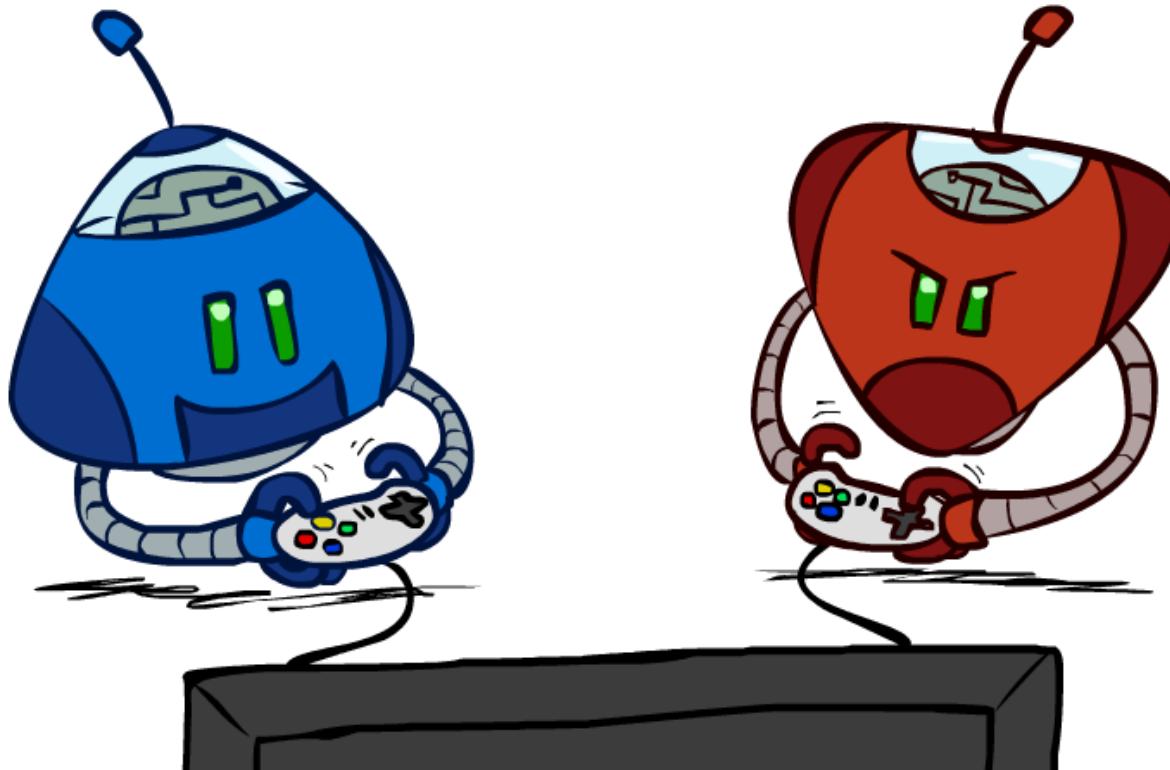


CS 5522: Artificial Intelligence II

Adversarial Search



Instructor: Wei Xu

Ohio State University

[These slides were adapted from CS188 Intro to AI at UC Berkeley.]

Machine Translation

Garcia and associates .

Garcia y asociados .

Carlos Garcia has three associates .

Carlos Garcia tiene tres asociados .

his associates are not strong .

sus asociados no son fuertes .

Garcia has a company also .

Garcia tambien tiene una empresa .

its clients are angry .

sus clientes estan enfadados .

the associates are also angry .

los asociados tambien estan enfadados .

the clients and the associates are enemies .

los clientes y los asociados son enemigos .

the company has three groups .

la empresa tiene tres grupos .

its groups are in Europe .

sus grupos estan en Europa .

the modern groups sell strong pharmaceuticals .

los grupos modernos venden medicinas fuertes .

the groups do not sell zanzanine .

los grupos no venden zanzaina .

the small groups are not modern .

los grupos pequenos no son modernos .

Word aligner

Garcia and associates .

\ \ /

Garcia y asociados .

Carlos Garcia has three associates .

\ | | | /

Carlos Garcia tiene tres asociados .

his associates are not strong .

| \ X /

sus asociados no son fuertes .

Garcia has a company also .

\ X X /

Garcia tambien tiene una empresa .

its clients are angry .

/ / | \

sus clientes estan enfadados .

the associates are also angry .

/ / X \

los asociados tambien estan enfadados .

the clients and the associates are enemies .

\ \ | / | / /

los clientes y los asociados son enemigos .

the company has three groups .

\ | | / /

la empresa tiene tres grupos .

its groups are in Europe .

/ | | \ /

sus grupos estan en Europa .

the modern groups sell strong pharmaceuticals .

\ X / \ X /

los grupos modernos venden medicinas fuertes .

the groups do not sell zanzanine .

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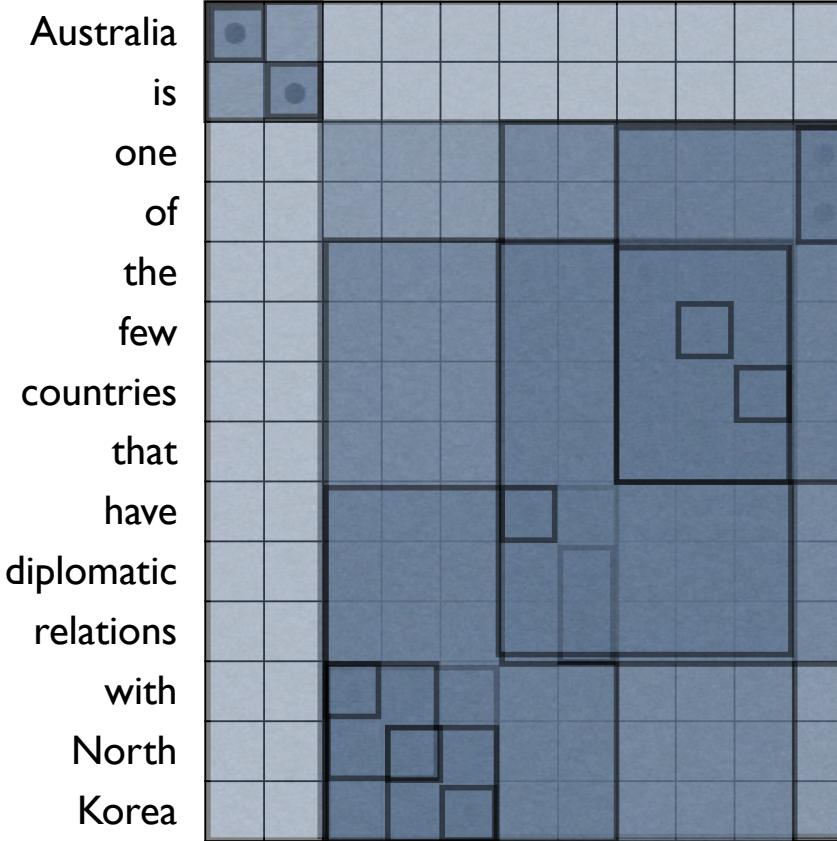
the small groups are not modern .

/ X \ X /

los grupos pequenos no son modernos .

Phrase Extractor

澳洲是与北韩有邦交的少数国家之一

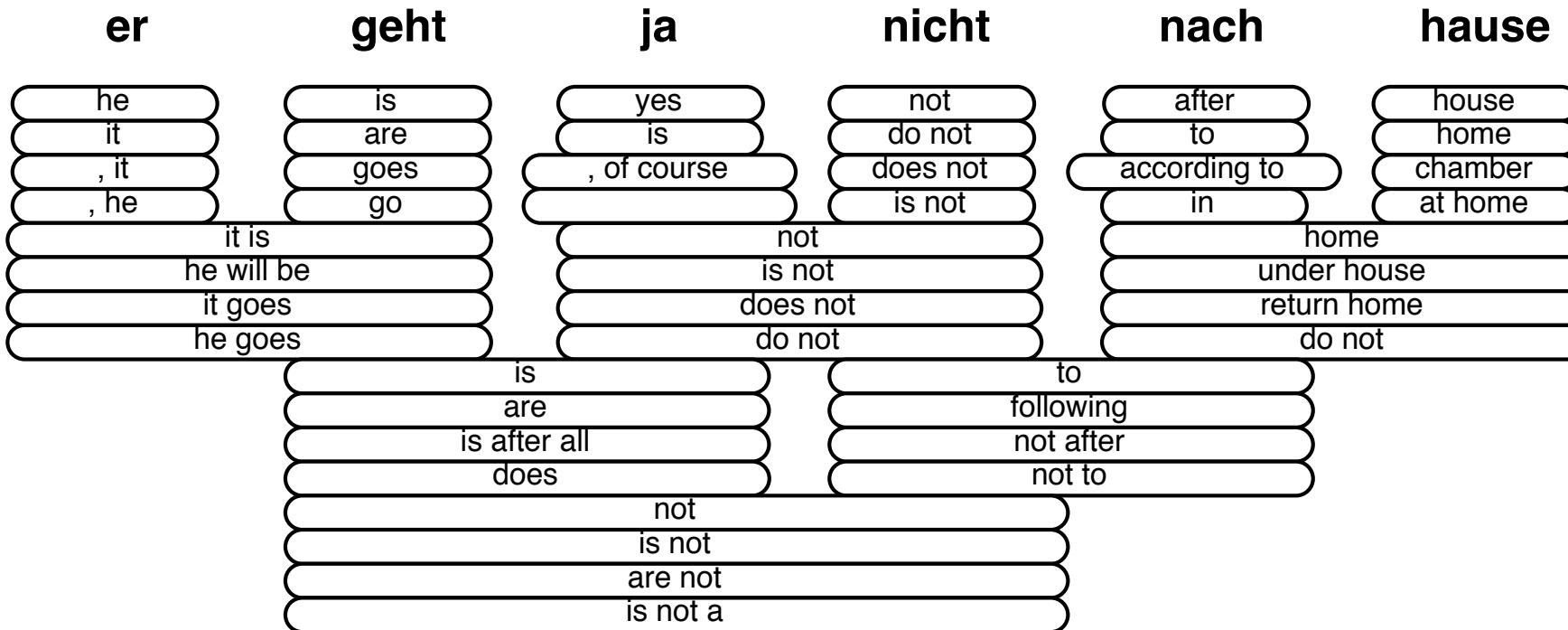


北韩
North Korea

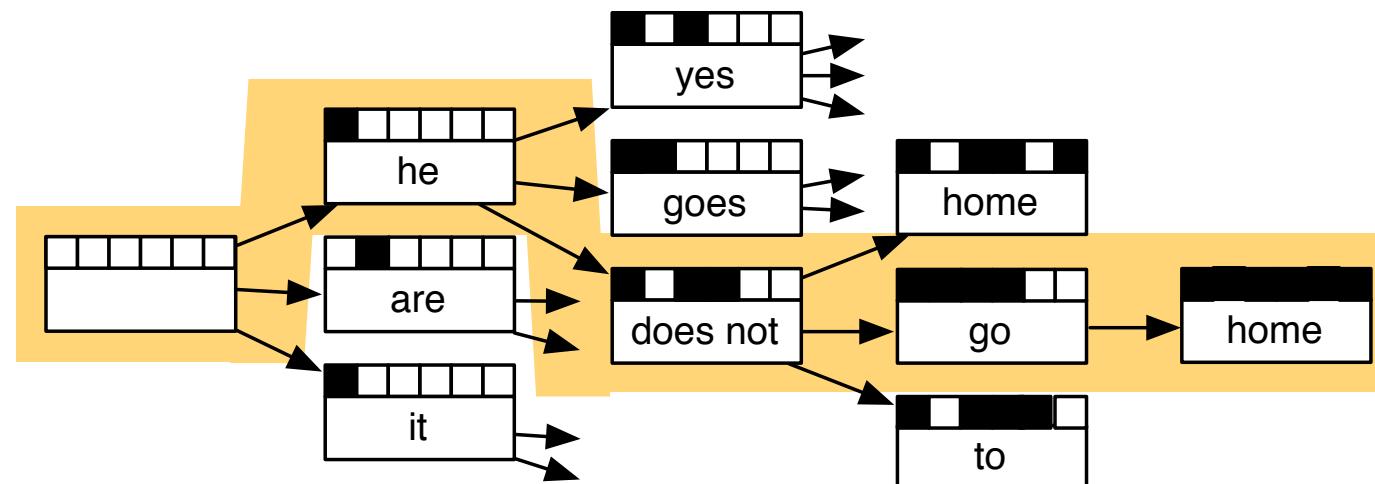
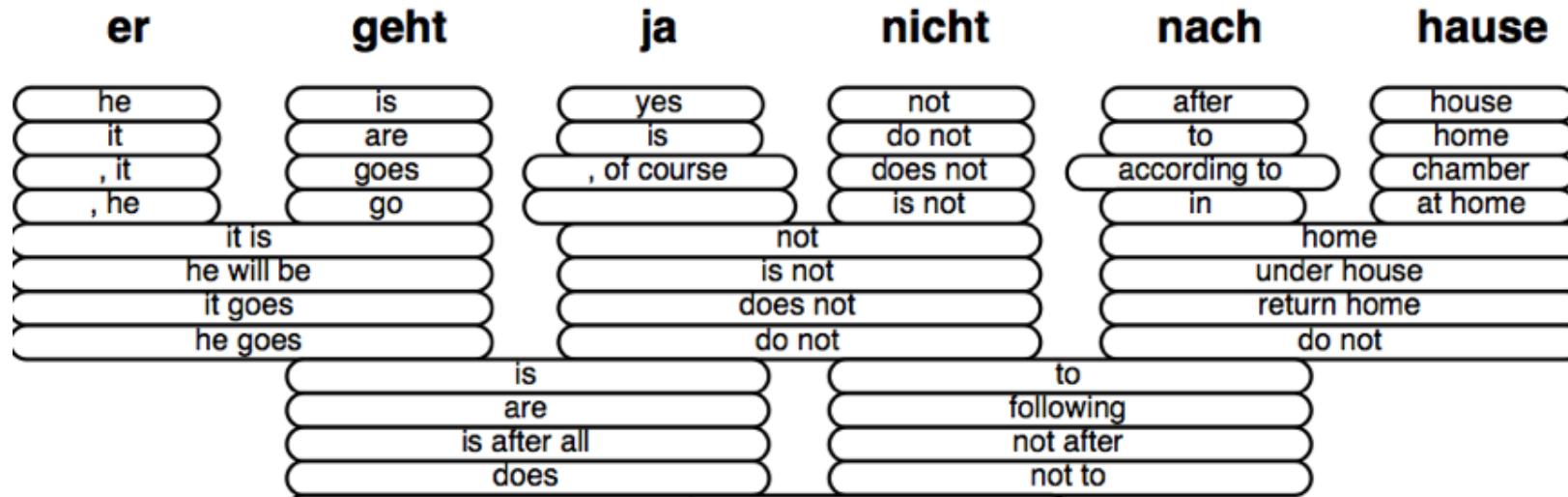
邦交
diplomatic relations

