Write answers in spaces provided.
**Partial Credit**: If you show your work and briefly describe your approach, we will happily give partial credit where possible. Answers without supporting work (or that are not clear/legible) may not be given credit. We also reserve the right to take off points for overly long answers.
**Pseudocode**: Pseudocode can be written at the level discussed in class and does not necessarily need to conform to any particular programming language or API.

| | |
|---|---|
| Q1-2. Search: Algorithms | /4 |
| Q3. Search: Heuristic Function Properties | /3 |
| Q4-7. Search: Adversarial Search | /6 |
| Q8. Games: Alpha-Beta Pruning | /3 |
| Q9. MDPs: Mini-Gridworld | /4 |
| Total | /20 |

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1. (**2 points**) The following implementation of graph search may be incorrect. Circle all the problems with the code.

```
function GRAPH-SEARCH(problem, fringe)
    closed ← an empty set,
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop
        if fringe is empty then
            return failure
        end if
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem,STATE[node]) then
            return node
        end if
        ADD STATE[node] TO closed
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end loop
end function
```

This algorithm is equivalent to tree search, rather than graph search.

In graph search, nodes added to the "closed" list should not be expanded again.

(a) Nodes may be expanded twice.

(b) The algorithm is no longer complete.

(c) The algorithm could return an incorrect solution.

(d) None of the above.

2. (**2 points**) The following implementation of A* graph search may be incorrect. You may assume that the algorithm is being run with a consistent heuristic. Circle all the problems with the code.

```
function A*-SEARCH(problem, fringe)
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop
        if fringe is empty then
            return failure
        end if
        node ← REMOVE-FRONT(fringe)
        if STATE[node] IS NOT IN closed then
            ADD STATE[node] TO closed
            for successor IN GETSUCCESSORS(problem, STATE[node]) do
                fringe ← INSERT(MAKE-NODE(successor), fringe)
                if GOAL-TEST(problem,successor) then
                    return successor
                end if
            end for
        end if
    end loop
end function
```

When should A* terminate?

Should we stop when we enqueue a goal?

No. only stop when we dequeue a goal.

Otherwise, like the stated algorithm here, it expands fewer nodes to find a goal.

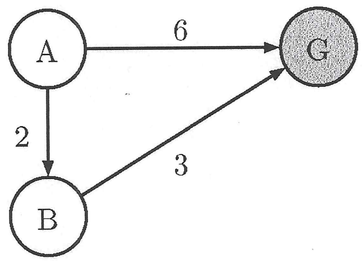
(a) Nodes may be expanded twice.

(b) The algorithm is no longer complete.

(c) The algorithm does not always find the optimal solution.

(d) None of the above.

3. For the following questions, consider the search problem shown on the left. It has only three states, and three directed edges. $A$ is the start node and $G$ is the goal node. To the right, four different heuristic functions are defined, numbered I through IV.




| | $h(A)$ | $h(B)$ | $h(G)$ |
|---|---|---|---|
| I | 4 | 1 | 0 |
| II | 5 | 4 | 0 |
| III | 4 | 3 | 0 |
| IV | 5 | 2 | 0 |

(a) **(2 points)** Admissibility and Consistency. For each heuristic function, circle whether it is admissible and whether it is consistent with respect to the search problem given above.

| | Admissible? | | Consistent? | |
|---|---|---|---|---|
| I | Yes | No | Yes | No |
| II | Yes | No | Yes | No |
| III | Yes | No | Yes | No |
| IV | Yes | No | Yes | No |

(b) **(1 points)** Function Domination. Recall that *domination* has a specific meaning when talking about heuristic functions.
Circle all true statements among the following.

i. Heuristic function III dominates IV.
ii. Heuristic function IV dominates III.
iii. Heuristic functions III and IV have no dominance relationship.
iv. Heuristic function I dominates IV.
v. Heuristic function IV dominates I.
vi. Heuristic functions I and IV have no dominance relationship.

(a) • Inadmissible heuristic overestimates the cost : $\overset{=4}{h_{II}(B)} > \overset{=3}{\text{cost}(B \to \text{Goal})}$

• For a heuristic to be consistent, ensure that for all paths : $h(\text{begin}) - h(\text{end}) \leq \text{path}(\text{begin} \to \text{end})$

For this small graph, that means $h(A) - h(B) \leq \underbrace{\text{path}(A \to B)}_{=2}$

(b) For one heuristic to dominate the other, <u>all</u> of its values must be greater than or equal to the corresponding values of the other heuristic.

Cont.

4. (**2 points**) Write down pseudocode for Minimax Search. Make sure to include the base case.
*Hint: You can define 3 functions:* MinValue($s$), MaxValue($s$), Value($s$)

Project #2 & lecture slides

5. **(2 points)** Write down psuedocode for Expectimax Search. Make sure to include the base case.
*Hint: You can define 3 functions:* ExpValue($s$), MaxValue($s$), Value($s$)

Project # 2 & lecture slides

6. **(1 points)** What is the $O(\ \ )$ number of states expanded by Expectimax search, assuming there are $b$ possible actions and the game ends after $m$ moves (here you can assume $m$ = number of agent moves + number of opponent moves)?

