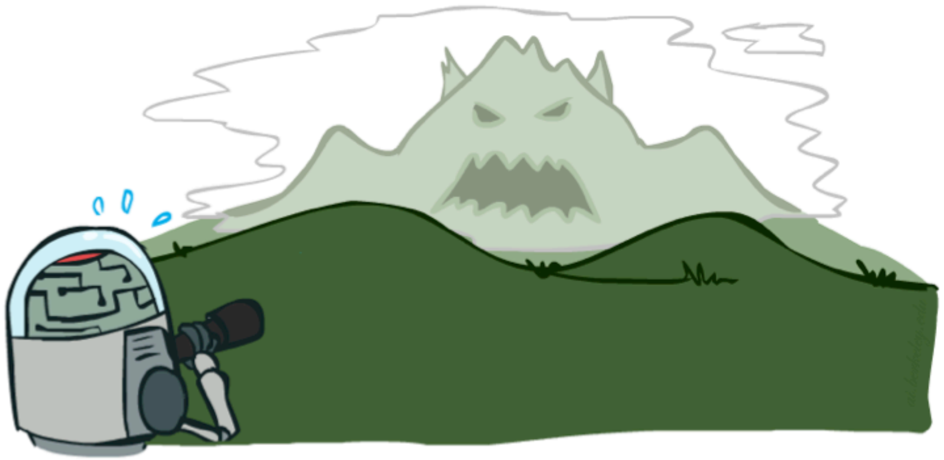
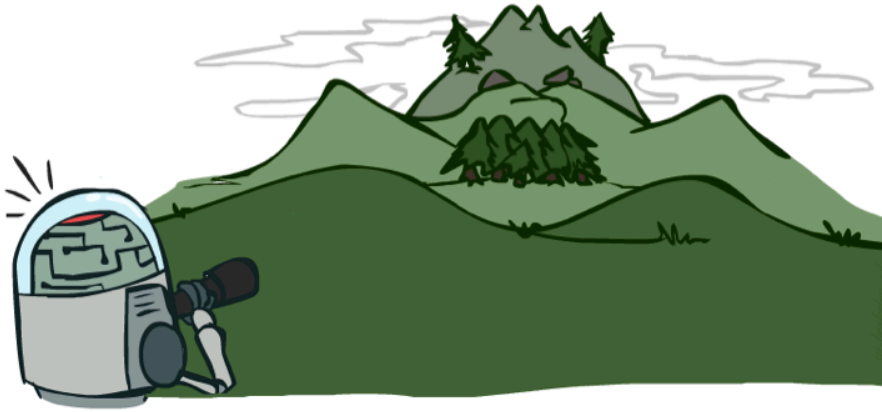
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use **iterative deepening** for an anytime algorithm



Depth Matters

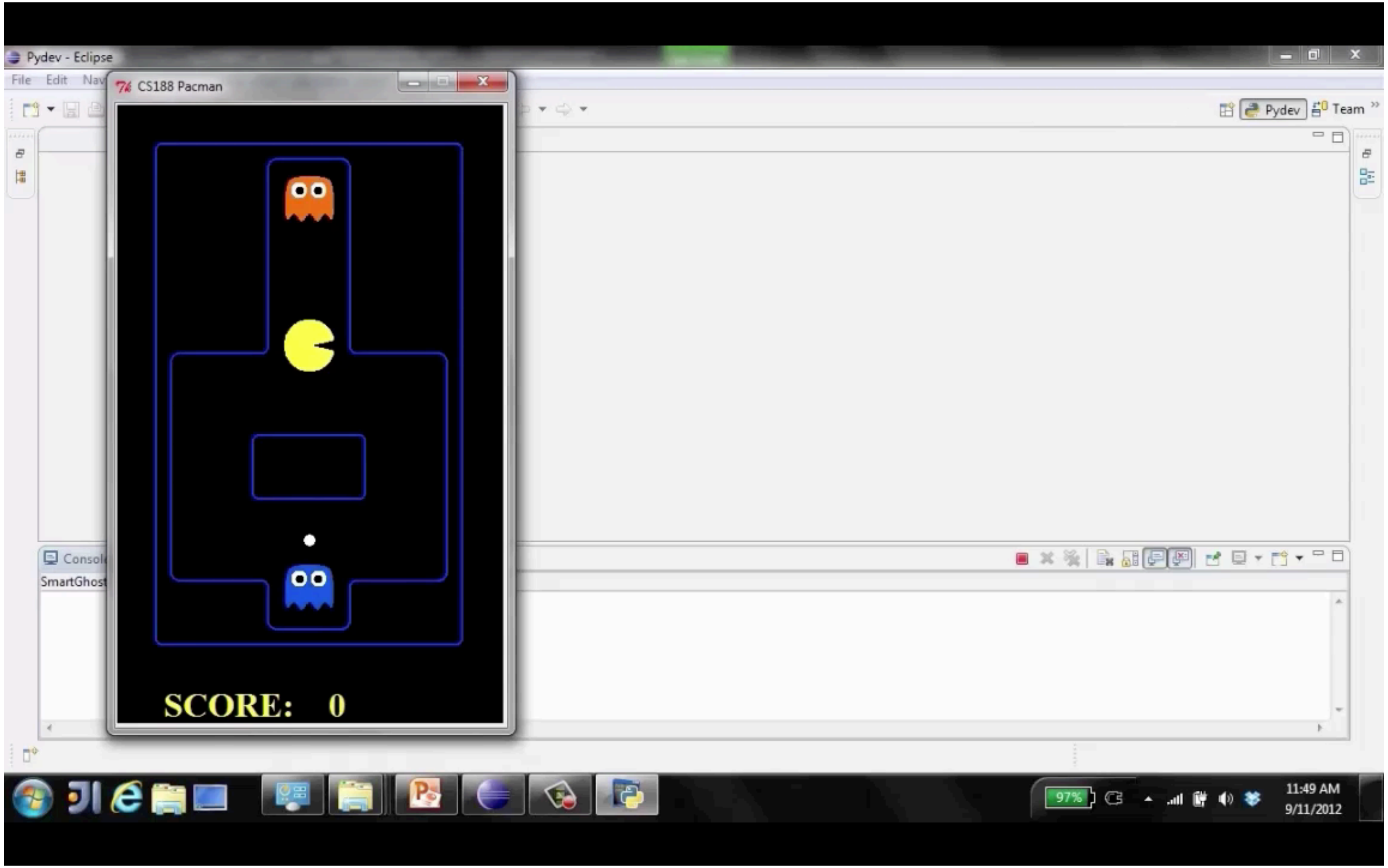
- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation





[Demo: depth limited (L6D4, L6D5)]

Video of Demo Limited Depth (2)



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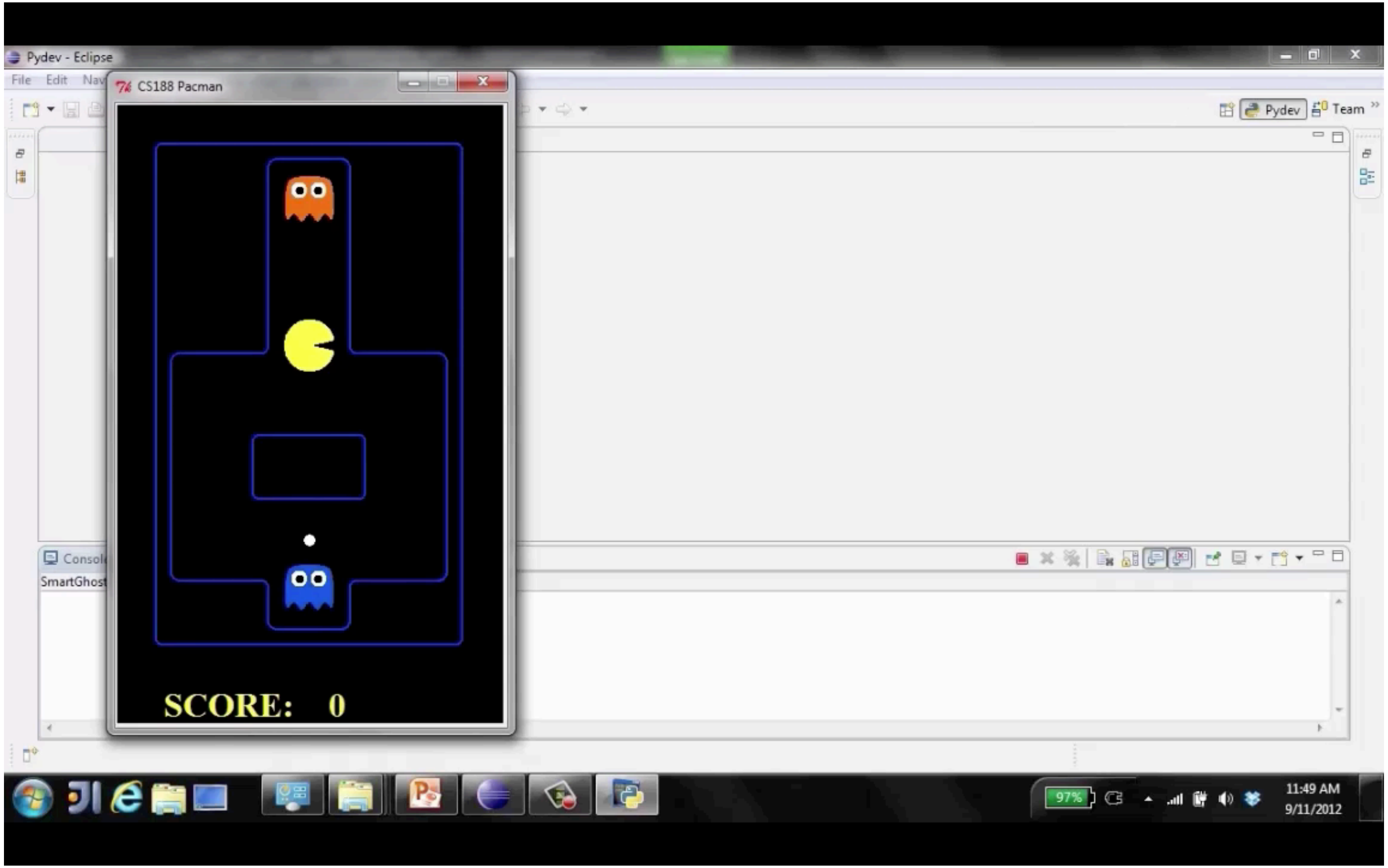

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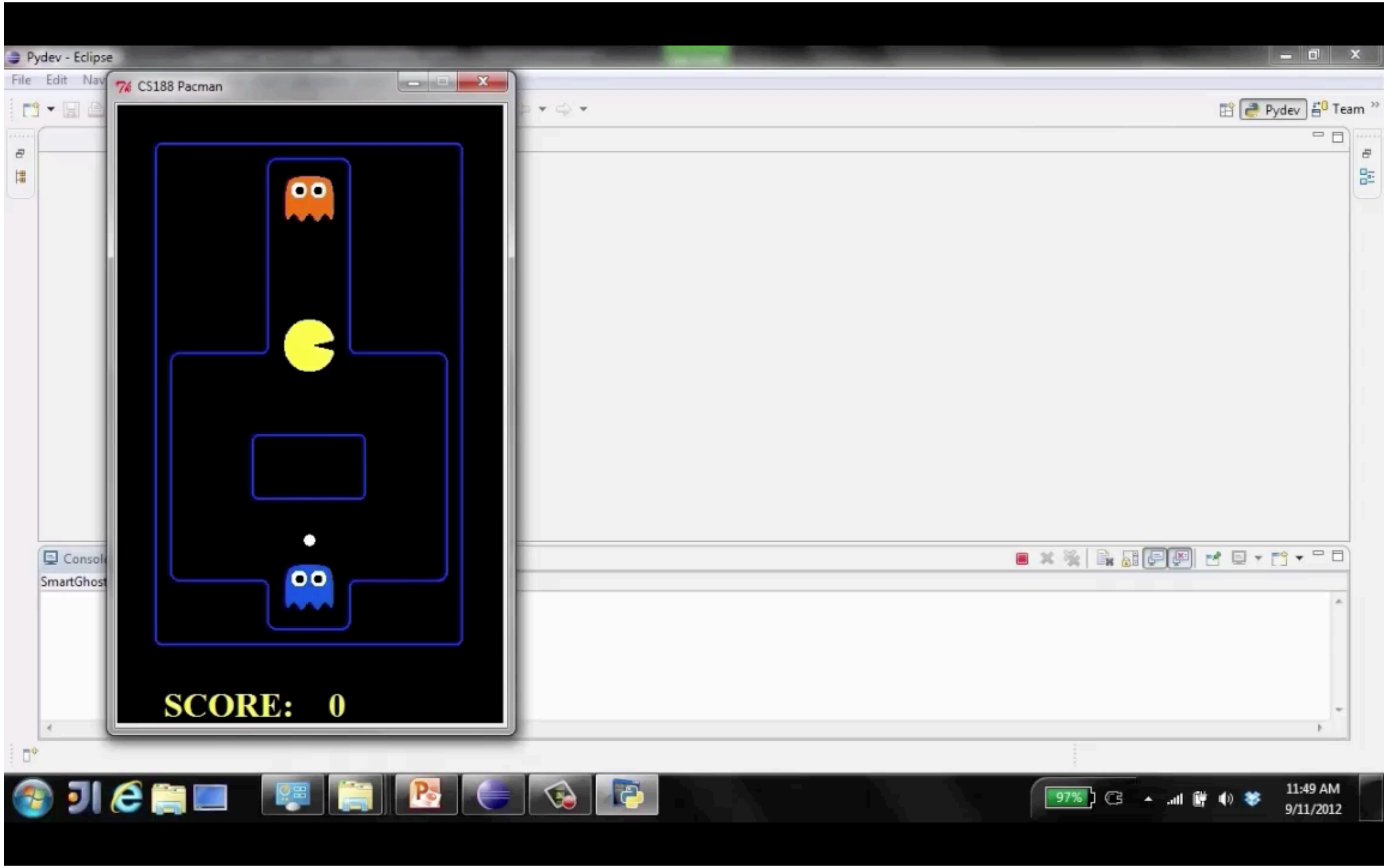

