Binary Classification

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(many slides from Greg Durrett and Vivek Srikumar)

Administrivia

- Readings on course website
- Homework 1 is due January 17.

This Lecture

- Linear classification fundamentals
- Naive Bayes, maximum likelihood in generative models
- Three discriminative models: logistic regression, perceptron, SVM
  - Different motivations but very similar update rules / inference!

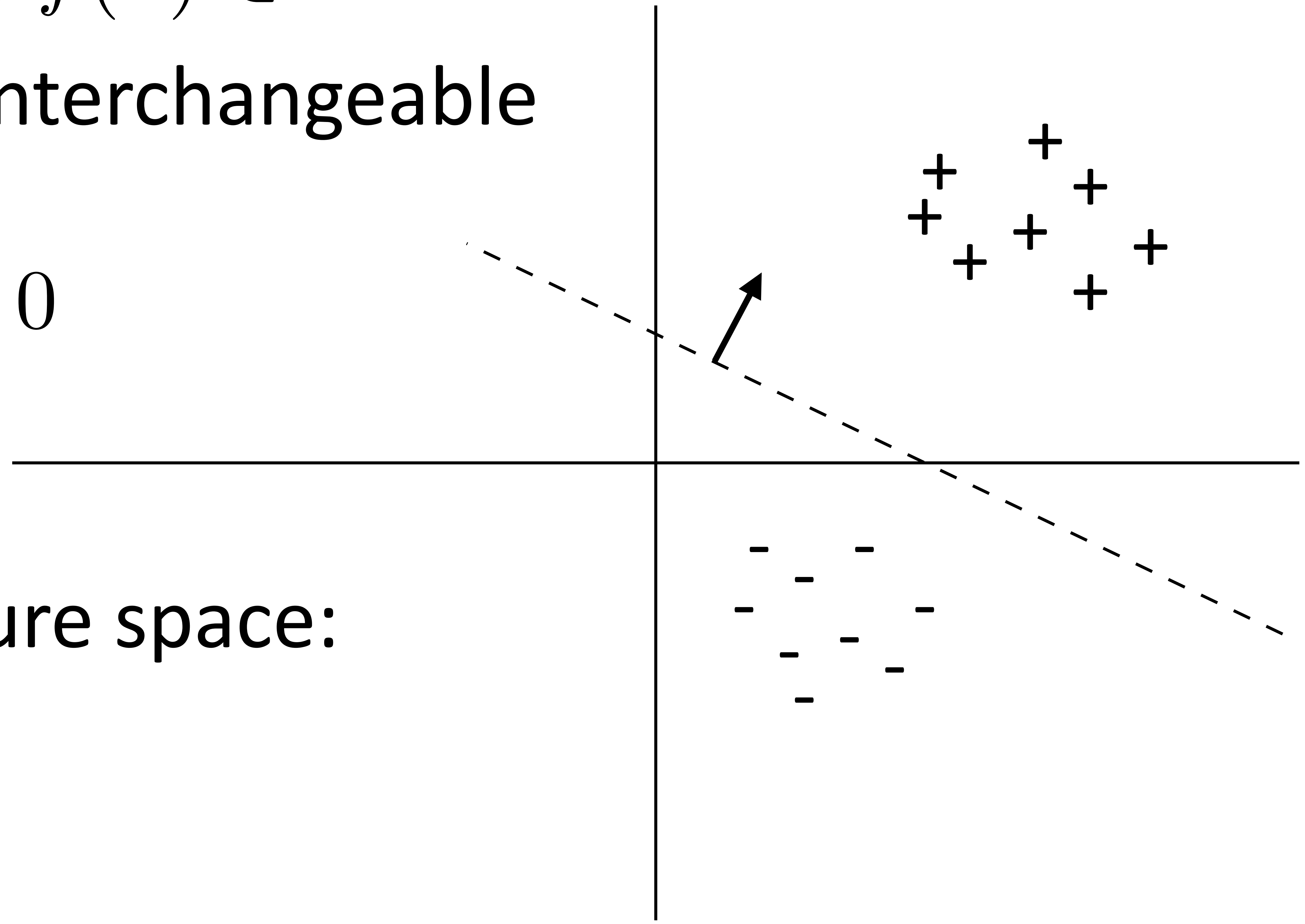