$O(b^m)$

7. **(1 points)** Is exhaustive search with Expectimax feasible for Pacman? If not, how did we deal with this in Project #2?

use evaluation function (e.g. a linear combination of features) to score non-terminal states.

8. **(3 points)** Assume we run $\alpha - \beta$ pruning expanding successors from left to right on a game with tree as shown in Figure 1 (a). Then we have that:

   (a) (*true* or *false*) For some choice of pay-off values, no pruning will be achieved (shown in Figure 1 (a)).

   (b) (*true* or *false*) For some choice of pay-off values, the pruning shown in Figure 1 (b) will be achieved.

   (c) (*true* or *false*) For some choice of pay-off values, the pruning shown in Figure 1 (c) will be achieved.

   (d) (*true* or *false*) For some choice of pay-off values, the pruning shown in Figure 1 (d) will be achieved.

   (e) (*true* or *false*) For some choice of pay-off values, the pruning shown in Figure 1 (e) will be achieved.

   (f) (*true* or *false*) For some choice of pay-off values, the pruning shown in Figure 1 (f) will be achieved.

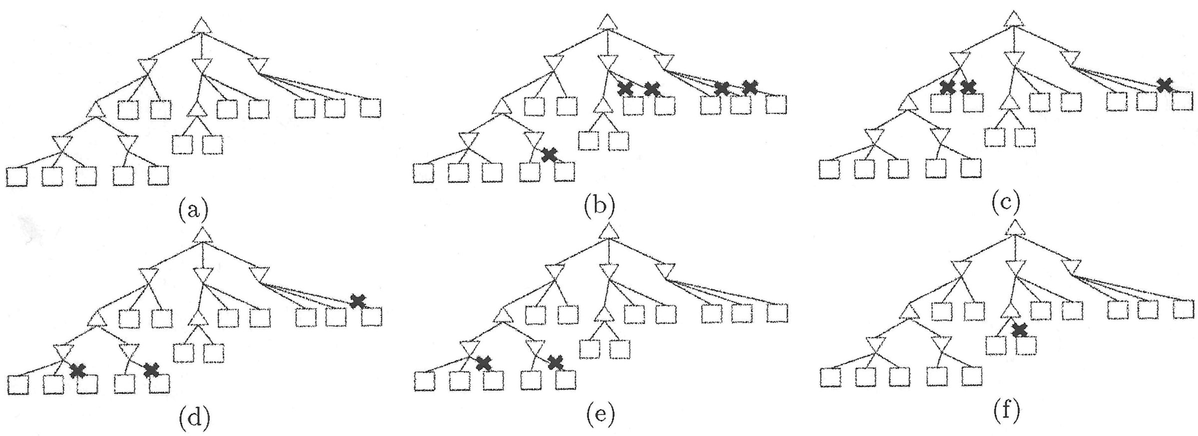



Figure 1: Game trees.

9. The following problems take place in various scenarios of the gridworld MDP. In all cases, $A$ is the start state and double-rectangle states are exit states. From an exit state, the only action available is *Exit*, which results in the listed reward and ends the game (by moving into a terminal state $X$, not shown). From non-exit states, the agent can choose either *Left* or *Right* actions, which move the agent in the corresponding direction. There are no living rewards; the only non-zero rewards come from exiting the grid. Throughout this problem, assume that value iteration begins with initial values $V_0(s) = 0$ for all states $s$.

First, consider the following mini-grid. For now, the discount is $\gamma = 1$ and legal movement actions will always succeed (and so the state transition function is deterministic).

| | | | | |
|---|---|---|---|---|
| +1 | A | | | +10 |

(a) **(0.5 points)** What is the optimal value $V^*(A)$?

$V^*(A) = 10$

(b) **(0.5 points)** When running value iteration, remember that we start with $V_0(s) = 0$ for all $s$. What is the first iteration $k$ for which $V_k(A)$ will be non-zero?

$k=2$ the fastest path to reward is a 2-step sequence: Left, exit

(c) **(0.5 points)** What will $V_k(A)$ be when it is first non-zero?

$V_2(A) = 1$

(d) **(0.5 points)** After how many iterations $k$ will we have $V_k(A) = V^*(A)$? If they will never become equal, write *never*.

$k=4$. $V_4(A) = 10$. the fastest path to maximal reward is a 4-step sequence: Right, Right, Right, exit

Now the situation is as before, but the discount $\gamma$ is less than 1.

(e) **(1 points)** If $\gamma = 0.5$, what is the optimal value $V^*(A)$? $\geq 1.25$

① Right, Right, Right, exit

$\gamma^0 \quad \gamma^1 \quad \gamma^2 \quad \gamma^3$

$0 \quad 0 \quad 0 \quad 10$

$(0.5)^3 \times 10 = 1.25$

② Left, exit

$\gamma^0 \quad \gamma^1$

$0 \quad 1$

$(0.5)^1 \times 1 = 0.5$

(f) **(1 points)** For what range of values $\gamma$ of the discount will it be optimal to go *Right* from $A$? Remember that $0 \leq \gamma \leq 1$. Write *all* or *none* if all or no legal values of $\gamma$ have this property.

$\gamma^3 \times 10 \geq \gamma^1 \times 1$

$\frac{1}{\sqrt{10}} \leq \gamma \leq 1$

The End.