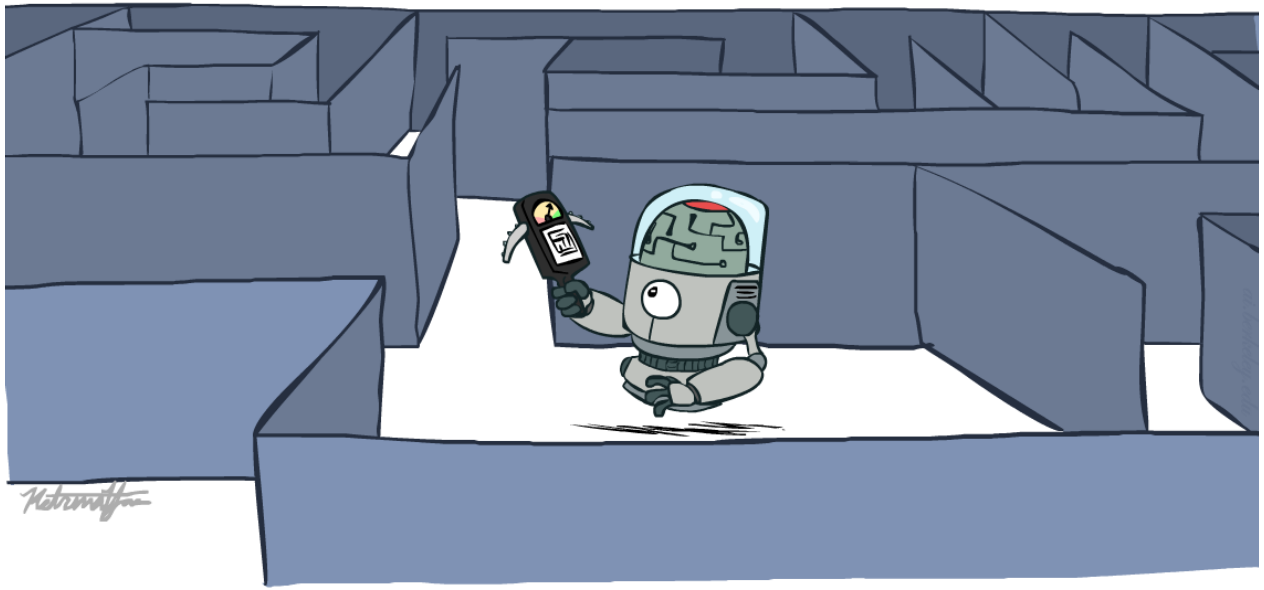
CS 5522: Artificial Intelligence II

A* Search, Graph Search, Their Completeness and Optimality



Instructor: Wei Xu

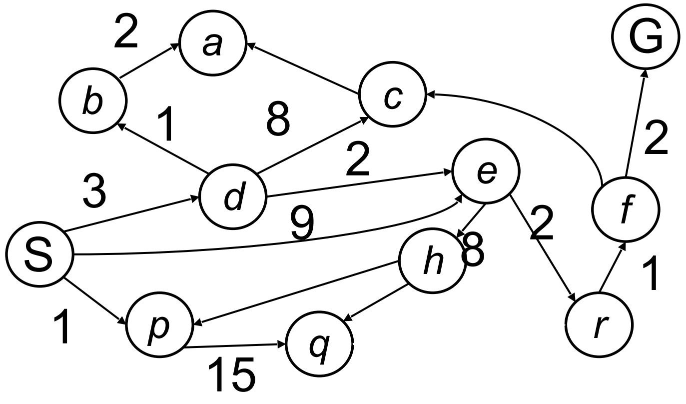
Ohio State University

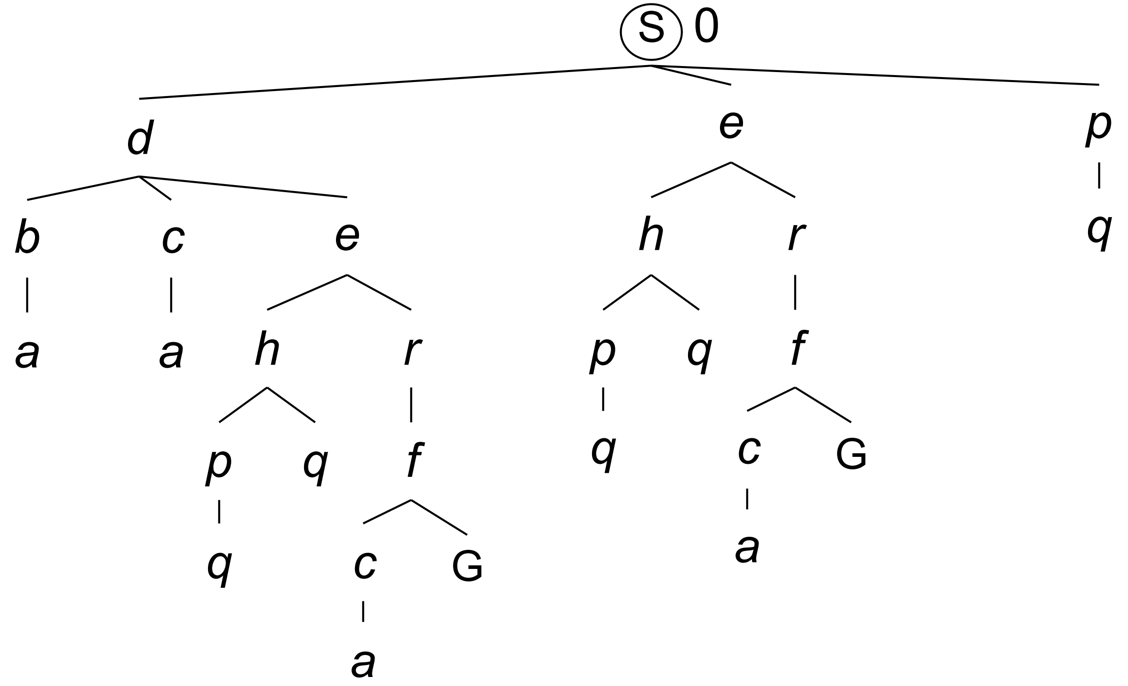
[These slides were adapted from CS188 Intro to AI at UC Berkeley]

Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

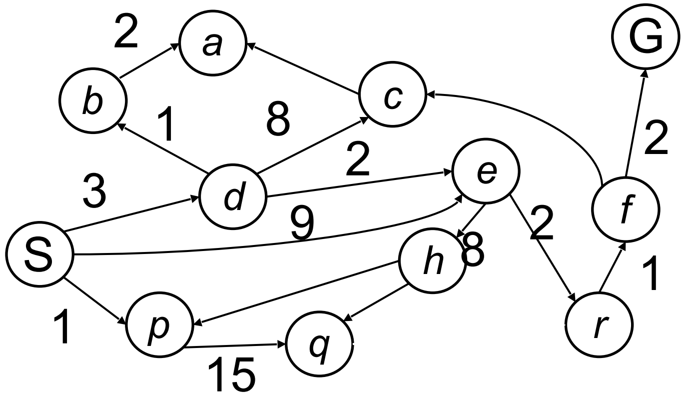


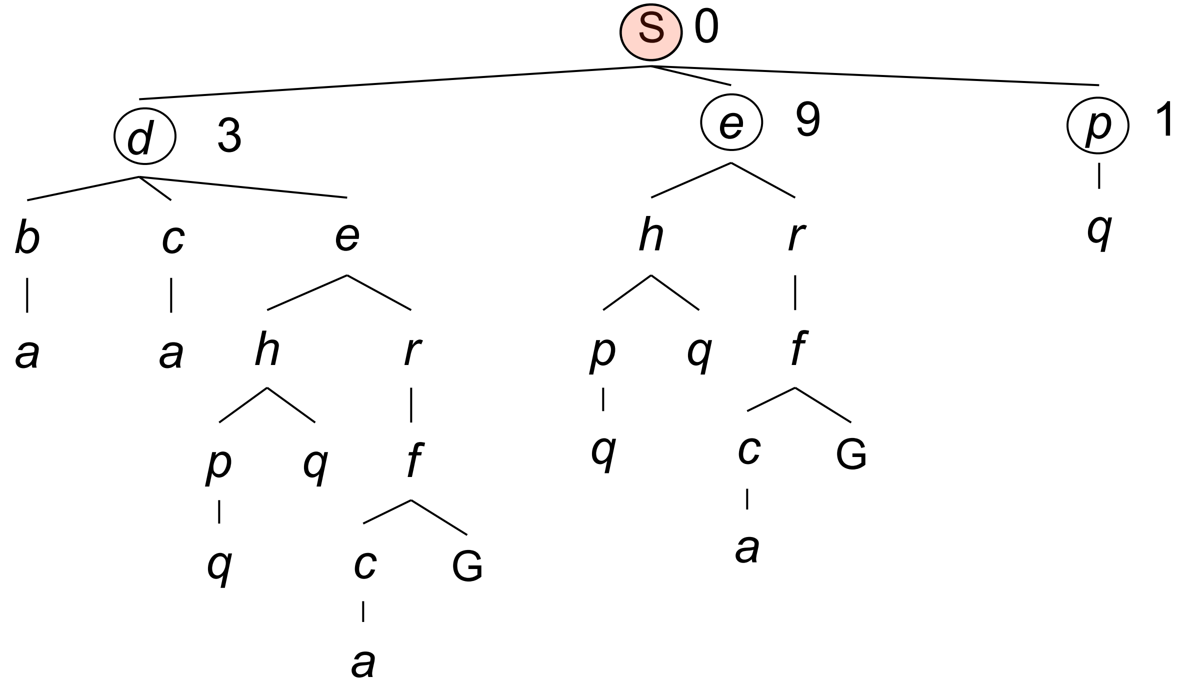


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

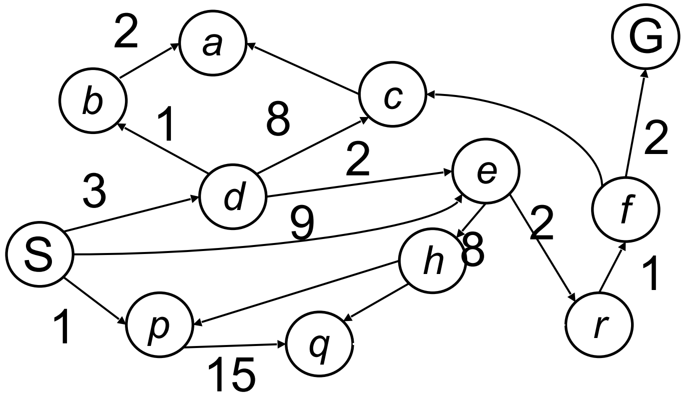


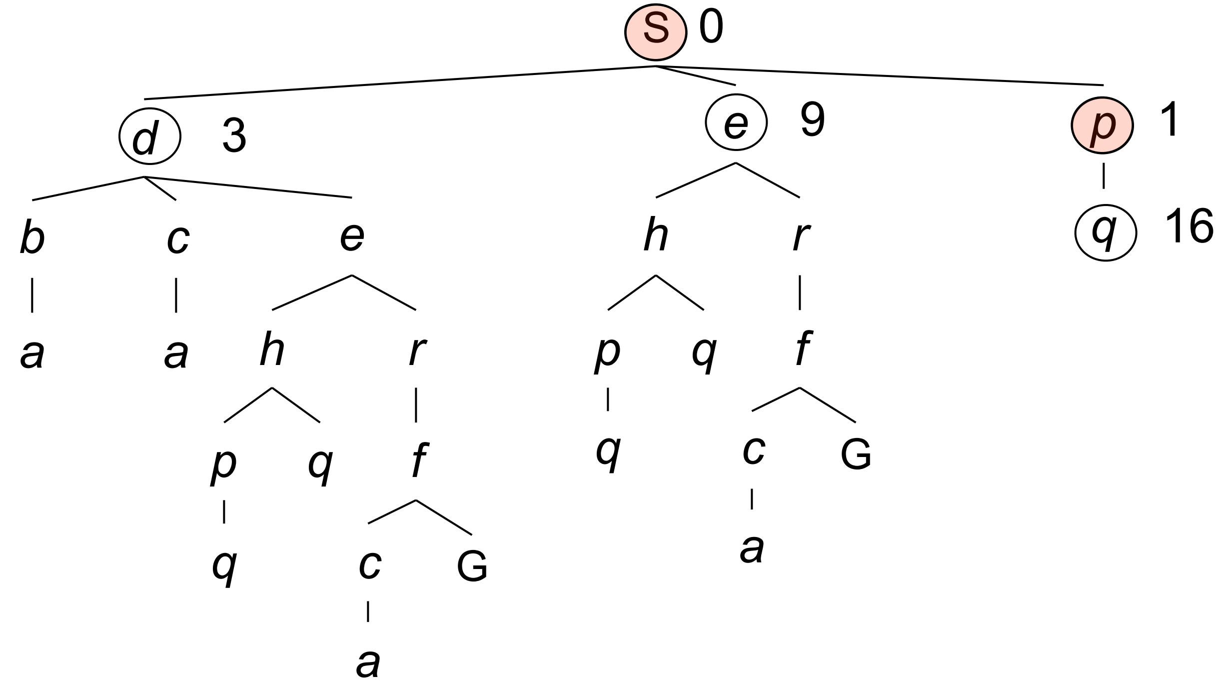


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

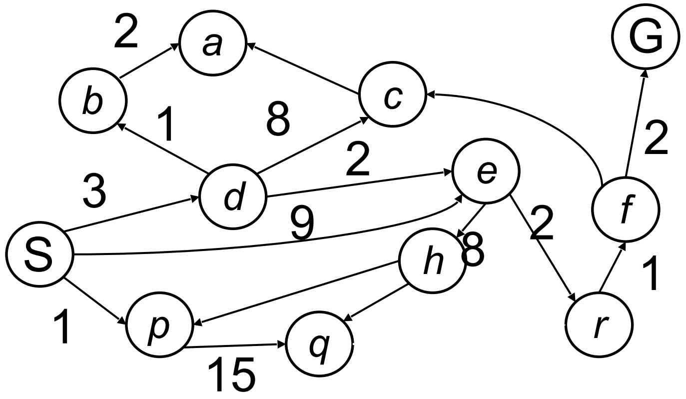


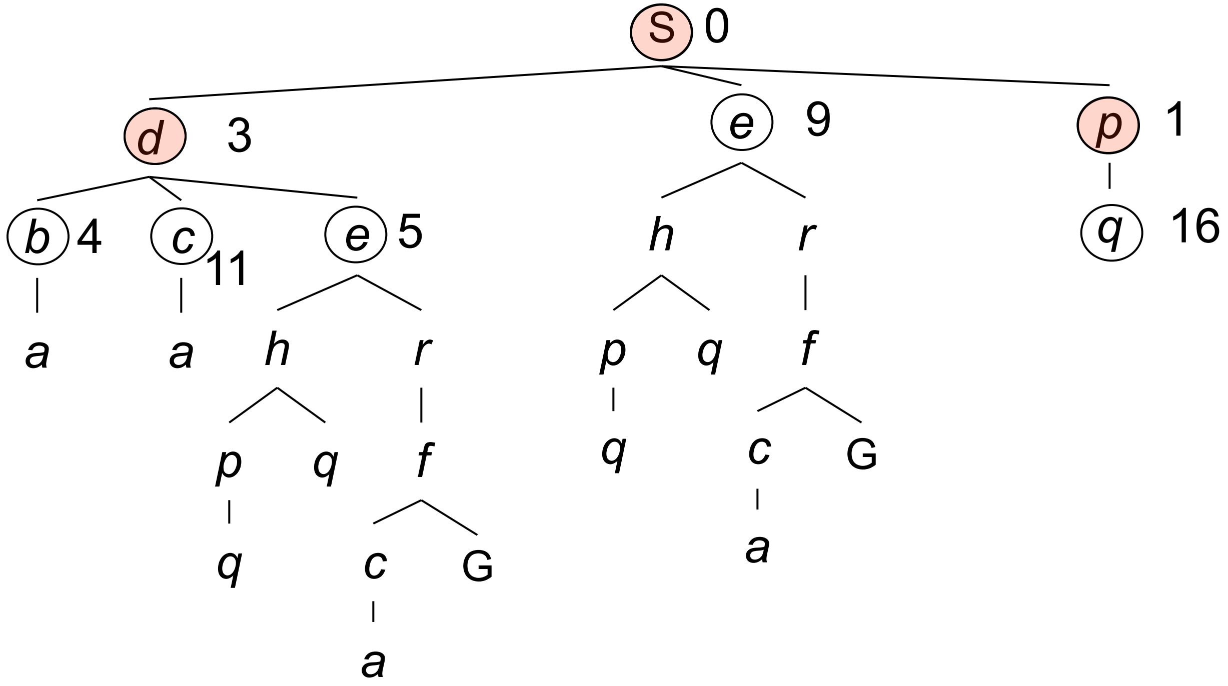


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

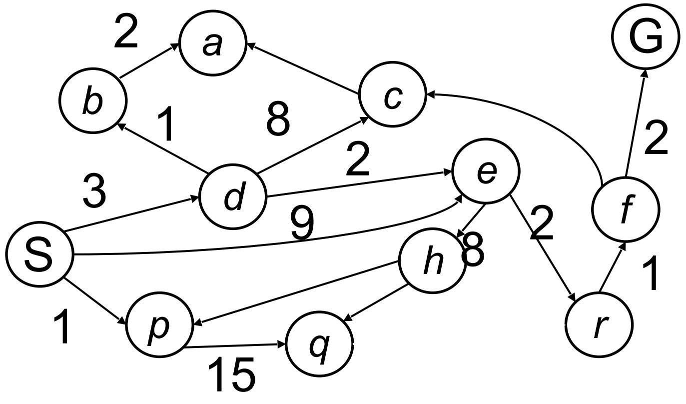


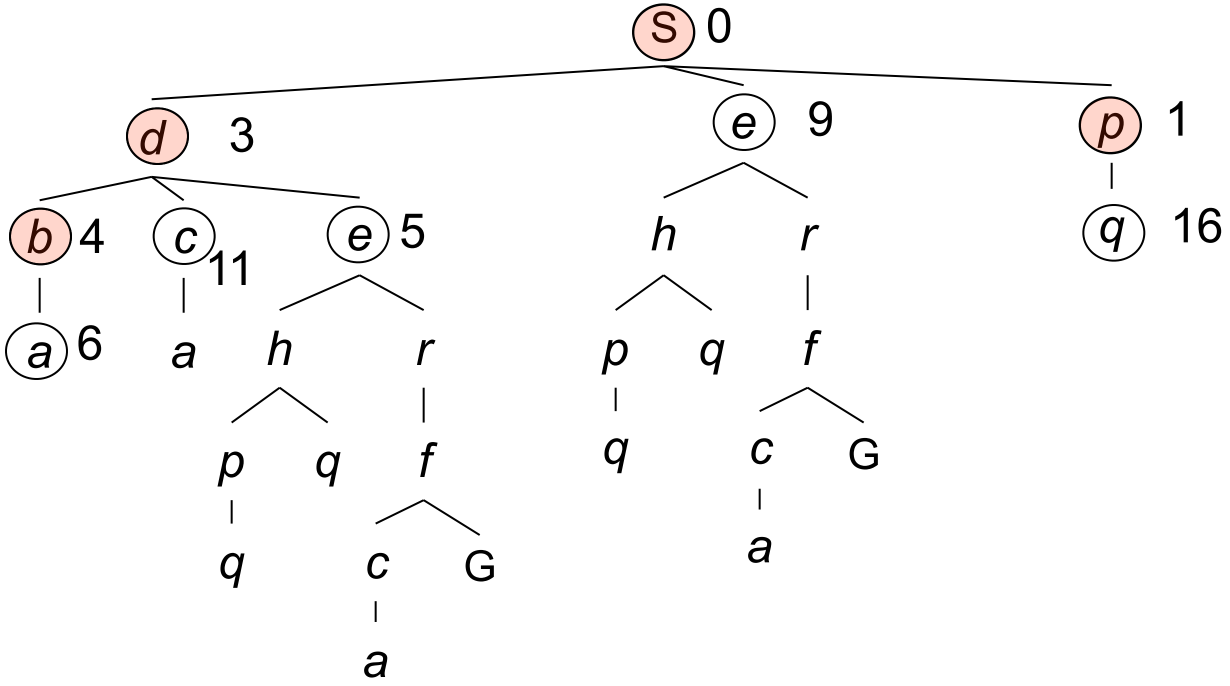


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

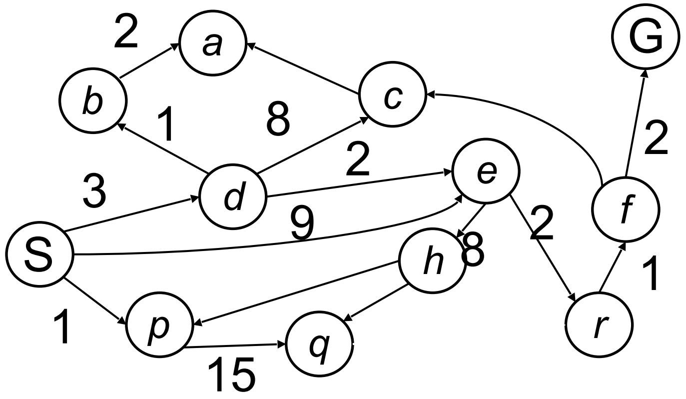


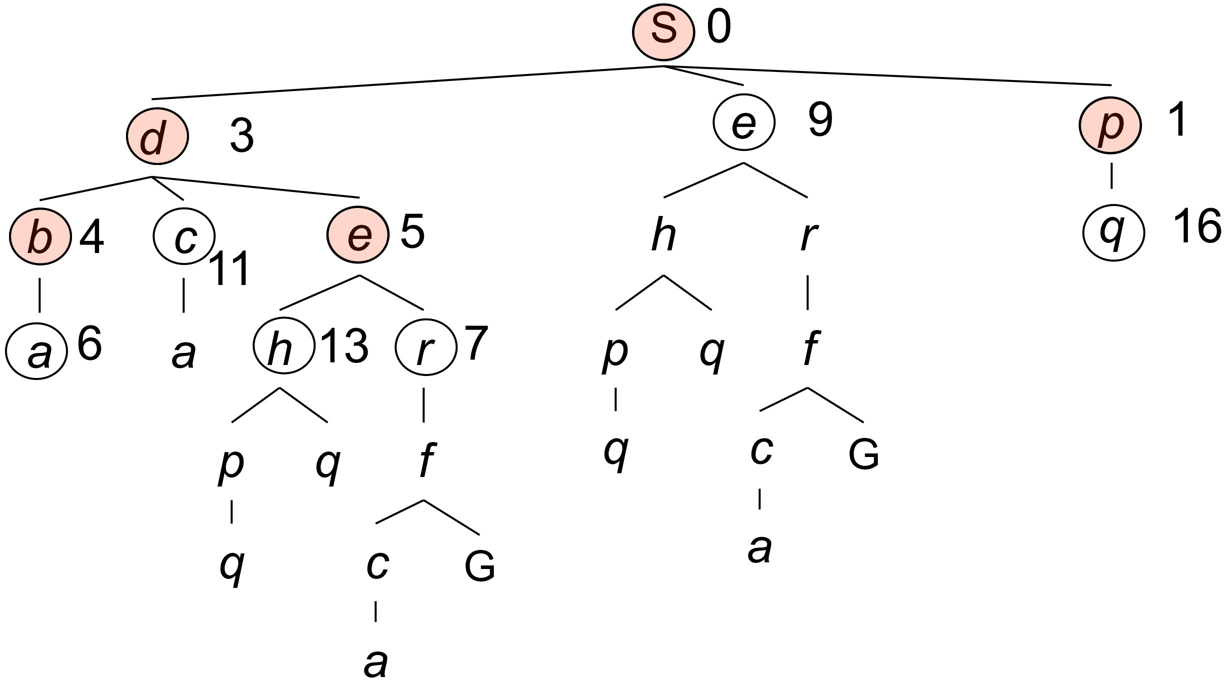


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

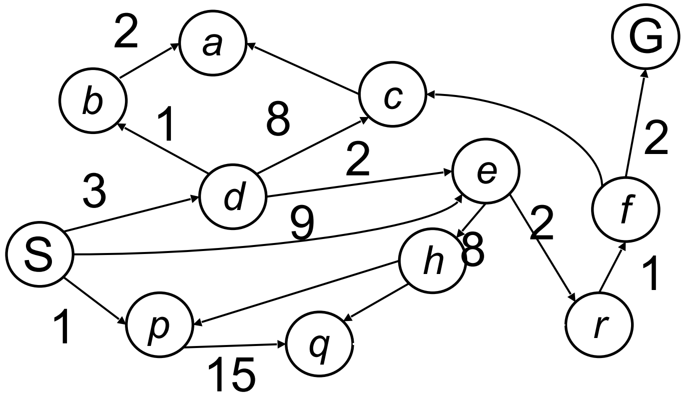


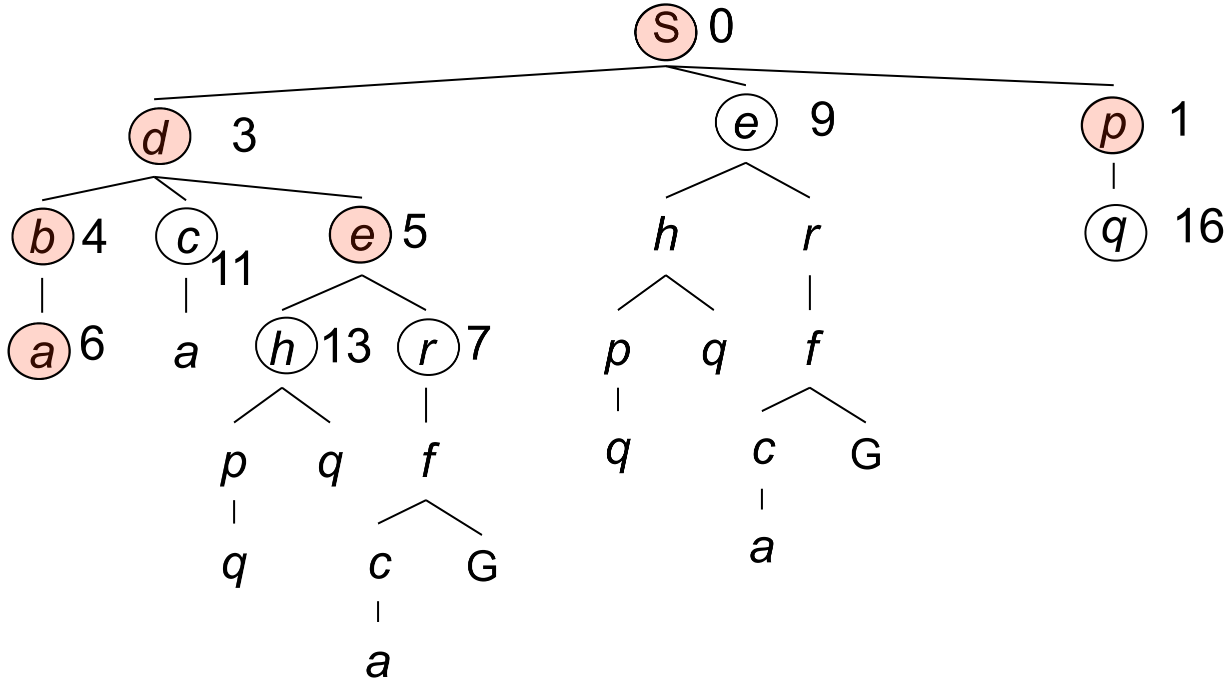


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

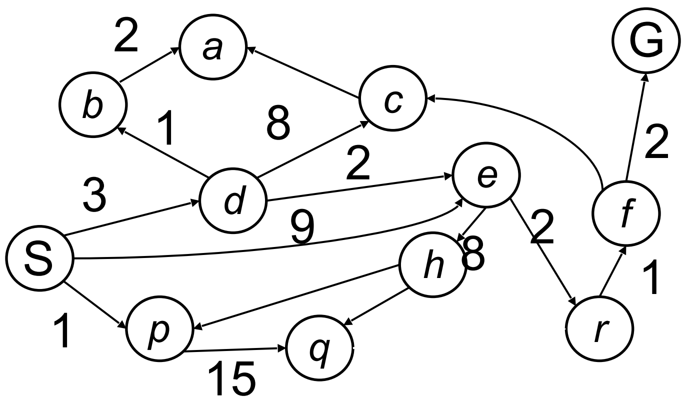


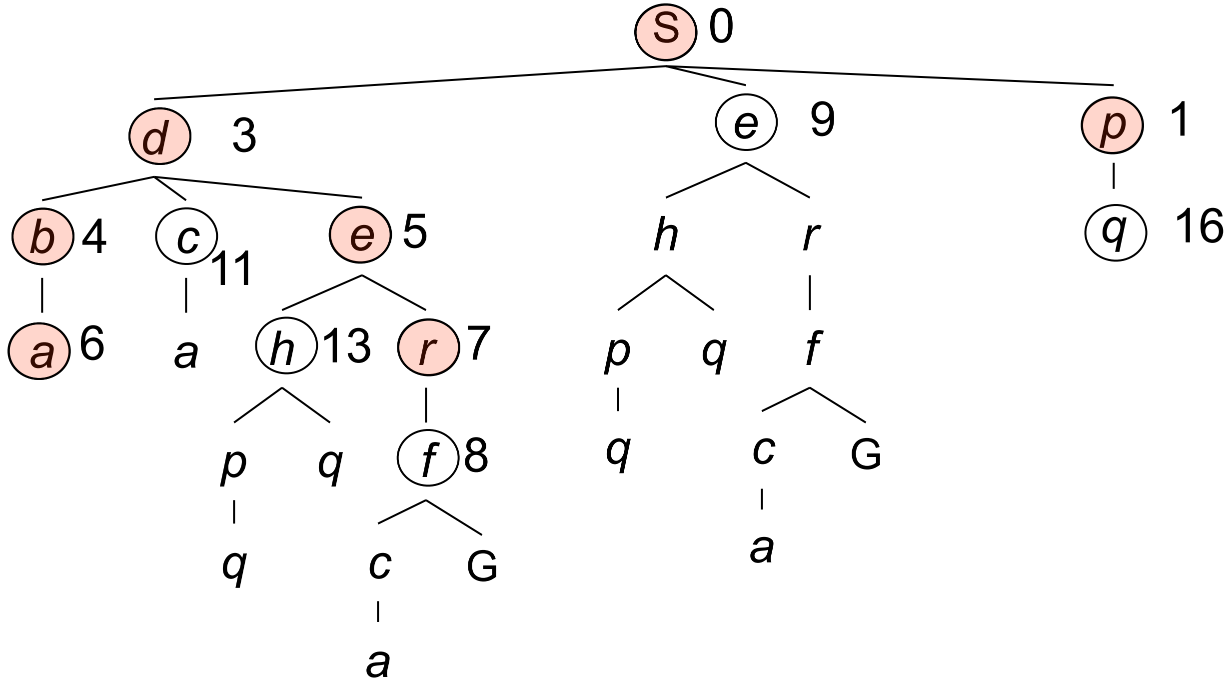


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

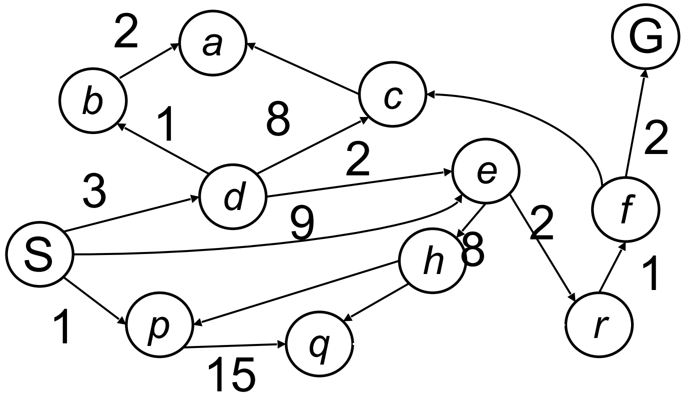


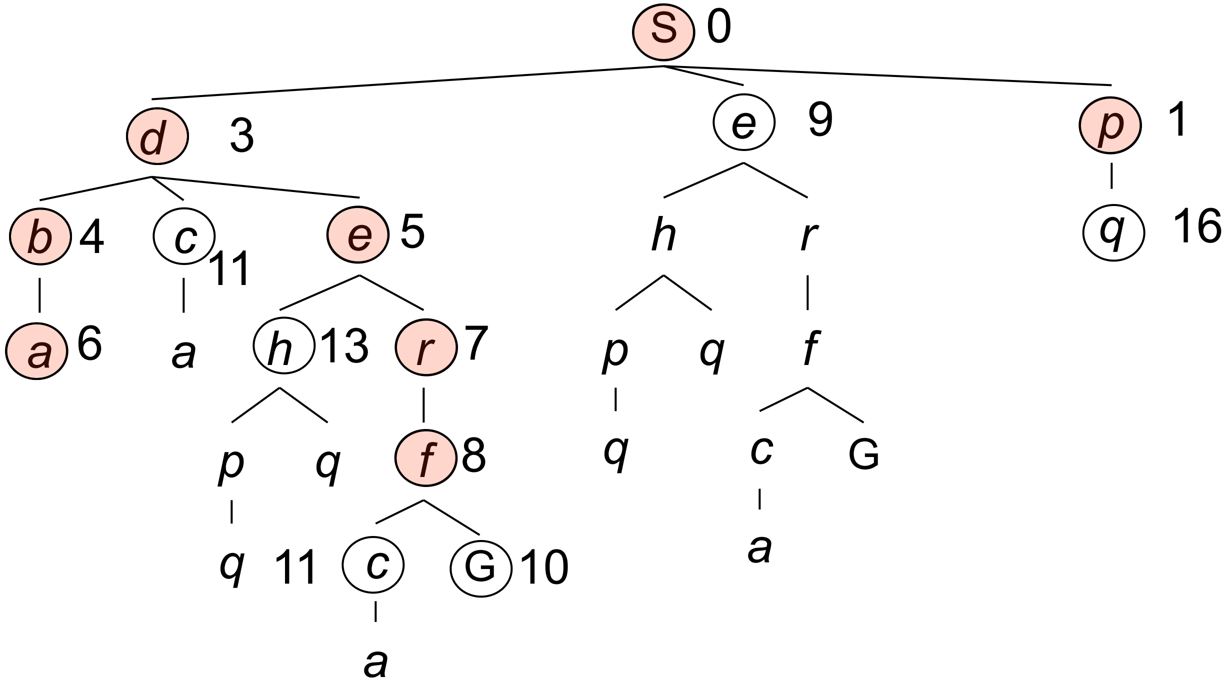


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

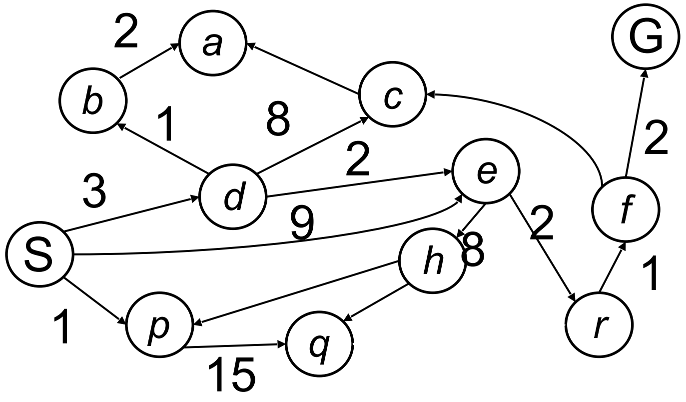


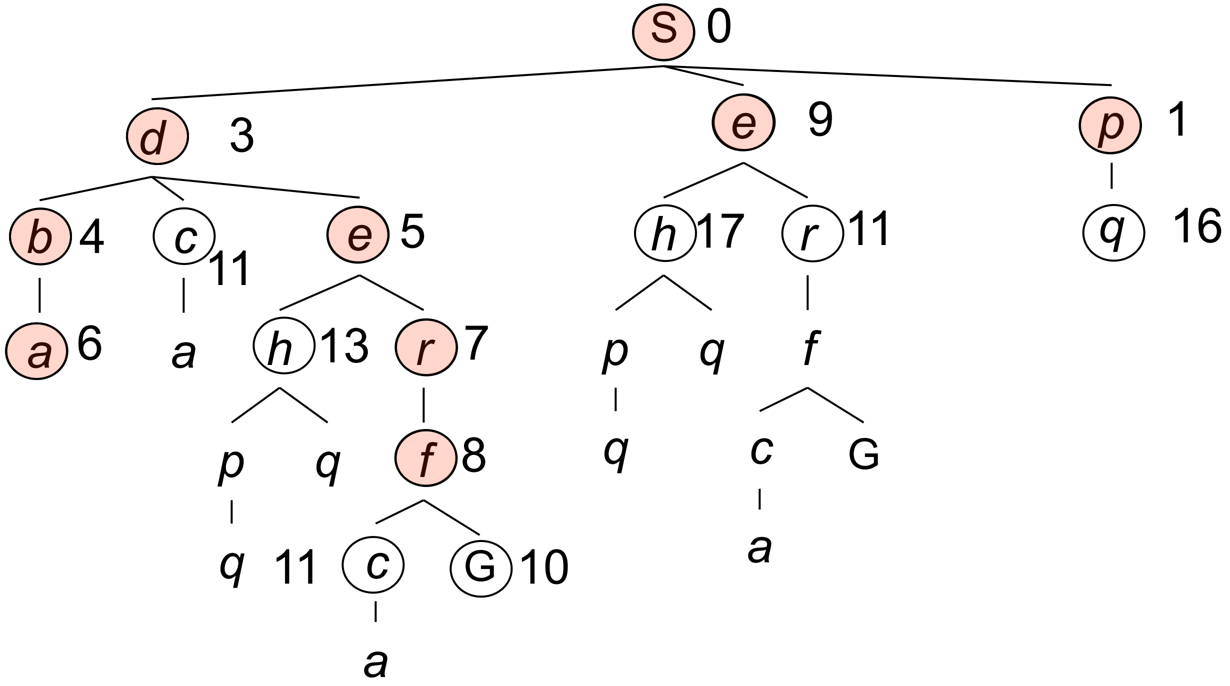


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

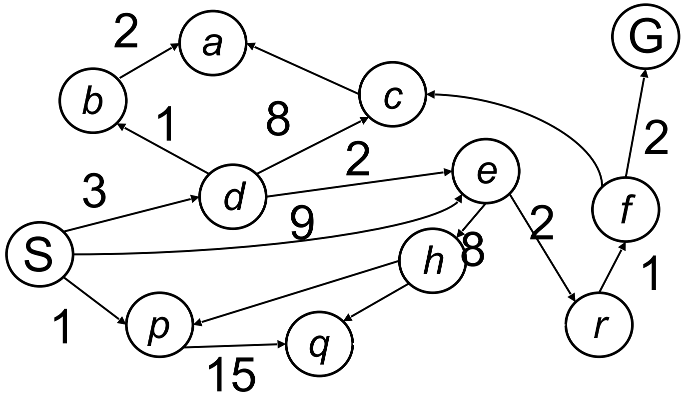


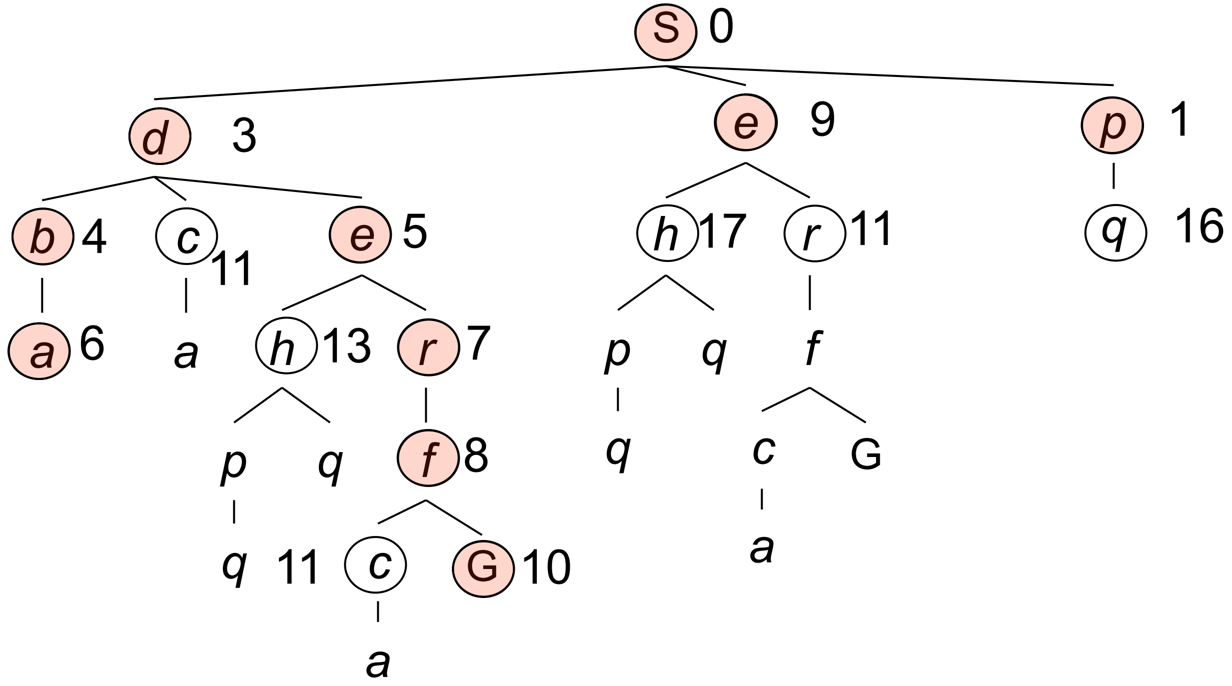


Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

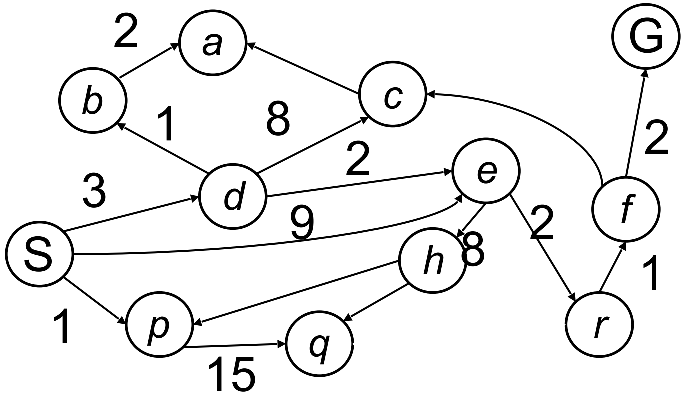




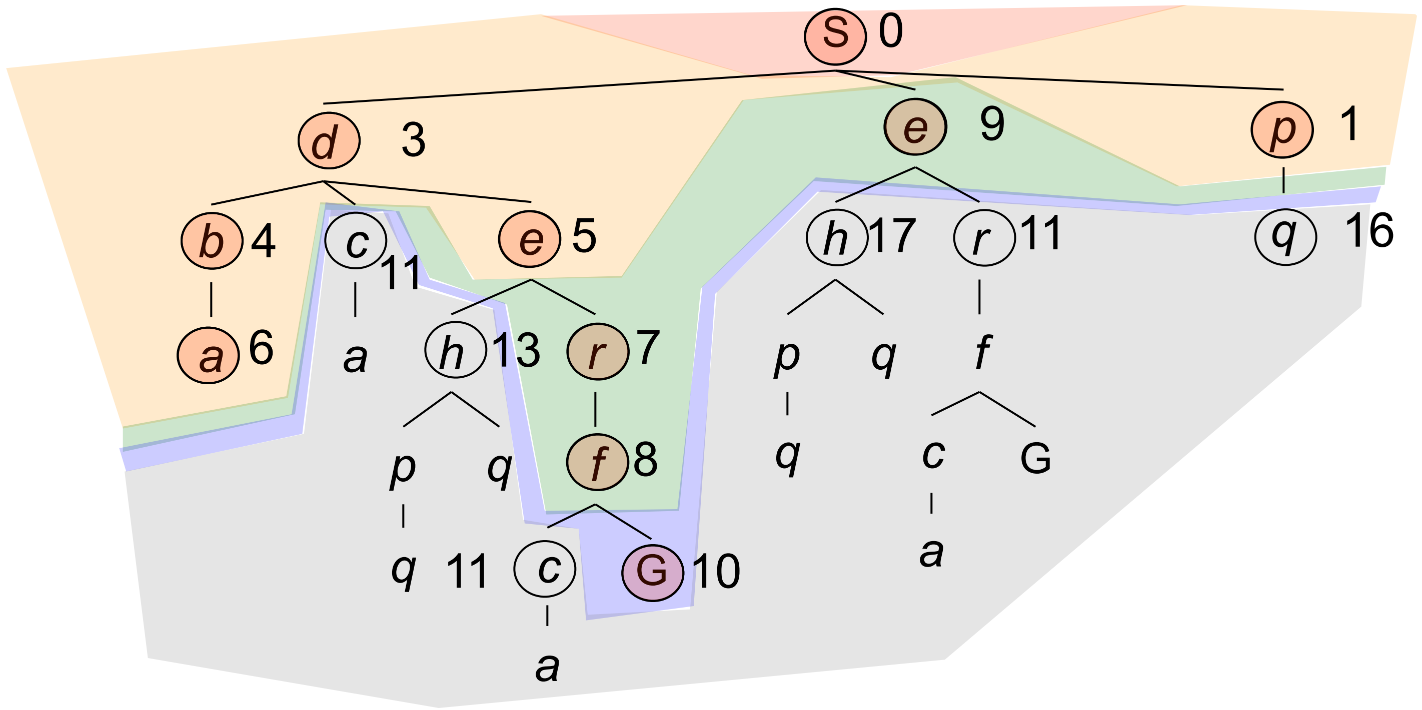
Uniform Cost Search (recap: uninformed search)

Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)

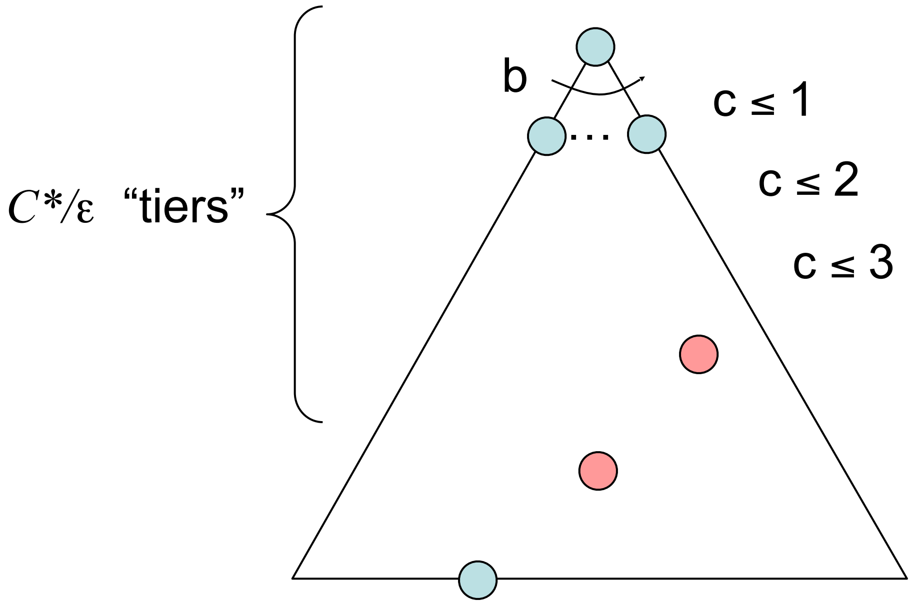


Cost contours



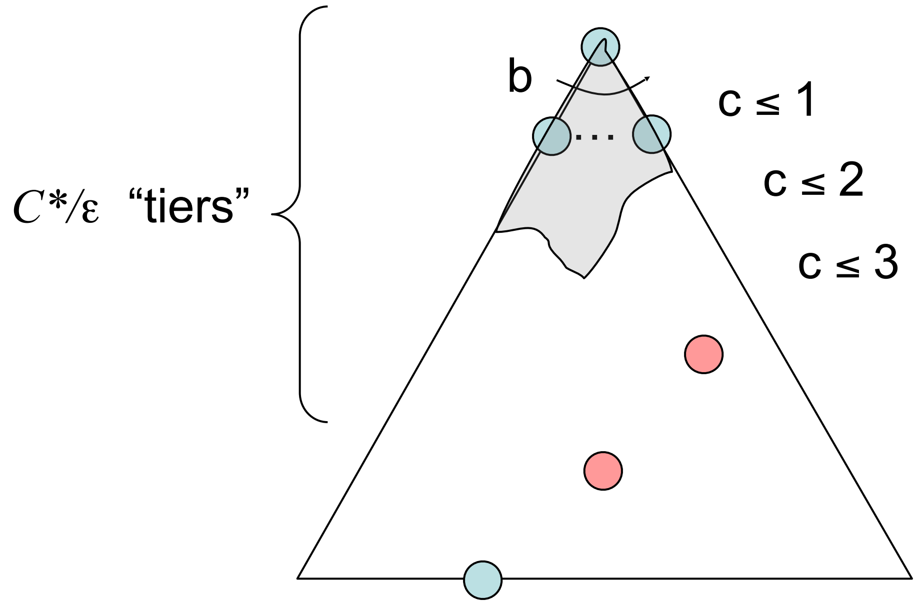
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?



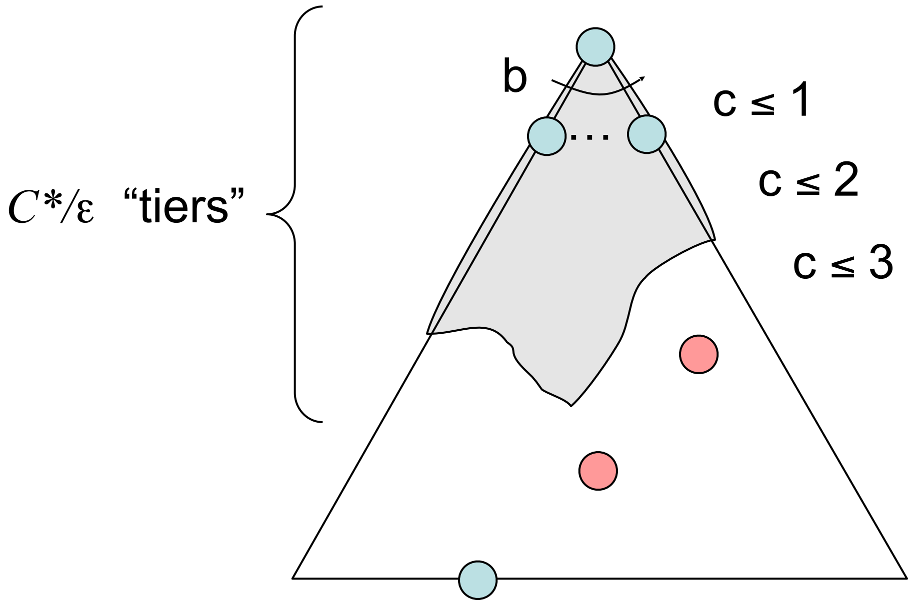
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?



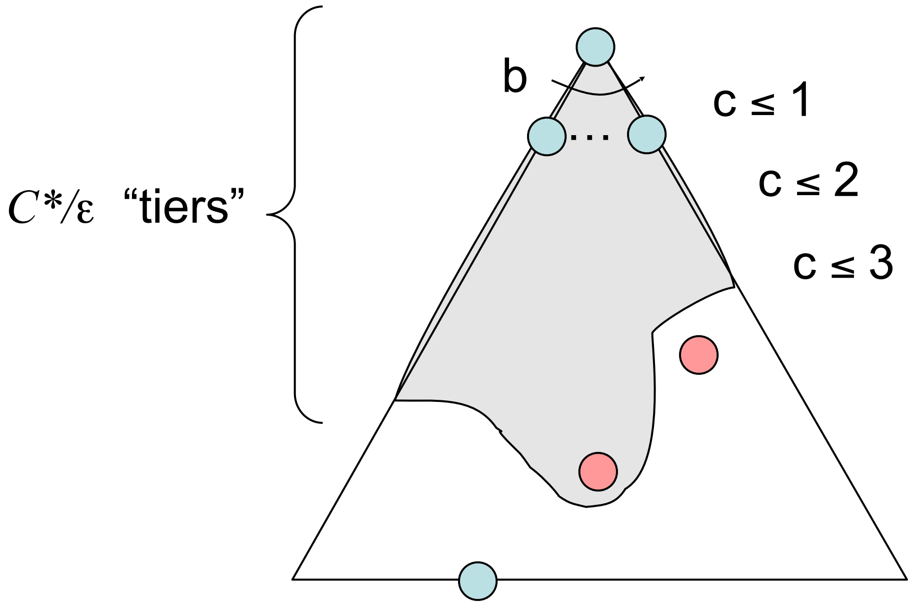
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?



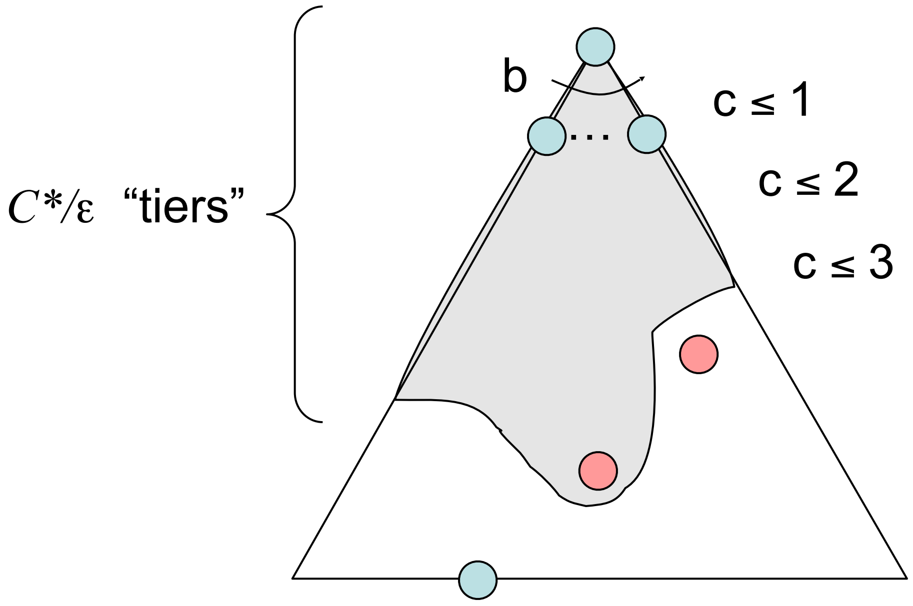
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?



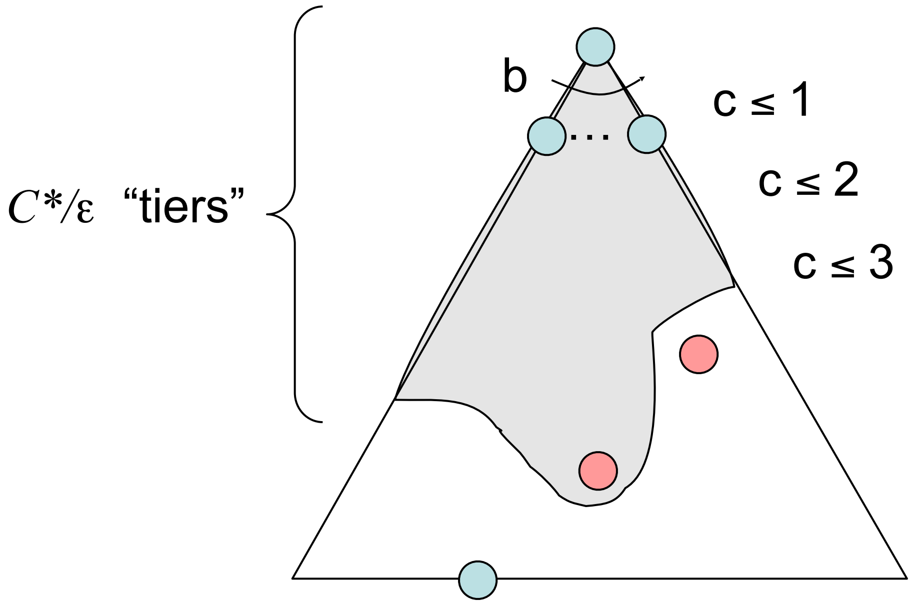
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!



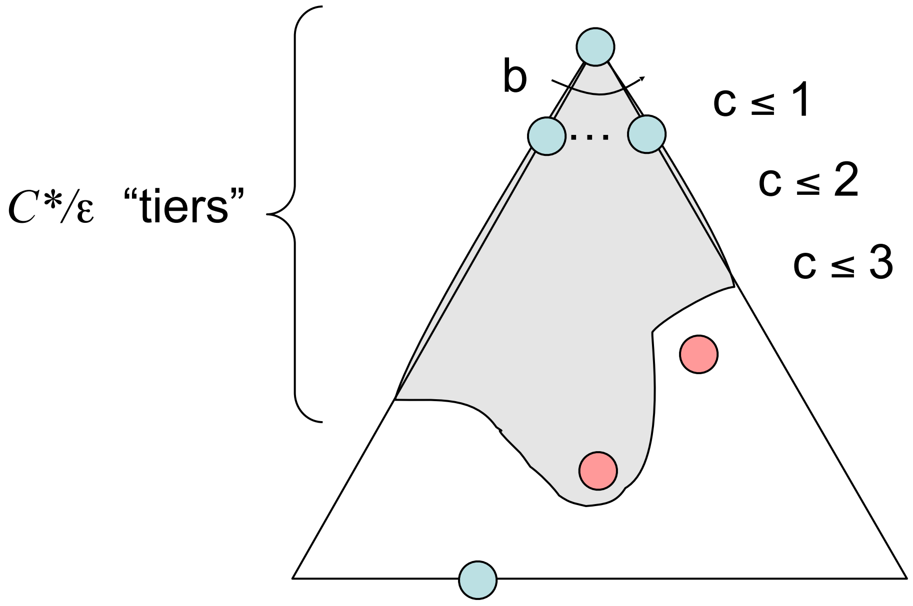
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε



Uniform Cost Search (UCS) Properties

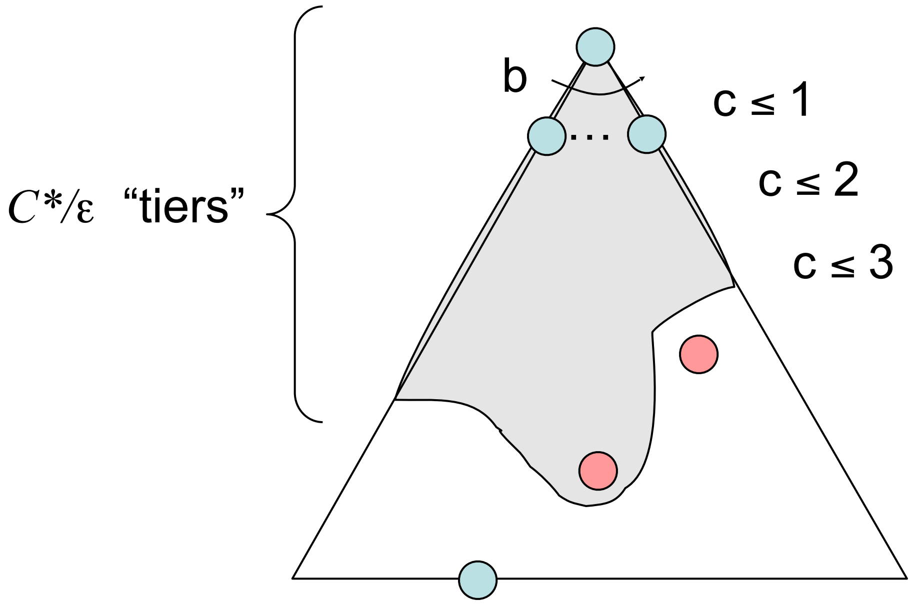
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)



Uniform Cost Search (UCS) Properties

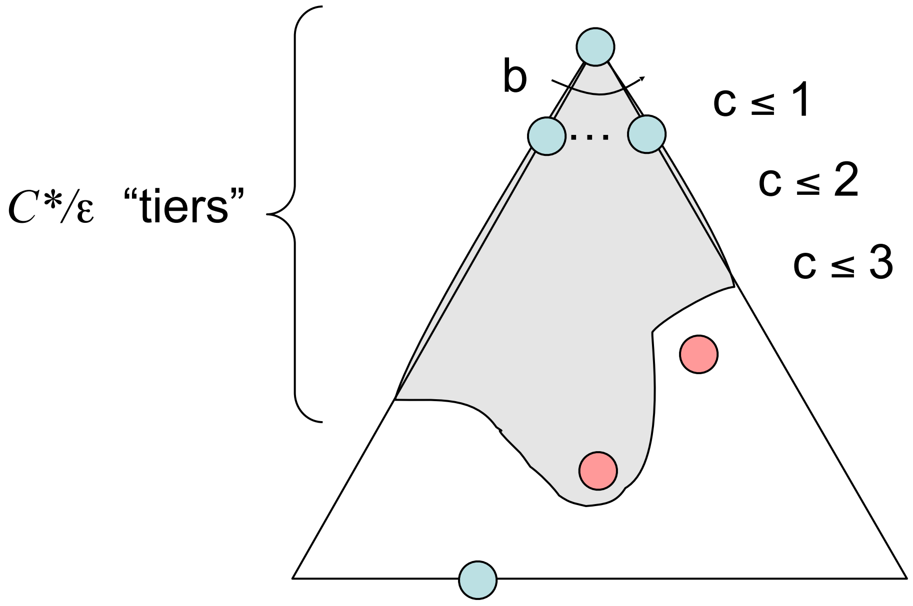
- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs $C*$ and arcs cost at least ε , then the "effective depth" is roughly $C*/\varepsilon$
  - Takes time $O(b^{C*/\varepsilon})$ (exponential in effective depth)

- How much space does the fringe take?



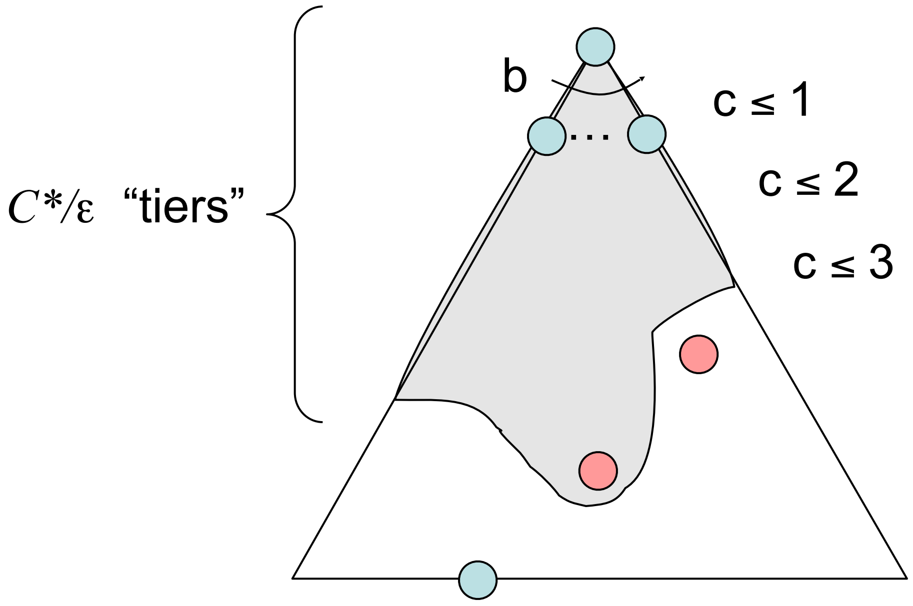
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$



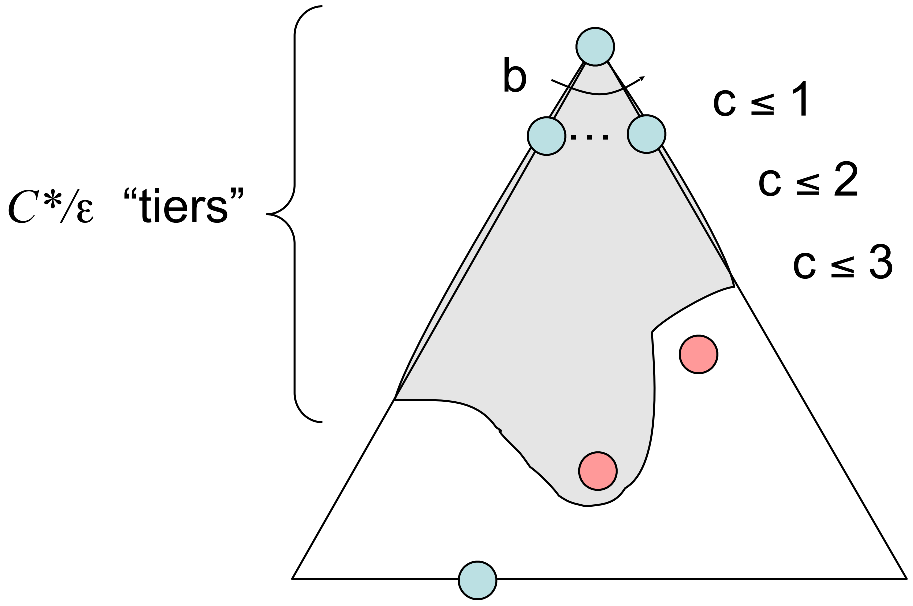
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?



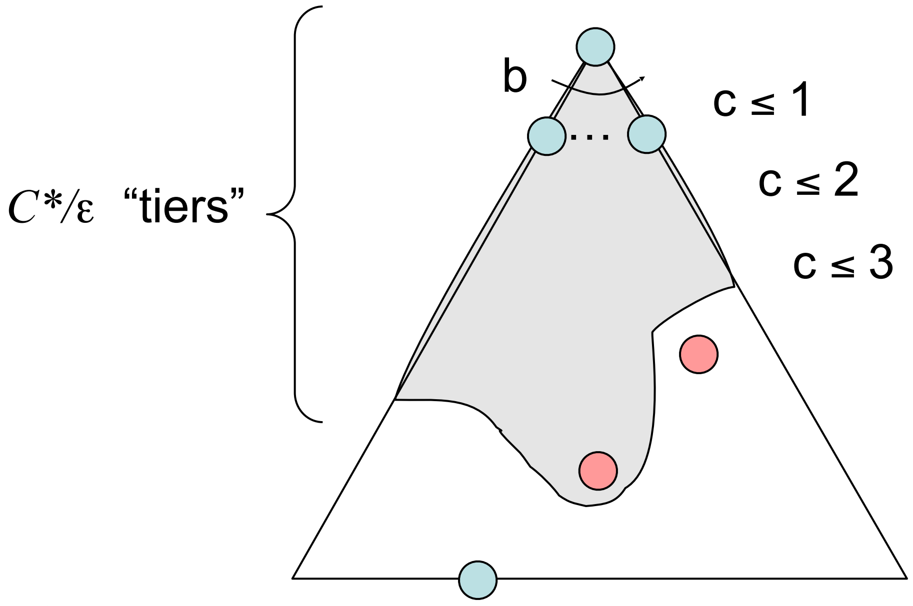
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!