Classification

Classification

- Datapoint x with label $y \in \{0, 1\}$

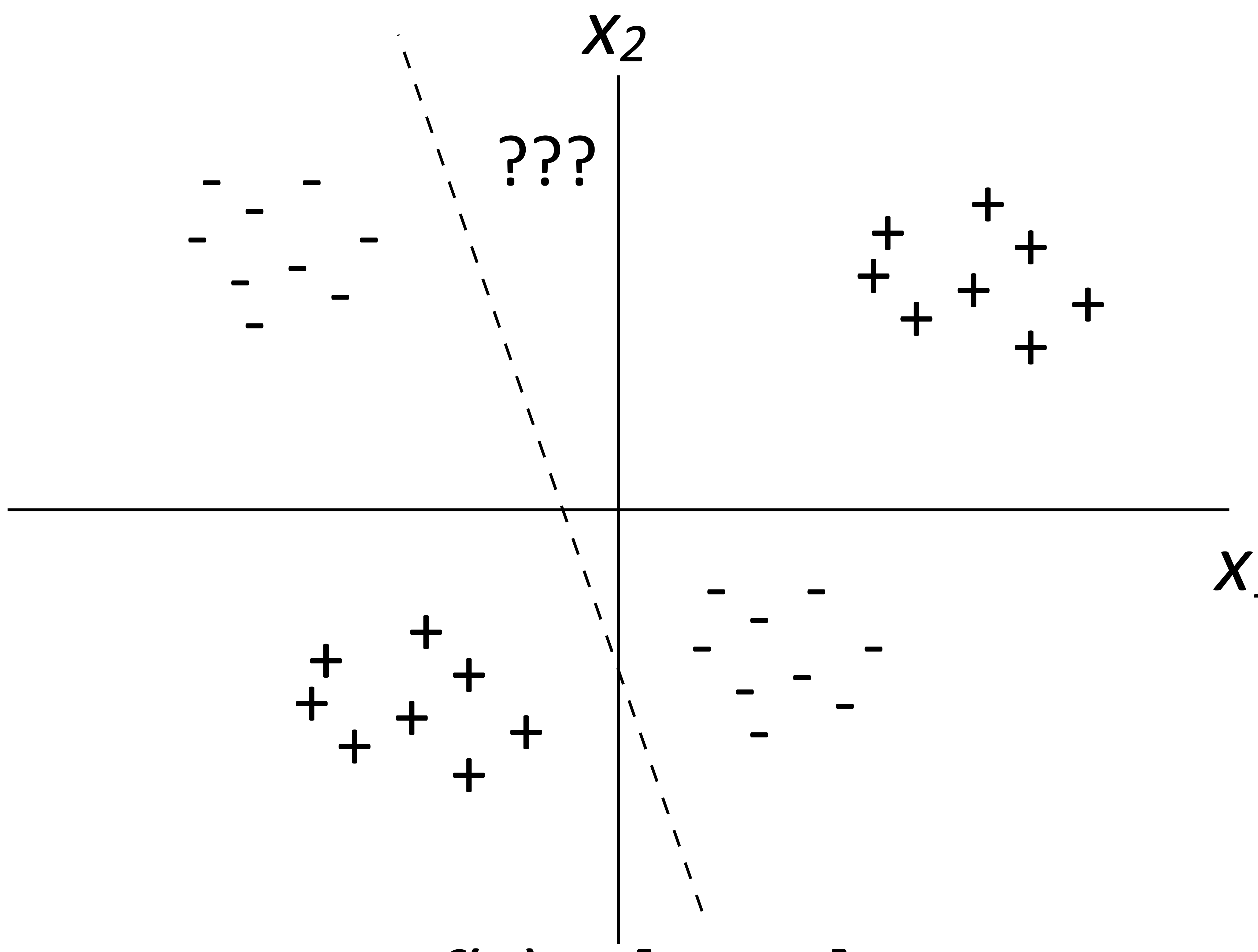
- Embed datapoint in a feature space $f(x) \in \mathbb{R}^n$
  but in this lecture $f(x)$ and x are interchangeable

- Linear decision rule: $w^\top f(x) + b > 0$
  $w^\top f(x) > 0$

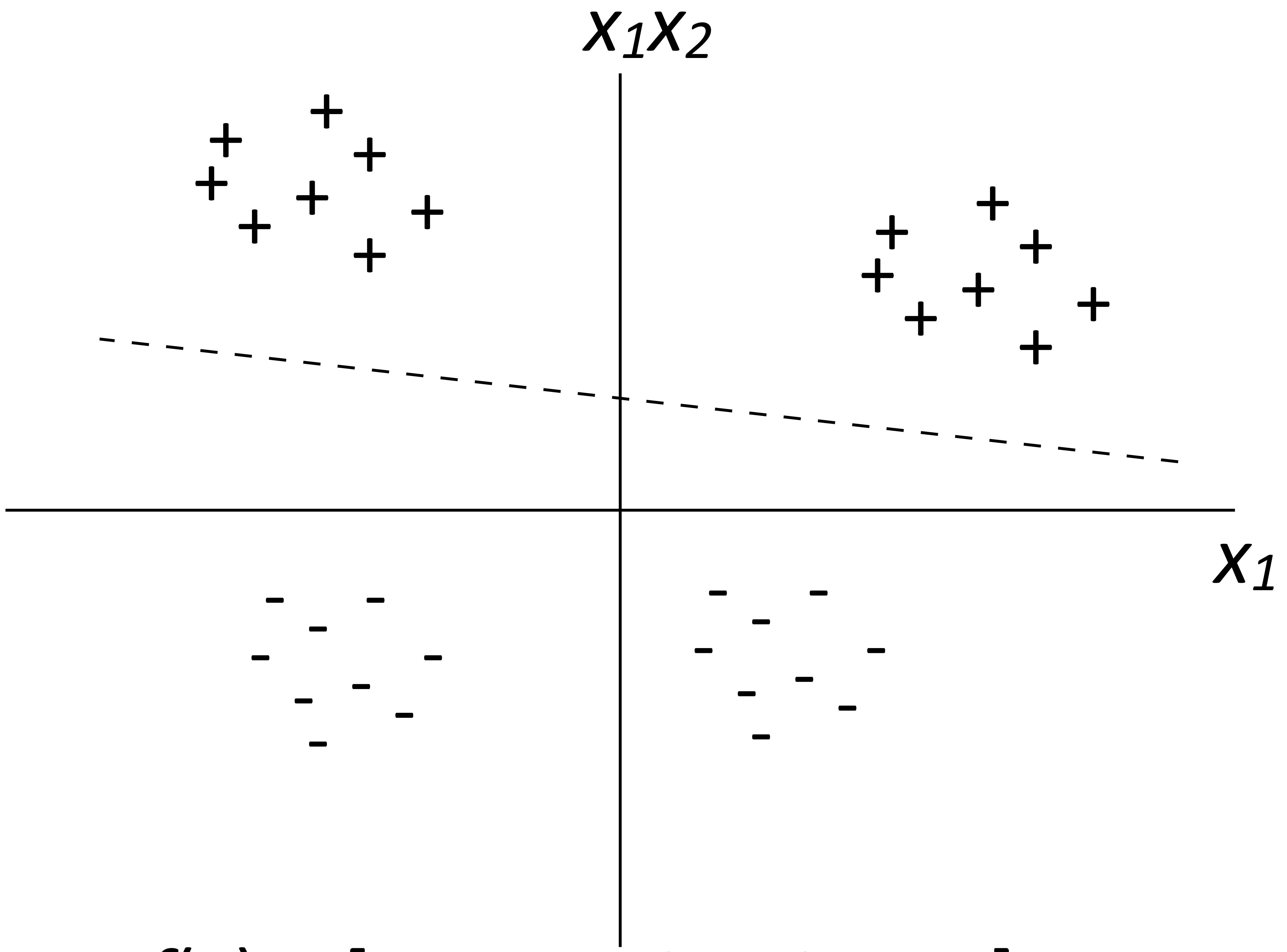


- Can delete bias if we augment feature space:
  $f(x)$ = [0.5, 1.6, 0.3]
  ↓
  [0.5, 1.6, 0.3, **1**]

Linear functions are powerful!