与北韩有邦交
have diplomatic relations
with North Korea

Phrase-based Decoder

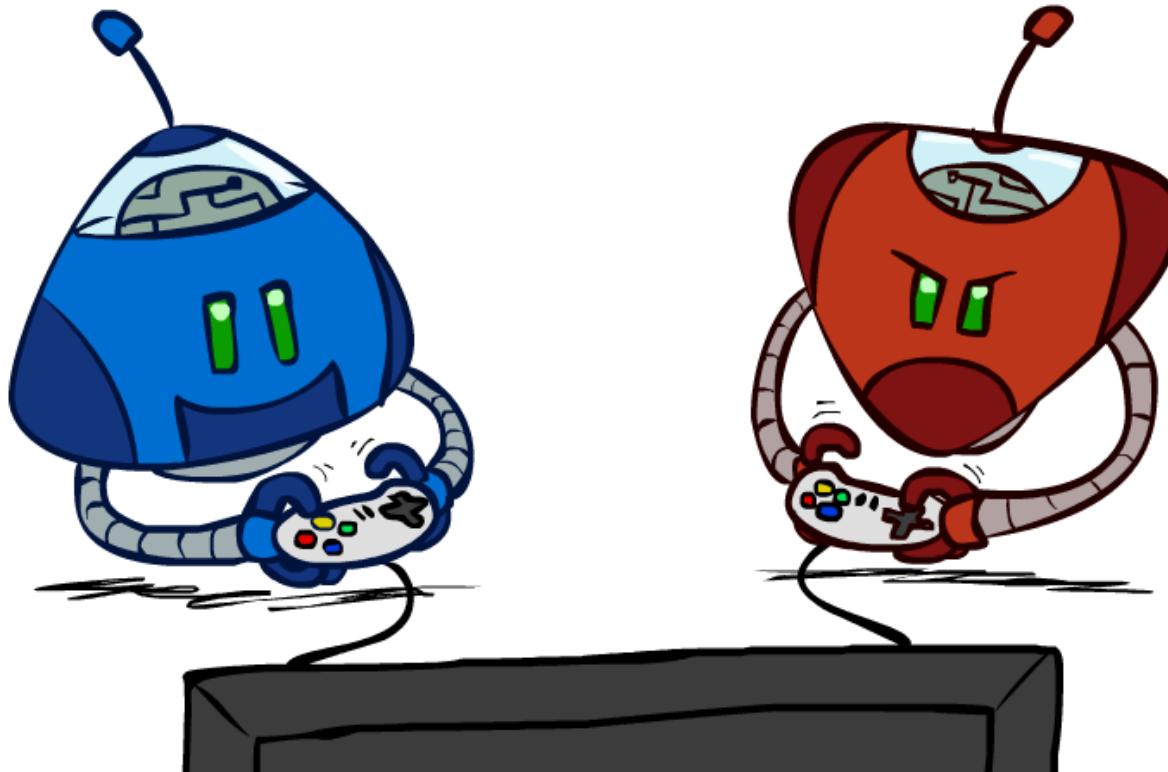


Phrase-based Decoder



CS 5522: Artificial Intelligence II

Adversarial Search

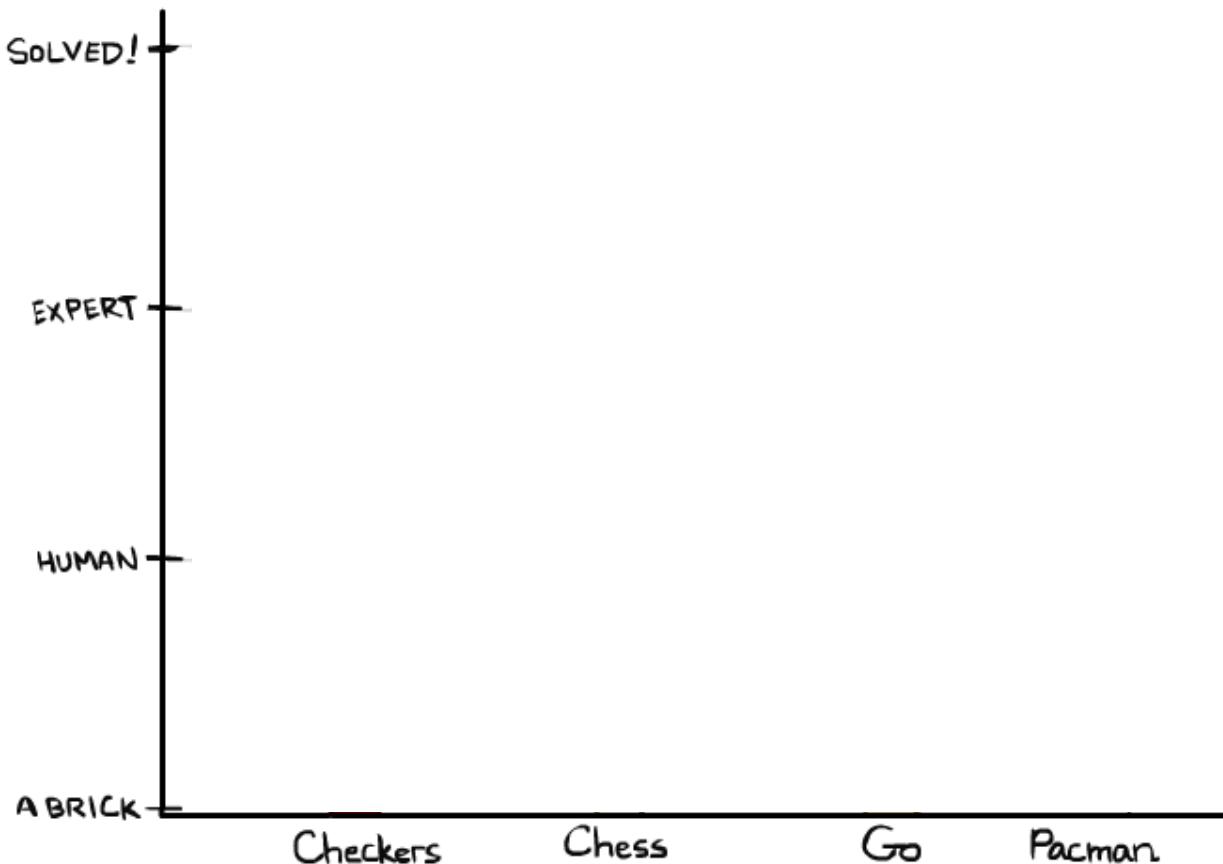


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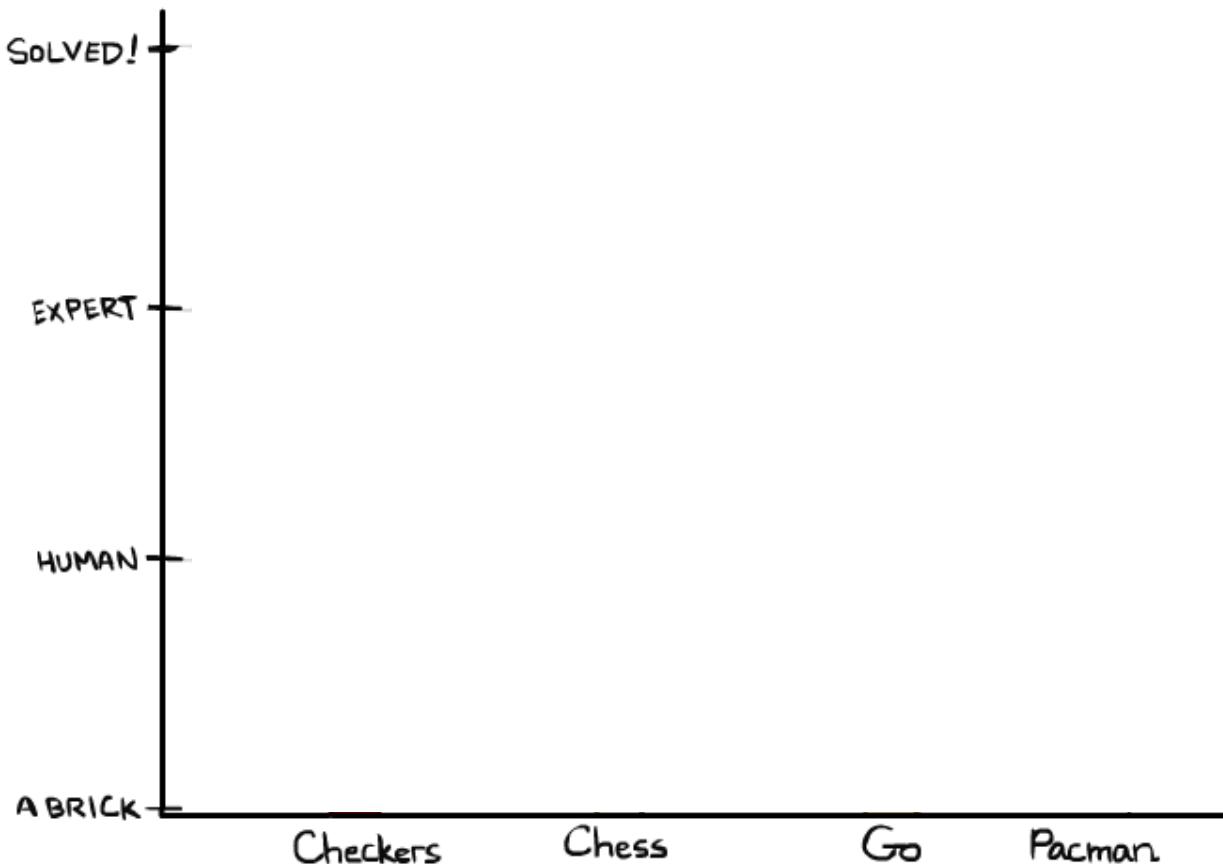
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Game Playing State-of-the-Art



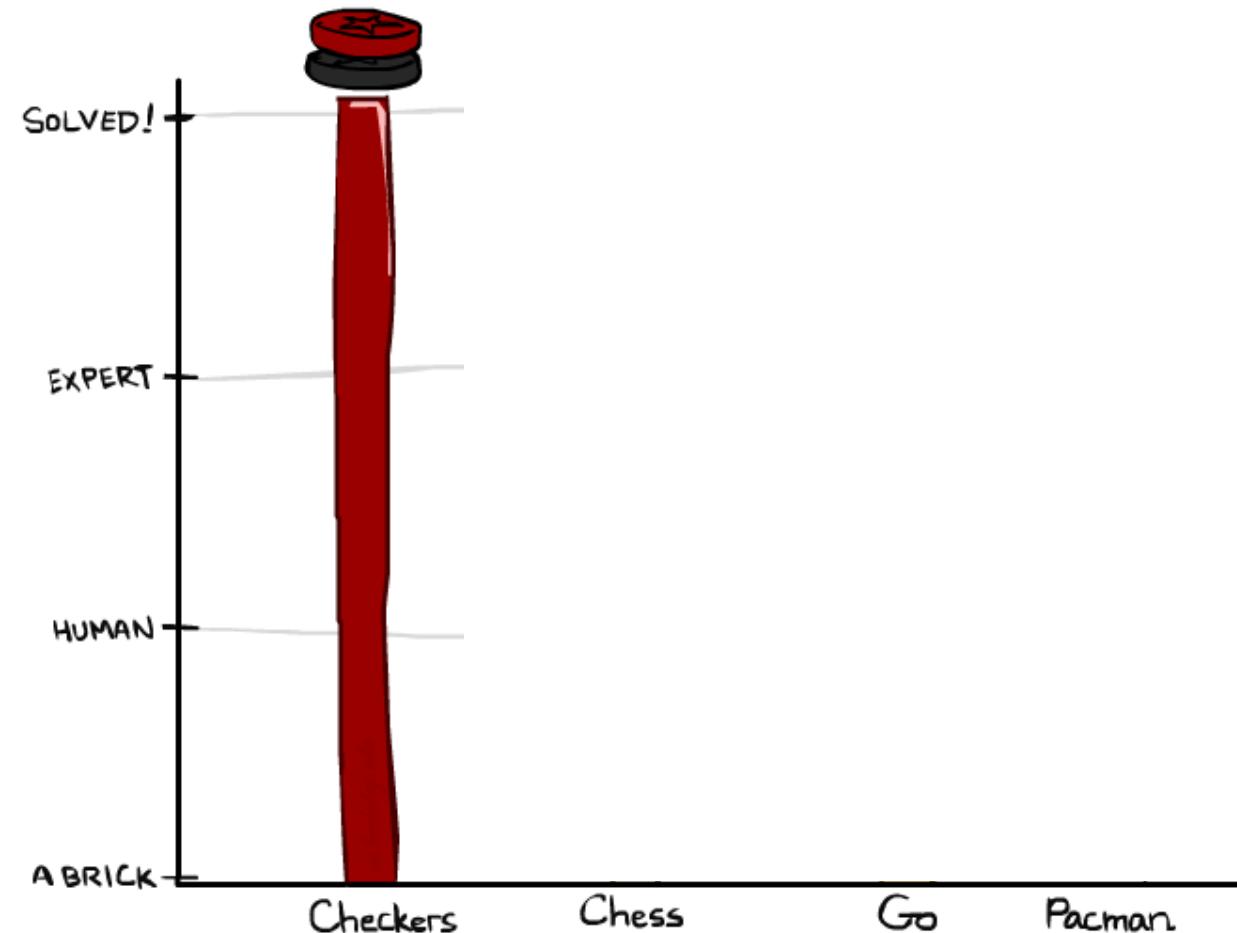
Game Playing State-of-the-Art

- **Checkers:** 1950: First computer player. 1994: First computer champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame. 2007: Checkers solved!



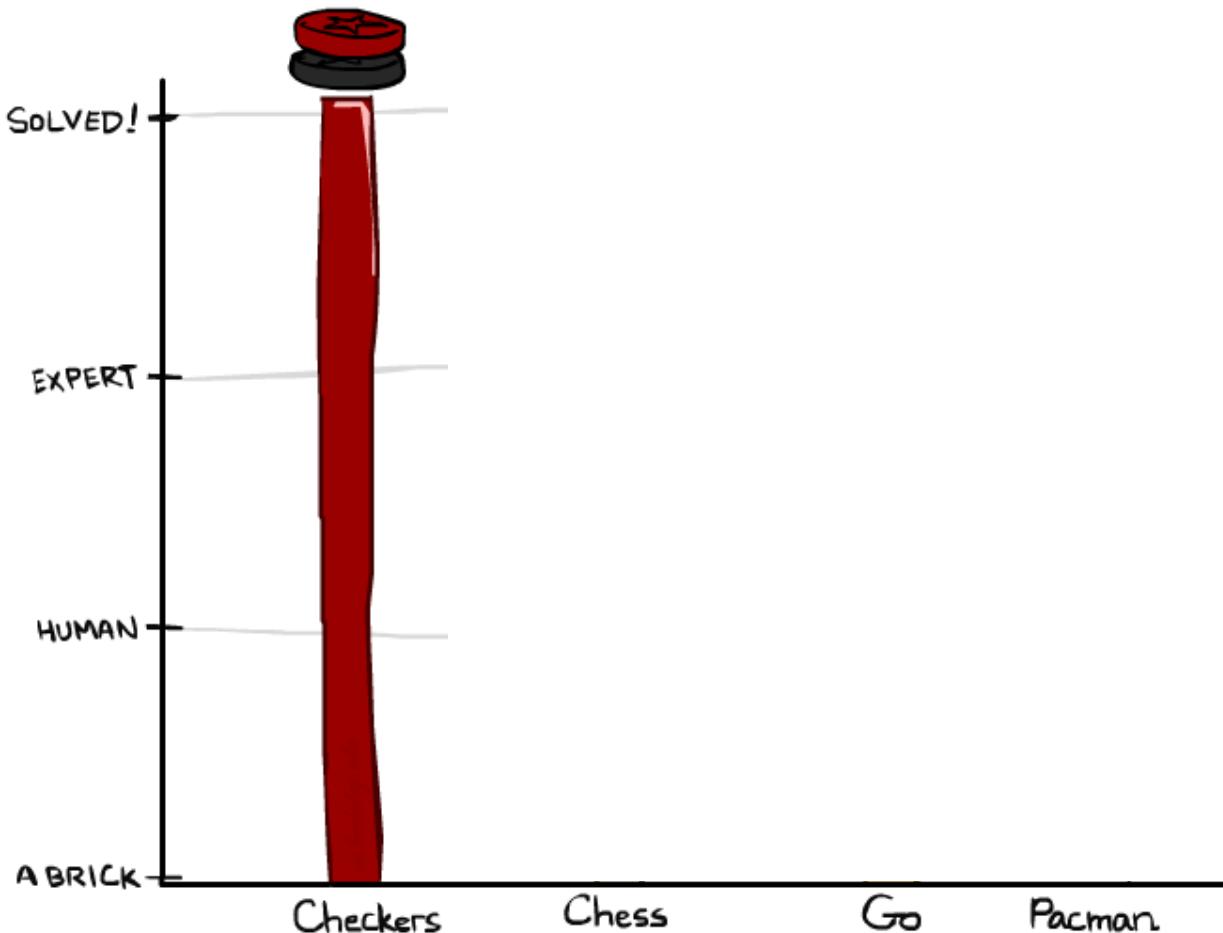
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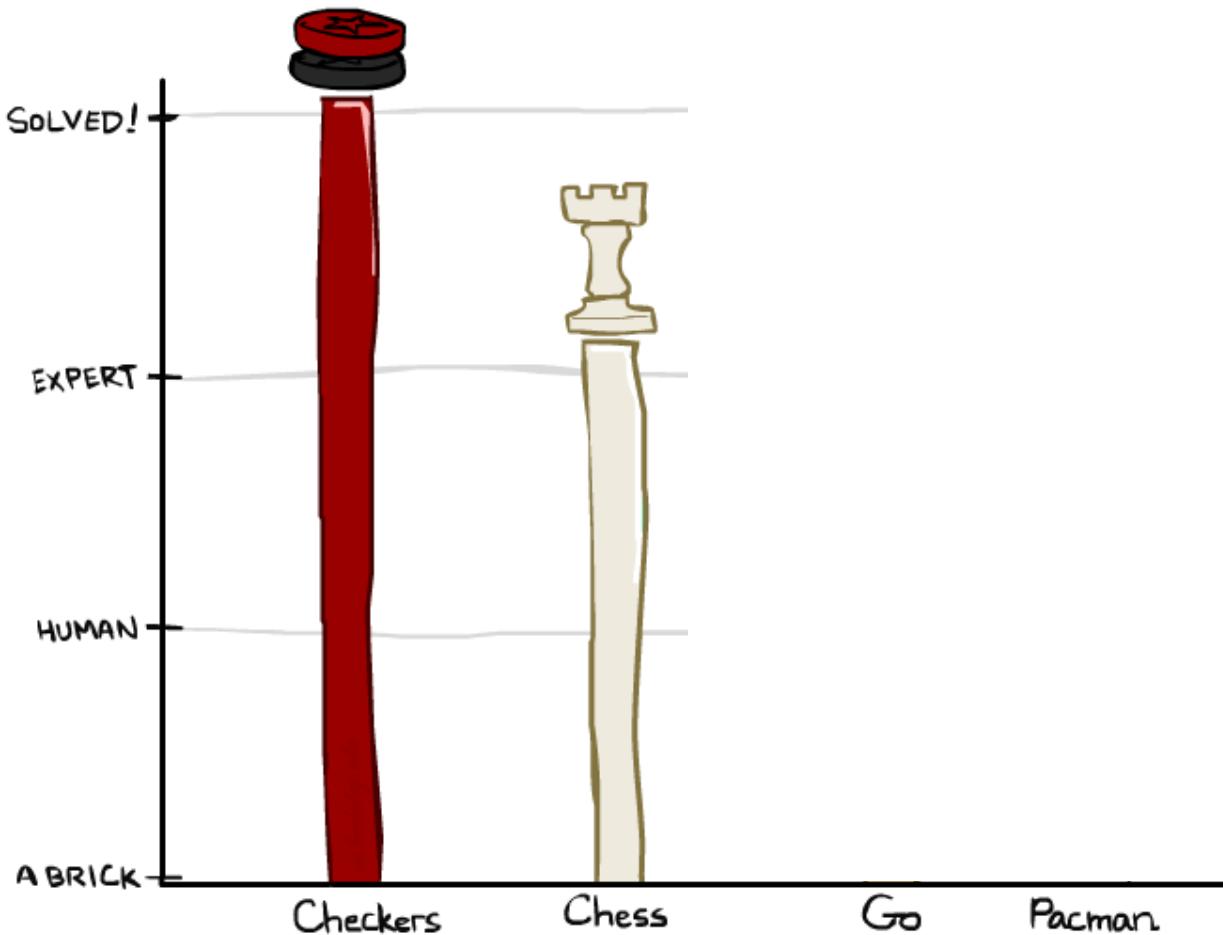
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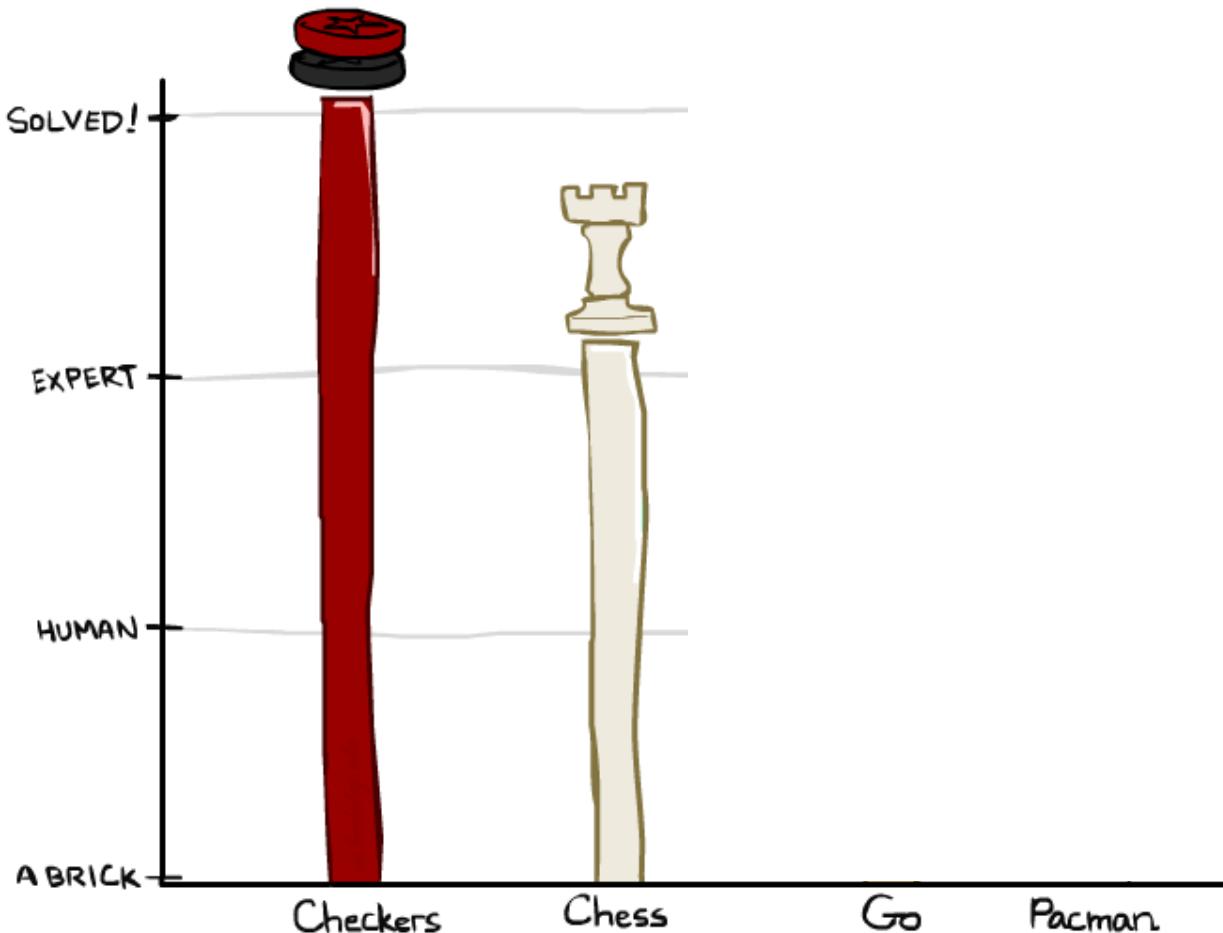
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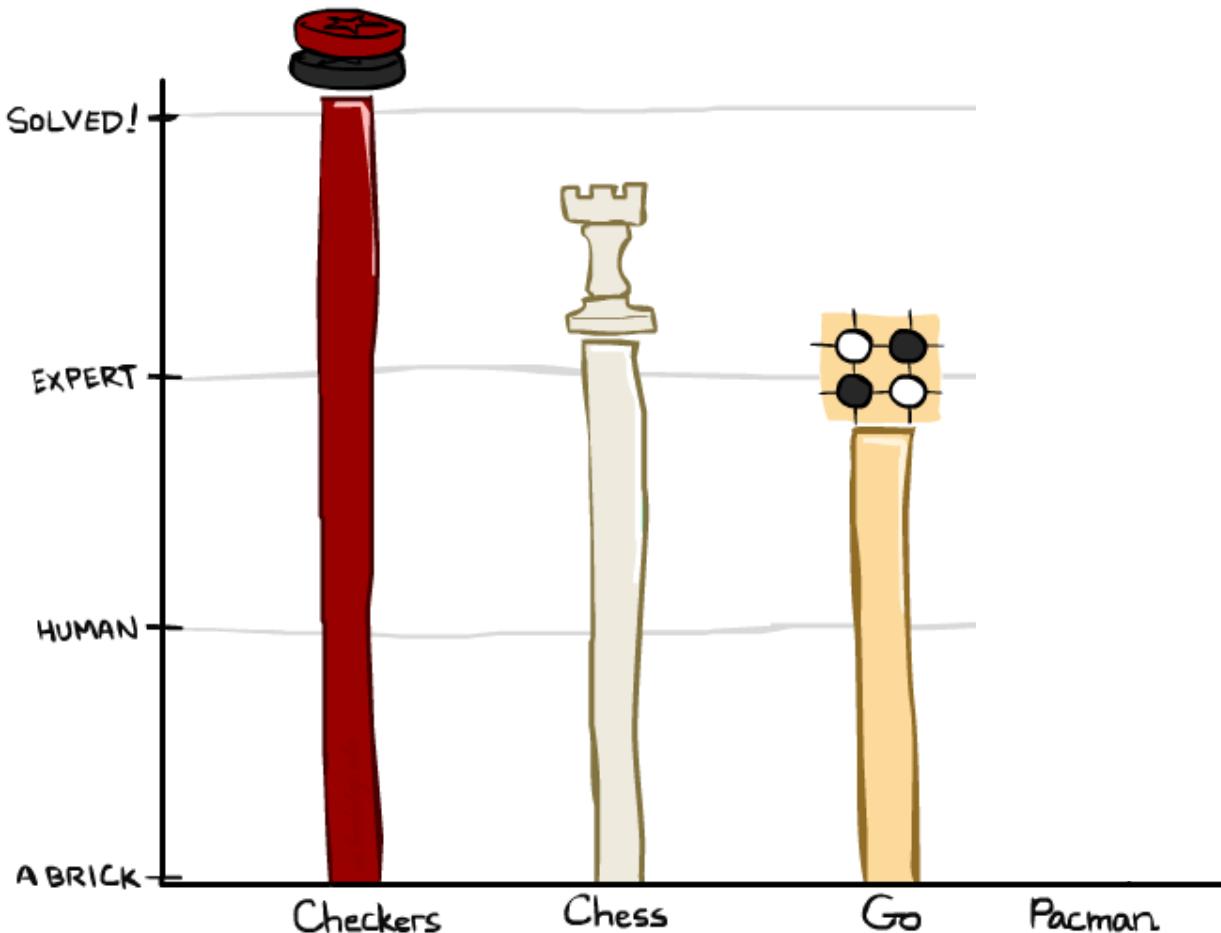
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- **Go:** Human champions are now starting to be challenged by machines, though the best humans still beat the best machines. In go, $b > 300!$ Classic programs use pattern knowledge bases, but big recent advances use Monte Carlo (randomized) expansion methods.