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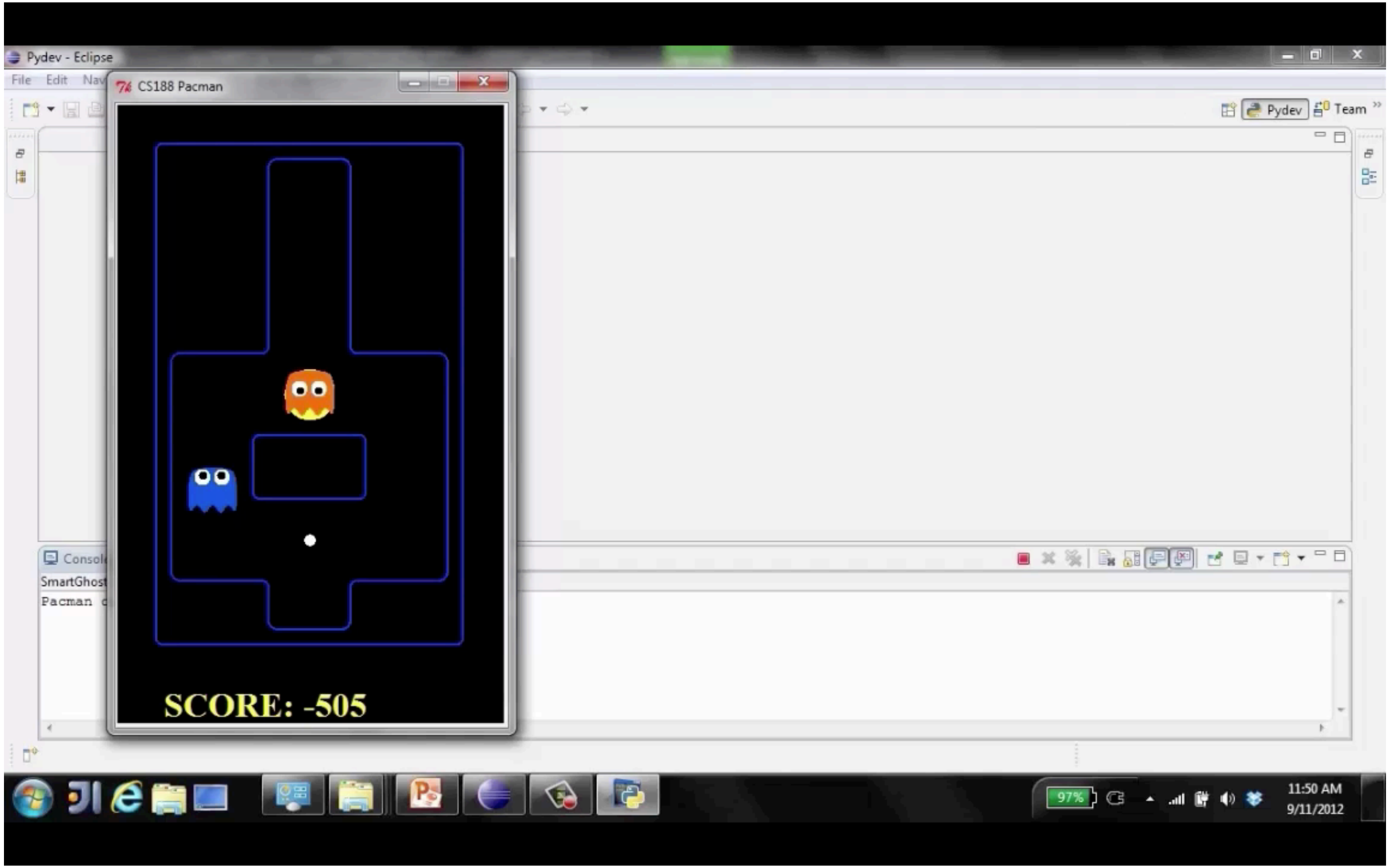
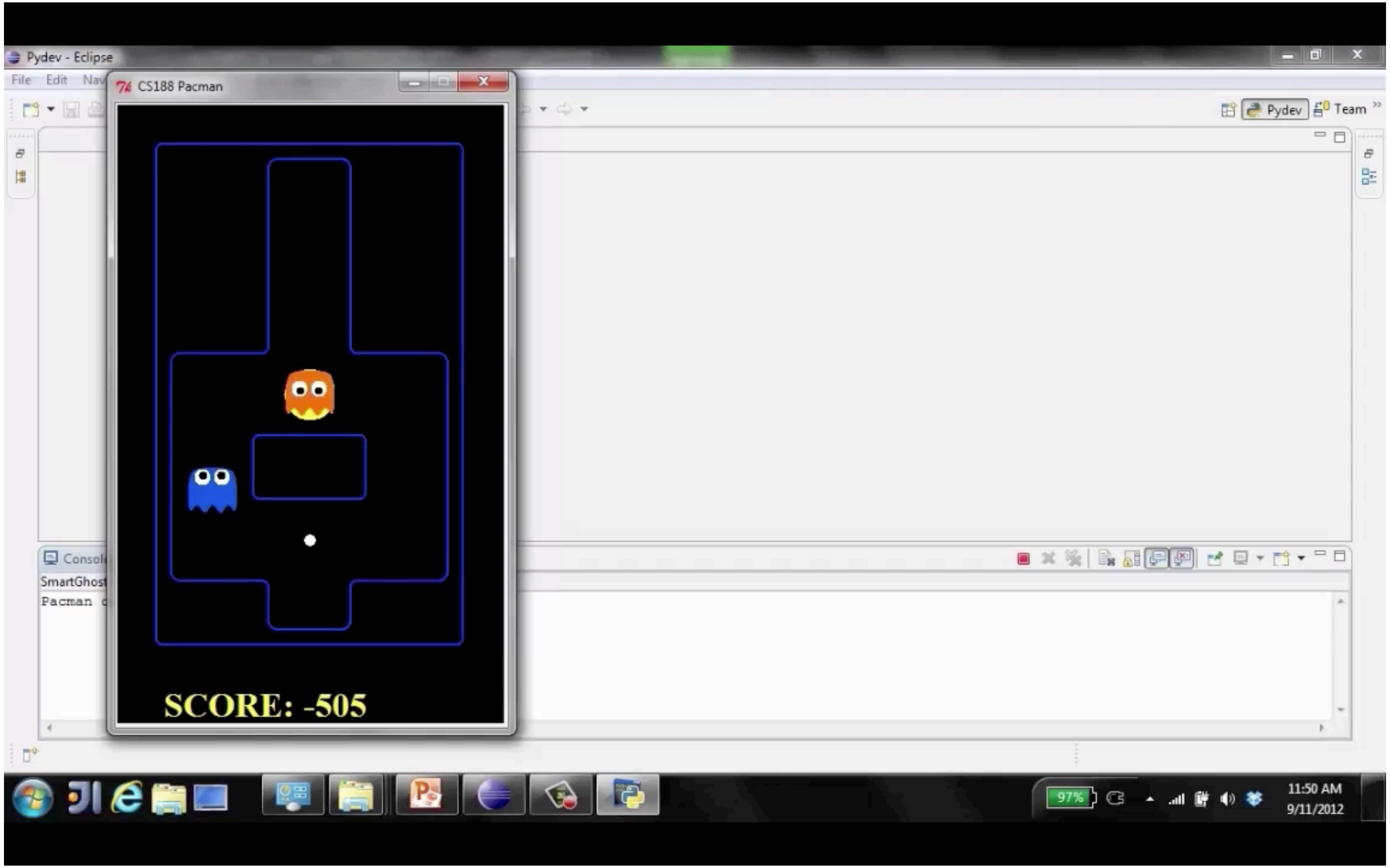


Video of Demo Limited Depth (10)



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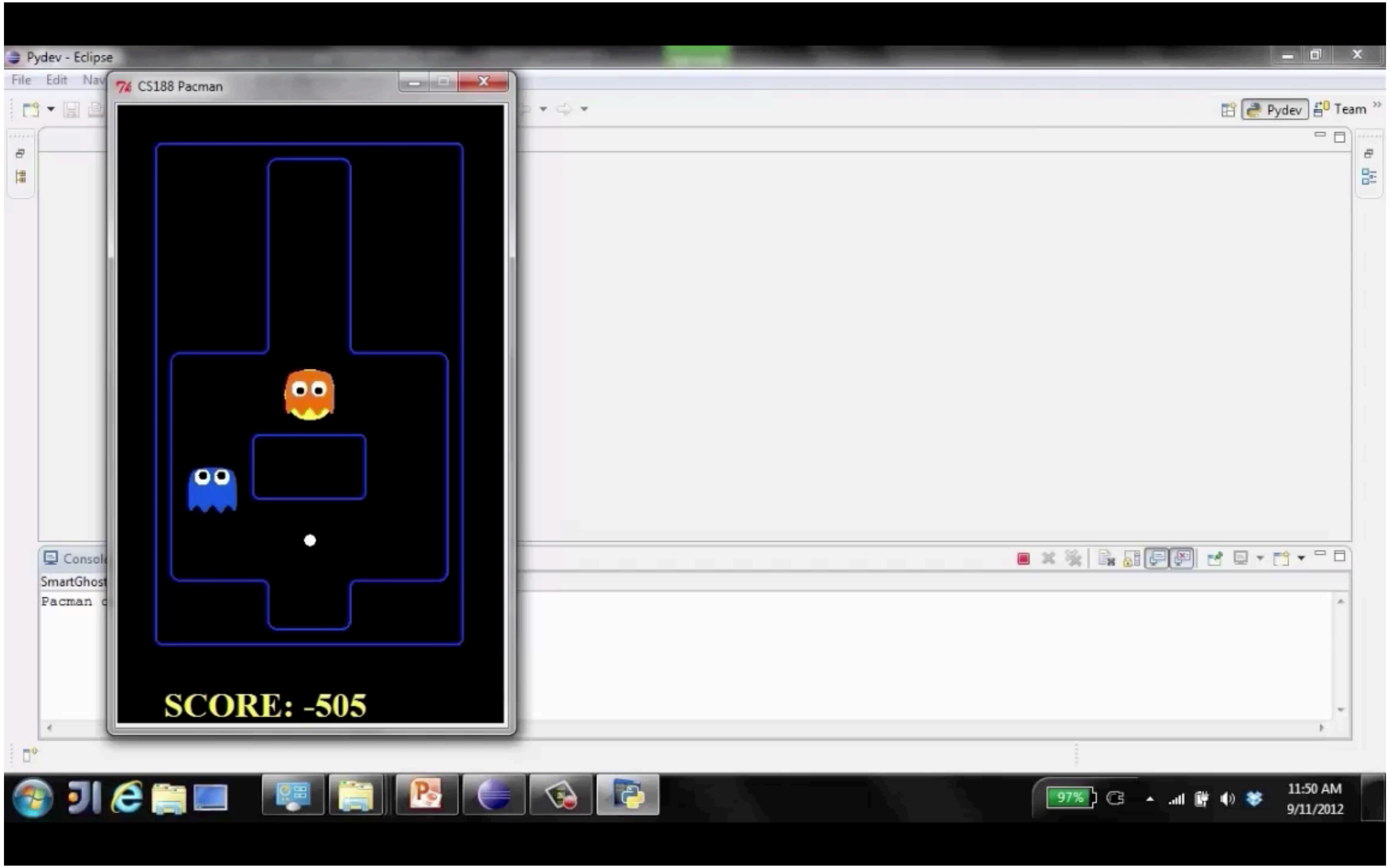
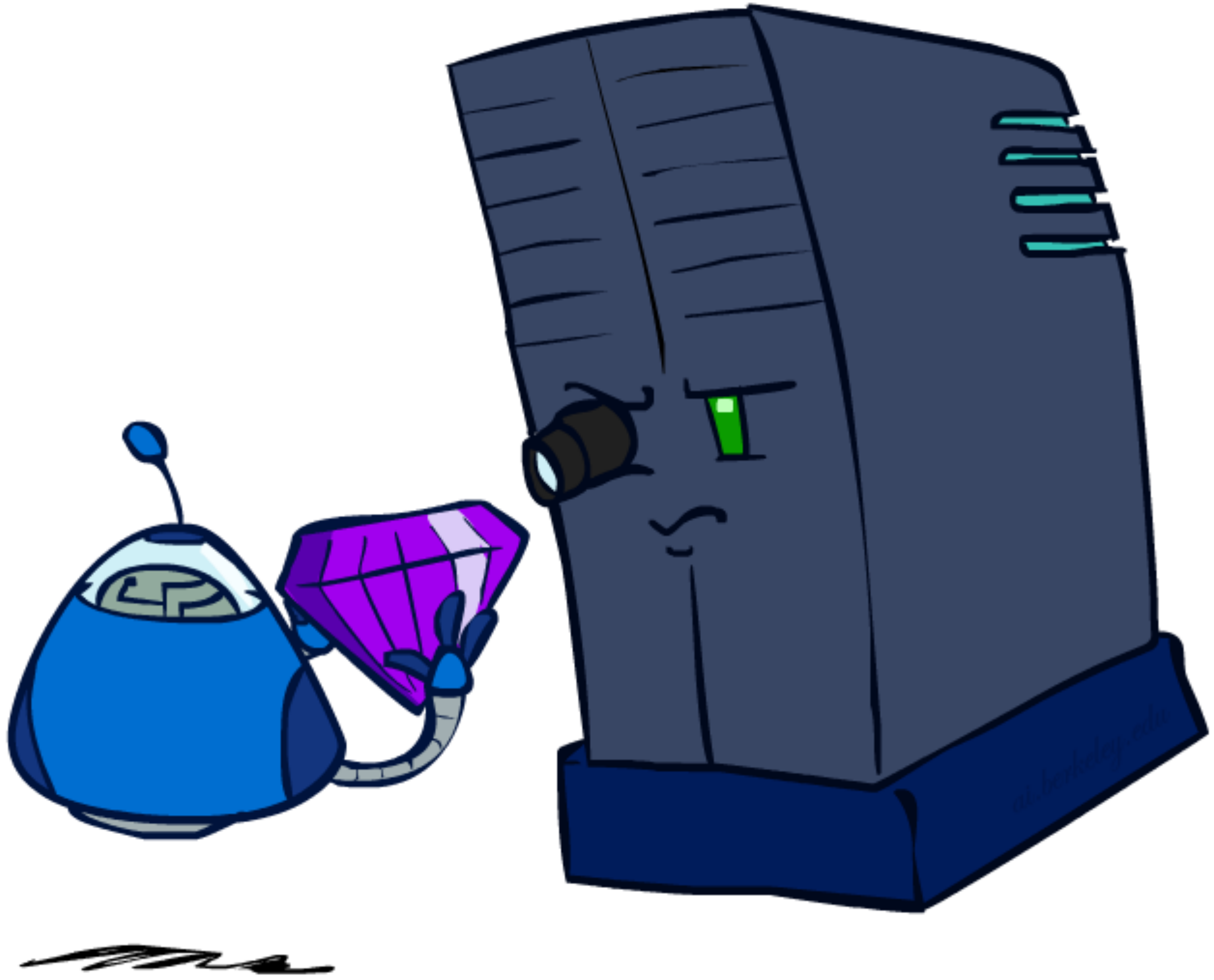
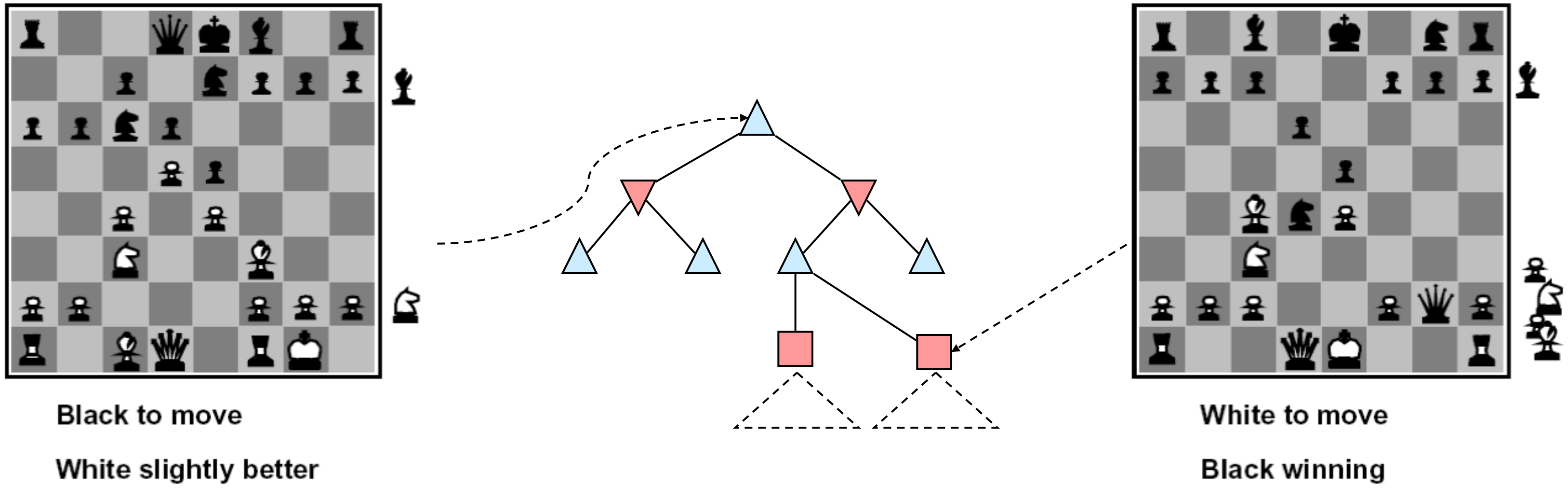


Evaluation Functions



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search

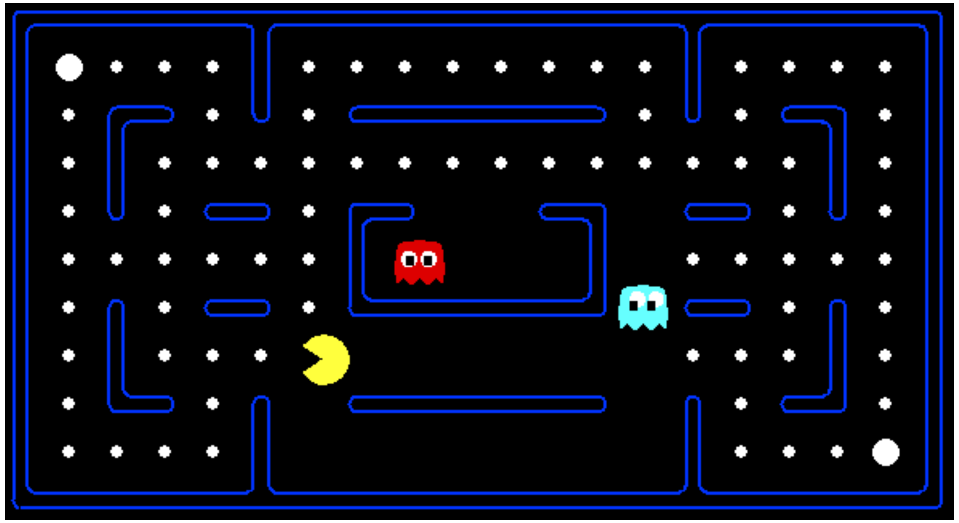


- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Evaluation for Pacman



[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate

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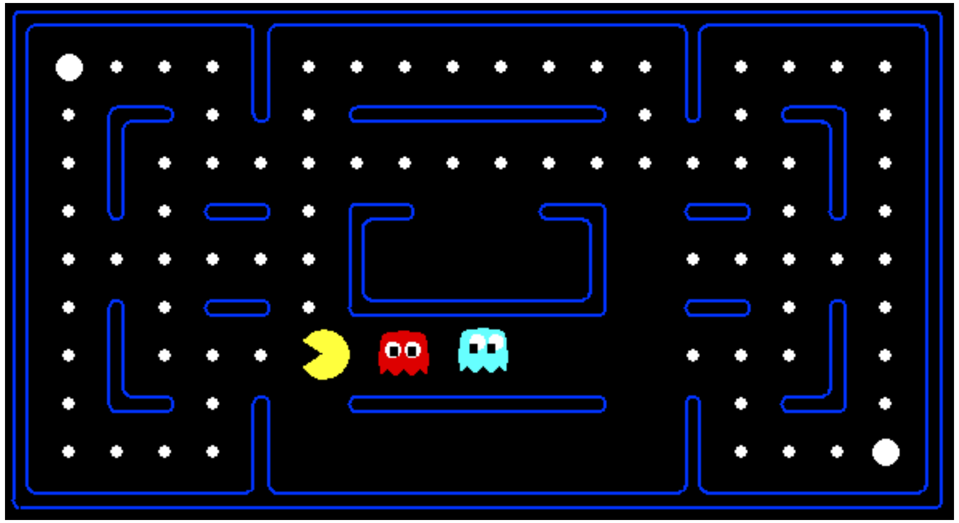