$f(x) = [x_1, x_2]$



$f(x) = [x_1, x_2, x_1^2, x_2^2, x_1x_2]$

- "Kernel trick" does this for "free," but is too expensive to use in NLP applications, training is $O(n^2)$ instead of $O(n \cdot (\text{num feats}))$

Classification: Sentiment Analysis

this movie was great! would watch again Positive

that film was awful, I'll never watch again Negative

- Surface cues can basically tell you what's going on here: presence or absence of certain words (*great*, *awful*)
- Steps to classification:
  - Turn examples like this into feature vectors
  - Pick a model / learning algorithm
  - Train weights on data to get our classifier

Feature Representation

this movie was great! would watch again Positive

- Convert this example to a vector using *bag-of-words features*

[contains *the*]	[contains *a*]	[contains *was*]	[contains *movie*]	[contains *film*]	...
position 0	position 1	position 2	position 3	position 4	
$f(x)$ = [0	0	1	1	0	...

- Very large vector space (size of vocabulary), sparse features
- Requires *indexing* the features (mapping them to axes)
- More sophisticated feature mappings possible (tf-idf), as well as lots of other features: character n-grams, parts of speech, lemmas, ...

Naive Bayes

Naive Bayes

- Data point $x = (x_1, ..., x_n)$, label $y \in \{0, 1\}$
- Formulate a probabilistic model that places a distribution $P(x, y)$
- Compute $P(y|x)$, predict $\text{argmax}_y P(y|x)$ to classify

$$P(y|x) = \frac{P(y)P(x|y)}{P(x)}$$

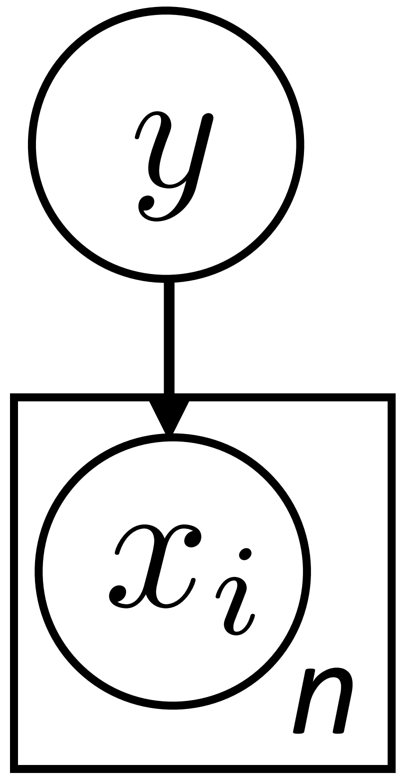
Bayes' Rule

constant: irrelevant for finding the max

$$\propto P(y)P(x|y)$$

"Naive" assumption:

$$= P(y) \prod_{i=1}^{n} P(x_i|y)$$



linear model!

$$\text{argmax}_y P(y|x) = \text{argmax}_y \log P(y|x) = \text{argmax}_y \left[\log P(y) + \sum_{i=1}^{n} \log P(x_i|y) \right]$$

Naive Bayes Example

it was great $\longrightarrow$ $P(y|x) \propto \left[\quad \right]$

$$P(y|x) \propto P(y) \prod_{i=1}^{n} P(x_i|y)$$

$$\text{argmax}_y \log P(y|x) = \text{argmax}_y \left[\log P(y) + \sum_{i=1}^{n} \log P(x_i|y) \right]$$

Maximum Likelihood Estimation

- Data points (x_j, y_j) provided (*j* indexes over examples)
- Find values of $P(y),\ \ P(x_i|y)$ that maximize data likelihood (generative):

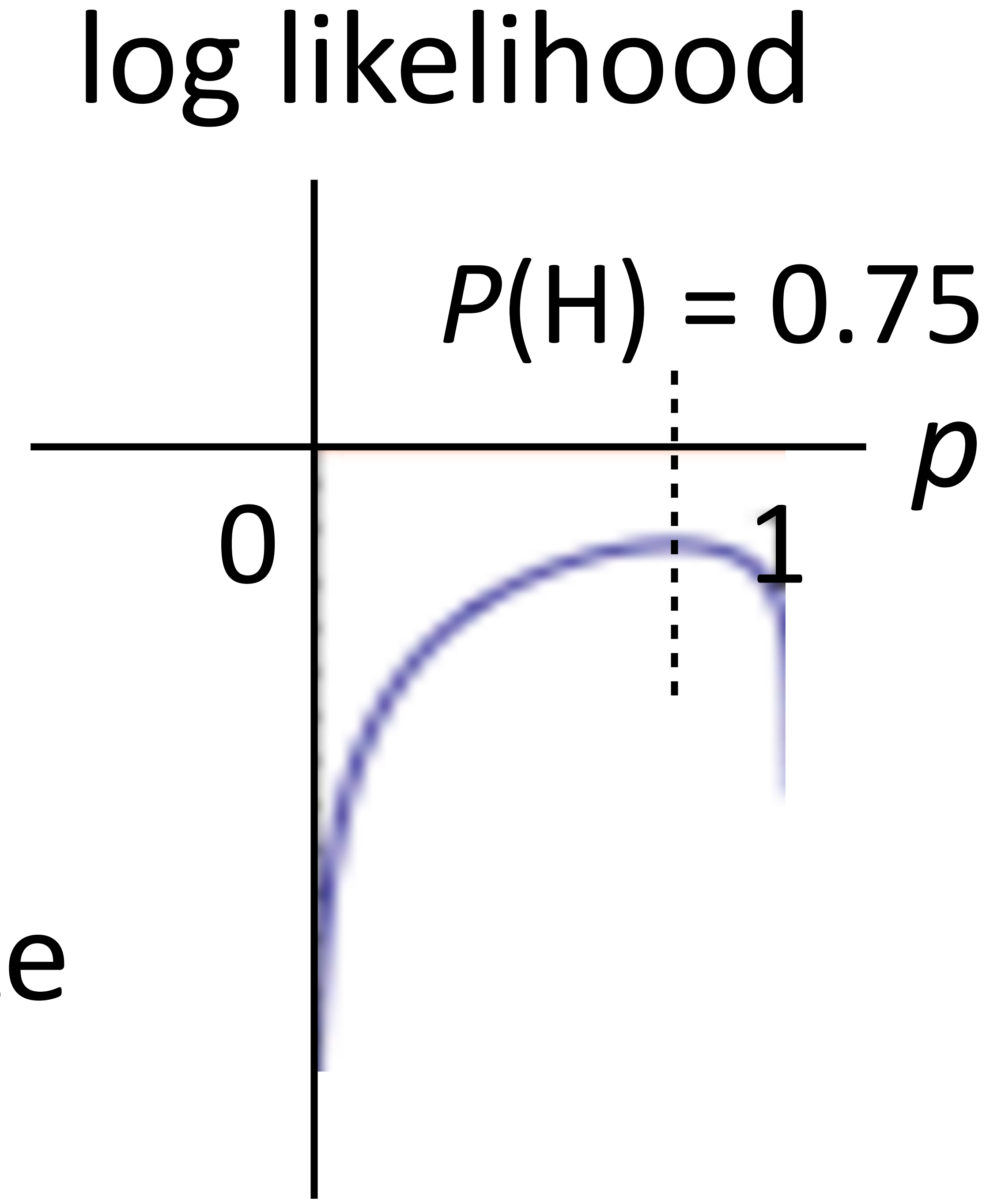
$$\prod_{j=1}^{m} P(y_j, x_j) = \prod_{j=1}^{m} P(y_j) \left[\prod_{i=1}^{n} P(x_{ji}|y_j)\right]$$