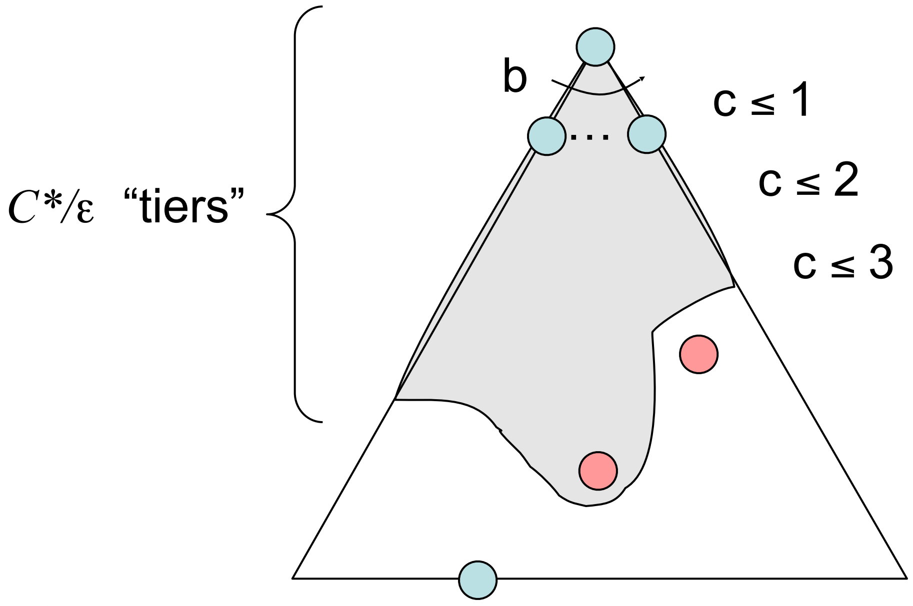
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?



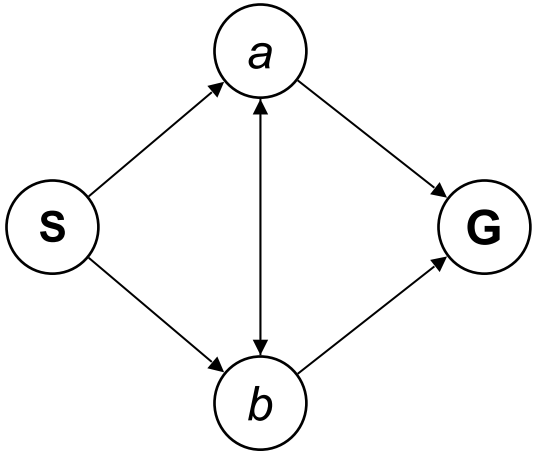
Uniform Cost Search (UCS) Properties

- What nodes does UCS expand?
  - Processes all nodes with cost less than cheapest solution!
  - If that solution costs C^* and arcs cost at least ε , then the "effective depth" is roughly C^*/ε
  - Takes time $O(b^{C^*/\varepsilon})$ (exponential in effective depth)
- How much space does the fringe take?
  - Has roughly the last tier, so $O(b^{C^*/\varepsilon})$
- Is it complete?
  - Assuming best solution has a finite cost and minimum arc cost is positive, yes!
- Is it optimal?
  - Yes! (Proof next lecture via A*)



Infinite Search Tree

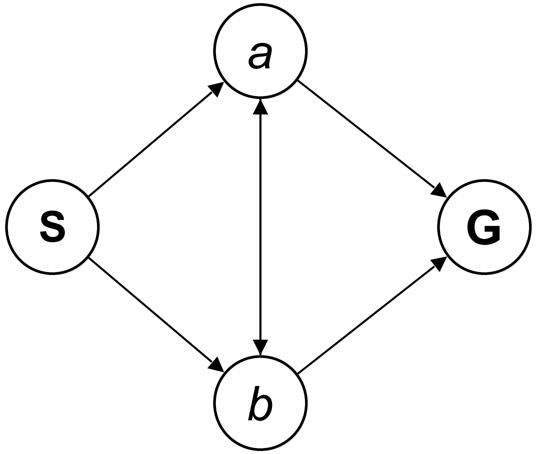
Consider this 4-state graph:



- best solution may have an infinite cost
  (e.g ride a sightseeing train for as long as possible)

Infinite Search Tree

Consider this 4-state graph:



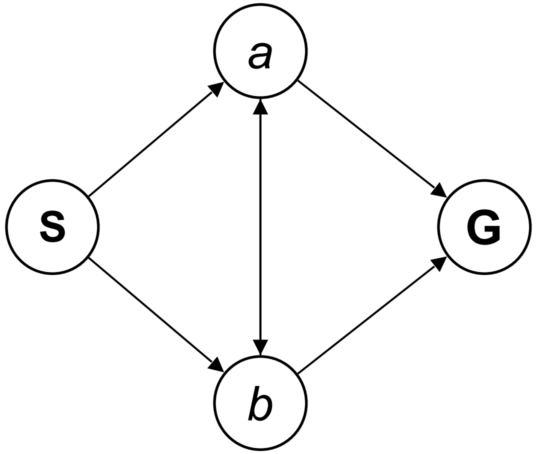
How big is its search tree (from S)?

- best solution may have an infinite cost

  (e.g ride a sightseeing train for as long as possible)

Infinite Search Tree

Consider this 4-state graph:



How big is its search tree (from S)?

$$\infty$$

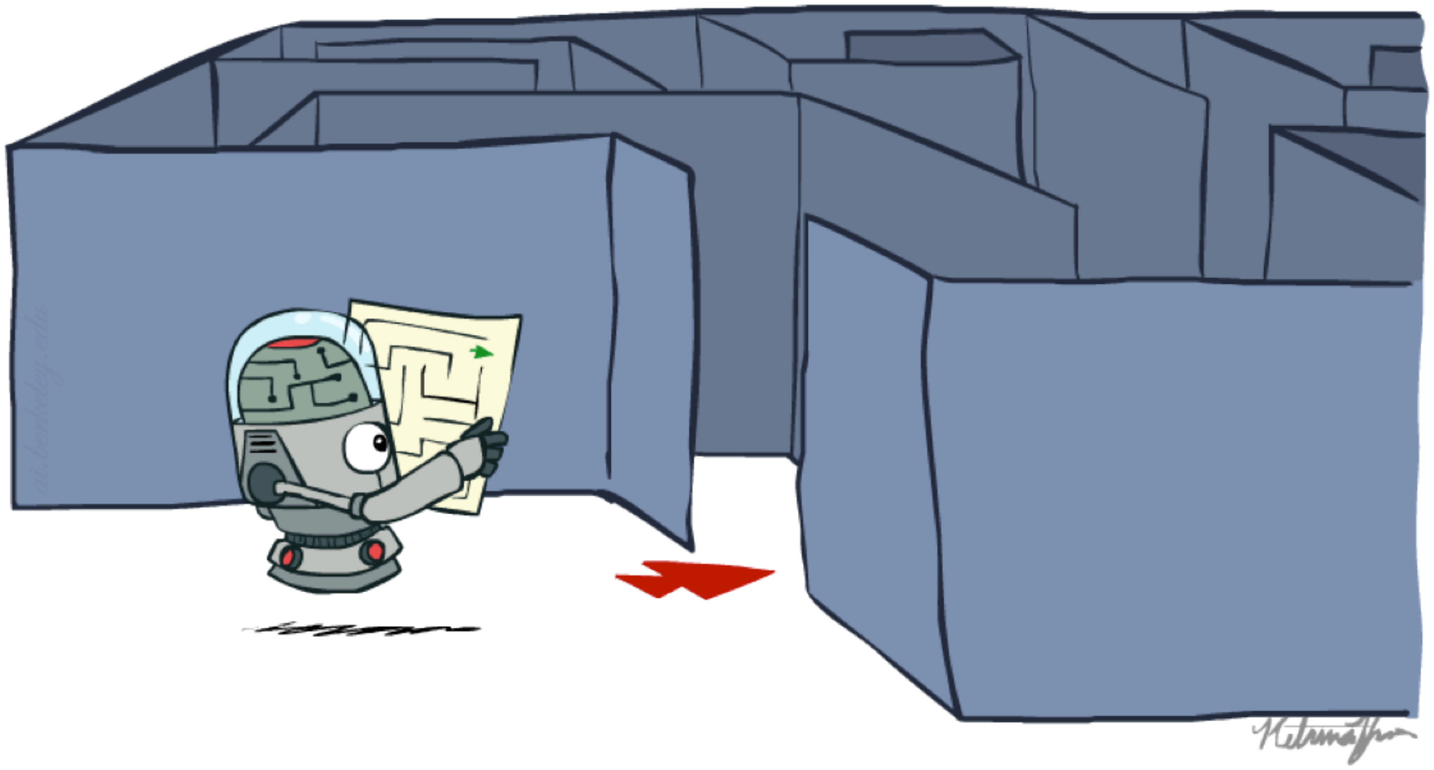
- best solution may have an infinite cost

  (e.g ride a sightseeing train for as long as possible)

Today

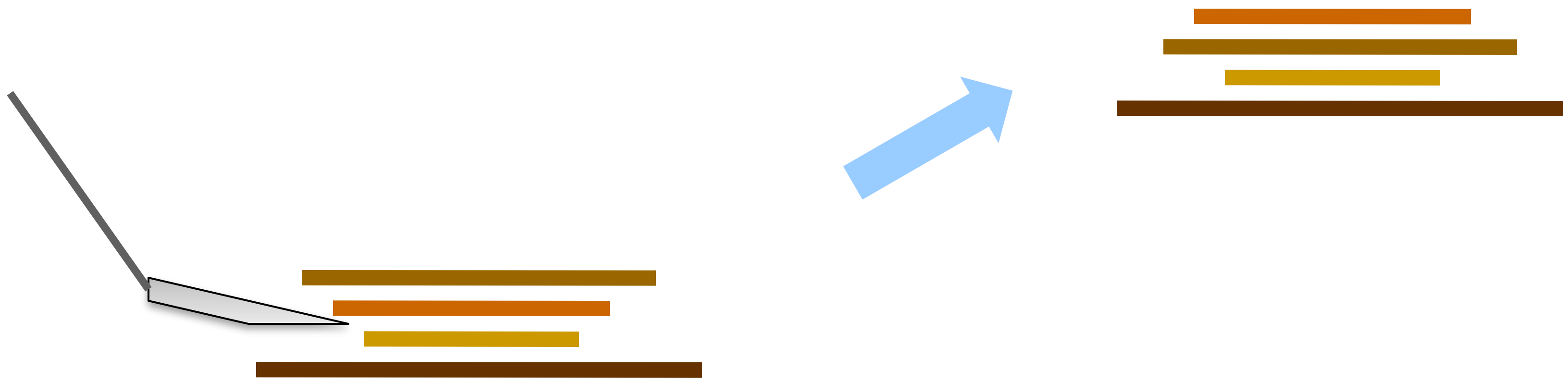
- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search
- Graph Search

Recap: Search

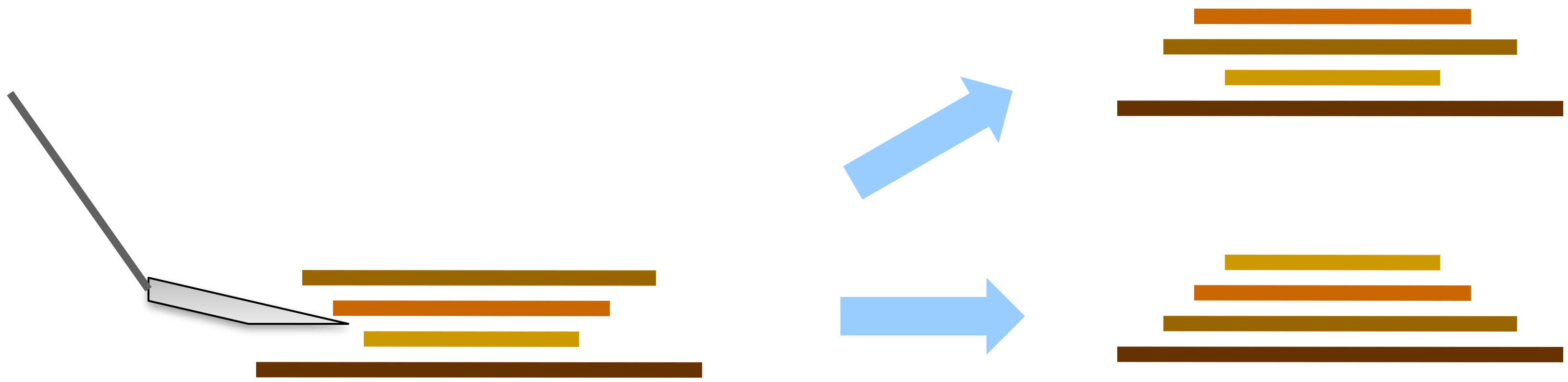


Example: Pancake Problem

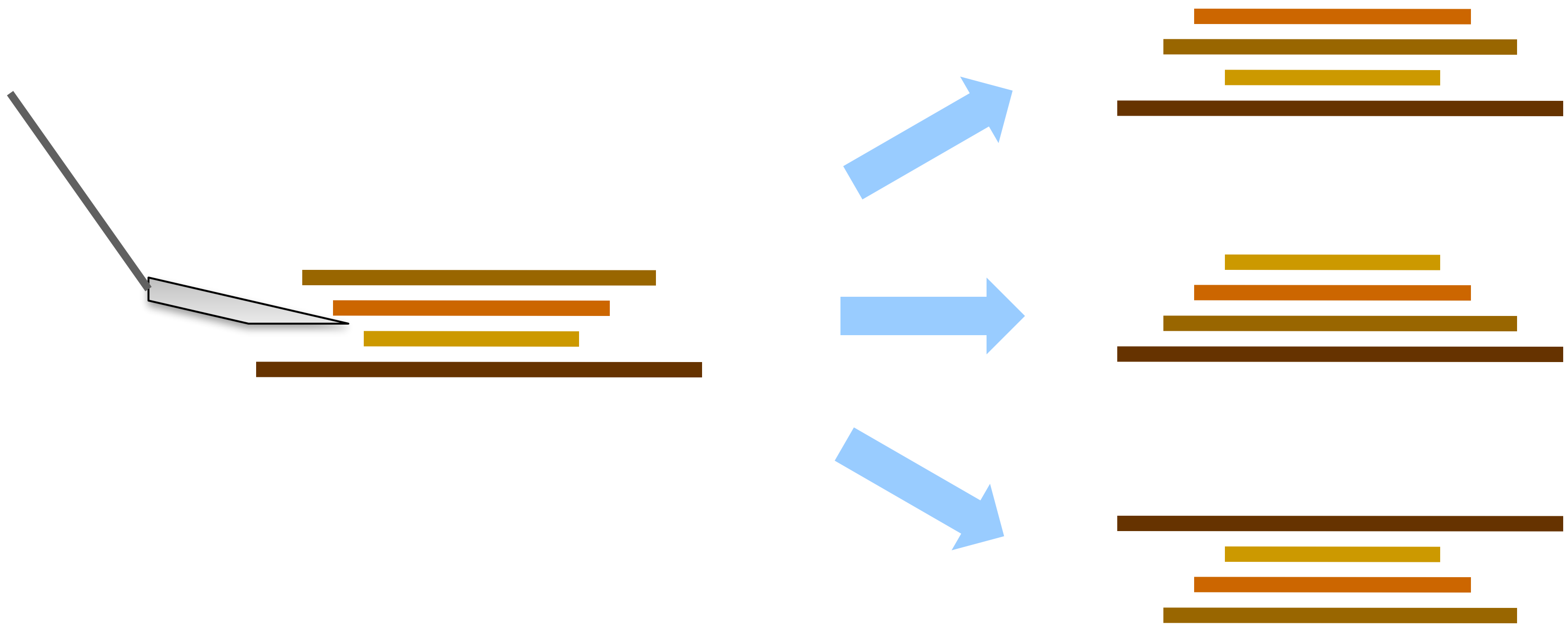
Example: Pancake Problem



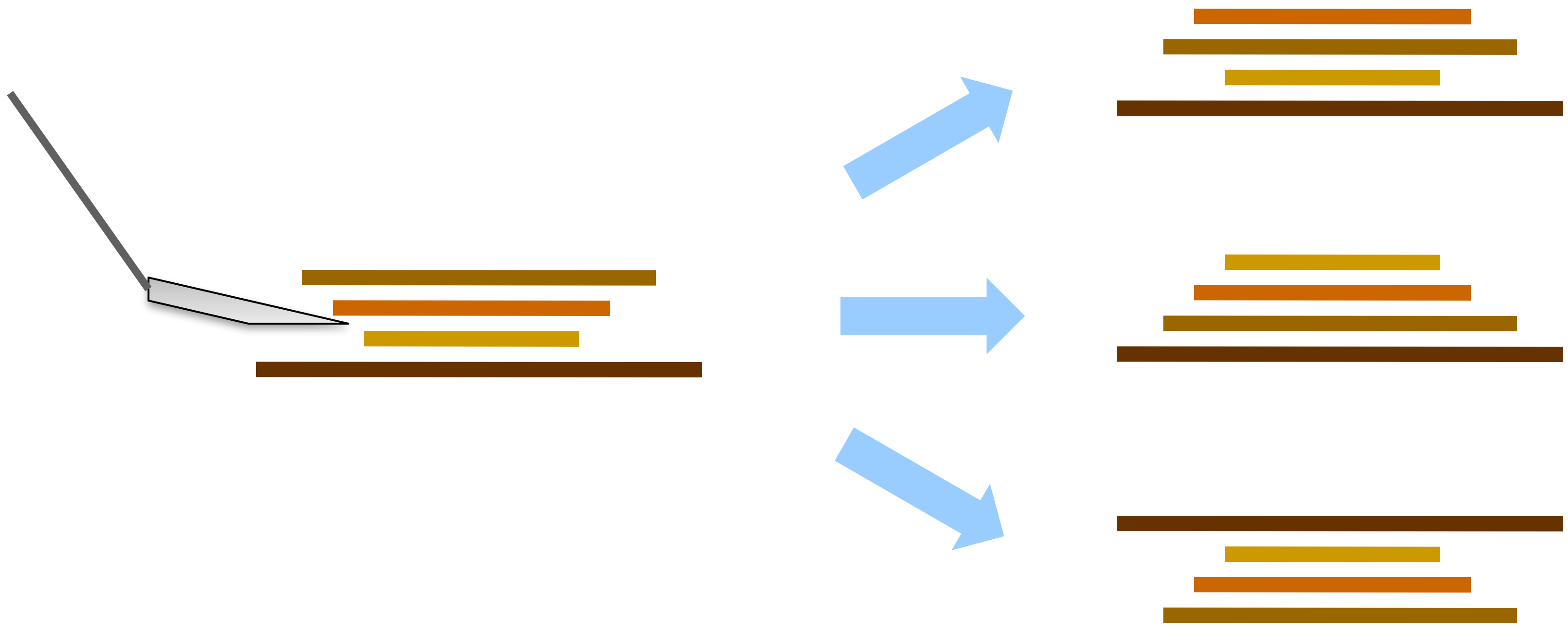
Example: Pancake Problem



Example: Pancake Problem

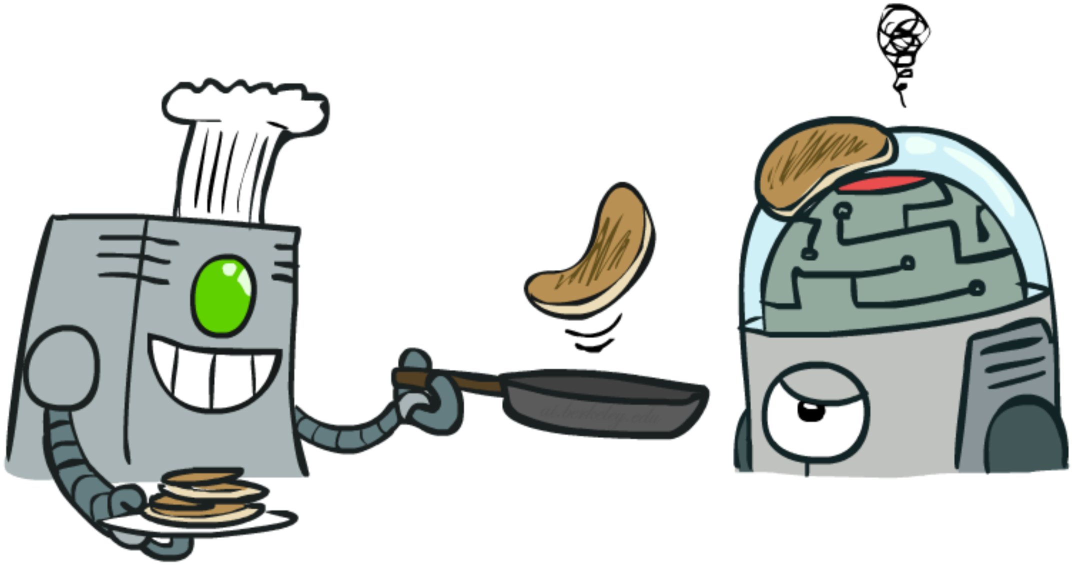


Example: Pancake Problem



Cost: Number of pancakes flipped

Example: Pancake Problem



Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

William H. GATES

Microsoft, Albuquerque, New Mexico

Christos H. PAPADIMITRIOU*†

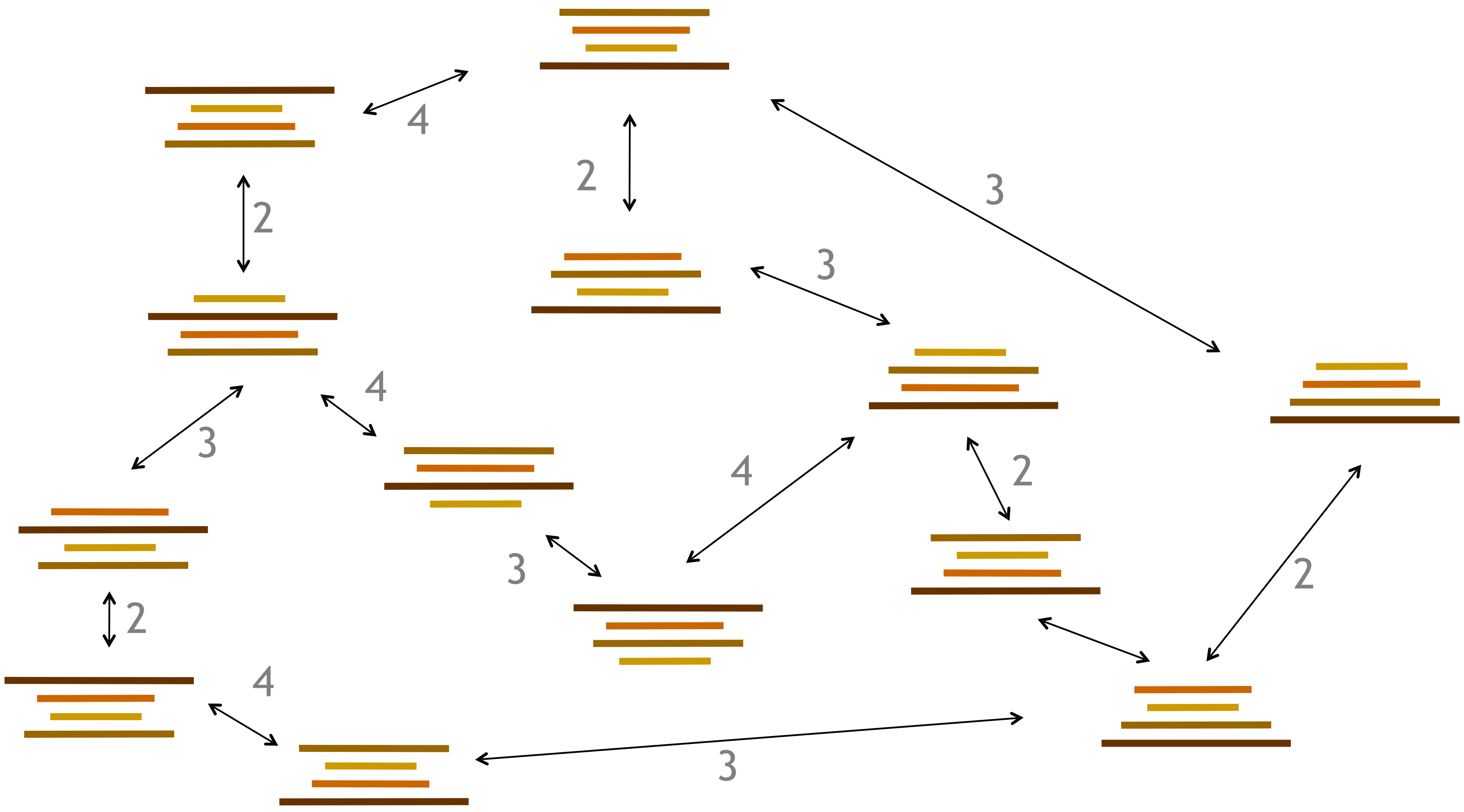
Department of Electrical Engineering, University of California, Berkeley, CA 94720, U.S.A.

Received 18 January 1978
Revised 28 August 1978

For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leqslant (5n+5)/3$, and that $f(n) \geqslant 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2 - 1 \leqslant g(n) \leqslant 2n+3$.

Example: Pancake Problem

State space graph with costs as weights

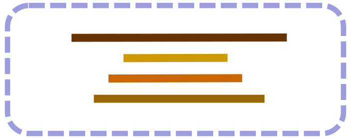


General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

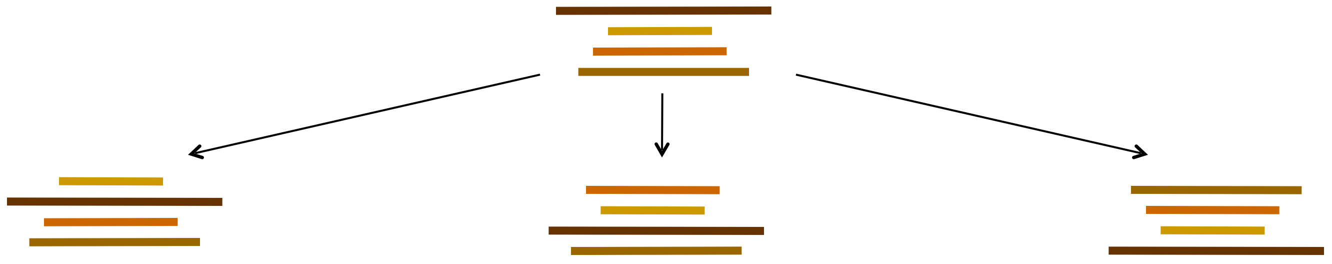
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```



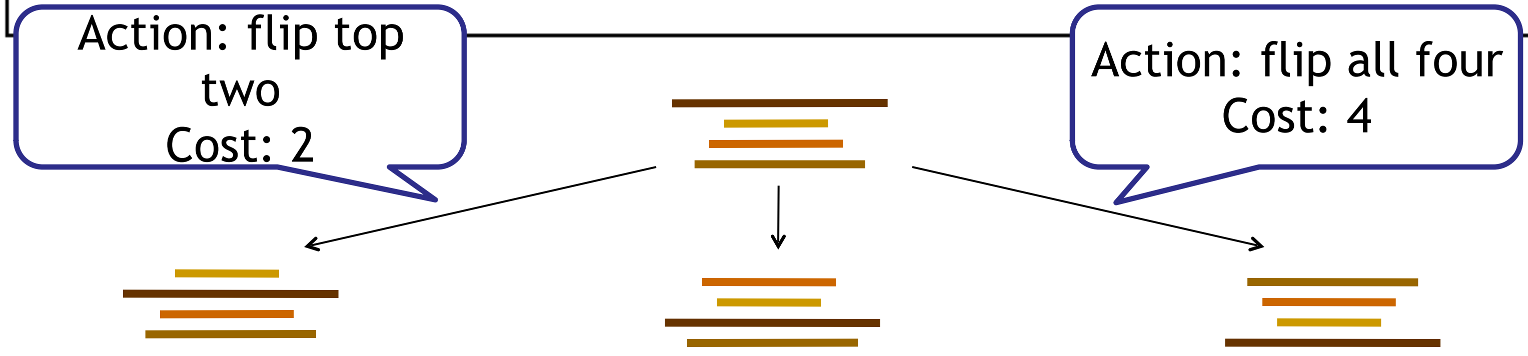
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```



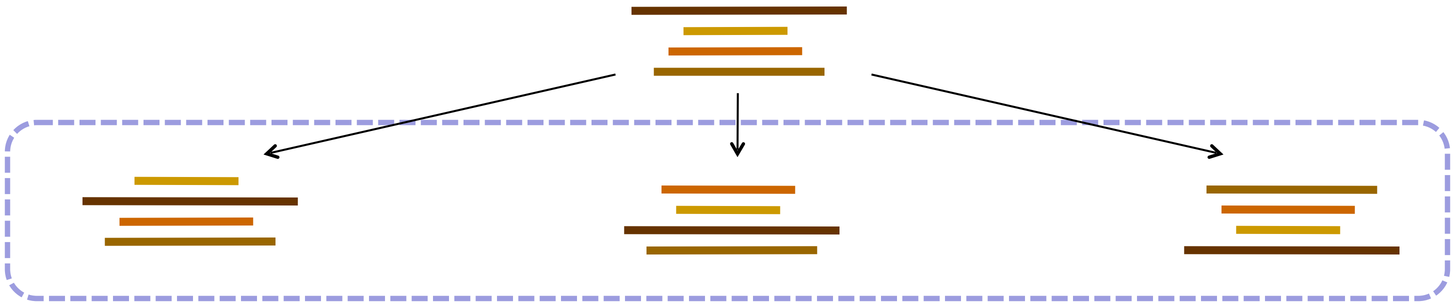
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```



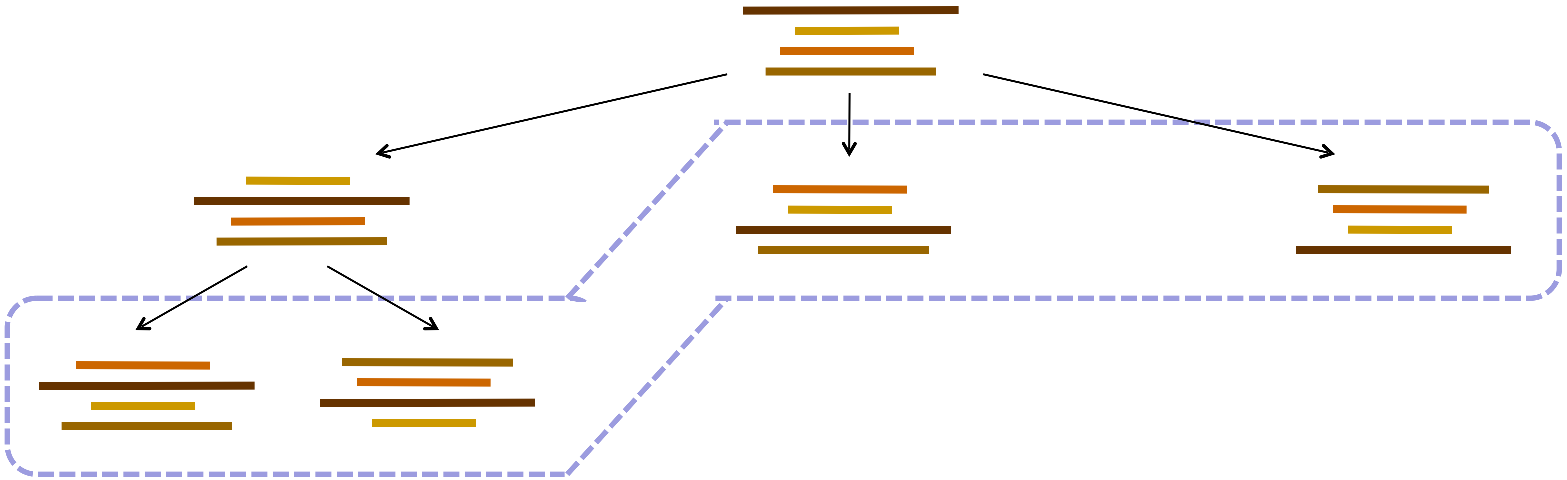
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```



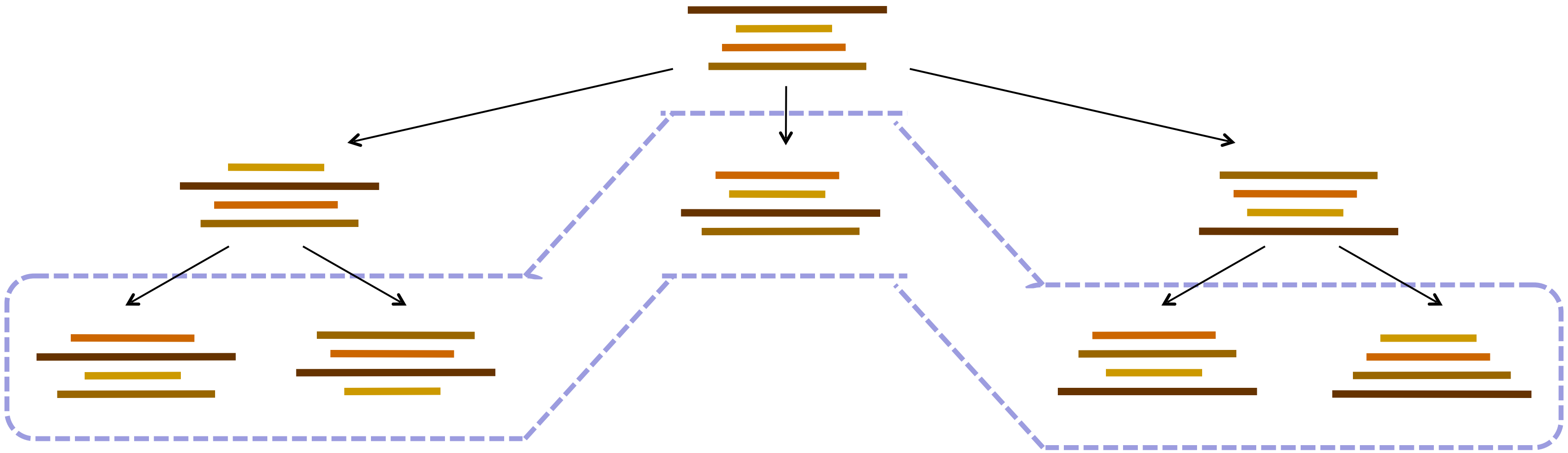
General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```



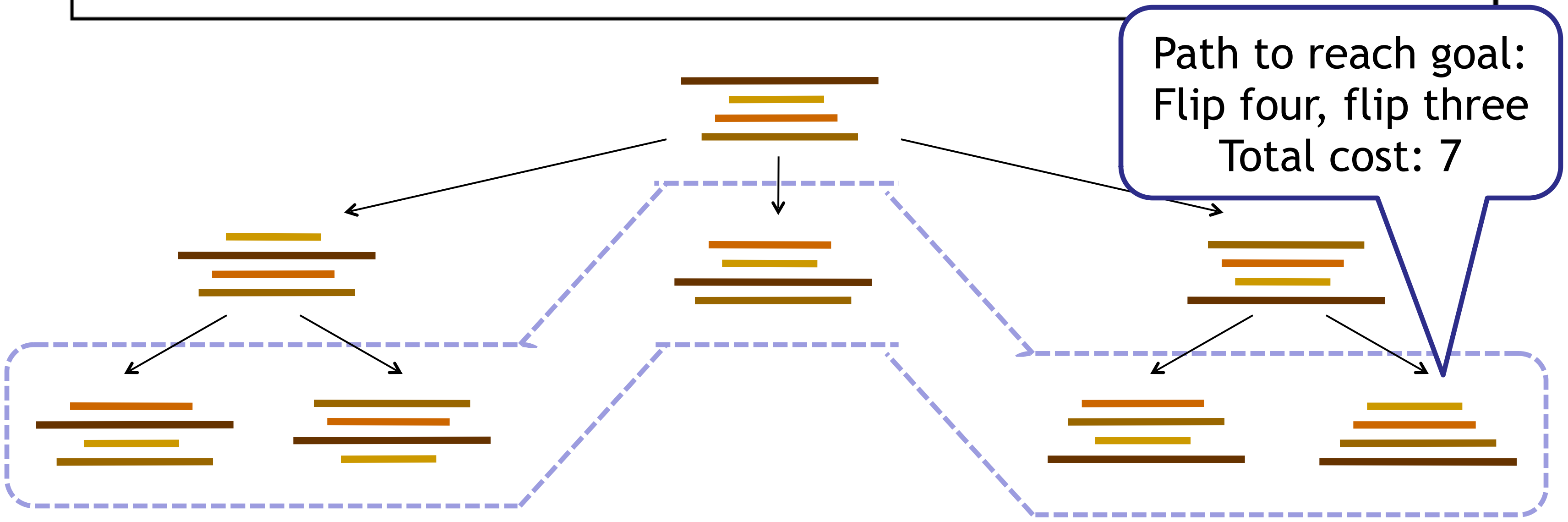
General Tree Search

function TREE-SEARCH(*problem*, *strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end



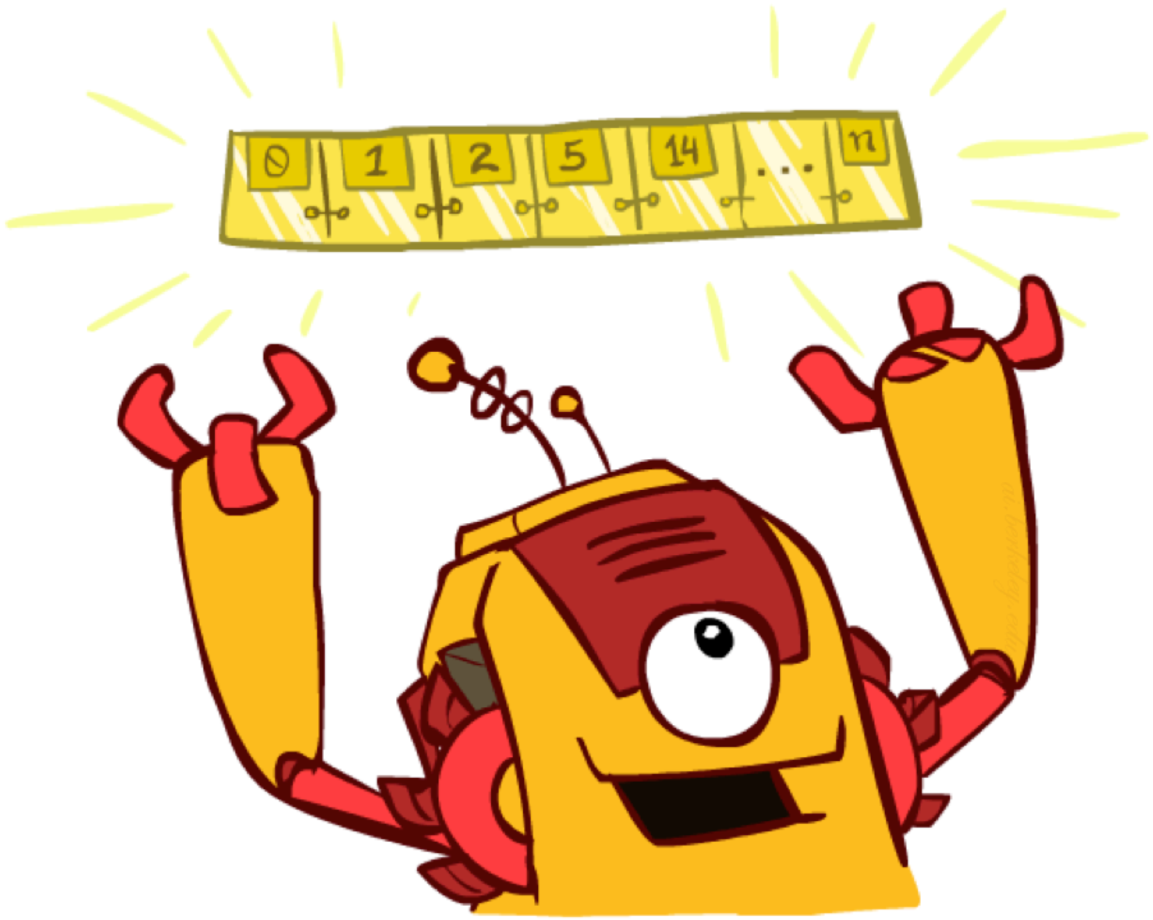
General Tree Search

function TREE-SEARCH(*problem, strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end



The One Queue

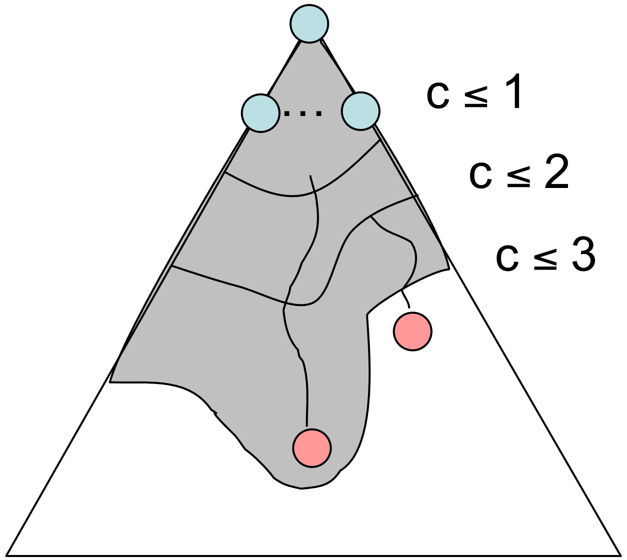
- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



Uninformed Search

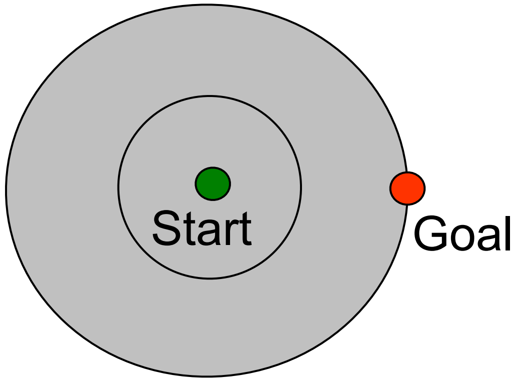
Uniform Cost Search

- Strategy: expand lowest path cost



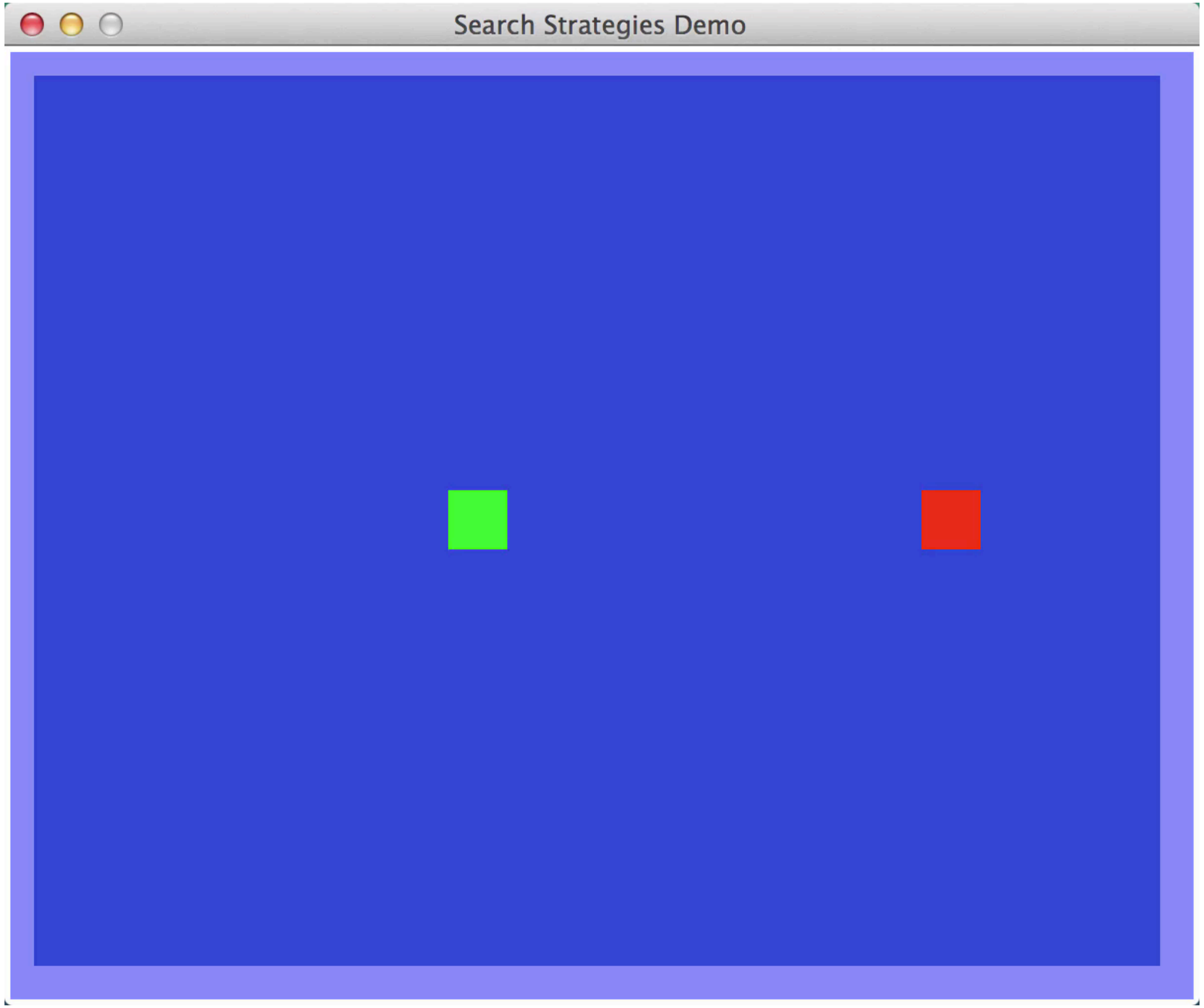
- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every "direction"
  - No information about goal location

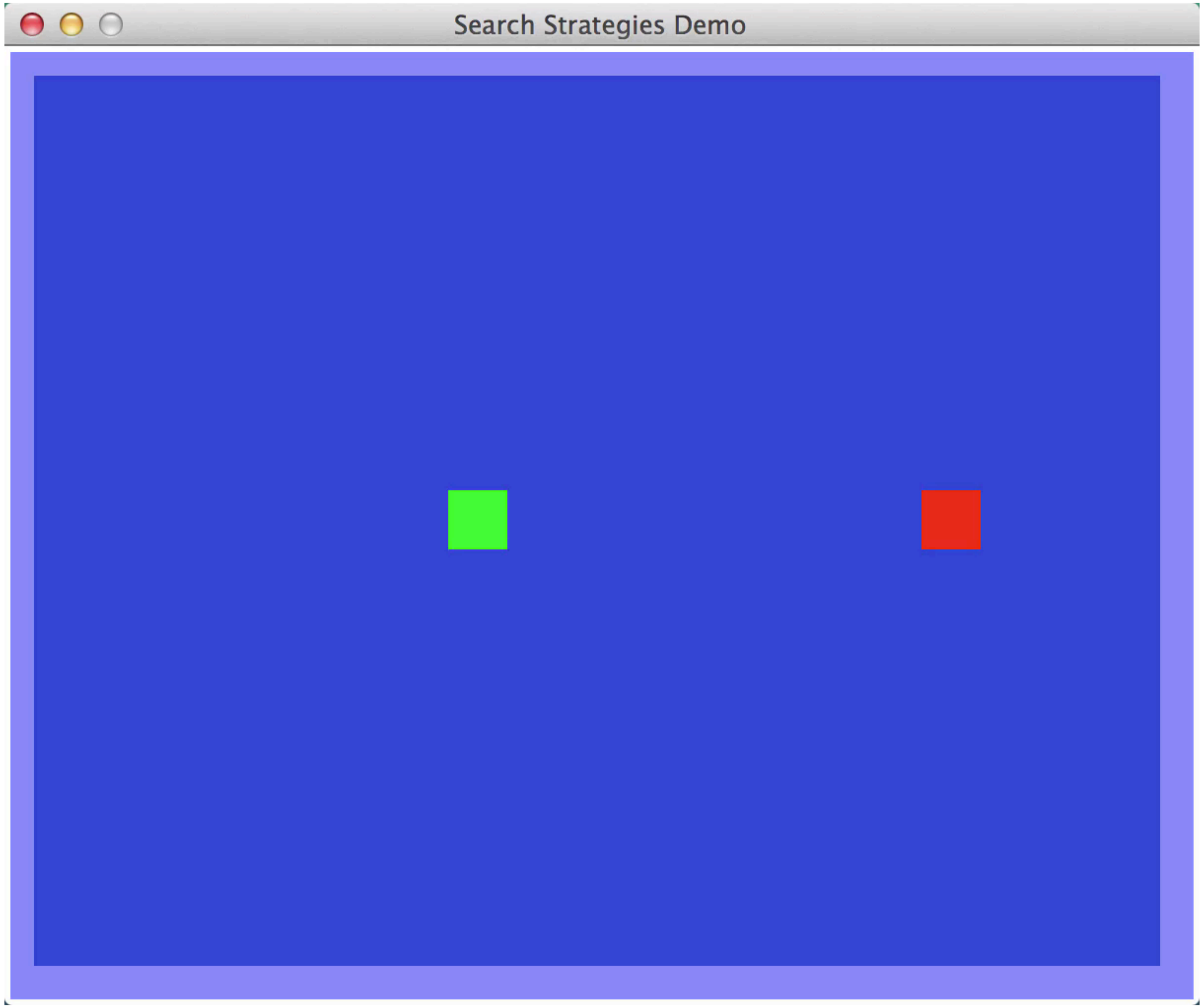


[Demo: contours UCS empty (L3D1)]
[Demo: contours UCS pacman small maze

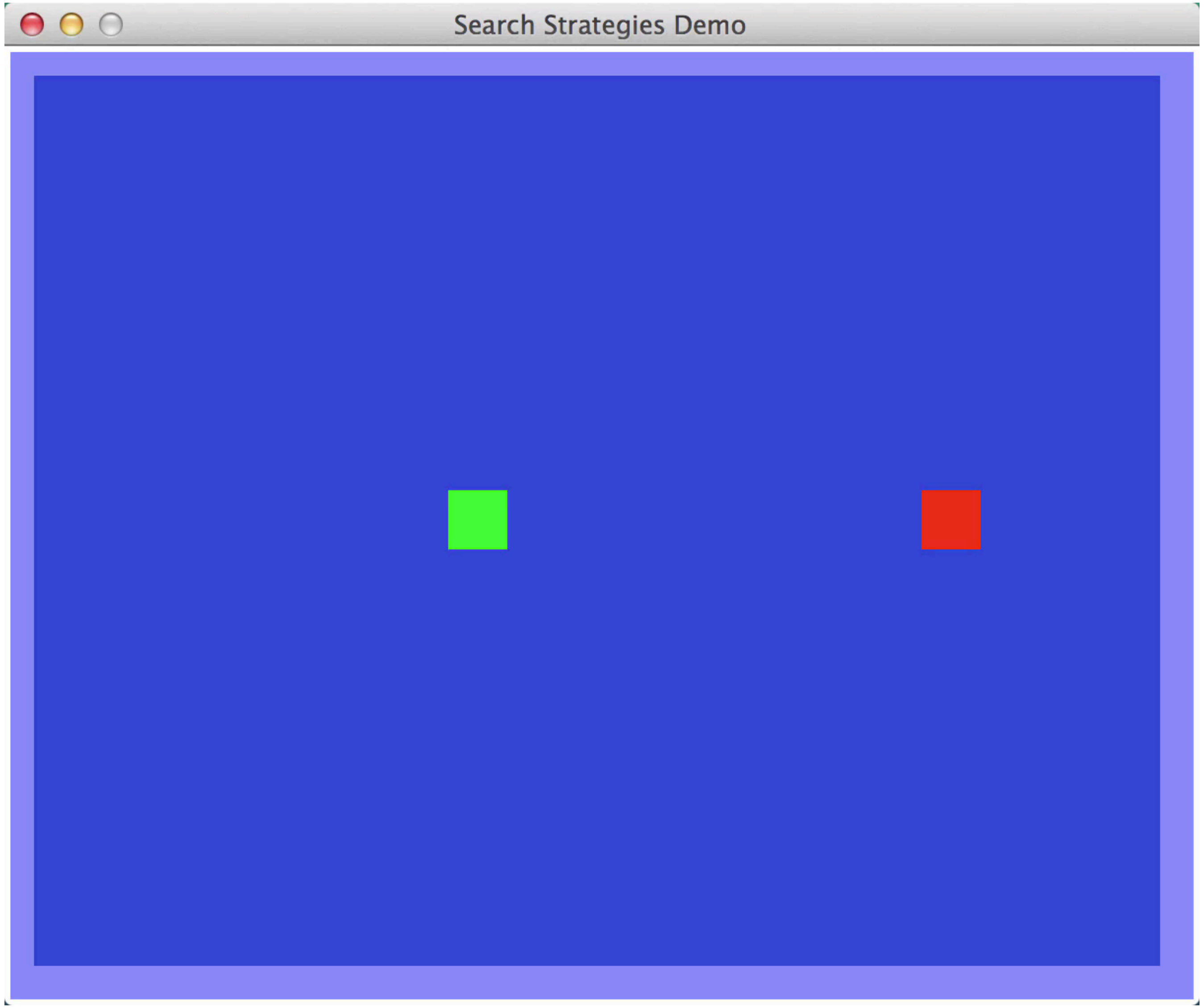
Video of Demo Contours UCS Empty



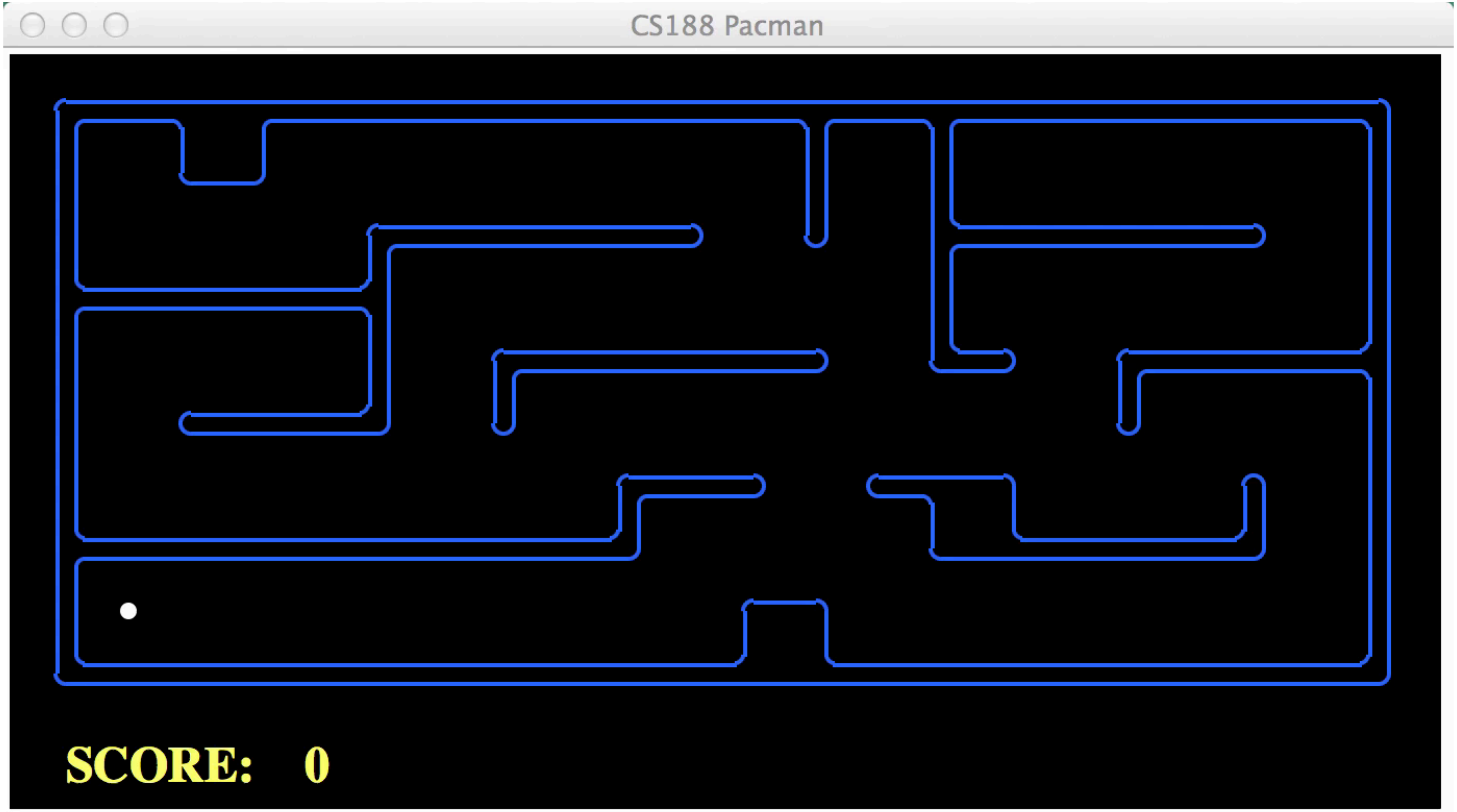
Video of Demo Contours UCS Empty



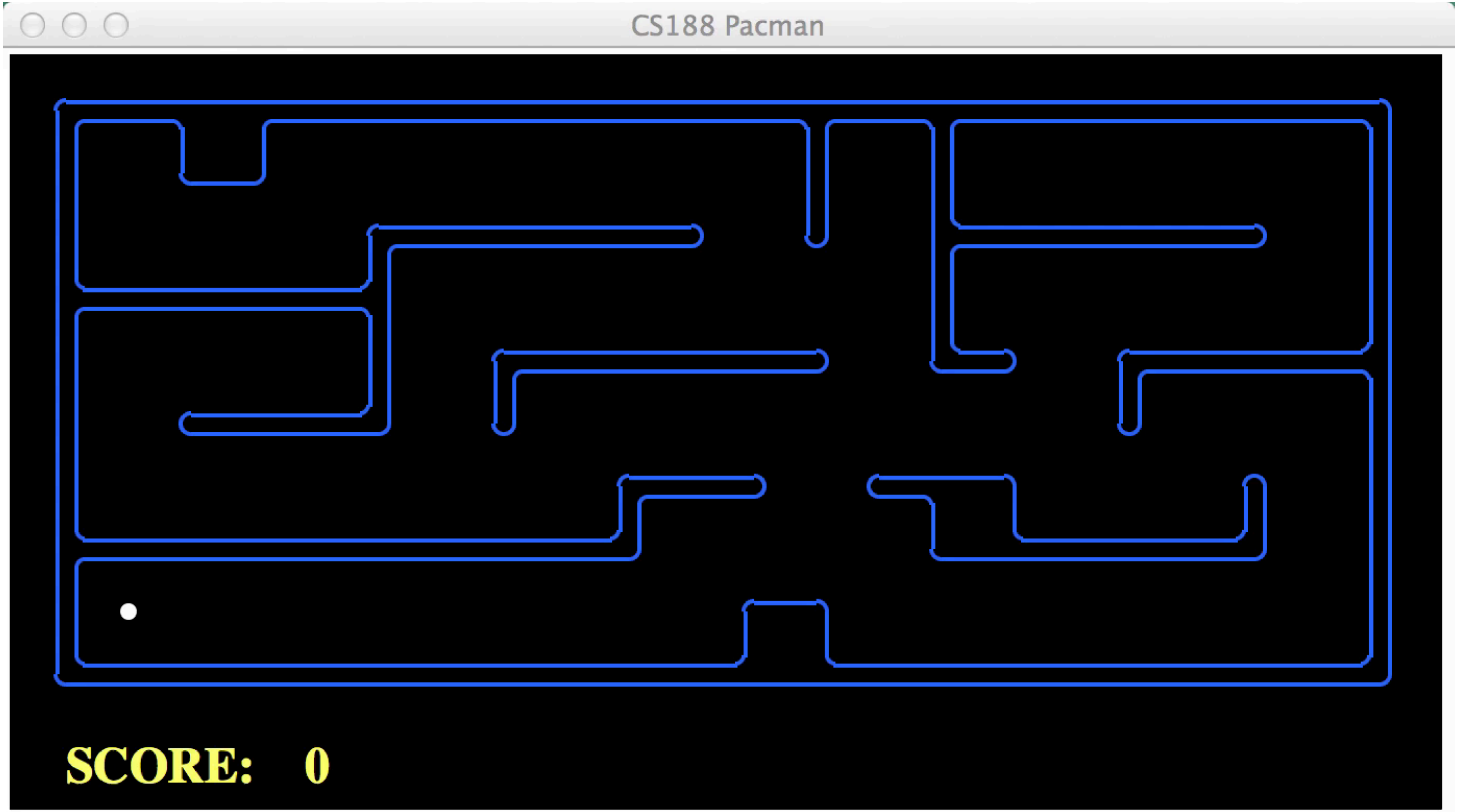
Video of Demo Contours UCS Empty



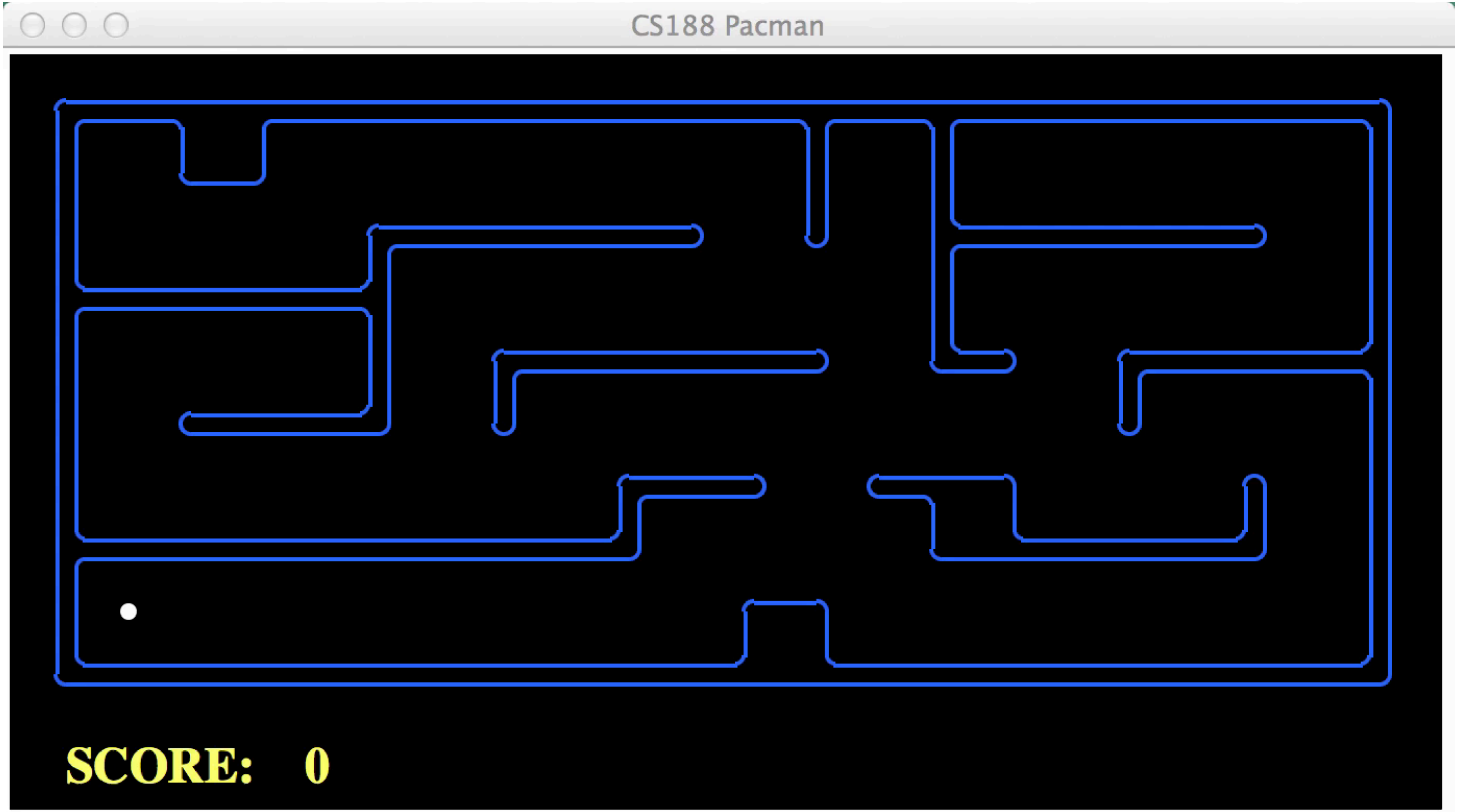
Video of Demo Contours UCS Pacman Small Maze