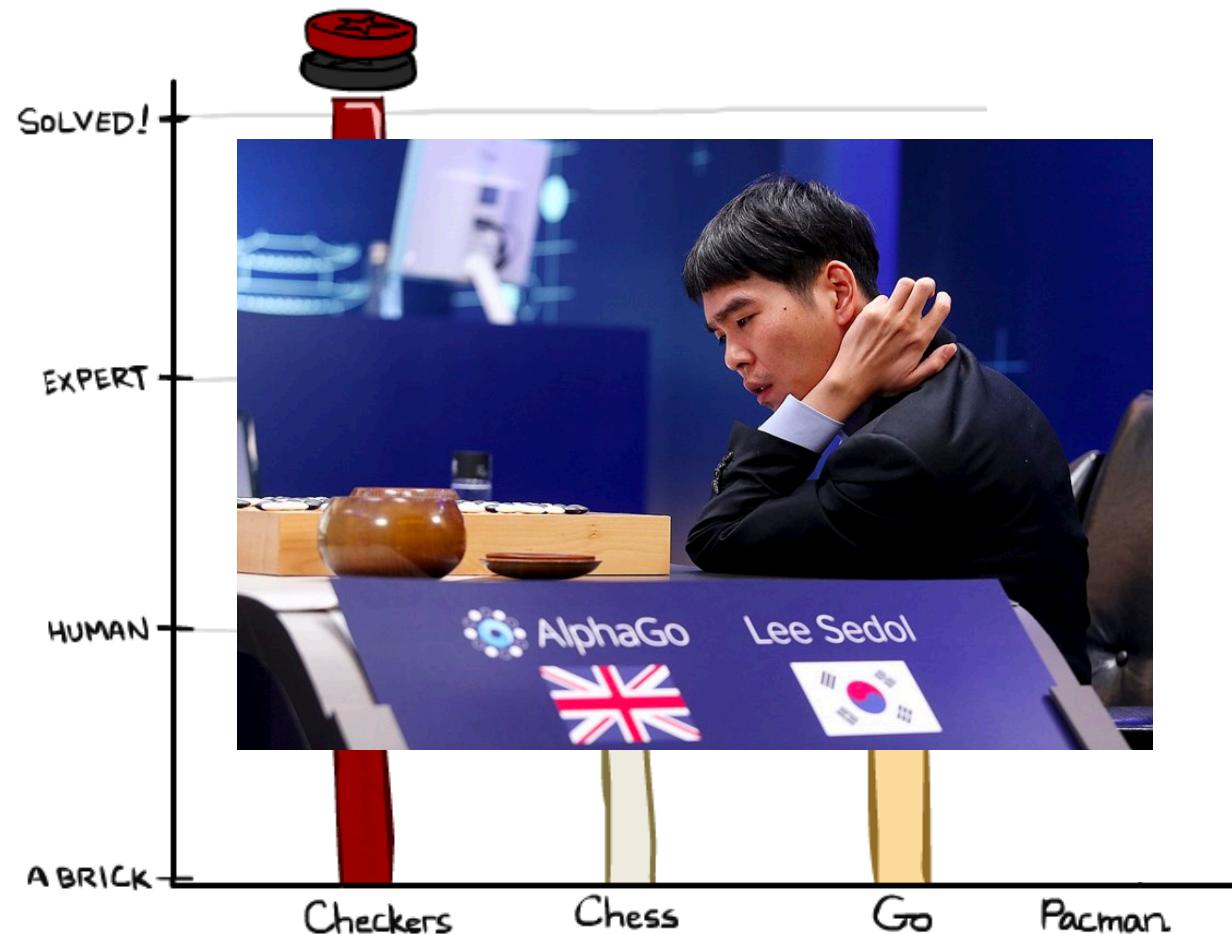
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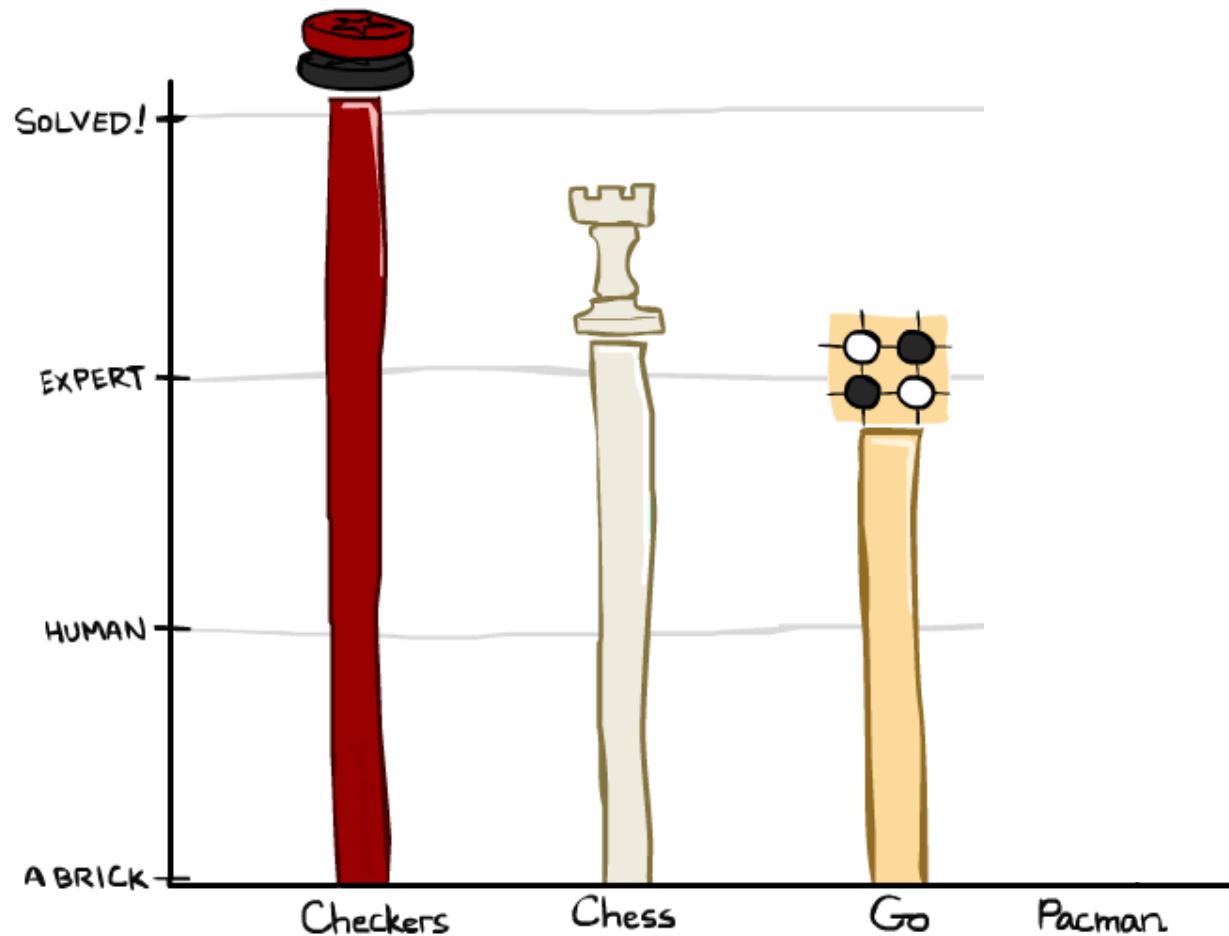
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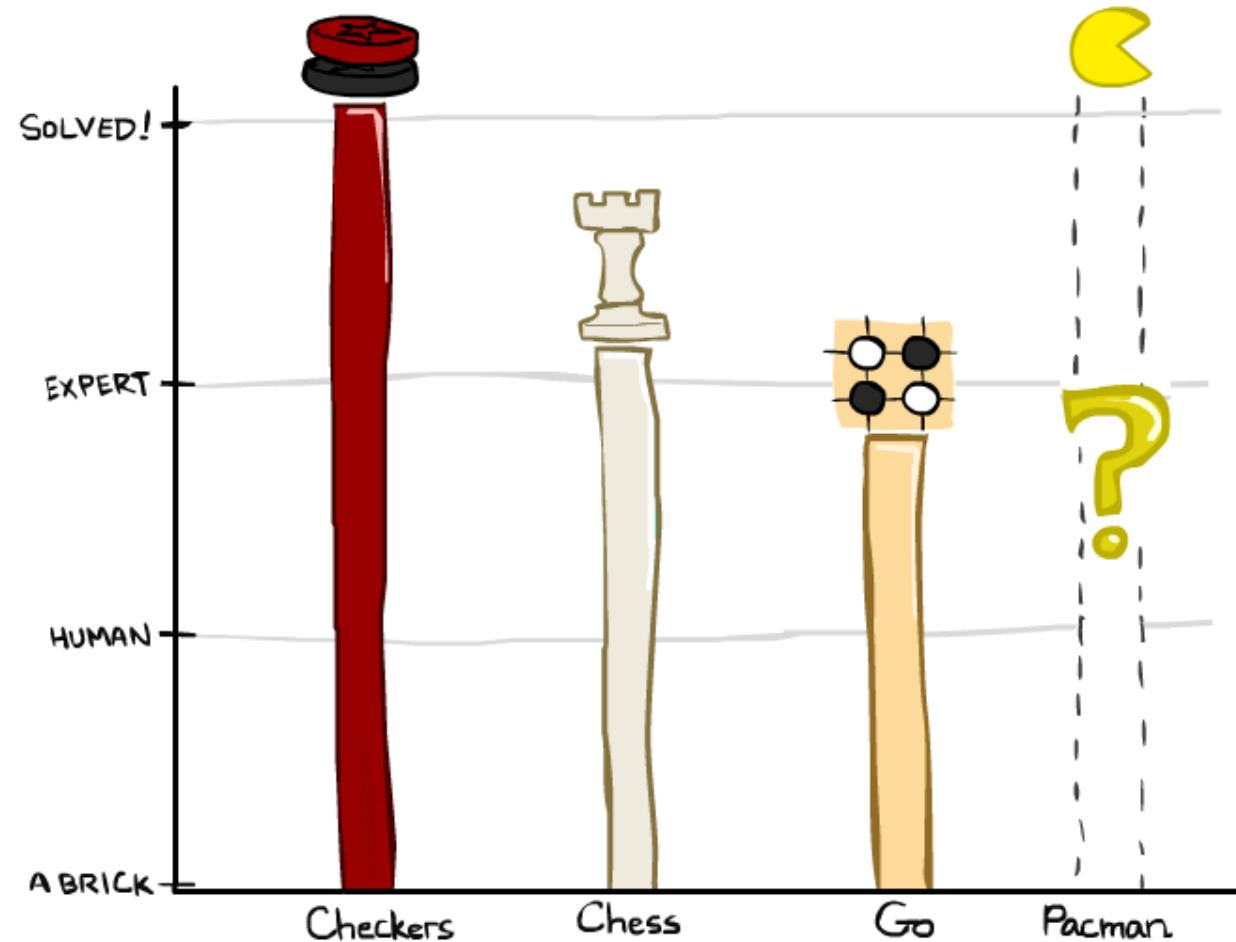
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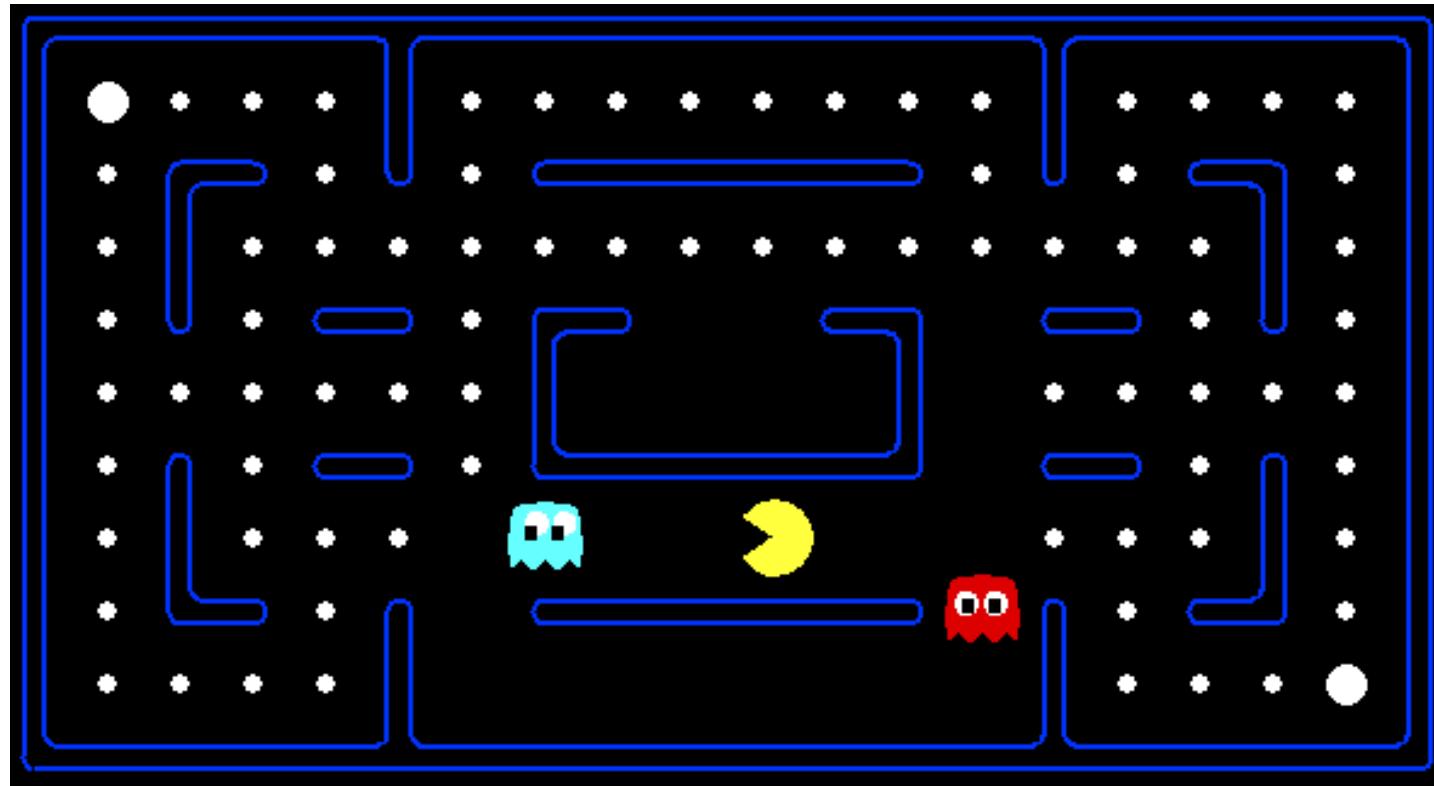


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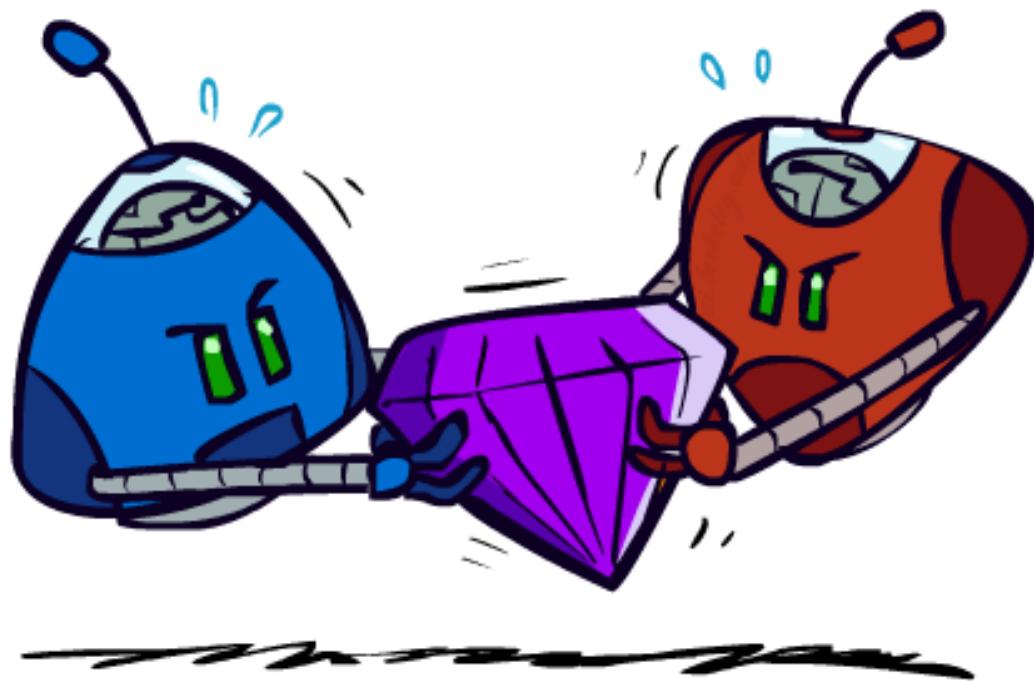


Behavior from Computation



[Demo: mystery pacman (L6D1)]

Adversarial Games



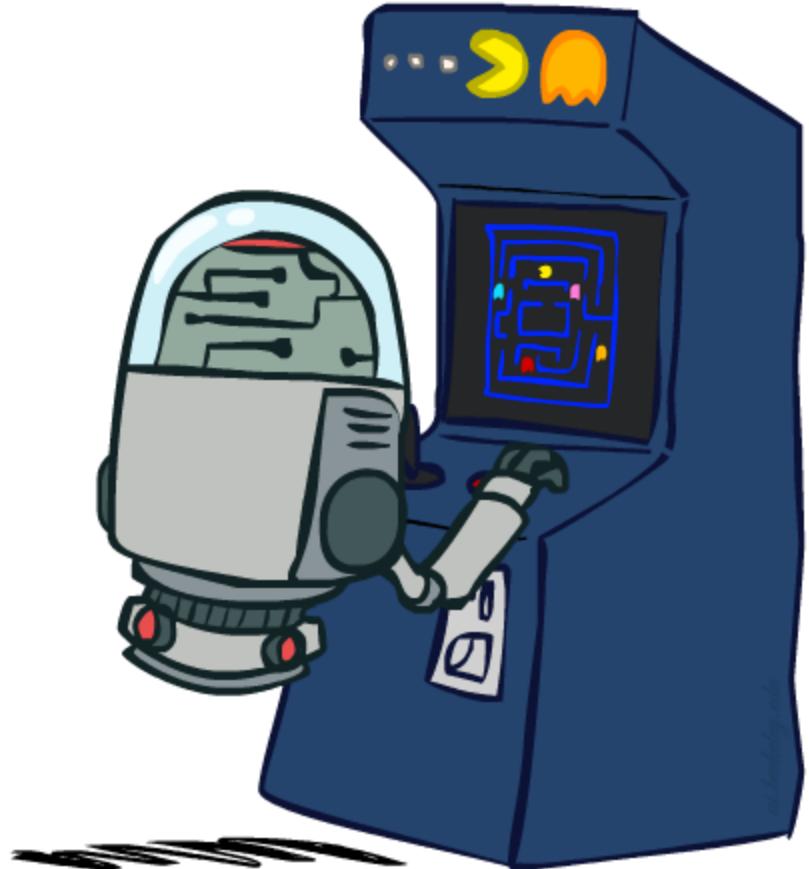
Types of Games

- Many different kinds of games!
- Axes:
 - Deterministic or stochastic?
 - One, two, or more players?
 - Zero sum?
 - Perfect information (can you see the state)?
- Want algorithms for calculating a **strategy (policy)** which recommends a move from each state