data points (*j*) features (*i*) *i*th feature of *j*th example

Maximum Likelihood Estimation

- Imagine a coin flip which is heads with probability *p*
- Observe (H, H, H, T) and maximize likelihood: $\prod_{j=1}^{m} P(y_j) = p^3(1-p)$
- Easier: maximize *log* likelihood

$$\sum_{j=1}^{m} \log P(y_j) = 3\log p + \log(1-p)$$



- Maximum likelihood parameters for binomial/ multinomial = read counts off of the data + normalize

Maximum Likelihood Estimation

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- Find values of $P(y),\ P(x_i|y)$ that maximize data likelihood (generative):

$$\prod_{j=1}^{m} P(y_j, x_j) = \prod_{j=1}^{m} P(y_j) \left[\prod_{i=1}^{n} P(x_{ji}|y_j) \right]$$

data points (*j*) features (*i*) *i*th feature of *j*th example

- Equivalent to maximizing logarithm of data likelihood:

$$\sum_{j=1}^{m} \log P(y_j, x_j) = \sum_{j=1}^{m} \left[\log P(y_j) + \sum_{i=1}^{n} \log P(x_{ji}|y_j) \right]$$

Maximum Likelihood for Naive Bayes

this movie was great! would watch again	+
I liked it well enough for an action flick	+
I expected a great film and left happy	+
brilliant directing and stunning visuals	+
that film was awful, I'll never watch again	—
I didn't really like that movie	—
dry and a bit distasteful, it misses the mark	—
great potential but ended up being a flop	—

$$P(+) = \frac{1}{2}$$

$$P(-) = \frac{1}{2}$$

$$P(\mathrm{great}|+) = \frac{1}{2}$$

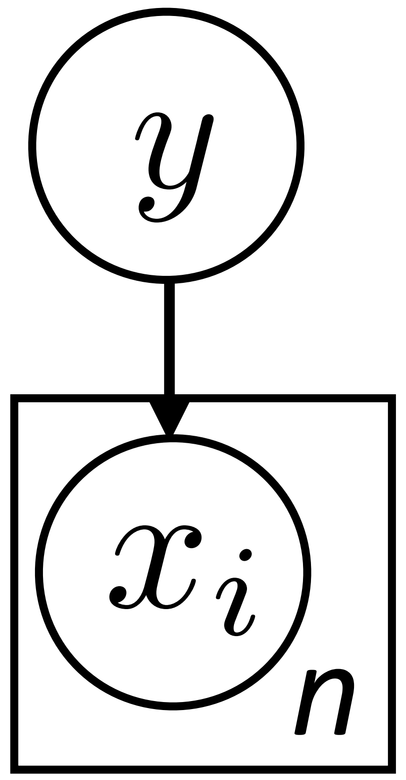
$$P(\mathrm{great}|-) = \frac{1}{4}$$

it was great $\longrightarrow$ $$P(y|x) \propto \begin{bmatrix} P(+)P(\mathrm{great}|+) \\ P(-)P(\mathrm{great}|-) \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/8 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

Naive Bayes: Summary

- Model

$$P(x,y) = P(y)\prod_{i=1}^{n} P(x_i|y)$$



- Inference

$$\mathrm{argmax}_y \log P(y|x) = \mathrm{argmax}_y \left[\log P(y) + \sum_{i=1}^{n} \log P(x_i|y)\right]$$

- Alternatively: $\log P(y=+|x) - \log P(y=-|x) > 0$

$$\Leftrightarrow \log \frac{P(y=+)}{P(y=-)} + \sum_{i=1}^{n} \log \frac{P(x_i|y=+)}{P(x_i|y=-)} > 0$$

- Learning: maximize $P(x,y)$ by reading counts off the data

Problems with Naive Bayes

the film was beautiful, stunning cinematography and gorgeous sets, but boring —

$$P(x_{\text{beautiful}}|+) = 0.1 \qquad P(x_{\text{beautiful}}|-) = 0.01$$

$$P(x_{\text{stunning}}|+) = 0.1 \qquad P(x_{\text{stunning}}|-) = 0.01$$

$$P(x_{\text{gorgeous}}|+) = 0.1 \qquad P(x_{\text{gorgeous}}|-) = 0.01$$

$$P(x_{\text{boring}}|+) = 0.01 \qquad P(x_{\text{boring}}|-) = 0.1$$

- Correlated features compound: *beautiful* and *gorgeous* are not independent!
- Naive Bayes is naive, but another problem is that it's *generative*: spends capacity modeling P(x,y), when what we care about is P(y|x)
- Discriminative models model P(y|x) directly (SVMs, most neural networks, …)