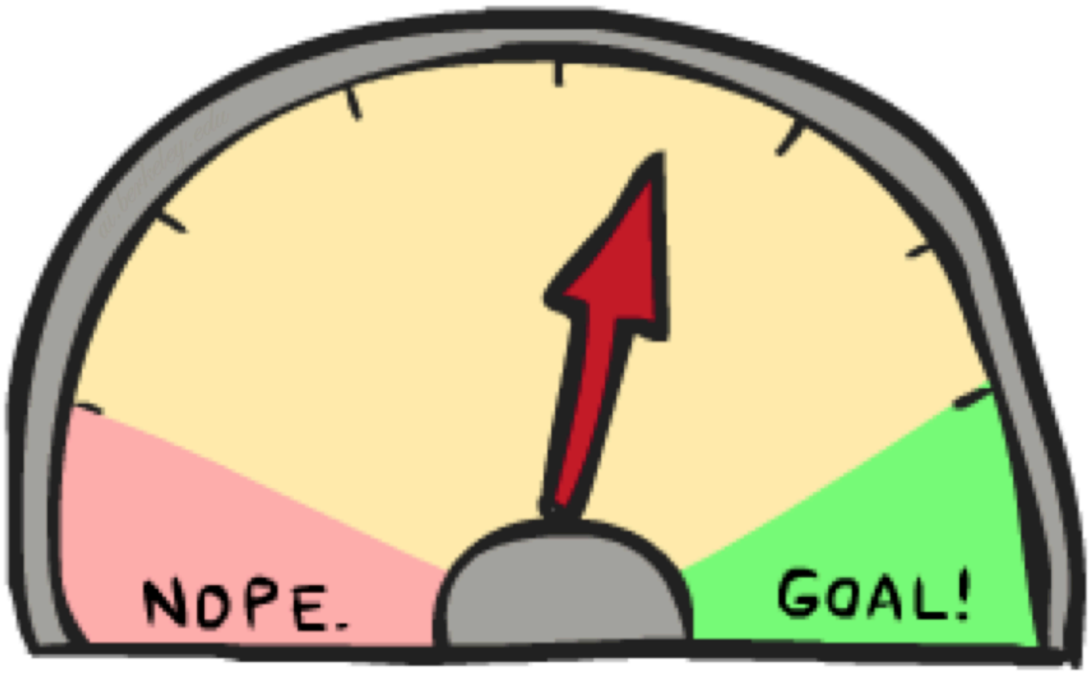
Video of Demo Contours UCS Pacman Small Maze



Video of Demo Contours UCS Pacman Small Maze



Informed Search



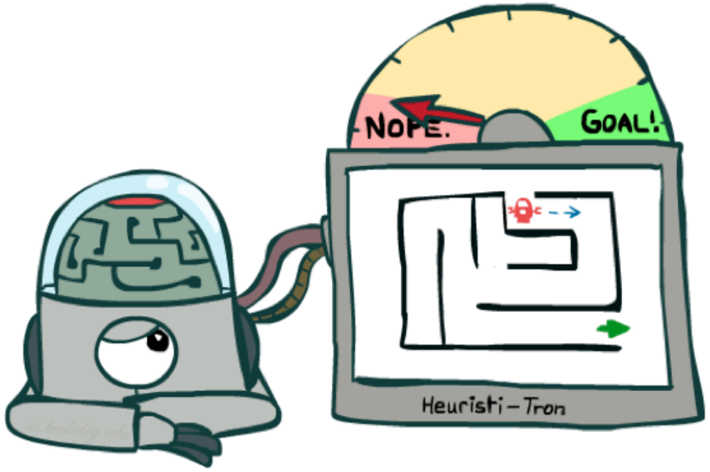
Search Heuristics

Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem

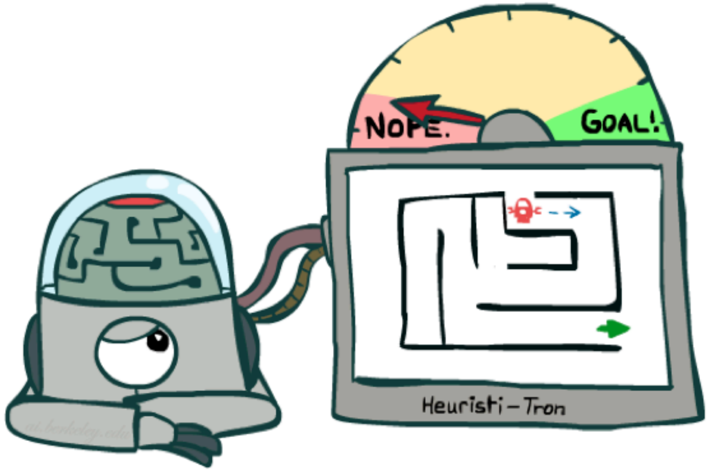
Search Heuristics

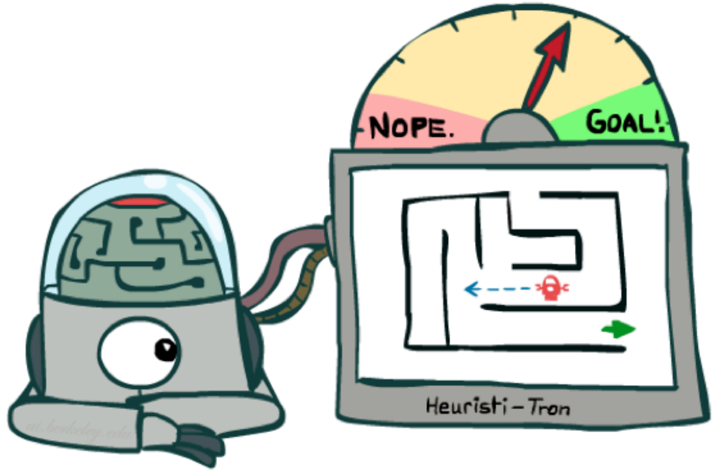
- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem



Search Heuristics

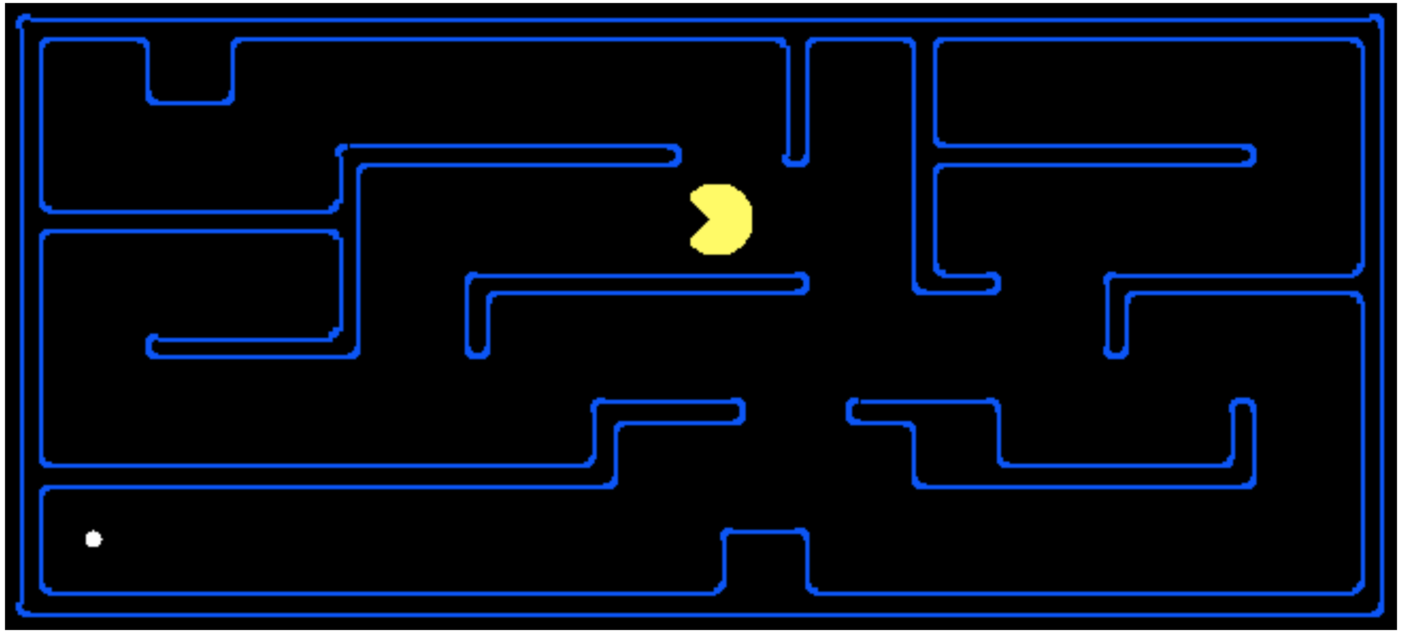
- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem

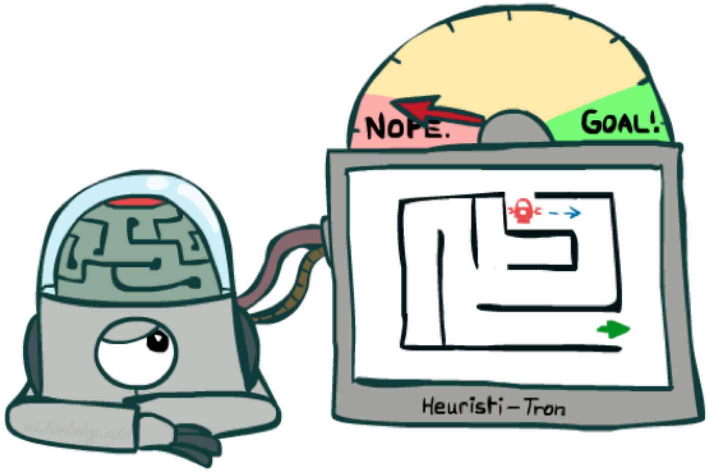


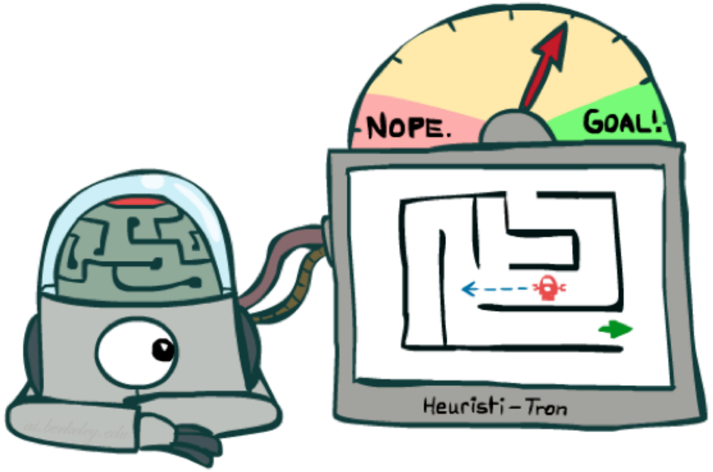


Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem

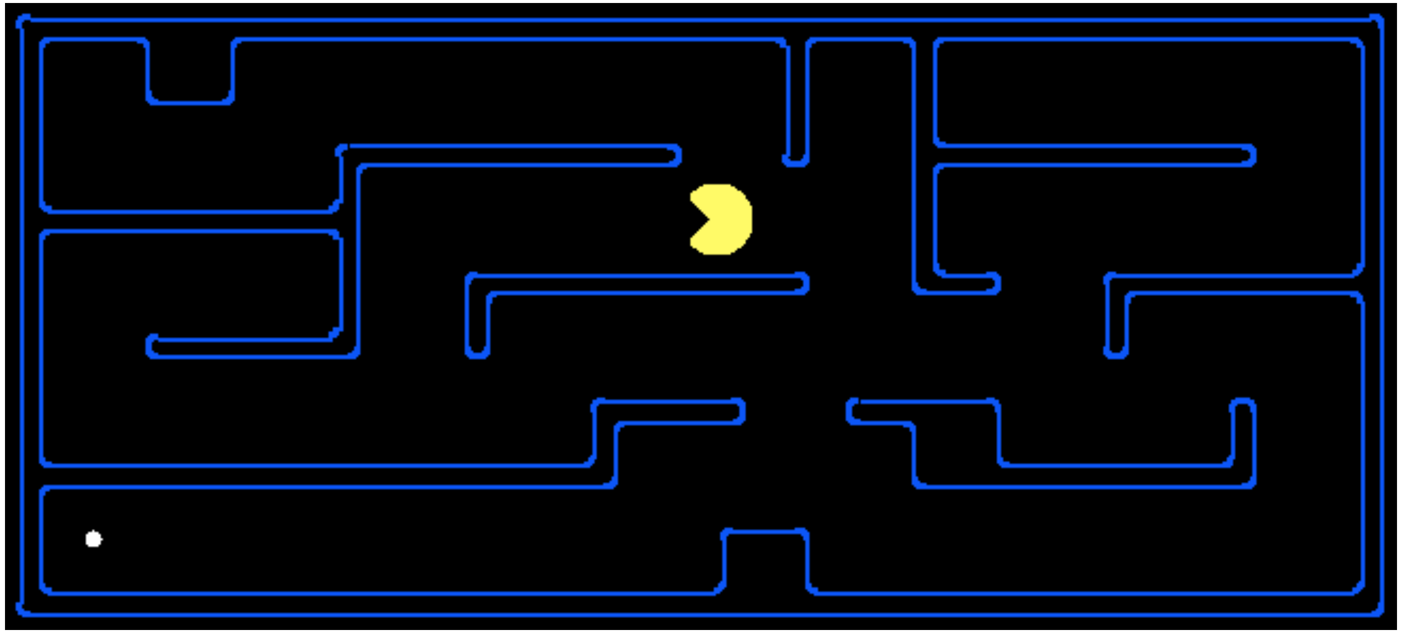


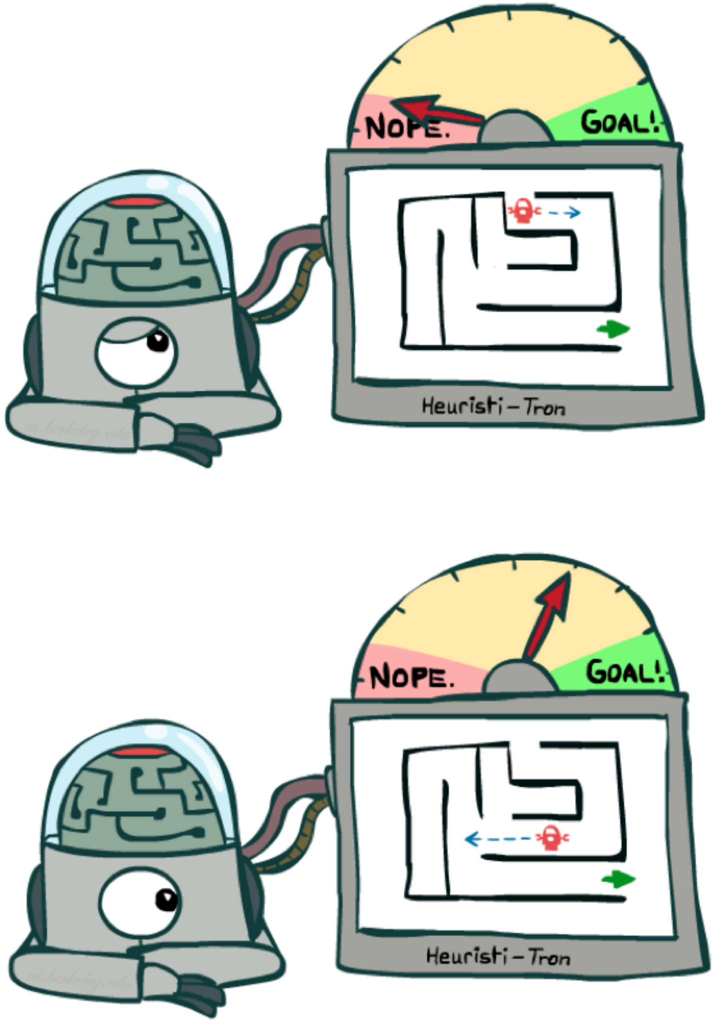




Search Heuristics

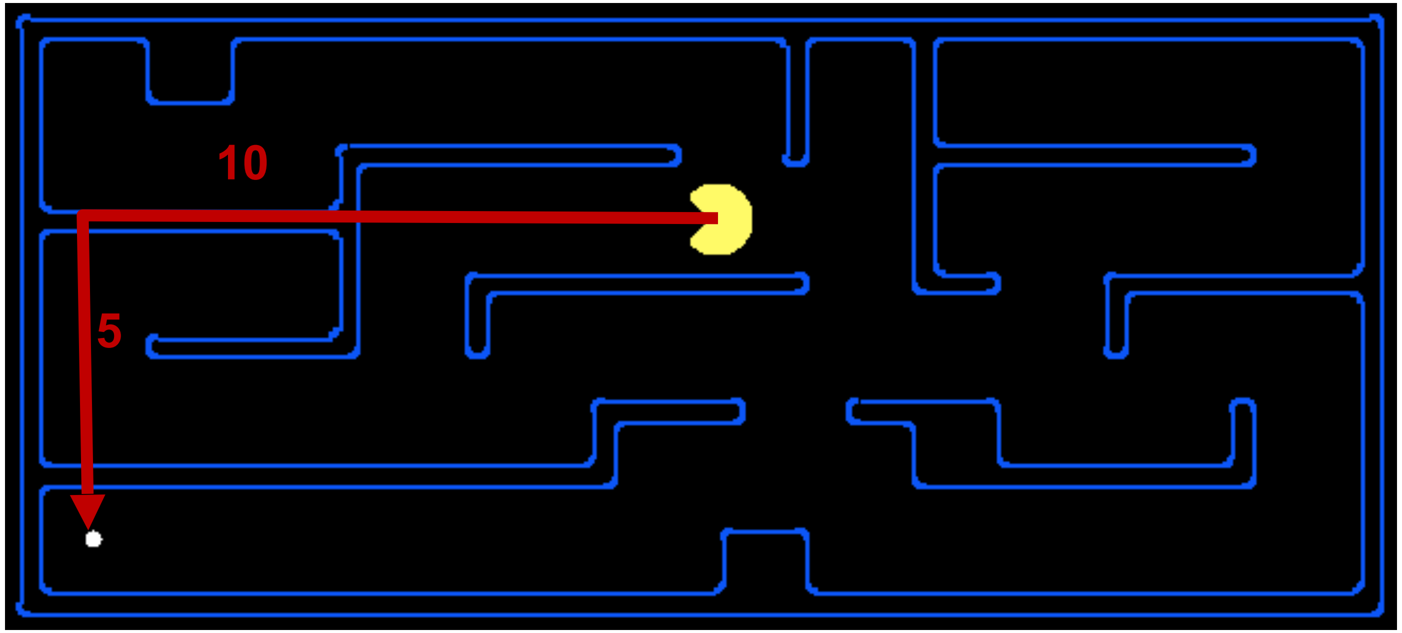
- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

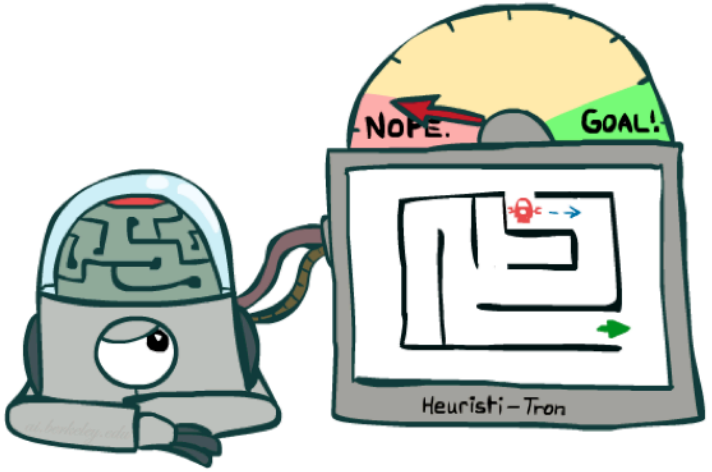


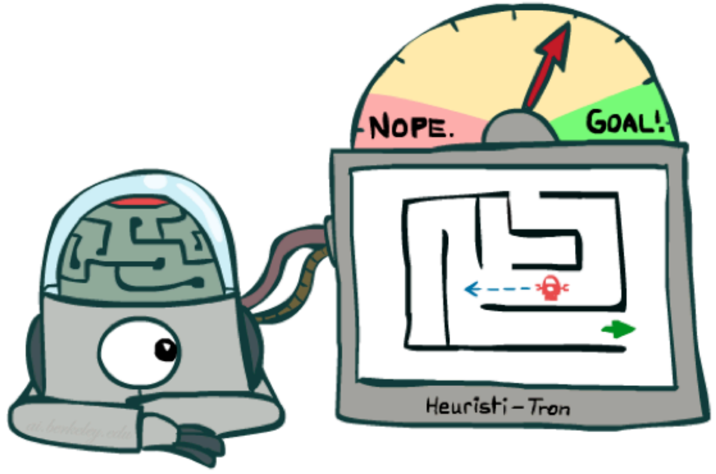


Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

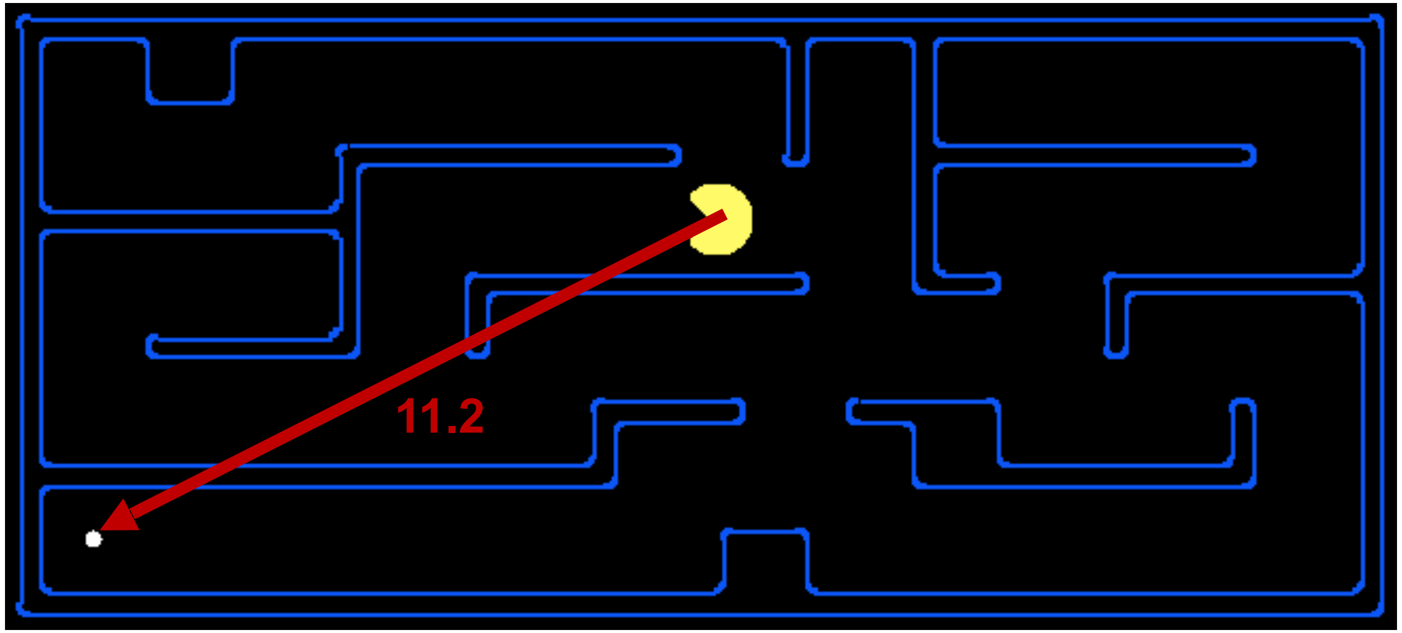


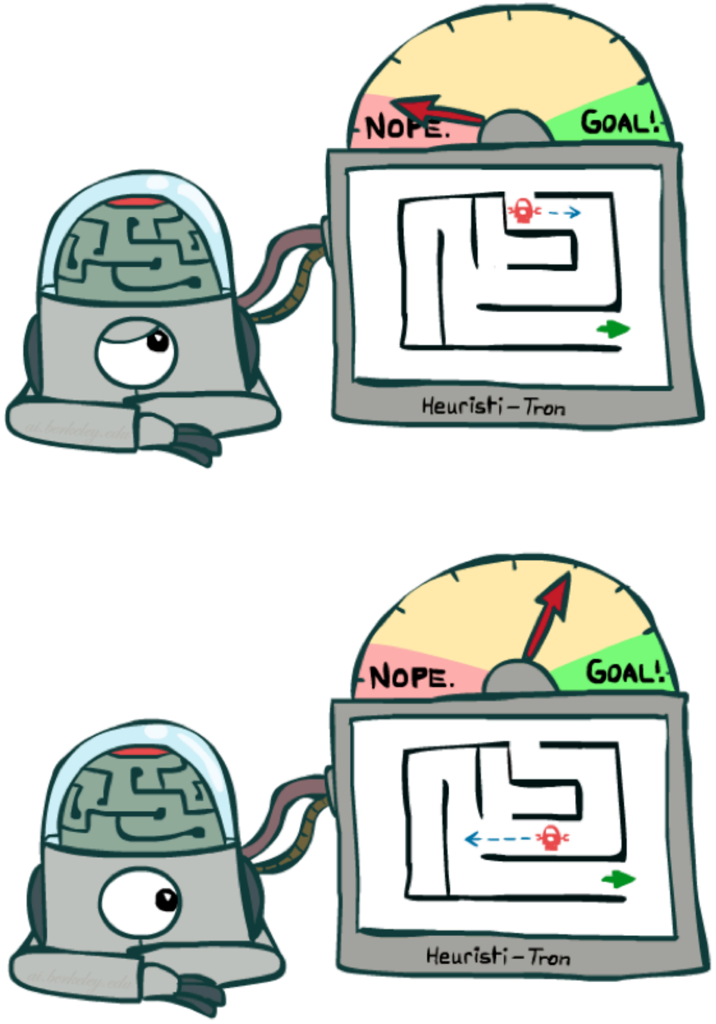




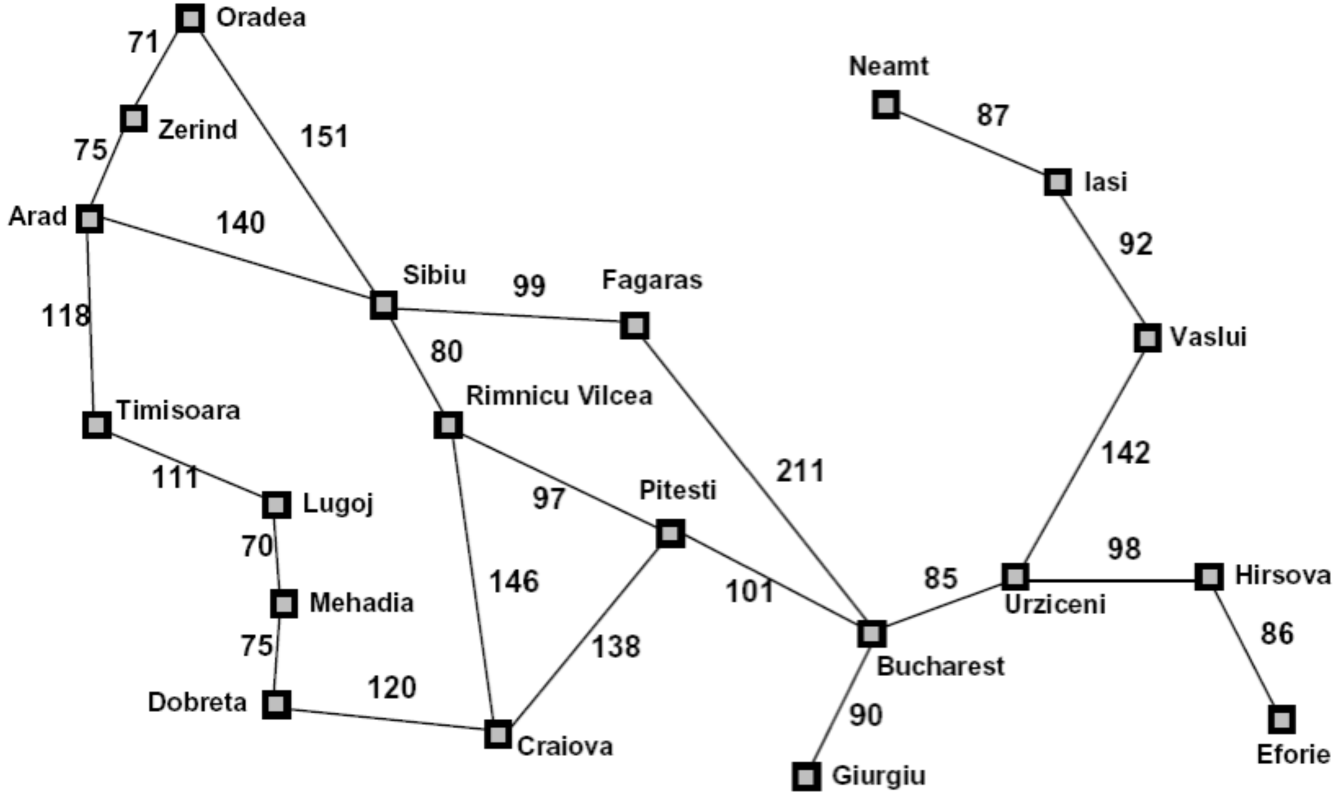
Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing





Example: Heuristic Function

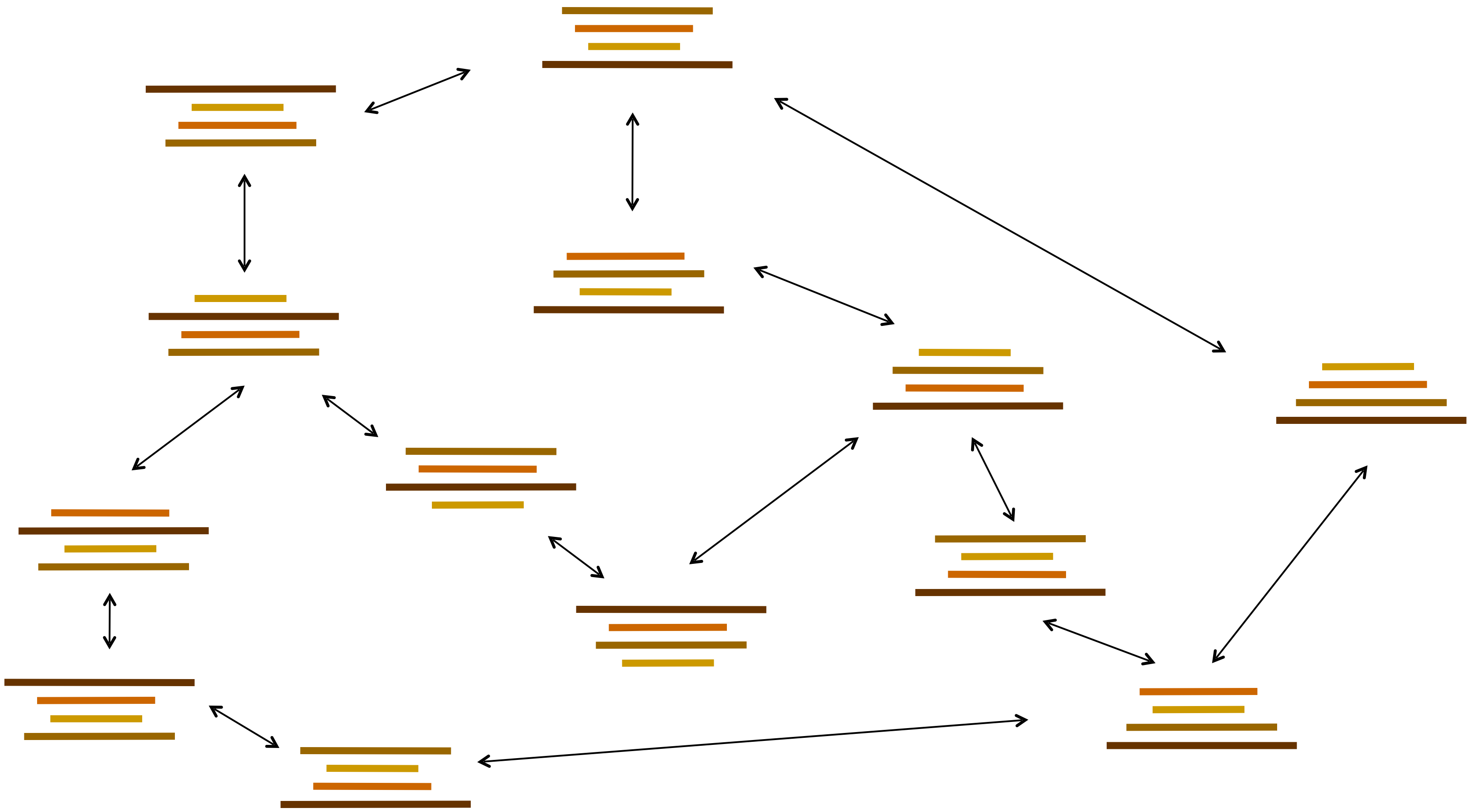


Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

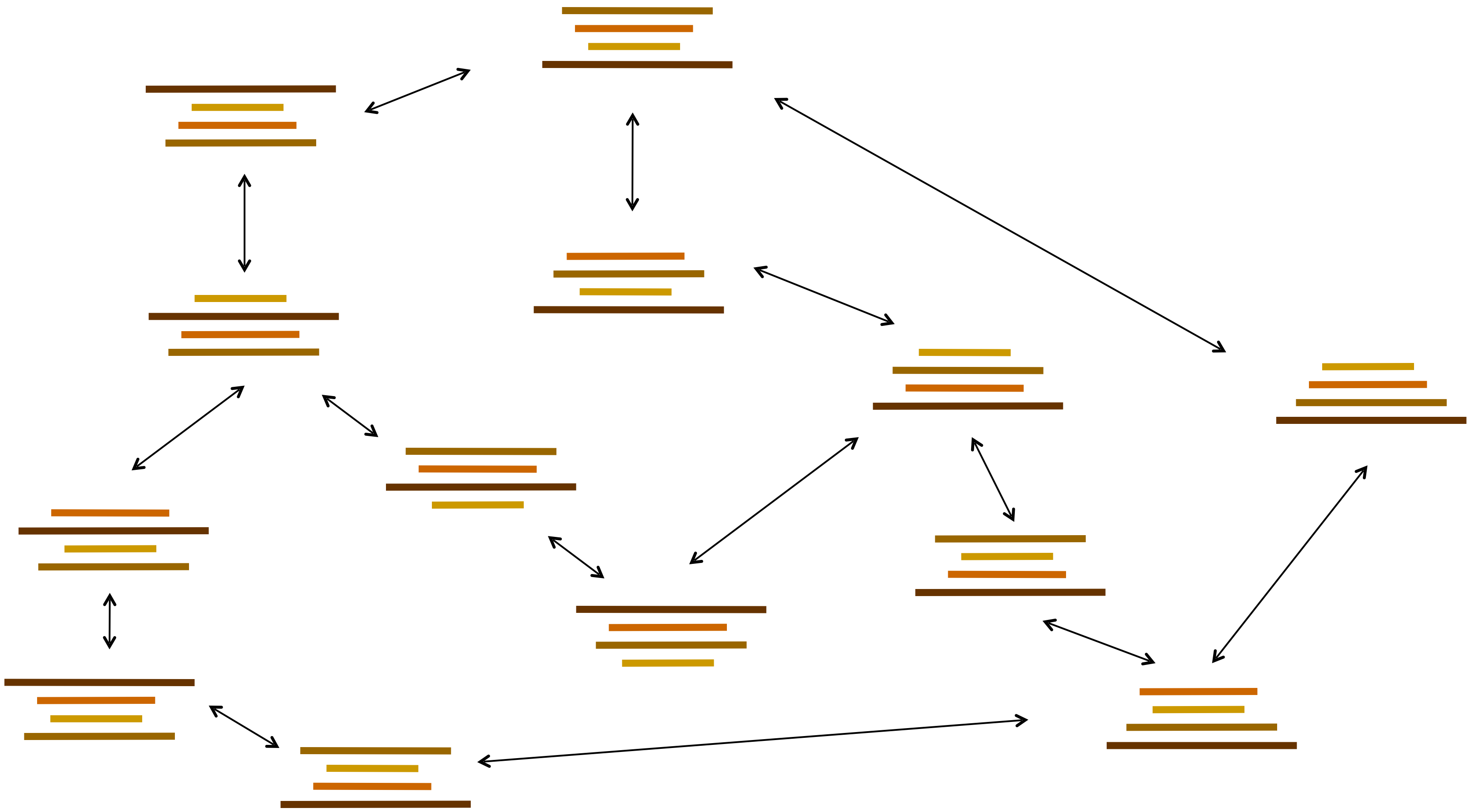
h(x)

Example: Heuristic Function



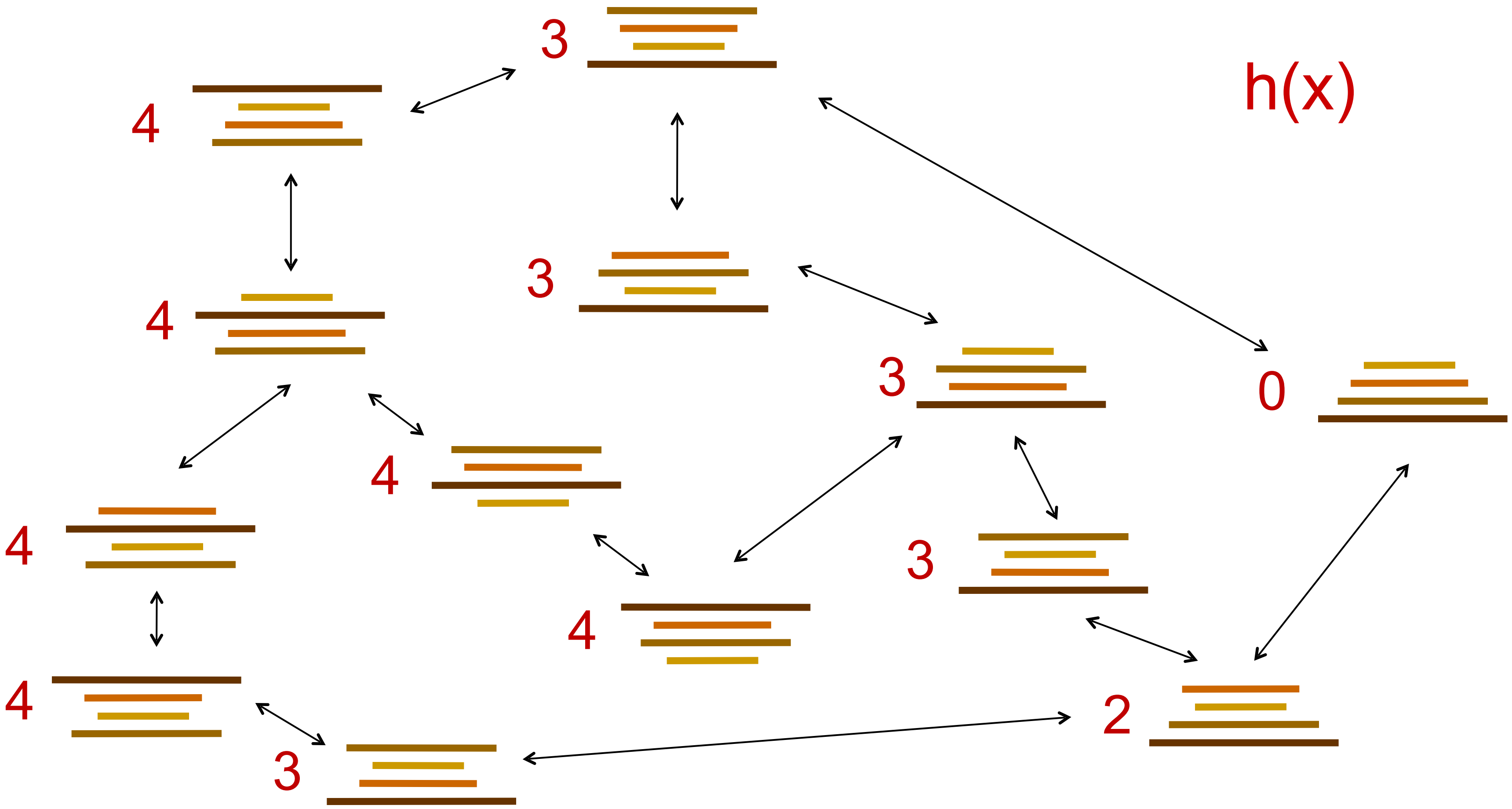
Example: Heuristic Function

Heuristic: (the size of) the largest pancake that is still out of place



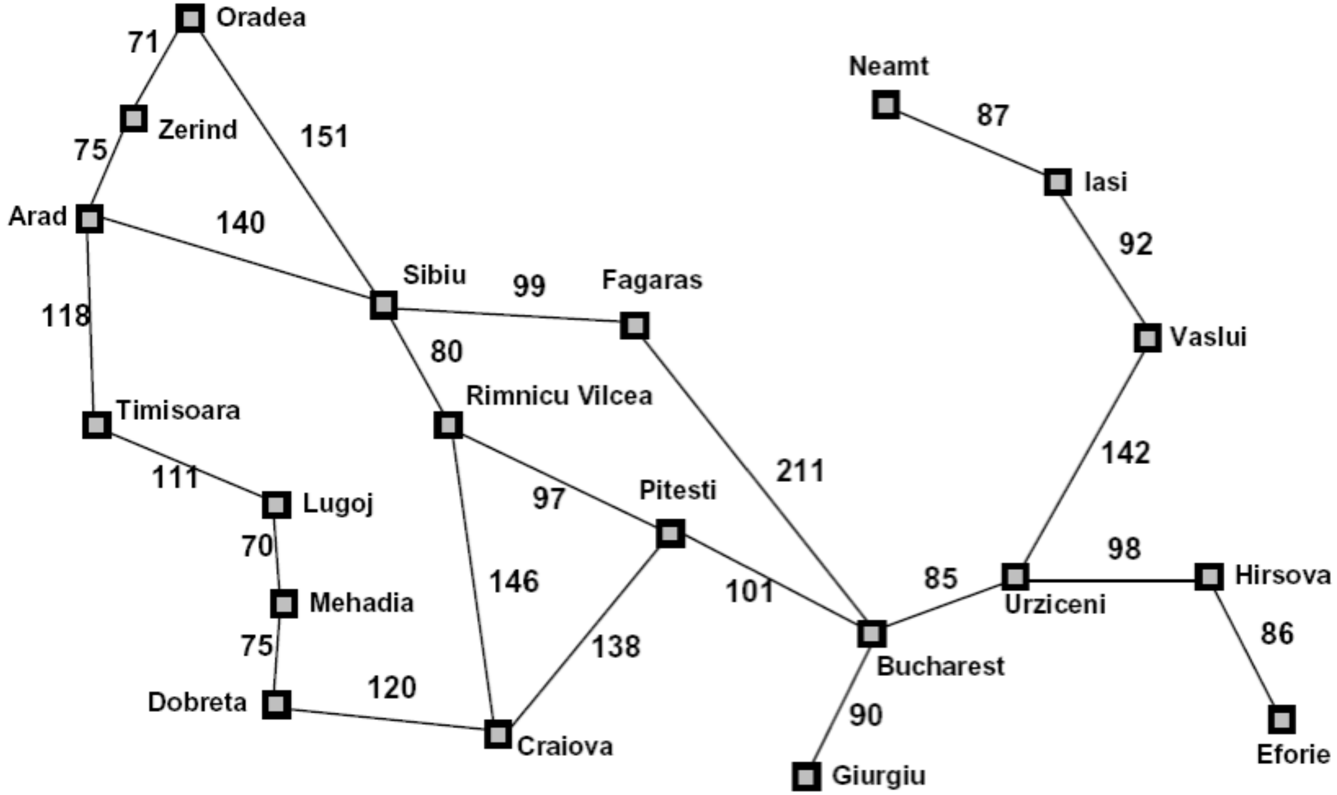
Example: Heuristic Function

Heuristic: (the size of) the largest pancake that is still out of place



Greedy Search

Example: Heuristic Function

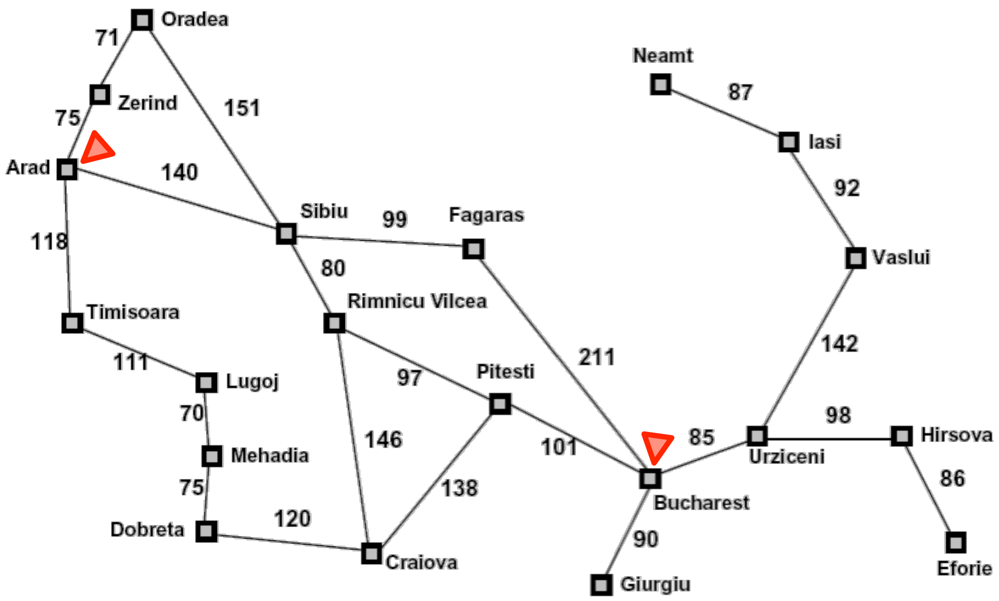


Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

$h(x)$

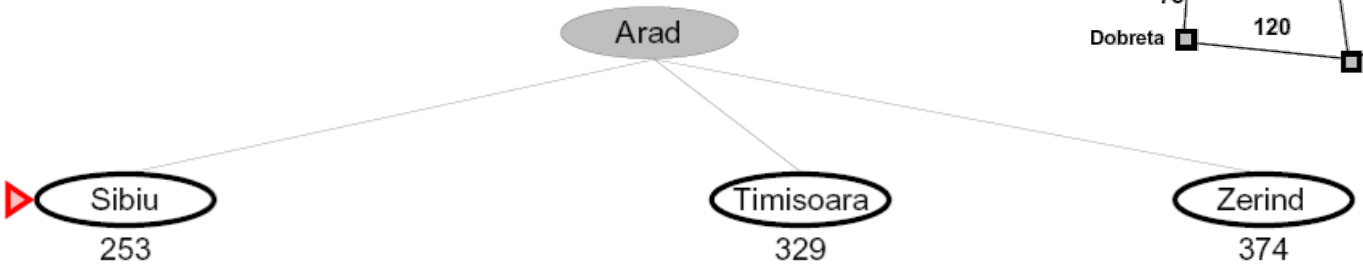
Greedy Search

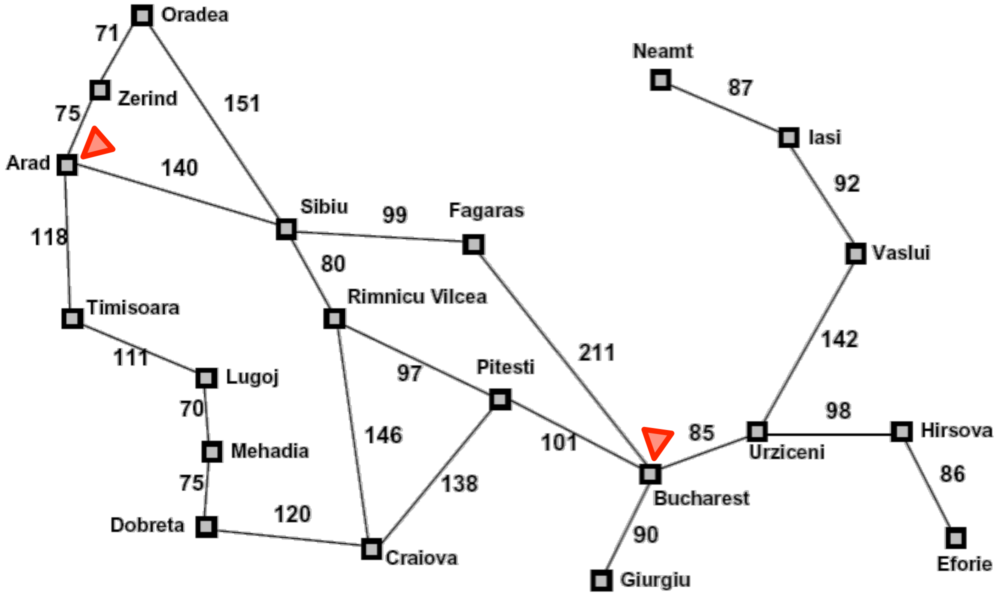


Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search

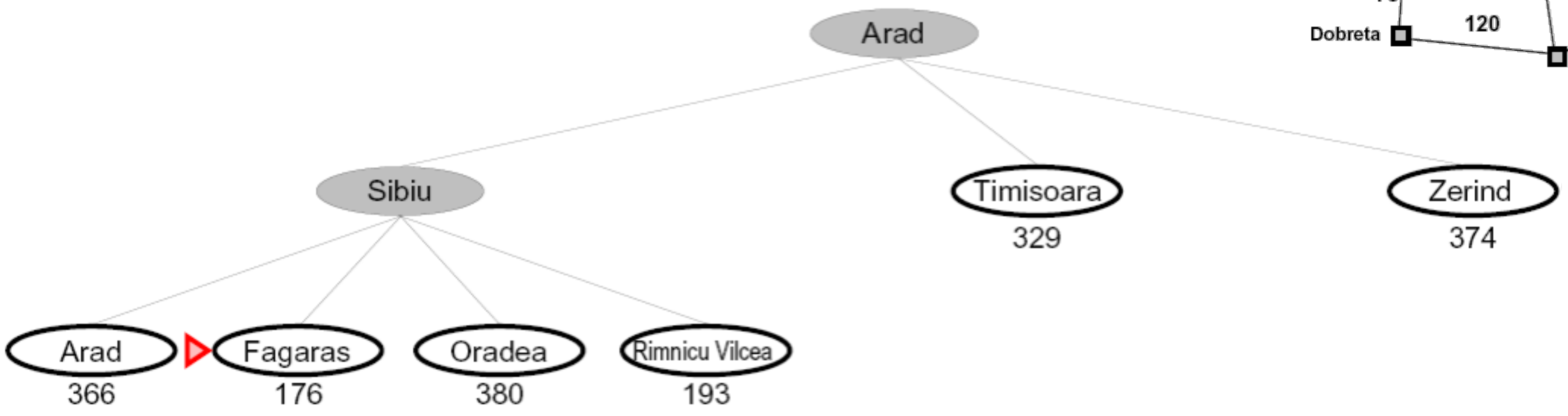


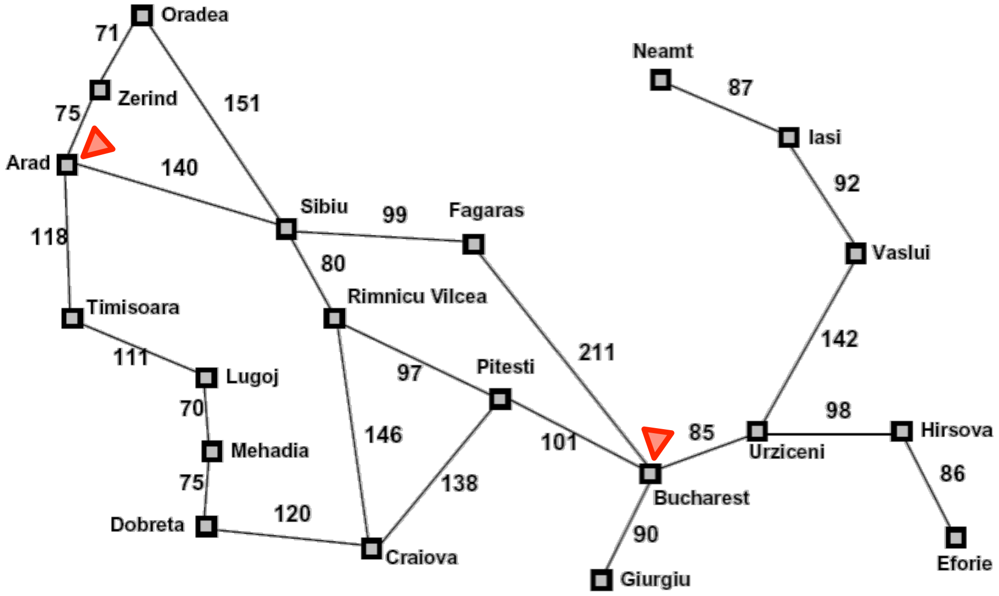


Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search

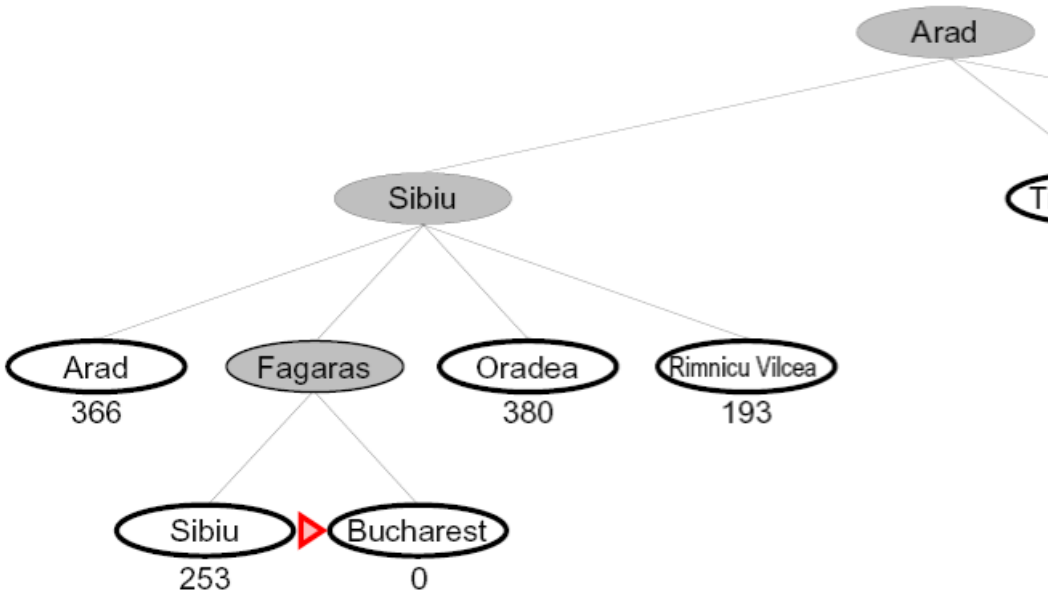


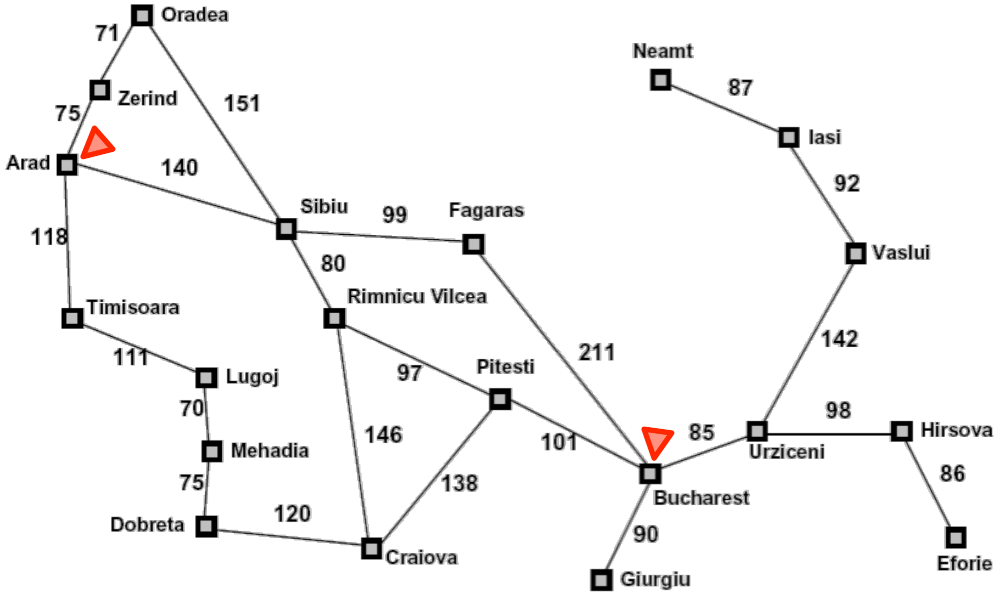


Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search



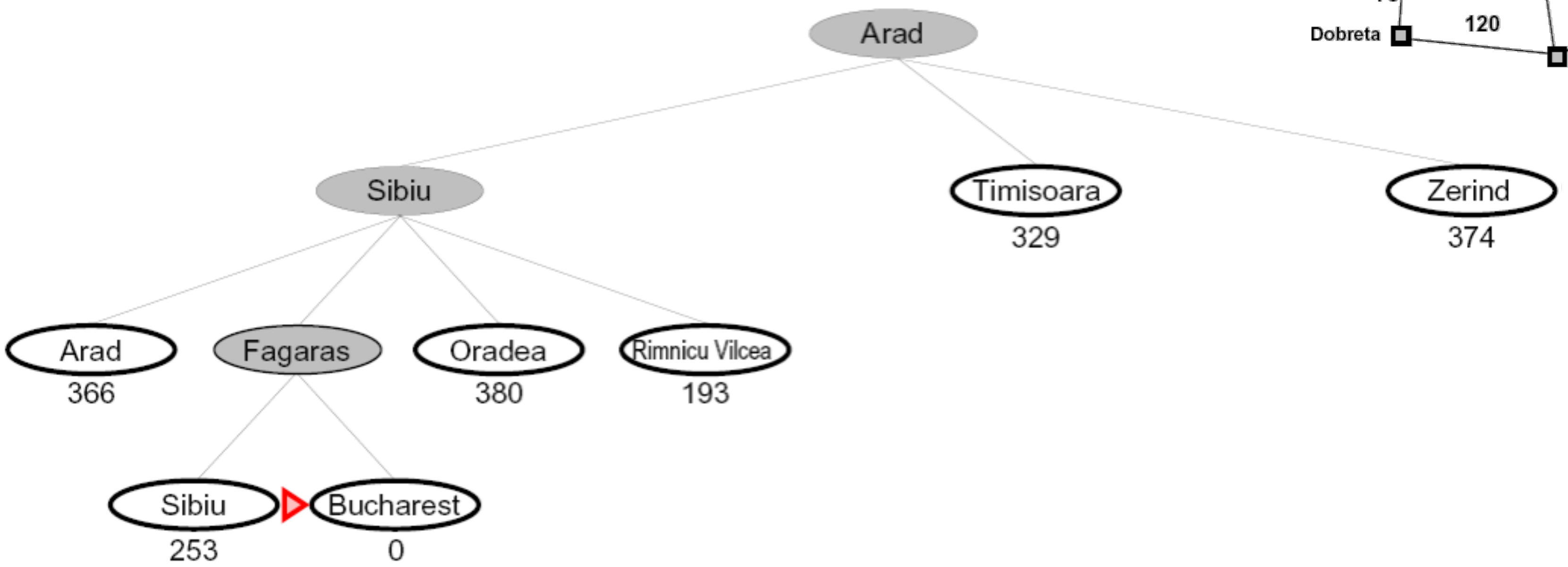


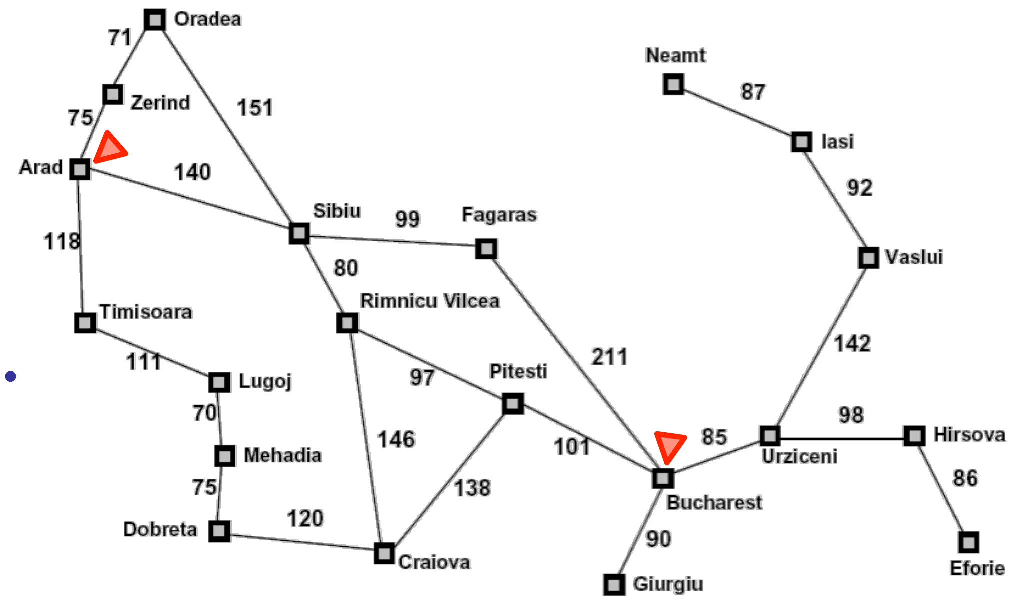
Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search

