Multi-Class Logistic Regression and Perceptron

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Some slides adapted from Dan Jurfasky, Brendan O'Connor and Marine Carpuat

MultiClass Classification

- Q: what if we have more than 2 categories?
  - Sentiment: Positive, Negative, Neutral
  - Document topics: Sports, Politics, Business, Entertainment, …

Q: How to easily do Multi-label classification?

Two Types of MultiClass Classification

- Multi-label Classification
  - each instance can be assigned more than one labels

- Multinominal Classification
  - each instance appears in exactly one class (classes are exclusive)

Multinominal Classification

- Pretty straightforward with Naive Bayes.

$$P(\text{spam}|D) \propto P(\text{spam}) \prod_{w \in D} P(w|\text{spam})$$

Log-Linear Models

$$P(y|x) \propto e^{w \cdot f(x,y)}$$

$$P(y|x) = \frac{1}{Z(w)} e^{w \cdot f(x,y)}$$

Multinominal Logistic Regression

$$P(y|x) \propto e^{w \cdot f(x,y)}$$

$$P(y|x) = \frac{1}{Z(w)} e^{w \cdot f(x,y)}$$

$$\boxed{P(y|x) = \frac{e^{w \cdot f(x,y)}}{\sum_{y' \in Y} e^{w \cdot f(x,y')}}}$$

Multinominal Logistic Regression

- Binary (two classes):
  - We have one feature vector that matches the size of the vocabulary
- Multi-class in practice:
  - one weight vector for each category

$$w_{\text{pos}} \qquad w_{\text{neg}} \qquad w_{\text{neut}}$$

Can represent this in practice with one giant weight vector and repeated features for each category.

Maximum Likelihood Estimation

$$w_{\mathrm{MLE}} = \mathrm{argmax}_w \log P(y_1, \ldots, y_n | x_1, \ldots, x_n; w)$$

$$= \mathrm{argmax}_w \sum_i \log P(y_i | x_i; w)$$

$$= \mathrm{argmax}_w \sum_i \log \frac{e^{w \cdot f(x_i, y_i)}}{\sum_{y' \in Y} e^{w \cdot f(x_i, y')}}$$

Multiclass LR Gradient

$$\frac{\partial \mathcal{L}}{\partial w_j} = \sum_{i=1}^{D} f_j(y_i, d_i) - \sum_{i=1}^{D} \sum_{y \in Y} f_j(y, d_i) P(y|d_i)$$

(a.k.a) Softmax Regression

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Softmax function

From Wikipedia, the free encyclopedia

In mathematics, the **softmax function**, or **normalized exponential function**,[1]:198 is a generalization of the logistic function that "squashes" a K-dimensional vector $\mathbf{z}$ of arbitrary real values to a K-dimensional vector $\sigma(\mathbf{z})$ of real values in the range (0, 1) that add up to 1. The function is given by

$$\sigma(\mathbf{z})_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}} \quad \text{for } j = 1, \ldots, K.$$

(a.k.a) Maximum Entropy Classifier

- or MaxEnt

- Math proof of “LR=MaxEnt”:
  - [Klein and Manning 2003]
  - [Mount 2011]

http://www.win-vector.com/dfiles/LogisticRegressionMaxEnt.pdf

Perceptron Algorithm

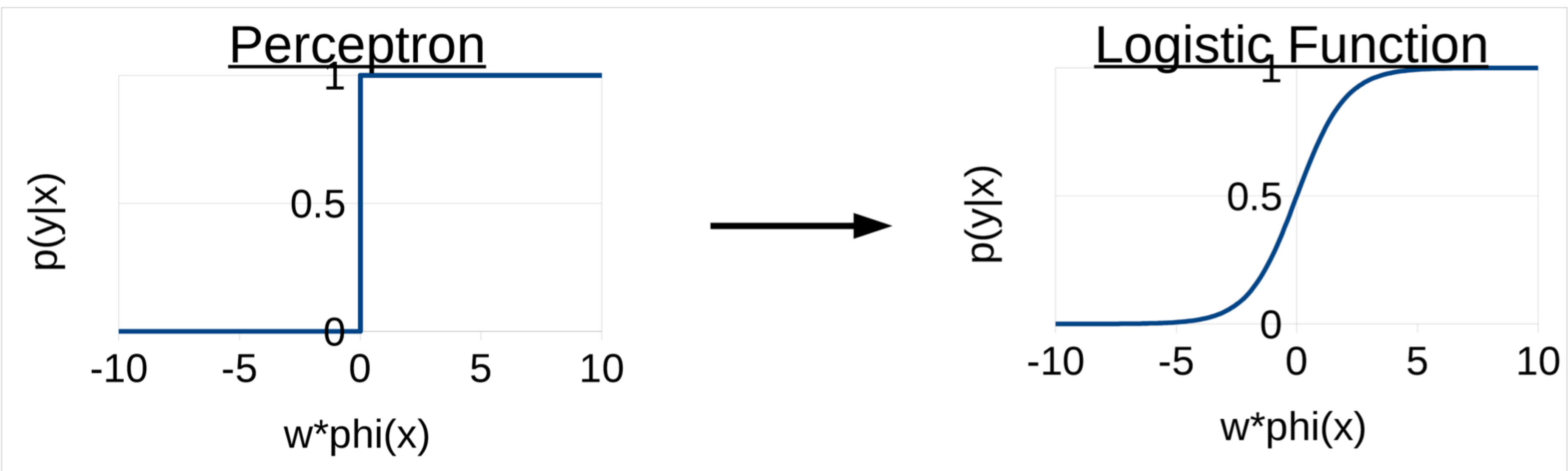
- Very similar to logistic regression
- Not exactly computing gradient