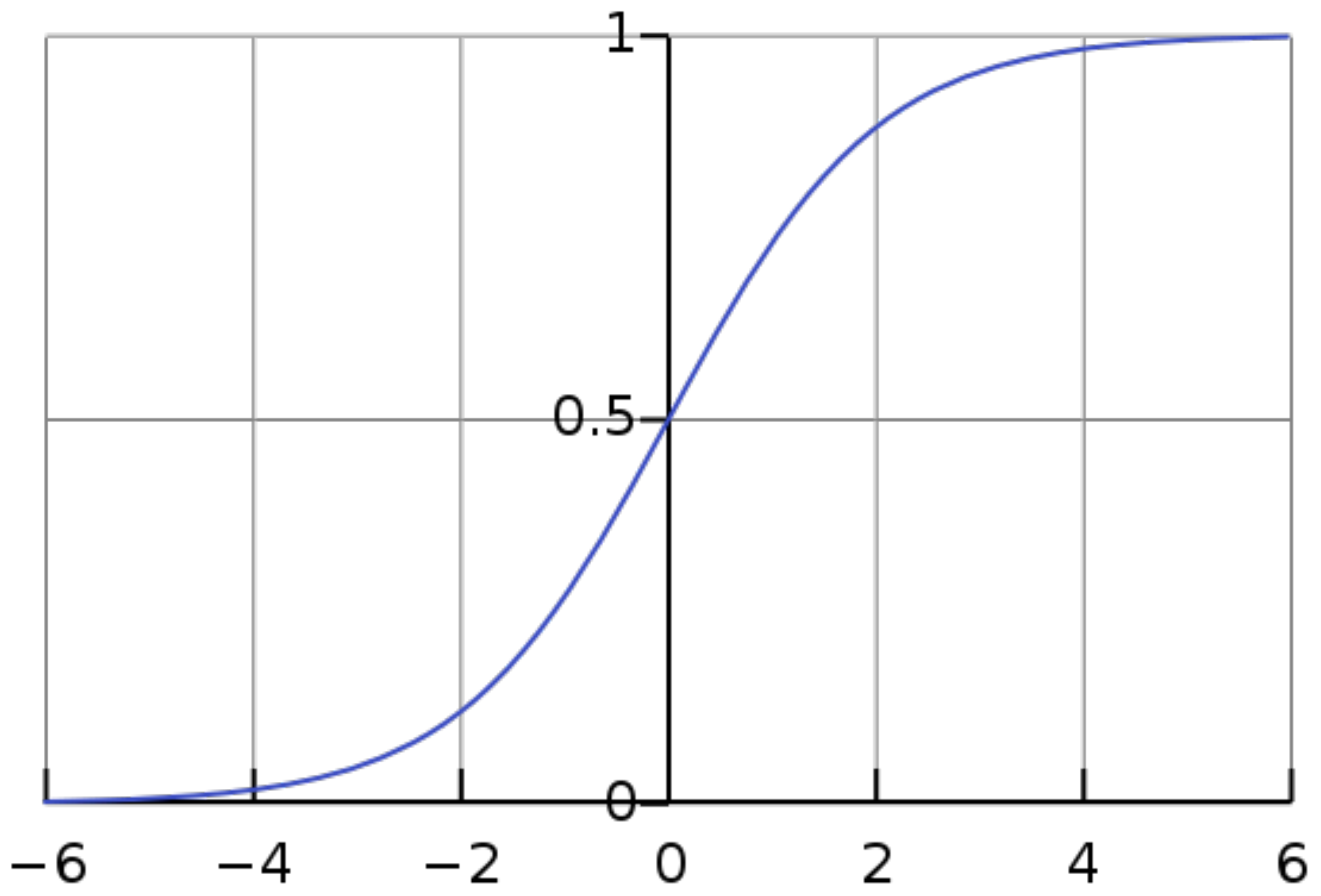
Homework 1 Demo (Numpy)

Logistic Regression

Logistic Regression

$$P(y=+|x) = \text{logistic}(w^\top x)$$

$$P(y=+|x) = \frac{\exp(\sum_{i=1}^{n} w_i x_i)}{1+\exp(\sum_{i=1}^{n} w_i x_i)}$$



- To learn weights: maximize discriminative log likelihood of data P(y|x)

$$\mathcal{L}(x_j, y_j = +) = \log P(y_j = +|x_j)$$

$$= \sum_{i=1}^{n} w_i x_{ji} - \log\left(1+\exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)\right)$$

sum over features

Logistic Regression

$$\mathcal{L}(x_j, y_j = +) = \log P(y_j = +|x_j) = \boxed{\sum_{i=1}^{n} w_i x_{ji}} - \log\left(1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)\right)$$

$$\frac{\partial \mathcal{L}(x_j, y_j)}{\partial w_i} = \boxed{x_{ji}} - \frac{\partial}{\partial w_i} \log\left(\boxed{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}\right)$$

$$= x_{ji} - \frac{1}{\boxed{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}} \frac{\partial}{\partial w_i}\left(\boxed{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}\right)$$

deriv of log

$$= x_{ji} - \frac{1}{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)} x_{ji} \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)$$

deriv of exp

$$= x_{ji} - x_{ji} \frac{\exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)}{1 + \exp\left(\sum_{i=1}^{n} w_i x_{ji}\right)} = x_{ji}(1 - P(y_j = +|x_j))$$

Logistic Regression

- Recall that $y_j = 1$ for positive instances, $y_j = 0$ for negative instances.
- Gradient of w_i on positive example $= x_{ji}(y_j - P(y_j = +|x_j))$

  If P(+) is close to 1, make very little update

  Otherwise make w_i look more like x_{ji}, which will increase P(+)

- Gradient of w_i on negative example $= x_{ji}(-P(y_j = +|x_j))$

  If P(+) is close to 0, make very little update

  Otherwise make w_i look less like x_{ji}, which will decrease P(+)