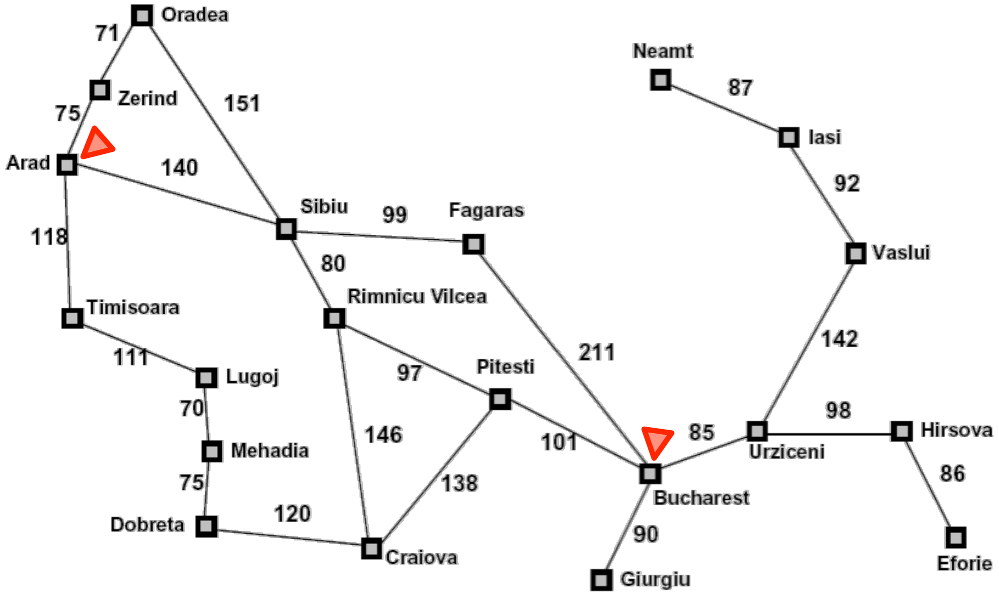
- Expand the node that seems closest…



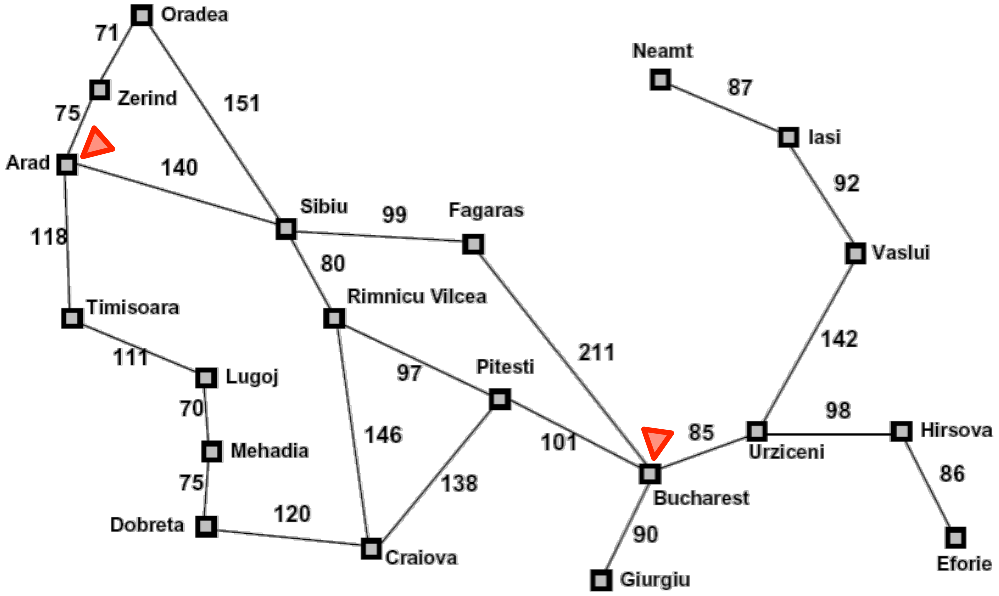


Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

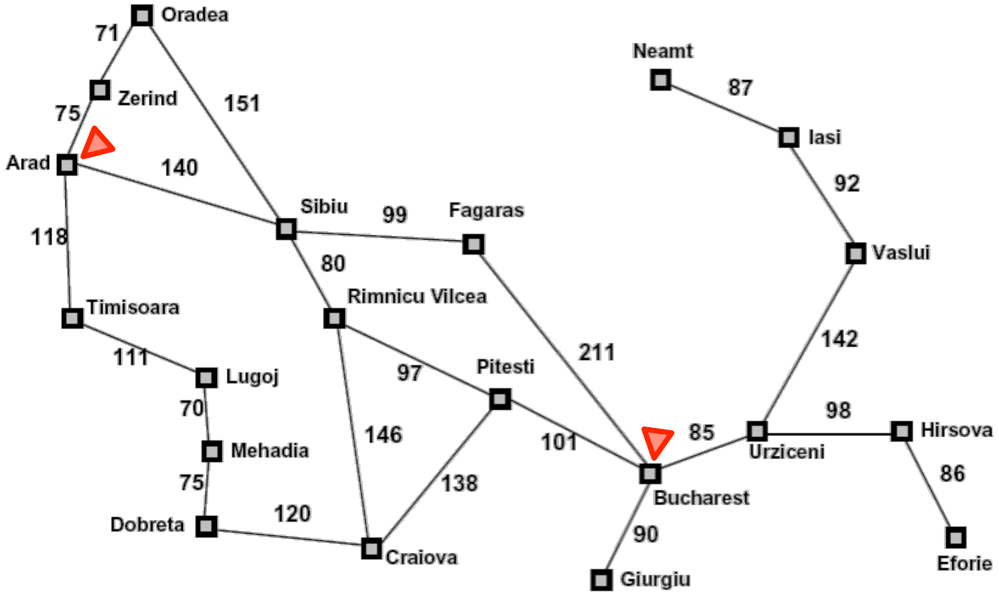
Greedy Search

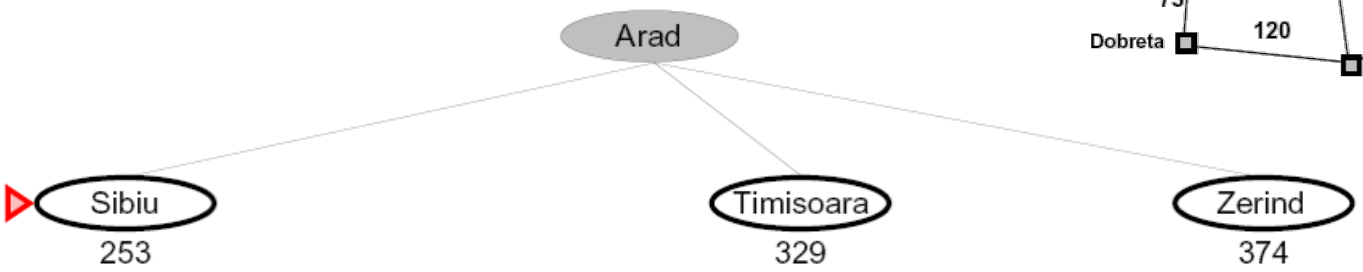


Greedy Search

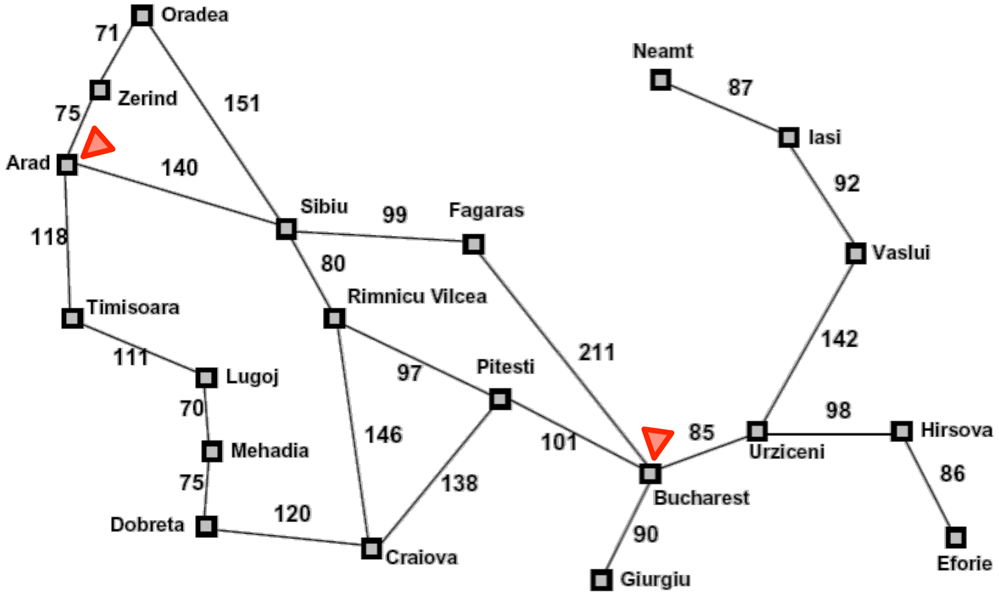


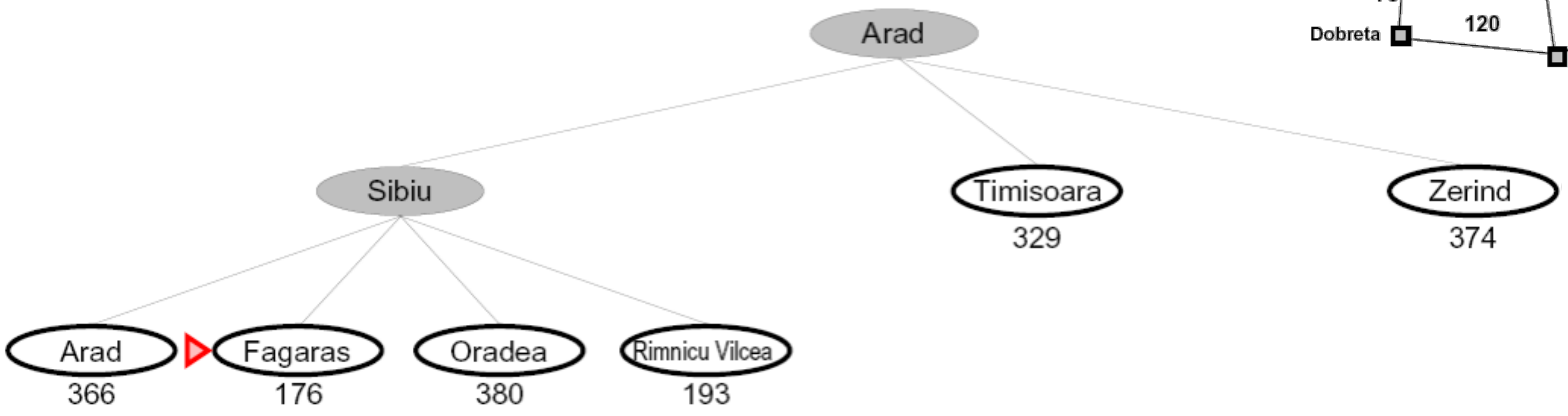
Greedy Search



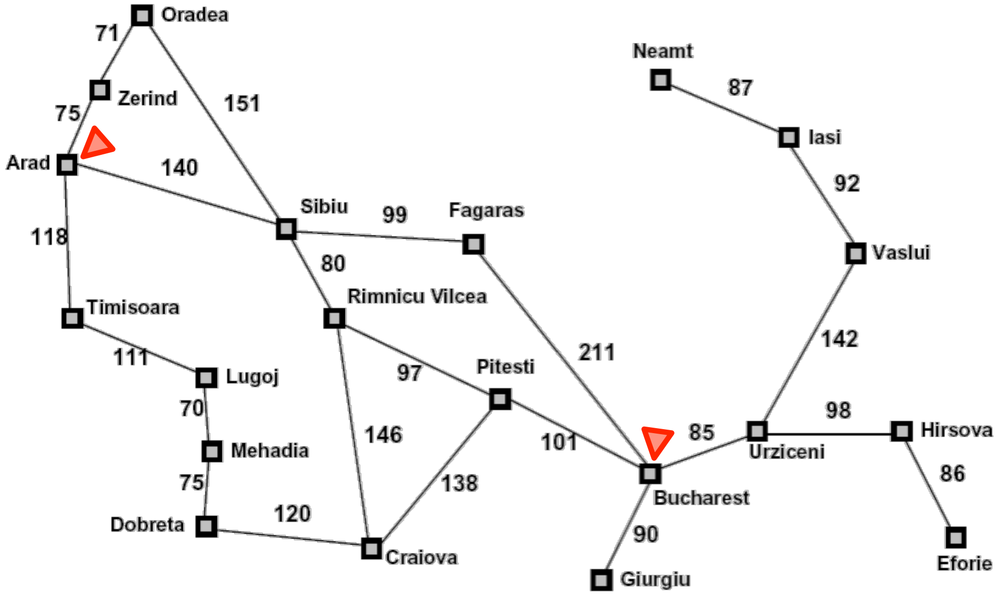


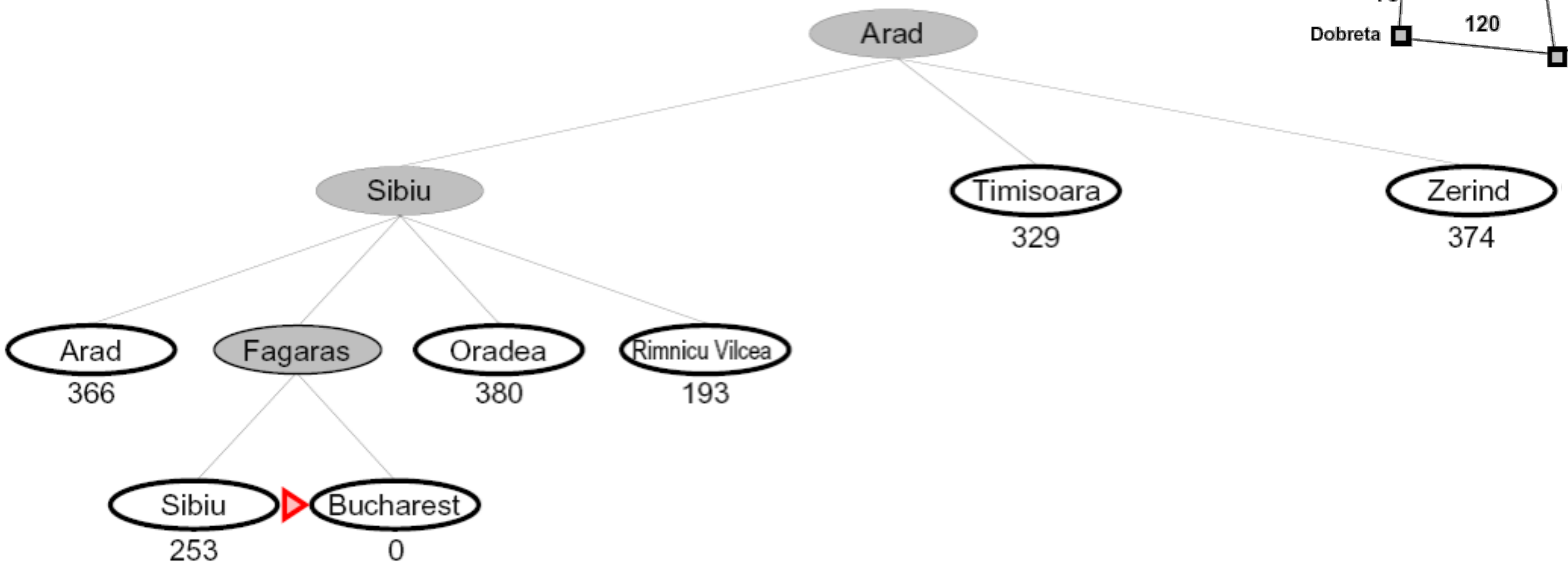
Greedy Search



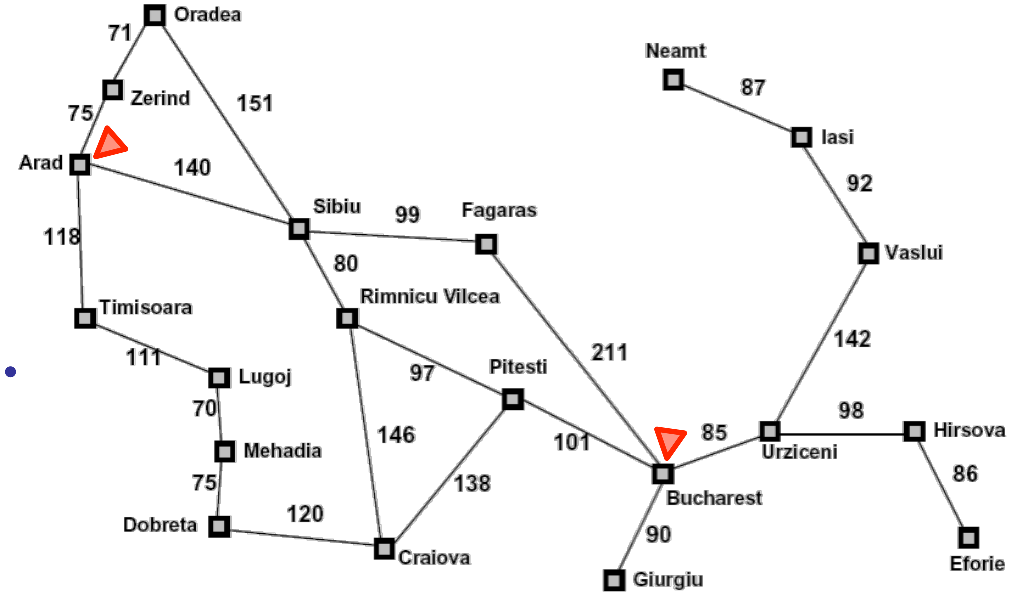


Greedy Search

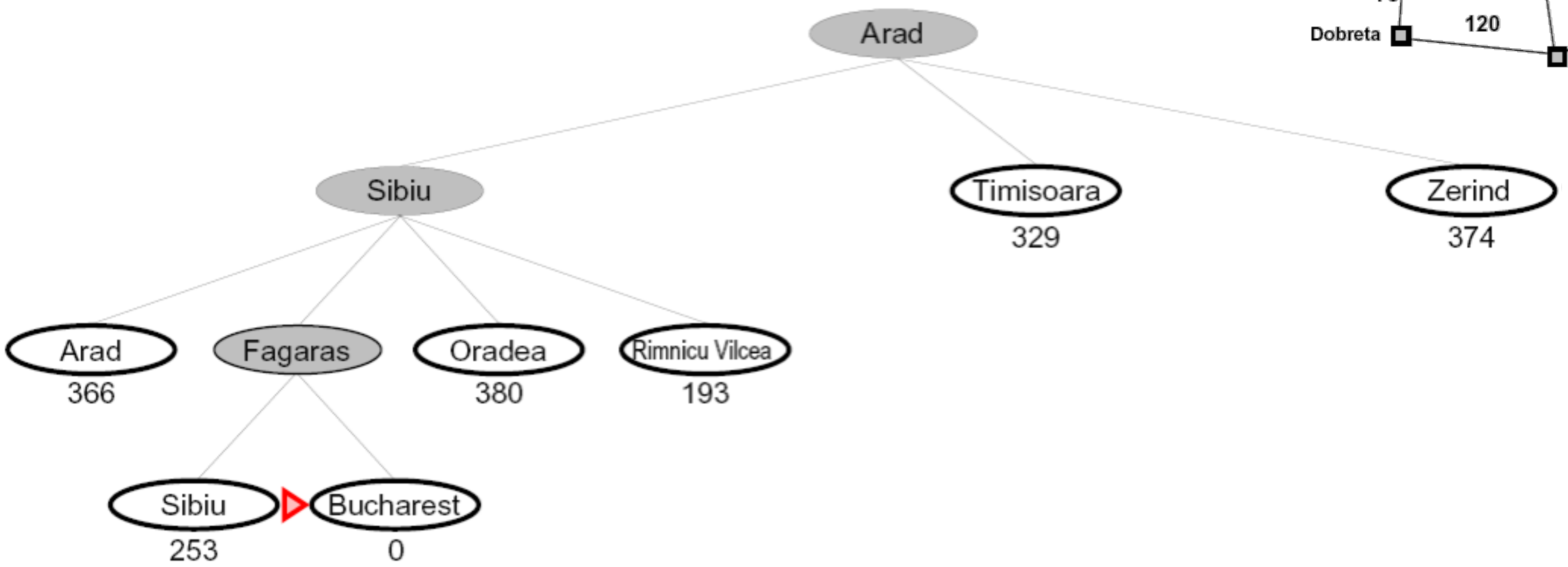




Greedy Search



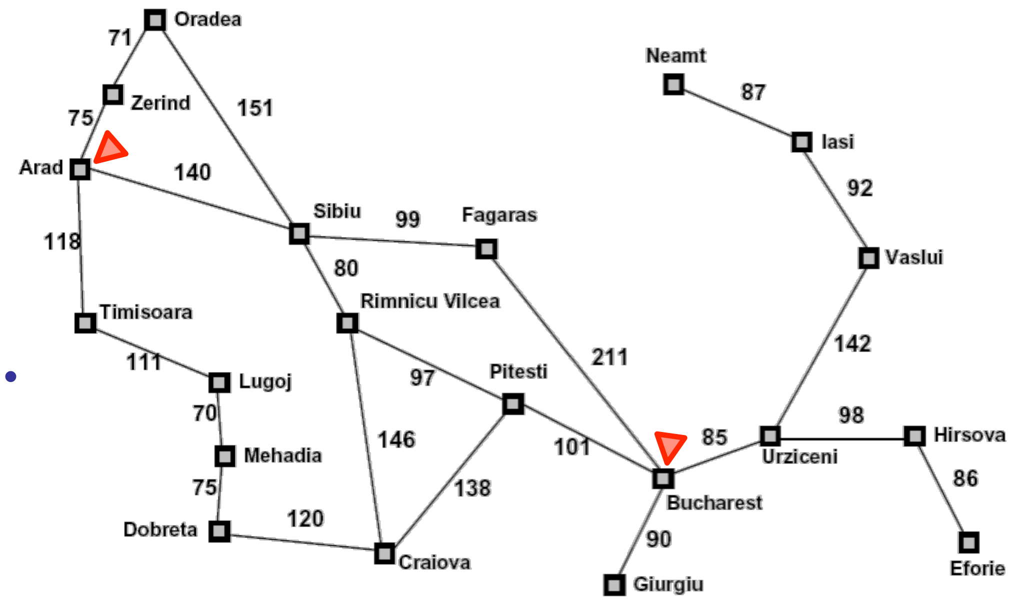
- Expand the node that seems closest…

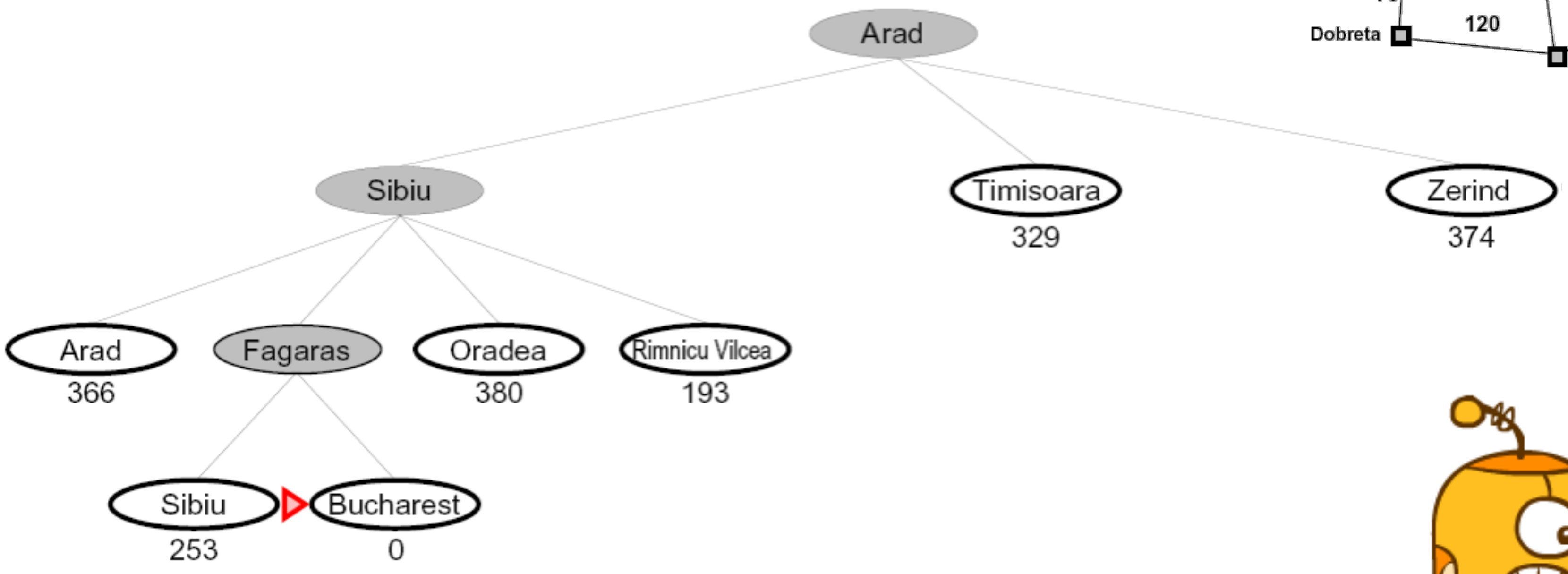


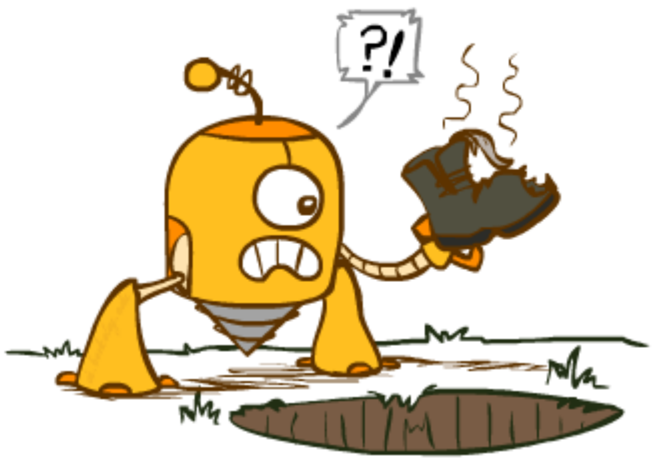
- What can go wrong?

Greedy Search

- Expand the node that seems closest…



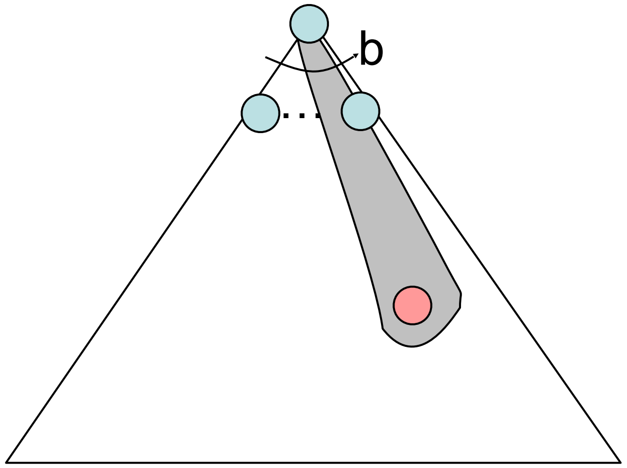




- What can go wrong?

Greedy Search

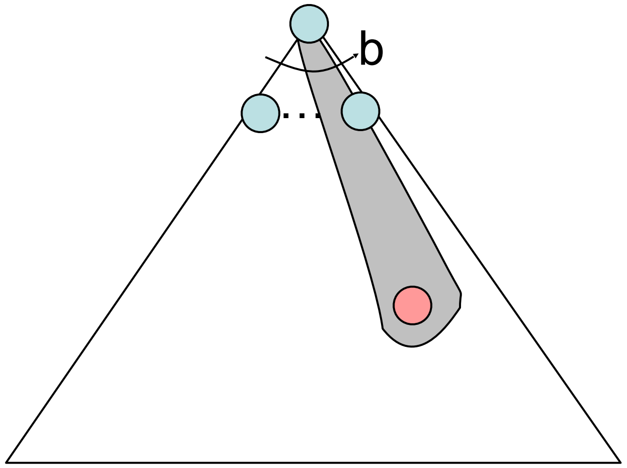
- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

Greedy Search

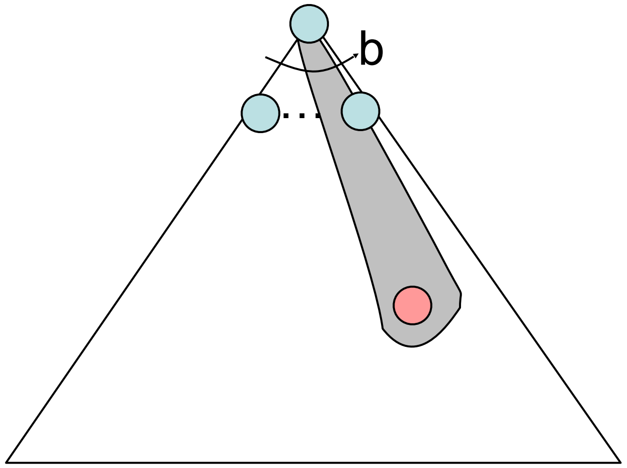
- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



- A common case:
  - Best-first takes you straight to the (wrong) goal

[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

Greedy Search



- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state

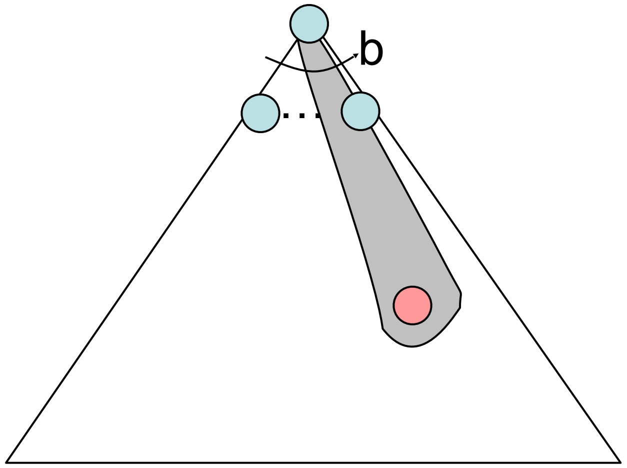
- A common case:
  - Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS

[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

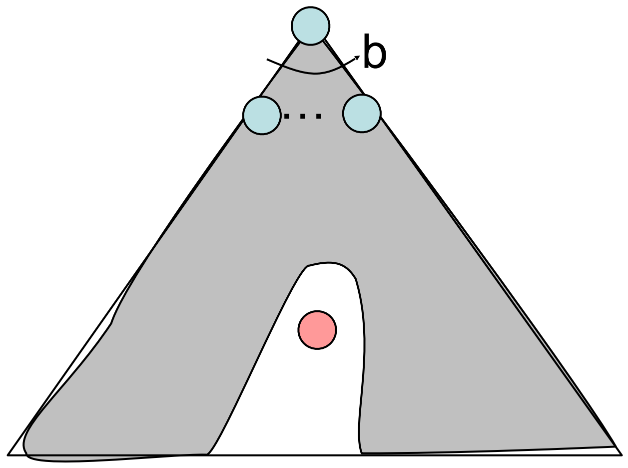
Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



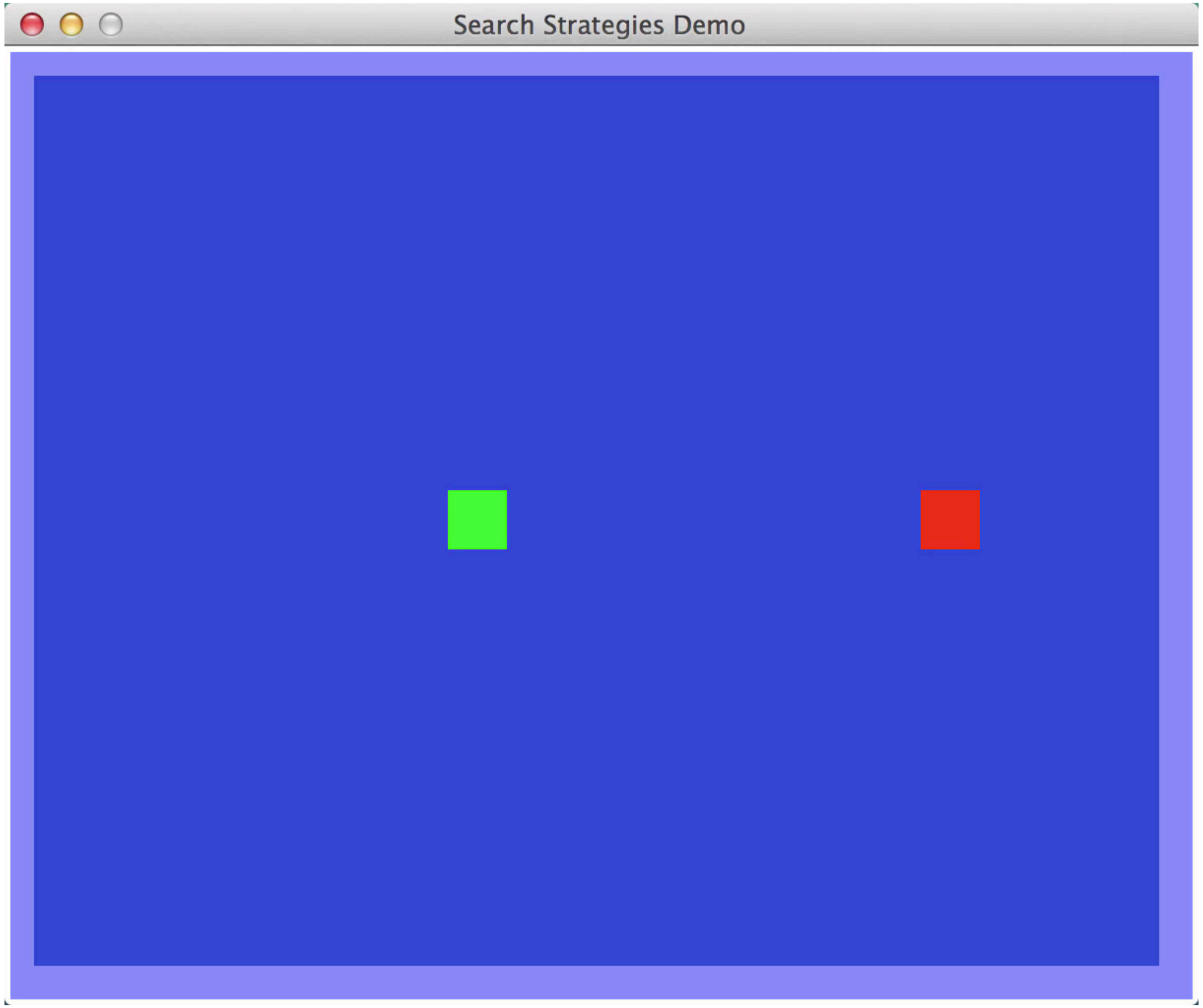
- A common case:
  - Best-first takes you straight to the (wrong) goal

- Worst-case: like a badly-guided DFS

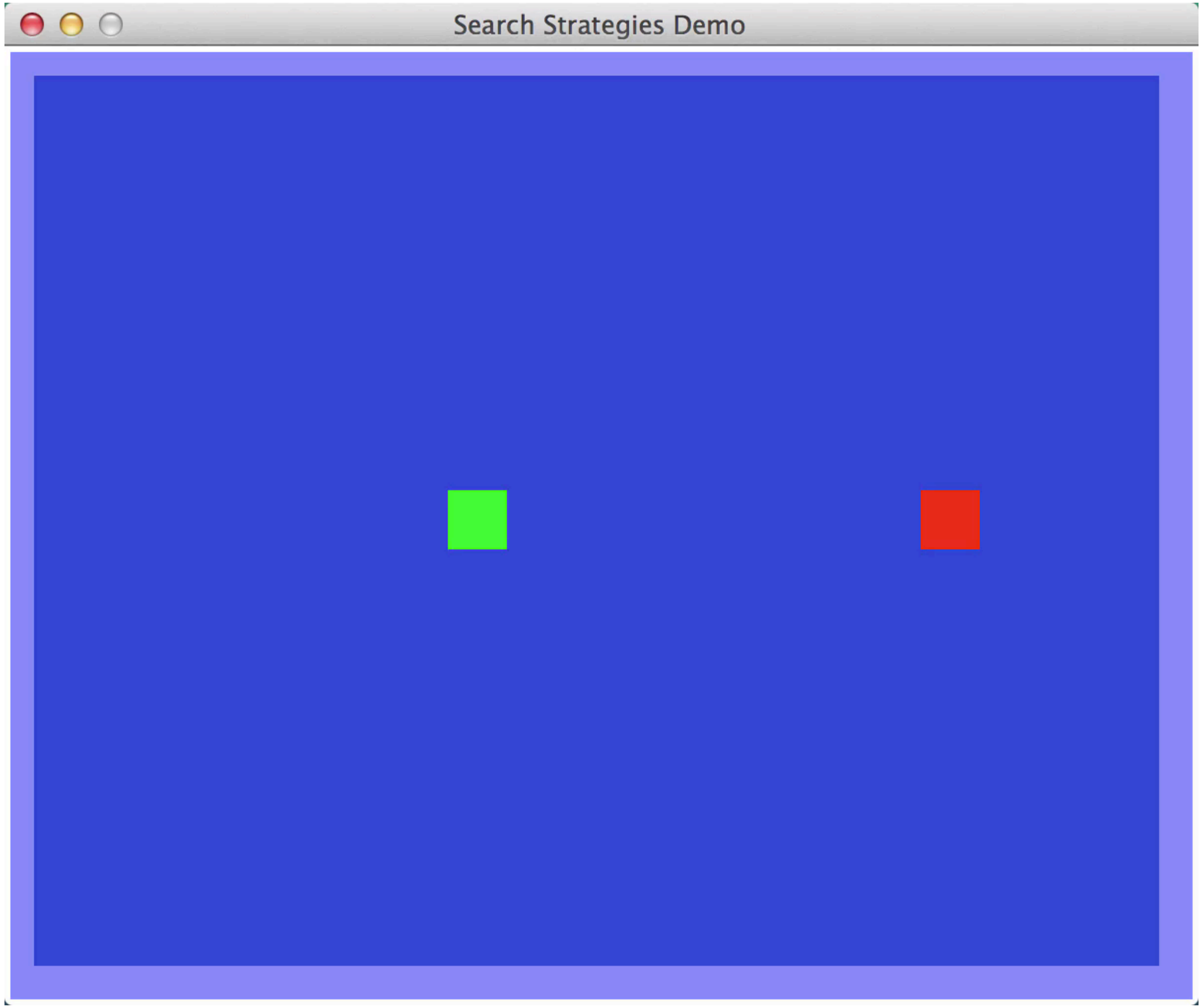


[Demo: contours greedy empty (L3D1)]
[Demo: contours greedy pacman small maze (L3D4)]

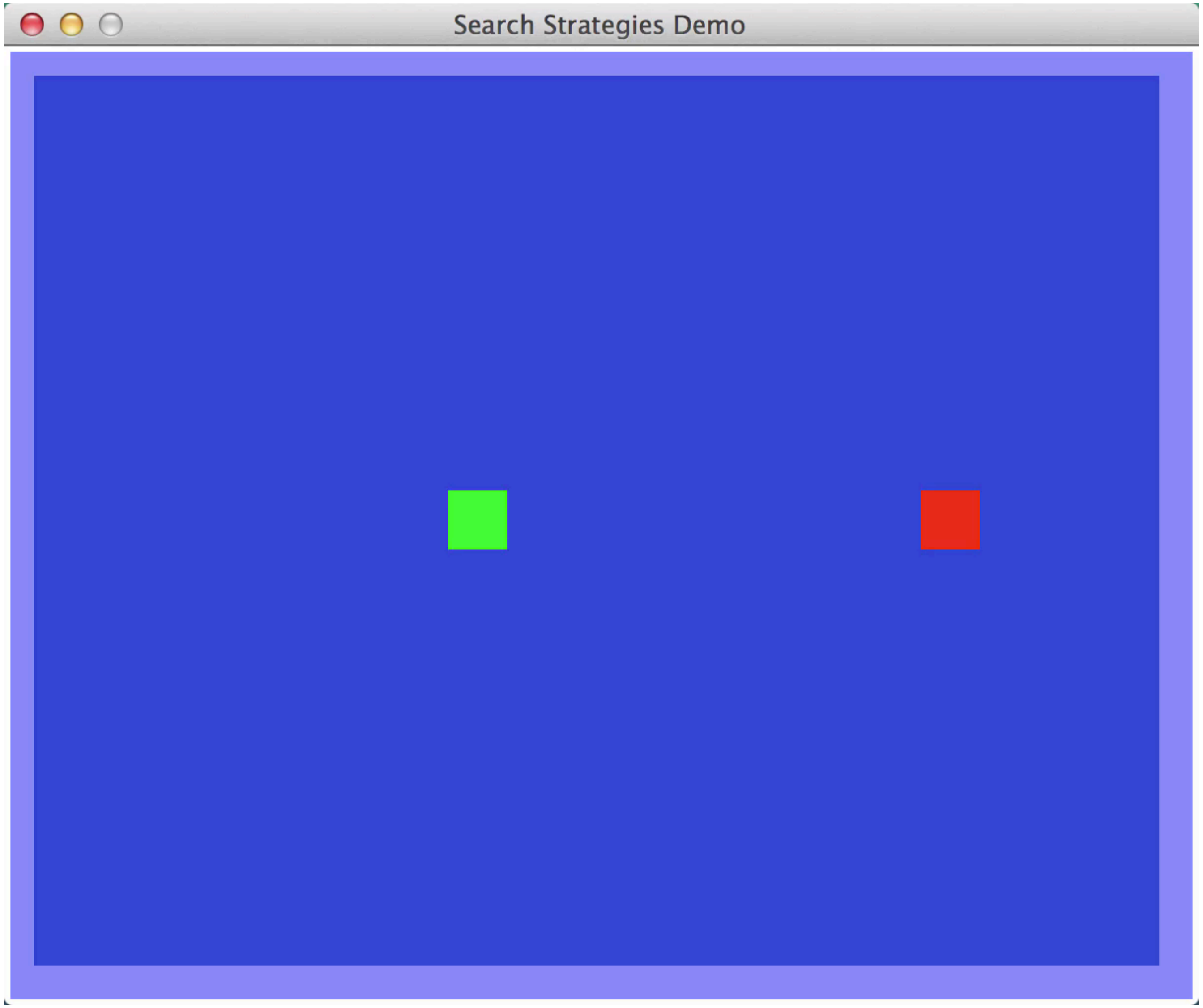
Video of Demo Contours Greedy (Empty)



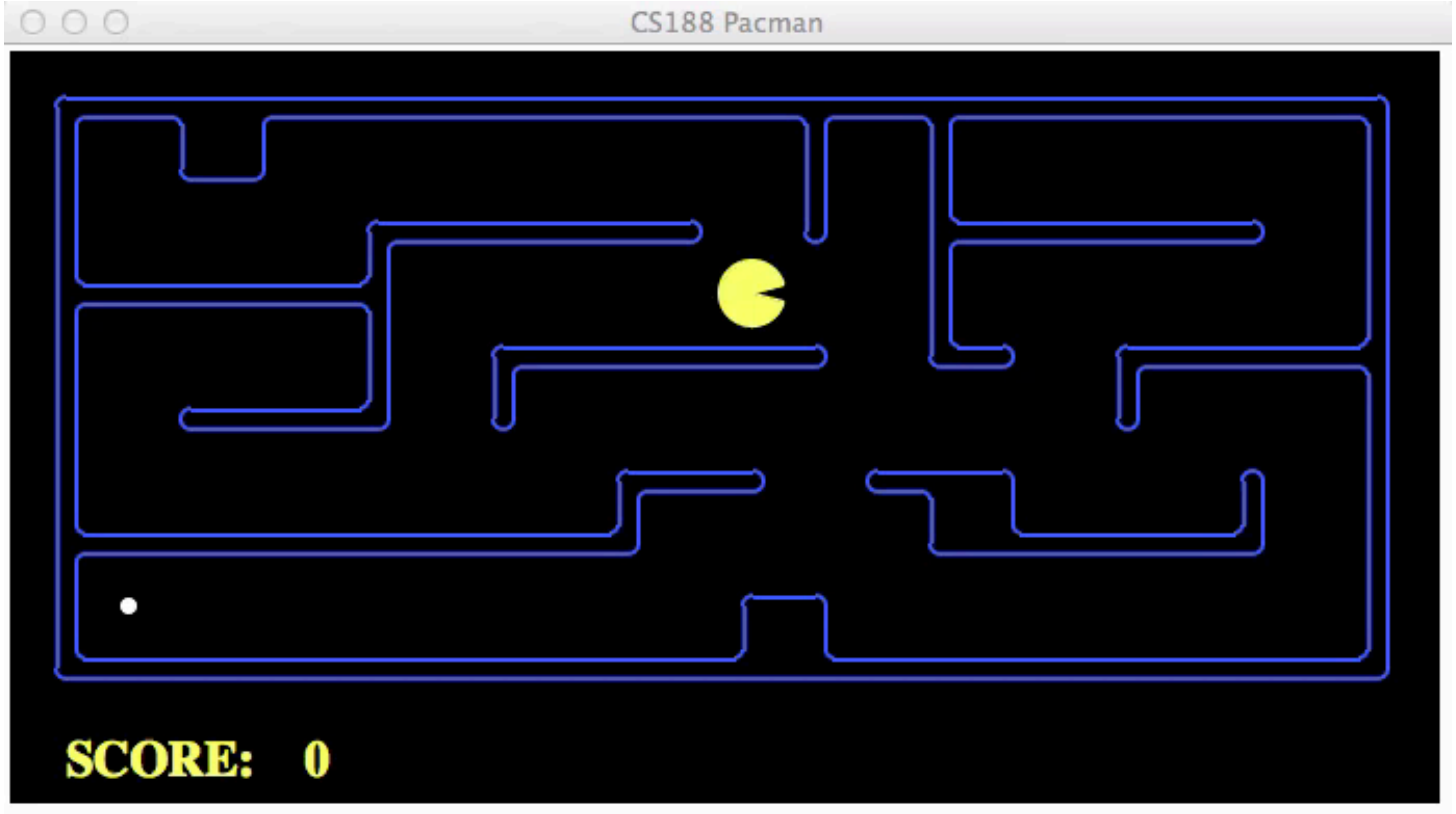
Video of Demo Contours Greedy (Empty)



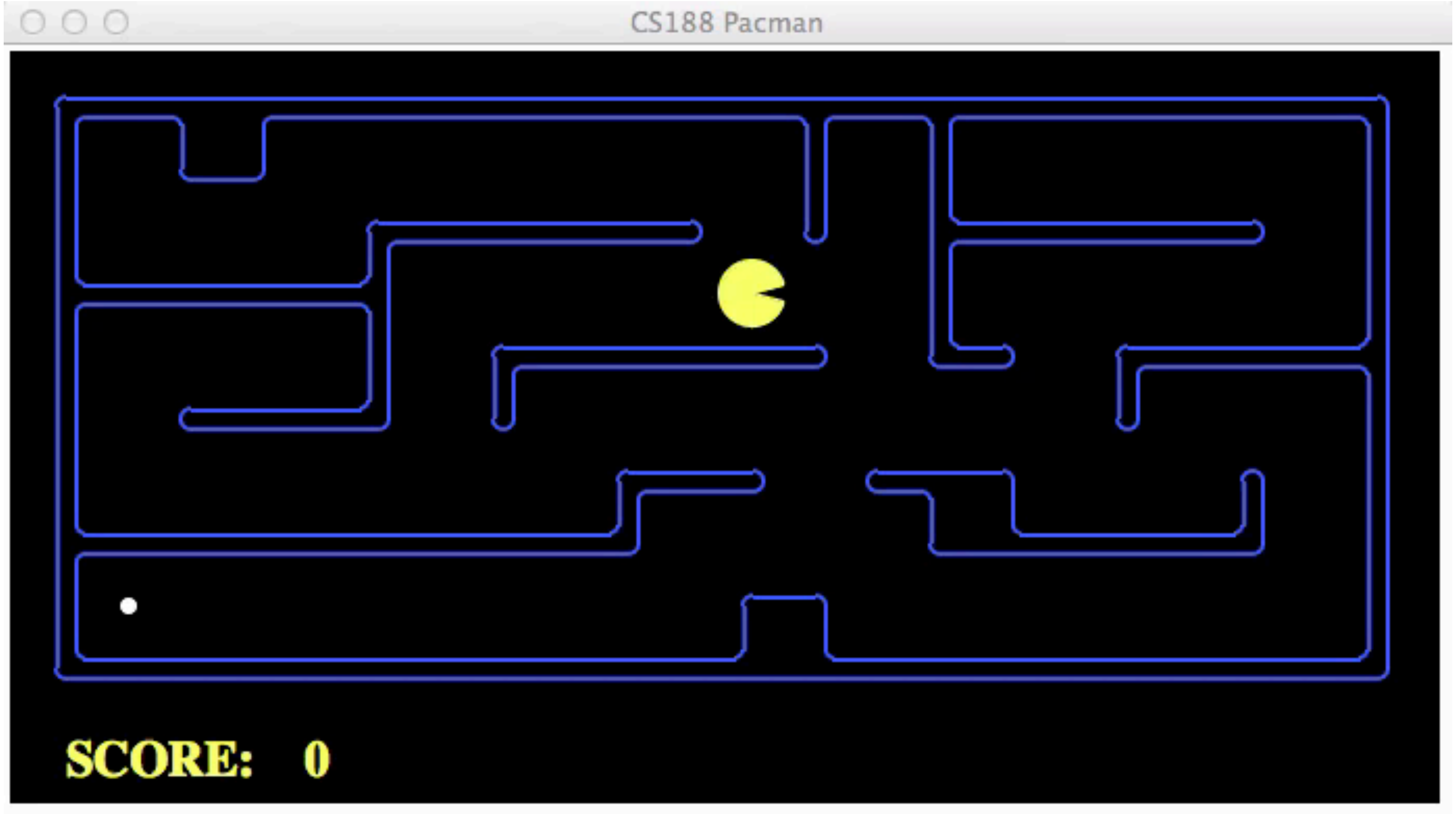
Video of Demo Contours Greedy (Empty)



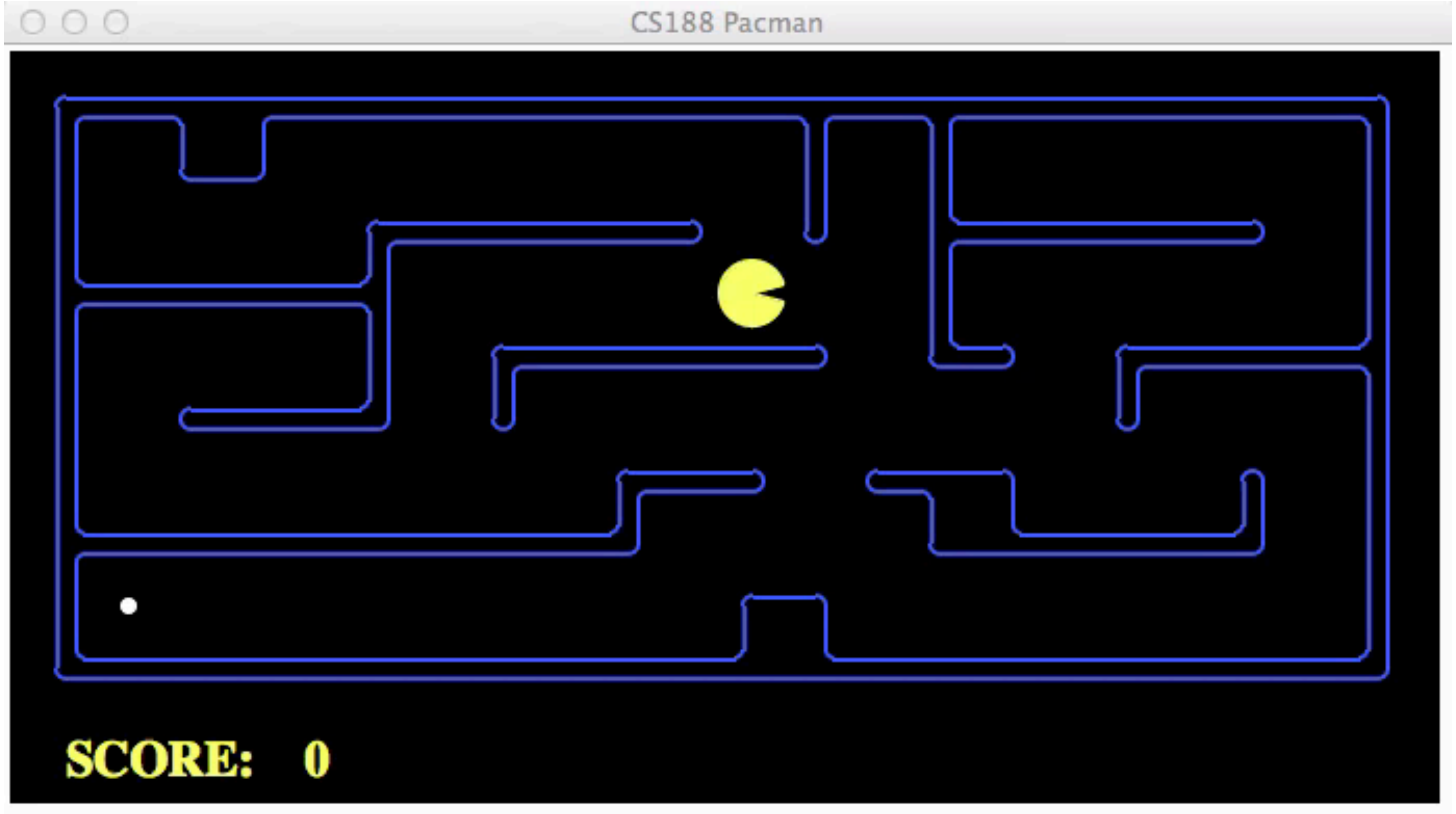
Video of Demo Contours Greedy (Pacman Small Maze)



Video of Demo Contours Greedy (Pacman Small Maze)



Video of Demo Contours Greedy (Pacman Small Maze)



A* Search

A* Search

A* Search

A* Search

UCS

A* Search

UCS

Greedy

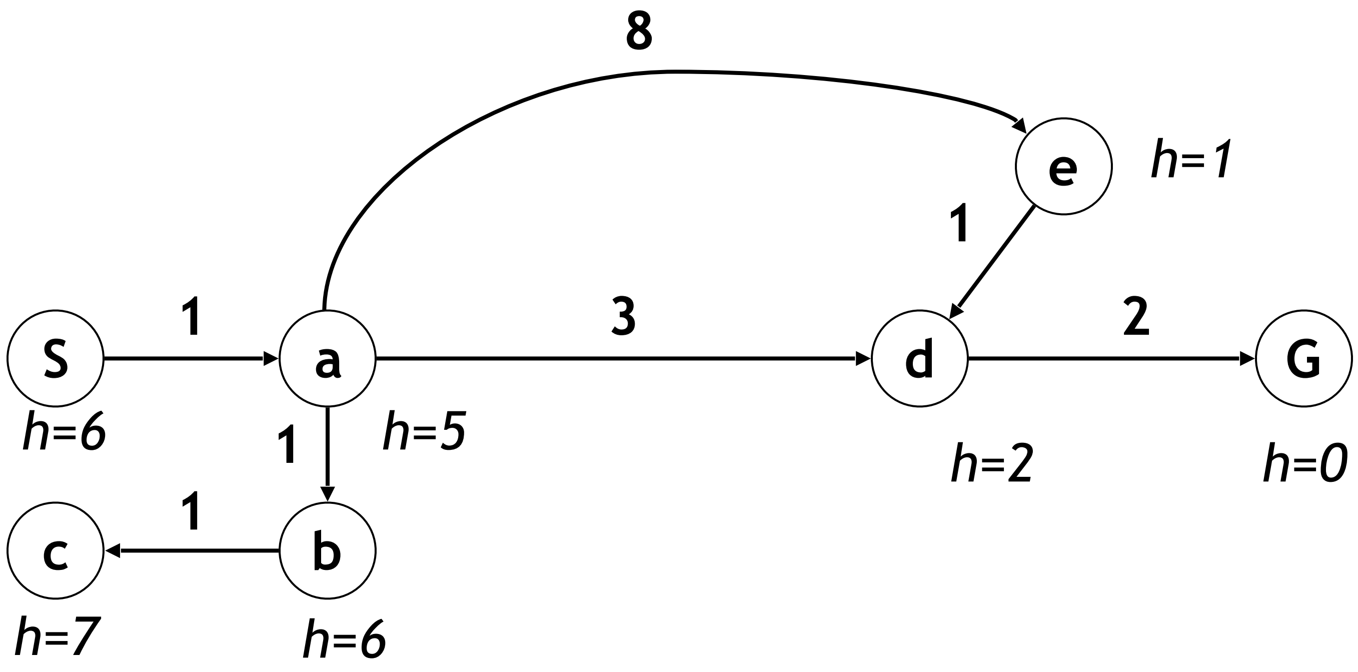
A* Search

UCS

Greedy

A*

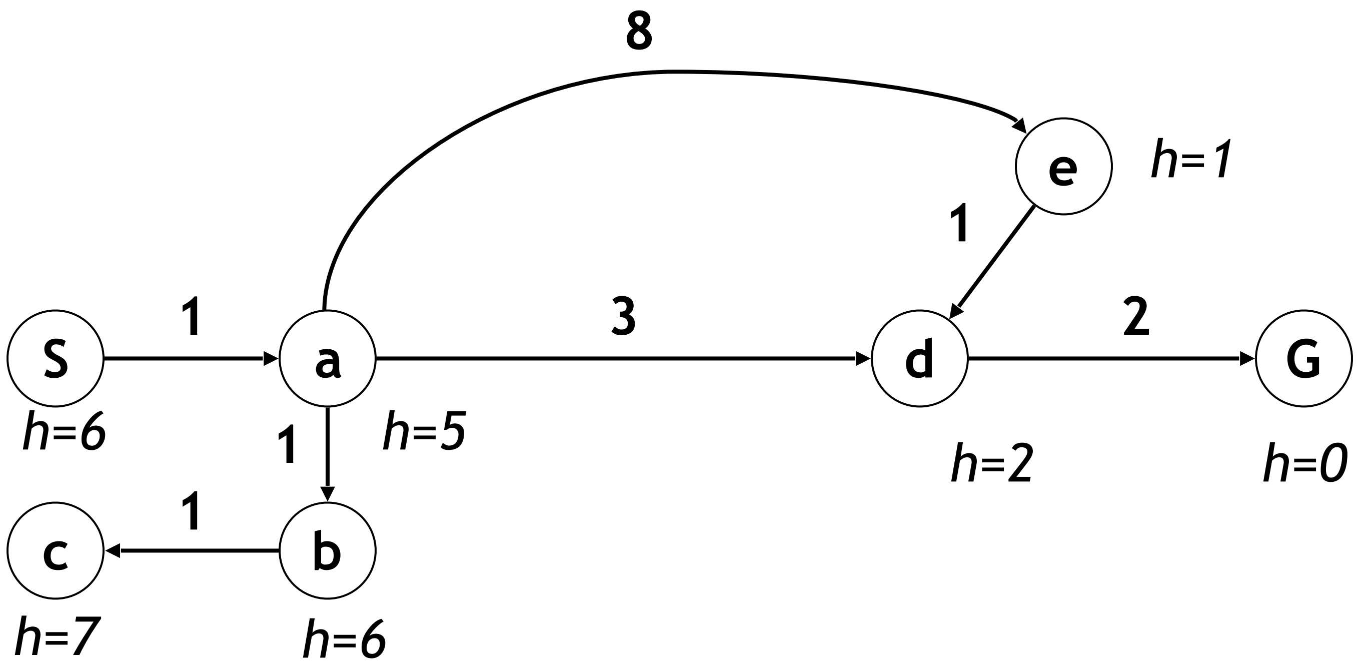
Combining UCS and Greedy



Example: Teg Grenager

Combining UCS and Greedy

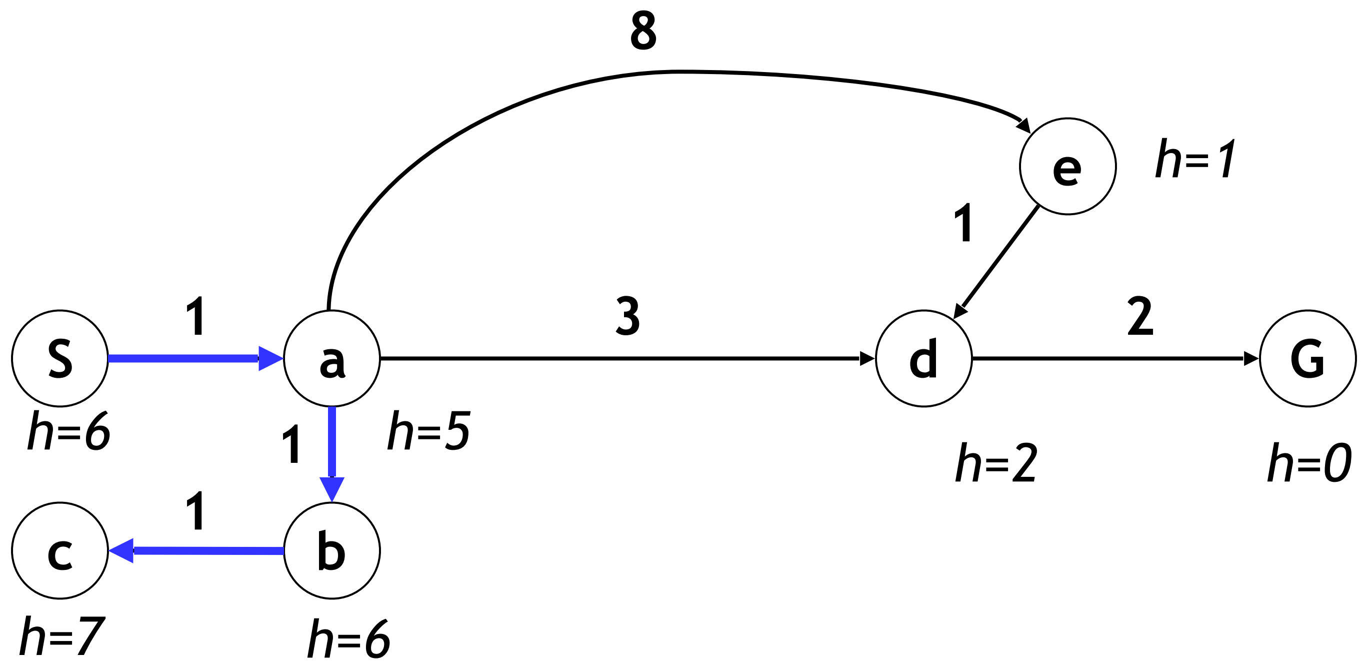
- Uniform-cost orders by path cost, or *backward cost* g(n)



Example: Teg Grenager

Combining UCS and Greedy

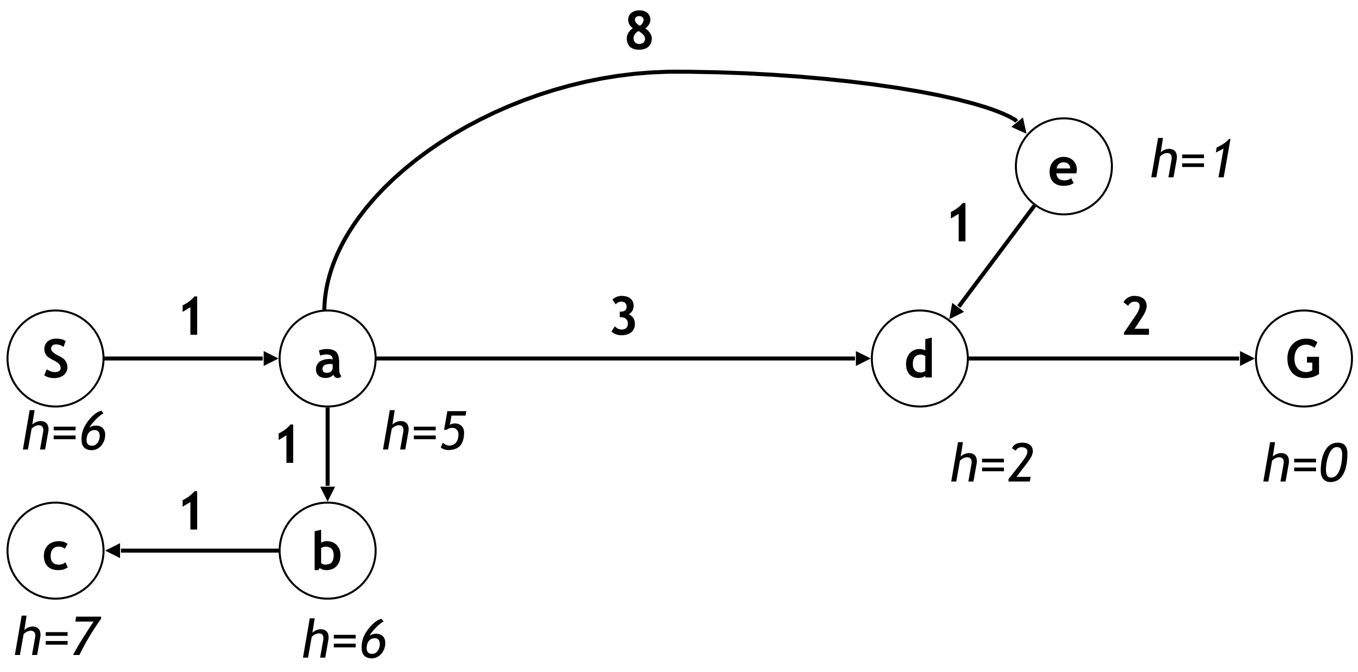
- Uniform-cost orders by path cost, or *backward cost* g(n)



Example: Teg Grenager

Combining UCS and Greedy

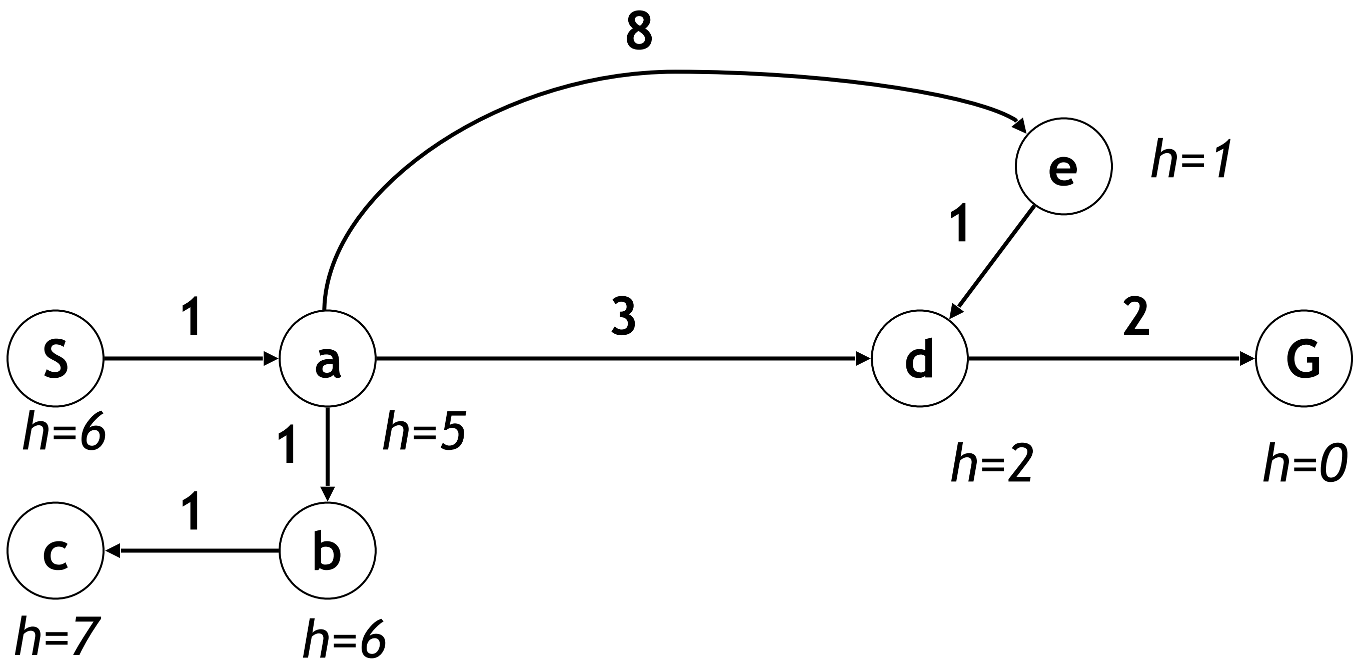
- Uniform-cost orders by path cost, or *backward cost* g(n)



Example: Teg Grenager

Combining UCS and Greedy

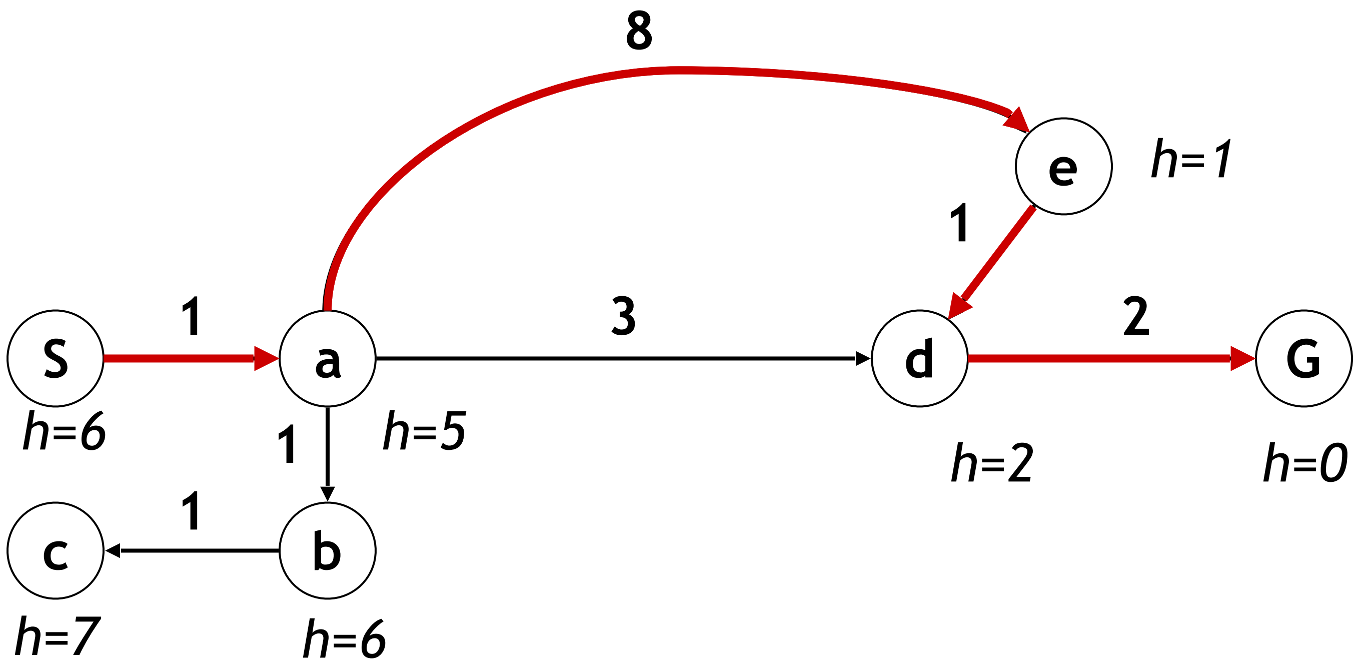
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