[Rosenblatt 1957]

http://www.peterasaro.org/writing/neural_networks.html

Perceptron Algorithm

- Very similar to logistic regression
- Not exactly computing gradient



$$P(y=1|x)=1 \text{ if } \mathbf{w}\cdot\boldsymbol{\varphi}(x)\geq 0$$
$$P(y=1|x)=0 \text{ if } \mathbf{w}\cdot\boldsymbol{\varphi}(x)<0$$

$$P(y=1|x)=\frac{e^{\mathbf{w}\cdot\boldsymbol{\varphi}(x)}}{1+e^{\mathbf{w}\cdot\boldsymbol{\varphi}(x)}}$$

Online Learning

- Update parameters for each training example (when predication is wrong)

```
for I iterations
    for each labeled pair x, y in the data
        phi = CREATE_FEATURES(x)
        y' = PREDICT_ONE(w, phi)
        if y' != y
            UPDATE_WEIGHTS(w, phi, y)
```

Online Learning

- The Perceptron is an online learning algorithm.
- Logistic Regression is not:

$$w_{\mathrm{MLE}} = \mathrm{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w)$$

Perceptron Algorithm

- Very similar to logistic regression
- Not exactly computing gradient

```
Initalize weight vector w = 0
Loop for K iterations
    Loop For all training examples x_i
        if sign(w * x_i) != y_i
            w += (y_i - sign(w * x_i)) * x_i
```

Perceptron Notes

- Guaranteed to converge if the data is linearly separable
- Only hyperparameter is maximum number of iterations
- Parameter averaging will greatly improve performance

Differences between LR and Perceptron

- Online learning vs. Batch

- Perceptron doesn’t always make updates

MAP-based learning (perceptron)

$$\frac{\partial \mathcal{L}}{\partial w_j} \approx \sum_{i=1}^{D} f_j(y_i, d_i) - \sum_{i=1}^{D} f_j(\arg\max_{y \in Y} P(y|d_i), d_i)$$

Online Learning (perceptron)

- Rather than making a full pass through the data, compute gradient and update parameters after each training example.

- Gradients will be less accurate, but the overall effect is to move in the right direction

- Often works well and converges faster than batch learning

MultiClass Perceptron Algorithm

```
Initalize weight vector w = 0
Loop for K iterations
    Loop For all training examples x_i
        y_pred = argmax_y w_y * x_i
        if y_pred != y_i
            w_y_gold += x_i
            w_y_pred -= x_i
```

Q: what if there are only 2 categories?

$$P(y = j | x_i) = \frac{e^{w_j \cdot x_i}}{\sum_k e^{w_k \cdot x_i}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_0 \cdot x + w_1 \cdot x - w_1 \cdot x} + e^{w_1 \cdot x}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_0 \cdot x - w_1 \cdot x} e^{w_1 \cdot x} + e^{w_1 \cdot x}}$$

Q: what if there are only 2 categories?

$$P(y=1|x) = \frac{e^{w_1 \cdot x}}{e^{w_1 \cdot x}(e^{w_0 \cdot x - w_1 \cdot x} + 1)}$$

Q: what if there are only 2 categories?

$$P(y = 1|x) = \frac{1}{e^{w_0 \cdot x - w_1 \cdot x} + 1}$$

Q: what if there are only 2 categories?

$$P(y = 1|x) = \frac{1}{e^{-w' \cdot x} + 1}$$

Sigmoid (logistic) function

Regularization

- Combating over fitting
- Intuition: don't let the weights get very large

$$w_{\mathrm{MLE}} = \mathrm{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w)$$

$$\mathrm{argmax}_w \log P(y_1, \ldots, y_d | x_1, \ldots, x_d; w) - \delta \sum_{i=1}^{V} w_i^2$$

Regularization in the Perceptron Algorithm

- Can’t directly include regularization in gradient

- # of iterations

- Parameter averaging