- Can combine these gradients as $x_j(y_j - P(y_j = 1|x_j))$

Regularization

- Regularizing an objective can mean many things, including an L2-norm penalty to the weights:

$$\sum_{j=1}^{m} \mathcal{L}(x_j, y_j) - \lambda \|w\|_2^2$$

- Keeping weights small can prevent overfitting
- For most of the NLP models we build, explicit regularization isn't necessary
  - Early stopping
  - Large numbers of sparse features are hard to overfit in a really bad way
  - For neural networks: dropout and gradient clipping

Logistic Regression: Summary

- Model

$$P(y=+|x) = \frac{\exp(\sum_{i=1}^{n} w_i x_i)}{1+\exp(\sum_{i=1}^{n} w_i x_i)}$$

- Inference

$\mathrm{argmax}_y P(y|x)$ fundamentally same as Naive Bayes

$$P(y=1|x) \geq 0.5 \Leftrightarrow w^\top x \geq 0$$

- Learning: gradient ascent on the (regularized) discriminative log-likelihood

Perceptron/SVM

Perceptron

- Simple error-driven learning approach similar to logistic regression

- Decision rule: $w^\top x > 0$
  - If incorrect: if positive, $w \leftarrow w + x$

    if negative, $w \leftarrow w - x$

Logistic Regression

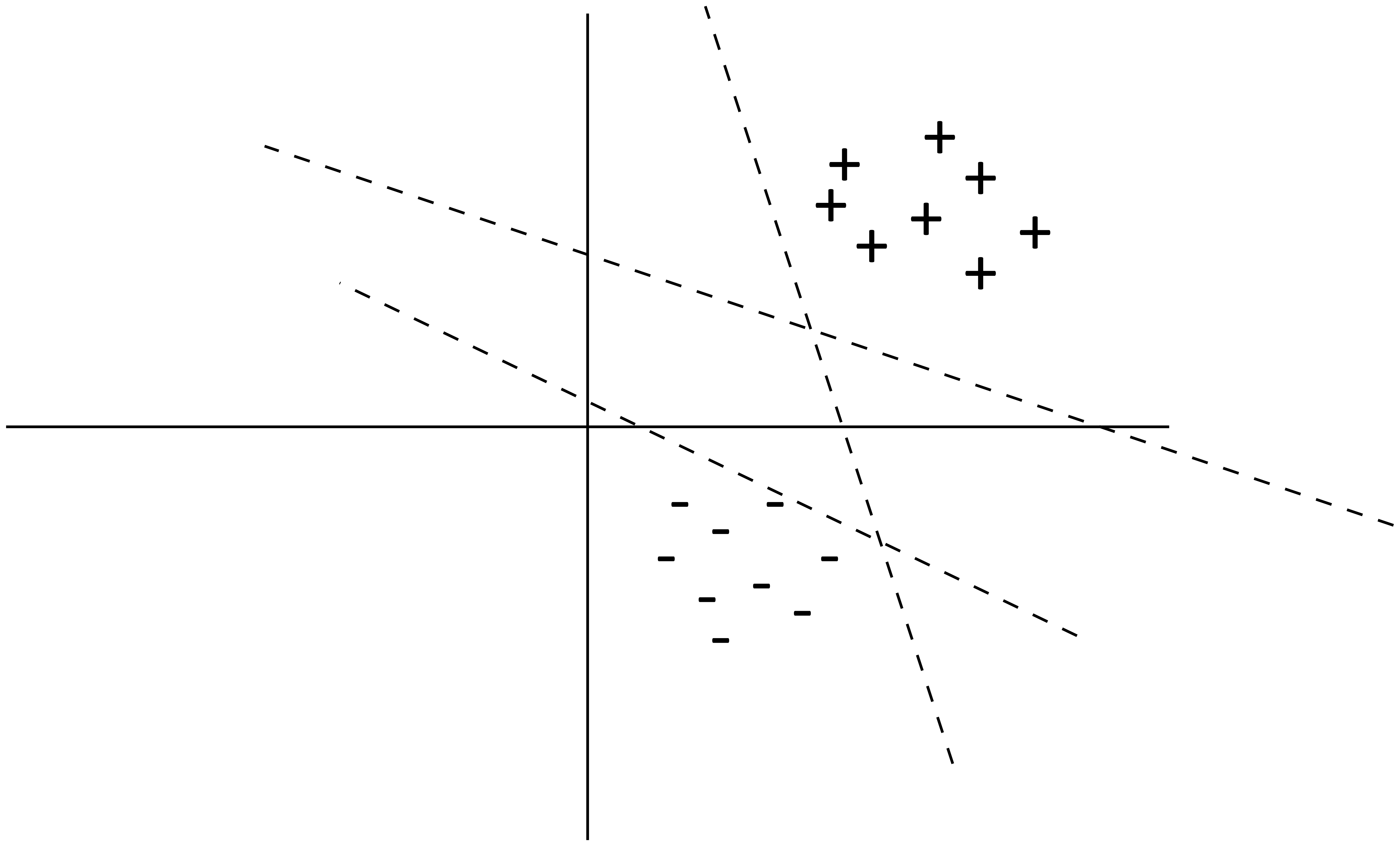
$$w \leftarrow w + x(1 - P(y = 1|x))$$

$$w \leftarrow w - xP(y = 1|x)$$