Example: Teg Grenager

Combining UCS and Greedy

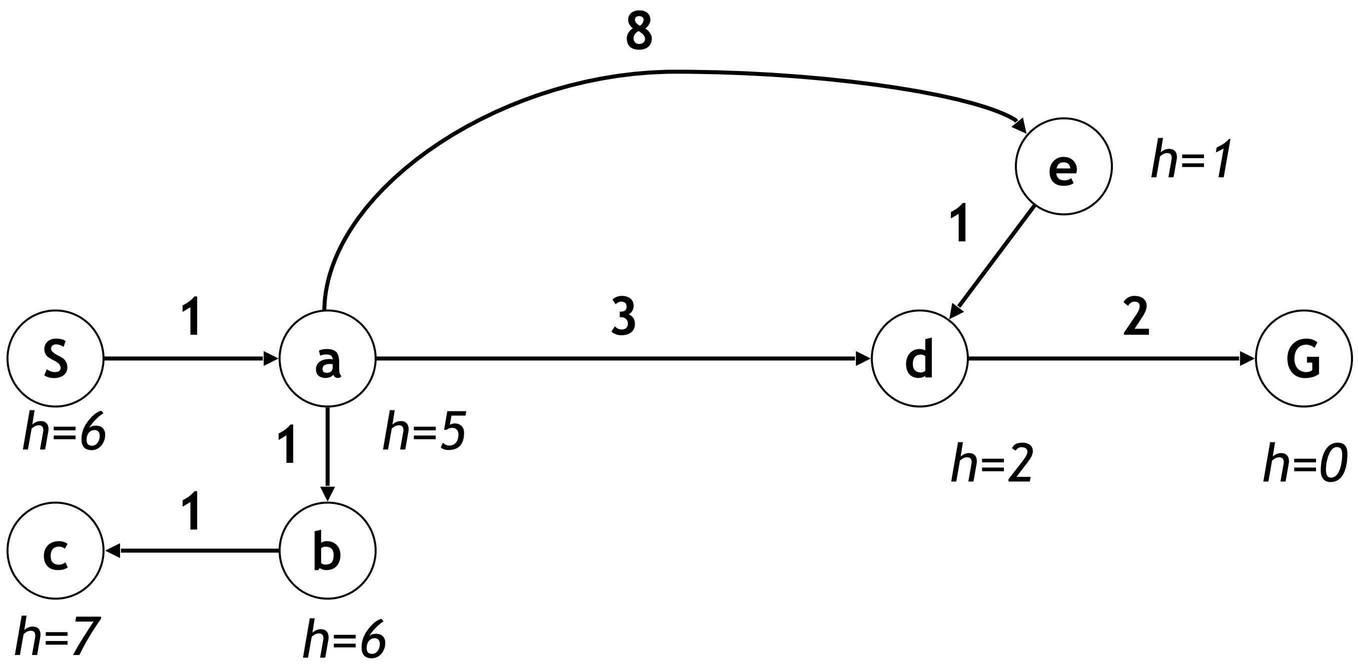
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



Example: Teg Grenager

Combining UCS and Greedy

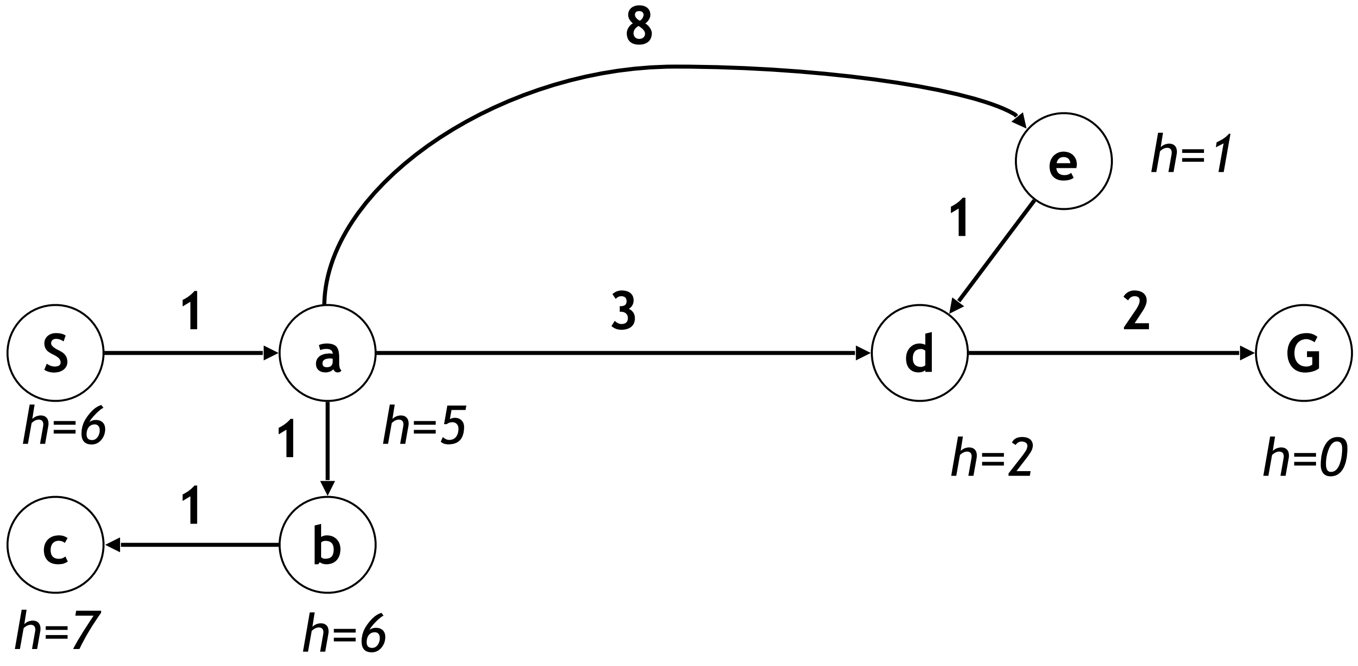
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



Example: Teg Grenager

Combining UCS and Greedy

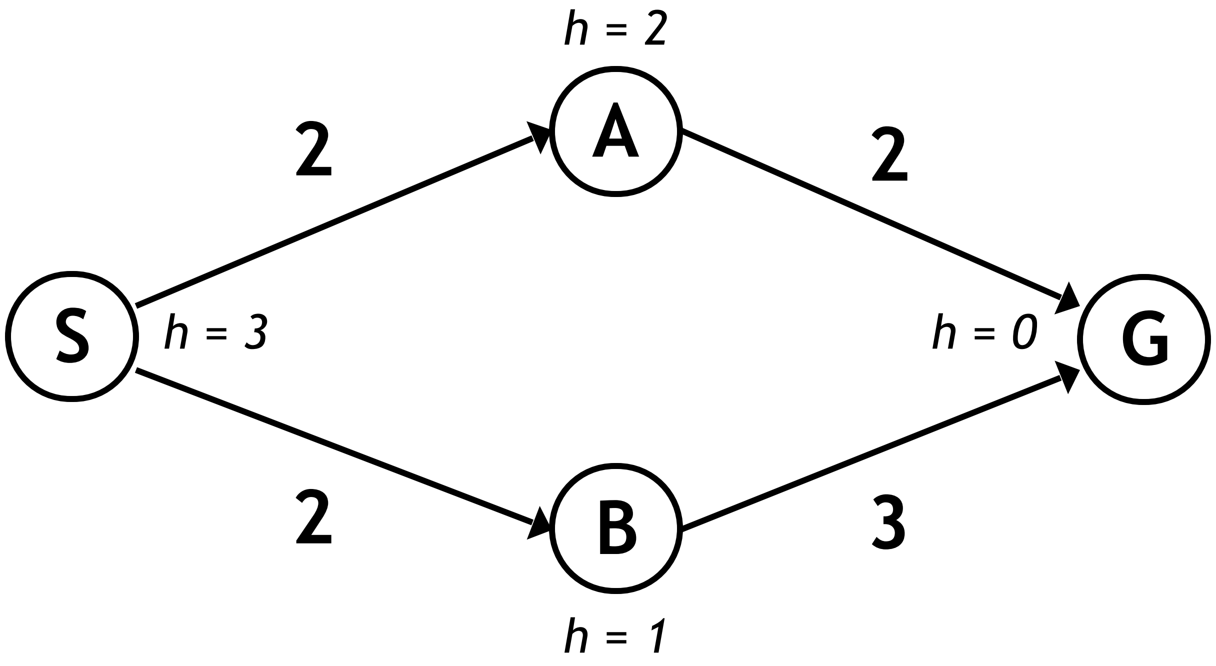
- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)



- A* Search orders by the sum: f(n) = g(n) + h(n)

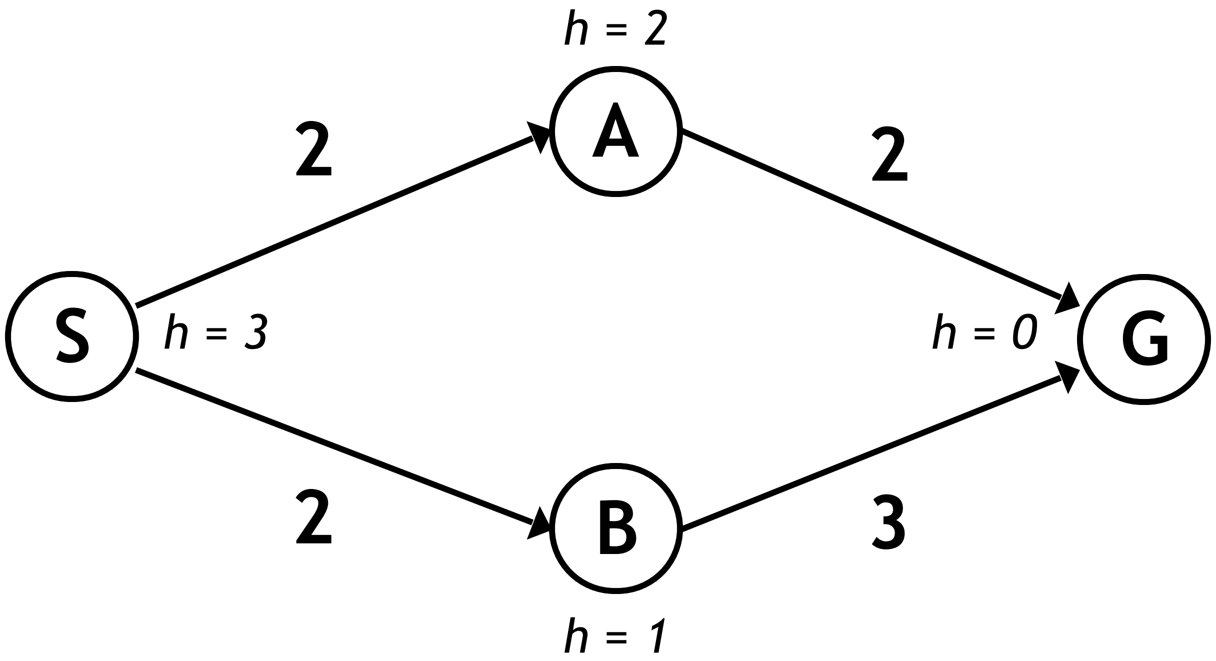
Example: Teg Grenager

When should A* terminate?



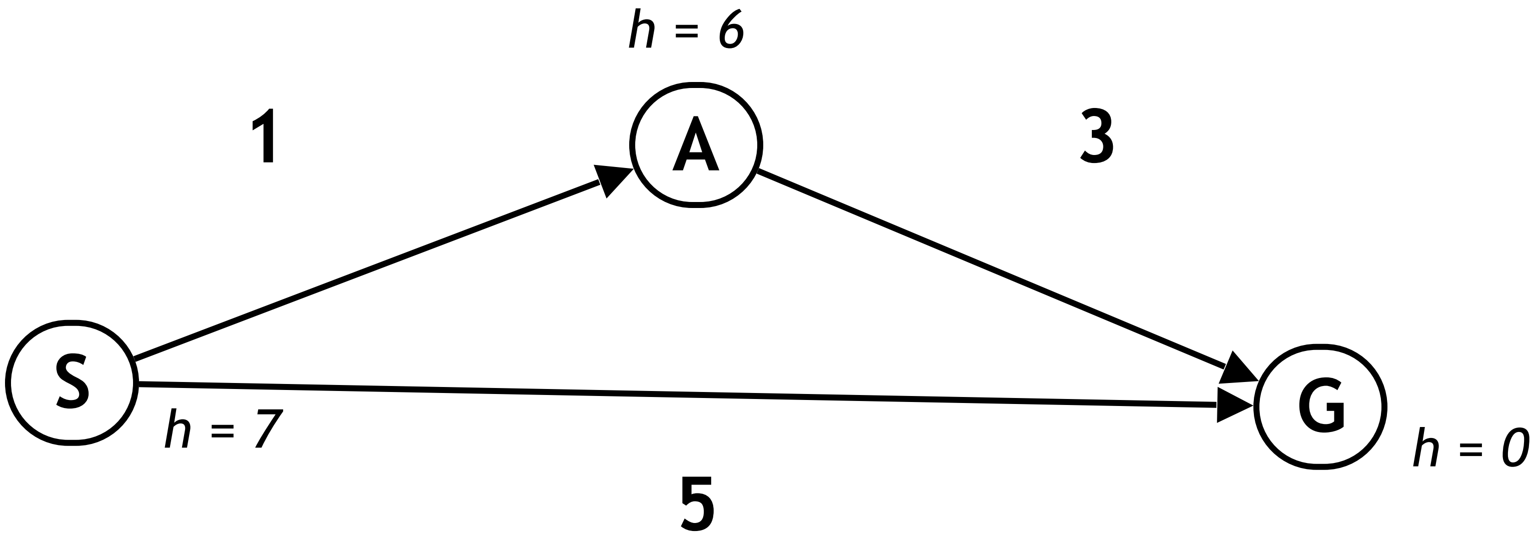
When should A* terminate?

- Should we stop when we enqueue a goal?



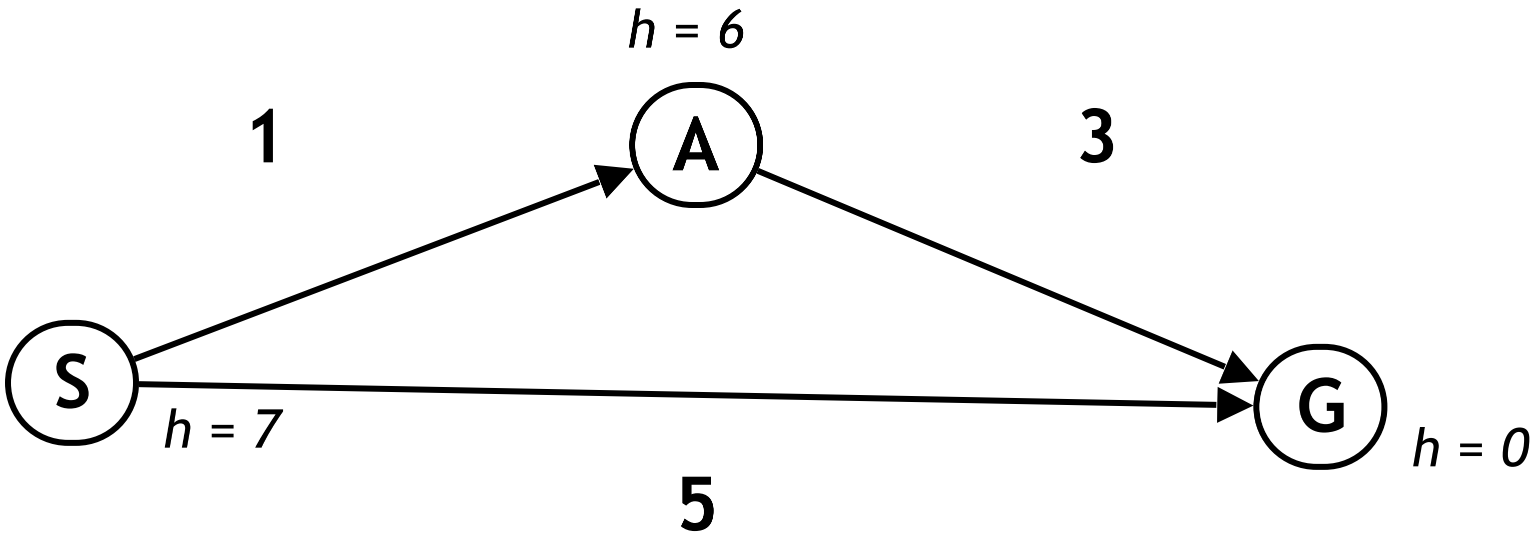
- No: only stop when we dequeue a goal

Is A* Optimal?



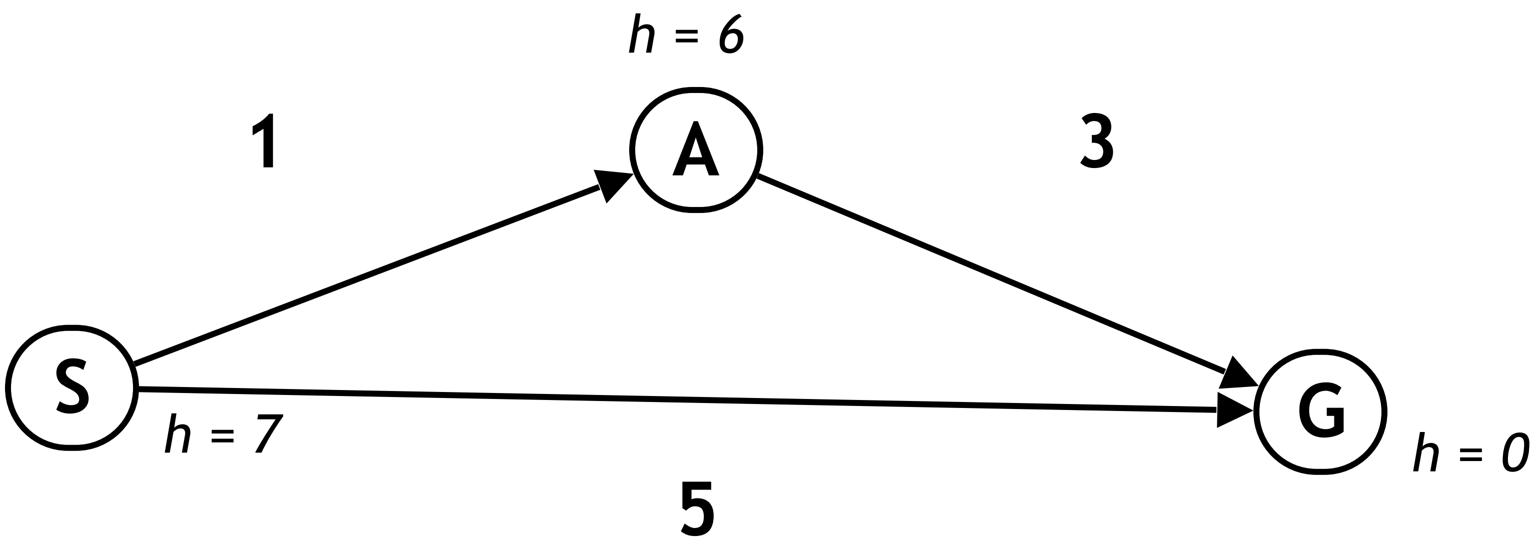
- What went wrong?

Is A* Optimal?



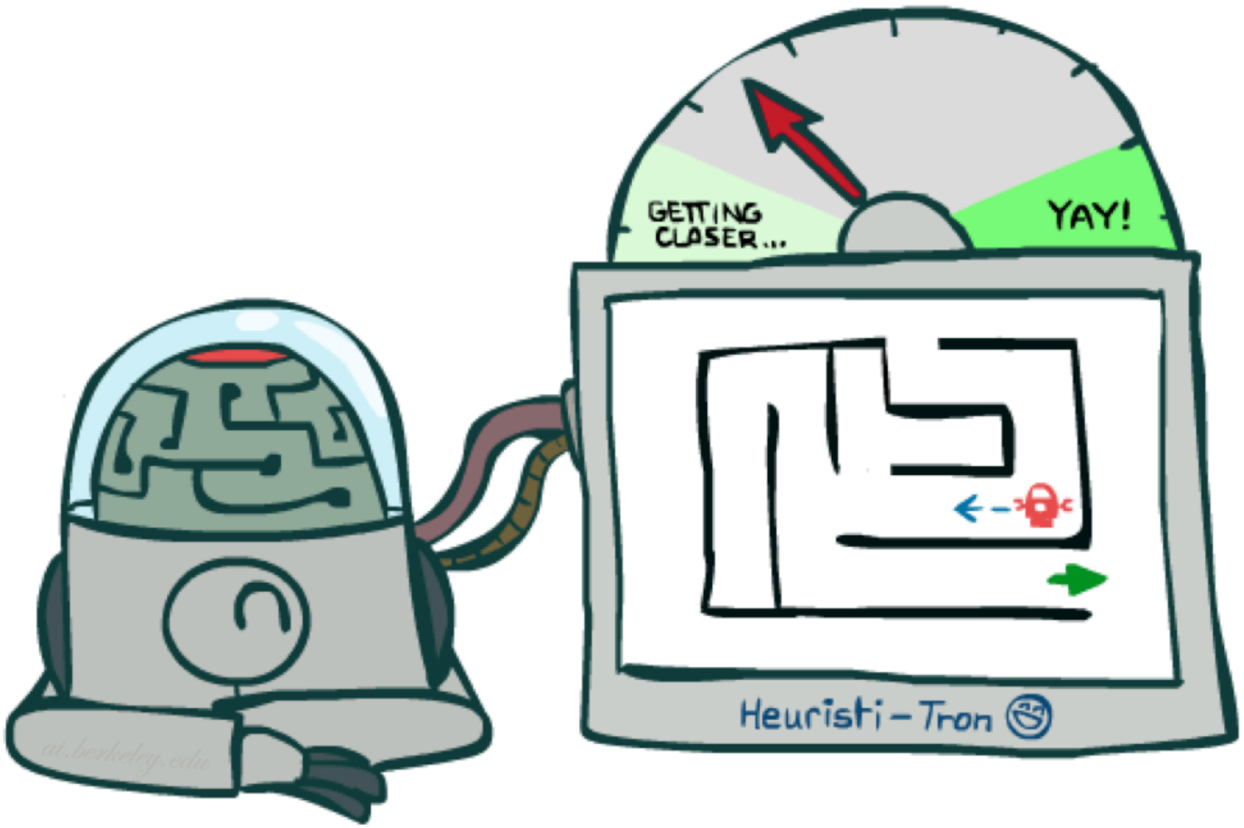
- What went wrong?
- Actual bad goal cost < estimated good goal cost

Is A* Optimal?

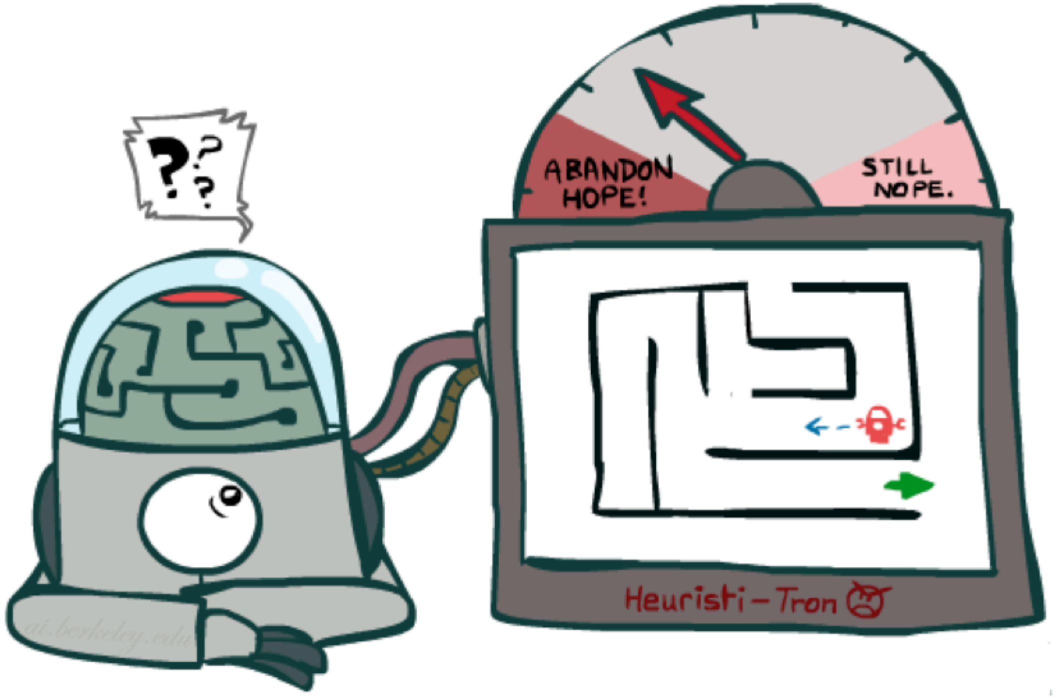


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

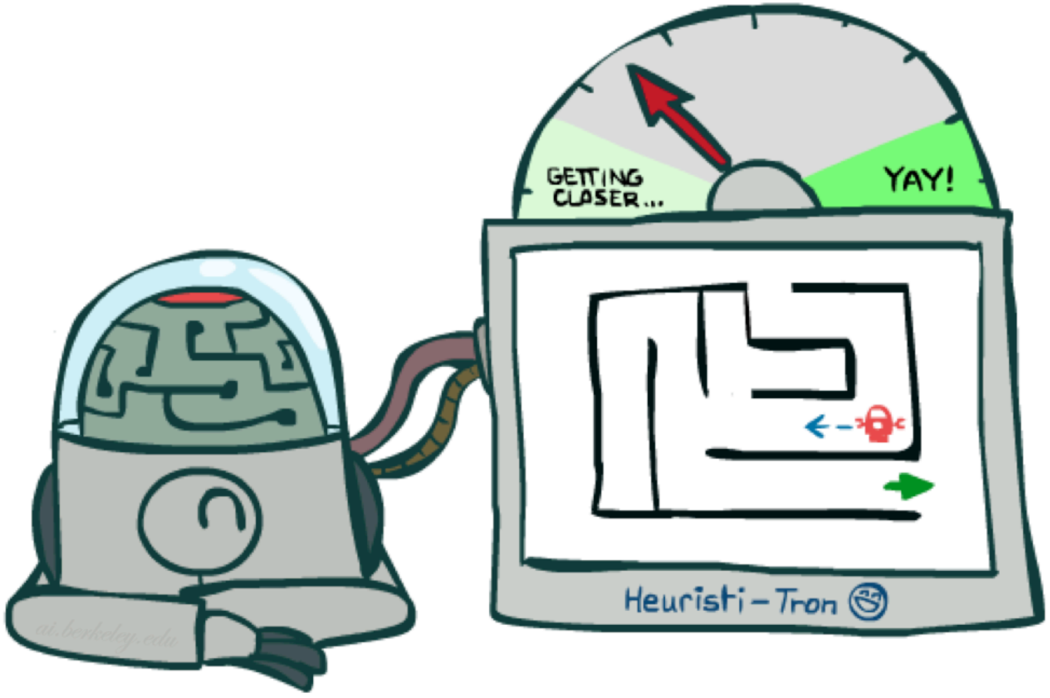
Admissible Heuristics



Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

- Examples:

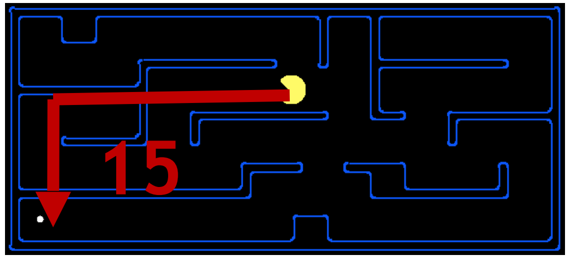
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



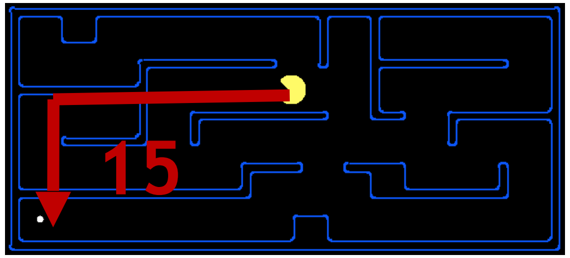
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



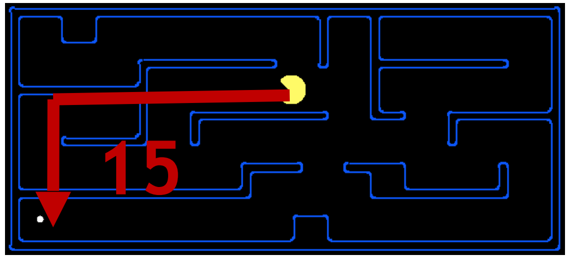
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

  where $h^*(n)$ is the true cost to a nearest goal

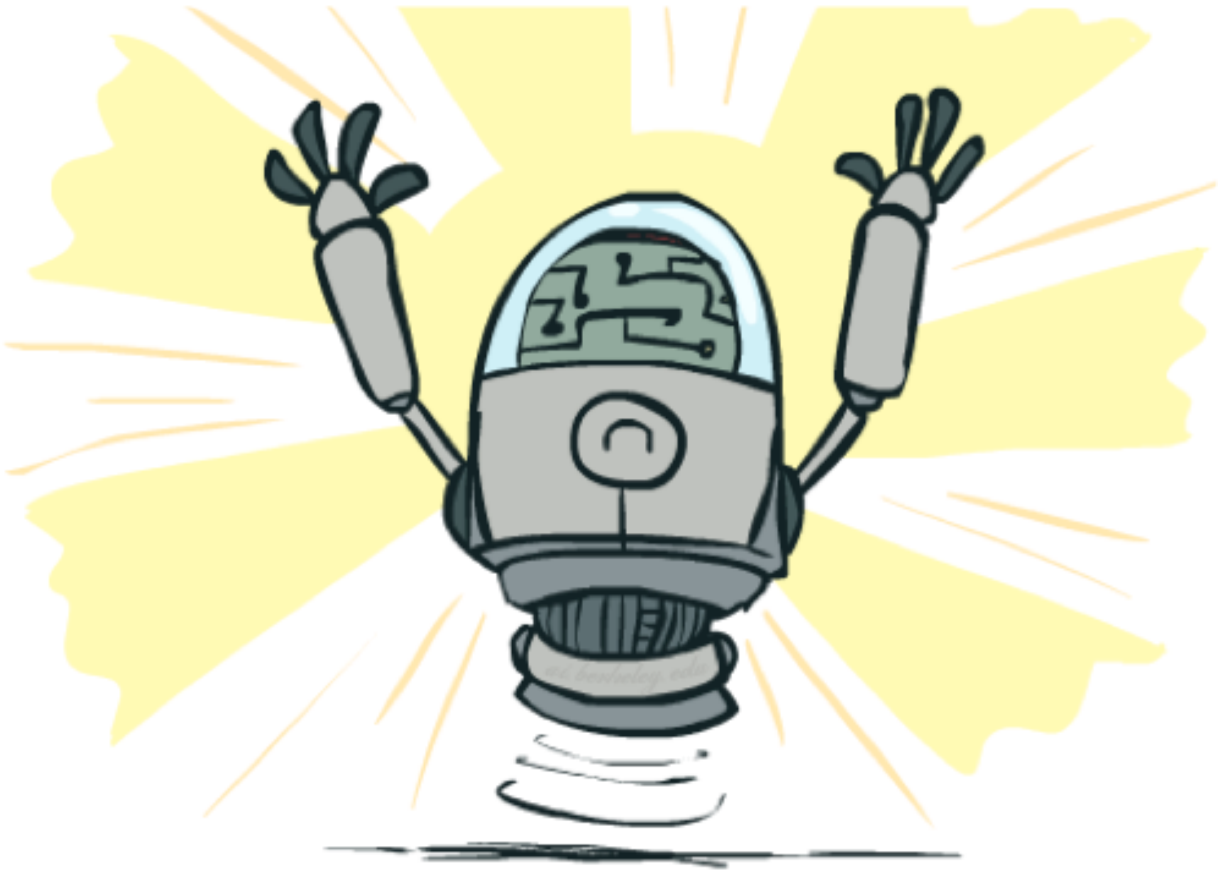
- Examples:



4

- Coming up with admissible heuristics is most of what's involved in using A* in practice.

Optimality of A* Tree Search



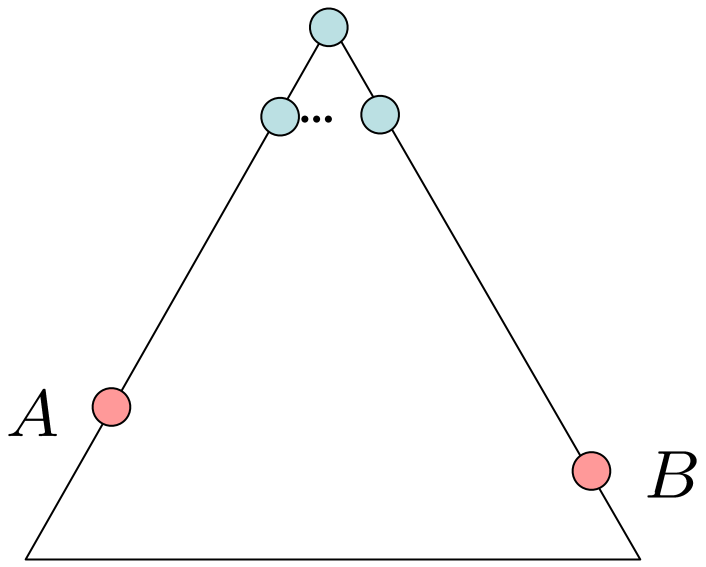
Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

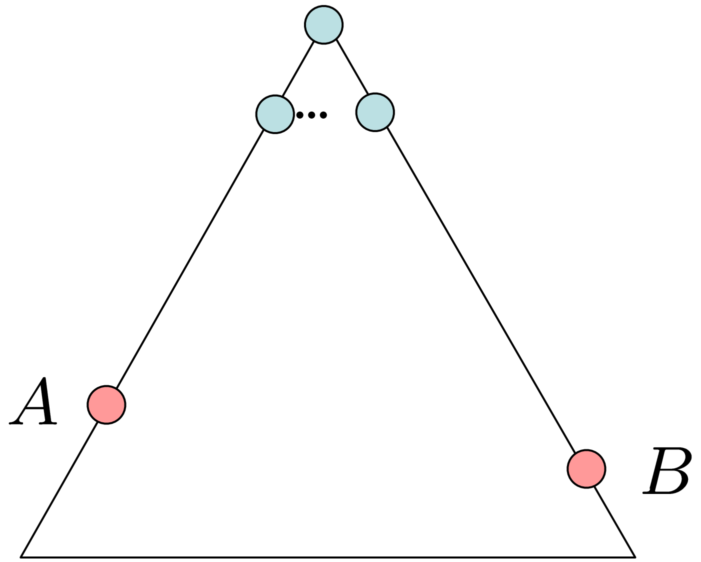
Claim:

- A will exit the fringe before B



Optimality of A* Tree Search: Blocking

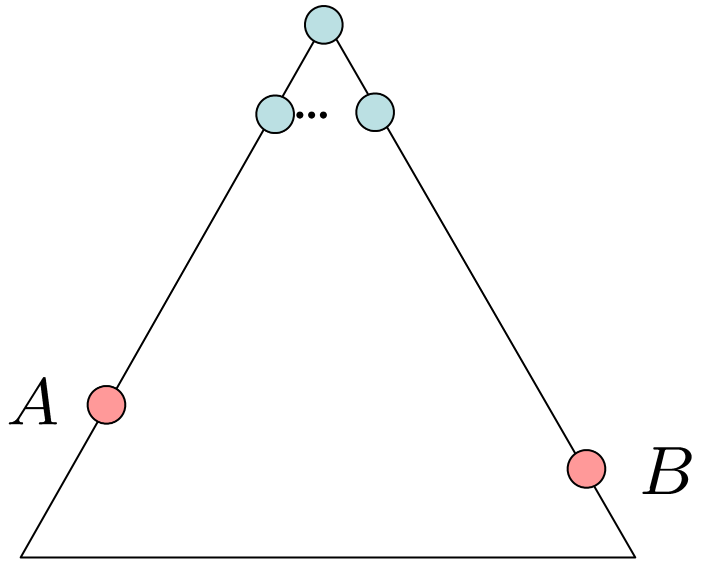
Proof:



Optimality of A* Tree Search: Blocking

Proof:

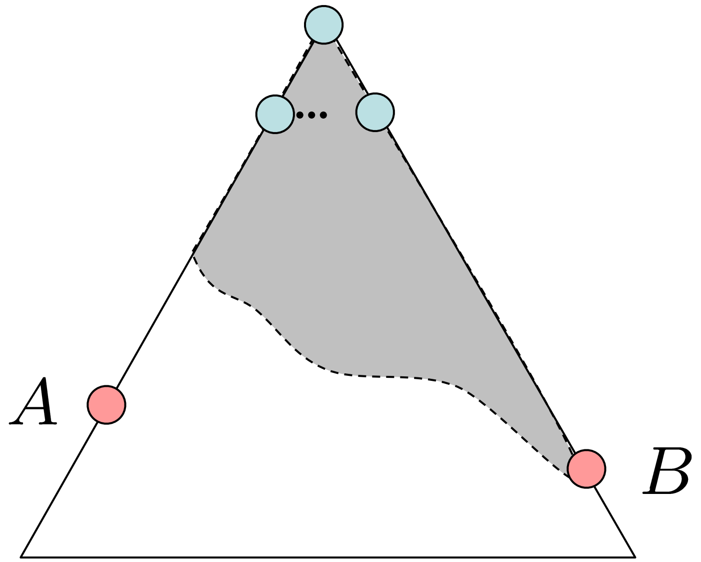
- Imagine B is on the fringe



Optimality of A* Tree Search: Blocking

Proof:

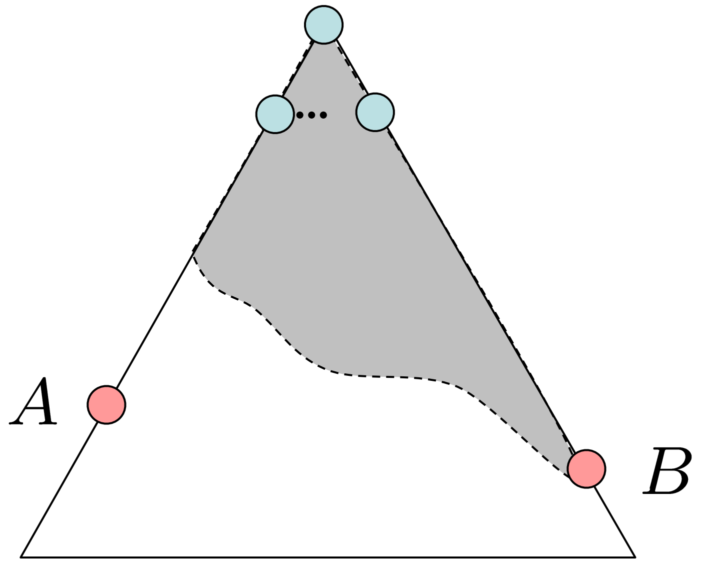
- Imagine B is on the fringe



Optimality of A* Tree Search: Blocking

Proof:

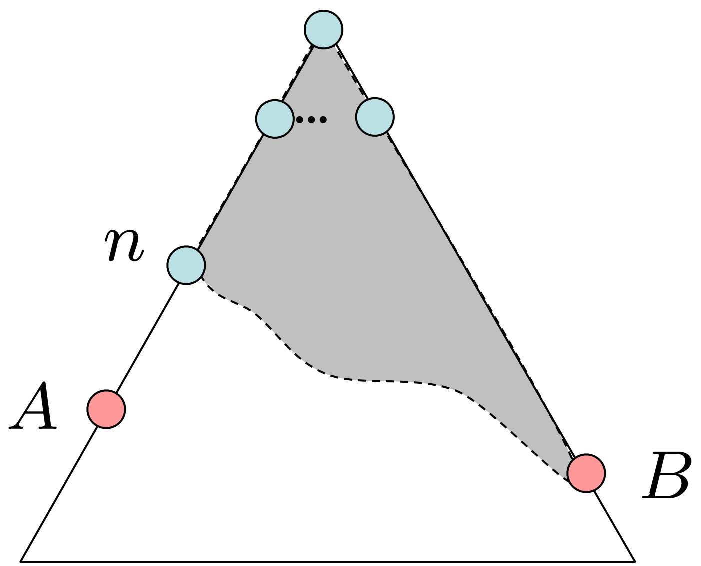
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)



Optimality of A* Tree Search: Blocking

Proof:

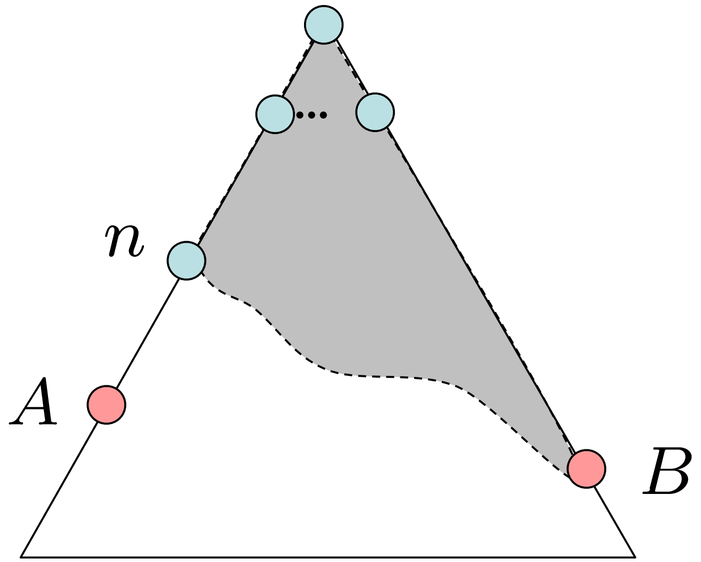
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)



Optimality of A* Tree Search: Blocking

Proof:

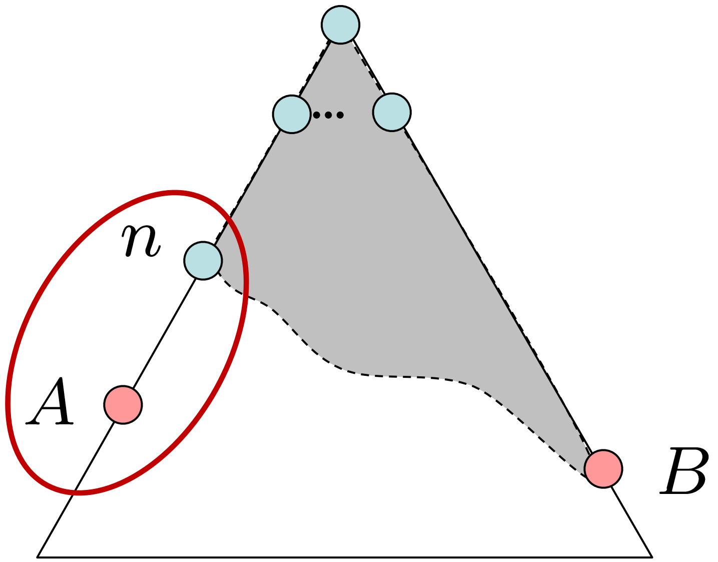
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B



Optimality of A* Tree Search: Blocking

Proof:

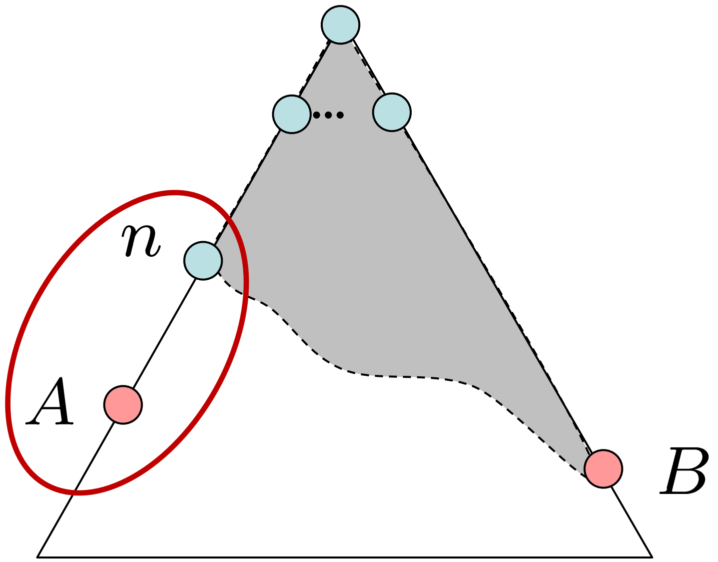
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B



Optimality of A* Tree Search: Blocking

Proof:

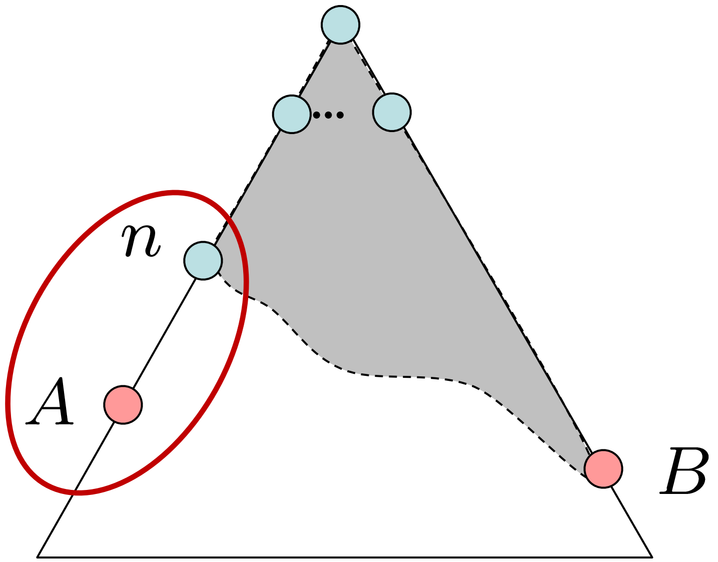
- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)



Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)



$$f(n) = g(n) + h(n)$$ Definition of f-cost

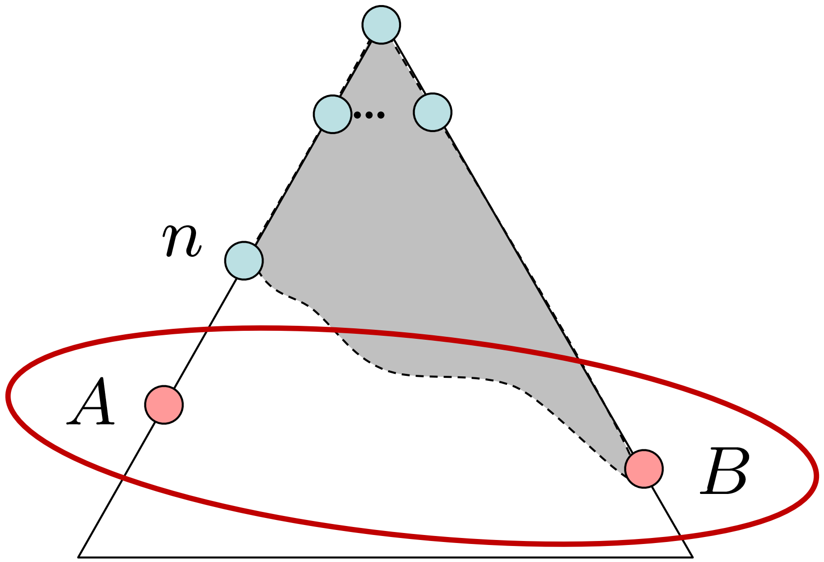
$$f(n) \leq \left(g(n) + h^*(n)\right) = g(A)$$ Admissibility of h

$$f(A) = \left(g(A) + h^*(A)\right) = g(A)$$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



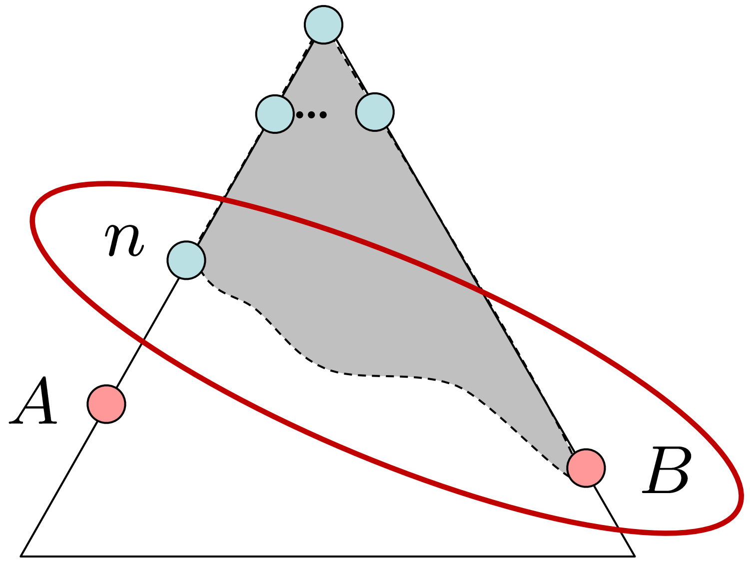
$g(A) < g(B)$ B is suboptimal

$f(A) < f(B)$ h = 0 at a goal

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)
  3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

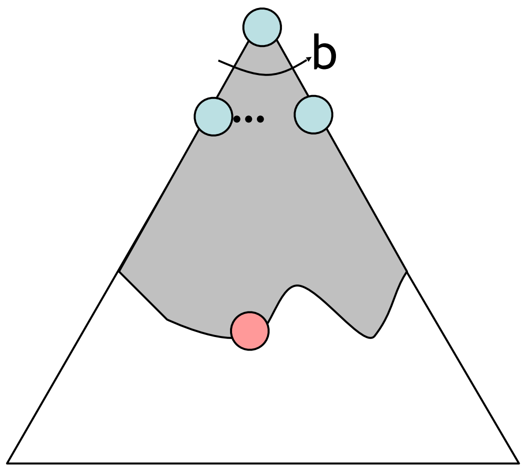


$$f(n) \leq f(A) < f(B)$$

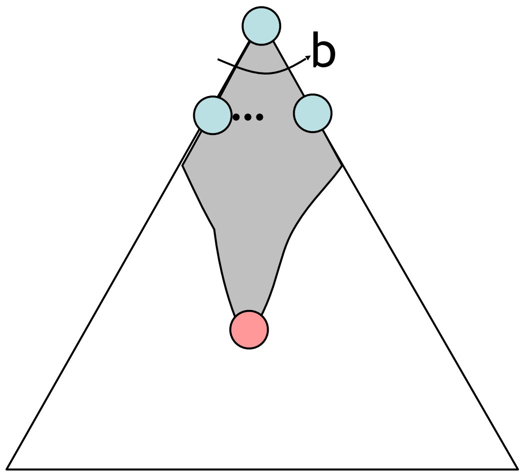
Properties of A*

Properties of A*

Uniform-Cost

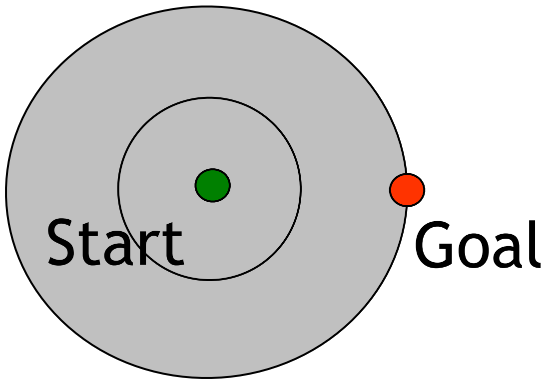


A*

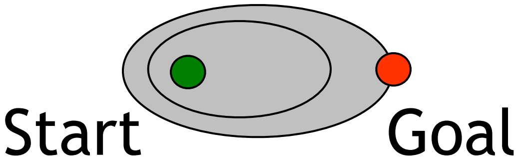


UCS vs A* Contours

- Uniform-cost expands equally in all “directions”

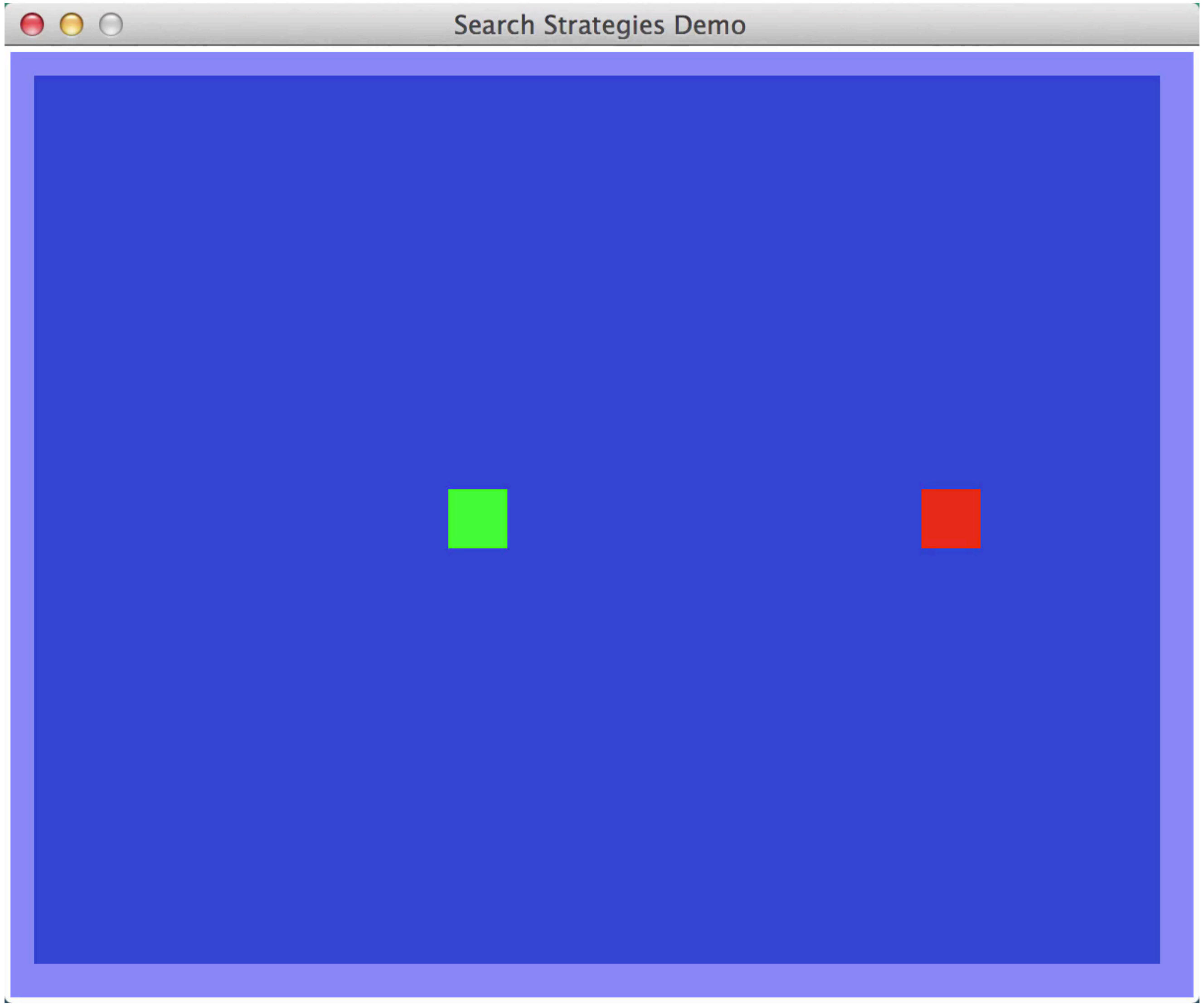


- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

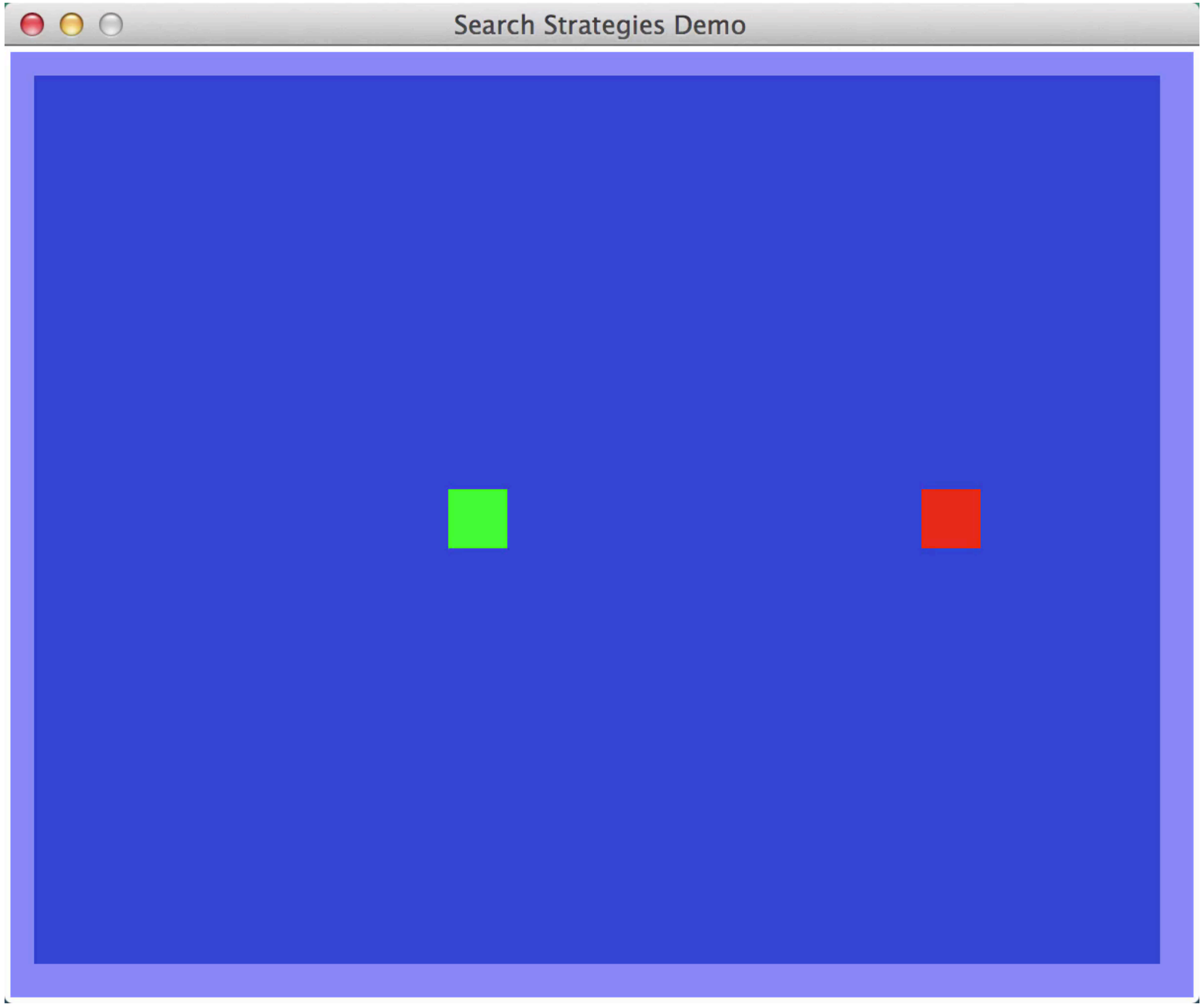


[Demo: contours UCS / greedy / A* empty (L3D1)]
[Demo: contours A* pacman small maze (L3D5)]