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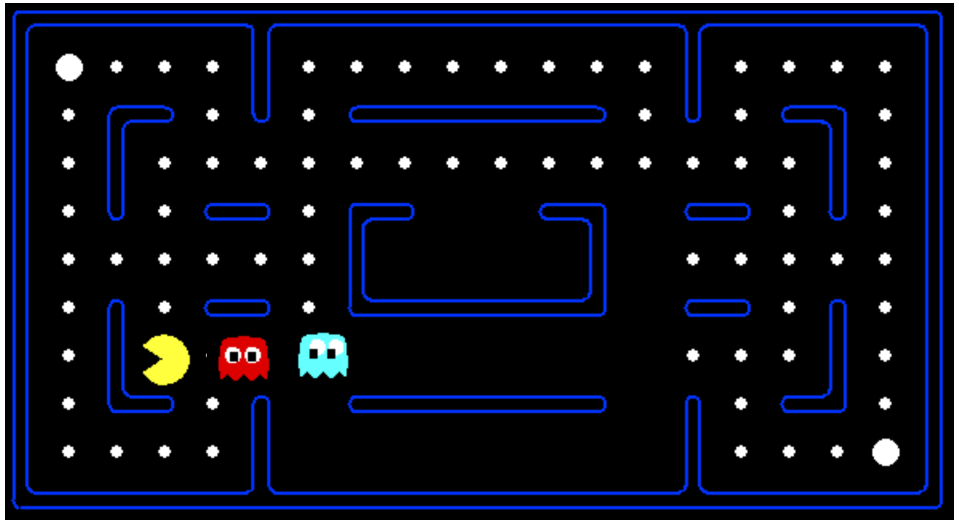

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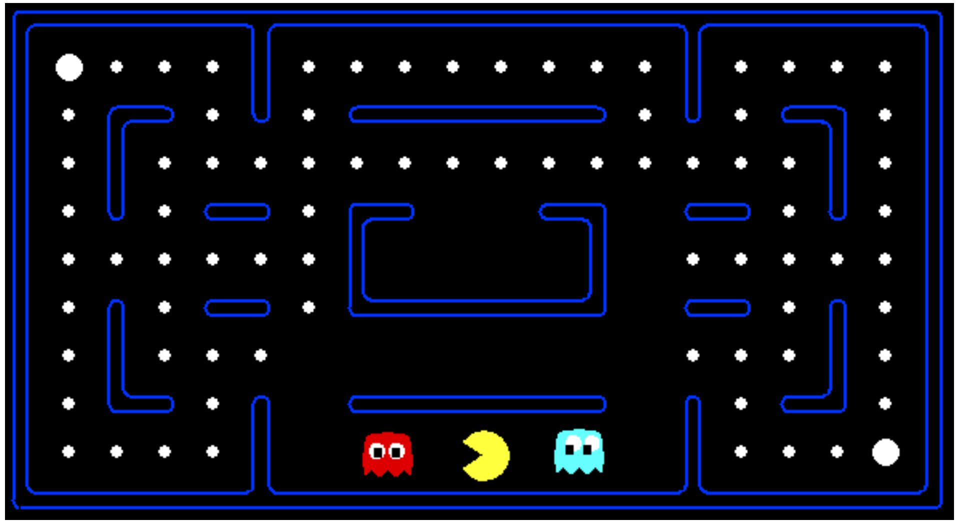

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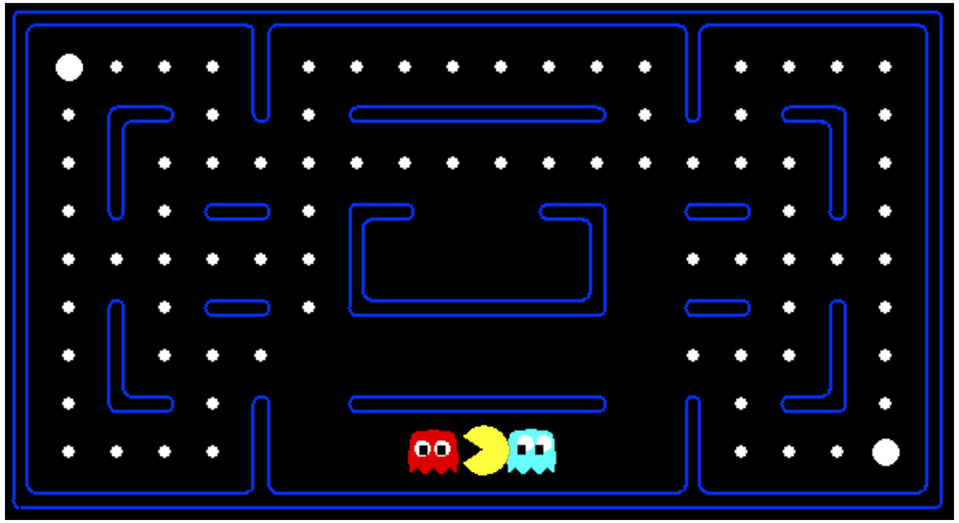

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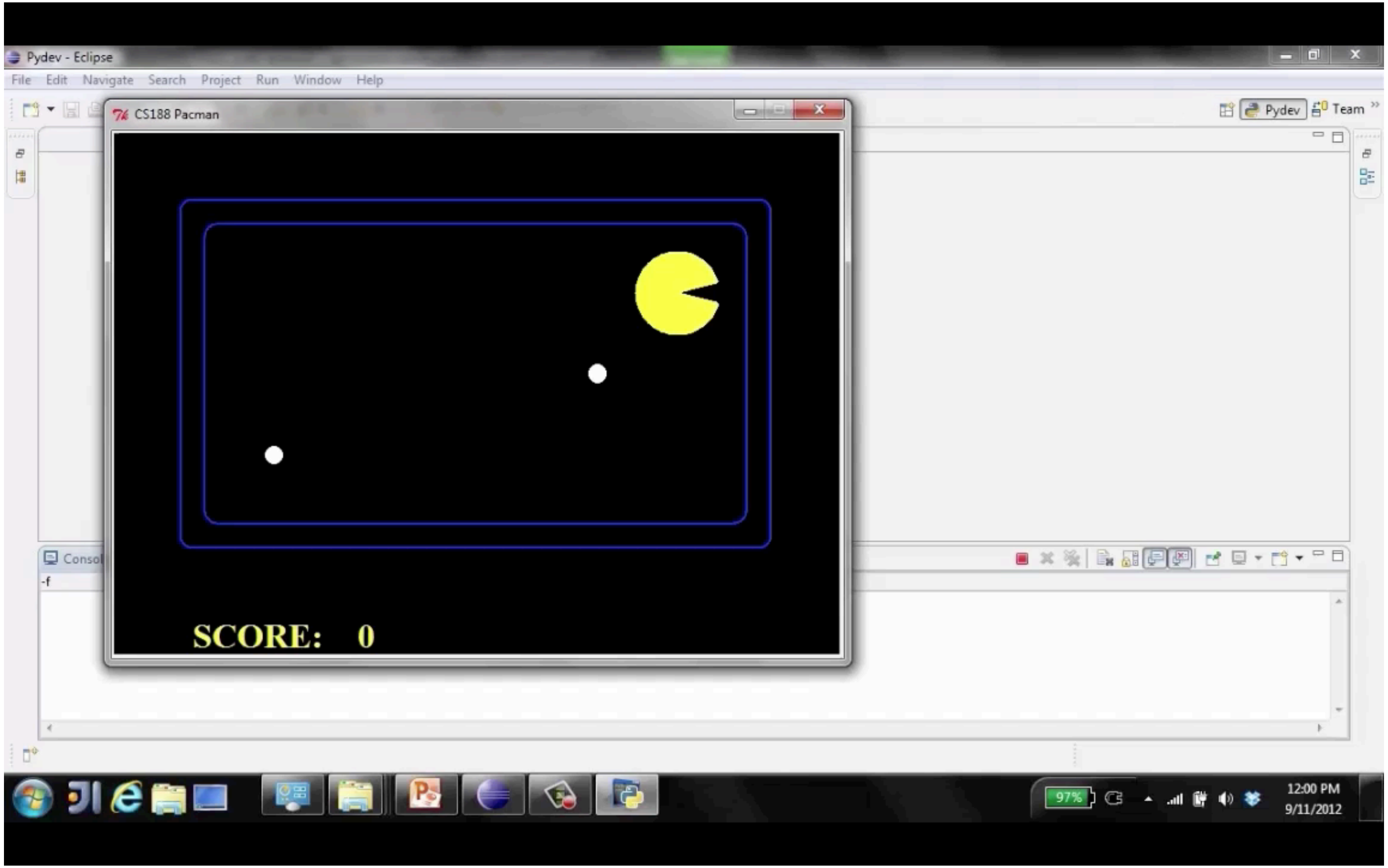
Video of Demo Thrashing (d=2)



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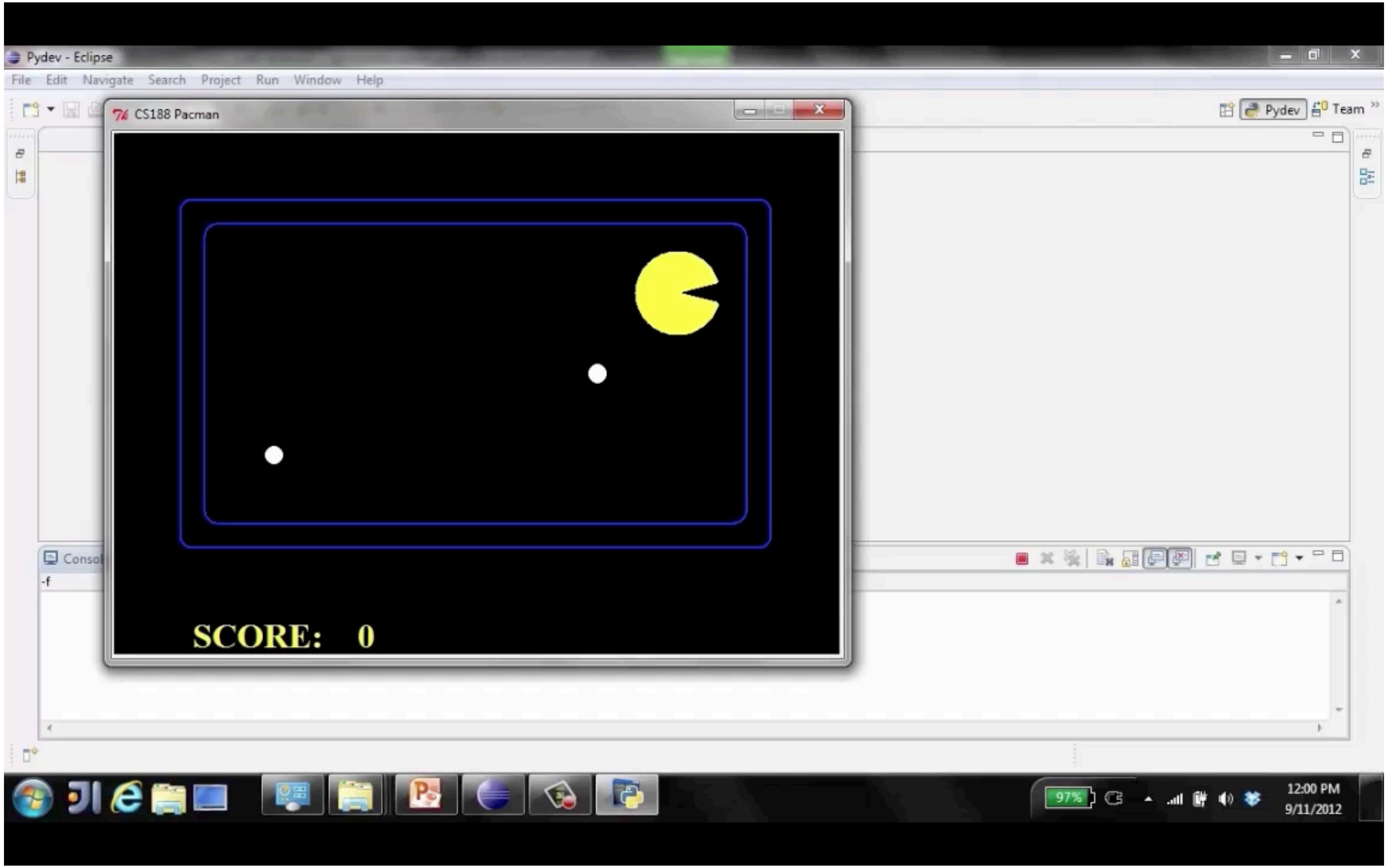

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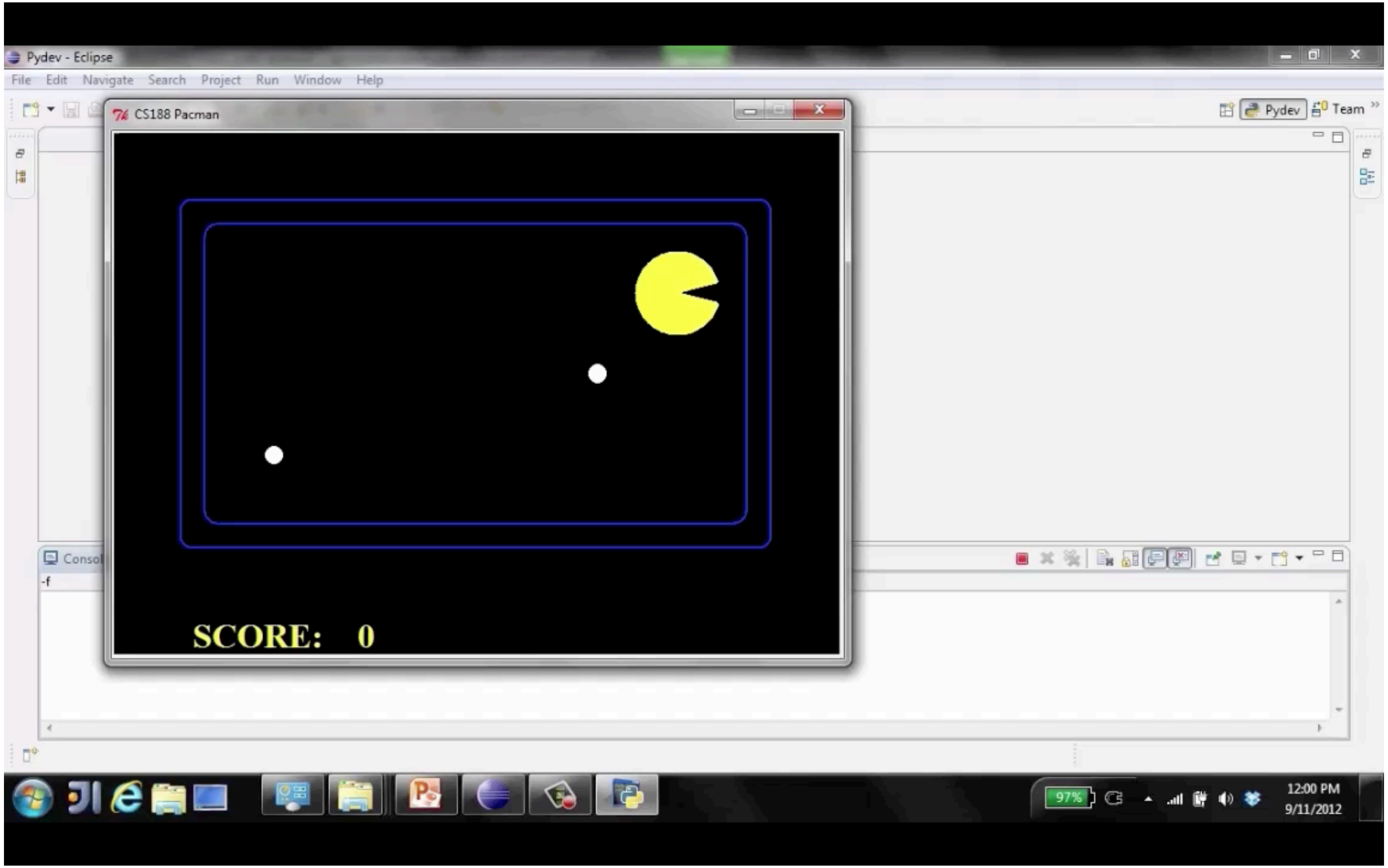