- Guaranteed to eventually separate the data if the data are separable

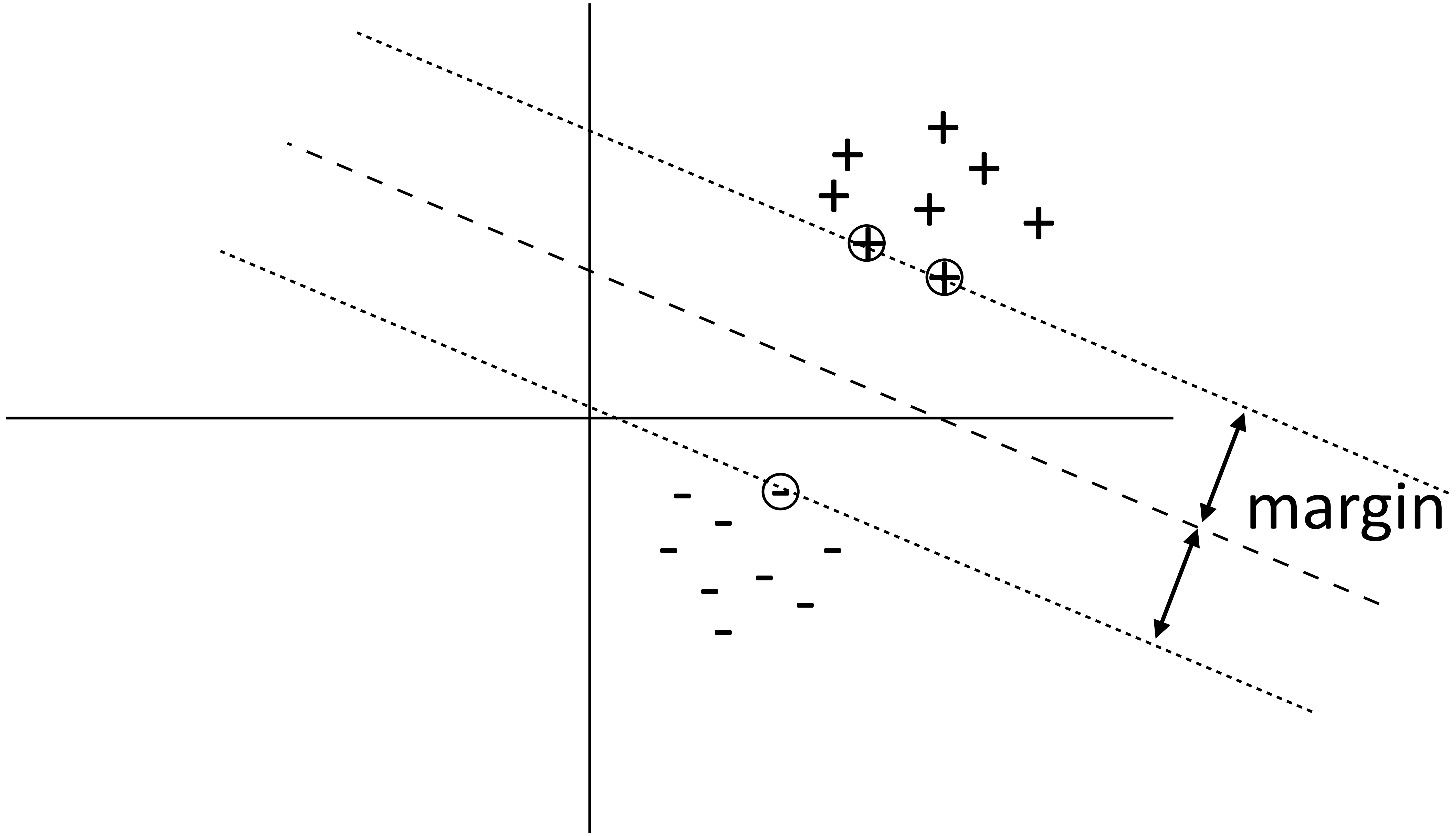
Support Vector Machines

- Many separating hyperplanes — is there a best one?



Support Vector Machines

- Many separating hyperplanes — is there a best one?



Support Vector Machines

- Constraint formulation: find *w* via following quadratic program:

$$\boxed{\begin{aligned} &\text{Minimize } \|w\|_2^2 \\ &\text{s.t. } \forall j \quad w^\top x_j \geq 1 \text{ if } y_j = 1 \\ &\qquad\qquad w^\top x_j \leq -1 \text{ if } y_j = 0 \end{aligned}}$$

minimizing norm with fixed margin <=> maximizing margin

As a single constraint:

$$\forall j \quad (2y_j - 1)(w^\top x_j) \geq 1$$

- Generally no solution (data is generally non-separable) — need slack!

N-Slack SVMs

Minimize $\lambda\|w\|_2^2 + \sum_{j=1}^{m} \xi_j$

s.t. $\forall j \quad (2y_j - 1)(w^\top x_j) \geq 1 - \xi_j \qquad \forall j \quad \xi_j \geq 0$

- The ξ_j are a "fudge factor" to make all constraints satisfied
- Take the gradient of the objective:

$$\frac{\partial}{\partial w_i}\xi_j = 0 \text{ if } \xi_j = 0 \qquad \frac{\partial}{\partial w_i}\xi_j = (2y_j - 1)x_{ji} \text{ if } \xi_j > 0$$
$$= x_{ji} \text{ if } y_j = 1, \; -x_{ji} \text{ if } y_j = 0$$