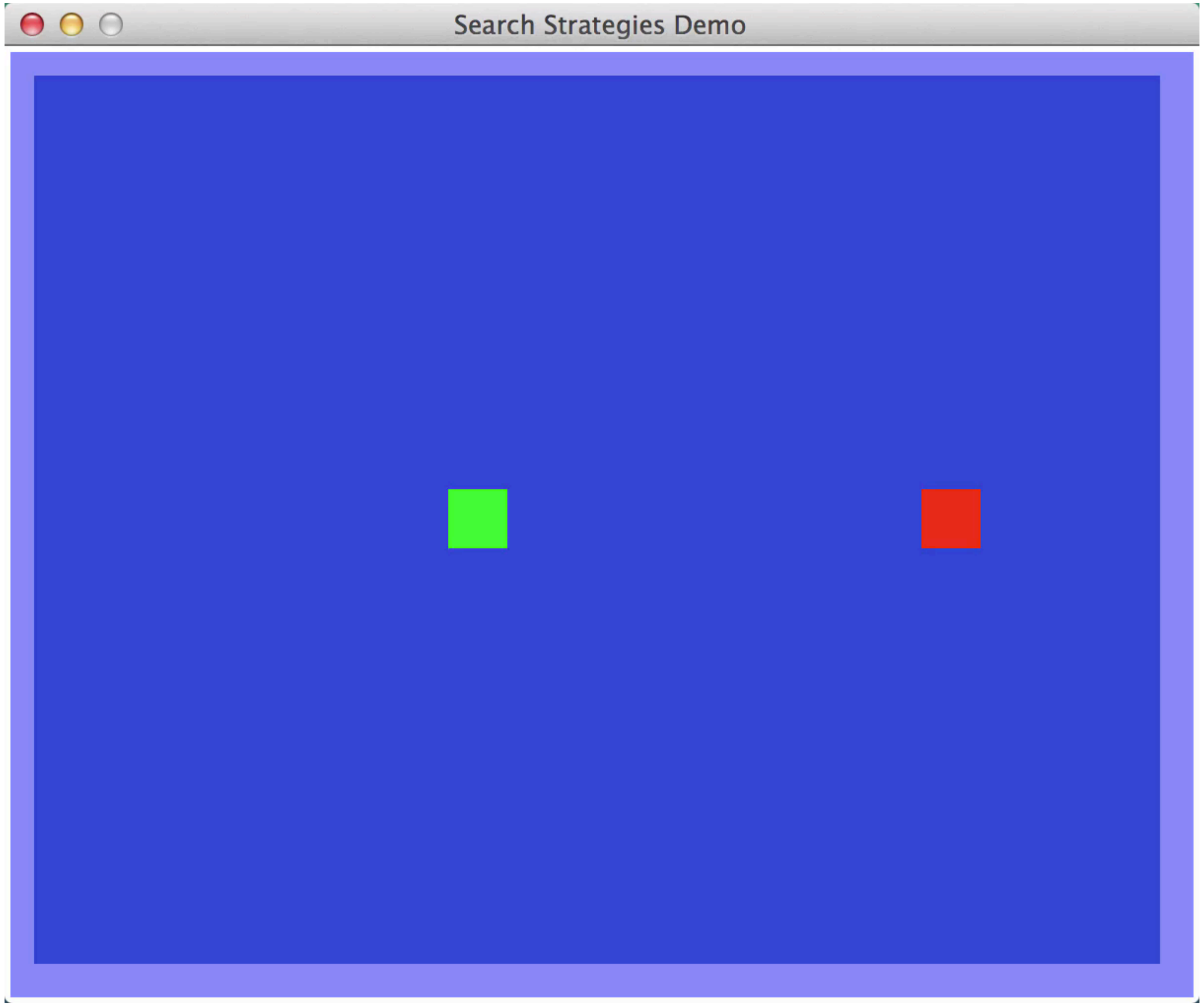
Video of Demo Contours (Empty) -- UCS



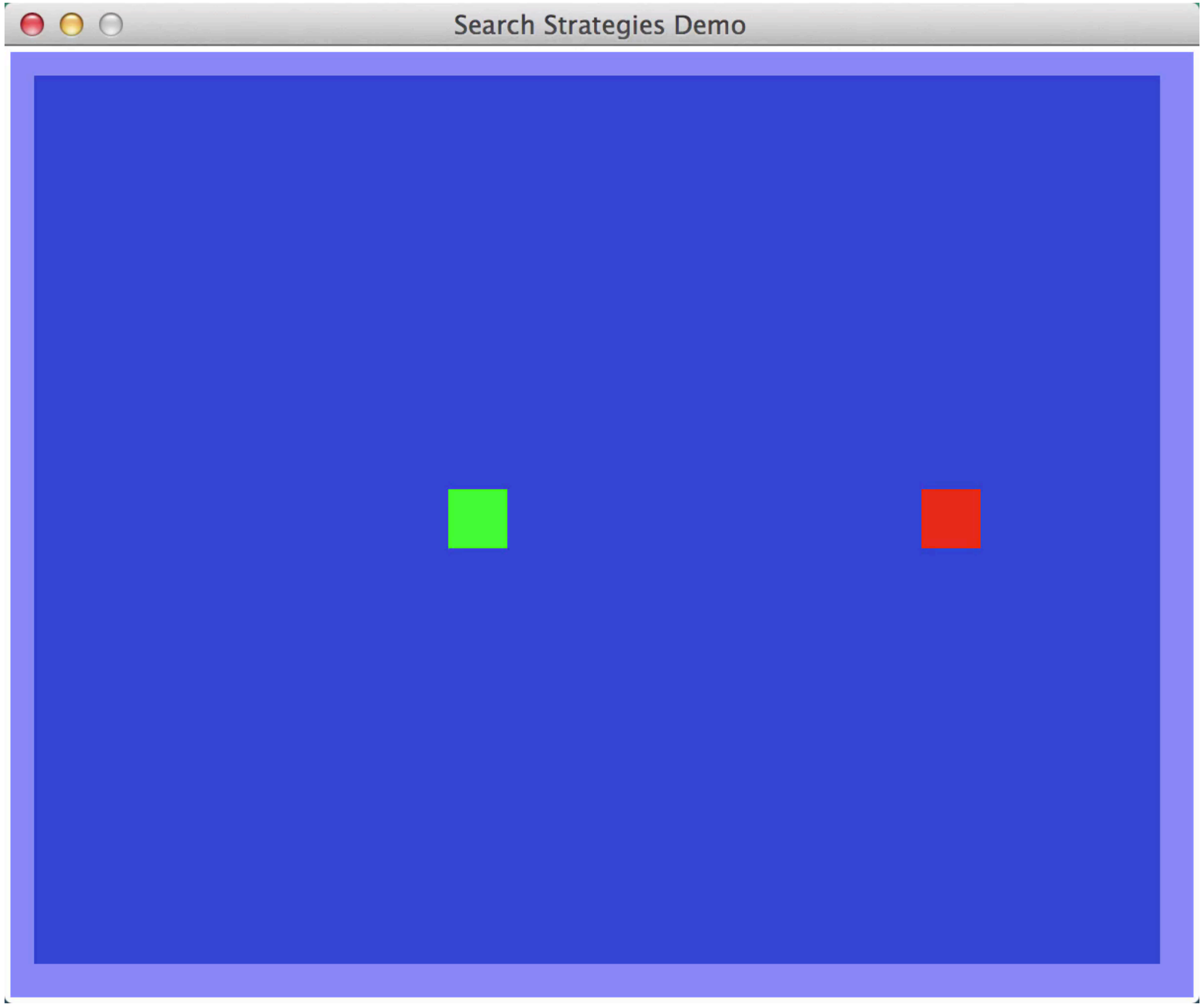
Video of Demo Contours (Empty) -- UCS



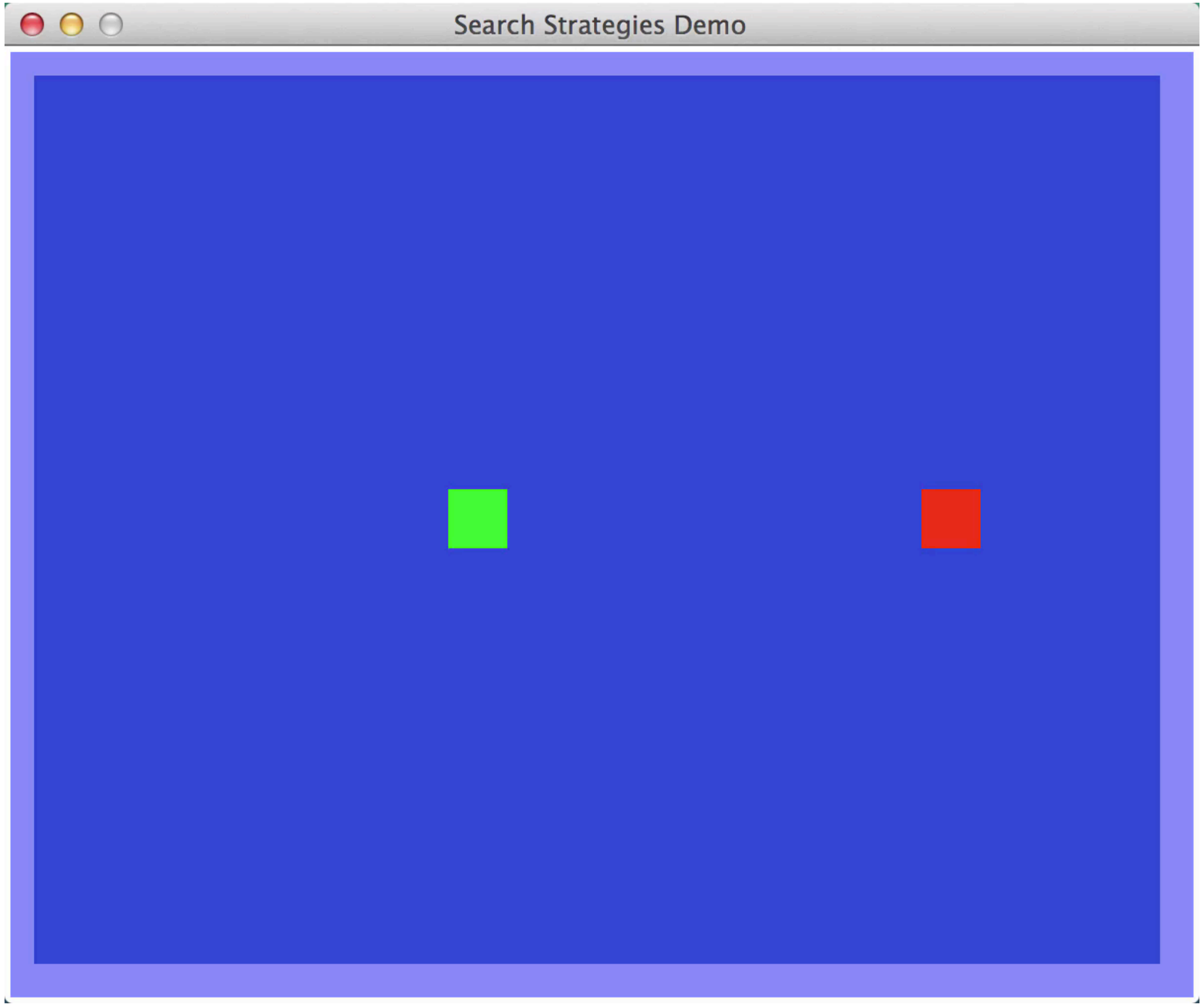
Video of Demo Contours (Empty) -- UCS



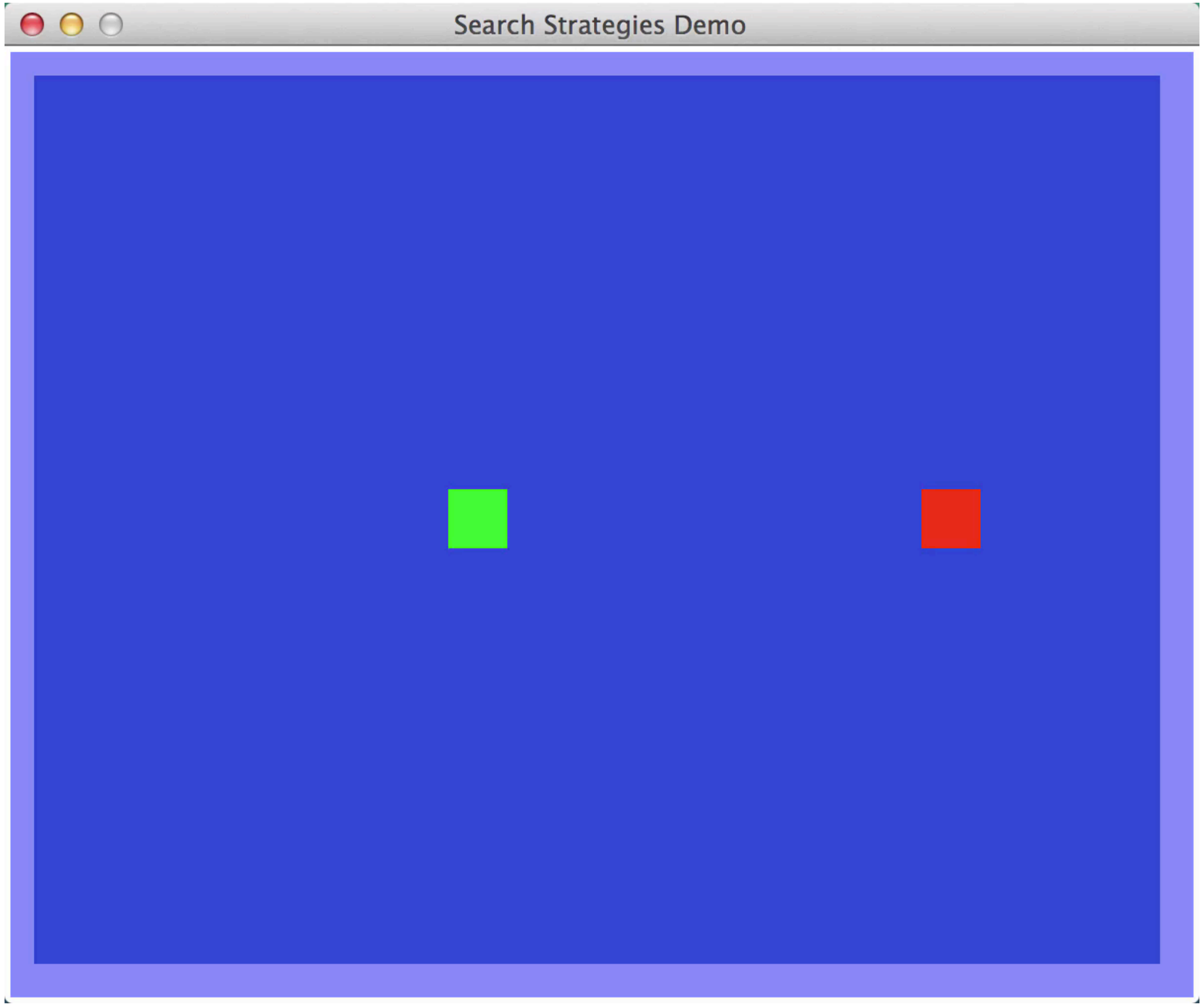
Video of Demo Contours (Empty) -- Greedy



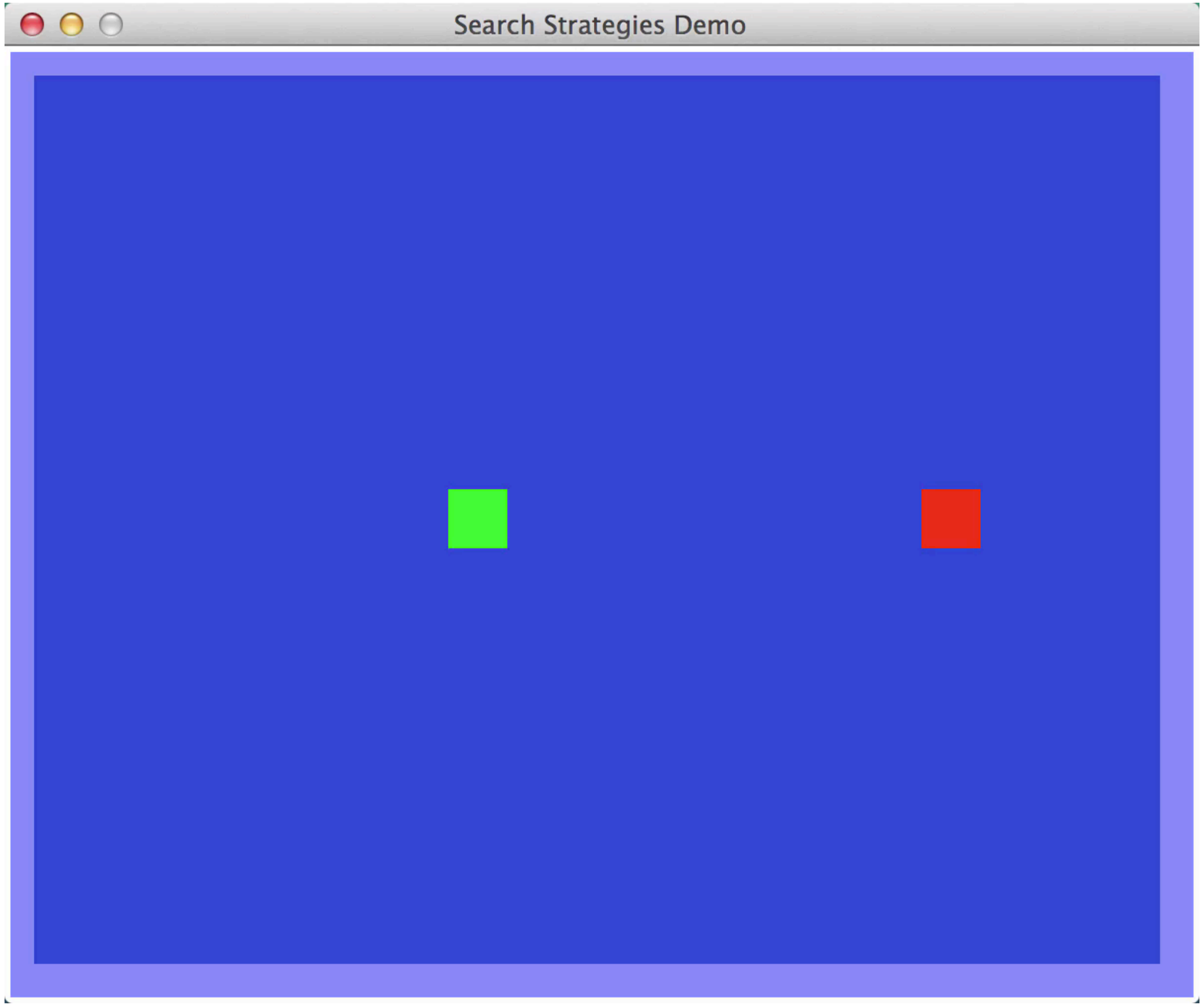
Video of Demo Contours (Empty) -- Greedy



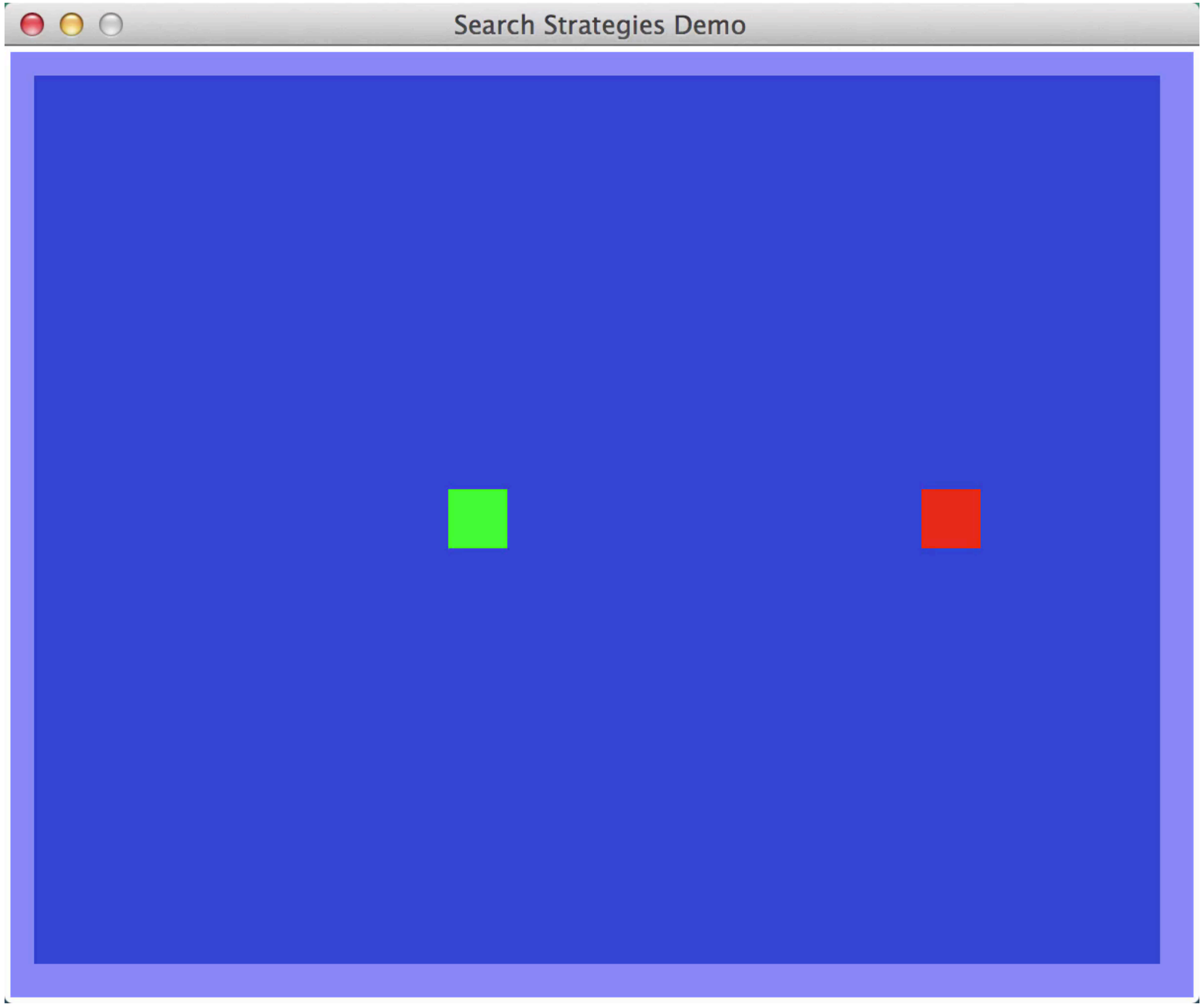
Video of Demo Contours (Empty) -- Greedy



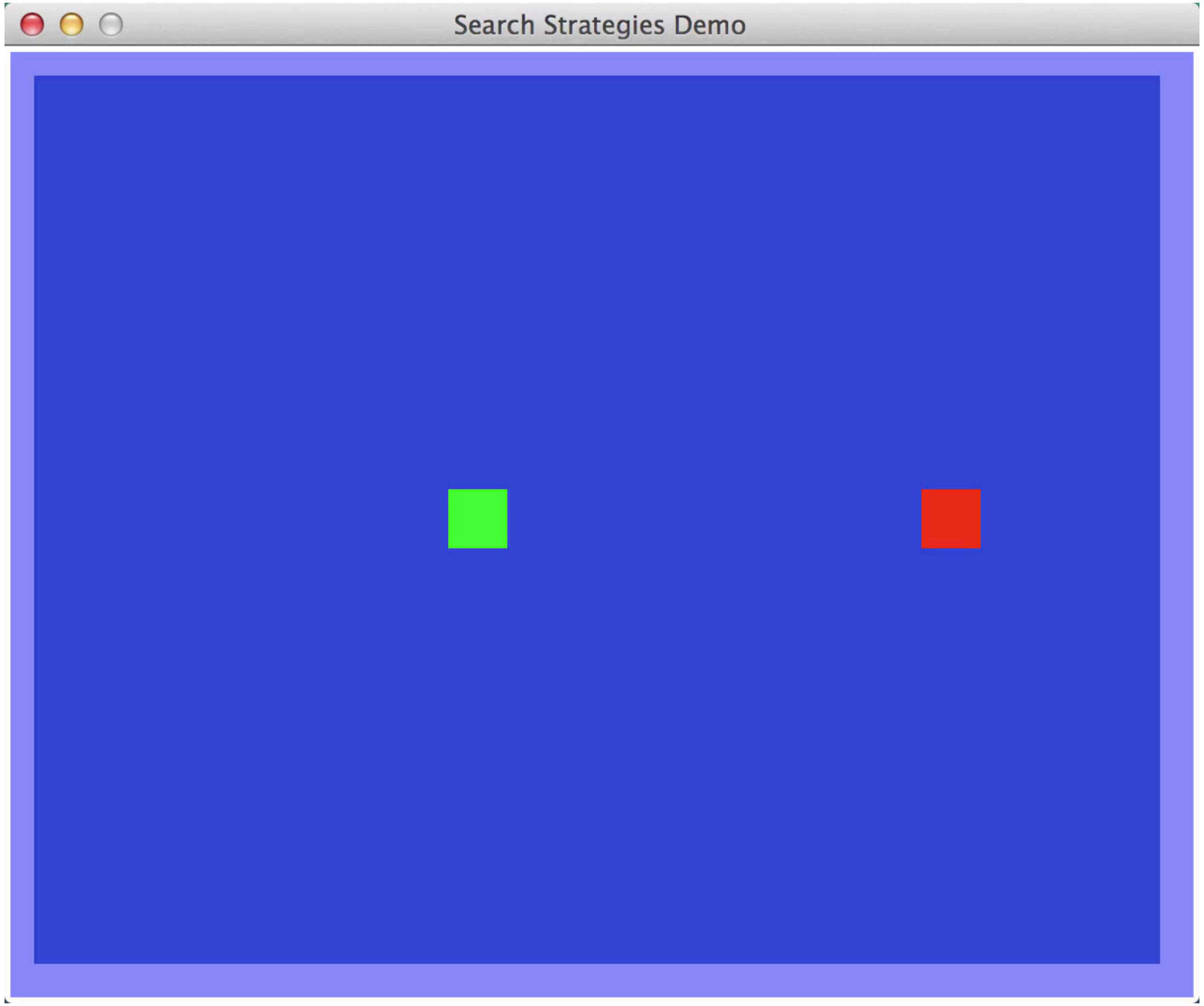
Video of Demo Contours (Empty) – A*



Video of Demo Contours (Empty) – A*

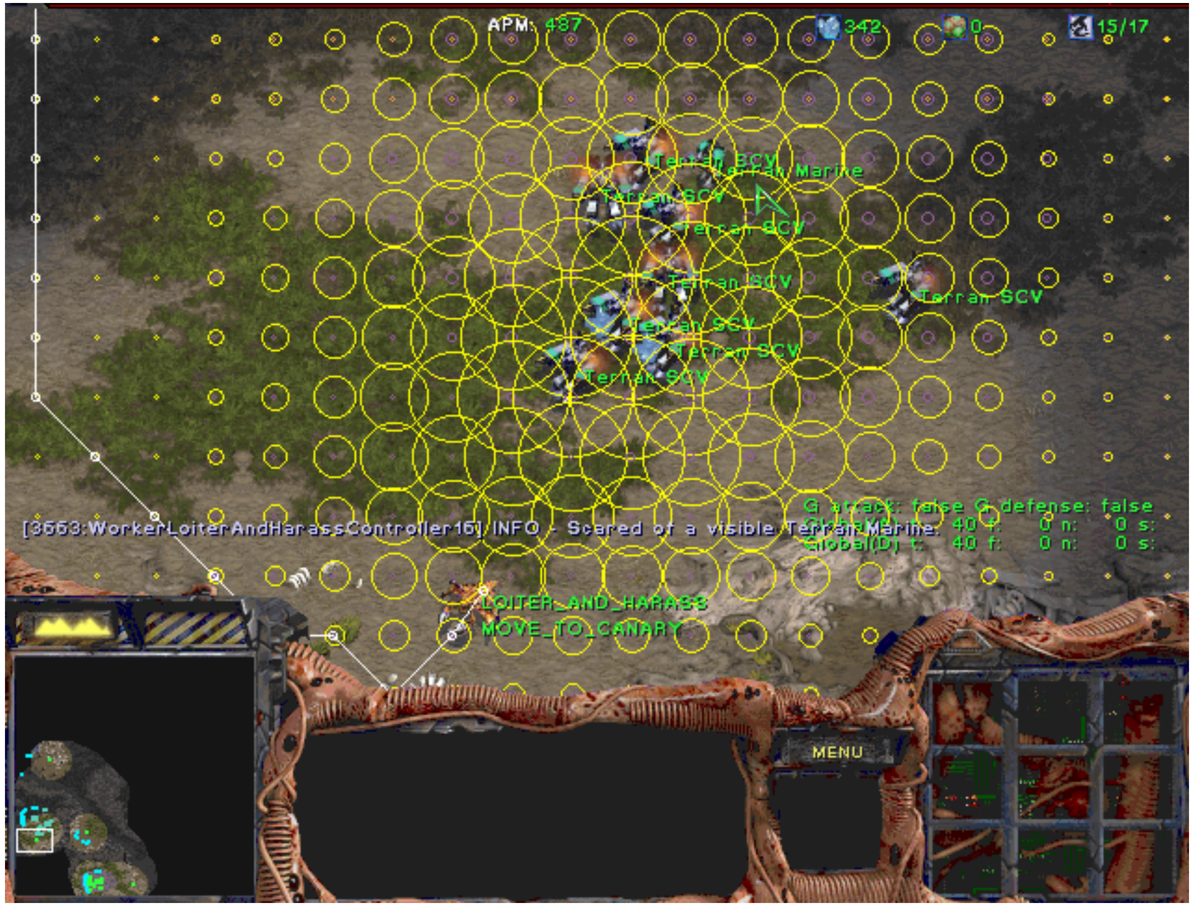


Video of Demo Contours (Empty) – A*



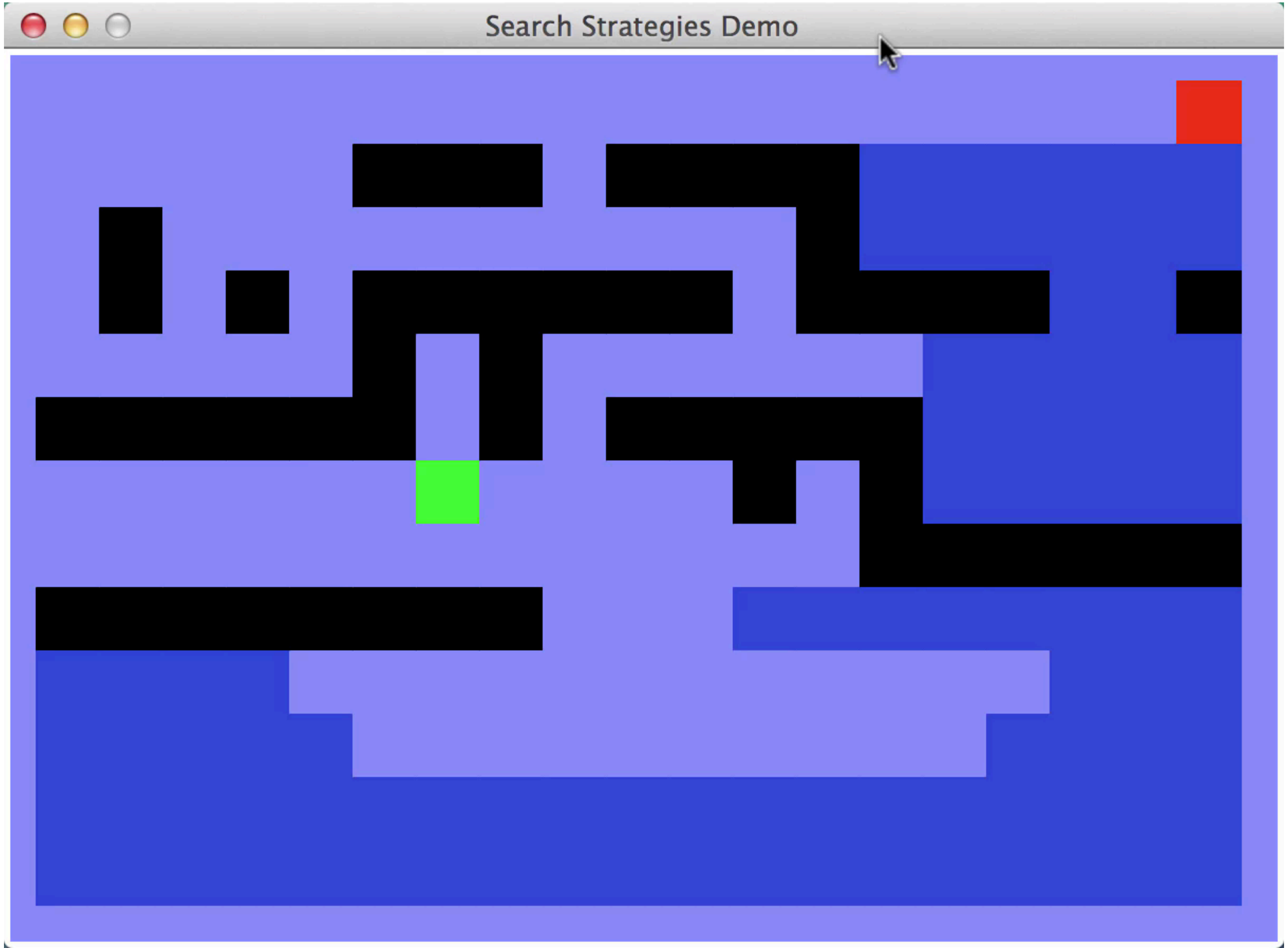
A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

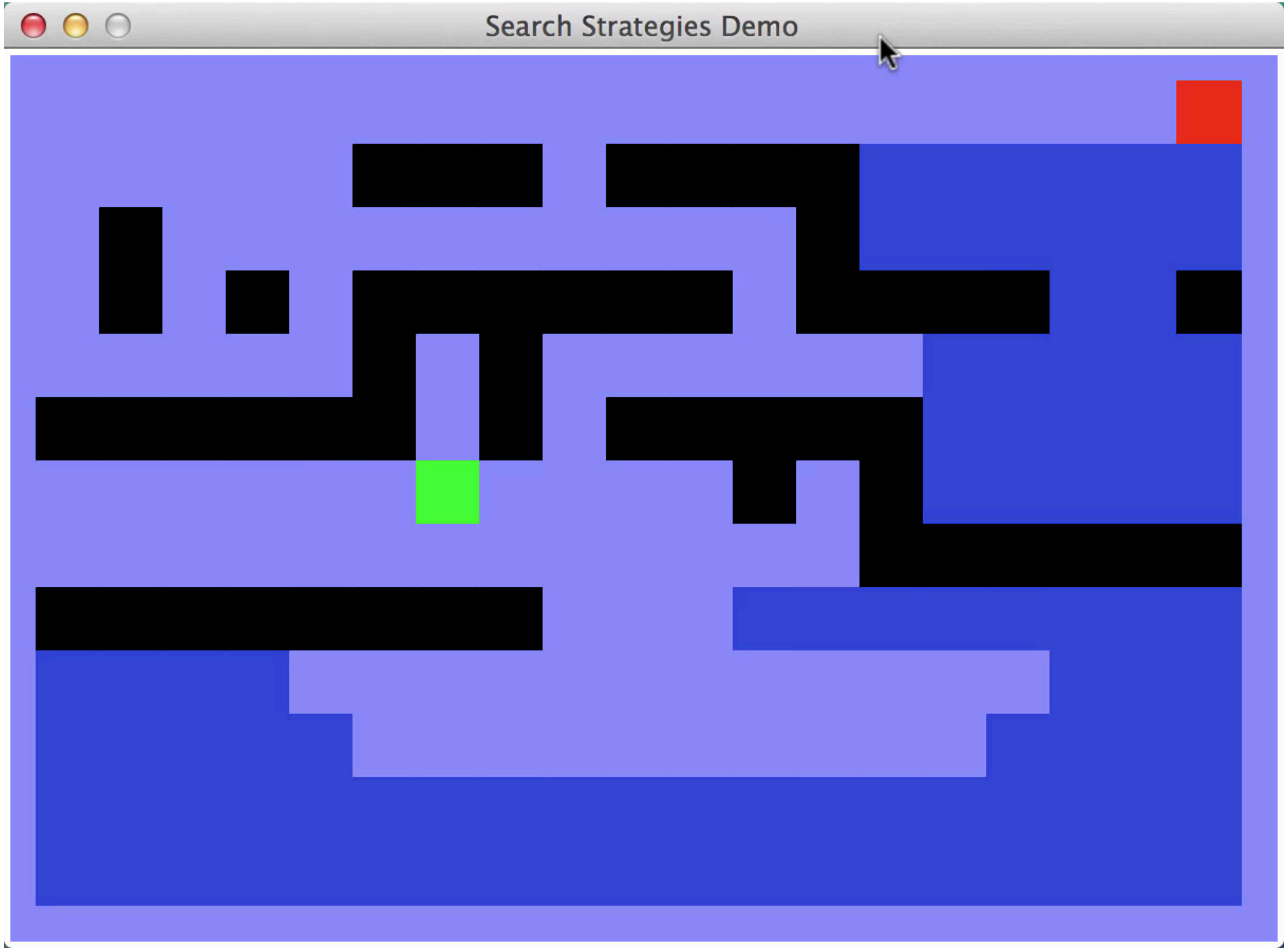


[Demo: UCS / A* pacman tiny maze (L3D6,L3D7)]
[Demo: guess algorithm Empty Shallow/Deep (L3D8)]

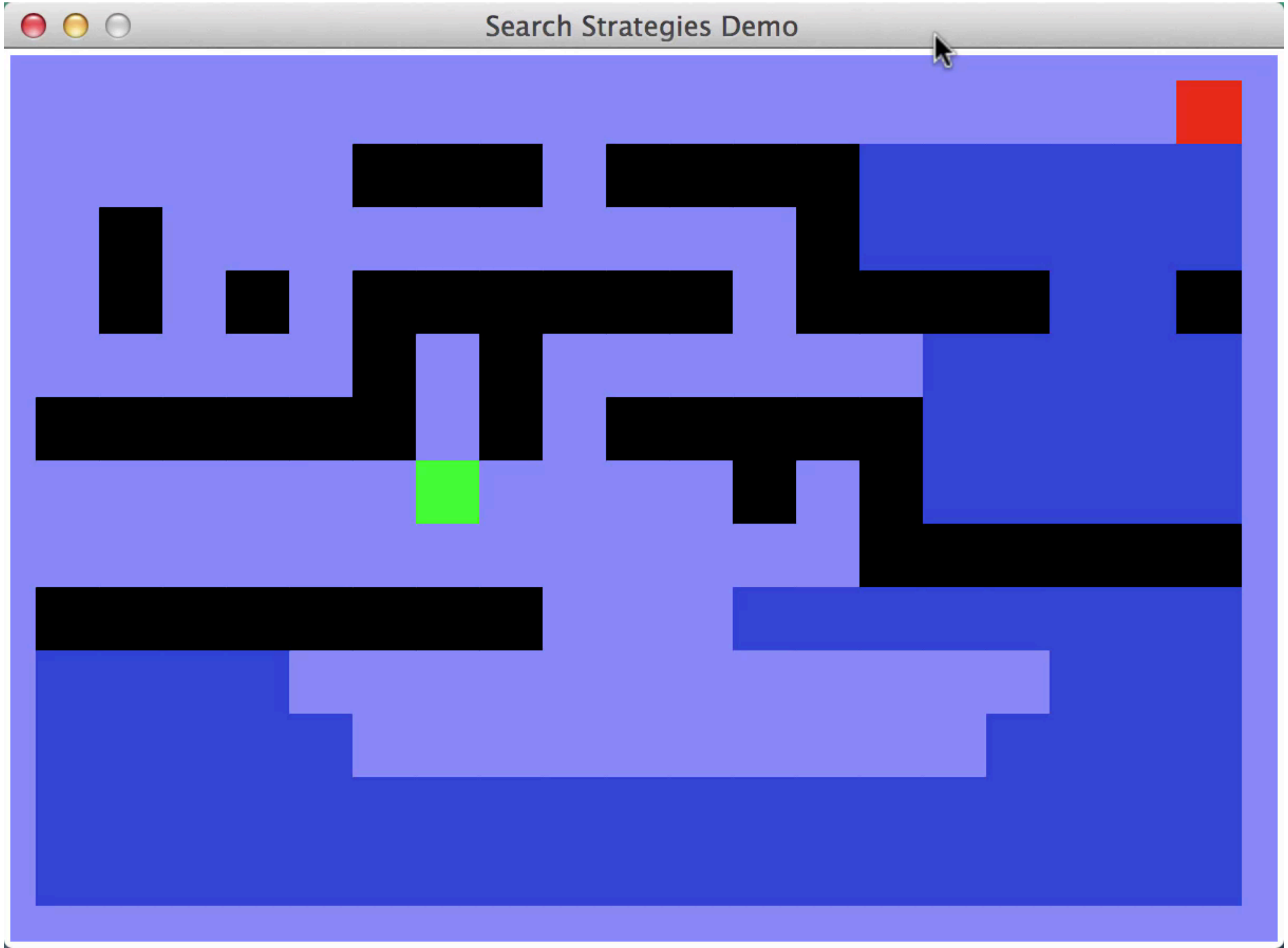
Video of Demo (Empty Shallow/Deep) -- UCS



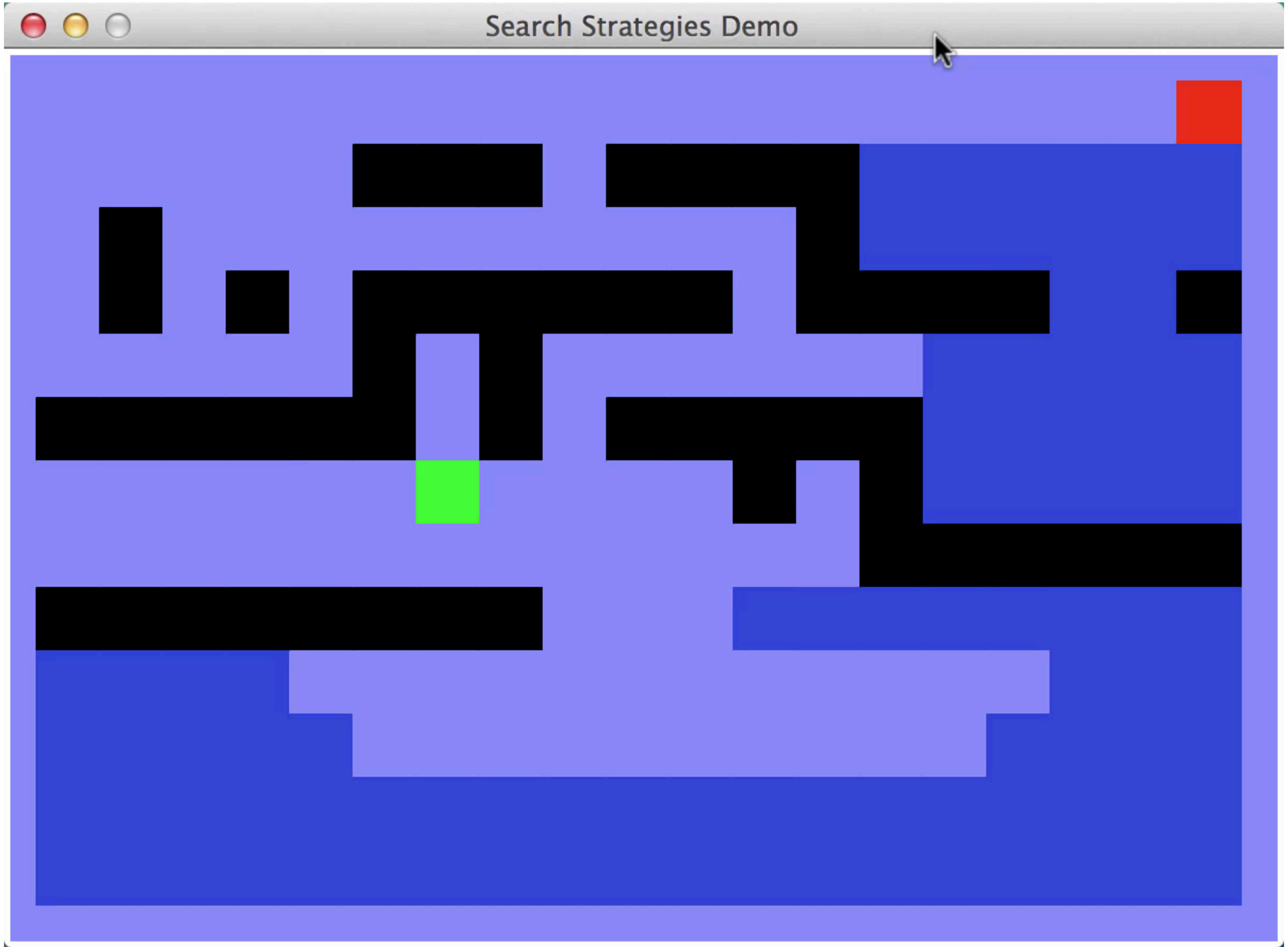
Video of Demo (Empty Shallow/Deep) -- UCS



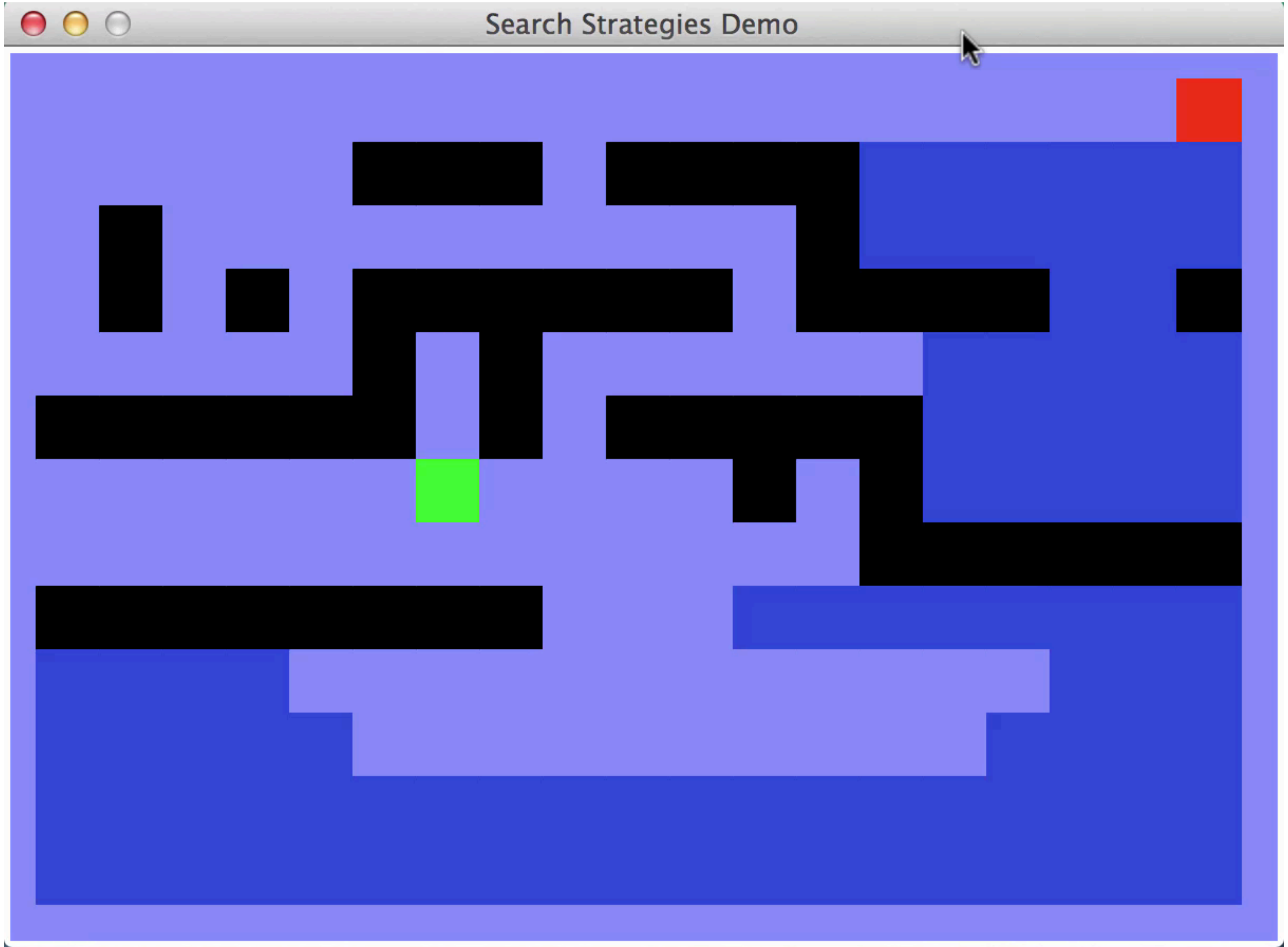
Video of Demo (Empty Shallow/Deep) – Greedy



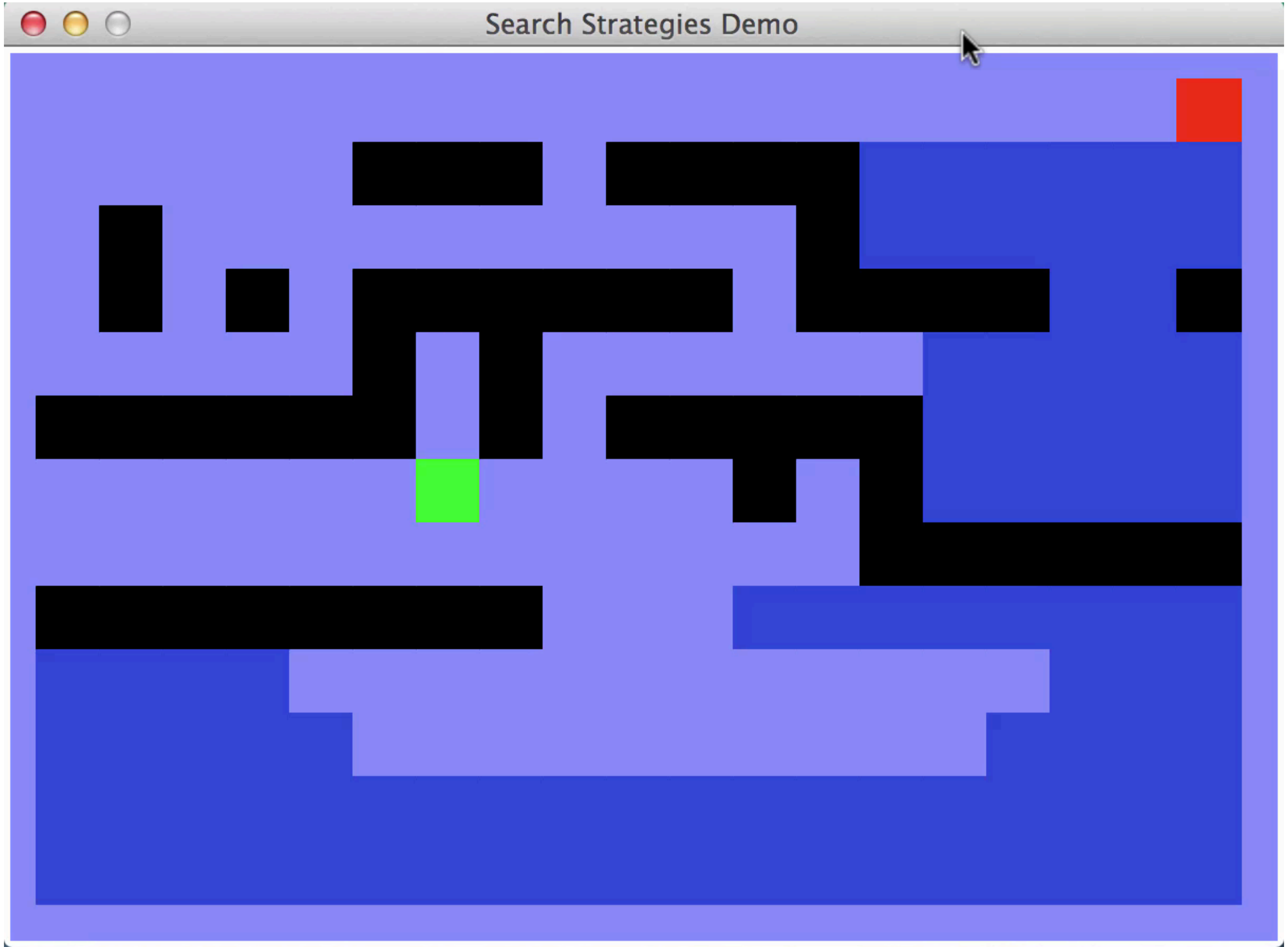
Video of Demo (Empty Shallow/Deep) – Greedy



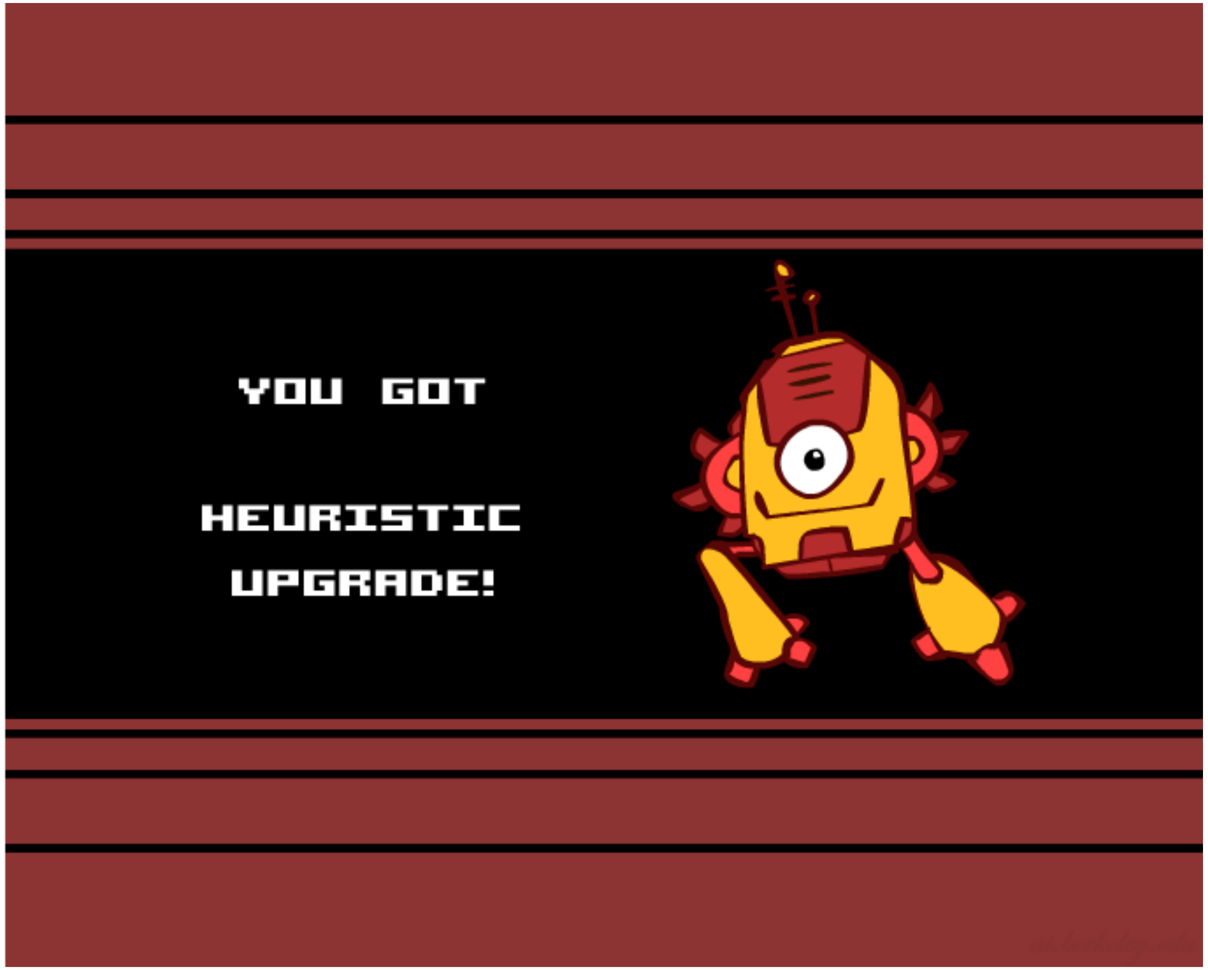
Video of Demo (Empty Shallow/Deep) – A*



Video of Demo (Empty Shallow/Deep) – A*



Creating Heuristics



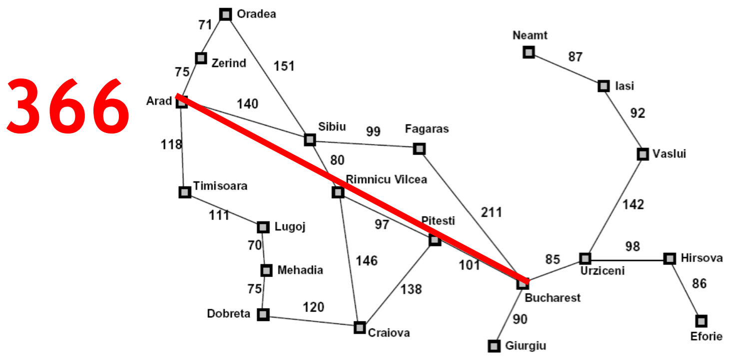
Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

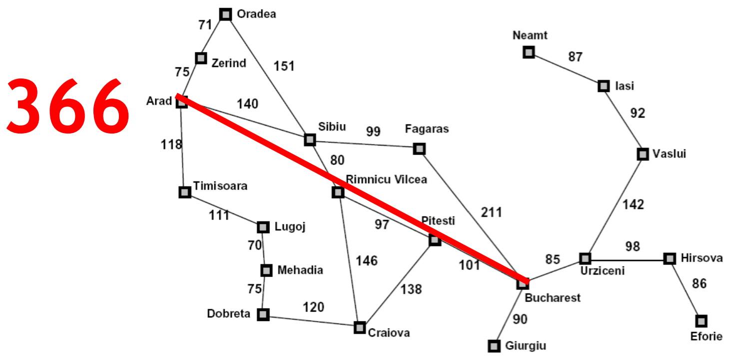
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

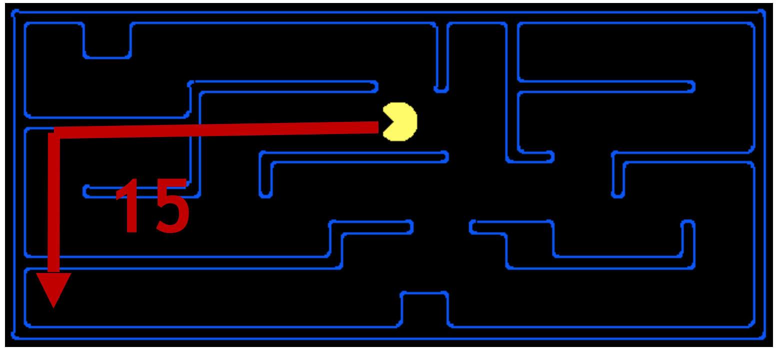


Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available

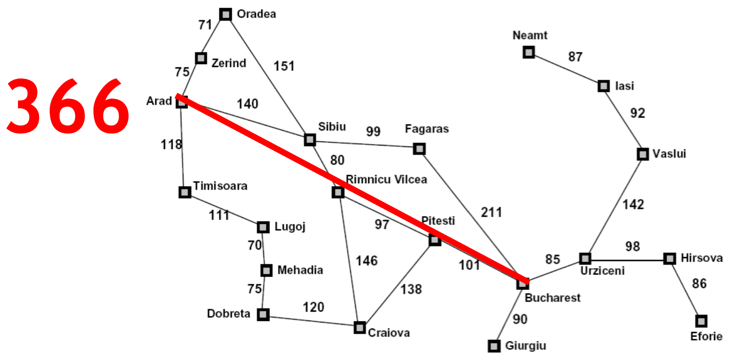


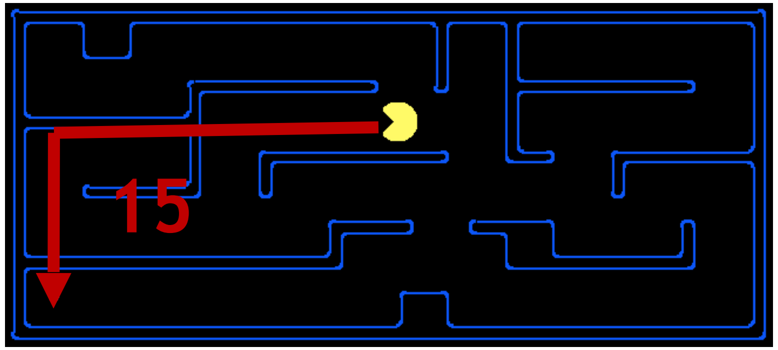


Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

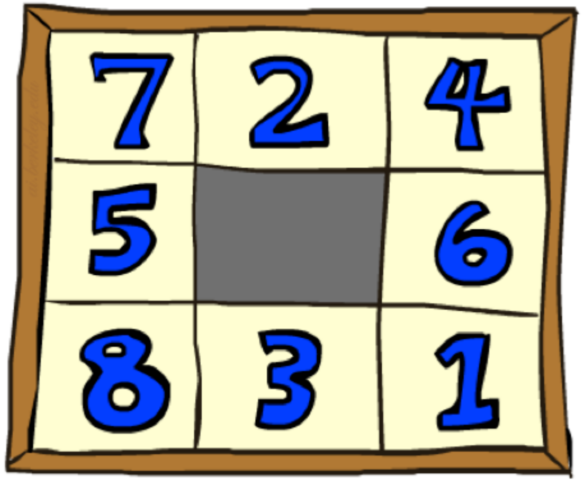
- Often, admissible heuristics are solutions to *relaxed problems*, where new actions are available



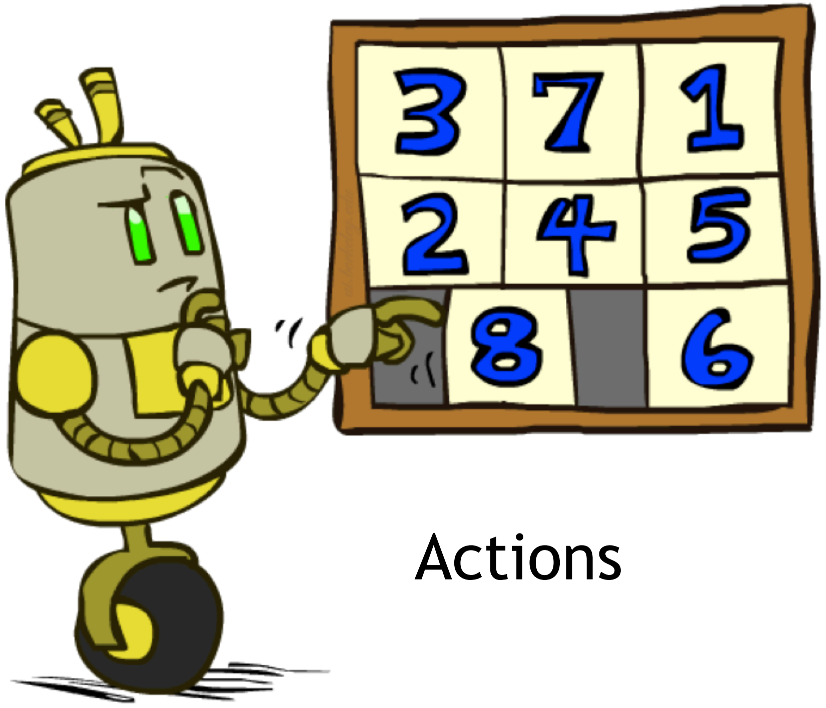


- Inadmissible heuristics are often useful too

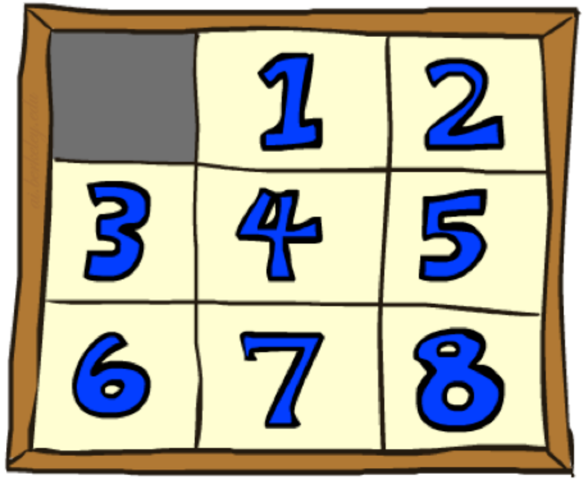
Example: 8 Puzzle



Start State



Actions

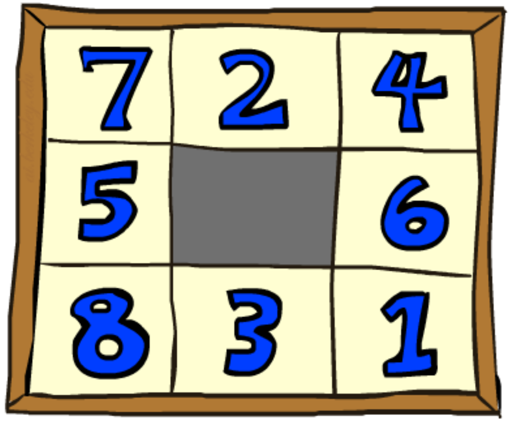


Goal State

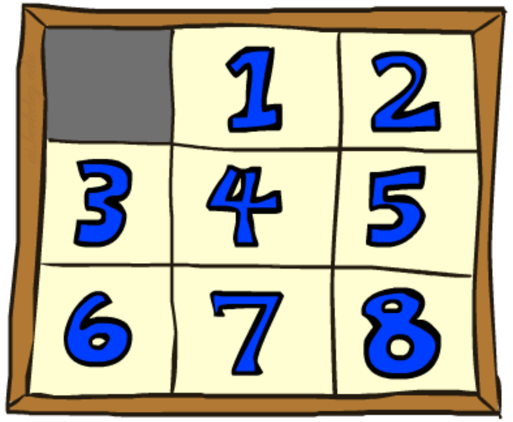
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

- Heuristic: Number of tiles misplaced



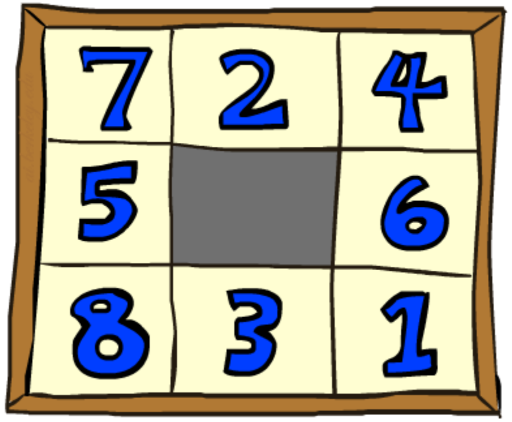
Start State



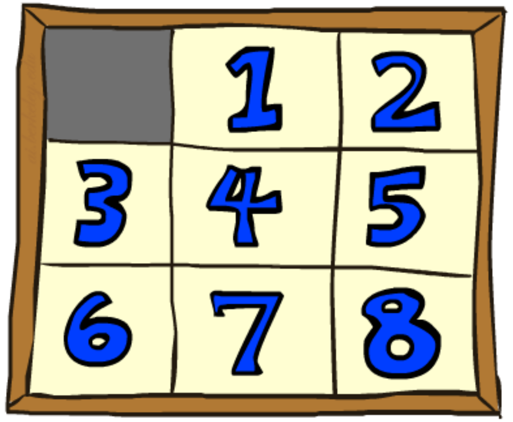
Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?