Why Pacman Starves

- A danger of replanning agents!
  - He knows his score will go up by eating the dot now (west, east)
  - He knows his score will go up just as much by eating the dot later (east, west)
  - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
  - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

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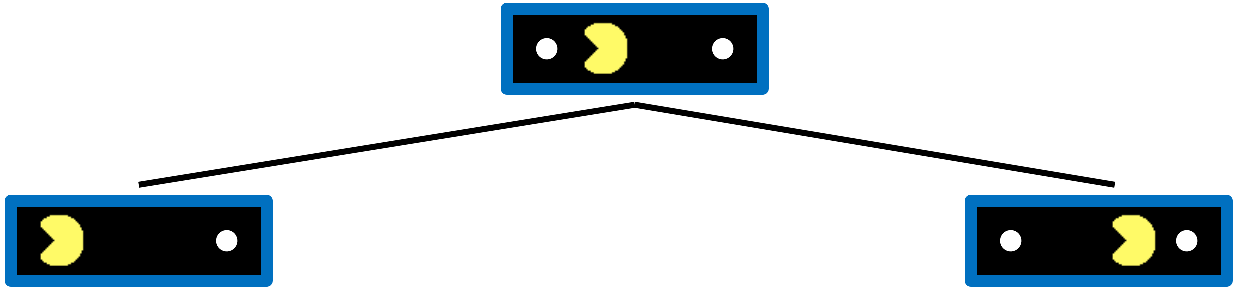

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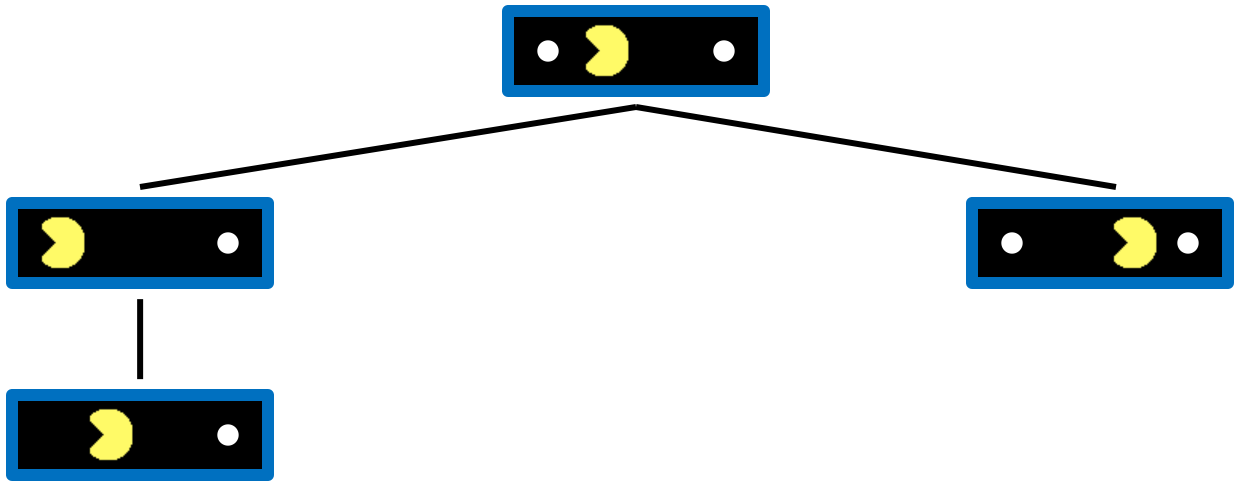

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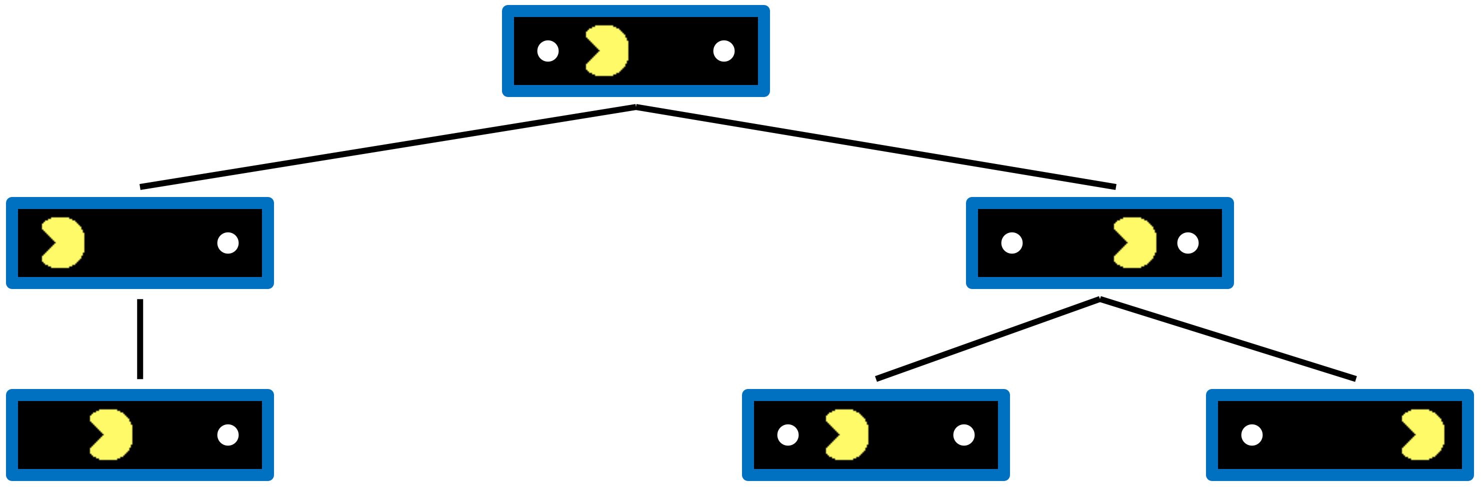

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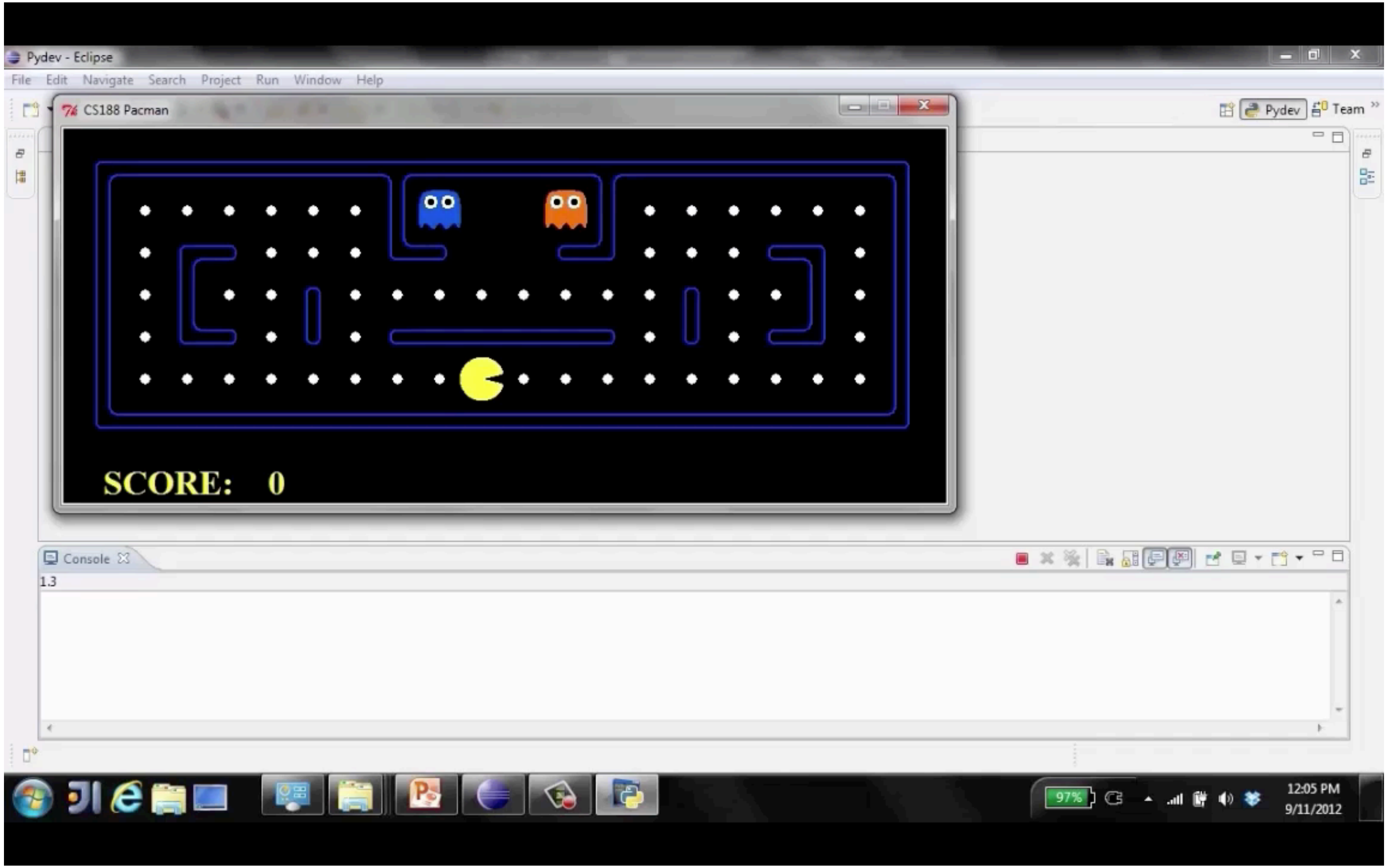
Video of Demo Smart Ghosts (Coordination)



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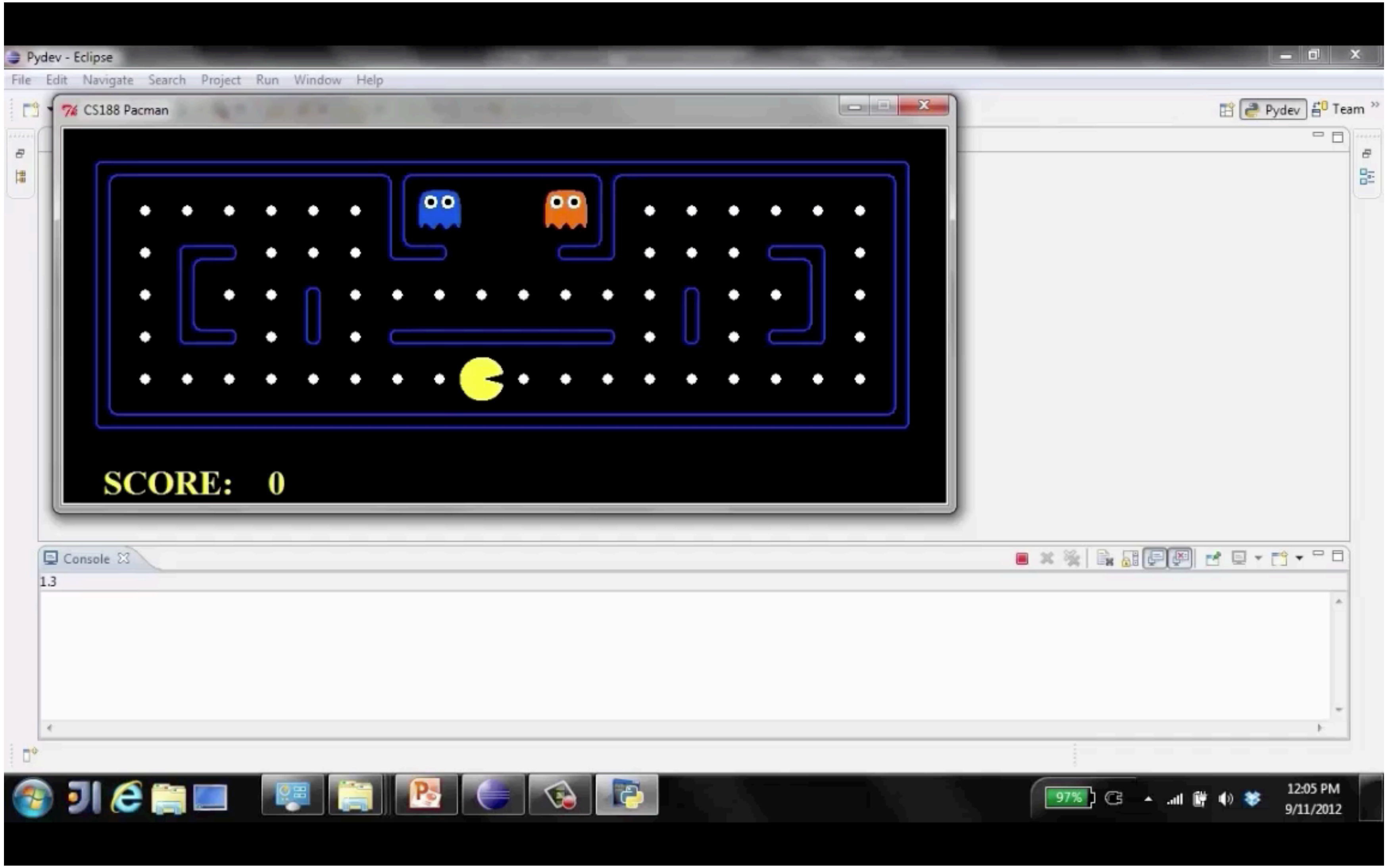