Deterministic Games

- Many possible formalizations, one is:
 - States: S (start at s_0)
 - Players: $P=\{1\dots N\}$ (usually take turns)
 - Actions: A (may depend on player / state)
 - Transition Function: $S \times A \rightarrow S$
 - Terminal Test: $S \rightarrow \{t, f\}$
 - Terminal Utilities: $S \times P \rightarrow R$
- Solution for a player is a **policy**: $S \rightarrow A$



Zero-Sum Games



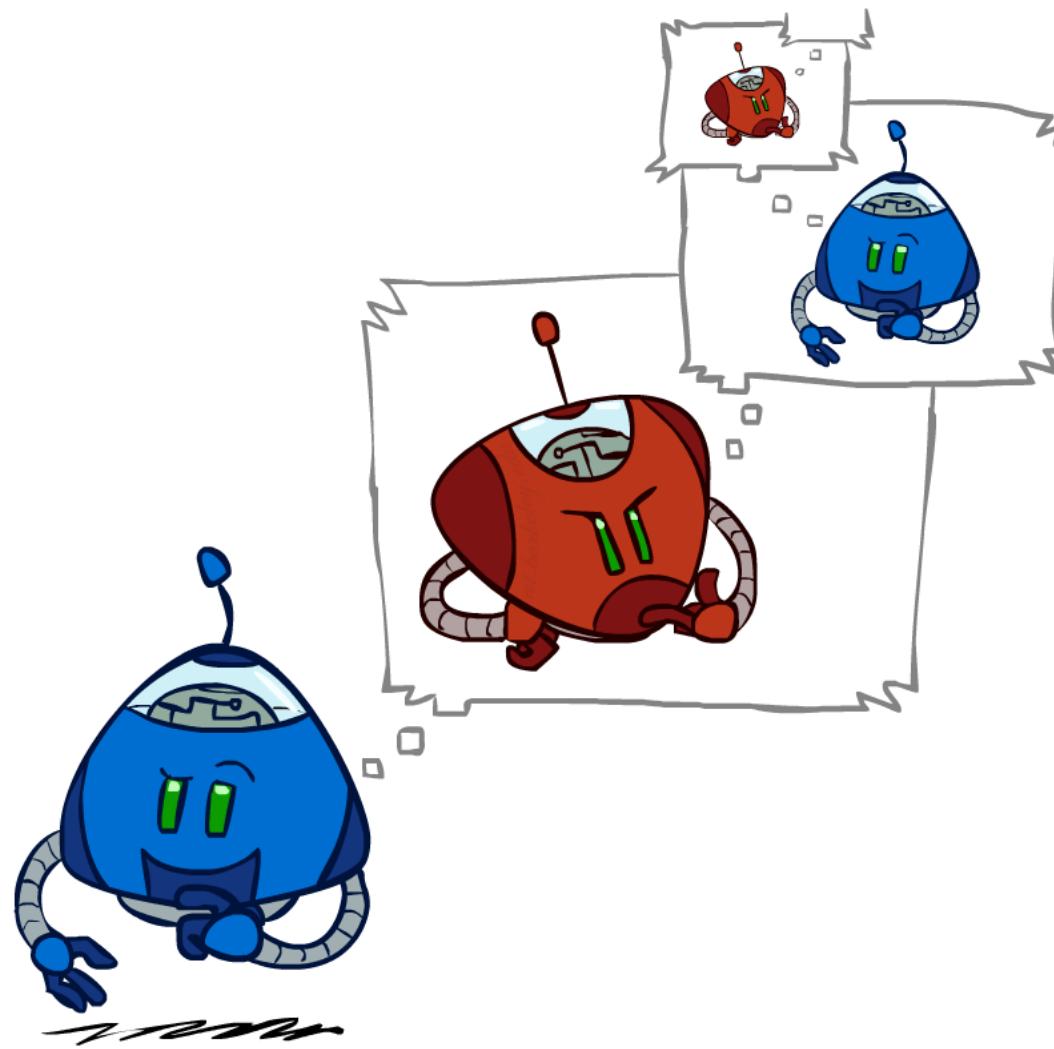
- **Zero-Sum Games**

- Agents have opposite utilities (values on outcomes)
- Lets us think of a single value that one maximizes and the other minimizes
- Adversarial, pure competition

- **General Games**

- Agents have independent utilities (values on outcomes)
- Cooperation, indifference, competition, and more are all possible
- More later on non-zero-sum games

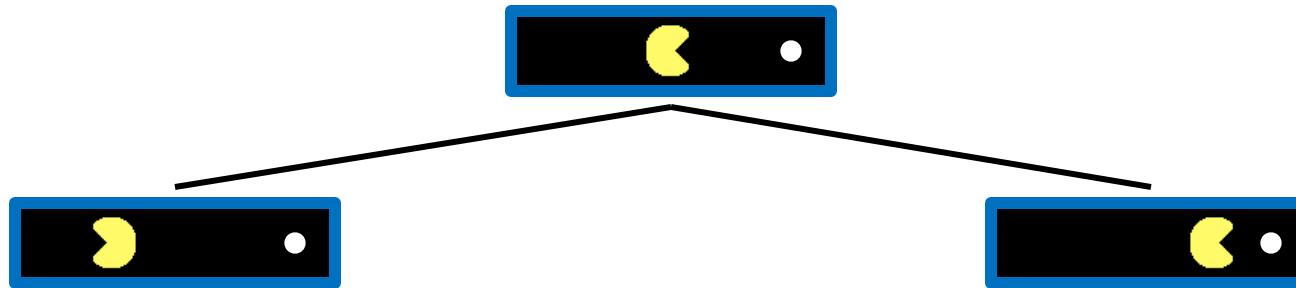
Adversarial Search



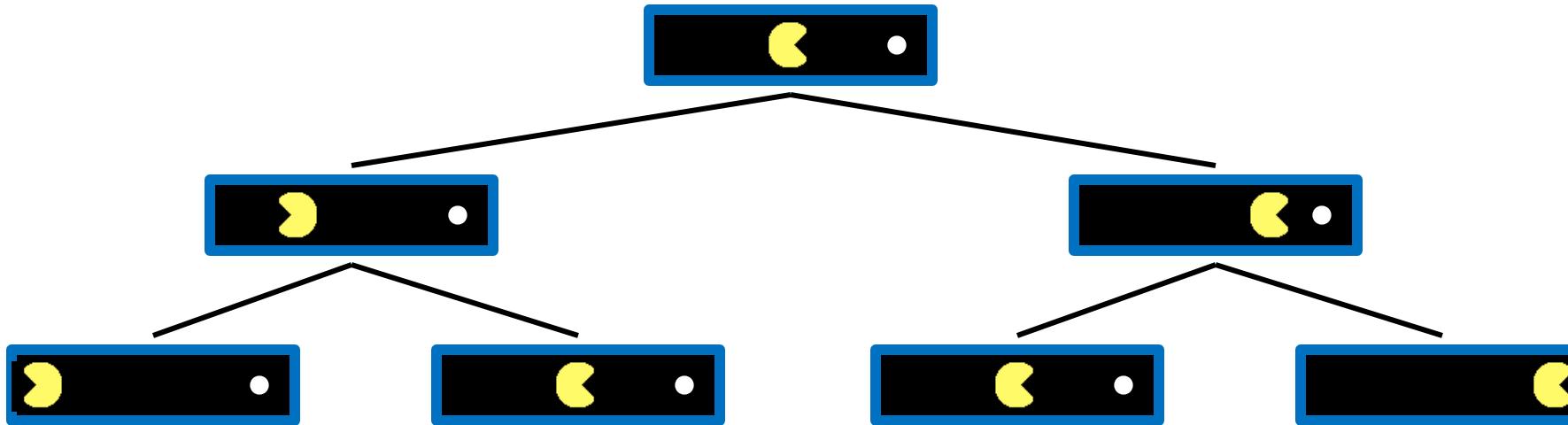
Single-Agent Trees



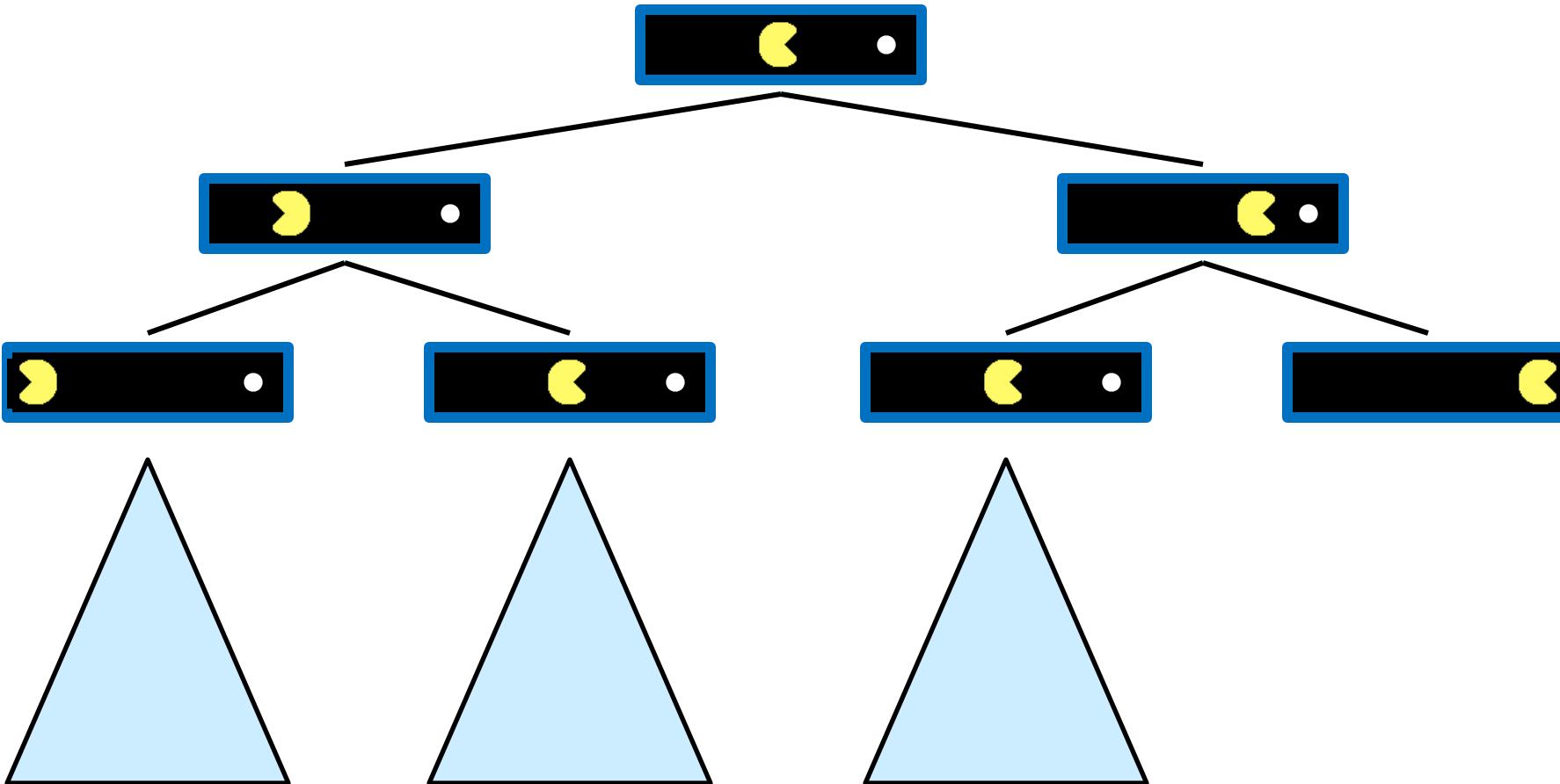
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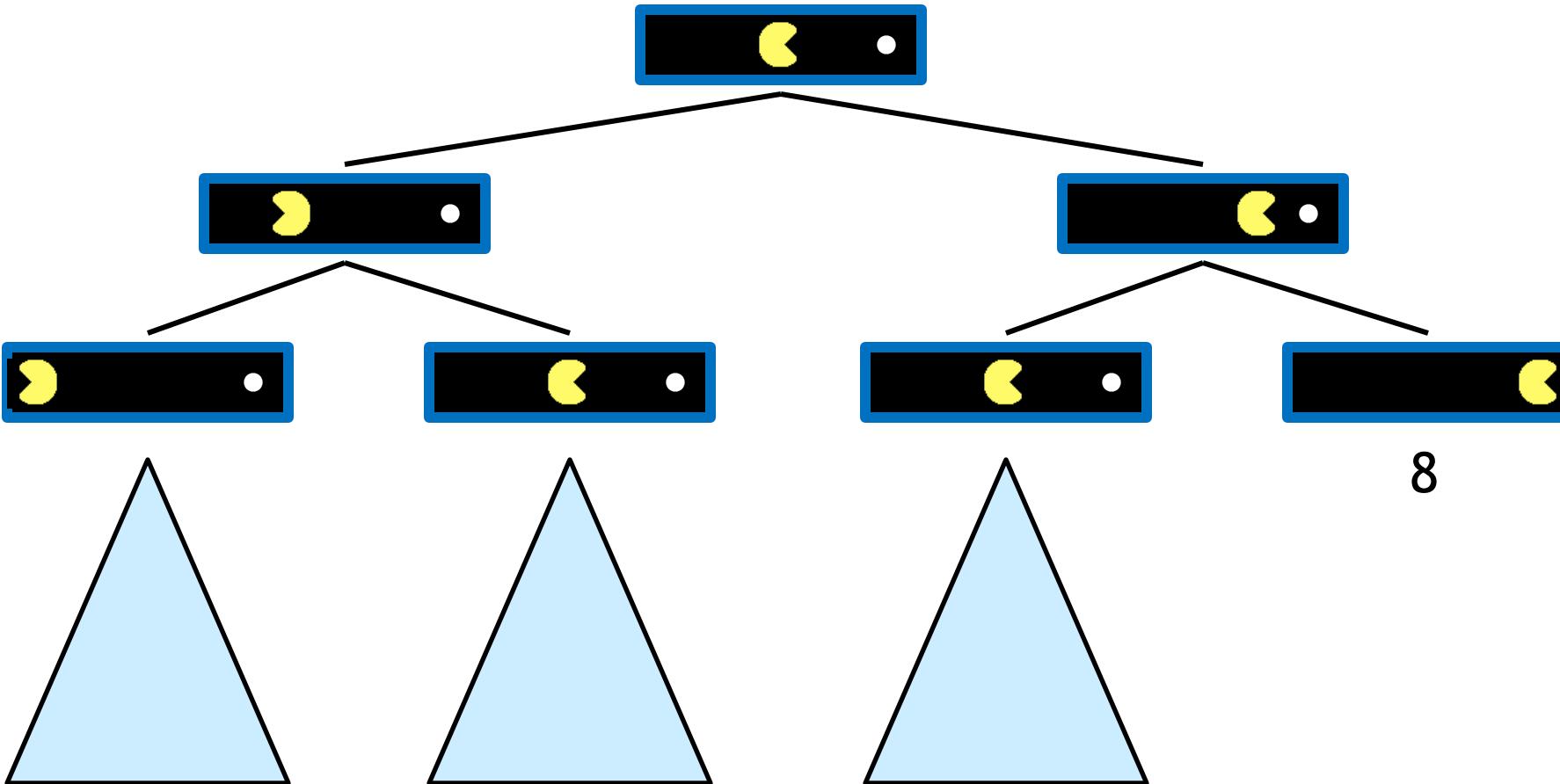
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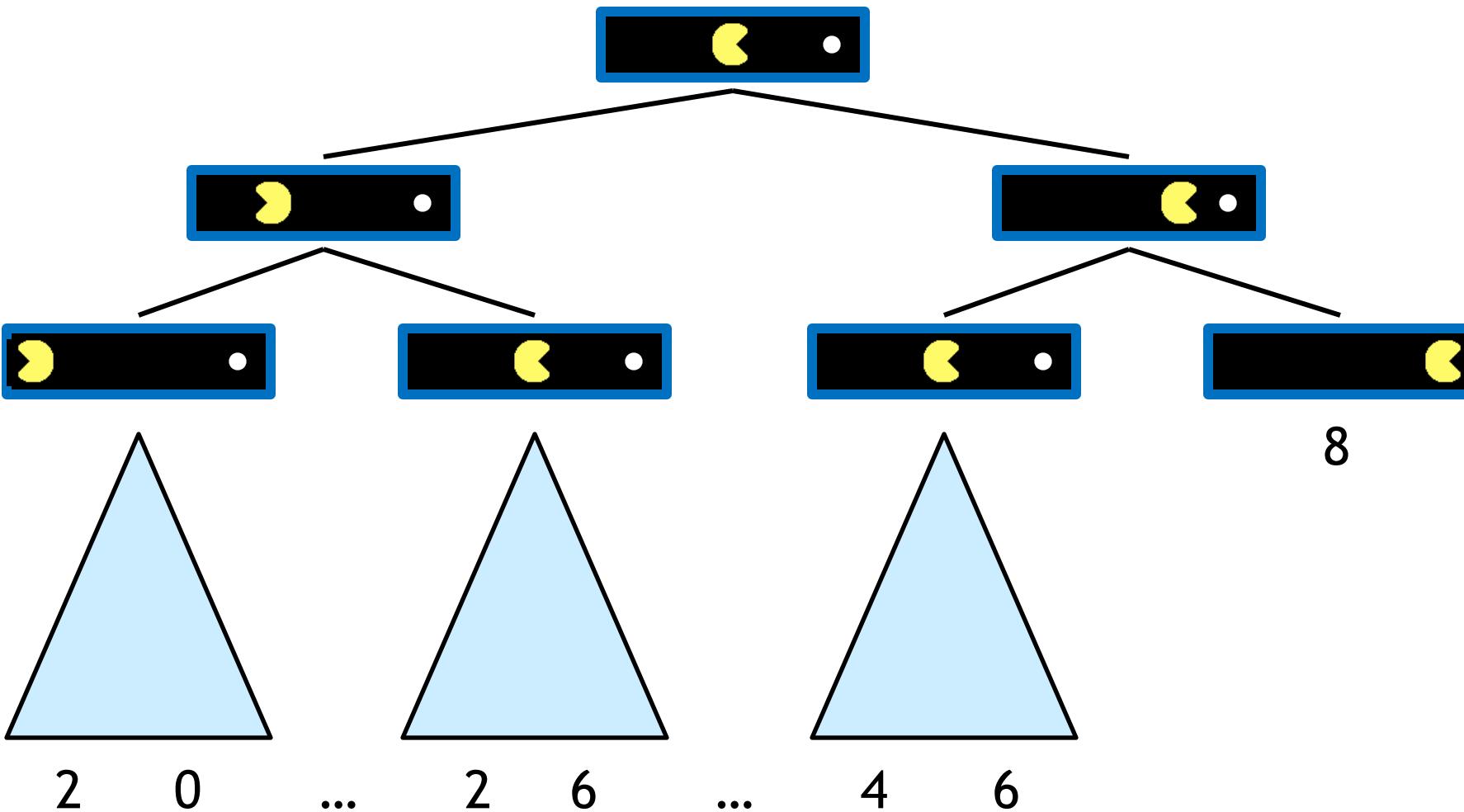
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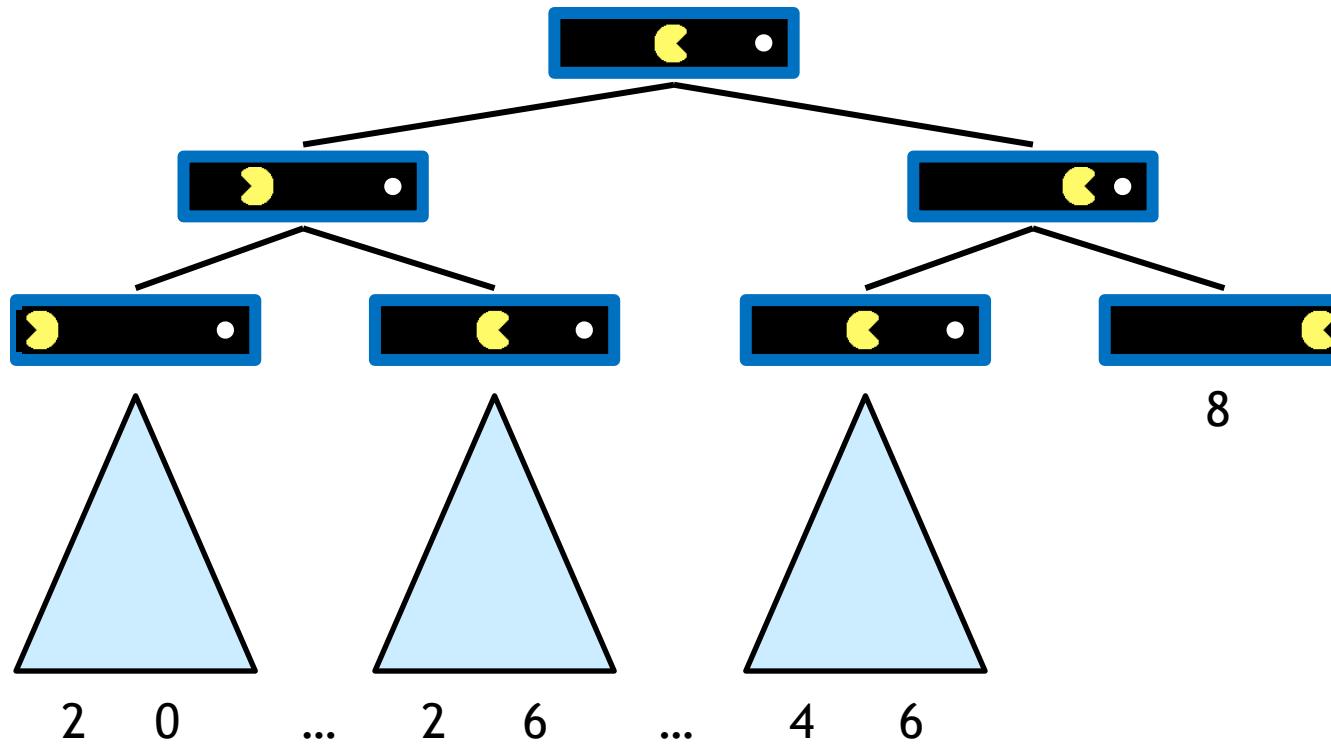
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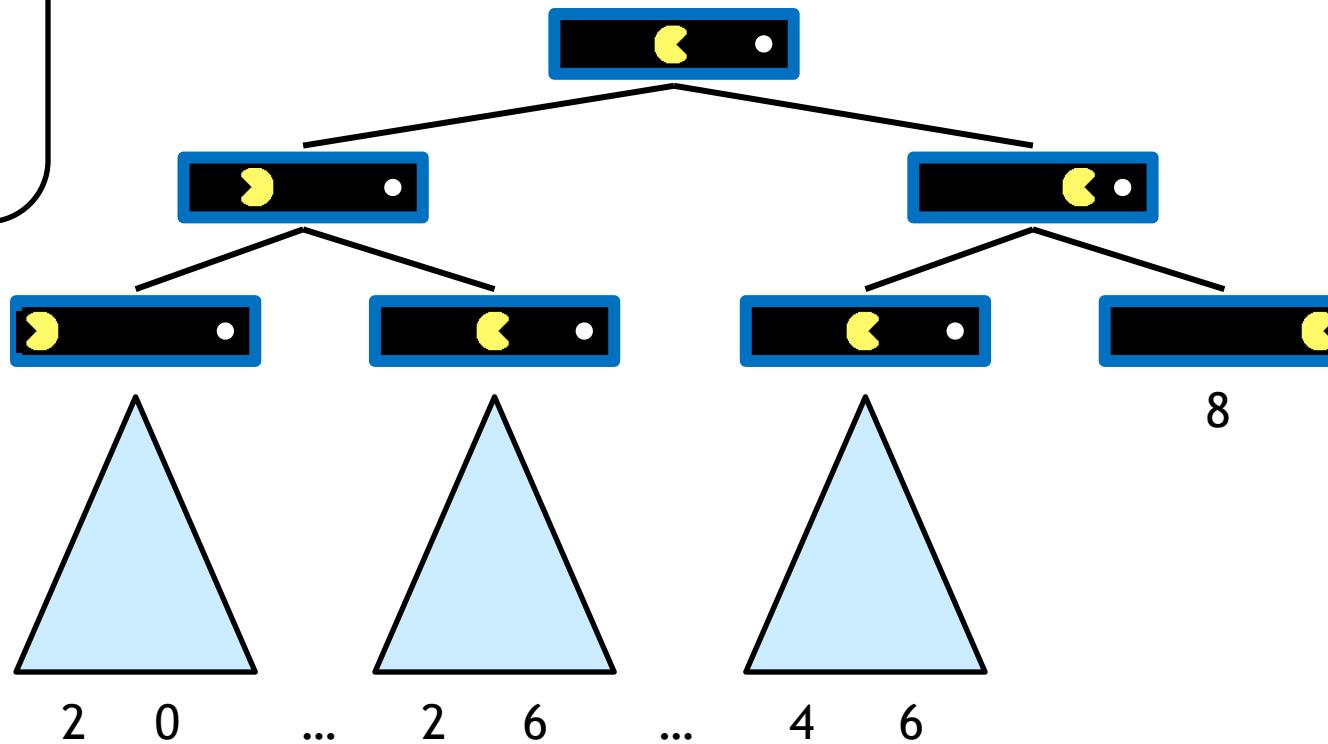


