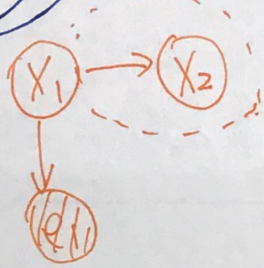
HMM



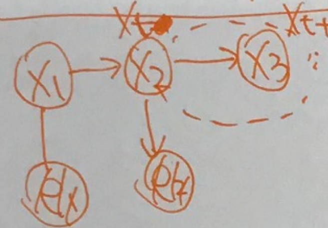
Given $P(X_t|e_{1:t})$

also $P(X_0)$. $P(X_{t+1}|X_t)$. $P(e_t|X_t)$

How to get $P(X_{t+1}|e_{1:t})$?

Base

$P(X_1|e_1) = \frac{P(X_1,e_1)}{P(e_1)} \propto P(X_1,e_1) = P(X_1)P(e_1|X_1)$ $\quad P(X_2) = \sum_{x_1} P(X_1,X_2) = \sum_{x_1} P(X_1)P(X_2|X_1)$



$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1}, x_t|\underline{e_{1:t}})$$

$$= \sum_{x_t} \underline{P(X_{t+1}|x_t, e_{1:t})}\, P(x_t|\underline{e_{1:t}})$$

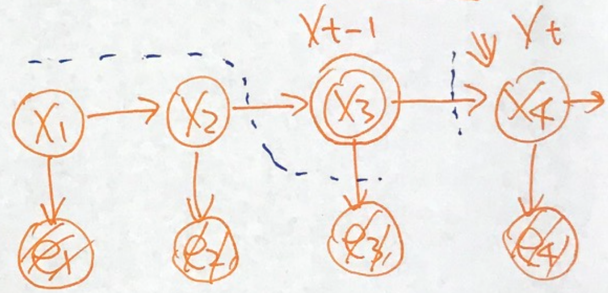
HMM independence assumption

$$= \sum_{x_t} \underline{P(X_{t+1}|x_t)}\, P(x_t|e_{1:t})$$

Independence

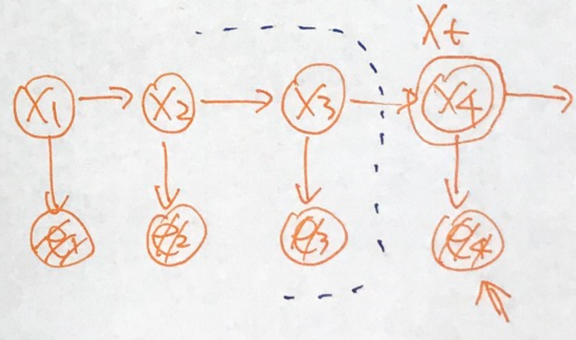
$X_t \perp\!\!\!\perp X_1, \cdots X_{t-2} \mid X_{t-1}$

$X_t \perp\!\!\!\perp E_1, \cdots E_{t-1} \mid X_{t-1}$



$E_t \perp\!\!\!\perp X_1, \cdots, X_{t-1} \mid X_t$

$E_t \perp\!\!\!\perp E_1, \cdots, E_{t-1} \mid X_t$



$P(X_t|X_{0:t-1}) = P(X_t|X_{t-1})$

$P(E_t|X_{0:t}; E_{0:t-1}) = P(E_t|X_t)$

Weather HMM

$$B_0(+r,-r) = P(R_0) = \langle 0.5, 0.5 \rangle \quad \text{initialization}$$

↓ time pass

$$B_1'(+r,-r) = P(R_1) = \sum_{r_0} P(R_1 | r_0) P(r_0)$$

$$= \langle 0.7, 0.3 \rangle \times 0.5 + \langle 0.3, 0.7 \rangle \times 0.5$$

$$= \langle 0.5, 0.5 \rangle$$

↓ observe evidence $u_1 =$ true (umbrella appears)

$$B_1(+r,-r) = P(R_1 | u_1) = \frac{P(R_1 | u_1)}{P(u_1)}$$

$$\propto_{R_1} P(R_1 | u_1)$$

$$= P(u_1 | R_1) P(R_1)$$

$$= \langle 0.9 . 0.2 \rangle \langle 0.5 . 0.5 \rangle$$

$$= \langle 0.45, 0.1 \rangle$$

normalize

$$\approx \langle 0.818, 0.182 \rangle$$

↓ time pass

$$B_2'(+r,-r) = P(R_2 | u_1) = \sum_{r_1} P(R_2 | r_1) P(r_1 | u_1)$$

$$= \langle 0.7, 0.3 \rangle \times 0.818 + \langle 0.3, 0.7 \rangle \times 0.182$$

$$\approx \langle 0.627, 0.373 \rangle$$