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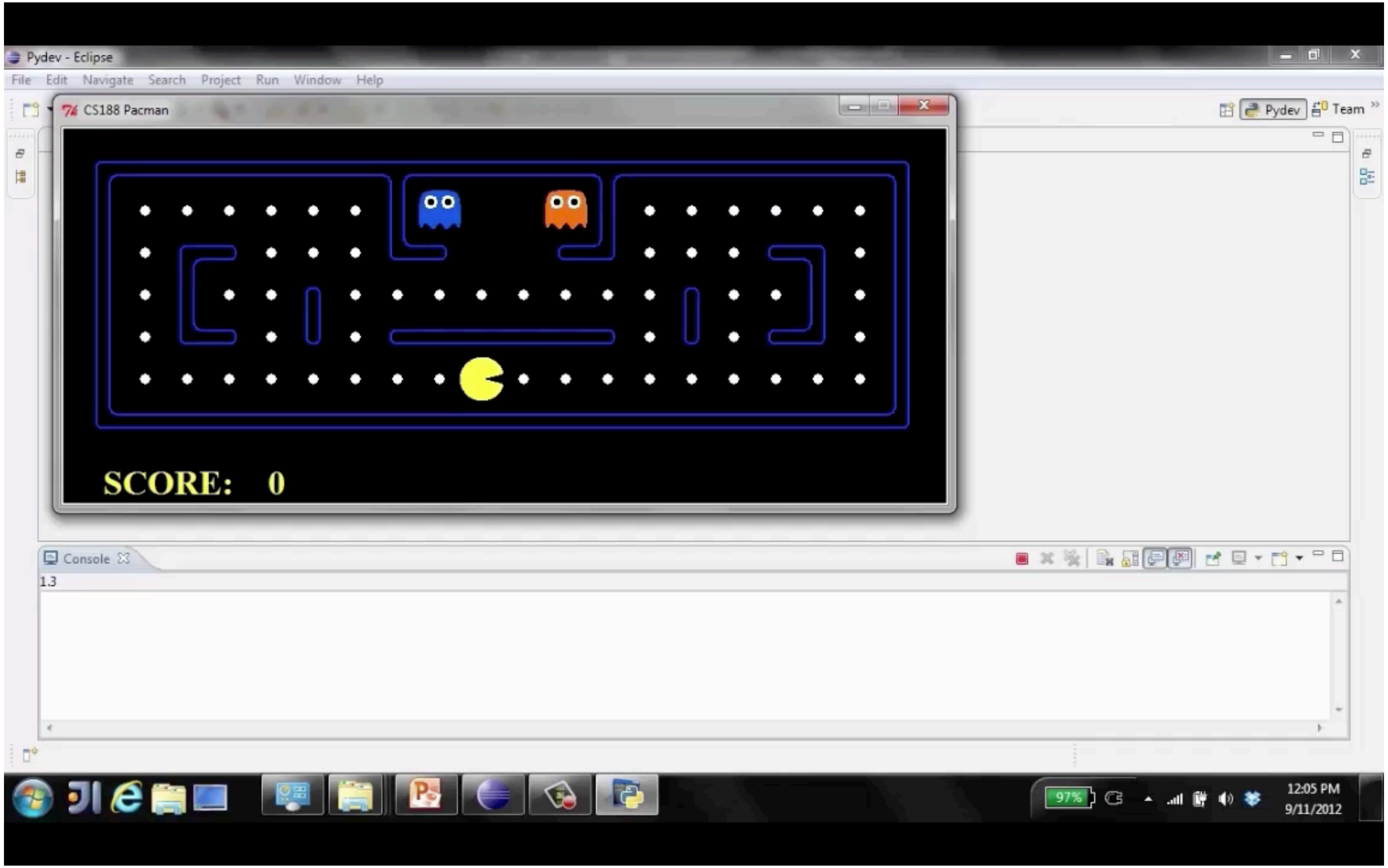
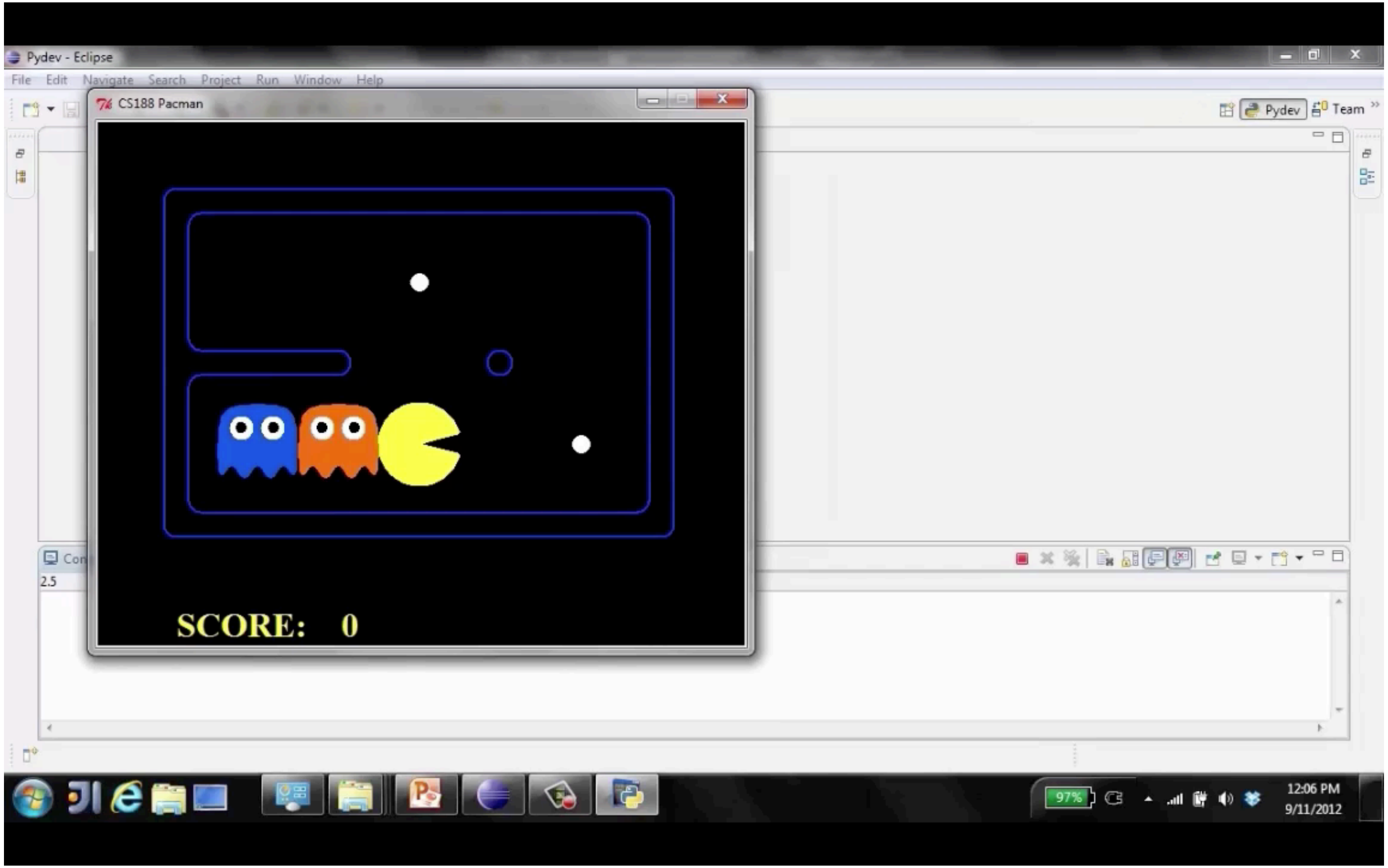
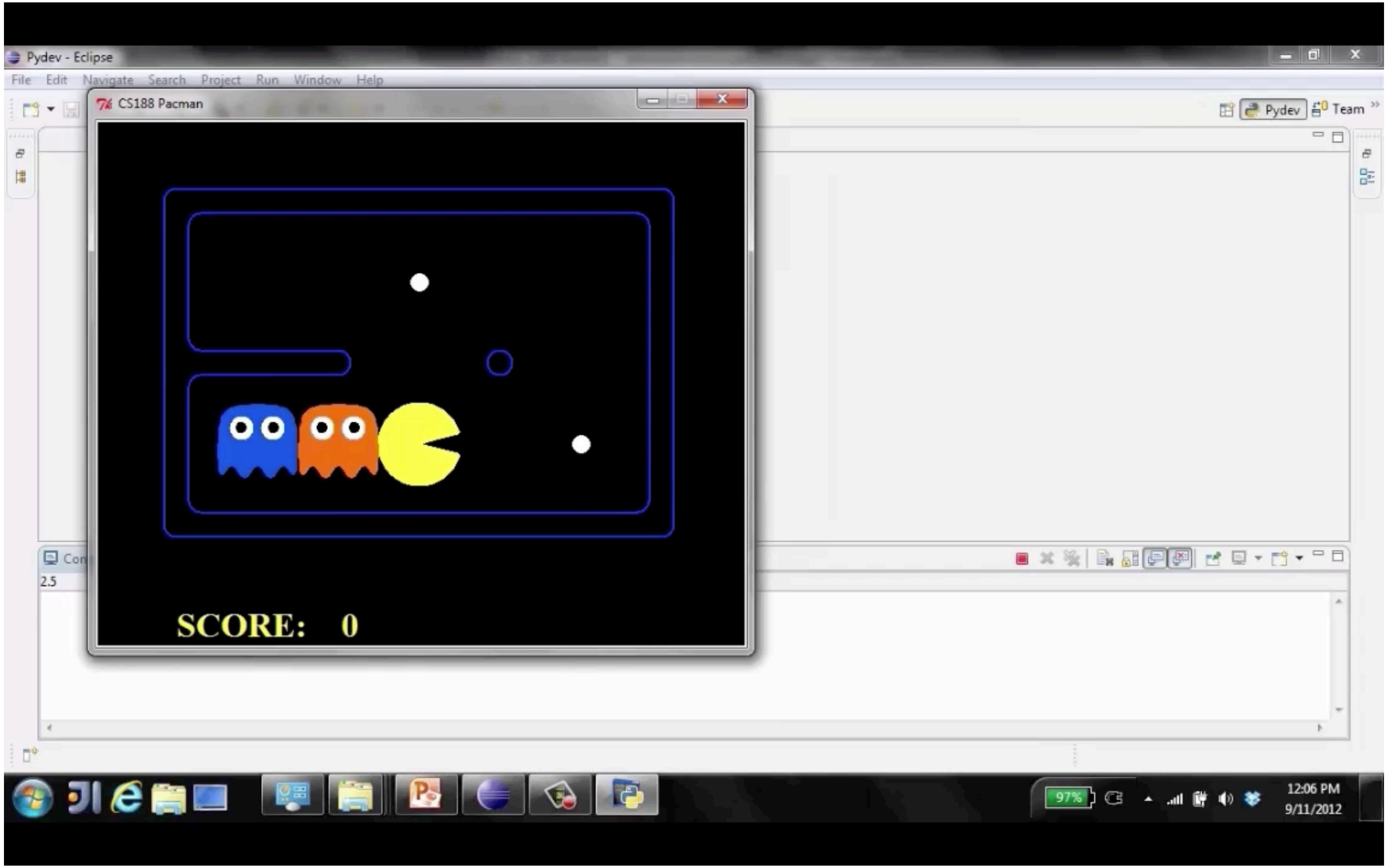


Video of Demo Smart Ghosts (Coordination) – Zoomed In



Video of Demo Smart Ghosts (Coordination) – Zoomed In



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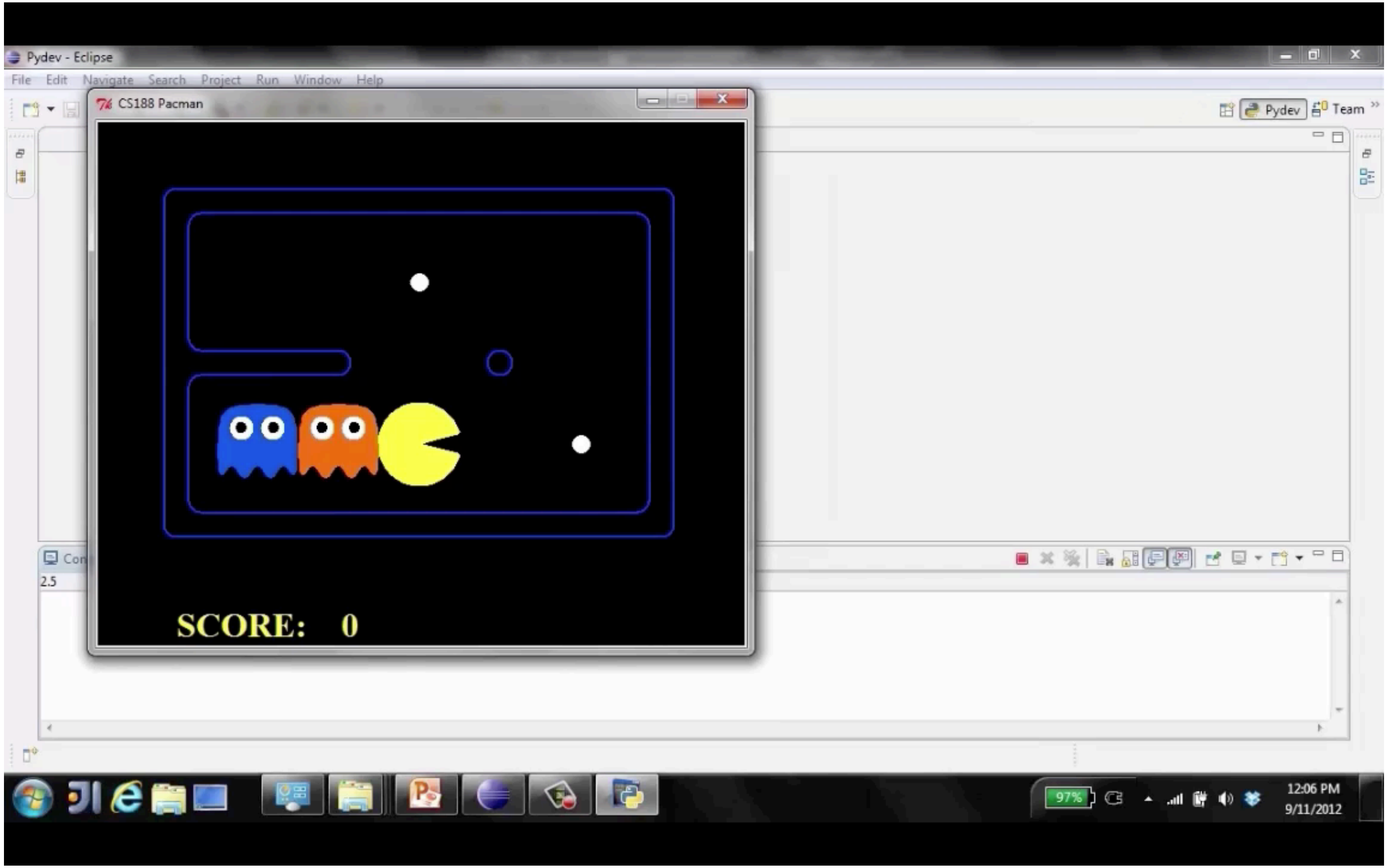
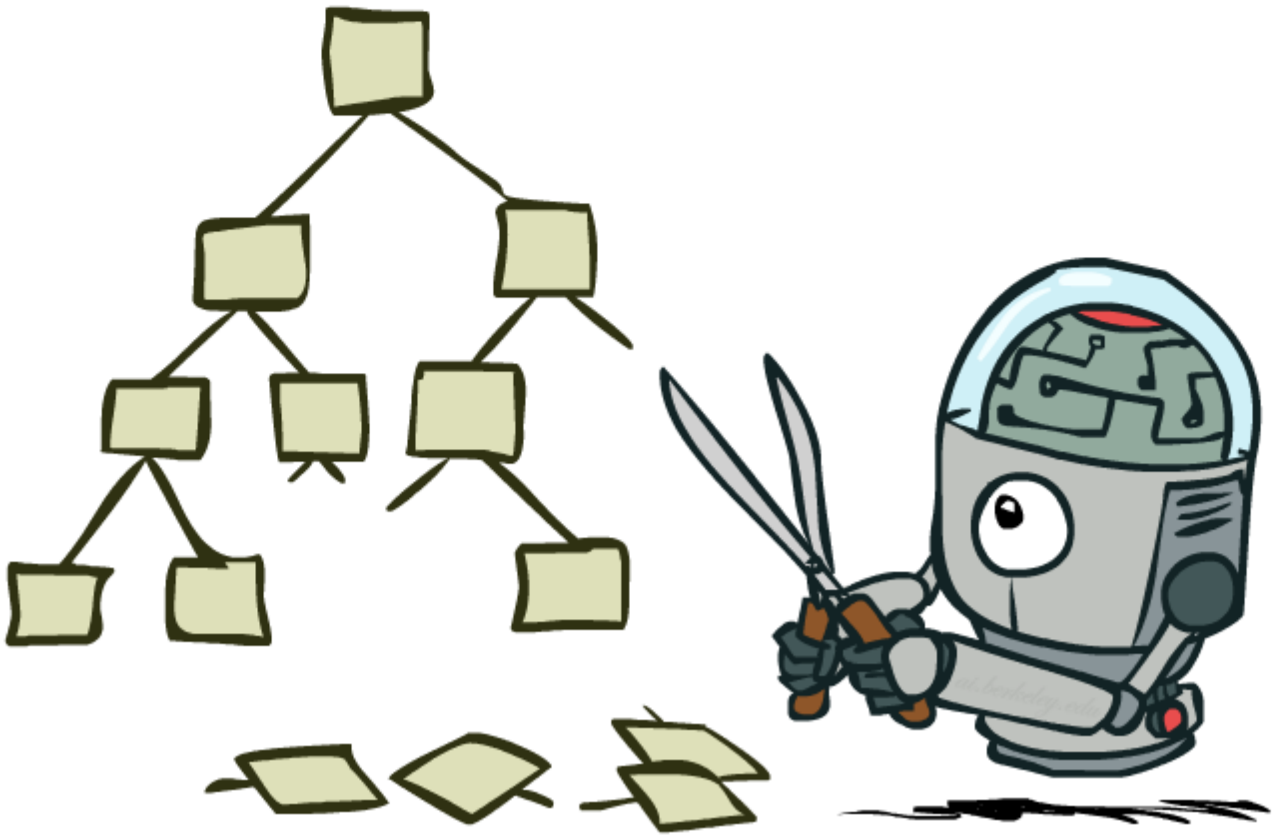
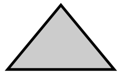


Game Tree Pruning



Minimax Example



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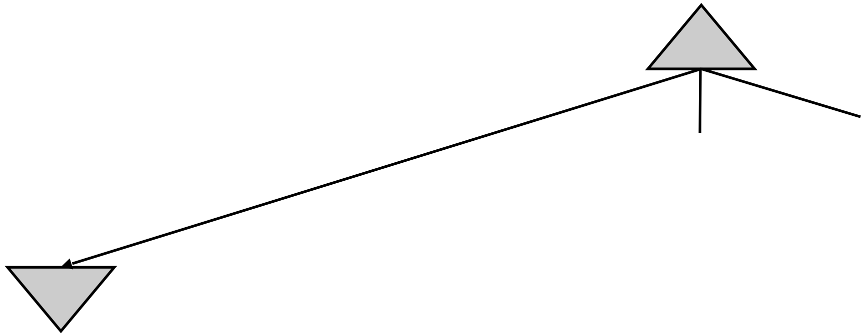

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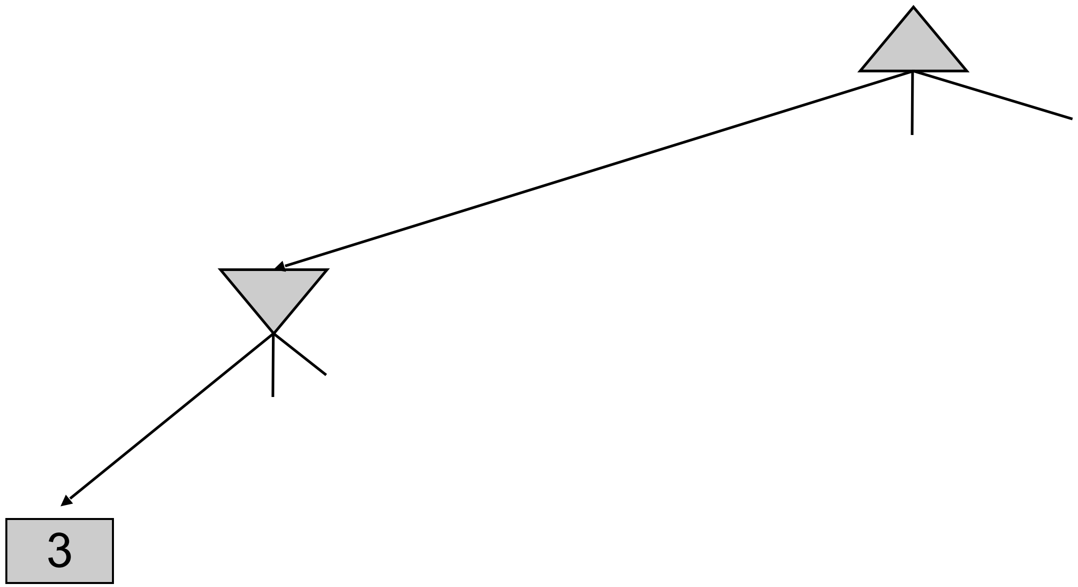