Value of a State



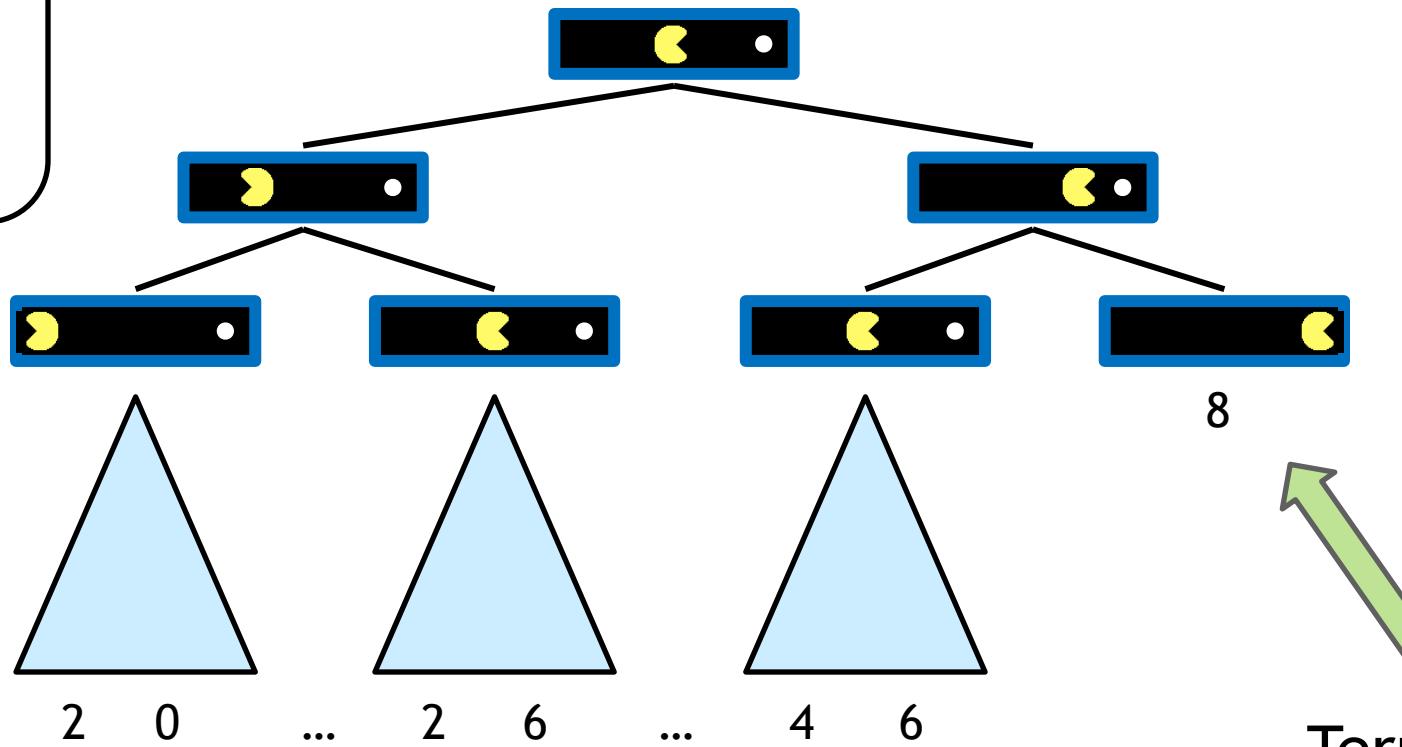
Value of a State

Value of a state:
The best
achievable
outcome (utility)
from that state



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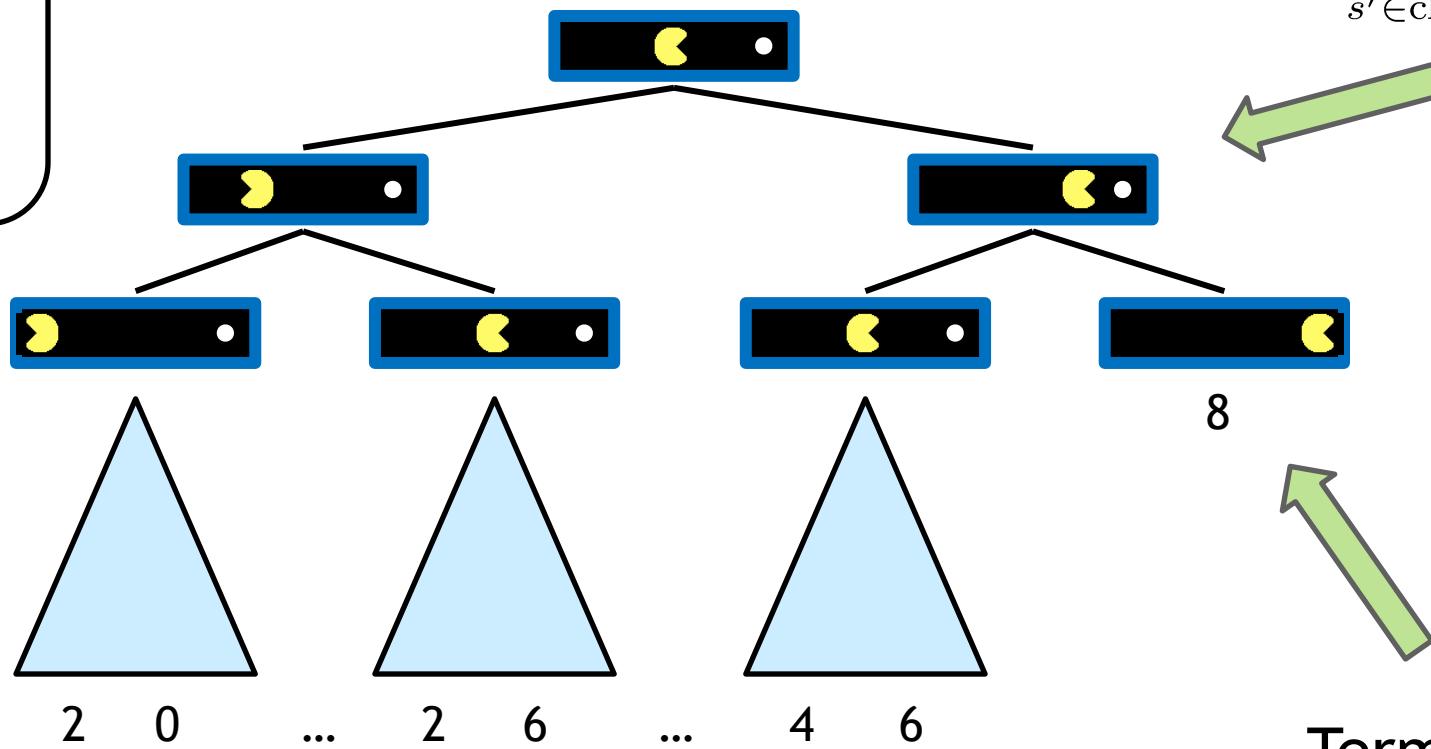


Terminal States:

$$V(s) = \text{known}$$

Value of a State

Value of a state:
The best
achievable
outcome (utility)
from that state



Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

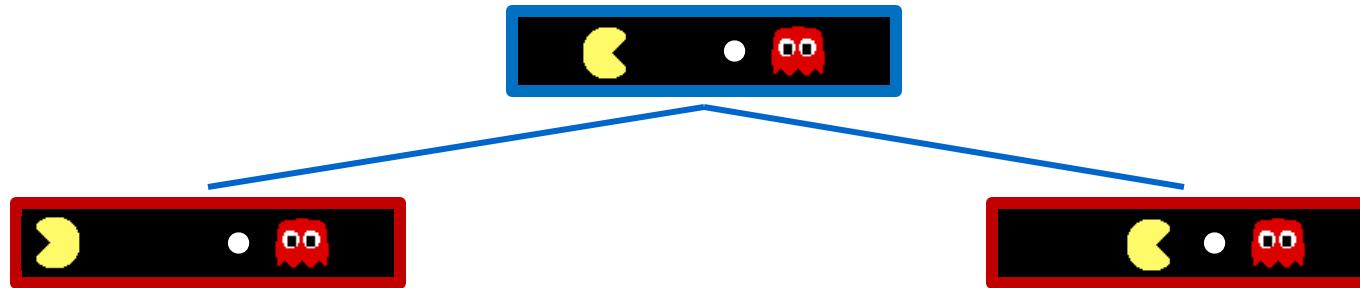
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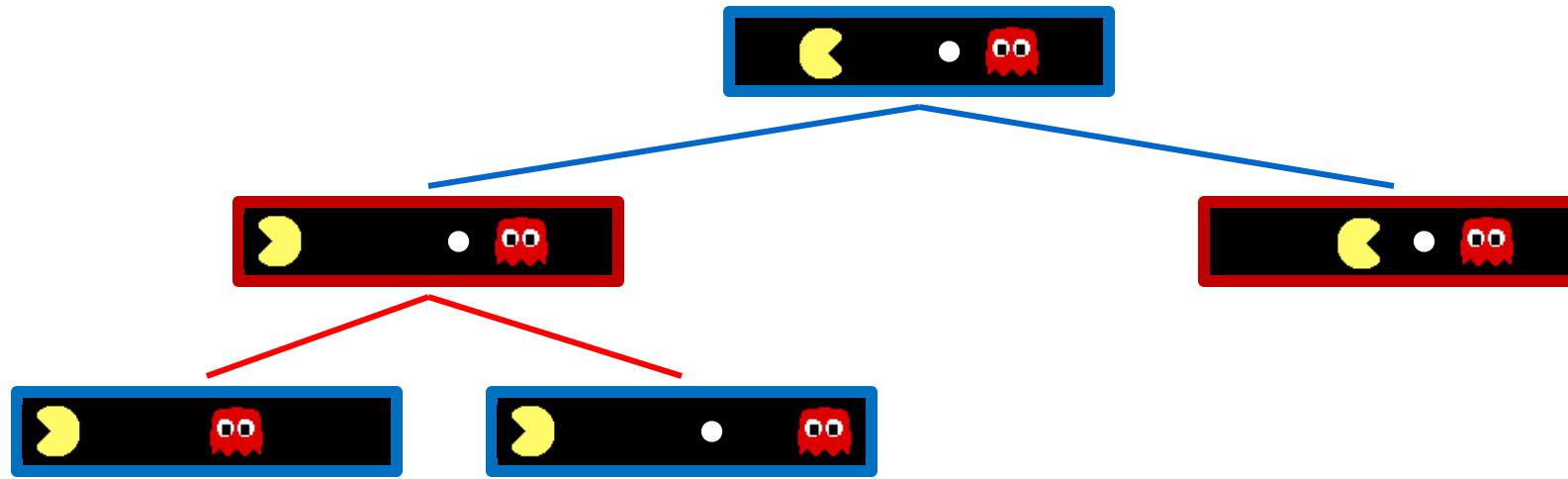
Adversarial Game Trees