- Looks like the perceptron! But updates more frequently

Gradients on Positive Examples

Logistic regression

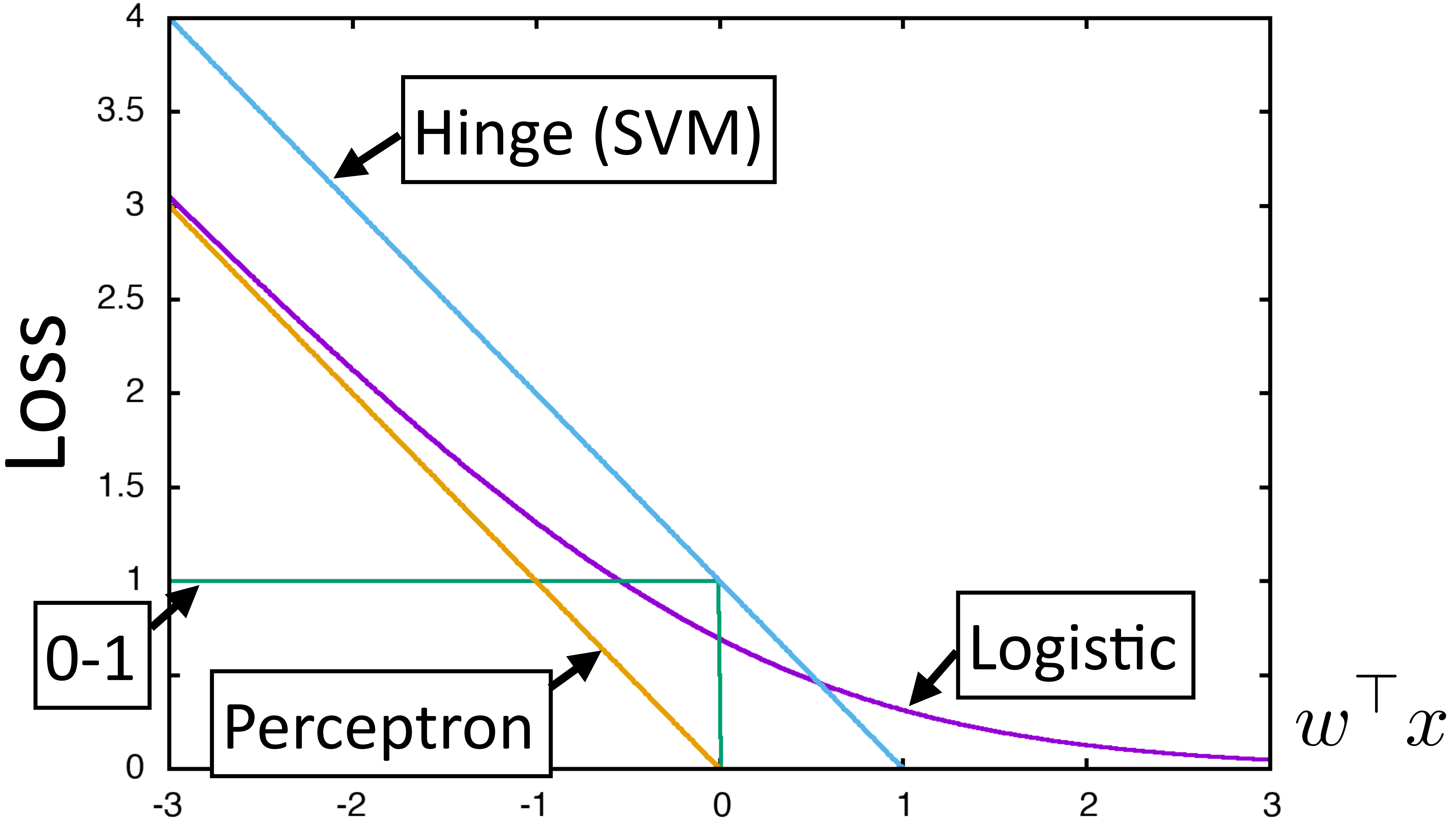
$$x(1 - \text{logistic}(w^\top x))$$

Perceptron

$$x \text{ if } w^\top x < 0, \text{ else } 0$$

SVM (ignoring regularizer)

$$x \text{ if } w^\top x < 1, \text{ else } 0$$



*gradients are for maximizing things, which is why they are flipped

Comparing Gradient Updates (Reference)

Logistic regression (unregularized)

$x(y - P(y = 1|x))$ = $x(y - \text{logistic}(w^\top x))$

Perceptron

$(2y - 1)x$ if classified incorrectly

0 else

SVM

$(2y - 1)x$ if not classified correctly with margin of 1

0 else

y = 1 for pos,
0 for neg

Optimization — next time…

- Range of techniques from simple gradient descent (works pretty well) to more complex methods (can work better)

- Most methods boil down to: take a gradient and a step size, apply the gradient update times step size, incorporate estimated curvature information to make the update more effective

Sentiment Analysis

this movie was great! would watch again +

the movie was gross and overwrought, but I liked it +

this movie was not really very enjoyable —

- Bag-of-words doesn't seem sufficient (discourse structure, negation)
- There are some ways around this: extract bigram feature for "*not* X" for all X following the *not*

Bo Pang, Lillian Lee, Shivakumar Vaithyanathan (2002)

Sentiment Analysis

	Features	# of features	frequency or presence?	NB	ME	SVM
(1)	unigrams	16165	freq.	**78.7**	N/A	72.8
(2)	unigrams	"	pres.	81.0	80.4	**82.9**
(3)	unigrams+bigrams	32330	pres.	80.6	80.8	**82.7**
(4)	bigrams	16165	pres.	77.3	**77.4**	77.1
(5)	unigrams+POS	16695	pres.	81.5	80.4	**81.9**
(6)	adjectives	2633	pres.	77.0	**77.7**	75.1
(7)	top 2633 unigrams	2633	pres.	80.3	81.0	**81.4**
(8)	unigrams+position	22430	pres.	81.0	80.1	**81.6**

- Simple feature sets can do pretty well!

Bo Pang, Lillian Lee, Shivakumar Vaithyanathan (2002)

Sentiment Analysis

Method	RT-s	MPQA
MNB-uni	77.9	85.3
MNB-bi	**79.0**	**86.3**
SVM-uni	76.2	86.1
SVM-bi	77.7	**86.7**
NBSVM-uni	**78.1**	85.3
NBSVM-bi	**79.4**	**86.3**
RAE	76.8	85.7
RAE-pretrain	77.7	**86.4**
Voting-w/Rev.	63.1	81.7
Rule	62.9	81.8
BoF-noDic.	75.7	81.8
BoF-w/Rev.	76.4	84.1
Tree-CRF	77.3	86.1
BoWSVM	–	–
Kim (2014) CNNs	**81.5**	**89.5**

Naive Bayes is doing well!

Ng and Jordan (2002) — NB can be better for small data

Before neural nets had taken off — results weren't that great

Wang and Manning (2012)

Recap

- Logistic regression: $P(y=1|x) = \frac{\exp\left(\sum_{i=1}^{n} w_i x_i\right)}{\left(1+\exp\left(\sum_{i=1}^{n} w_i x_i\right)\right)}$

  Decision rule: $P(y=1|x) \geq 0.5 \Leftrightarrow w^\top x \geq 0$

  Gradient (unregularized): $x(y - P(y=1|x))$

- SVM:

  Decision rule: $w^\top x \geq 0$

  (Sub)gradient (unregularized): 0 if correct with margin of 1, else $x(2y-1)$

Recap

- Logistic regression, SVM, and perceptron are closely related
- SVM and perceptron inference require taking maxes, logistic regression has a similar update but is “softer” due to its probabilistic nature
- All gradient updates: “make it look more like the right thing and less like the wrong thing”