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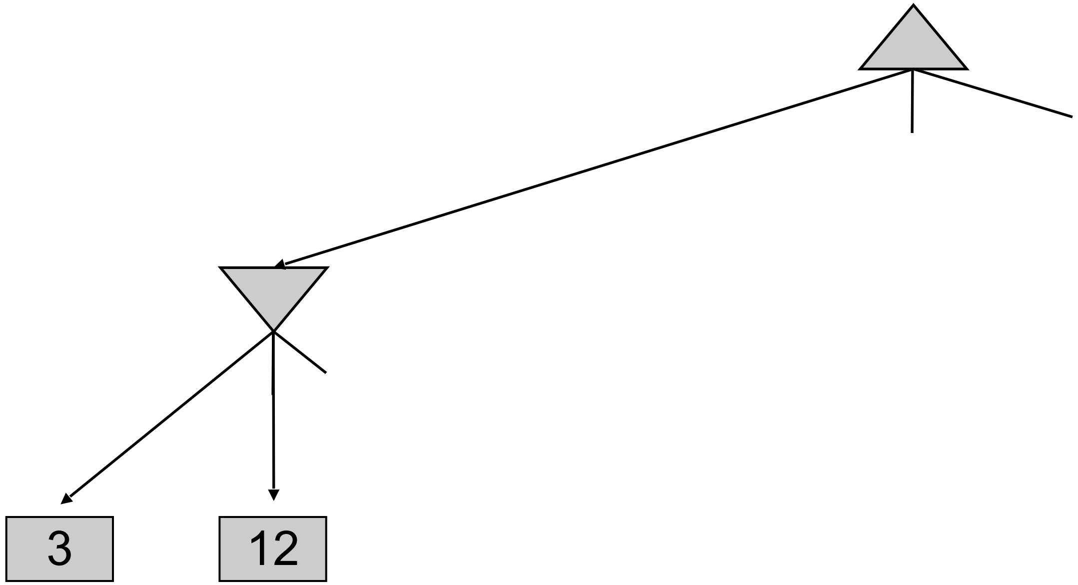
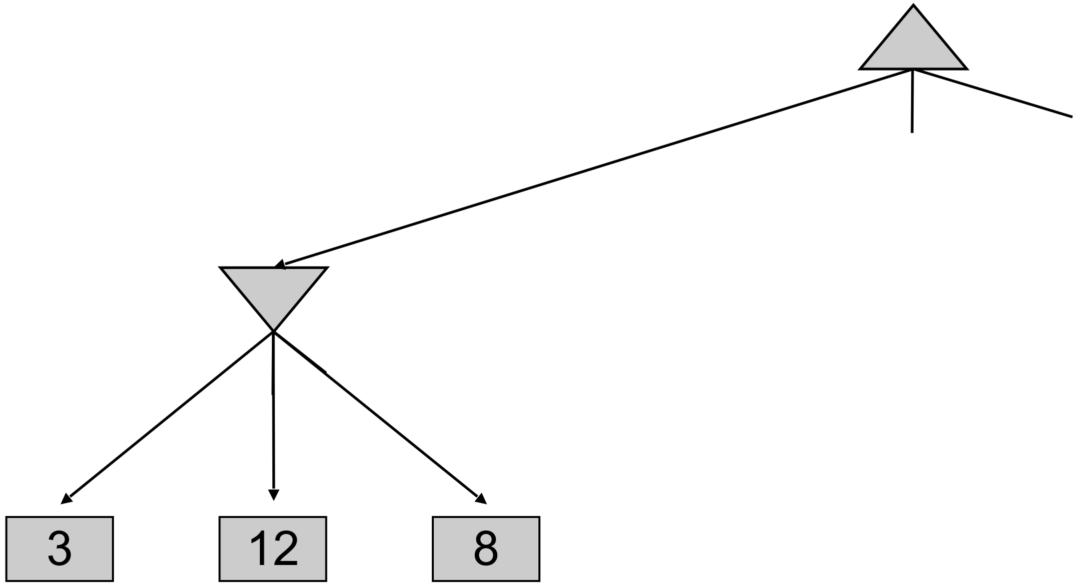


Minimax Example



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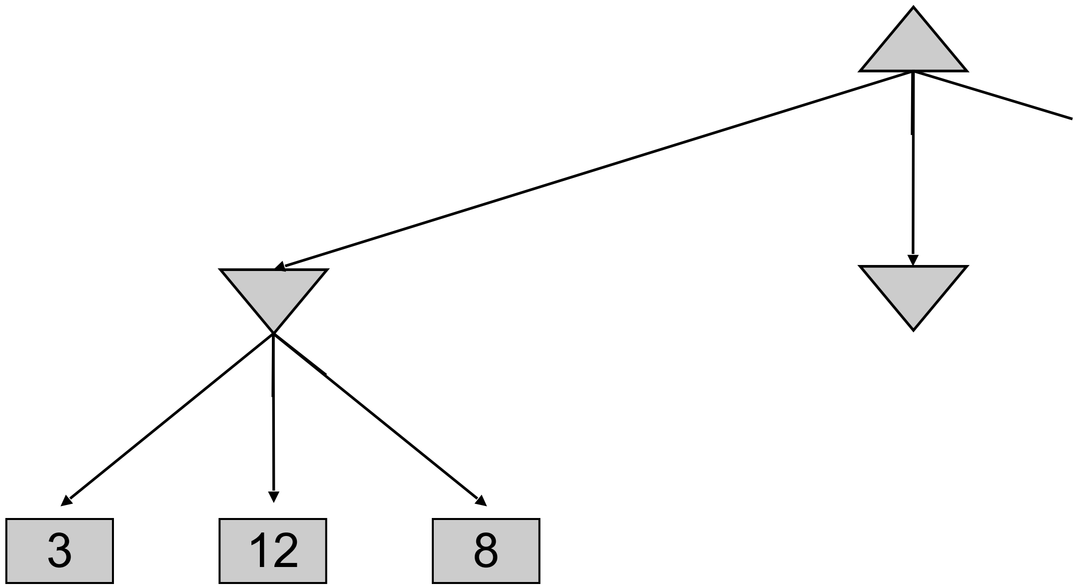

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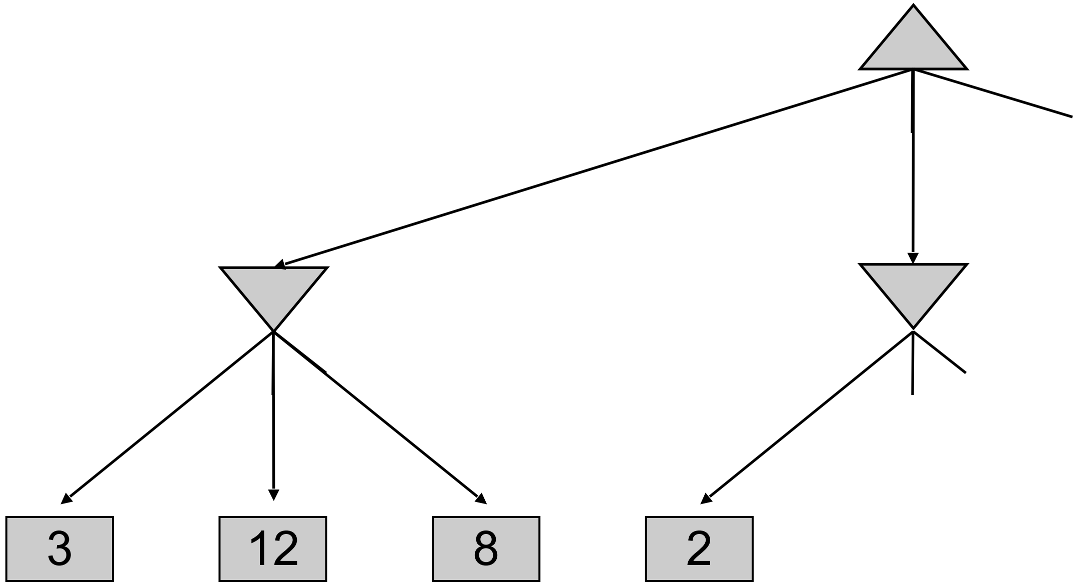

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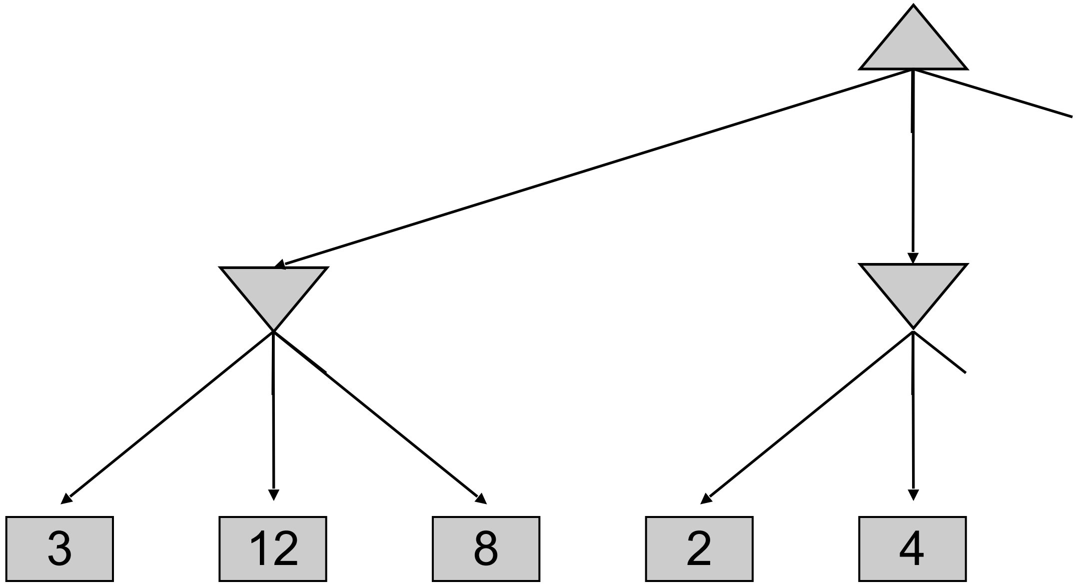

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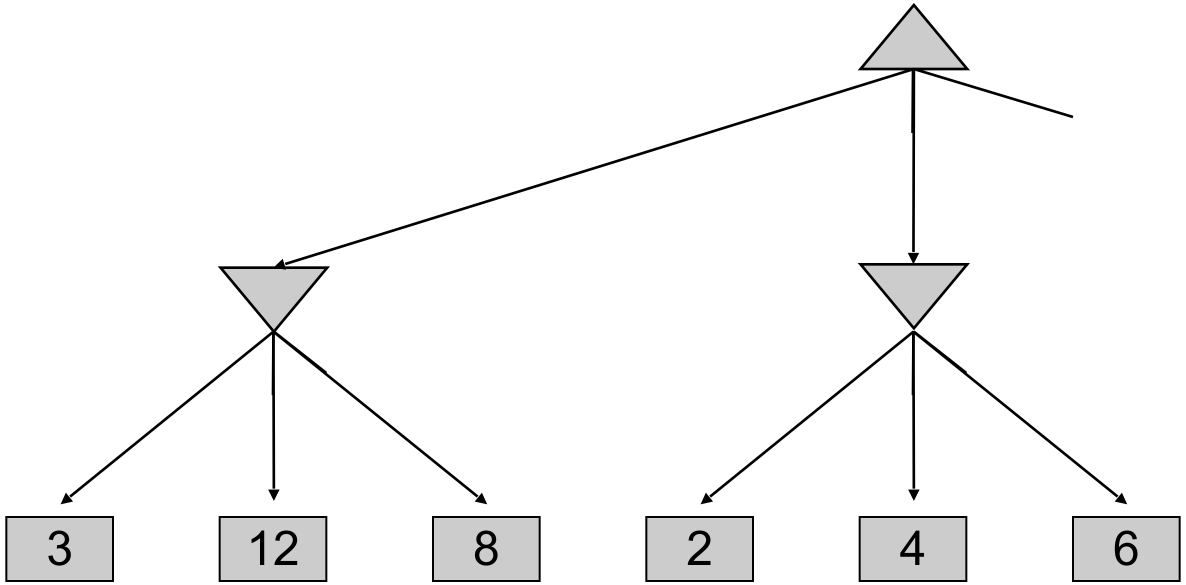

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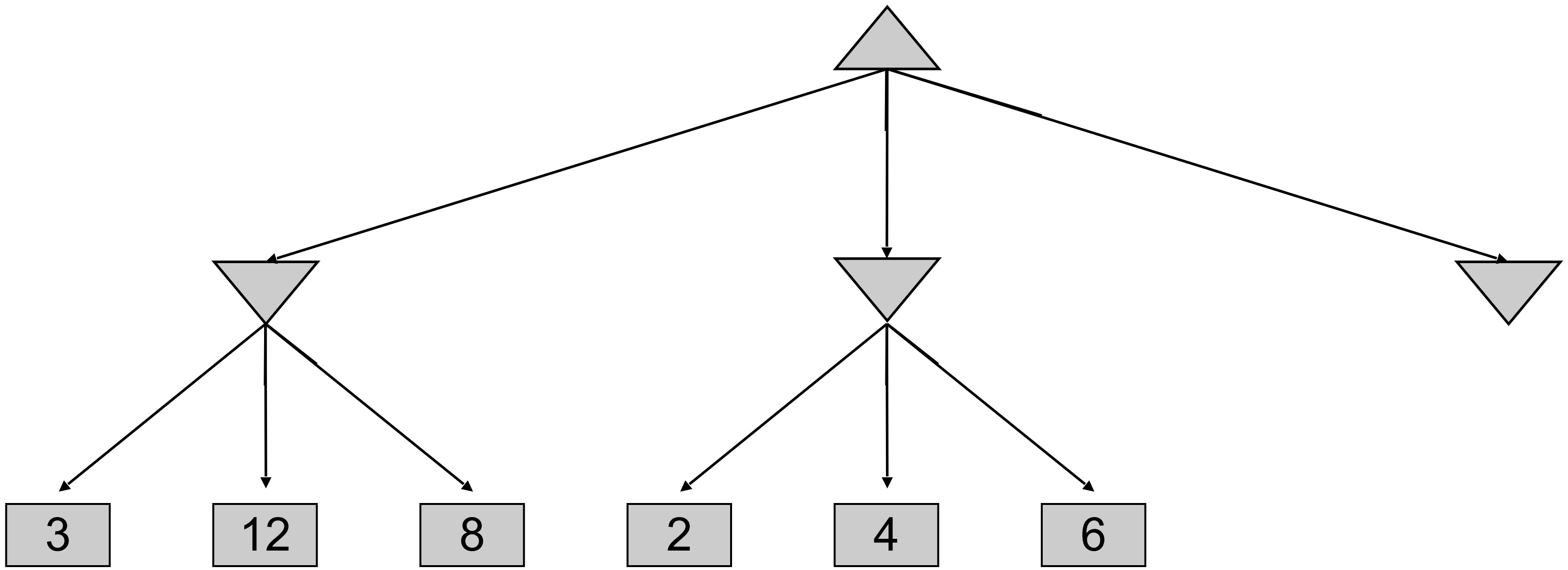