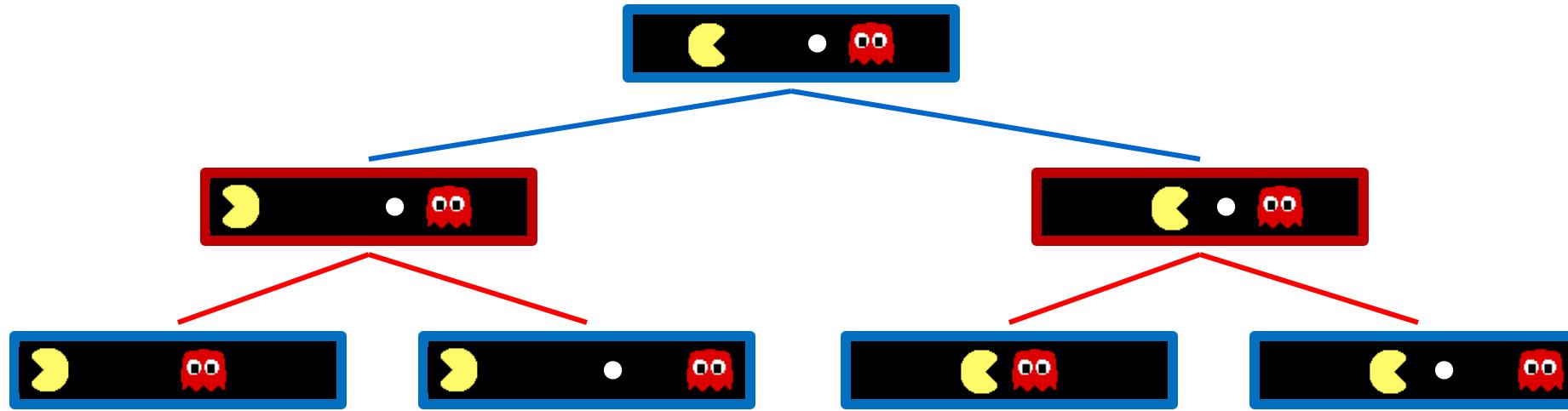
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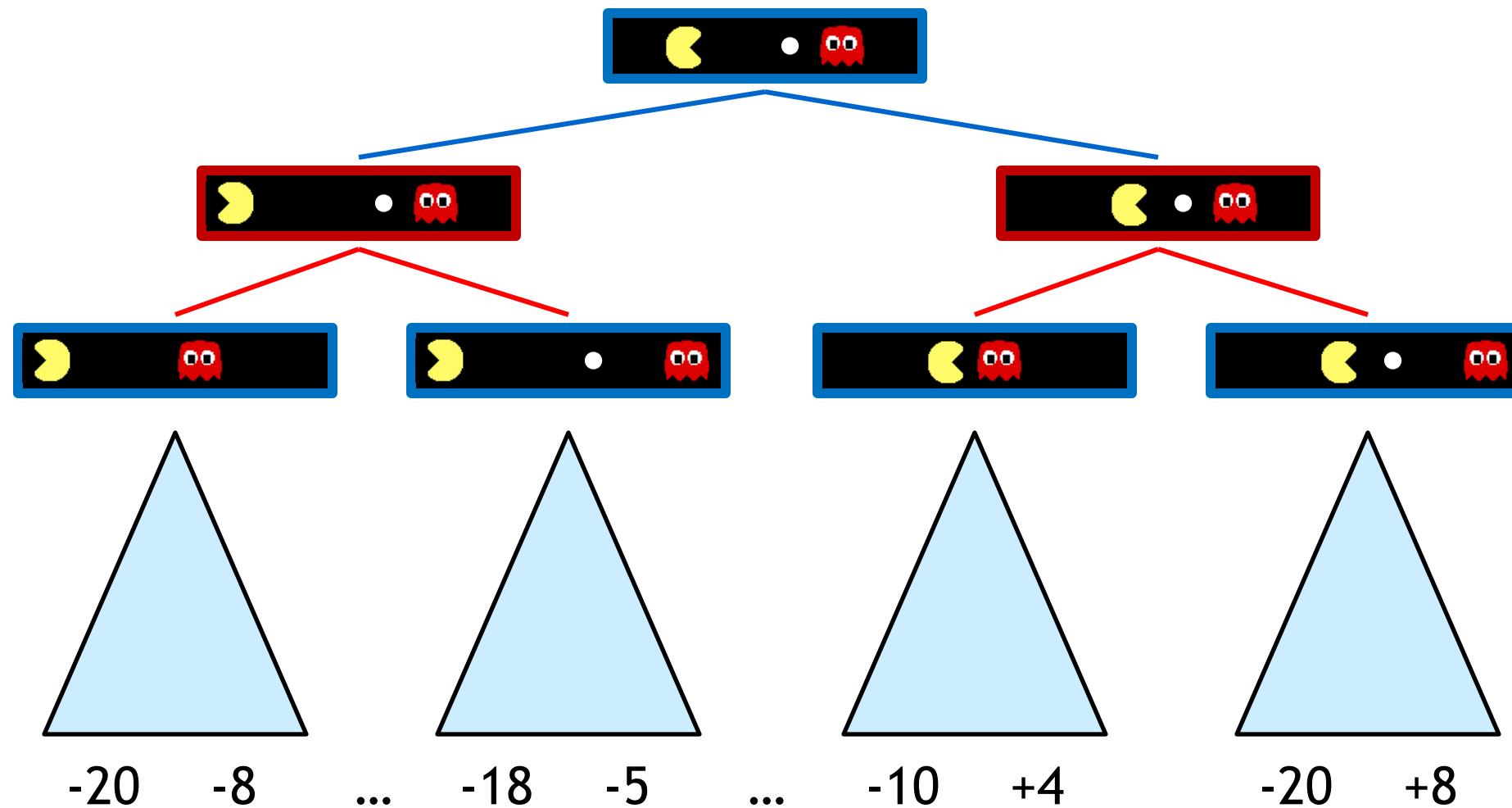
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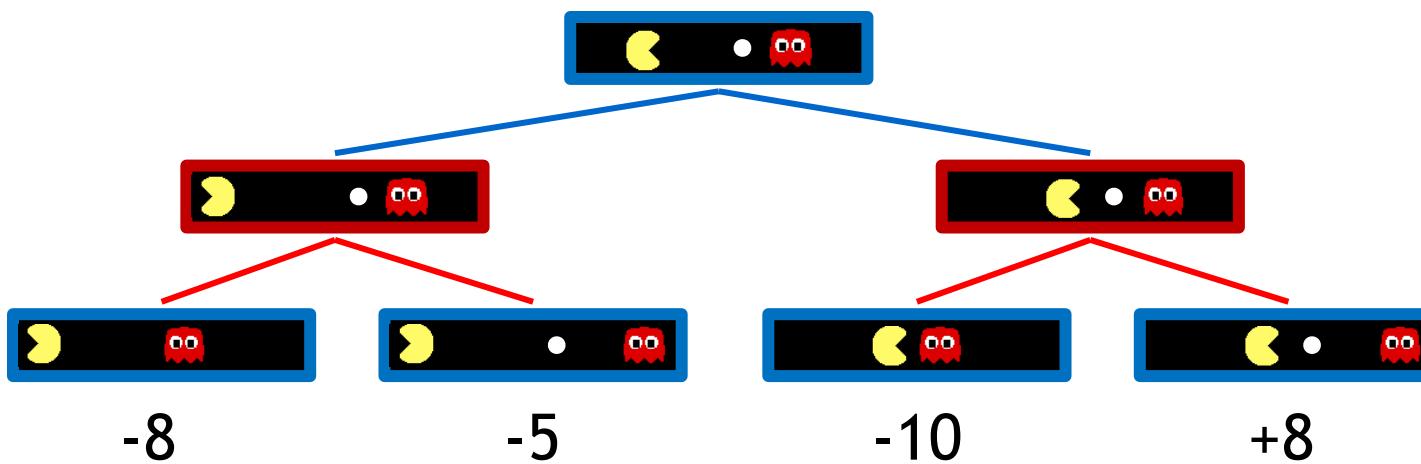
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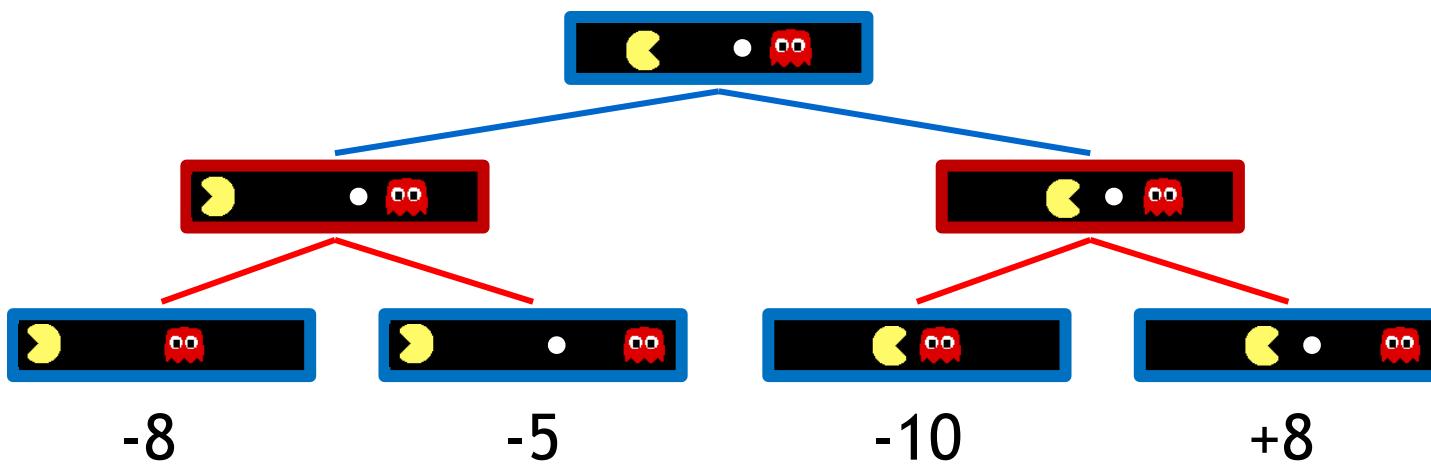
Adversarial Game Trees



Minimax Values



Minimax Values



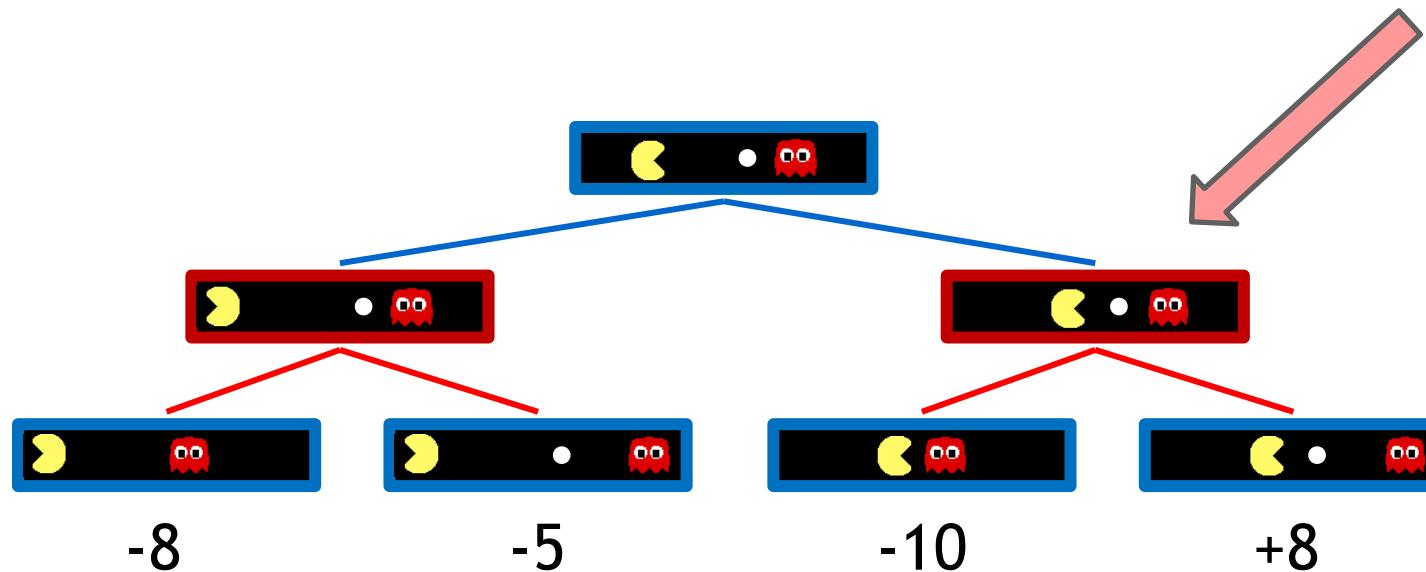
Terminal States:

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Minimax Values

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



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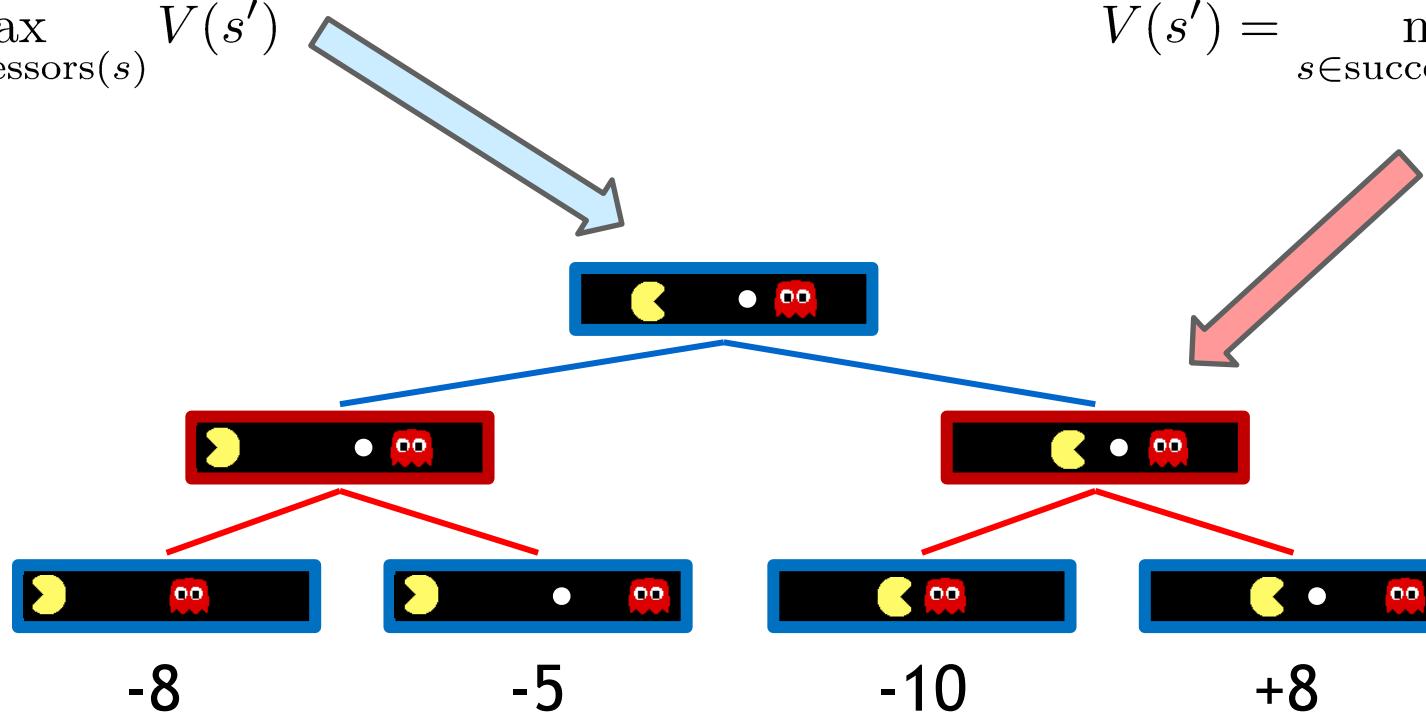
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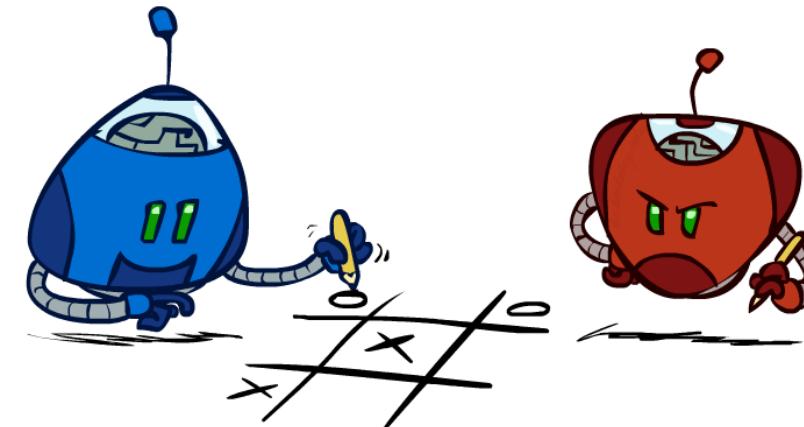
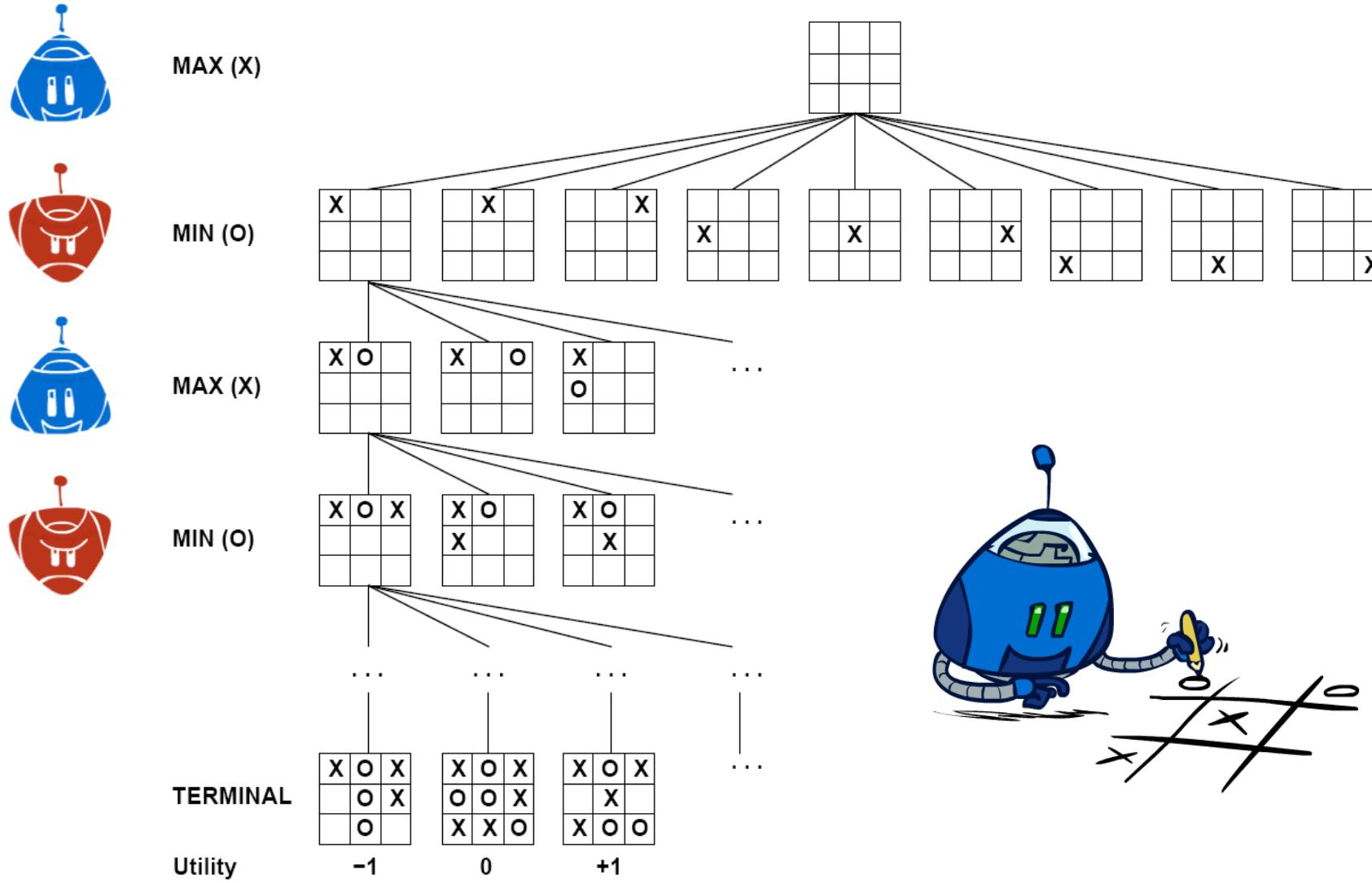
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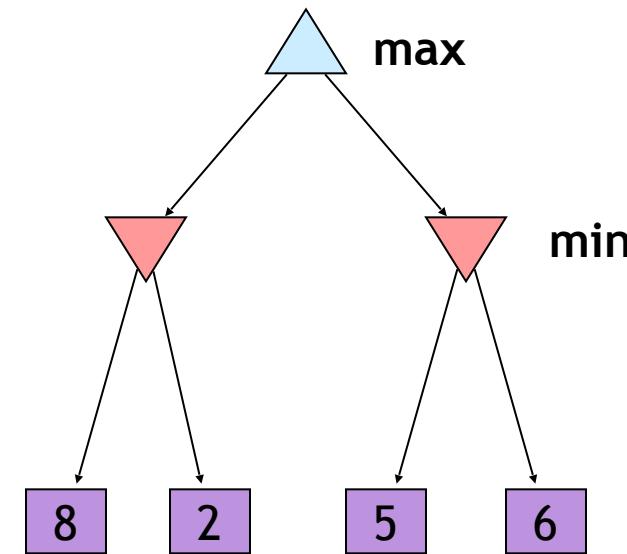
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Tic-Tac-Toe Game Tree



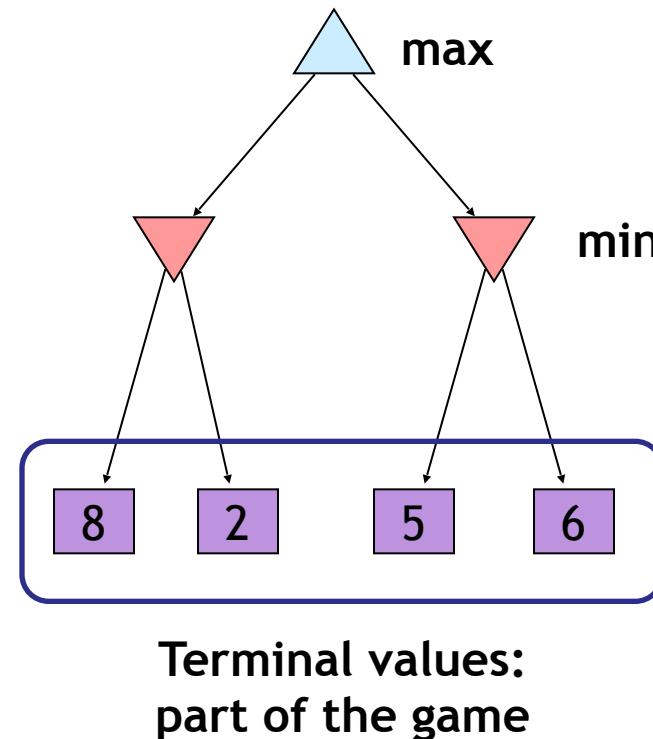
Adversarial Search (Minimax)

- Deterministic, zero-sum games:
 - Tic-tac-toe, chess, checkers
 - One player maximizes result
 - The other minimizes result
- Minimax search:
 - A state-space search tree
 - Players alternate turns
 - Compute each node's **minimax value**:
the best achievable utility against a rational (optimal) adversary



Adversarial Search (Minimax)

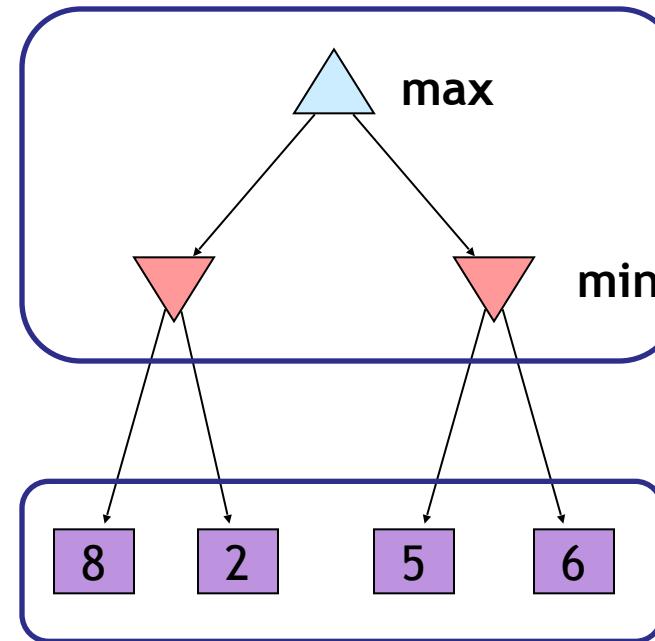
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Minimax values:
computed recursively