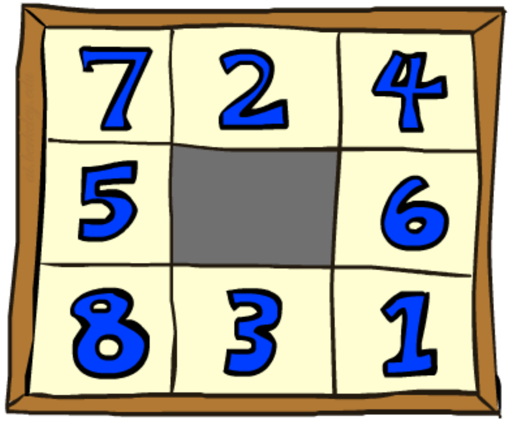
Start State



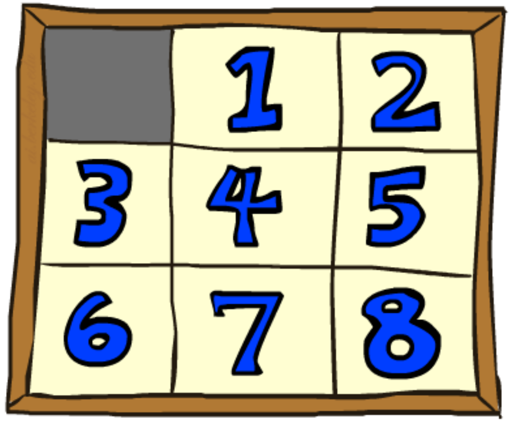
Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =



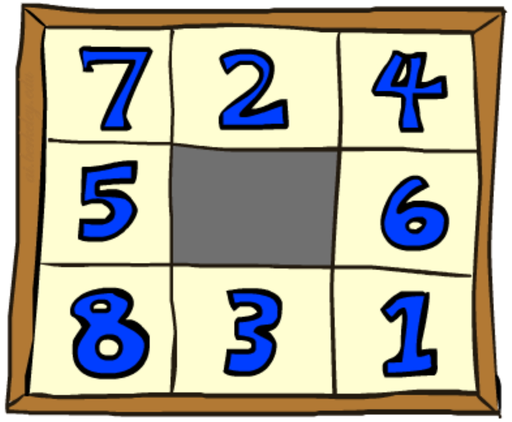
Start State



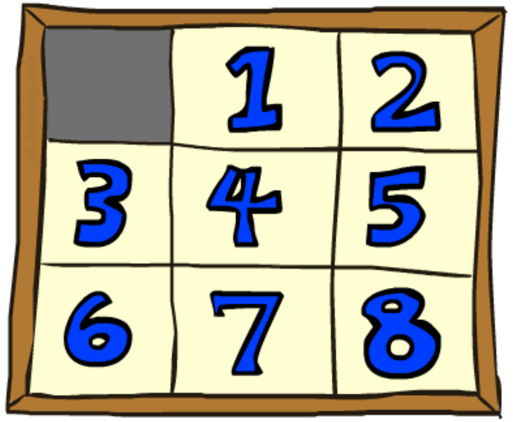
Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =8



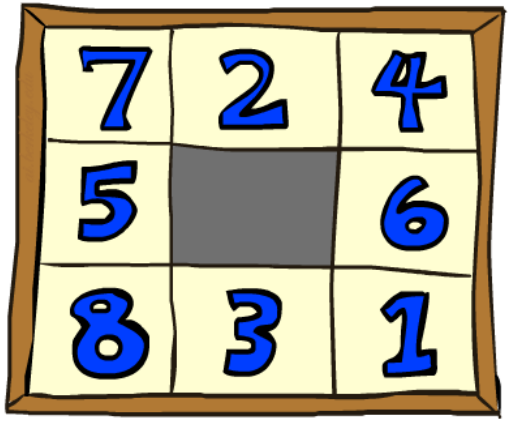
Start State



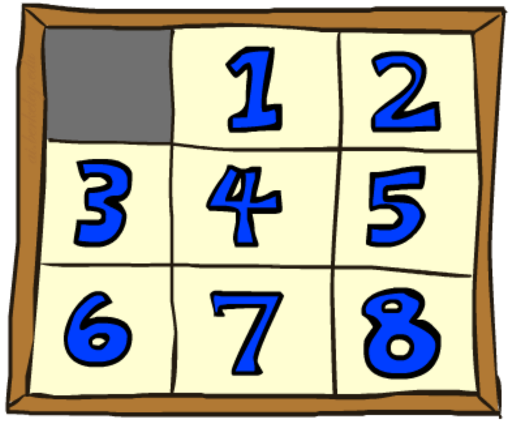
Goal State

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =8



Start State



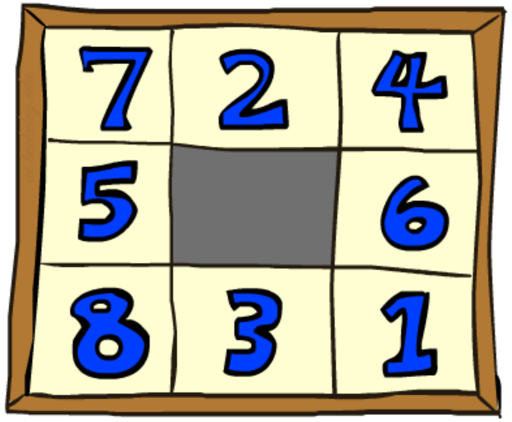
Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

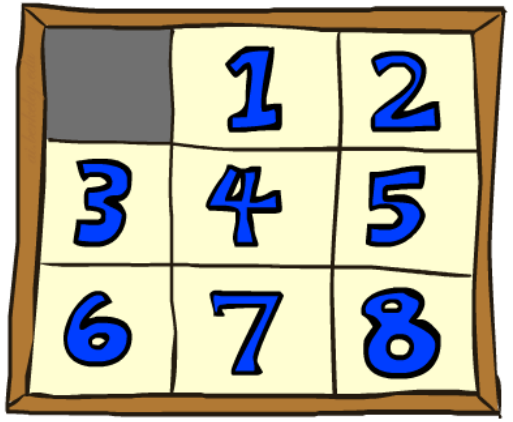
Statistics from Andrew Moore

8 Puzzle I

- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =8
- This is a *relaxed-problem* heuristic



Start State



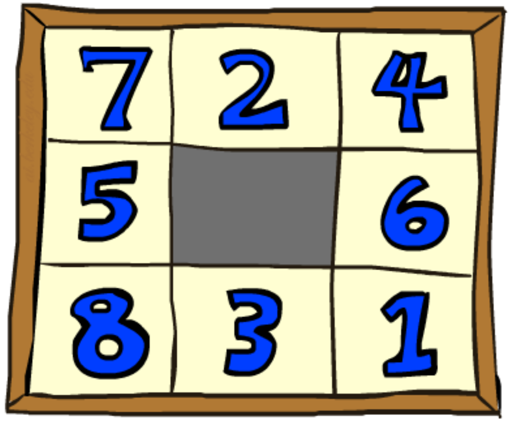
Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

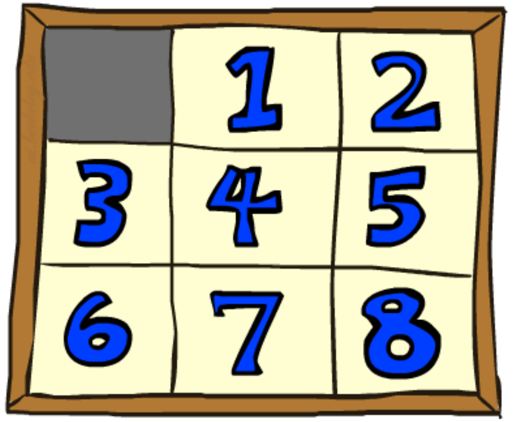
Statistics from Andrew Moore

8 Puzzle I

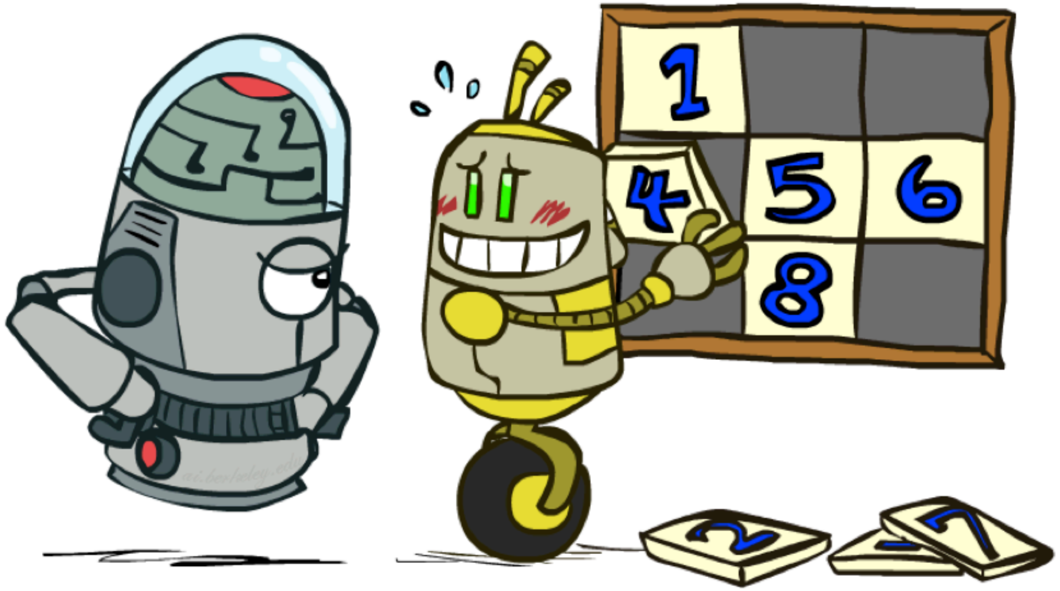
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) =8
- This is a *relaxed-problem* heuristic



Start State



Goal State

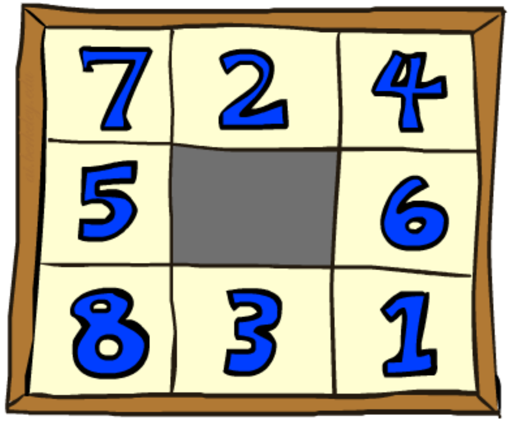


	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

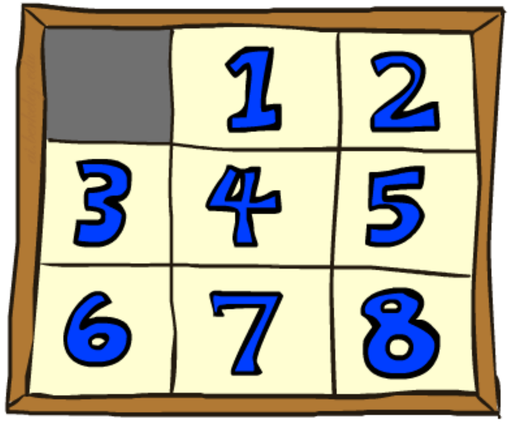
Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?



Start State

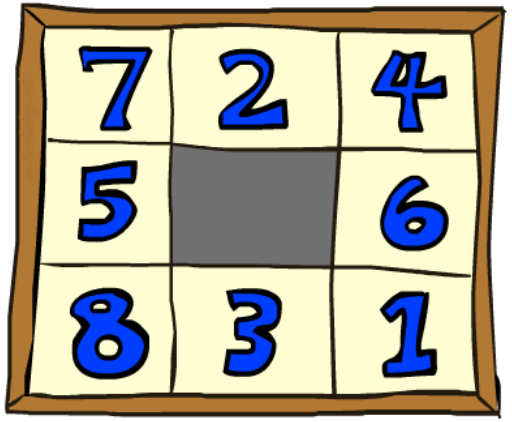


Goal State

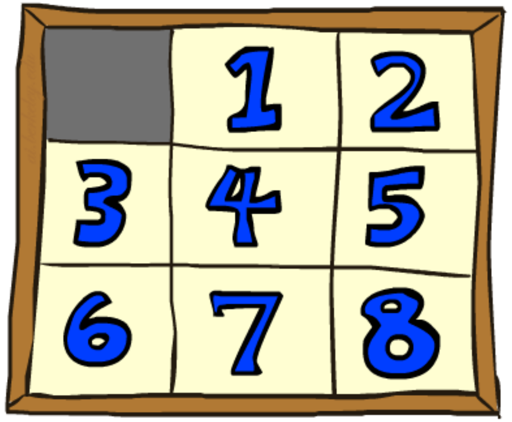
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

- Total *Manhattan* distance



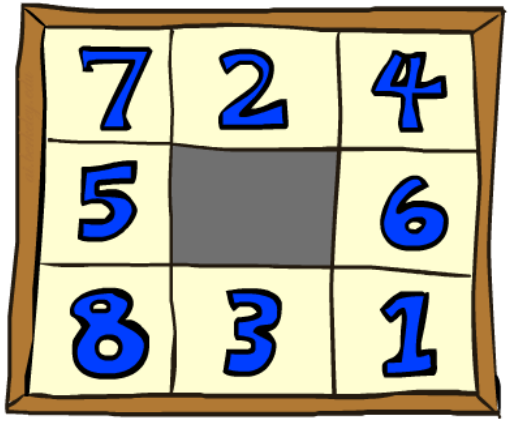
Start State



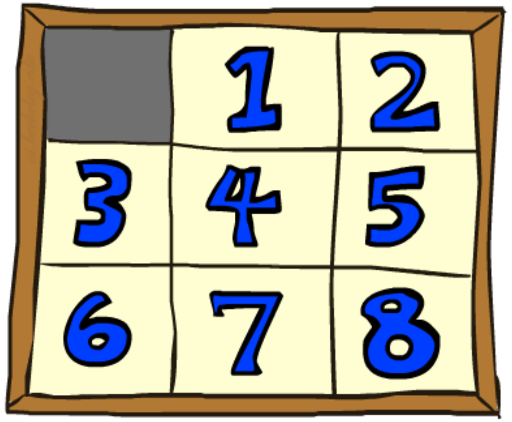
Goal State

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?



Start State



Goal State

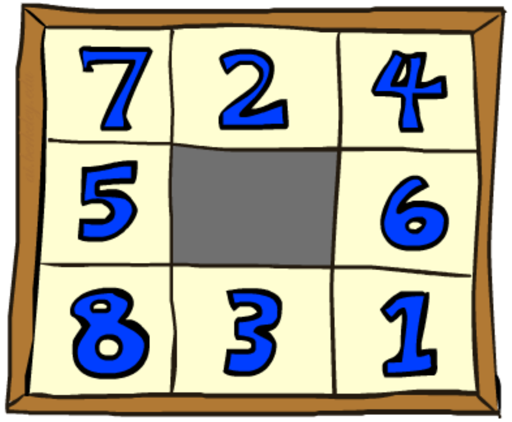
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

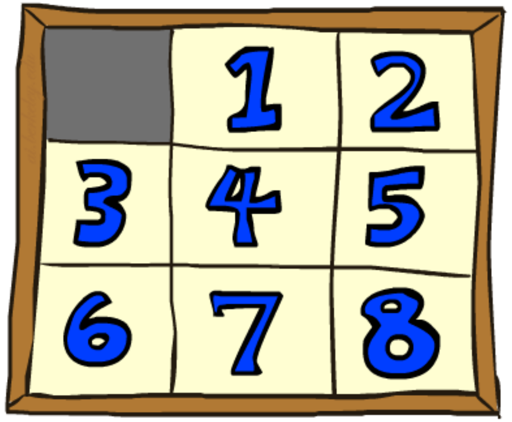
- Total *Manhattan* distance

- Why is it admissible?

- h(start) =



Start State



Goal State

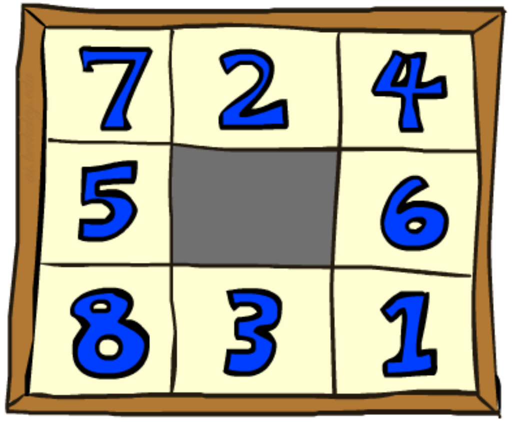
8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?

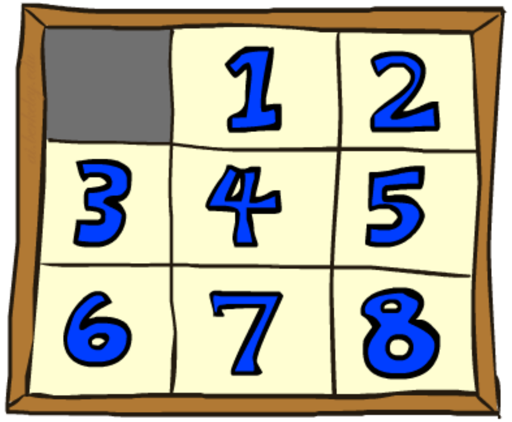
- Total *Manhattan* distance

- Why is it admissible?

- h(start) = 3 + 1 + 2 + … = 18



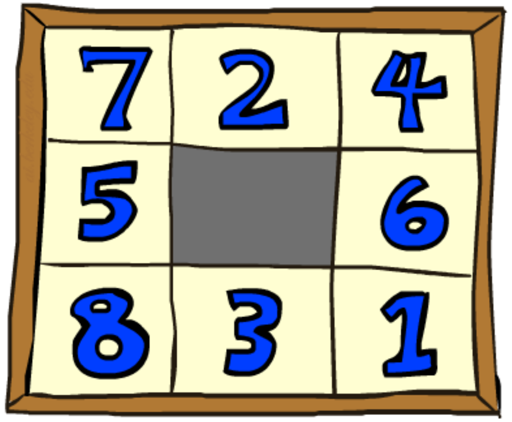
Start State



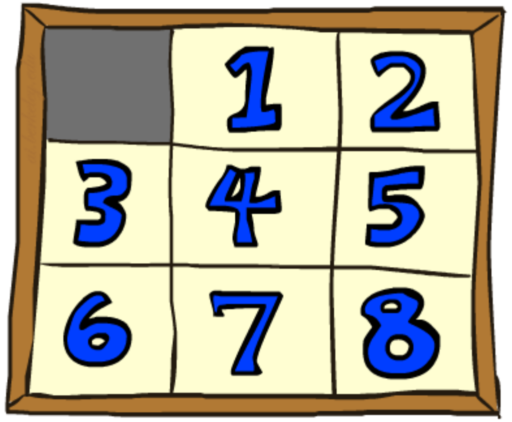
Goal State

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State



Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What’s wrong with it?

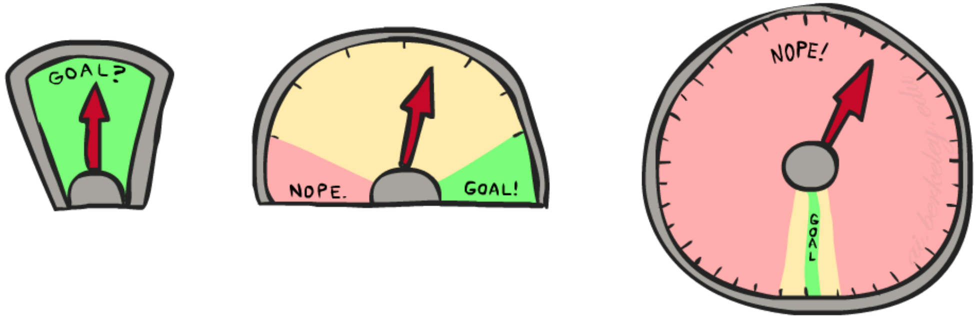
8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?

- With A*: a trade-off between quality of estimate and work per node

8 Puzzle III

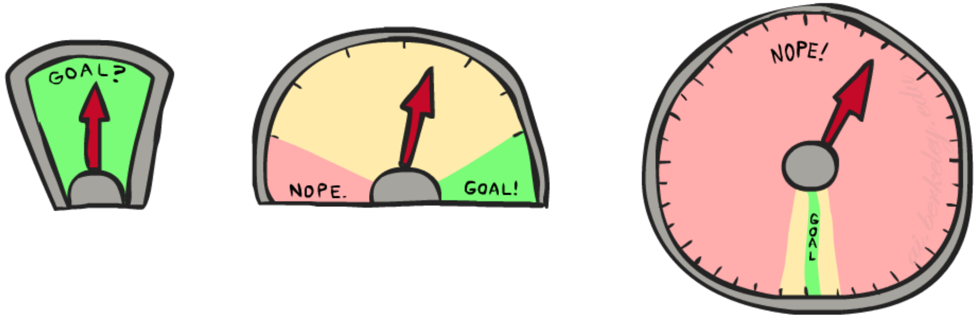
- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Semi-Lattice of Heuristics

Trivial Heuristics, Dominance

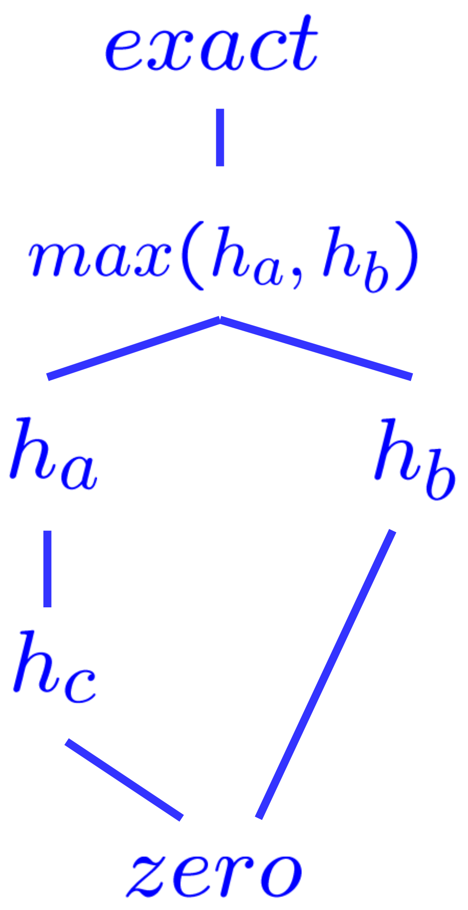
- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

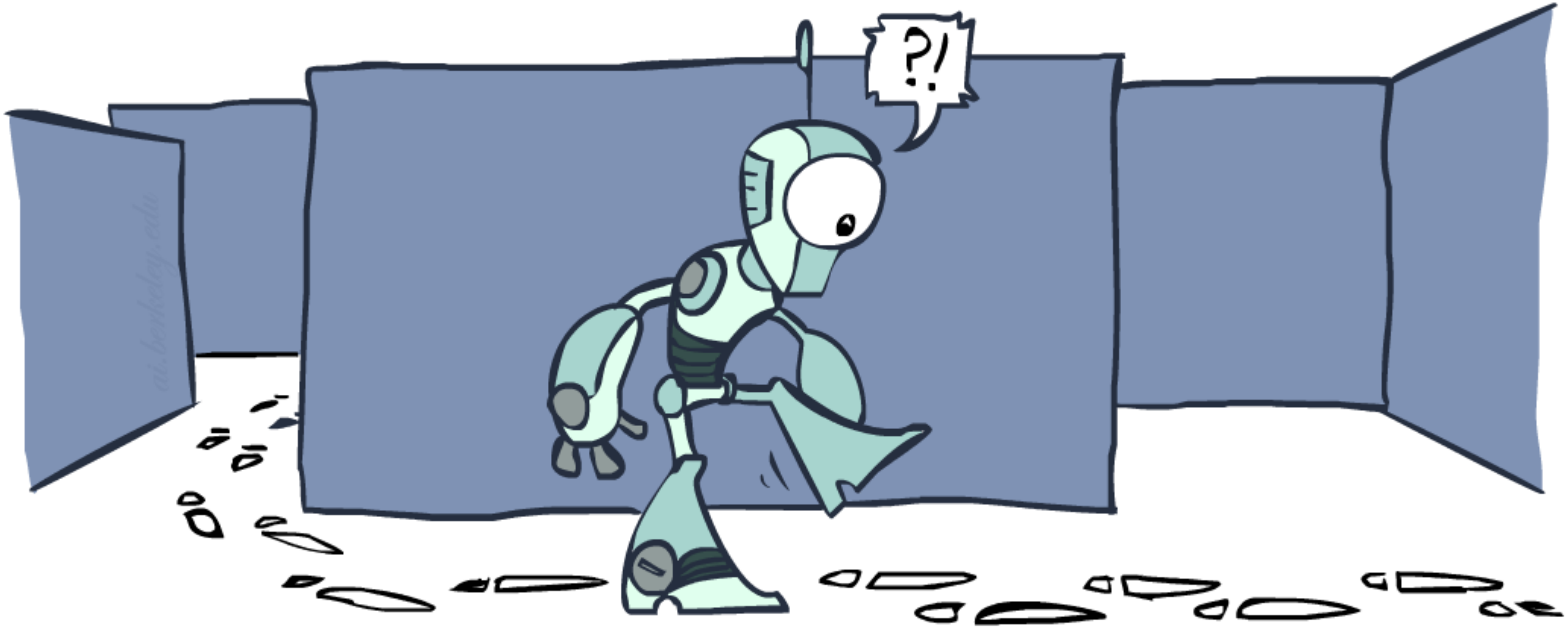
- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic



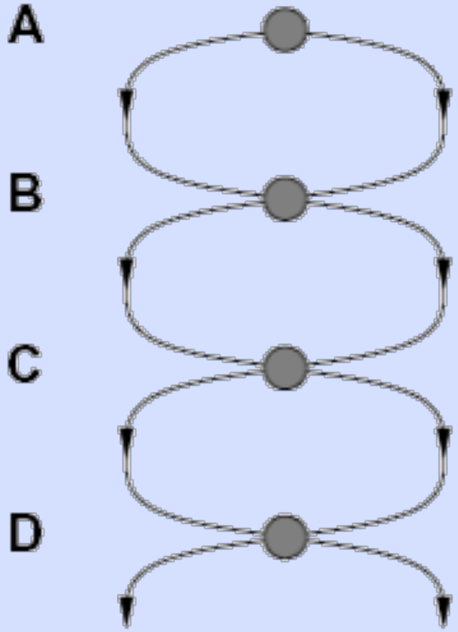
Graph Search



Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.

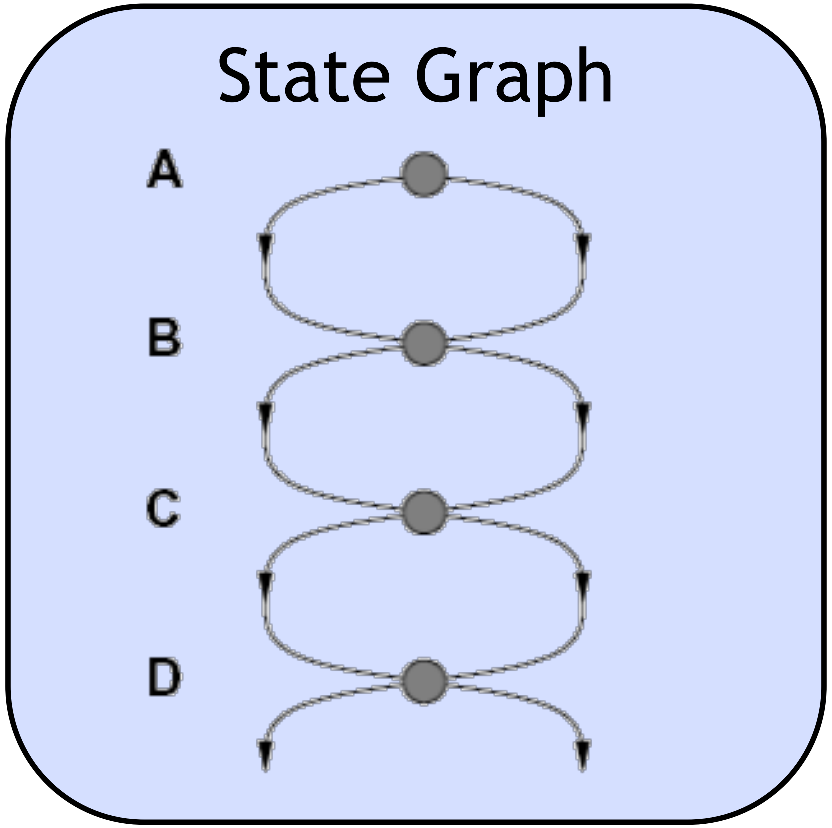
State Graph

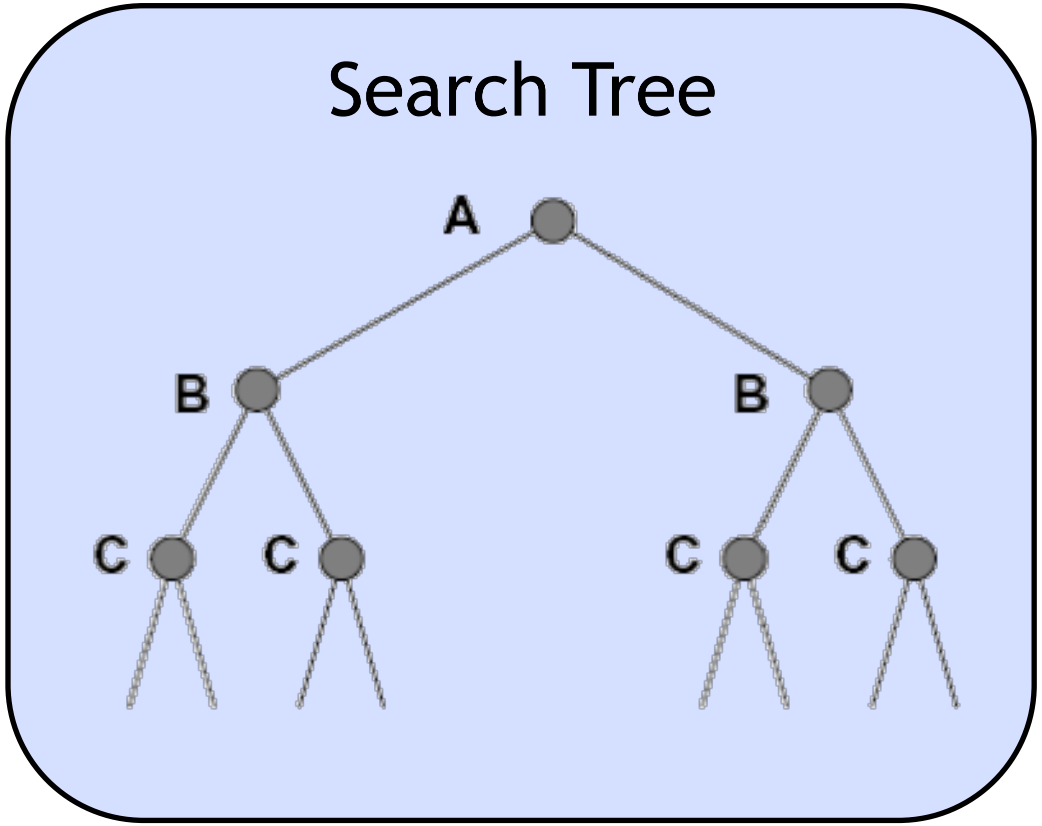


Search Tree

Tree Search: Extra Work!

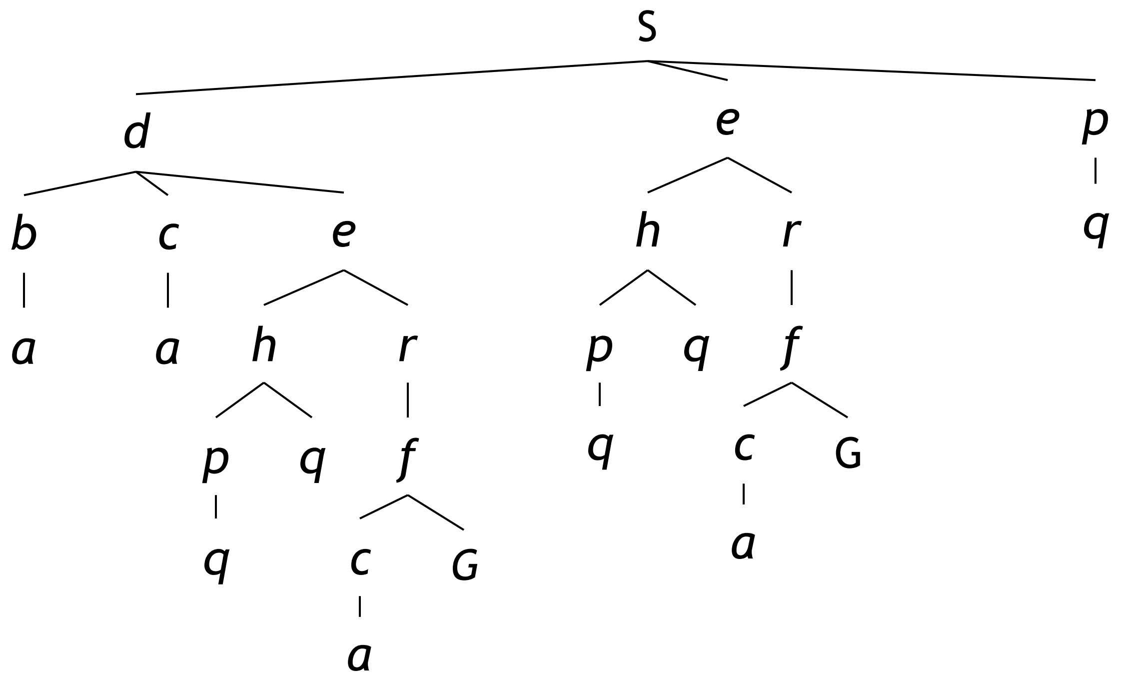
- Failure to detect repeated states can cause exponentially more work.





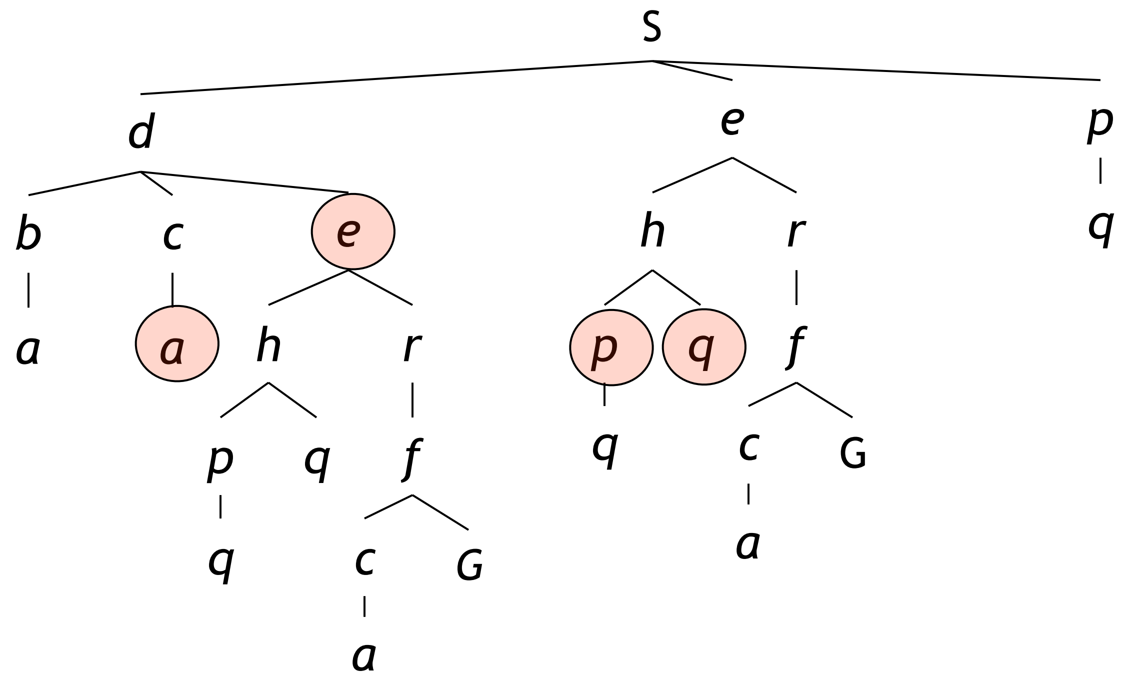
Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

- Idea: never expand a state twice

Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set

Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list

Graph Search

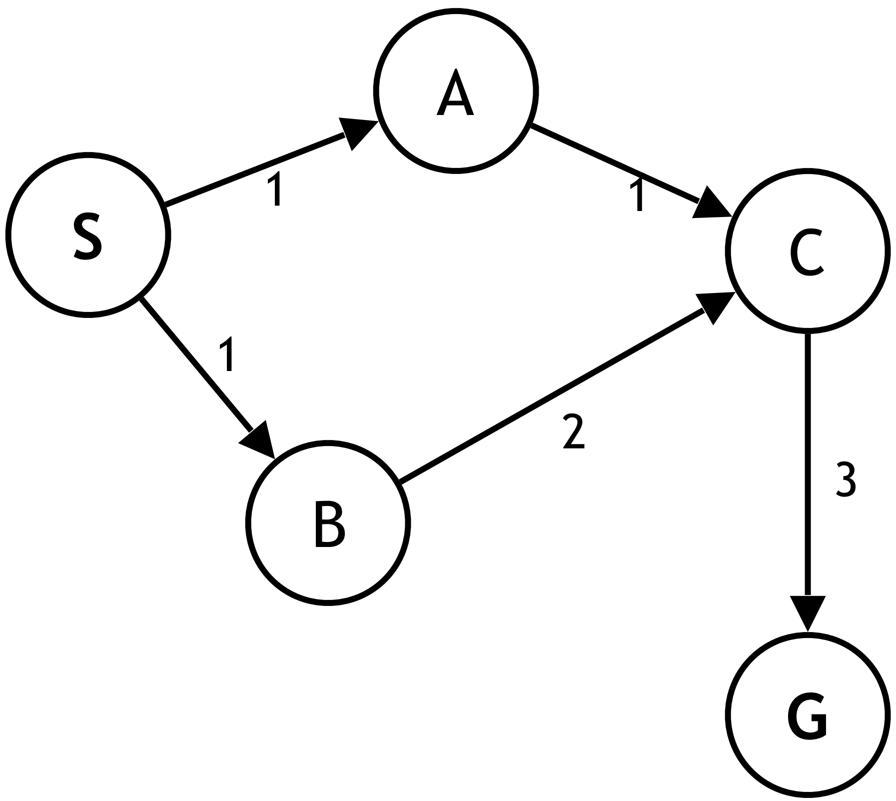
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?

Graph Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states (“closed set”)
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

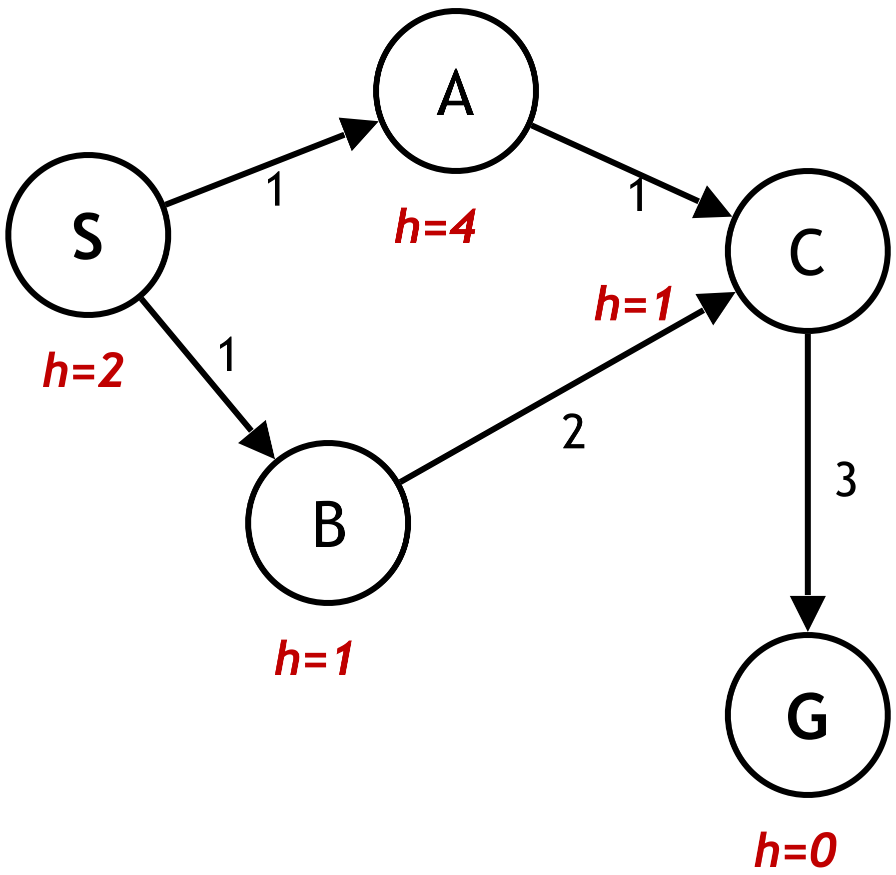
A* Graph Search Gone Wrong?

State space graph



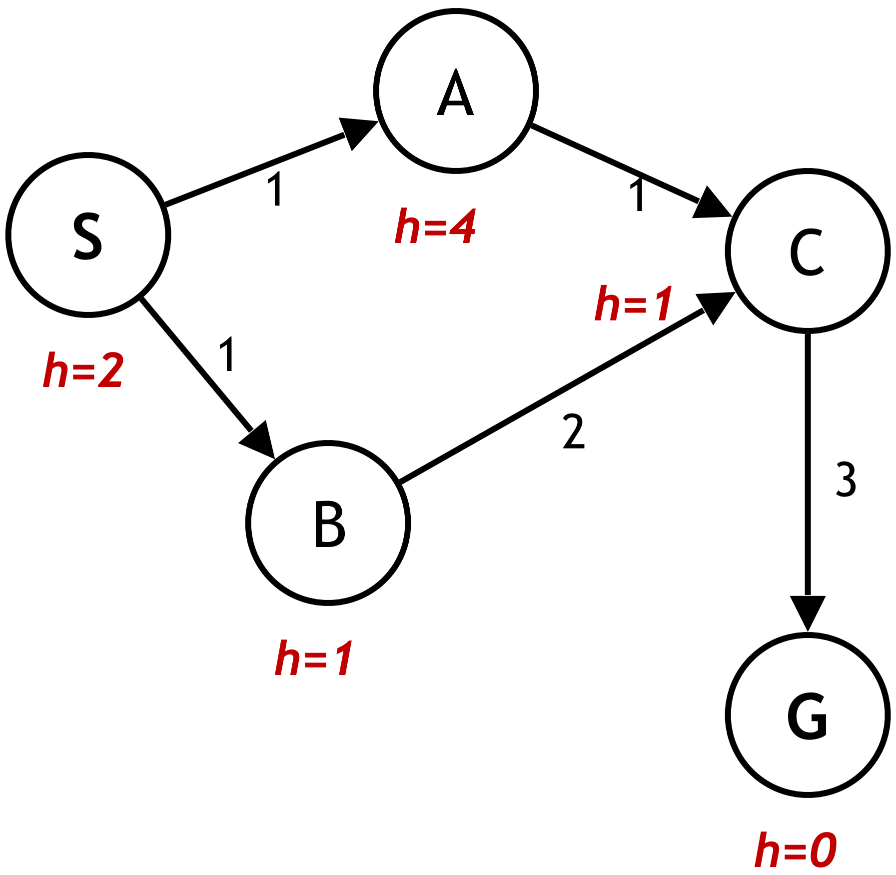
A* Graph Search Gone Wrong?

State space graph



A* Graph Search Gone Wrong?

State space graph

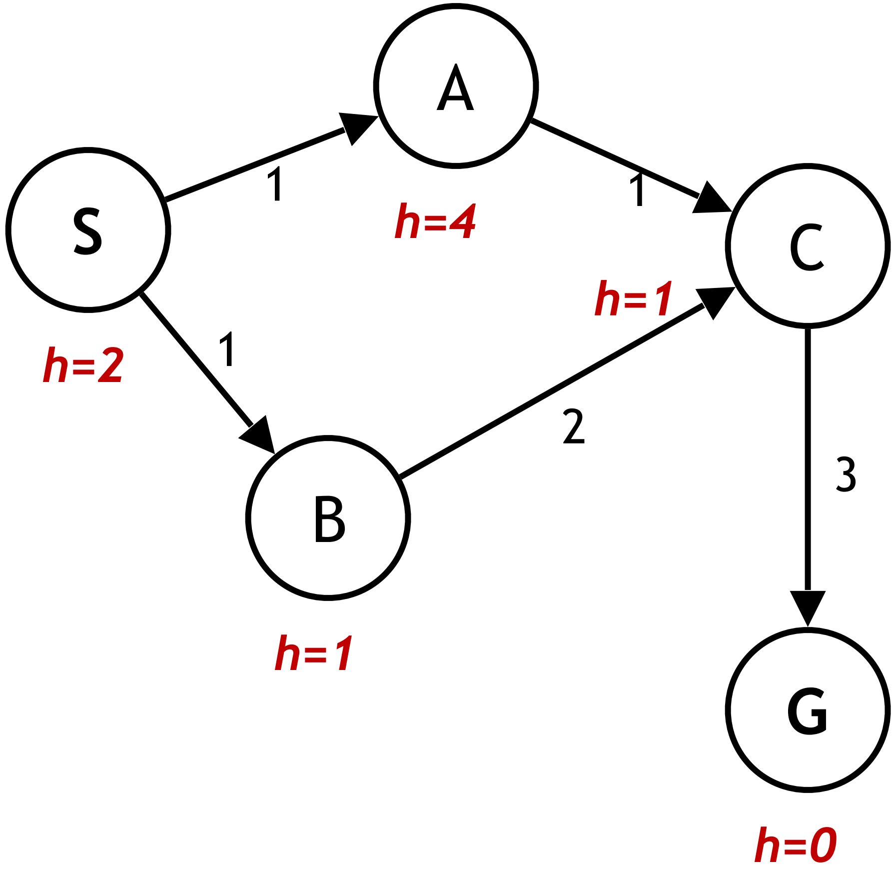


Search tree

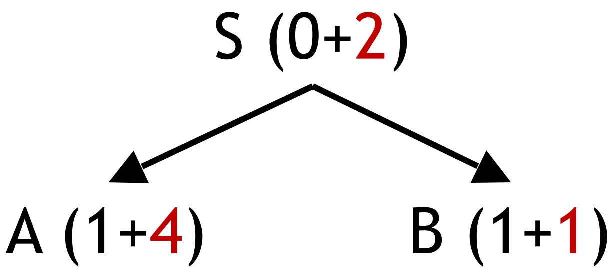
S (0+2)

A* Graph Search Gone Wrong?

State space graph

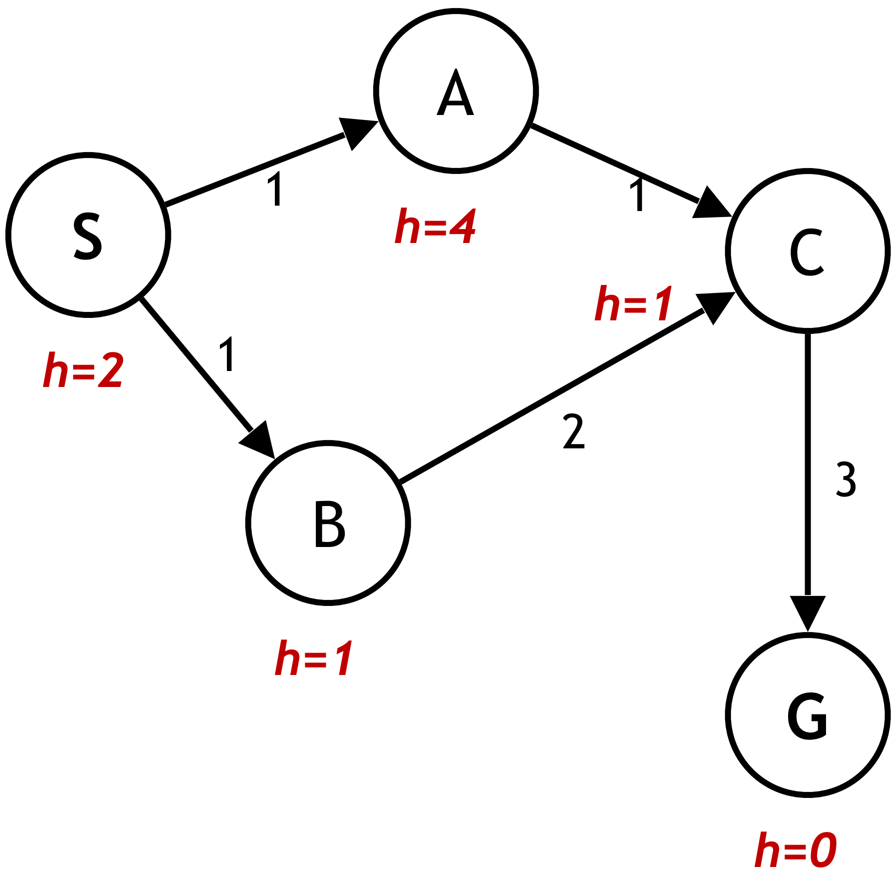


Search tree

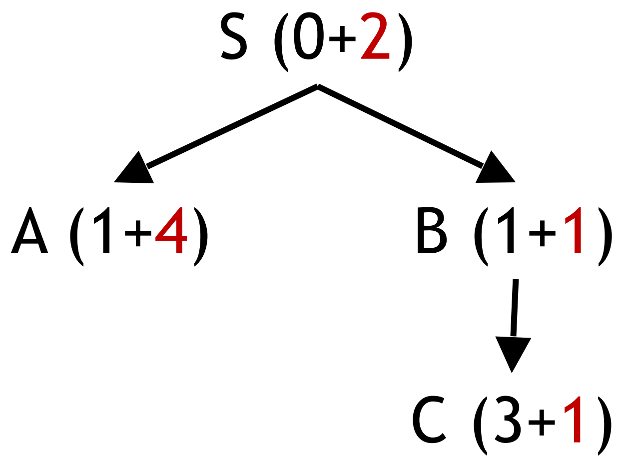


A* Graph Search Gone Wrong?

State space graph

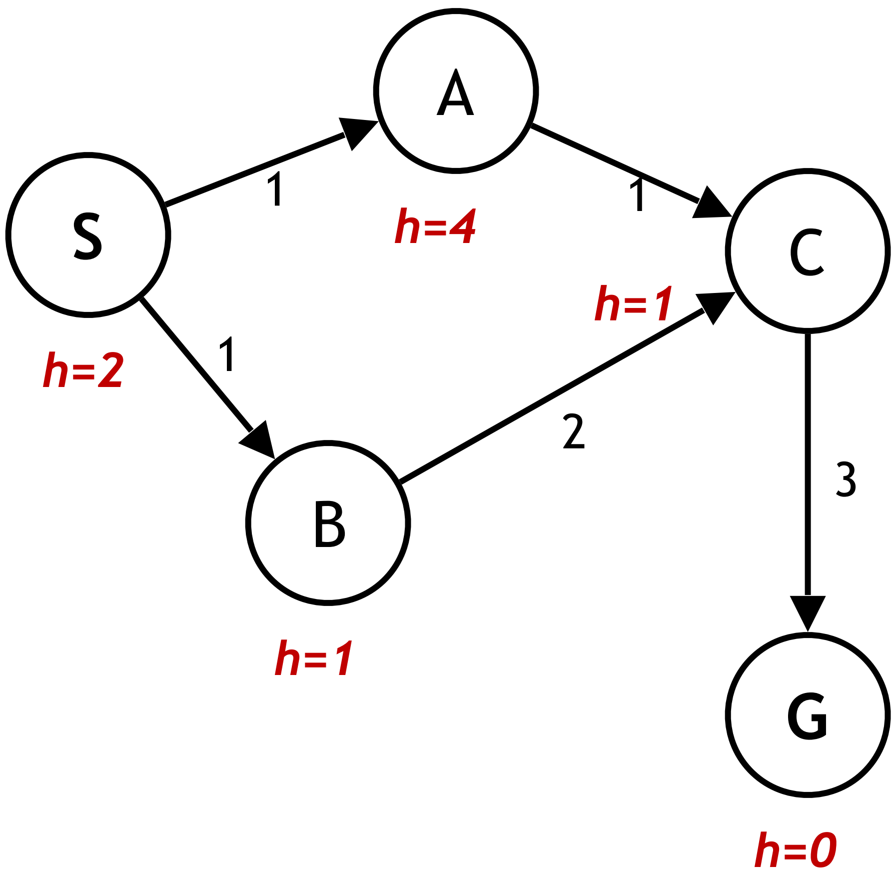


Search tree

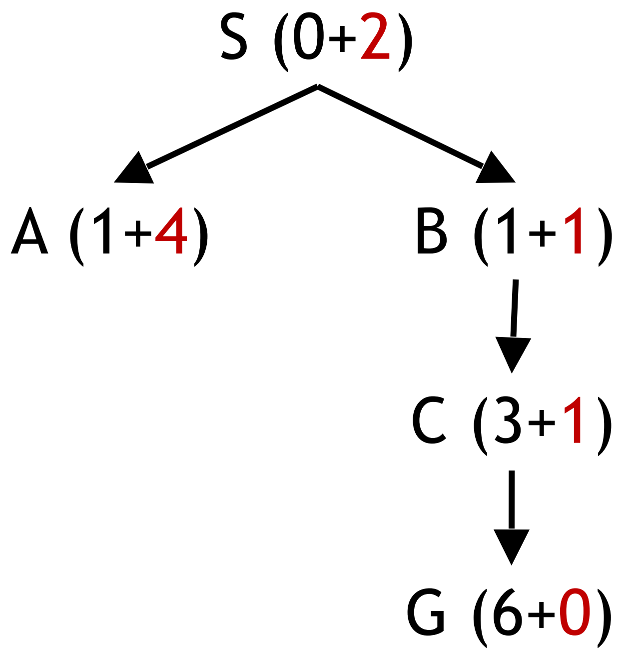


A* Graph Search Gone Wrong?

State space graph

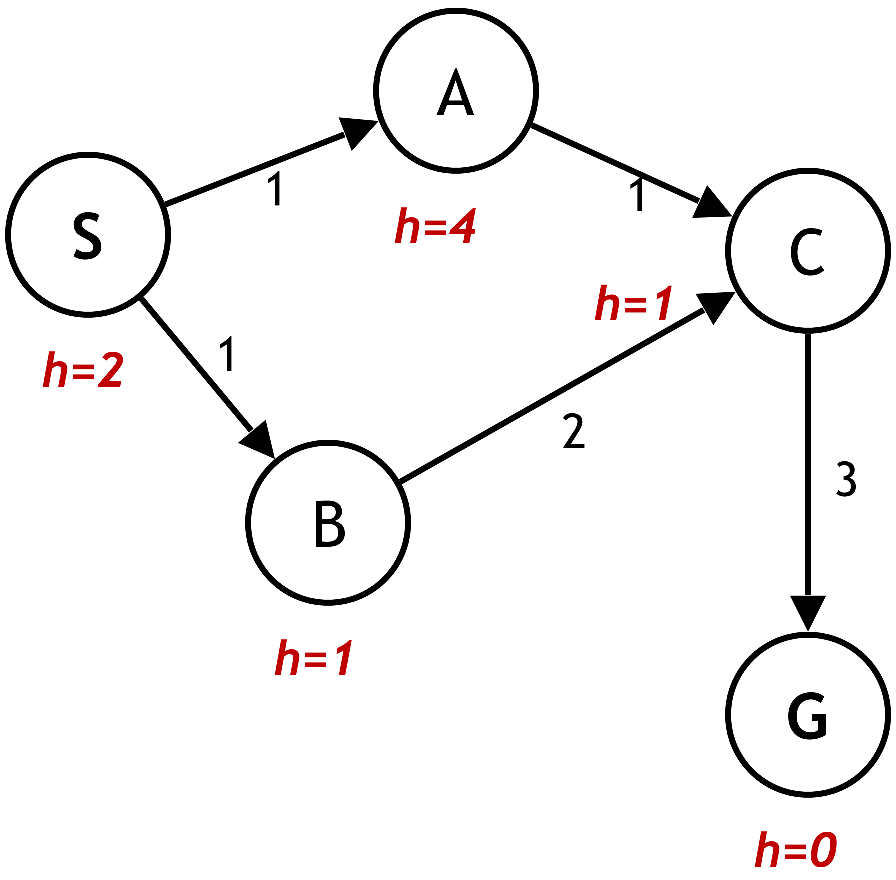


Search tree

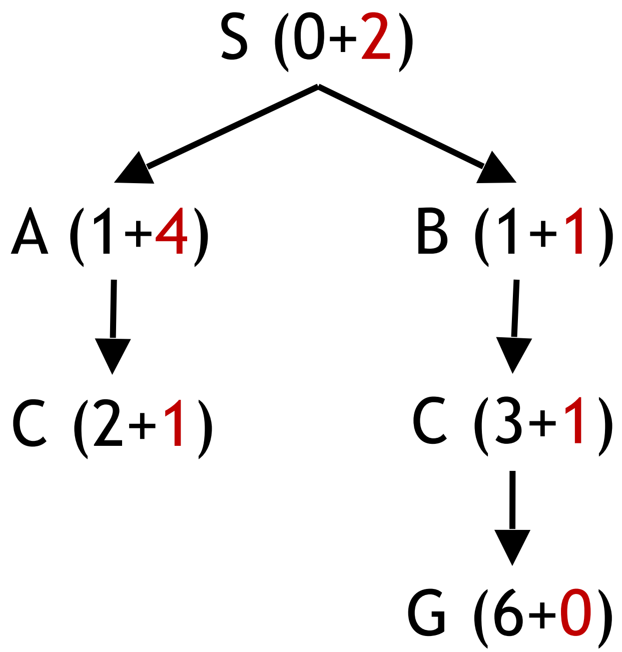


A* Graph Search Gone Wrong?

State space graph

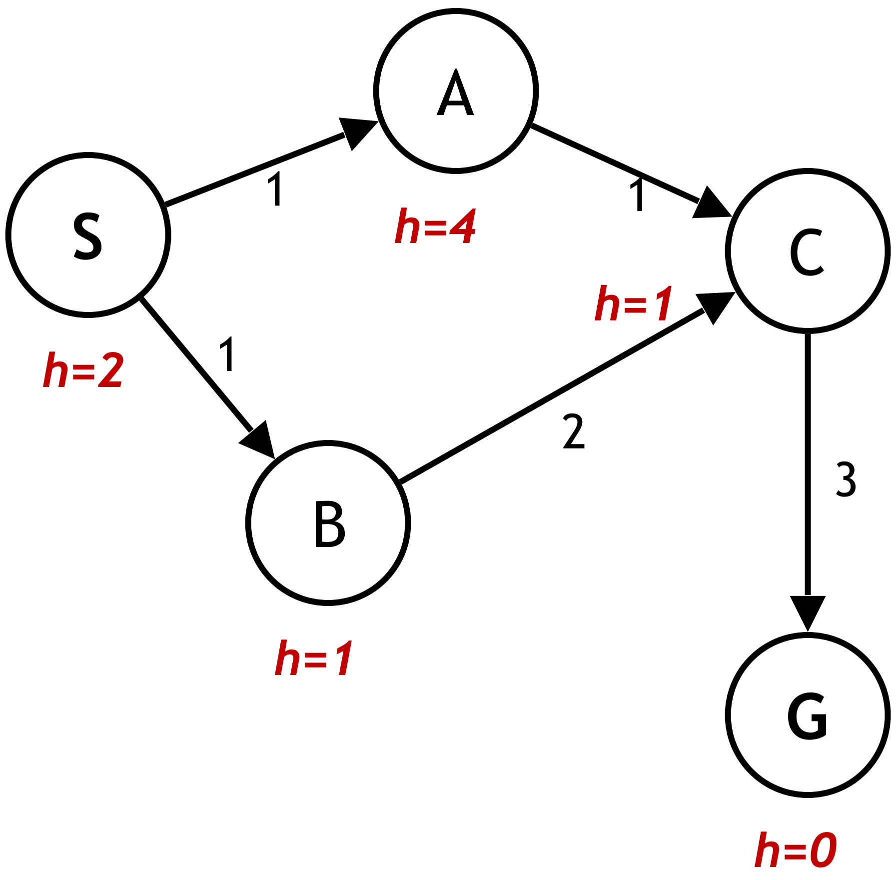


Search tree

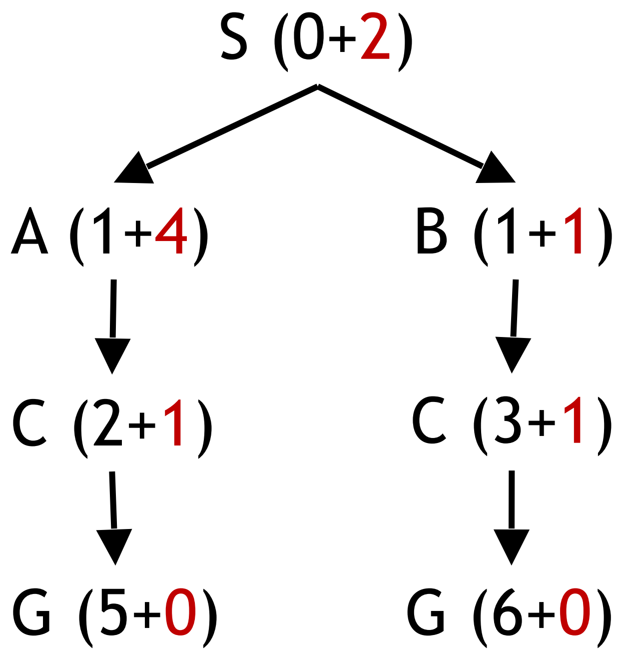


A* Graph Search Gone Wrong?