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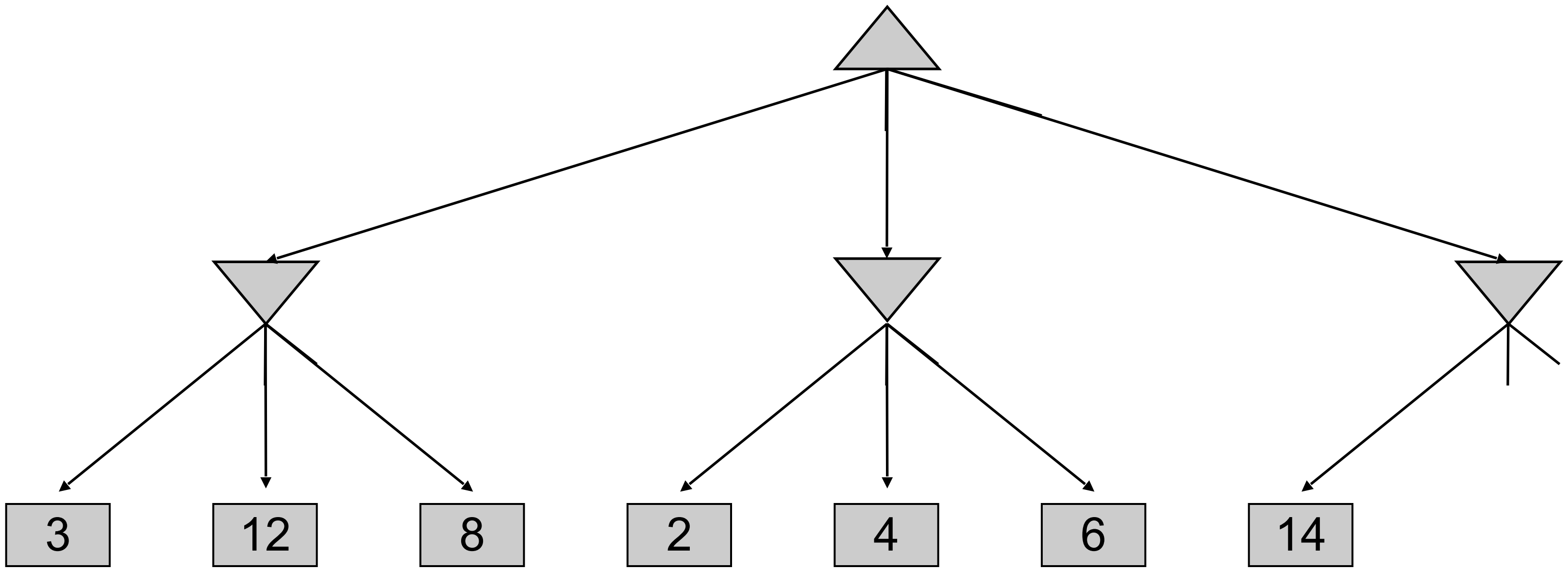

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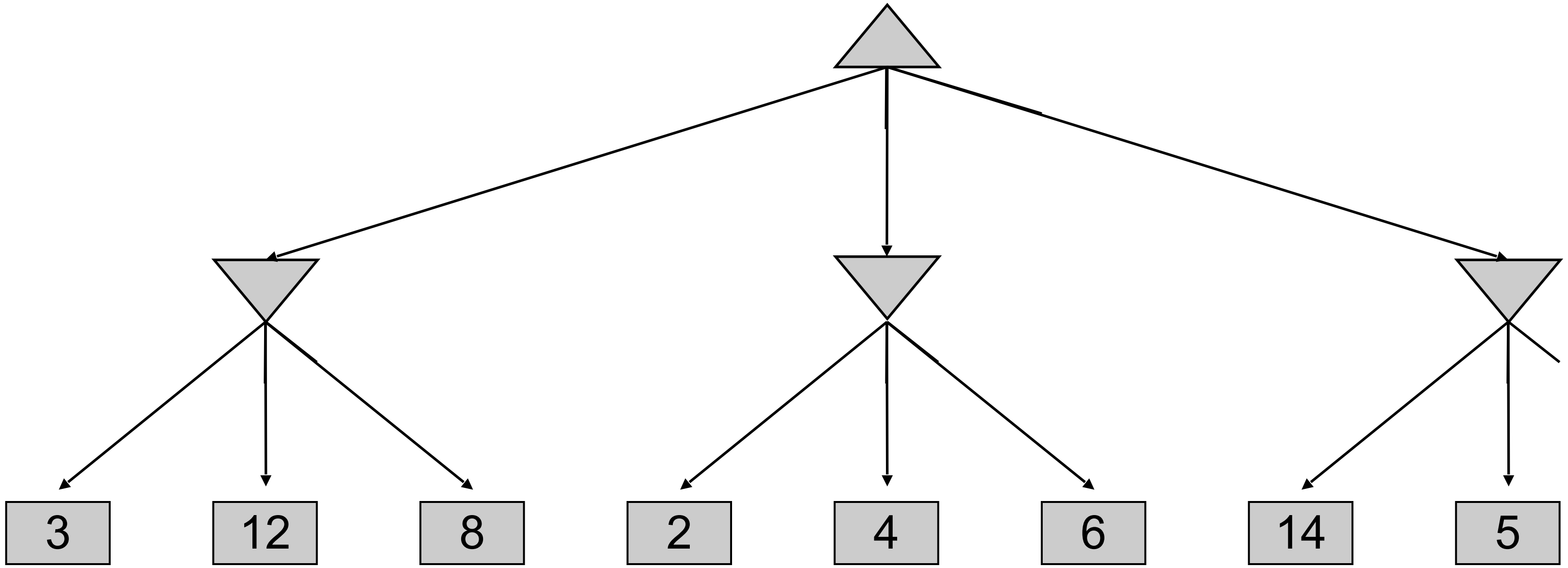

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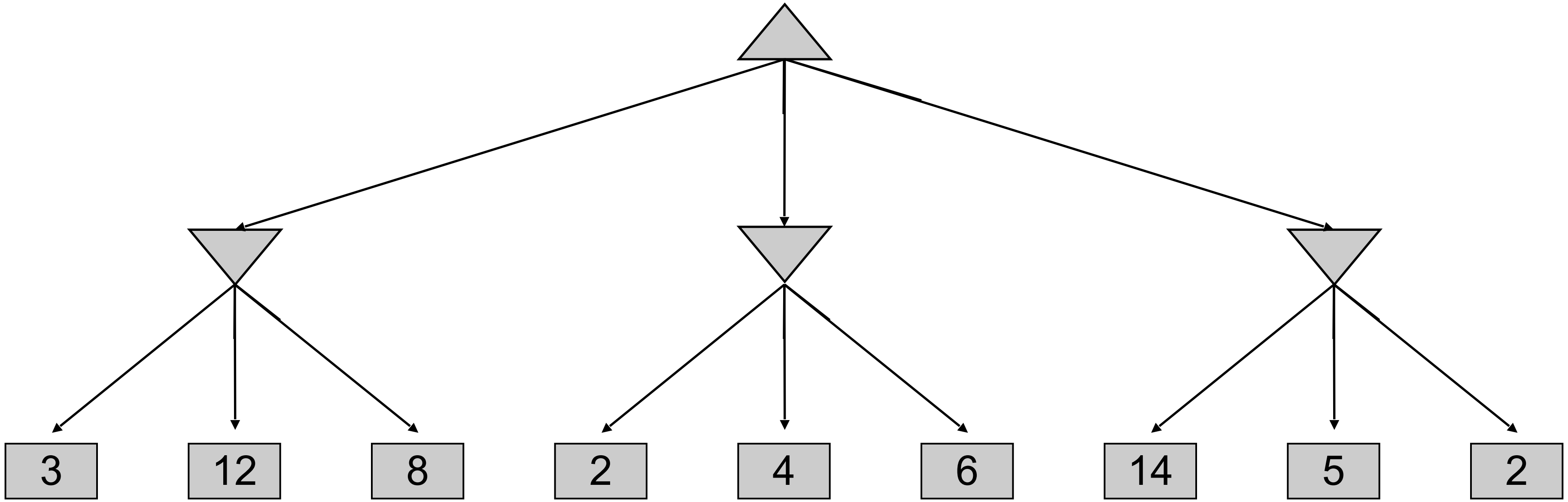
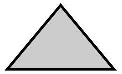


Minimax Pruning



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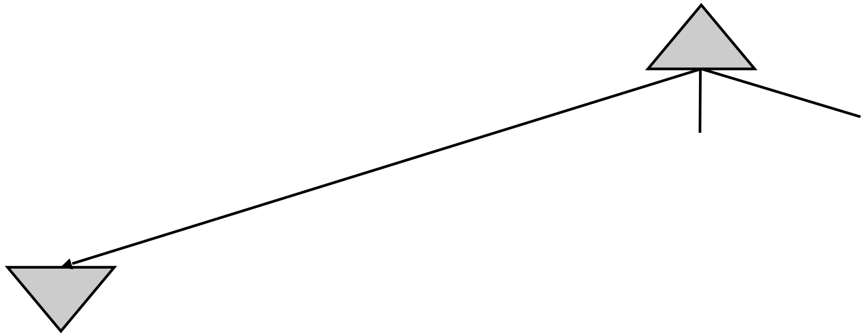
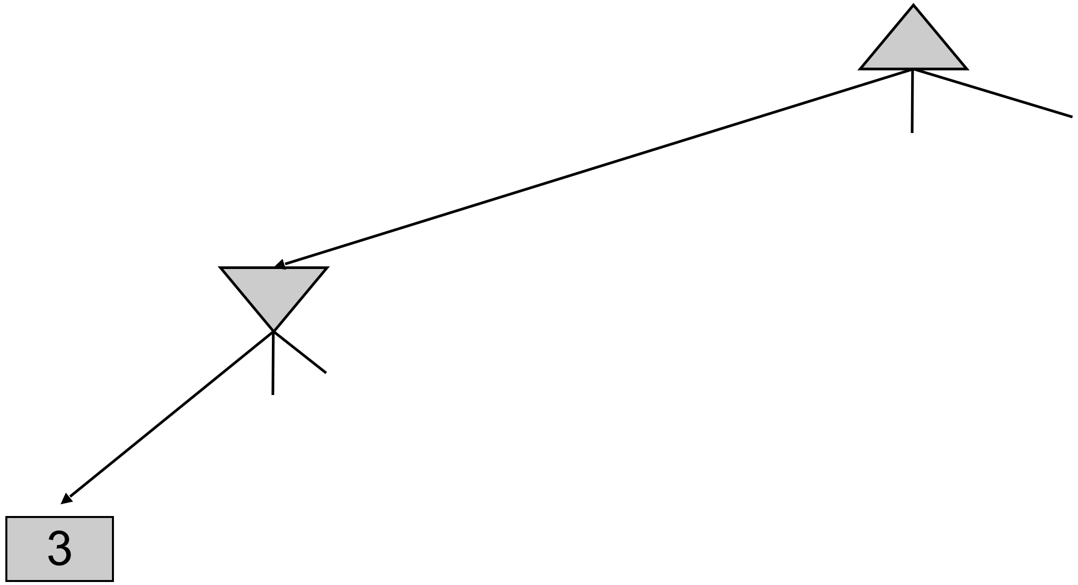


Minimax Pruning



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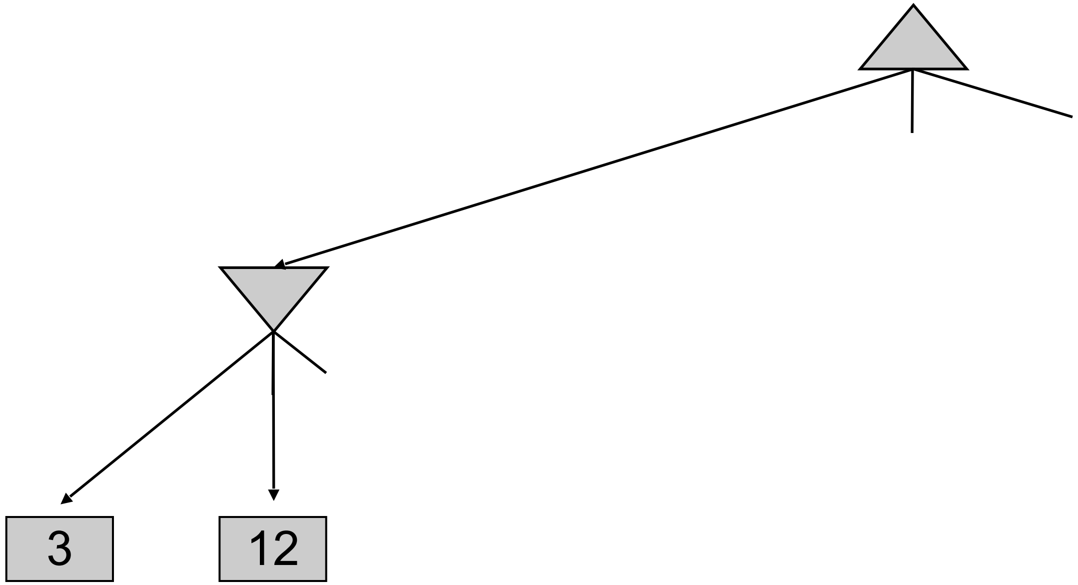

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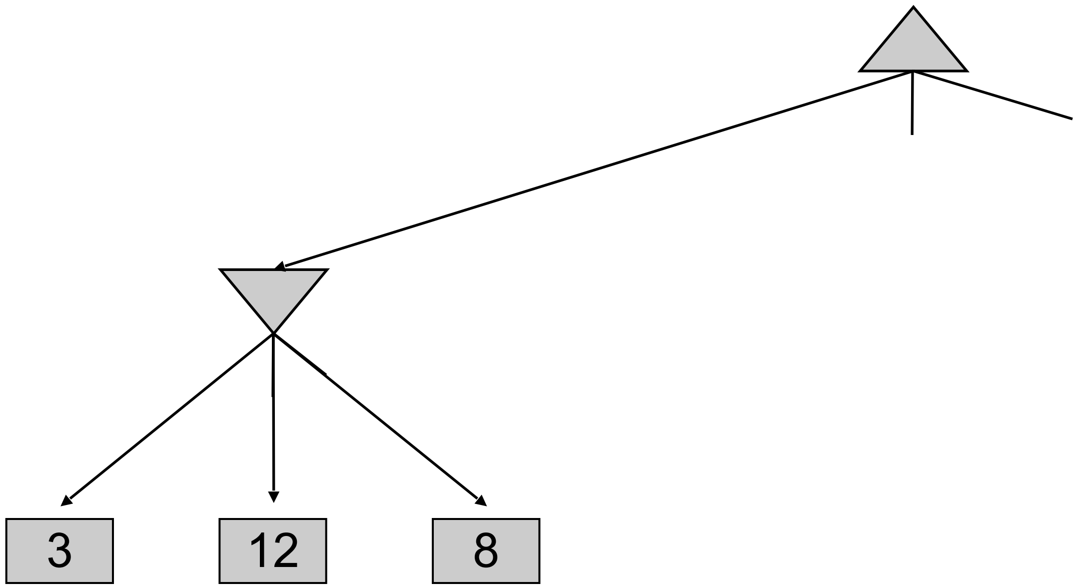

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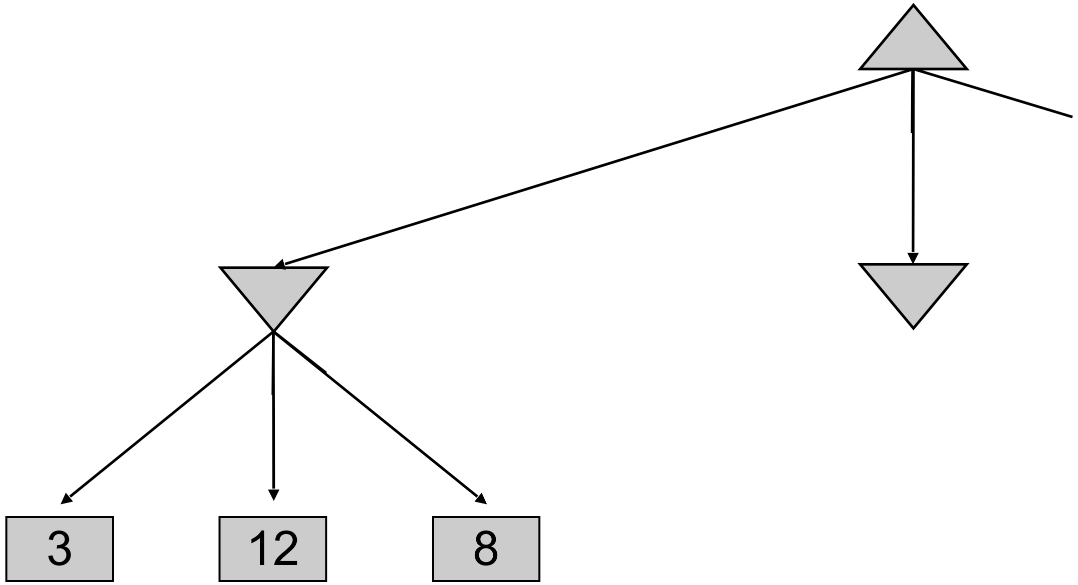

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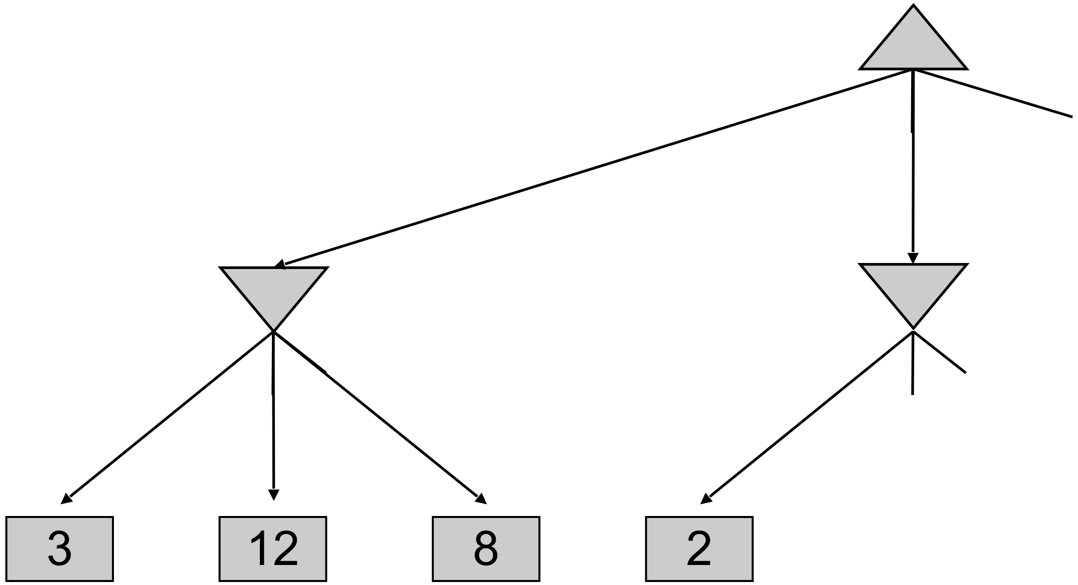