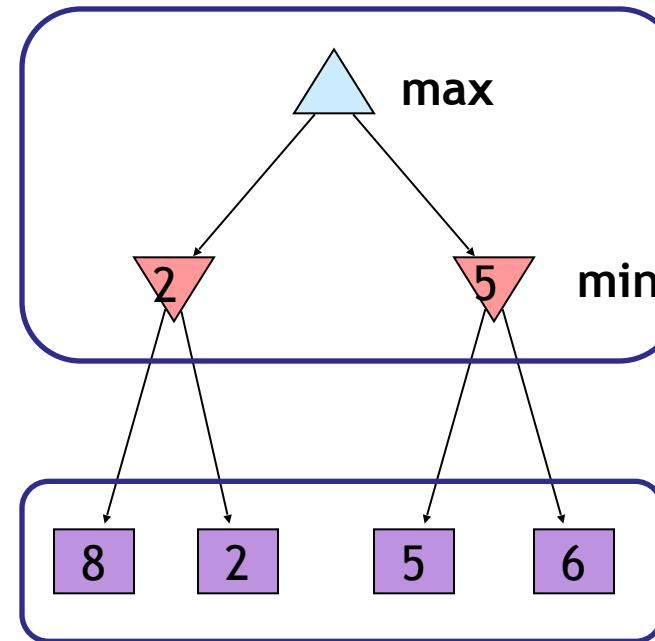


Terminal values:
part of the game

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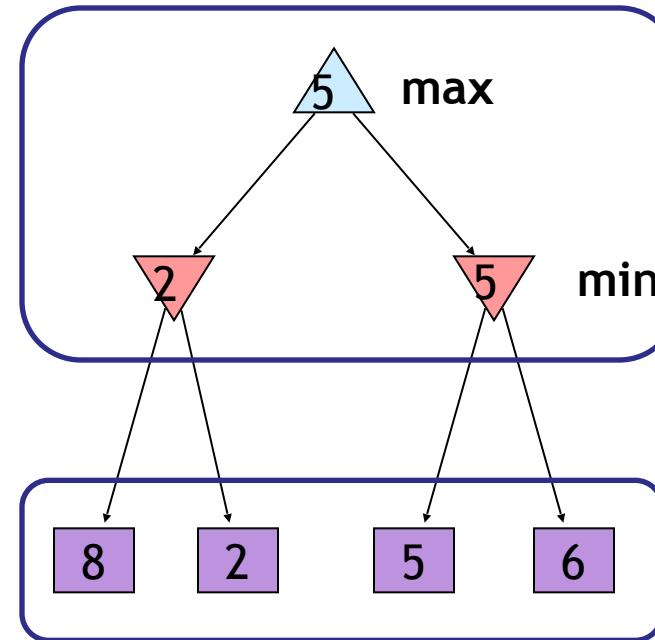


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Terminal values:
part of the game

Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

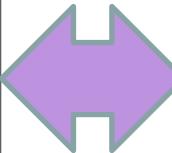
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def min-value(state):
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$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation (Dispatch)

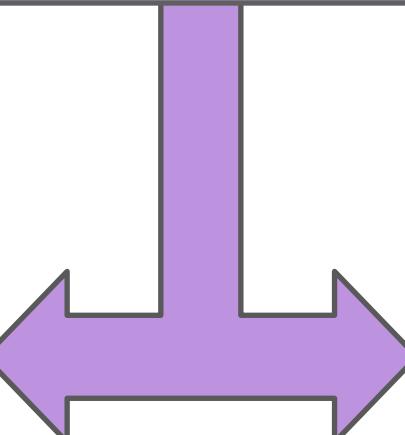
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

Minimax Implementation (Dispatch)

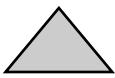
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    return v
```

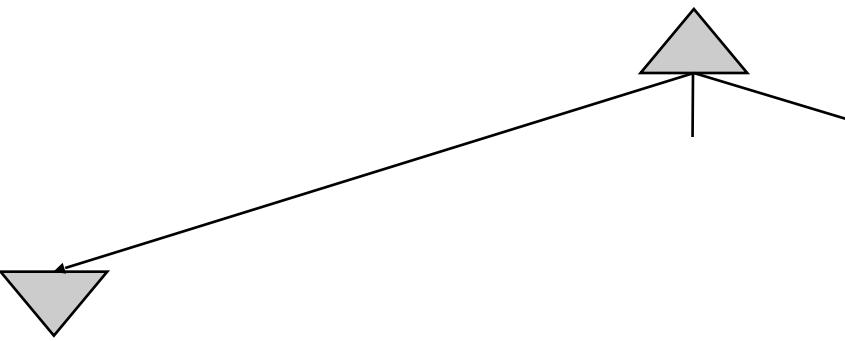
```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```



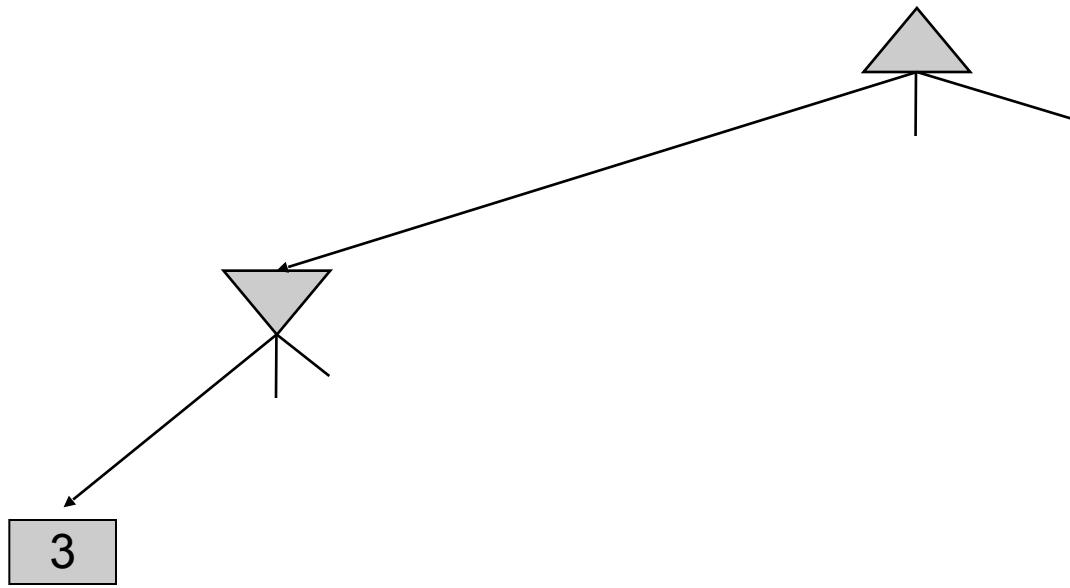
Minimax Example



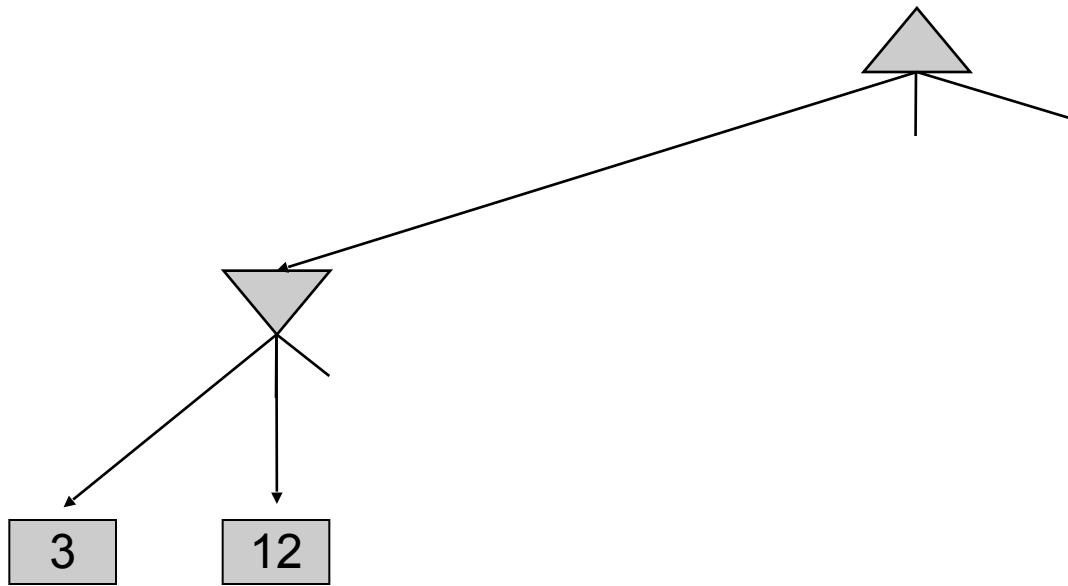
Minimax Example



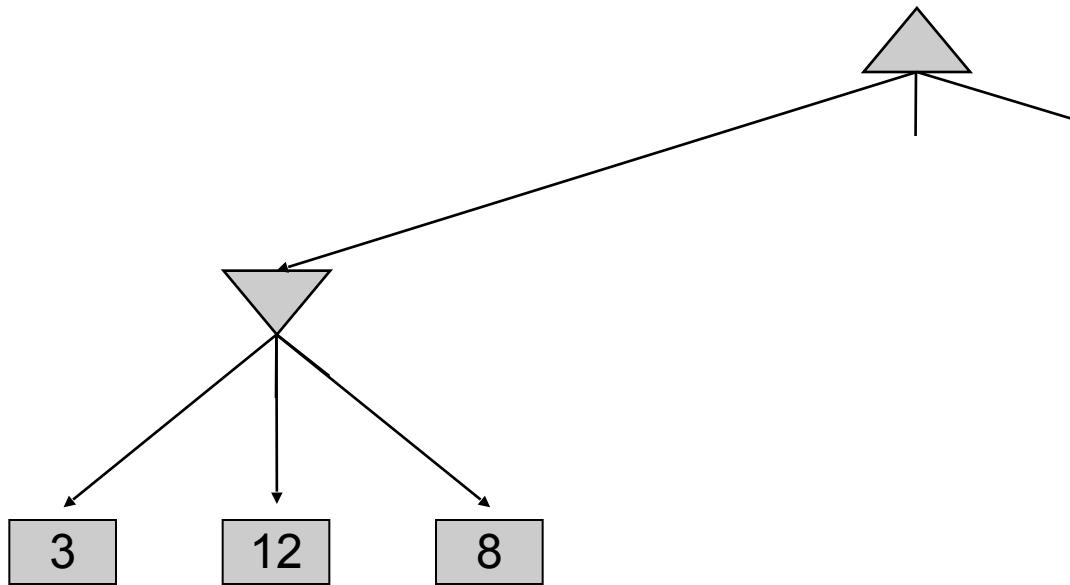
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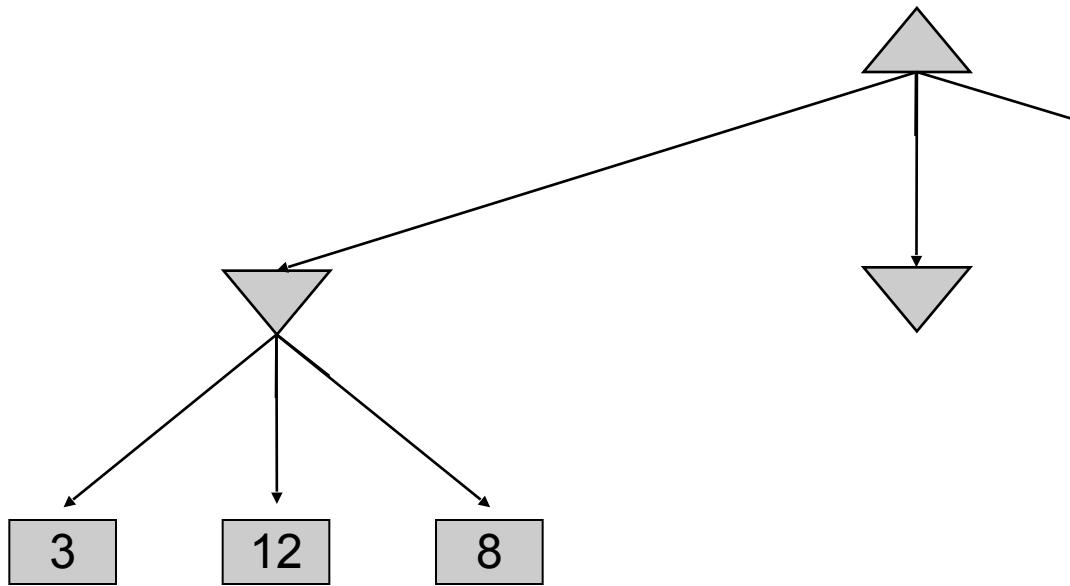
Minimax Example