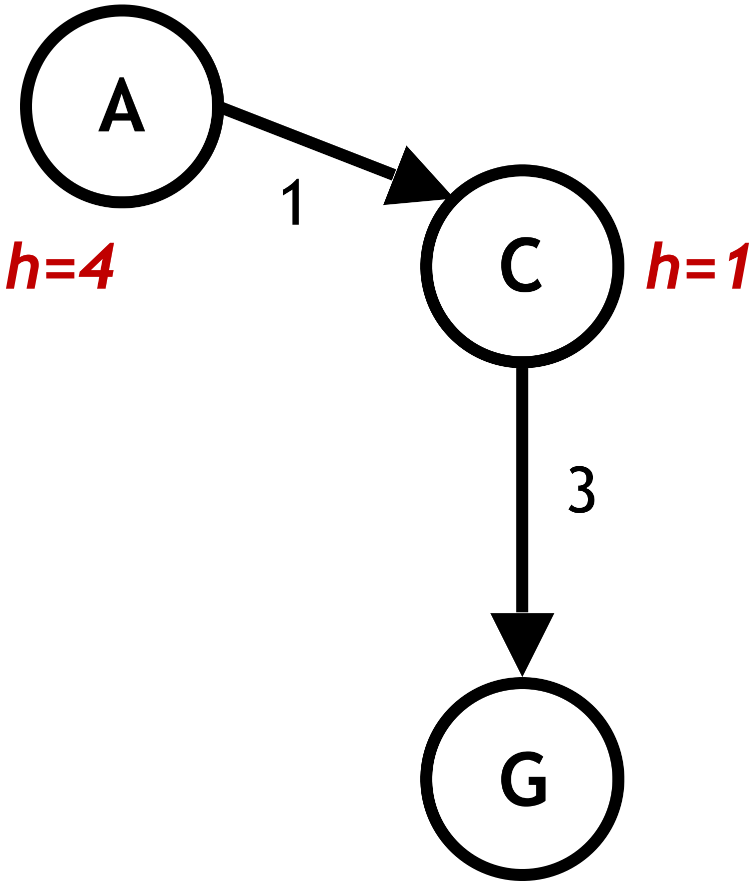
State space graph



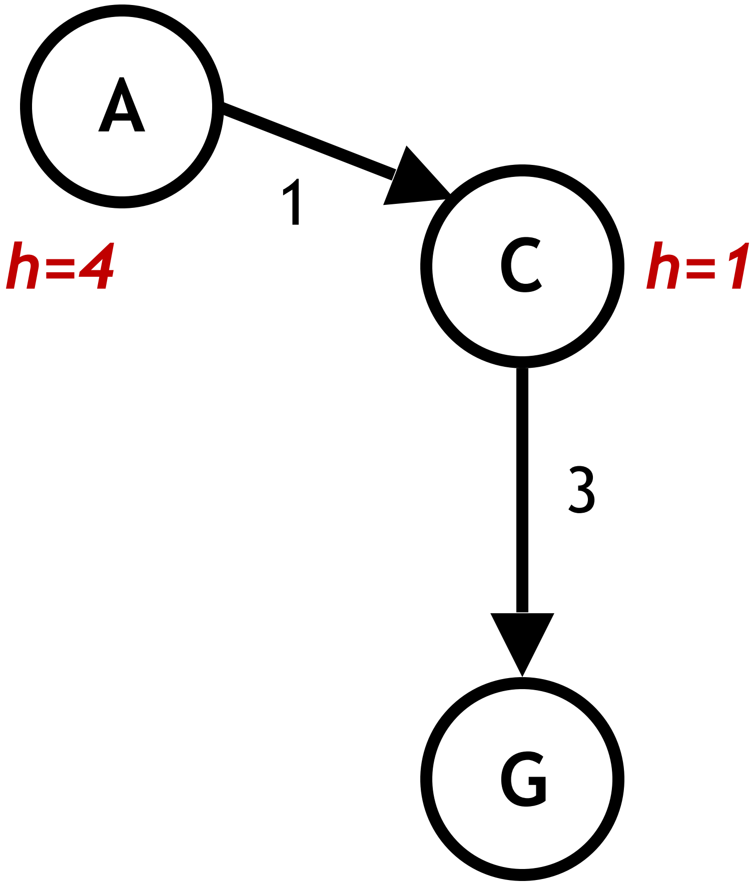
Search tree



Consistency of Heuristics



Consistency of Heuristics



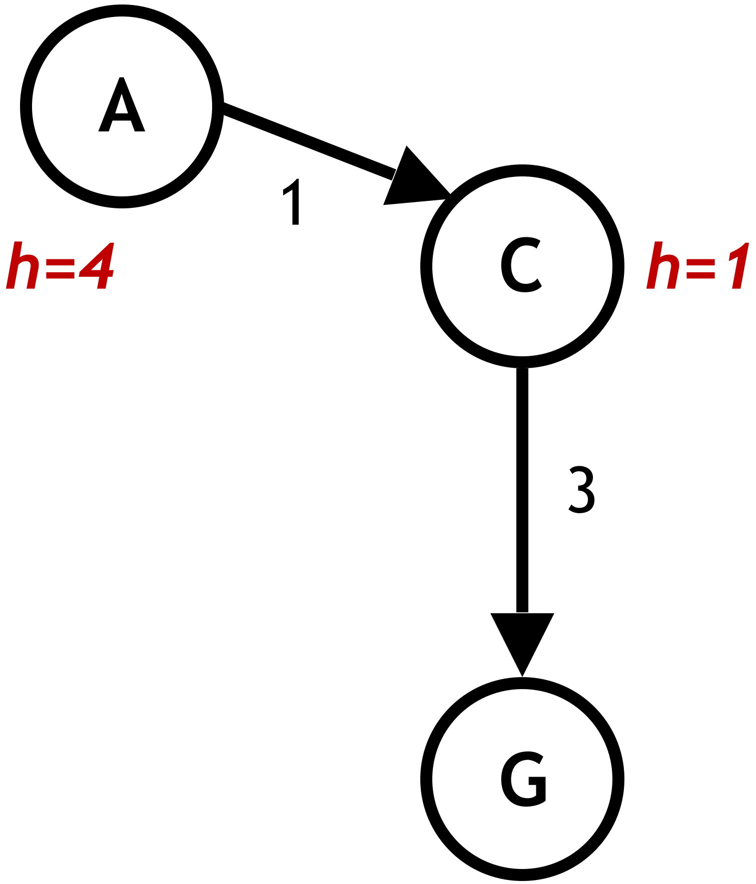
- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

Consistency of Heuristics



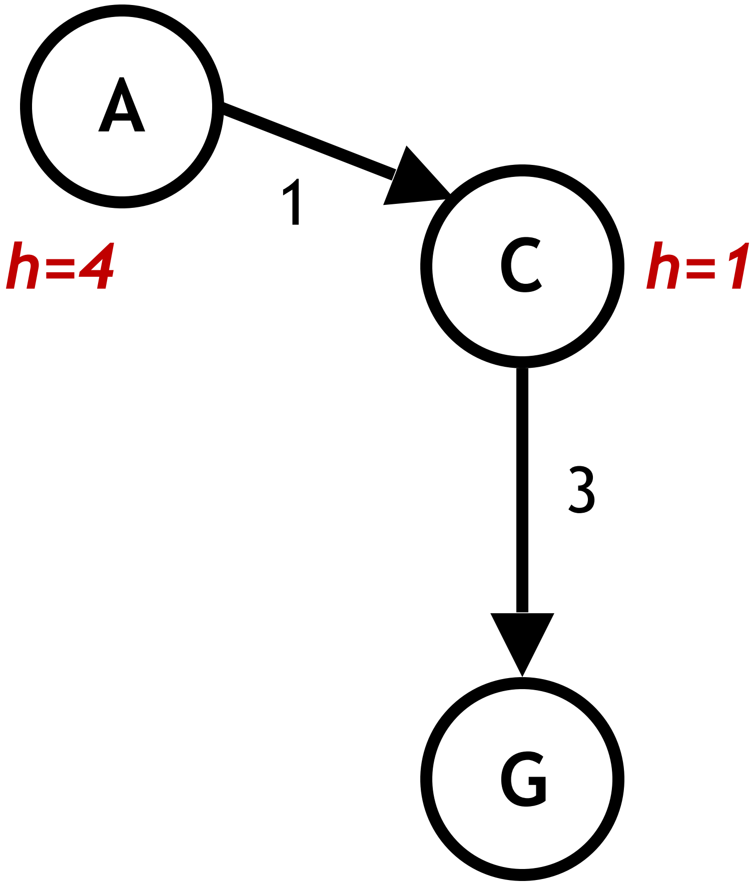
- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic “arc” cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic “arc” cost ≤ actual cost for each arc

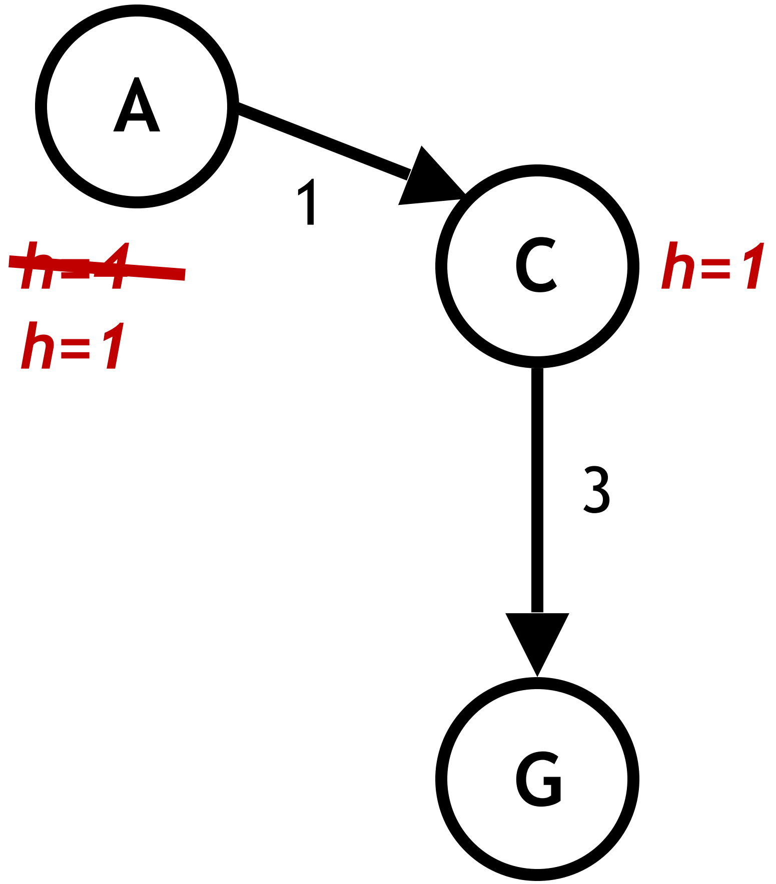
    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

  - A* graph search is optimal

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

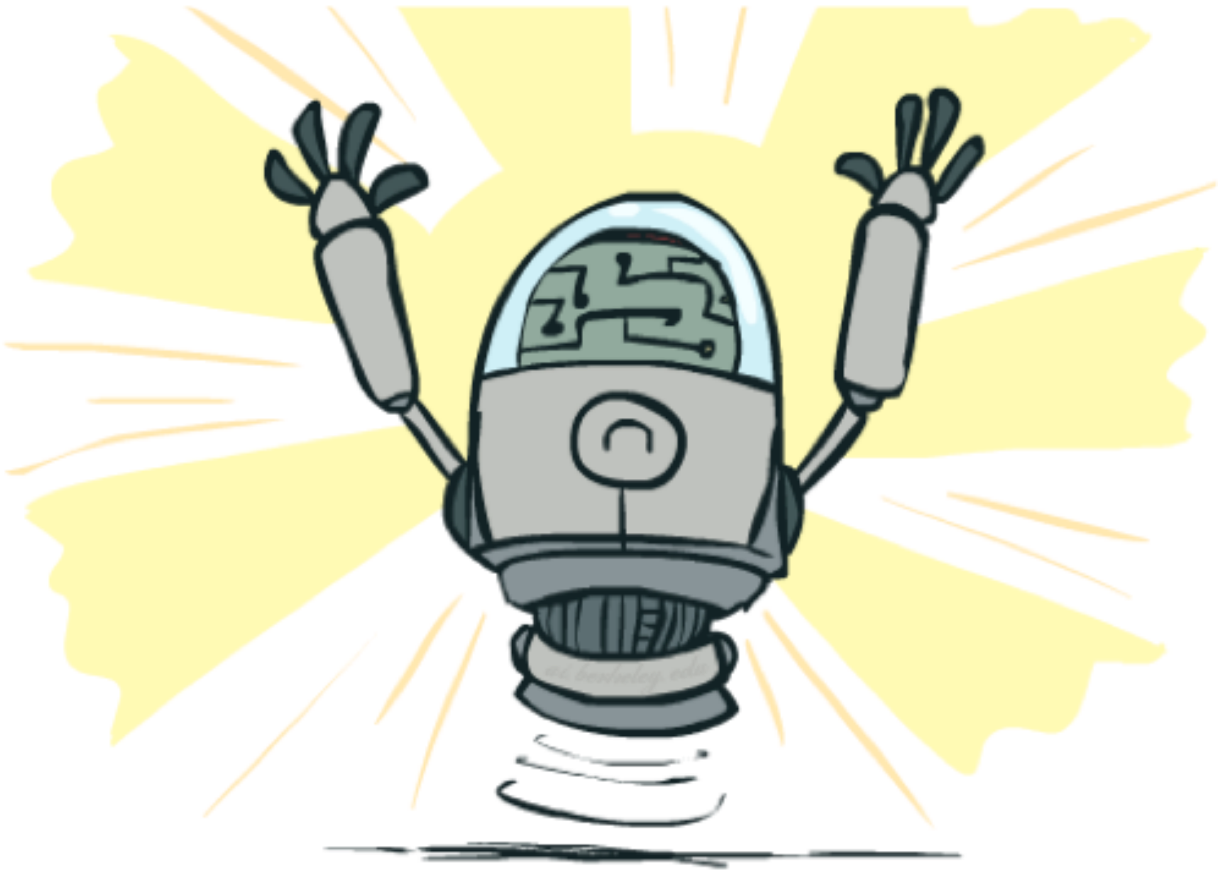
    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

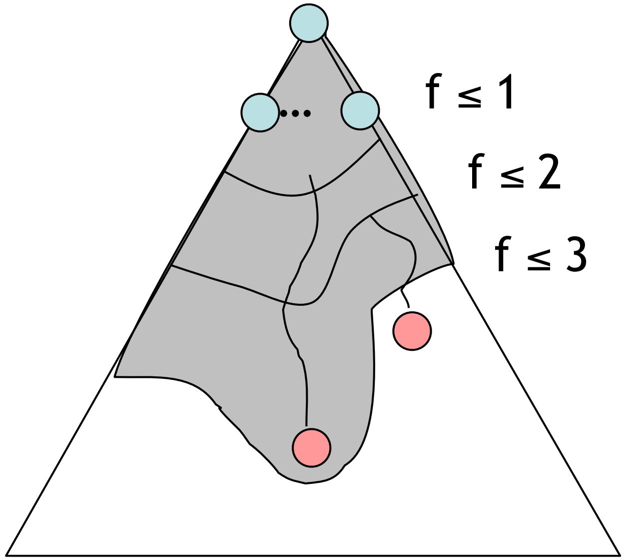
  - A* graph search is optimal

Optimality of A* Graph Search



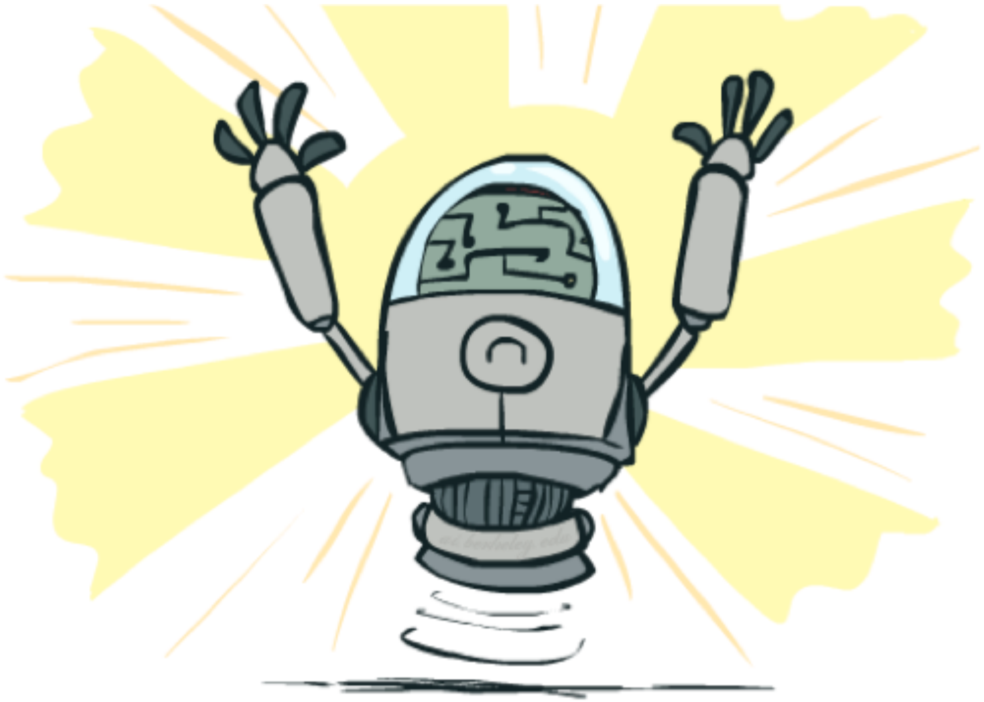
Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



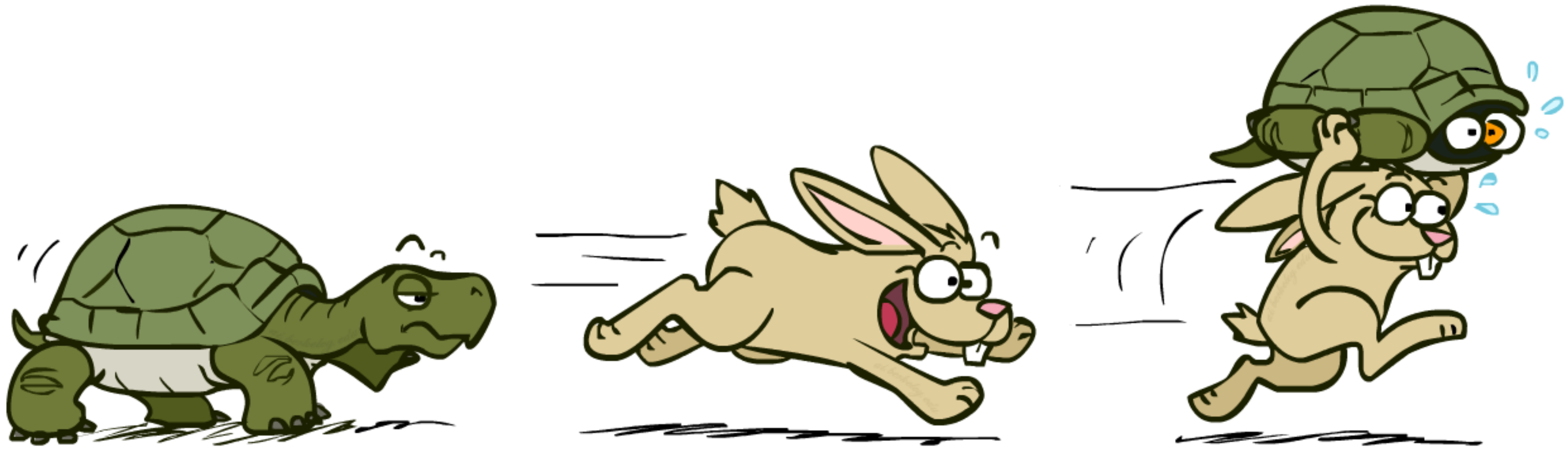
Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems



A*: Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems



Tree Search Pseudo-Code

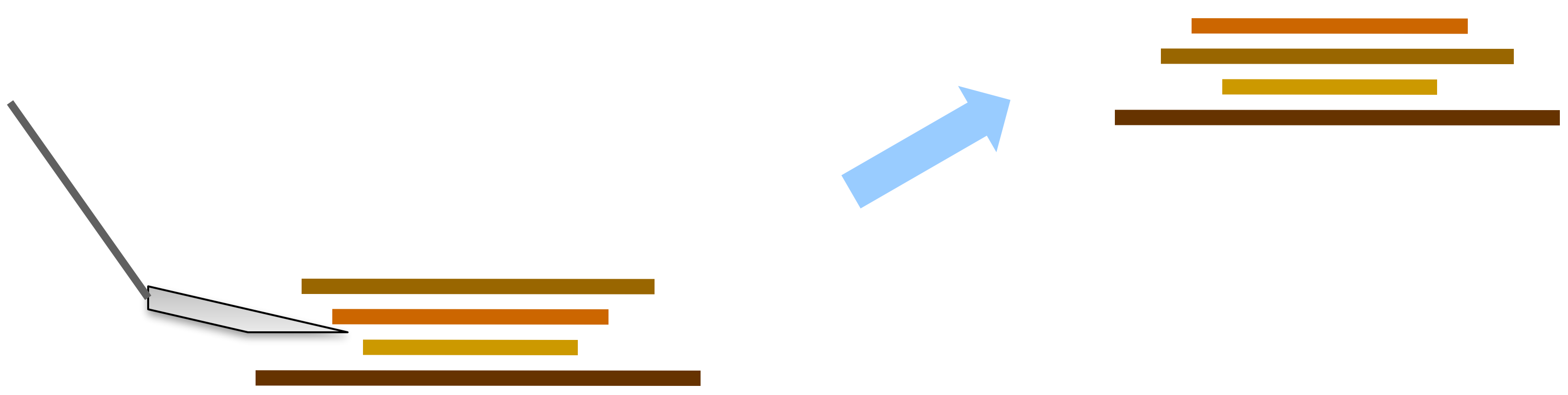
```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

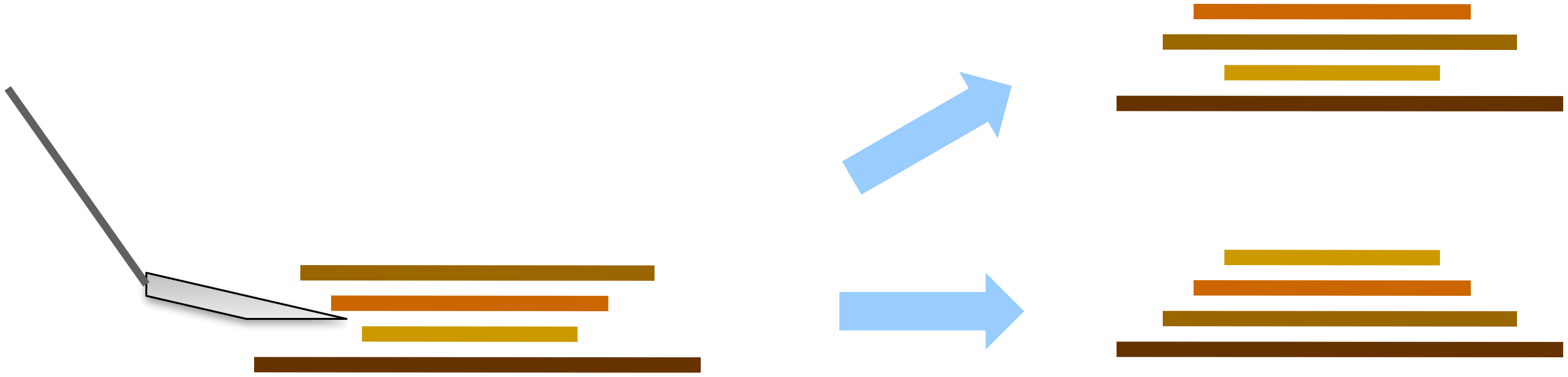
```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

Example: Pancake Problem

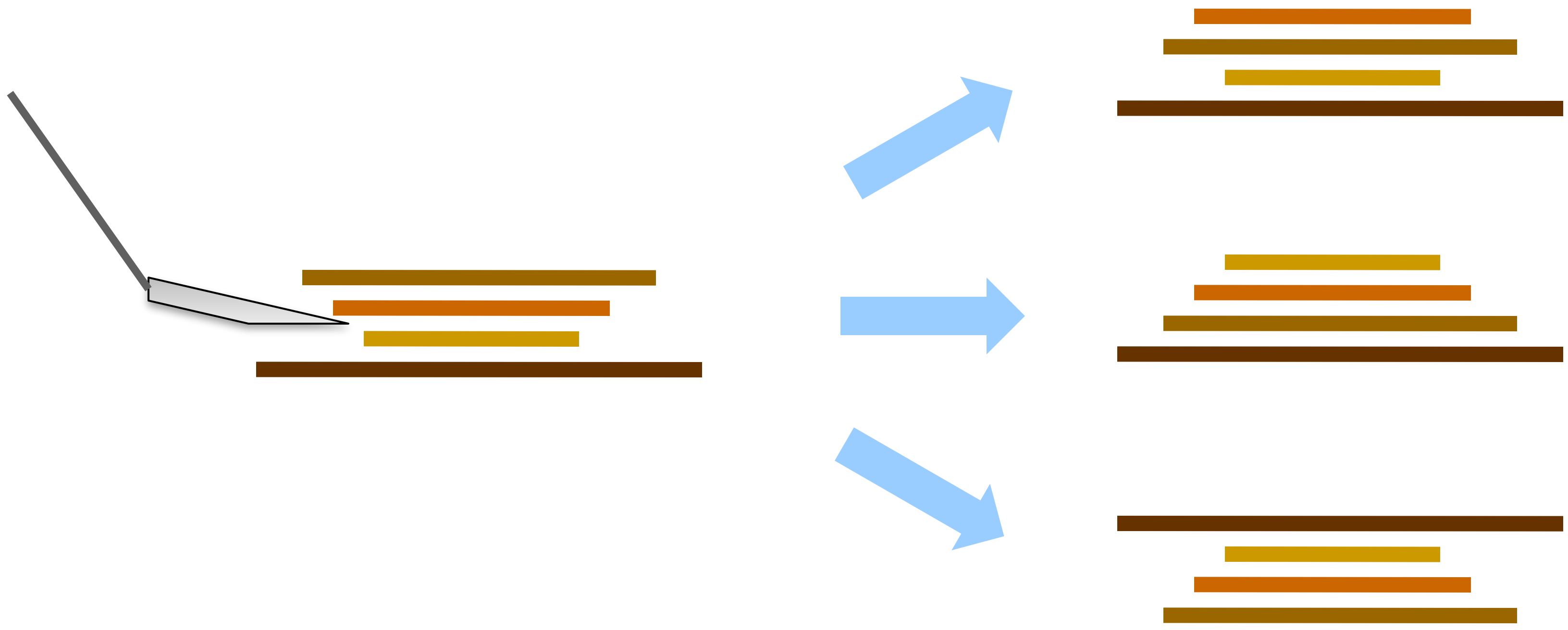
Example: Pancake Problem



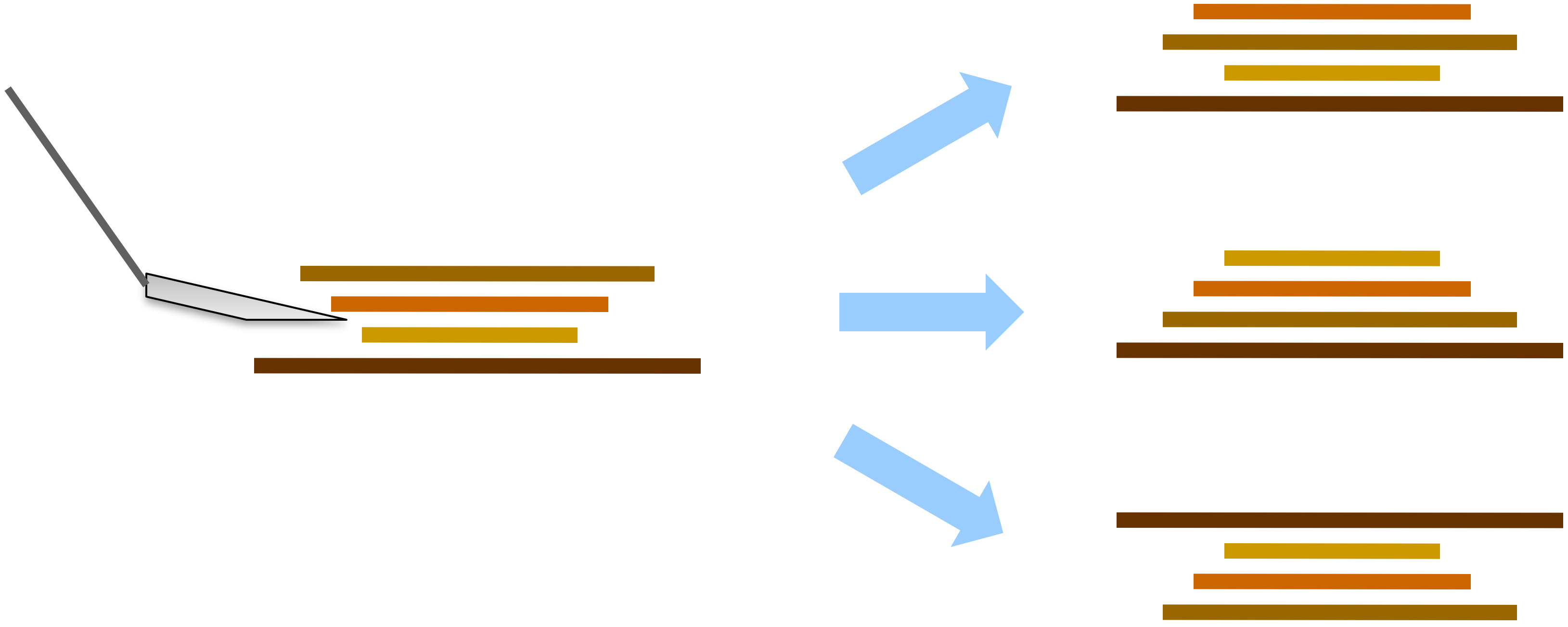
Example: Pancake Problem



Example: Pancake Problem



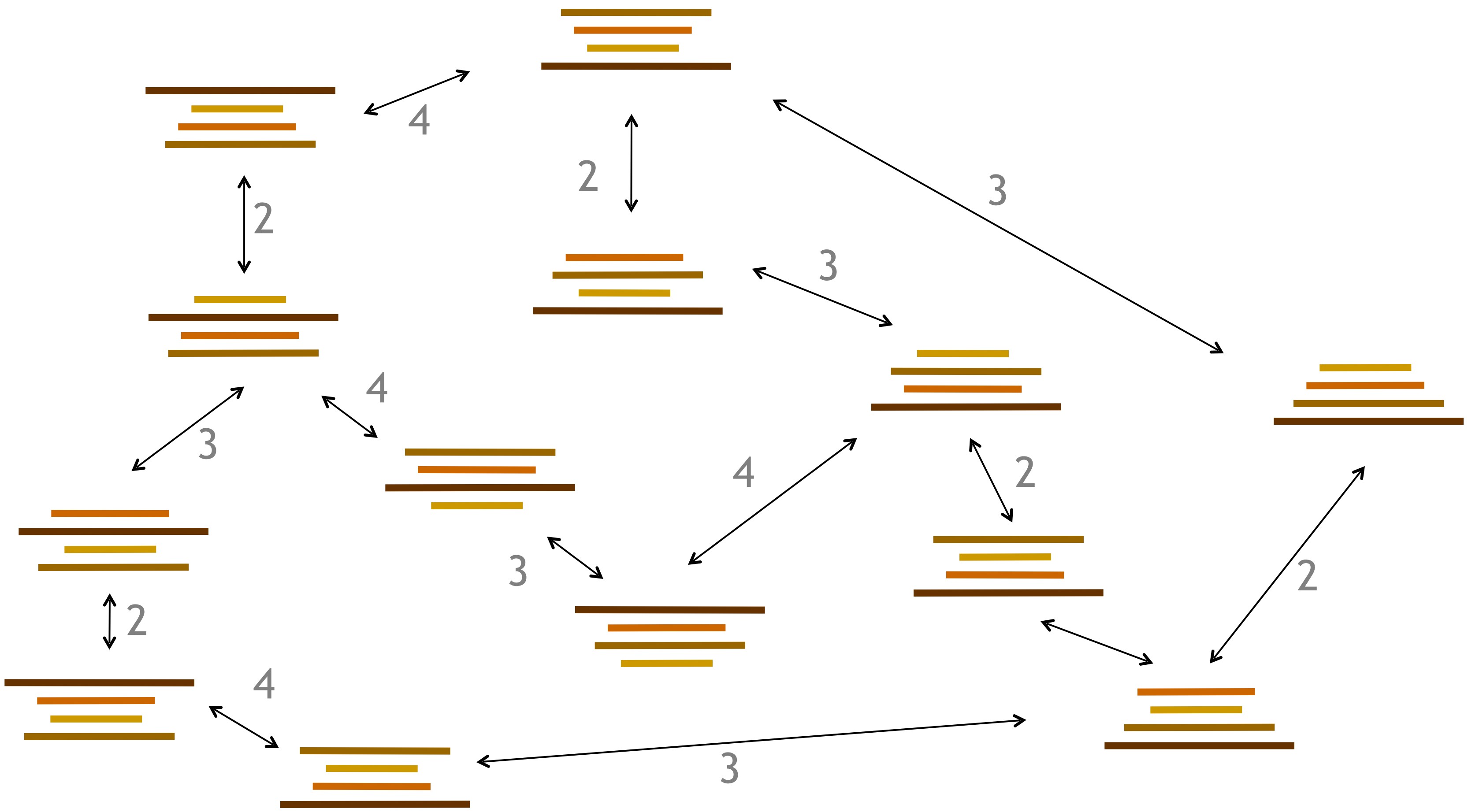
Example: Pancake Problem



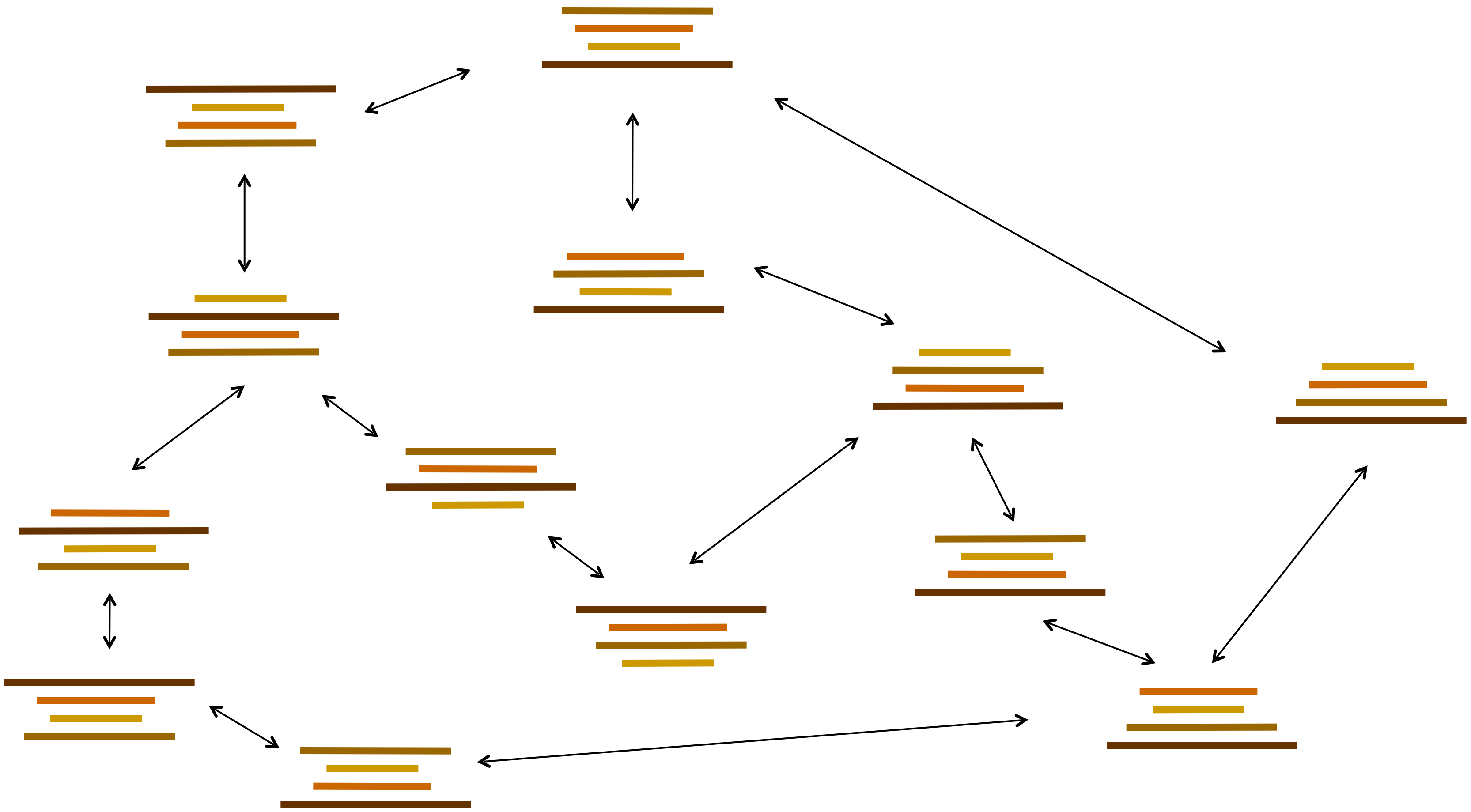
Cost: Number of pancakes flipped

Example: Pancake Problem

State space graph with costs as weights

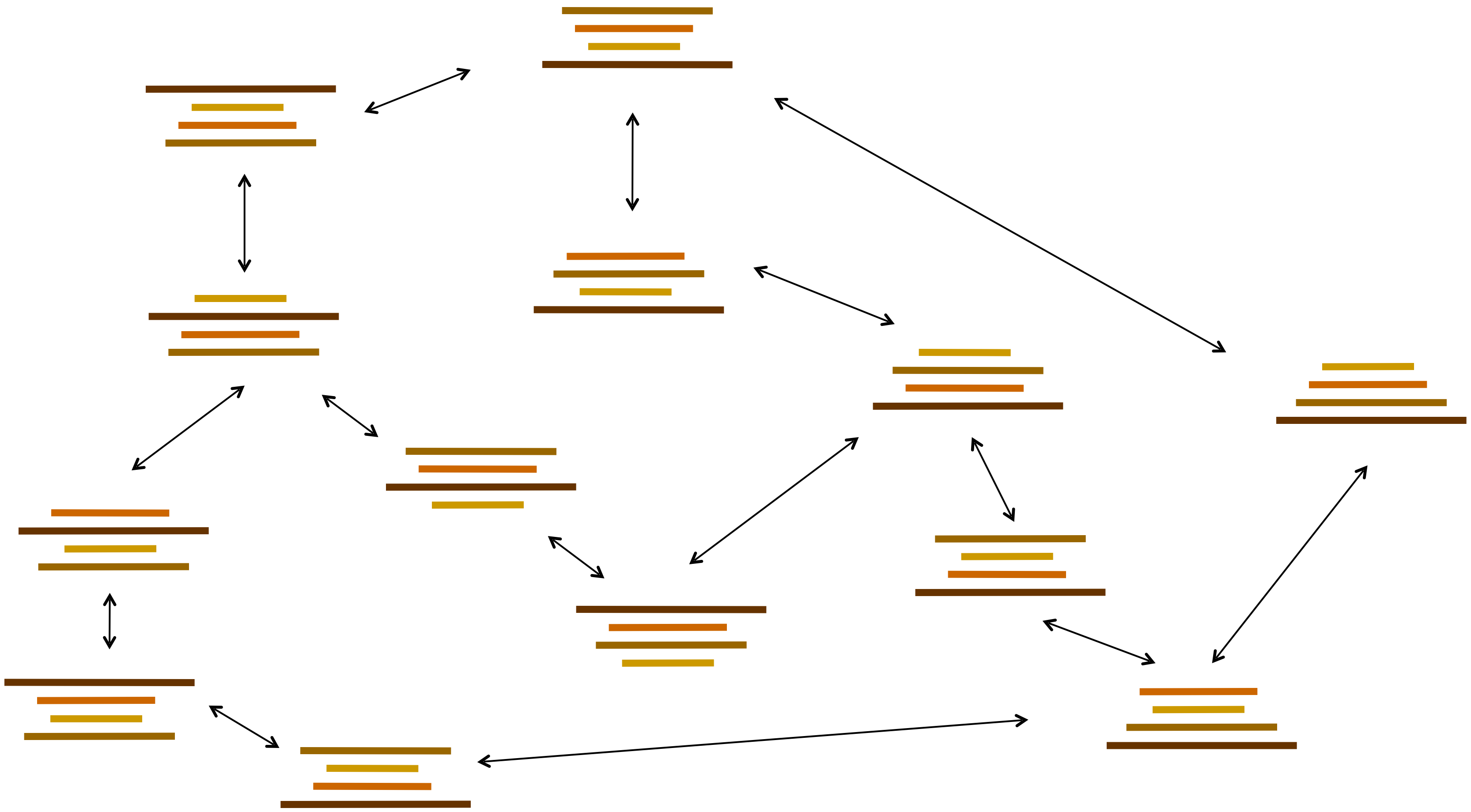


Example: Heuristic Function



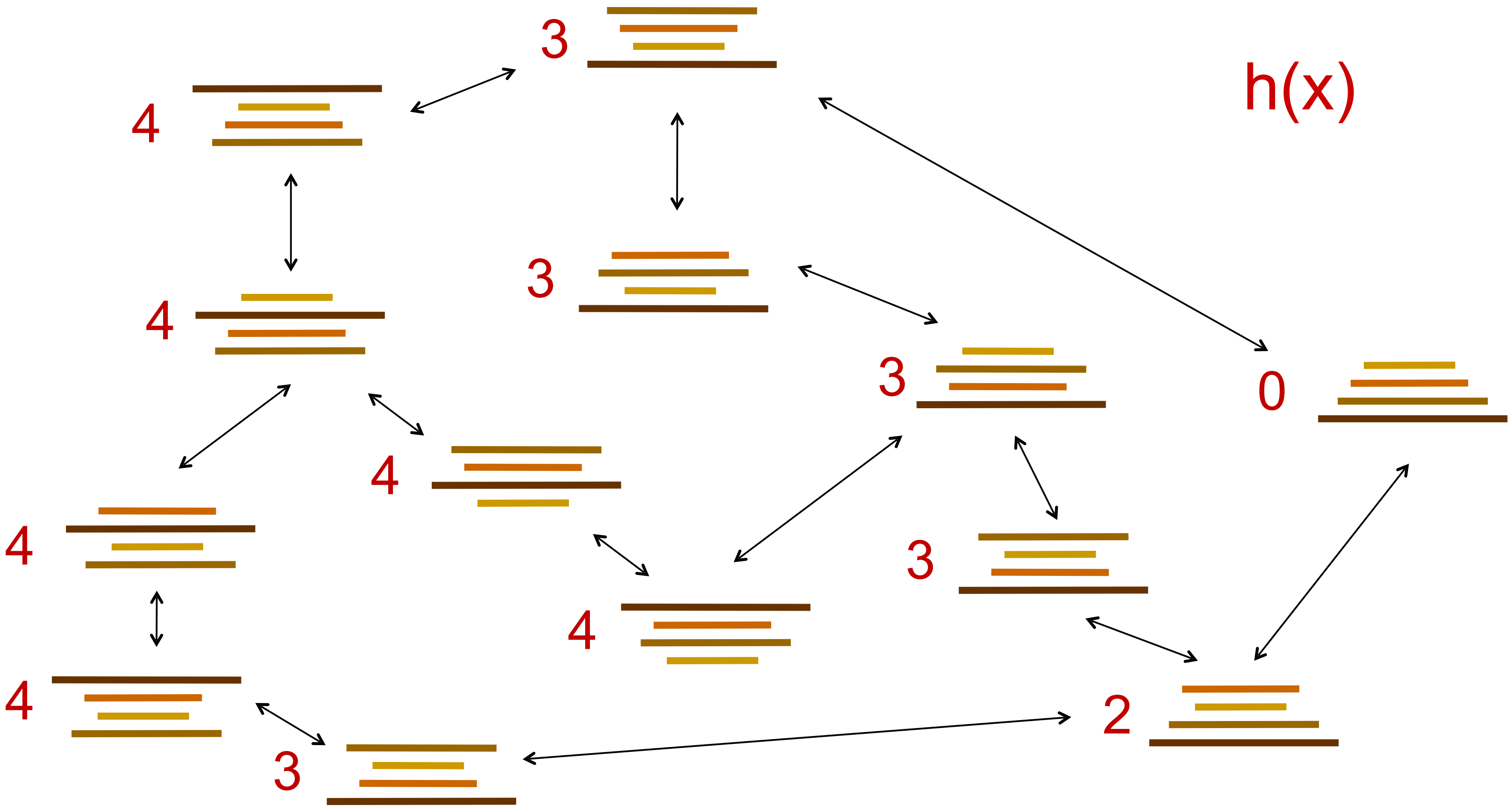
Example: Heuristic Function

Heuristic h_a: the largest pancake that is still out of place



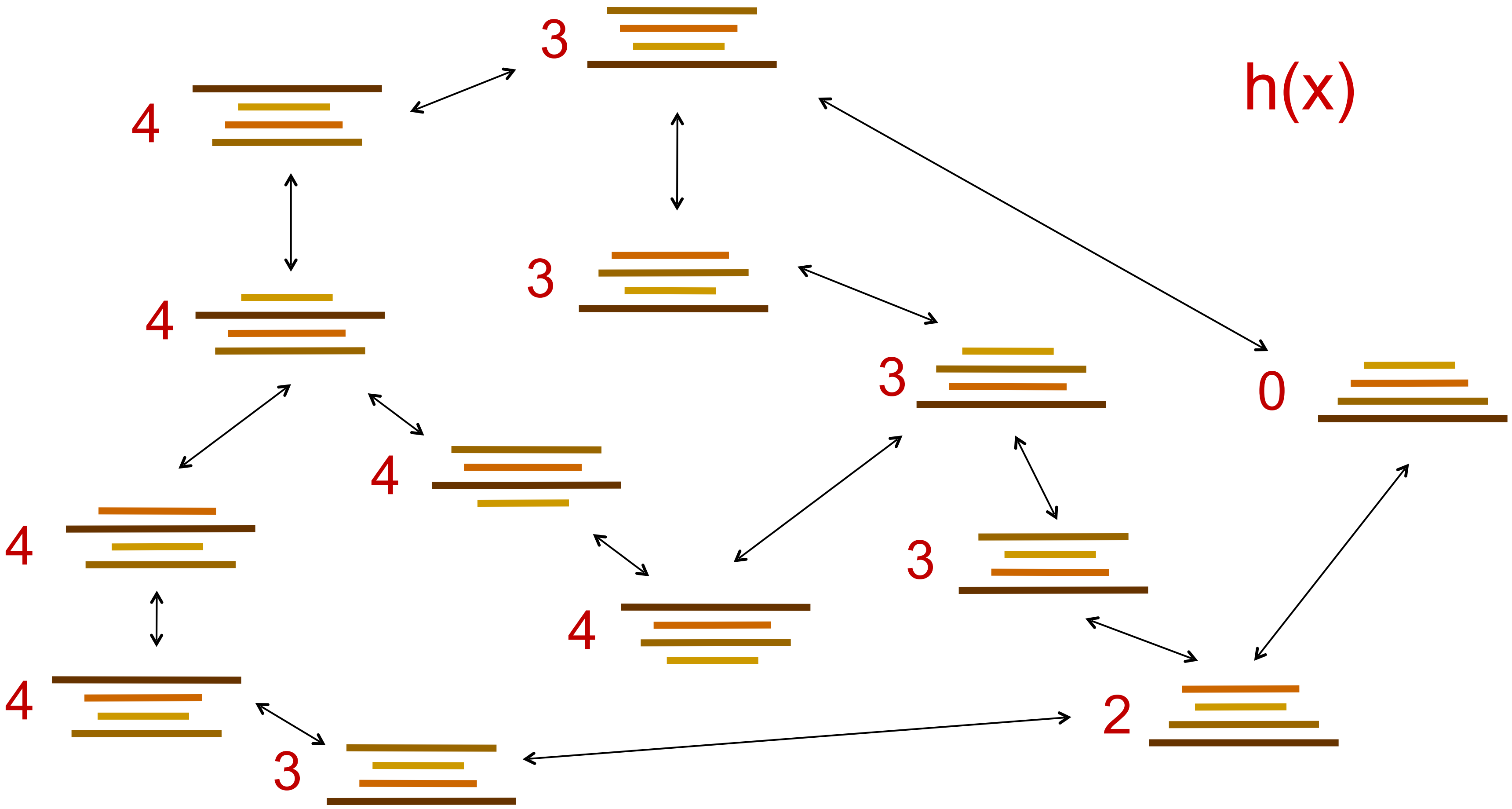
Example: Heuristic Function

Heuristic h_a: the largest pancake that is still out of place



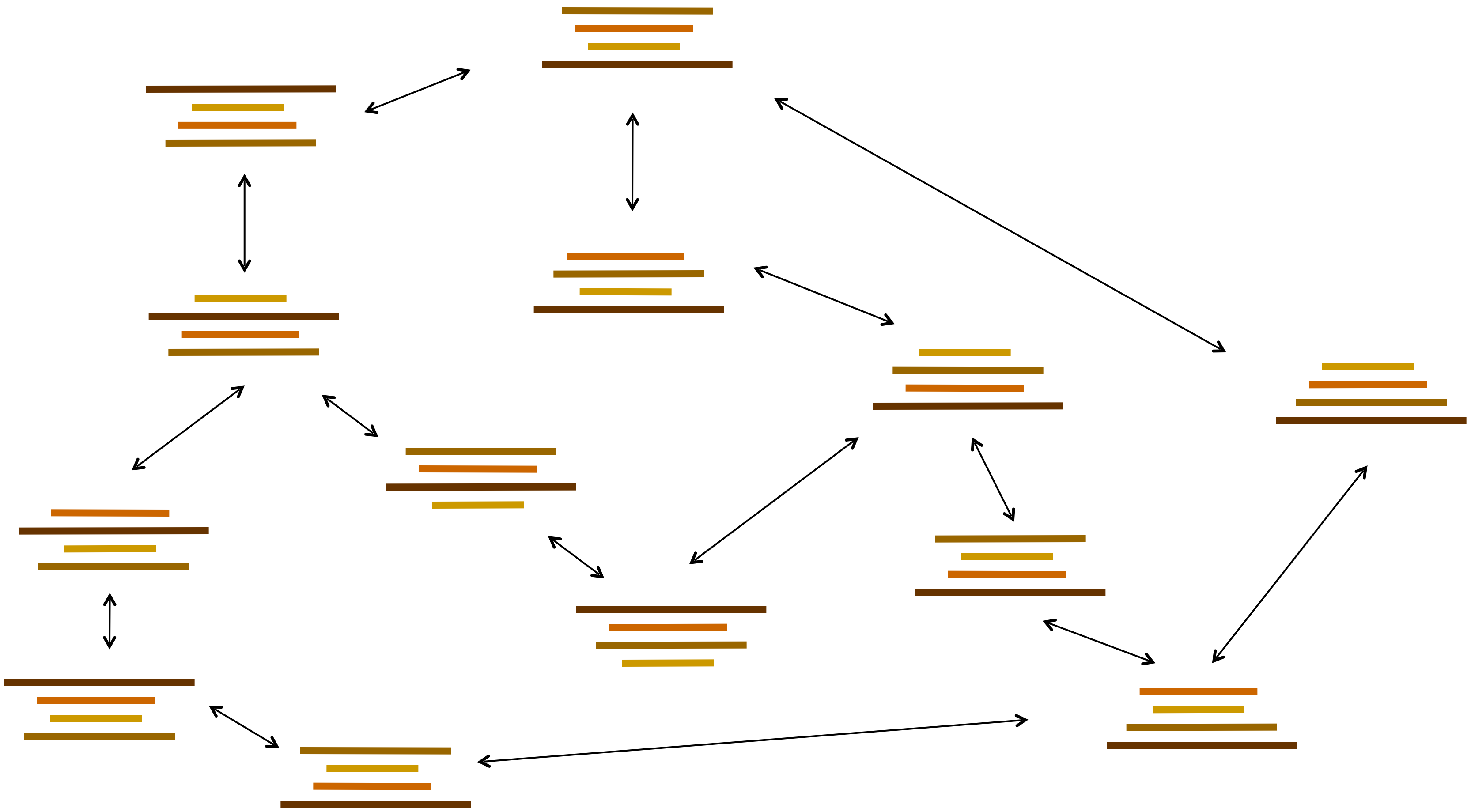
Example: Heuristic Function

Heuristic h_a: the largest pancake that is still out of place



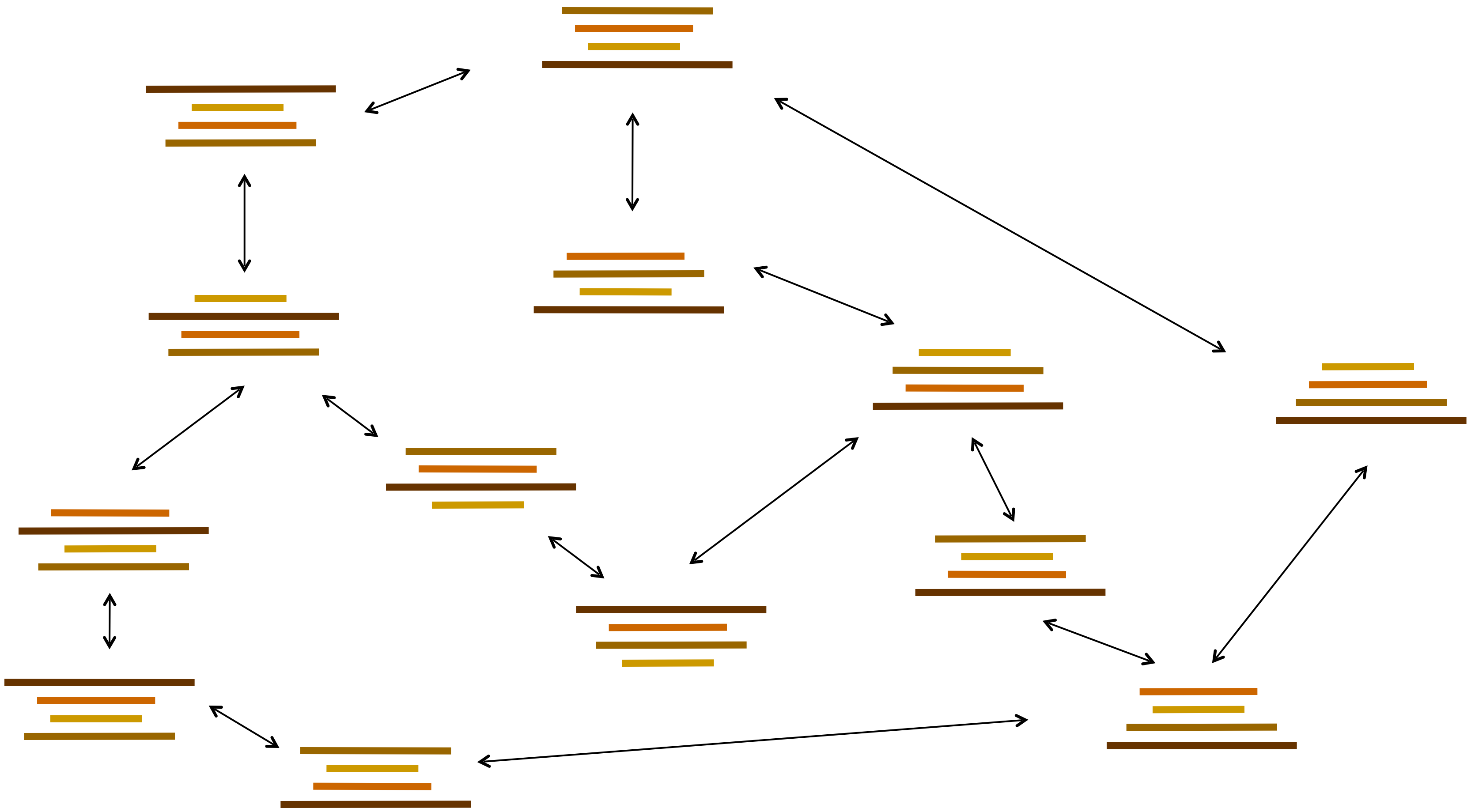
h_a is admissible, because putting the pancake *i* into place has a cost of at least *i*.

Example: Heuristic Function



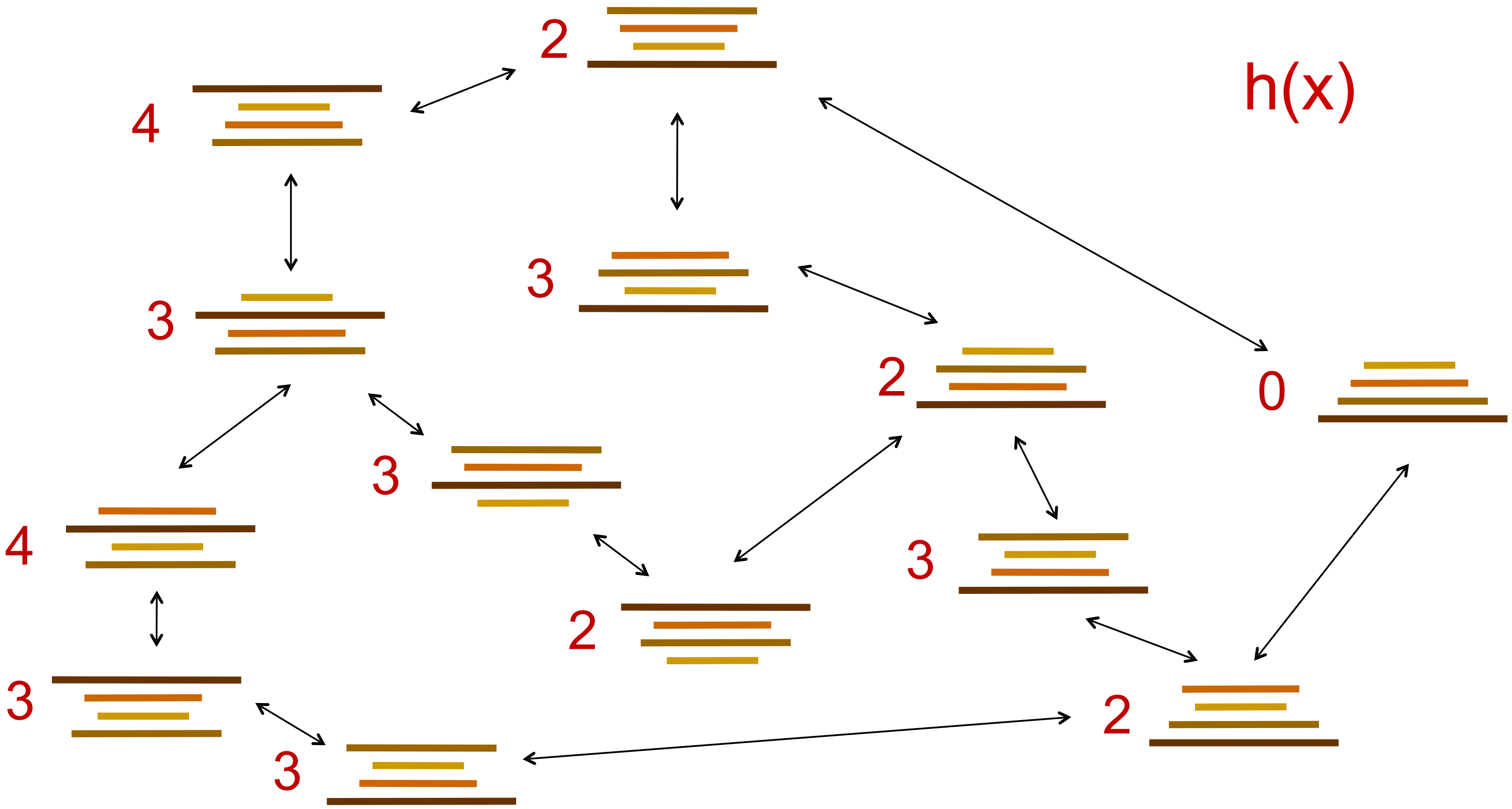
Example: Heuristic Function

Heuristic h_c: the number of the pancake that is still out of place



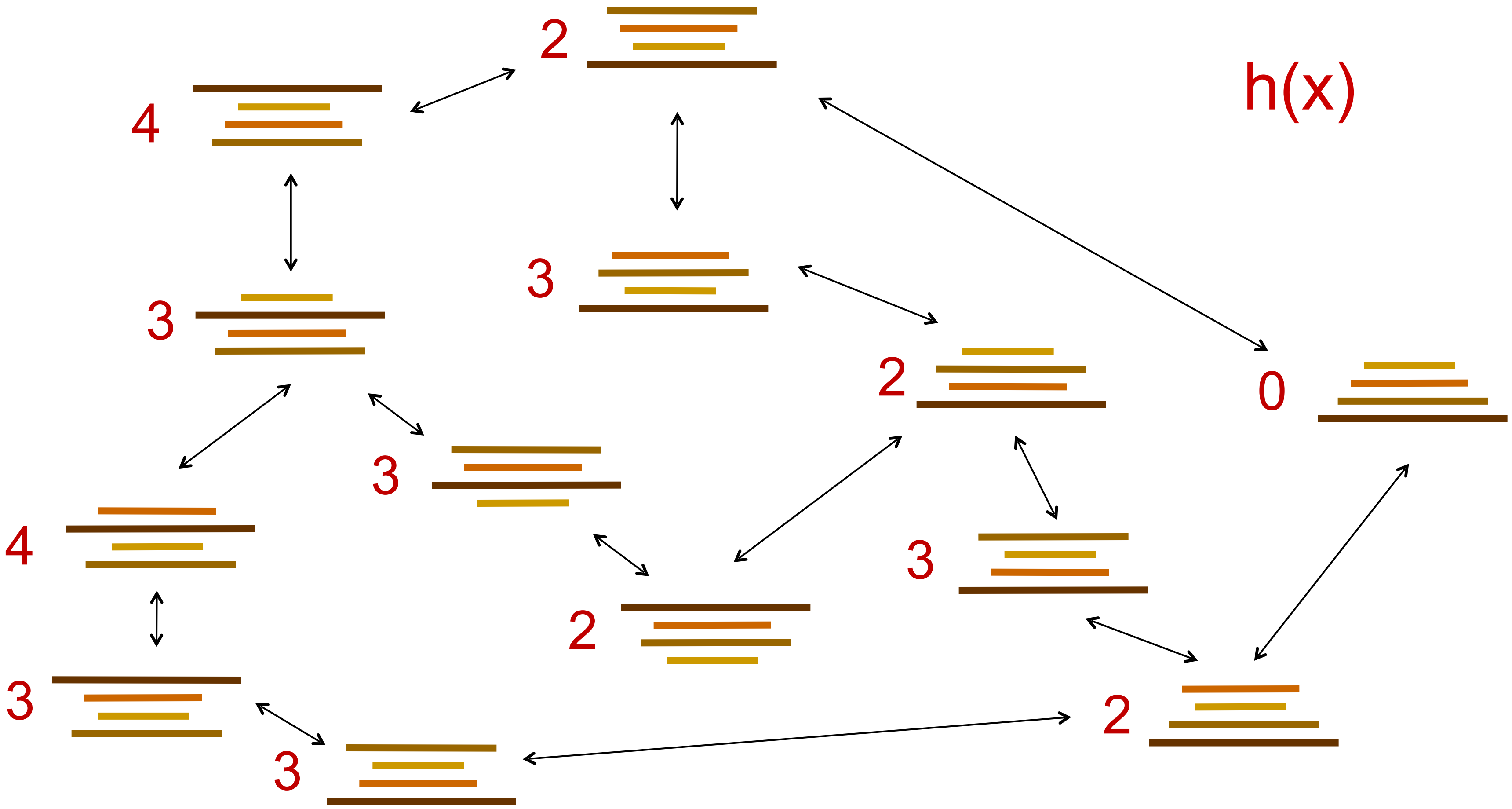
Example: Heuristic Function

Heuristic h_c: the number of the pancake that is still out of place



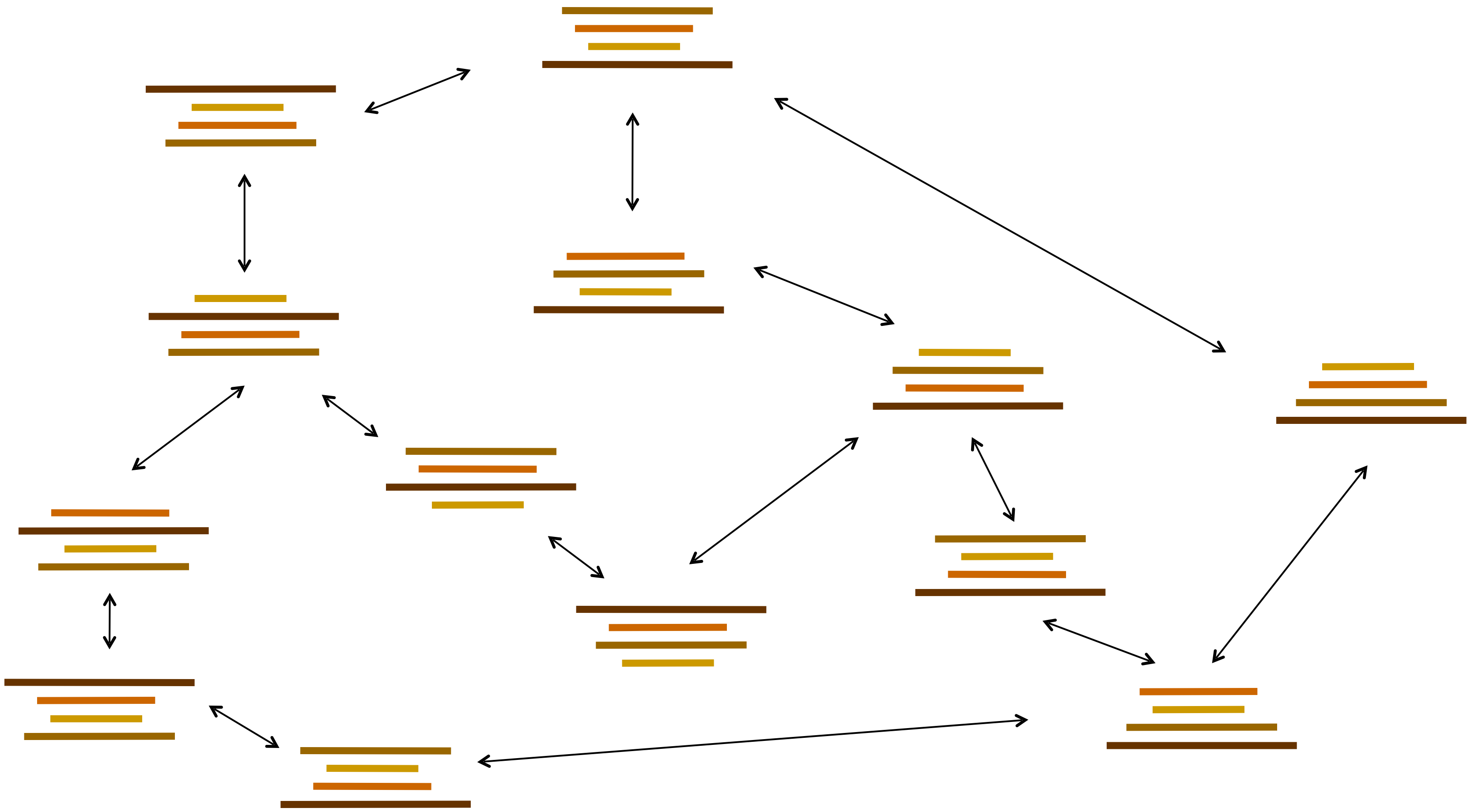
Example: Heuristic Function

Heuristic h_c: the number of the pancake that is still out of place



h_c is admissible, because a flip of size k can put at most put k pancakes into position.

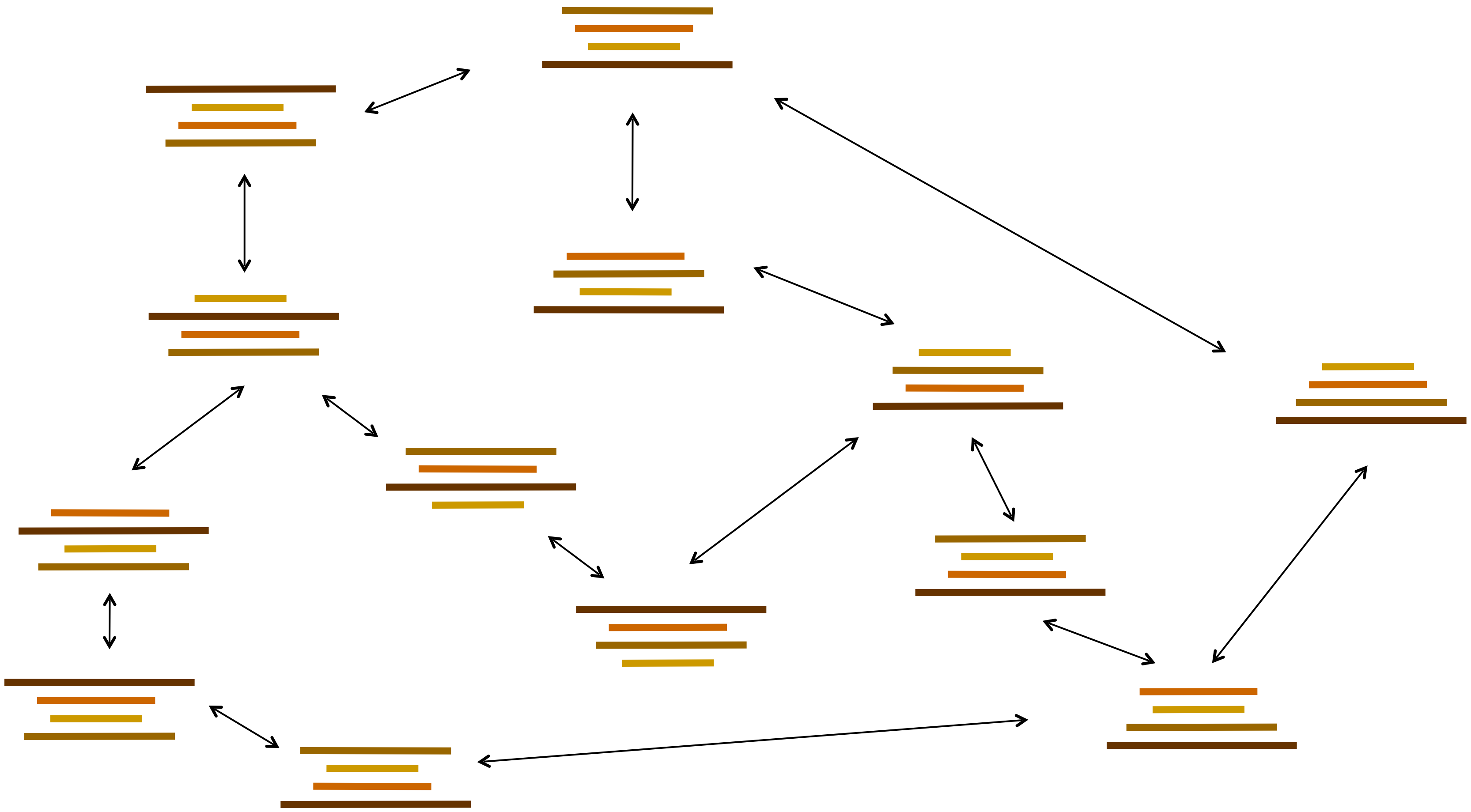
Example: Heuristic Function



h_d is admissible, because of the size of flip needed to move the top pancake into position.

Example: Heuristic Function

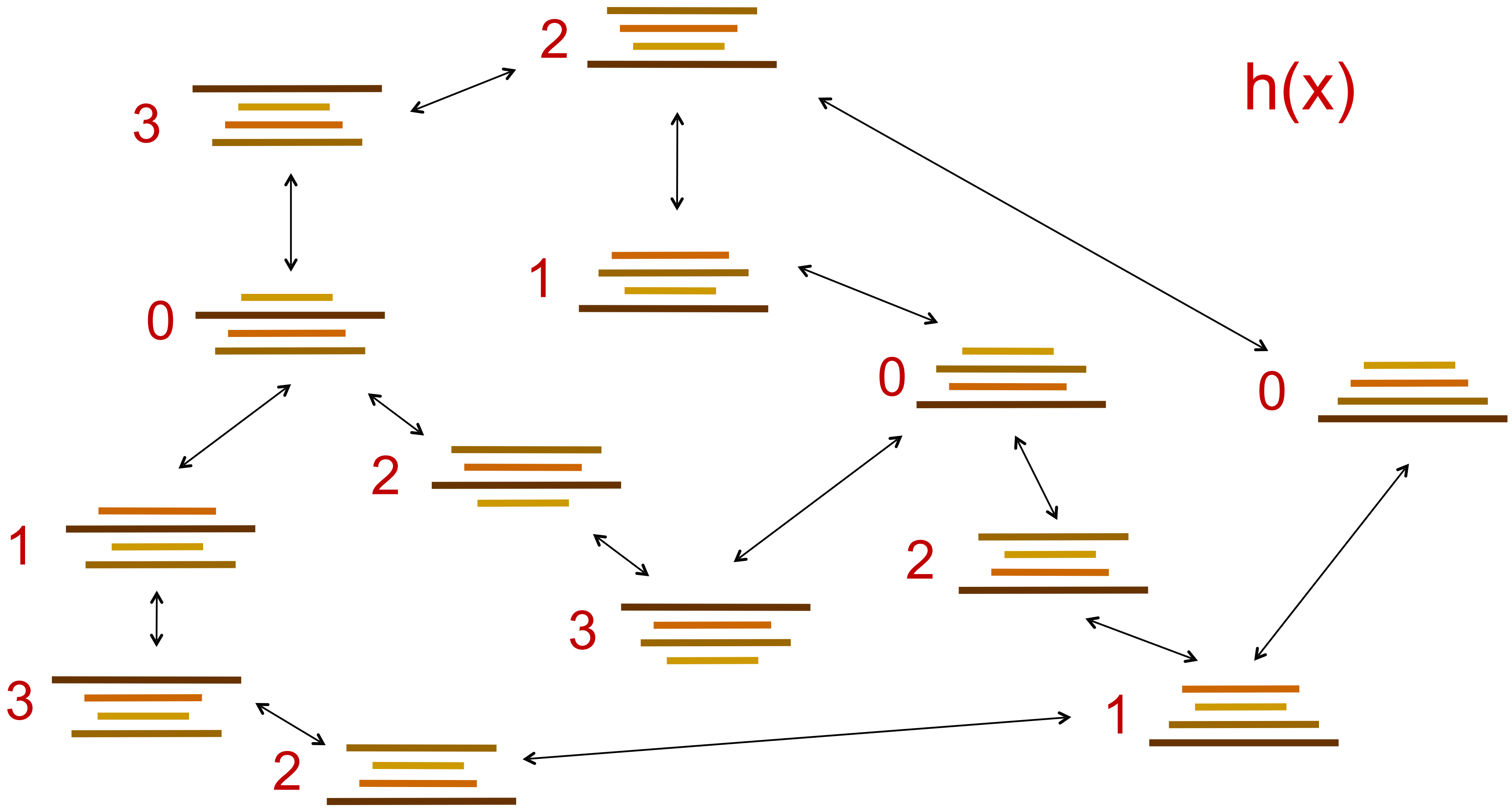
Heuristic h_d: one less than the size of the pancake at the top



h_d is admissible, because of the size of flip needed to move the top pancake into position.

Example: Heuristic Function

Heuristic h_d: one less than the size of the pancake at the top



h_d is admissible, because of the size of flip needed to move the top pancake into position.