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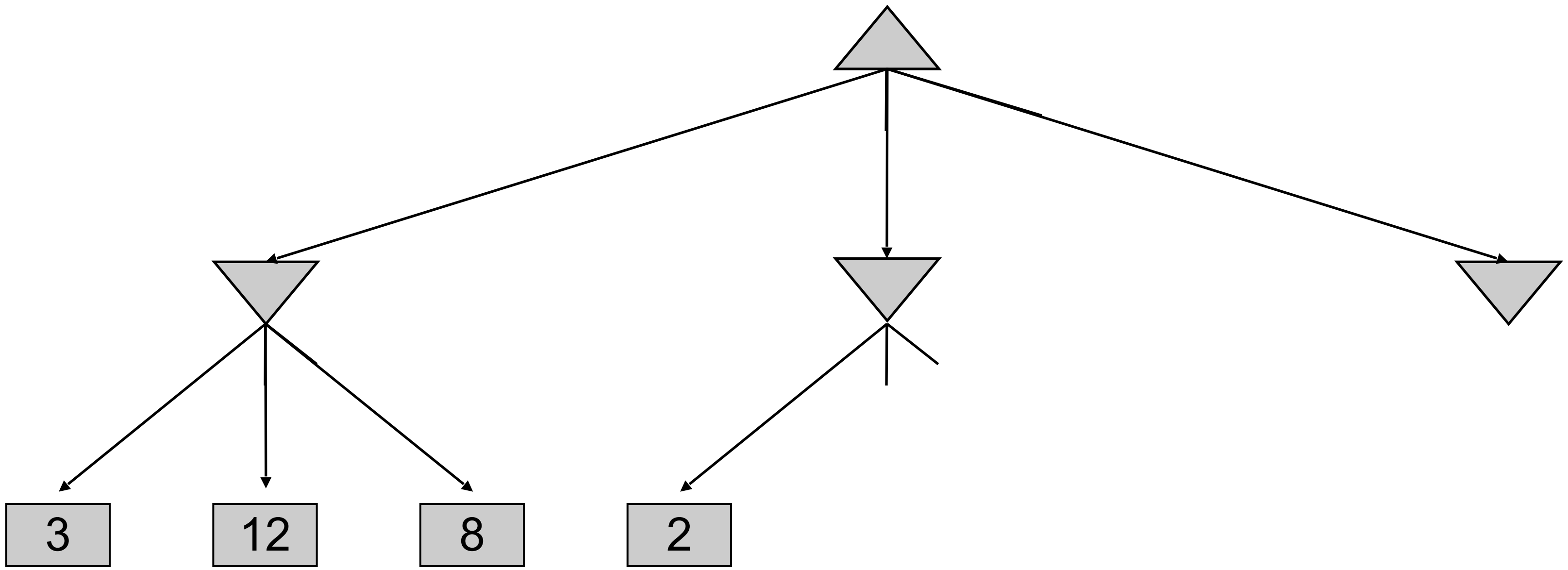
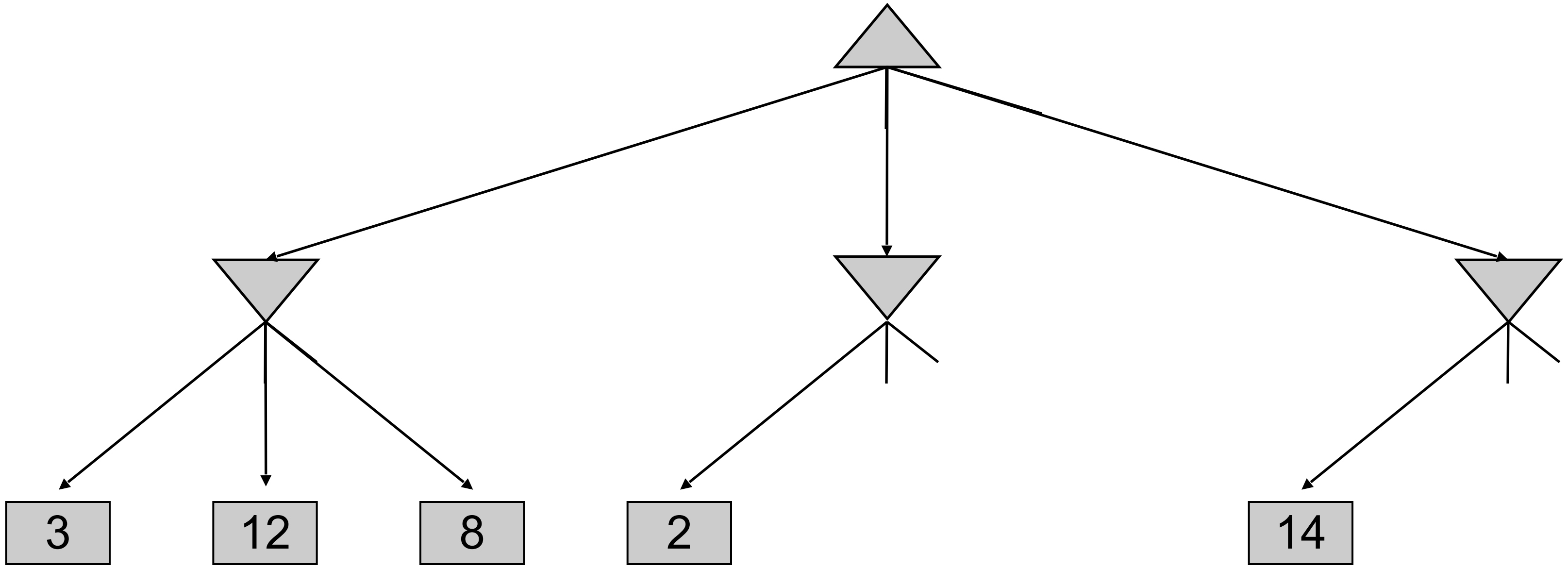


Minimax Pruning



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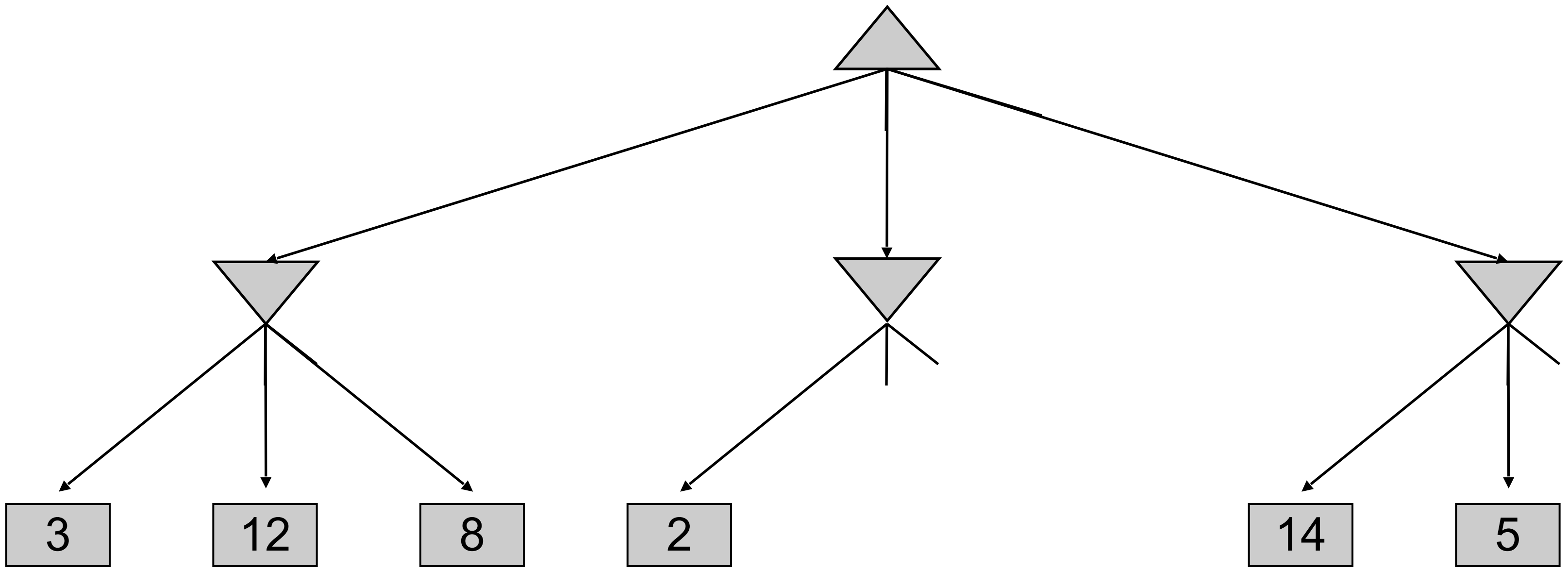
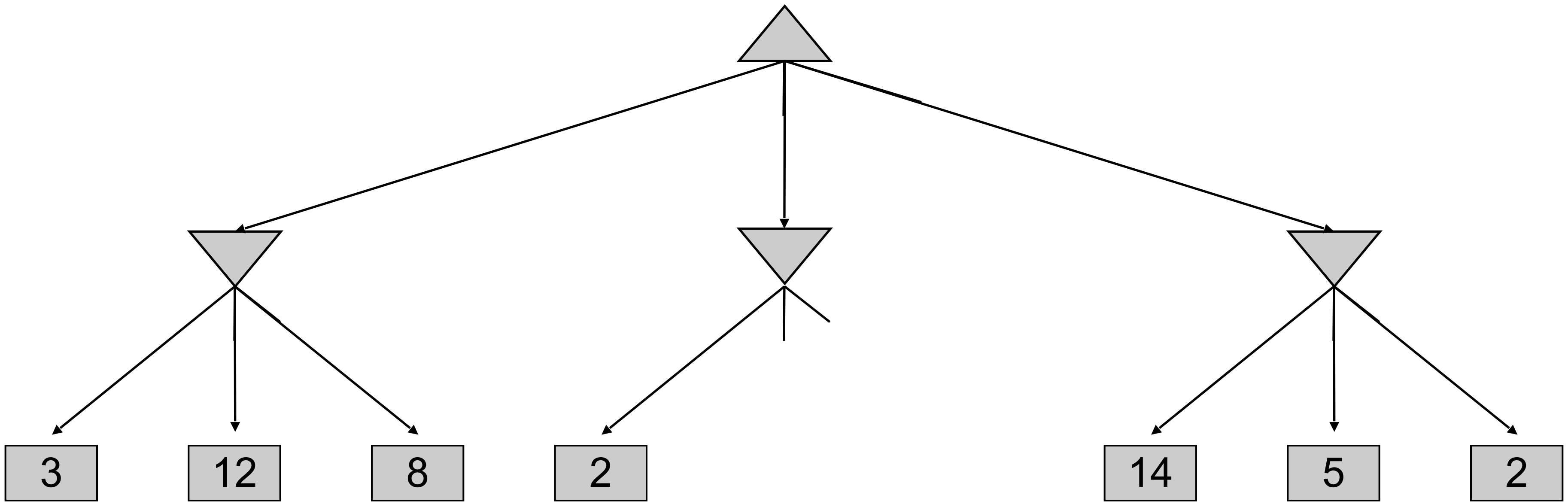


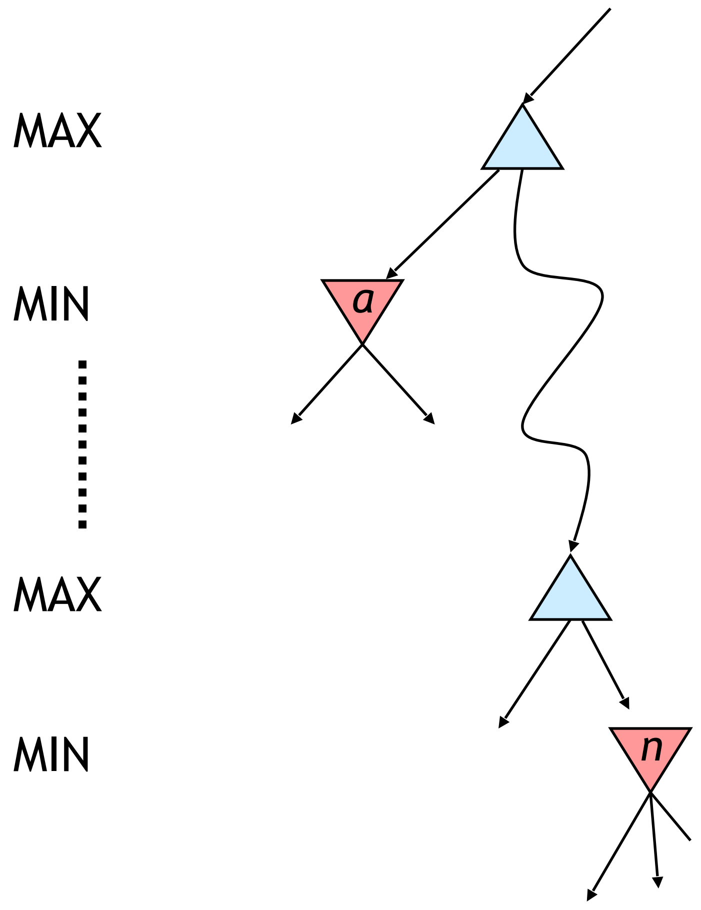
Minimax Pruning



Alpha-Beta Pruning

- General configuration (MIN version)
  - We're computing the MIN-VALUE at some node *n*
  - We're looping over *n*'s children
  - *n*'s estimate of the childrens' min is dropping
  - Who cares about *n*'s value? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If *n* becomes worse than *a*, MAX will avoid it, so we can stop considering *n*'s other children (it's already bad enough that it won't be played)

- MAX version is symmetric



Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

Minimax Implementation (Recap)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

Minimax Implementation (Recap)

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

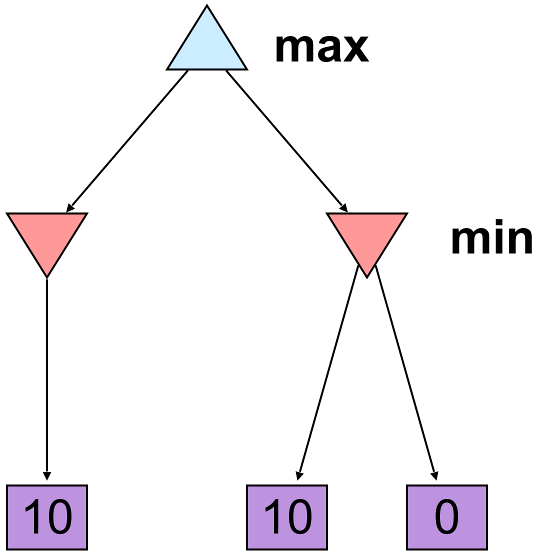
```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

Alpha-Beta Pruning Properties

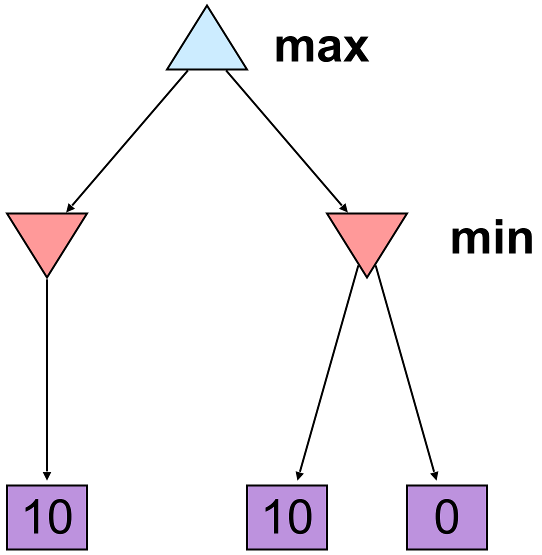
- This pruning has no effect on minimax value computed for the root!

- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection



Alpha-Beta Pruning Properties

- This pruning has no effect on minimax value computed for the root!
- Values of intermediate nodes might be wrong
  - Important: children of the root may have the wrong value
  - So the most naïve version won't let you do action selection
- Good child ordering improves effectiveness of pruning
- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth!
  - Full search of, e.g. chess, is still hopeless…
- This is a simple example of metareasoning (computing about what to compute)



Alpha-Beta Quiz