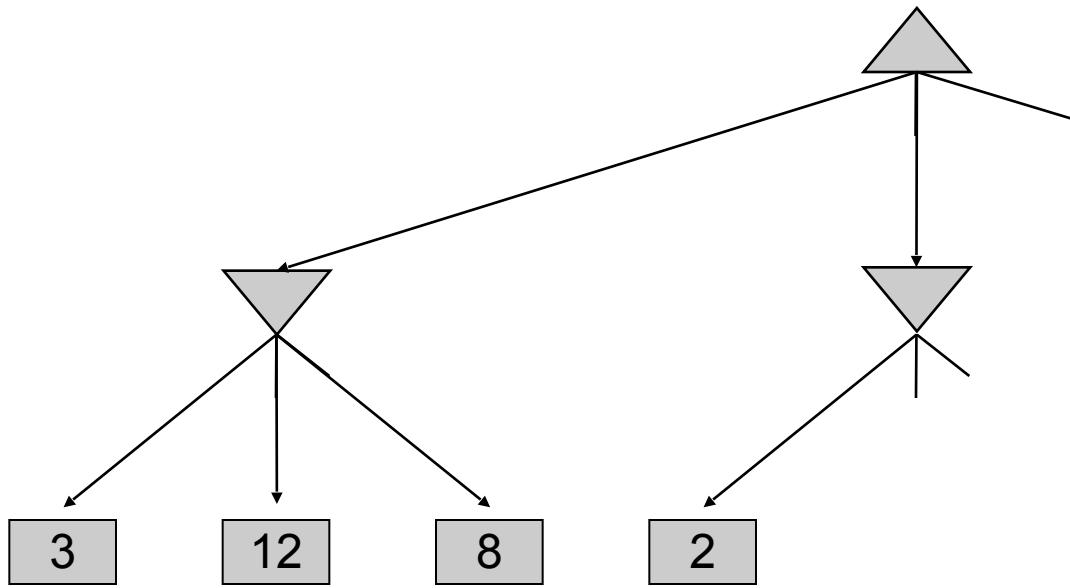
Minimax Example



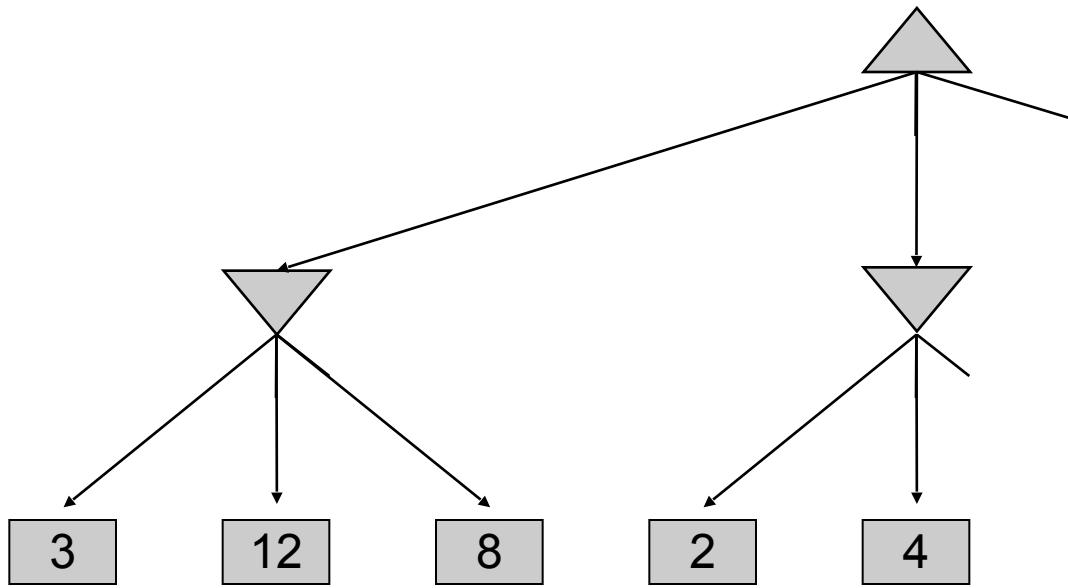
Minimax Example



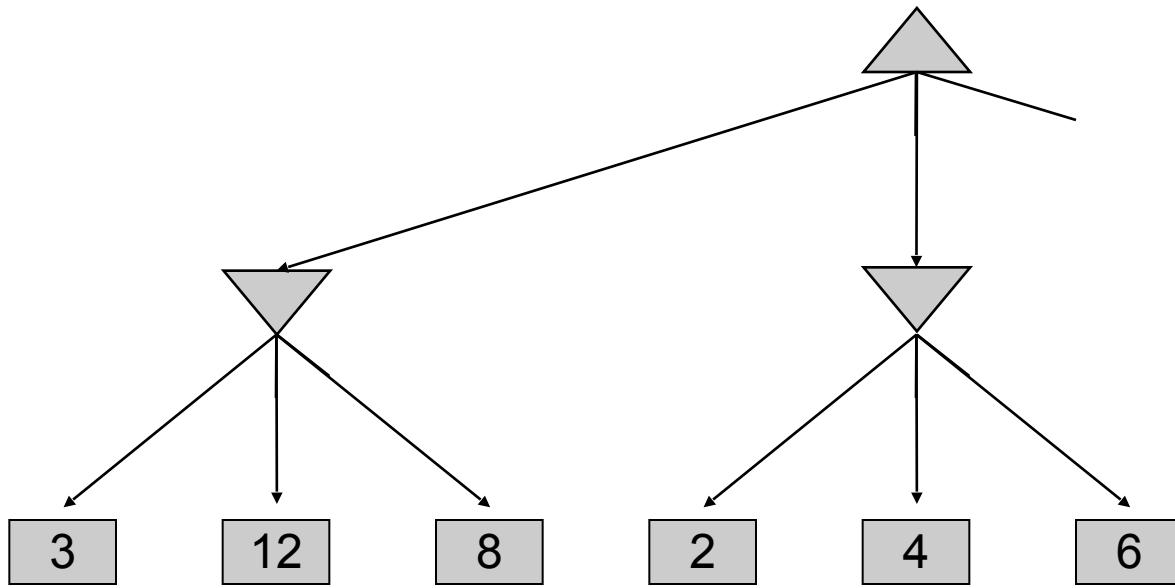
Minimax Example



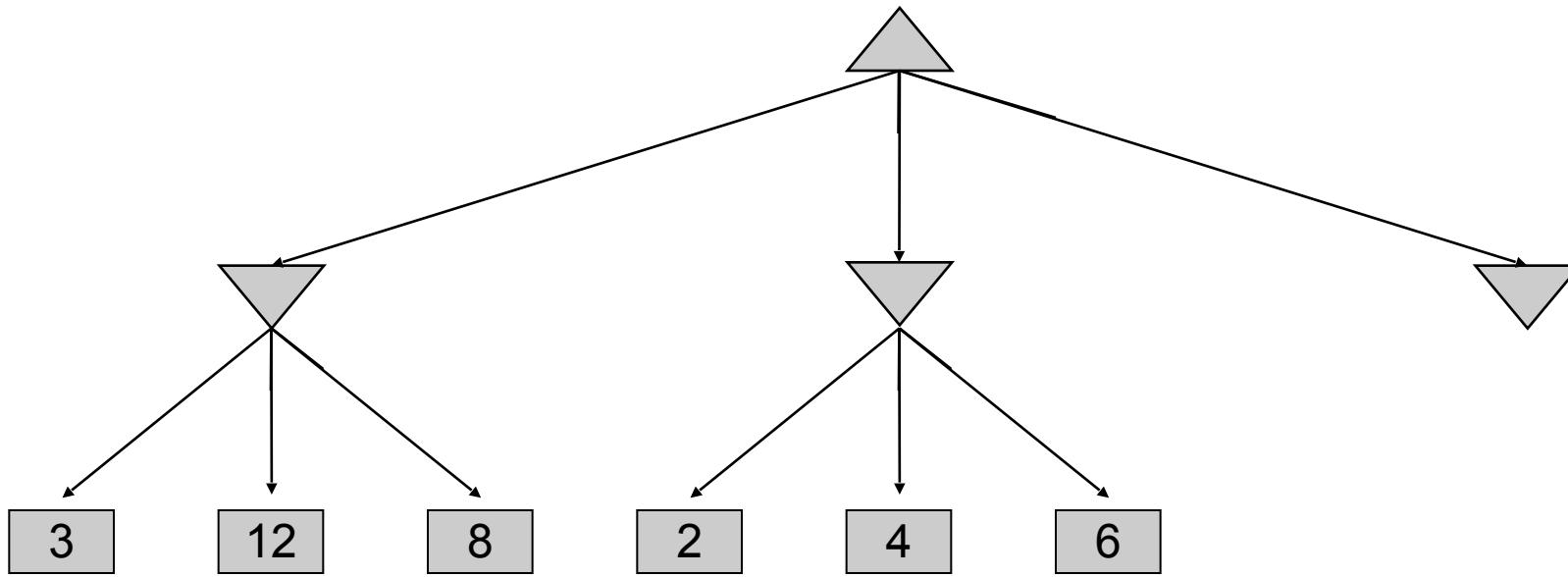
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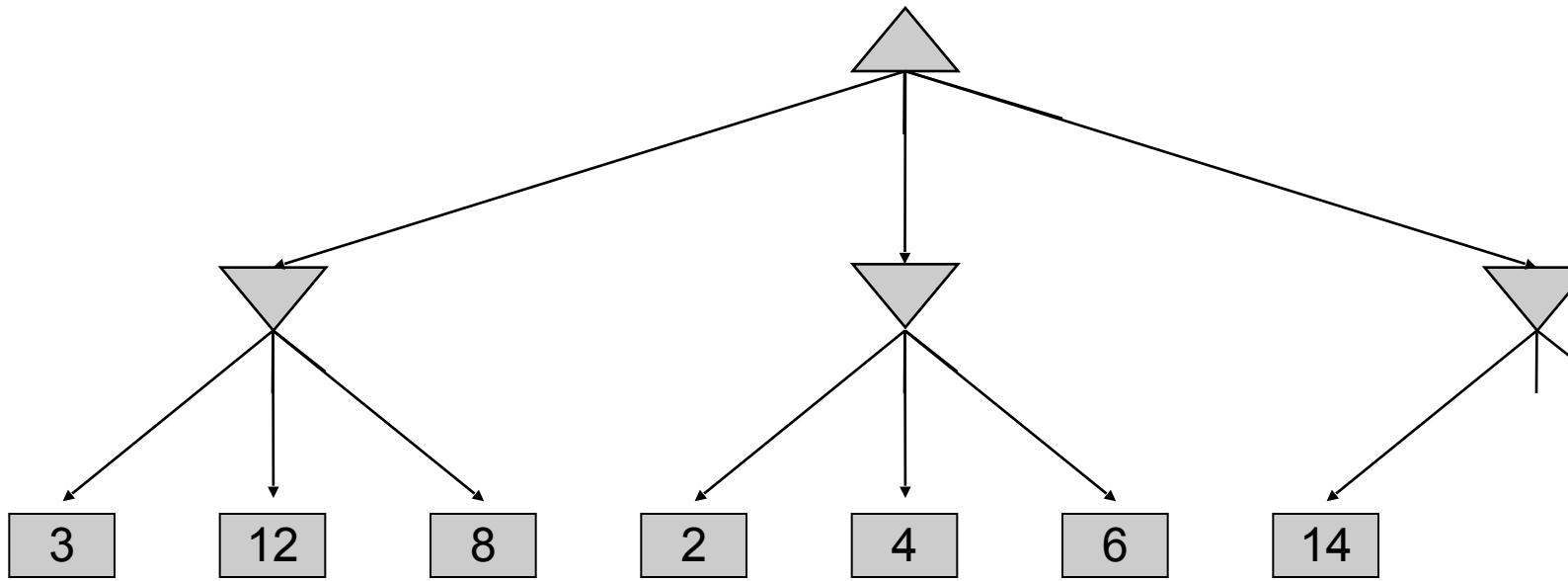
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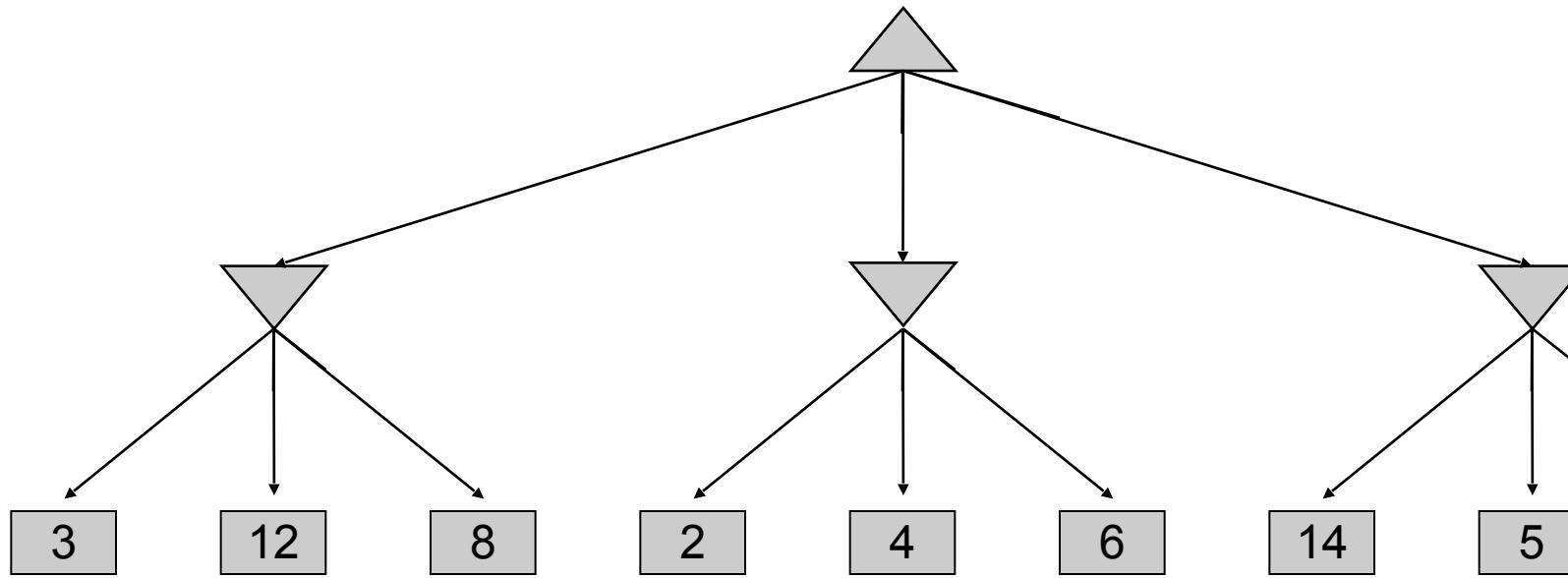
Minimax Example



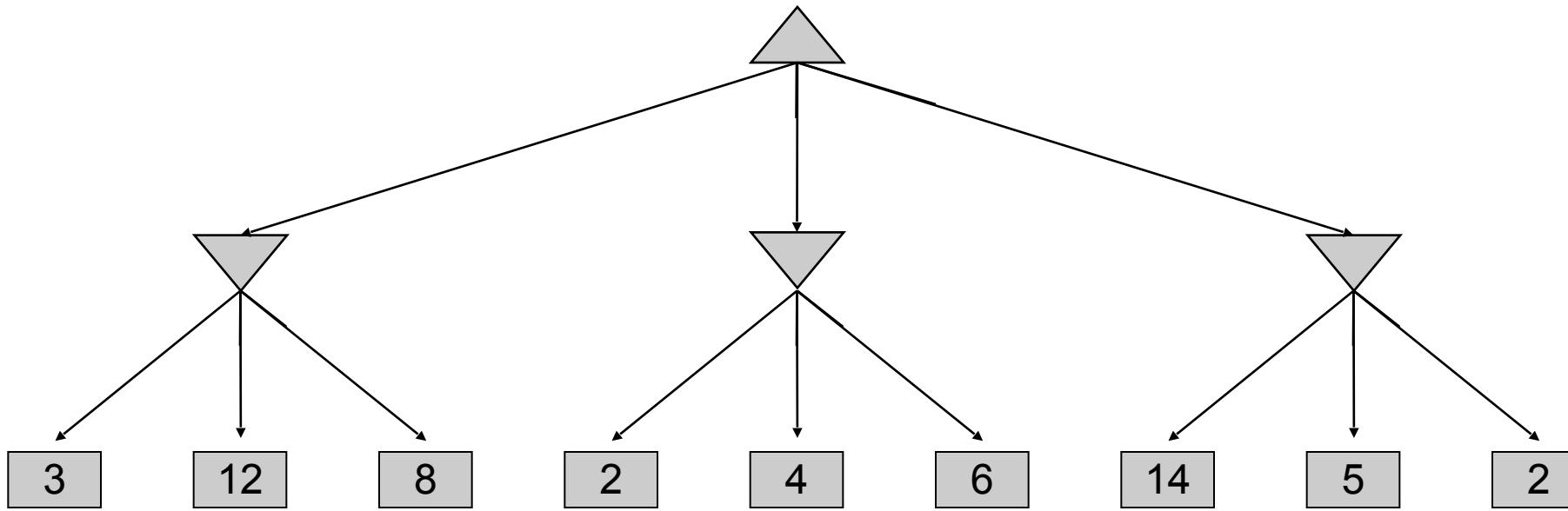
Minimax Example



Minimax Example

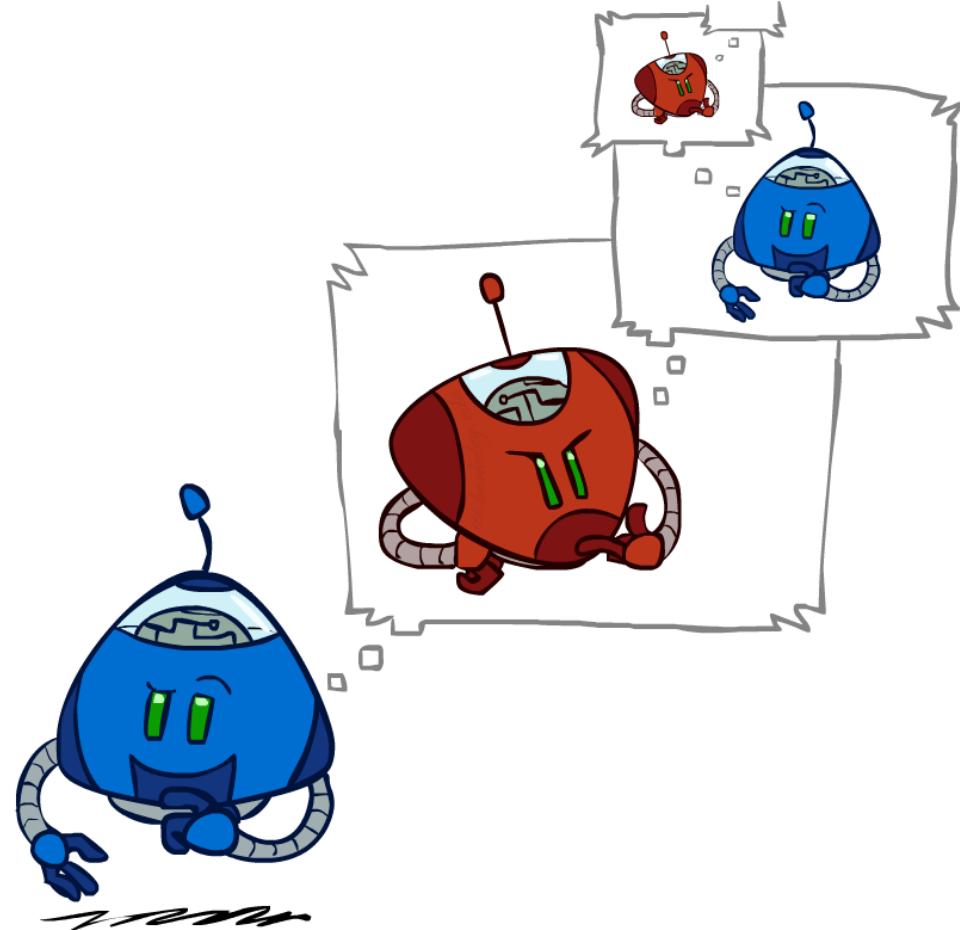


Minimax Example

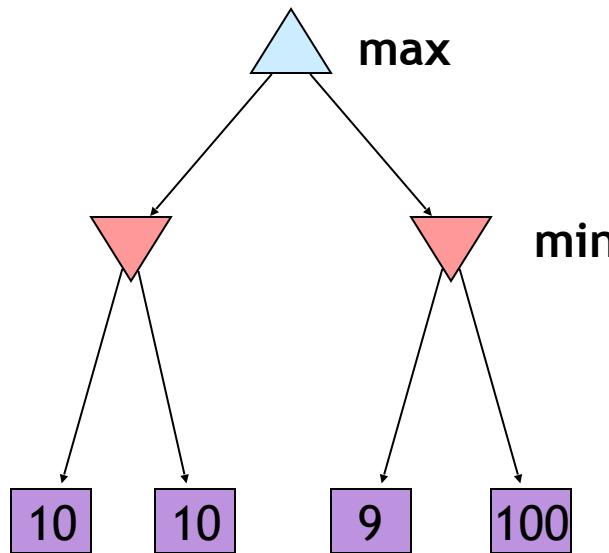


Minimax Efficiency

- How efficient is minimax?
 - Just like (exhaustive) DFS
 - Time: $O(b^m)$
 - Space: $O(bm)$
- Example: For chess, $b \approx 35$, $m \approx 100$
 - Exact solution is completely infeasible
 - But, do we need to explore the whole tree?

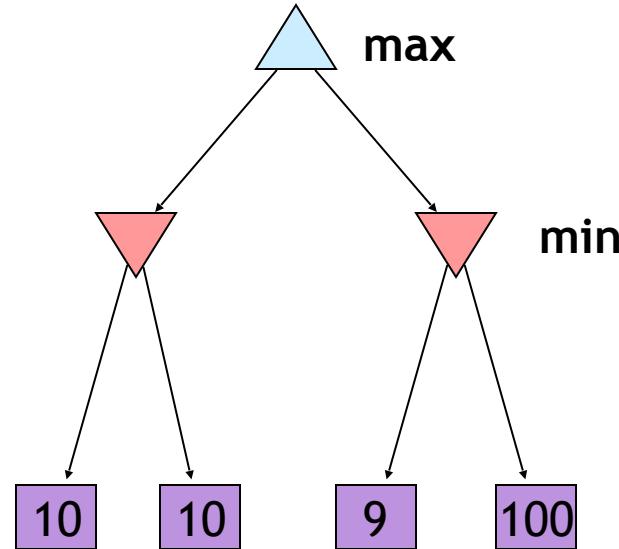
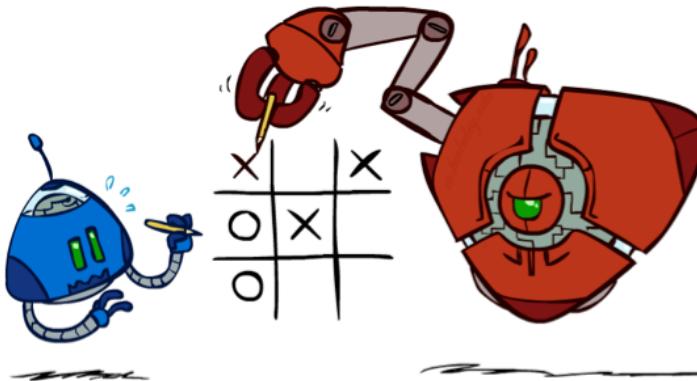


Minimax Properties



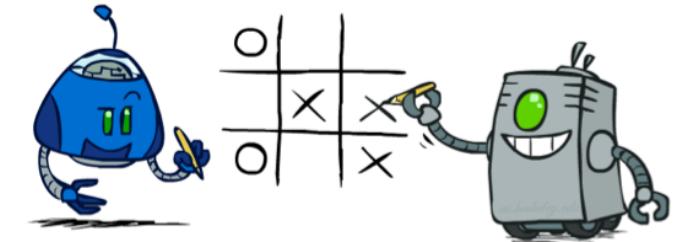
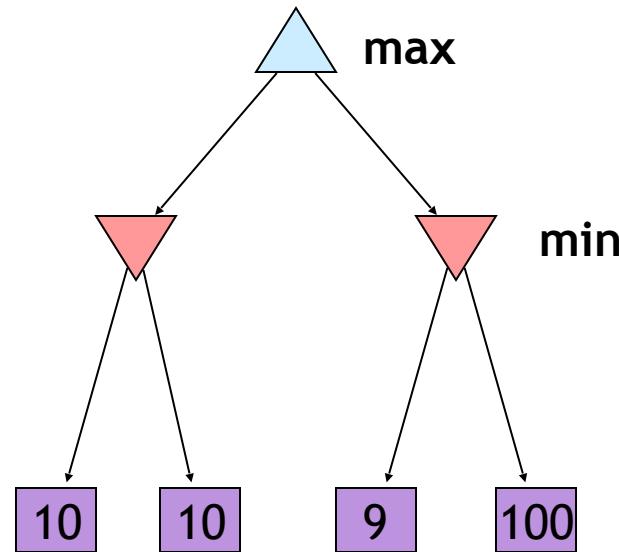
Optimal against a perfect player. Otherwise?

Minimax Properties



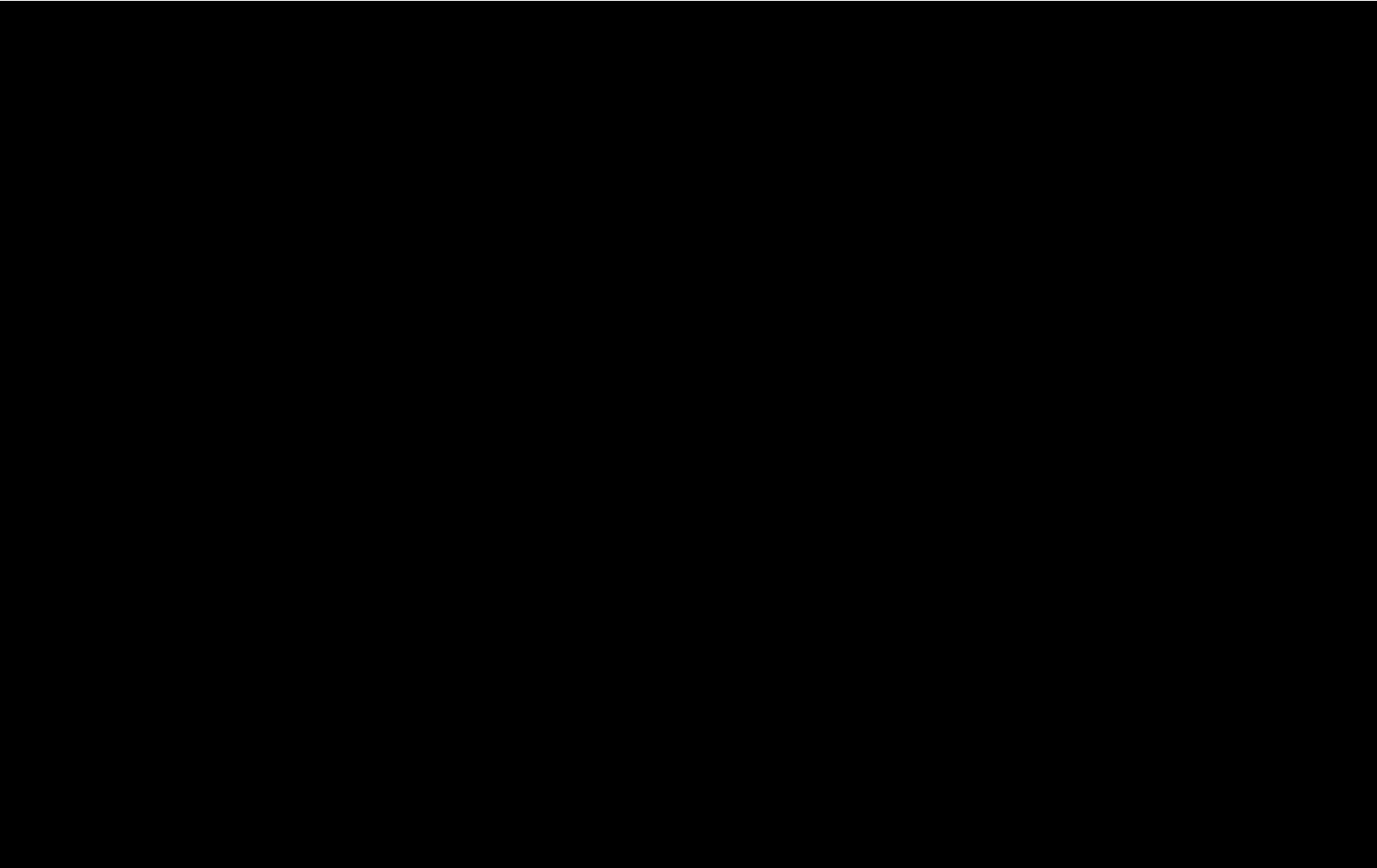
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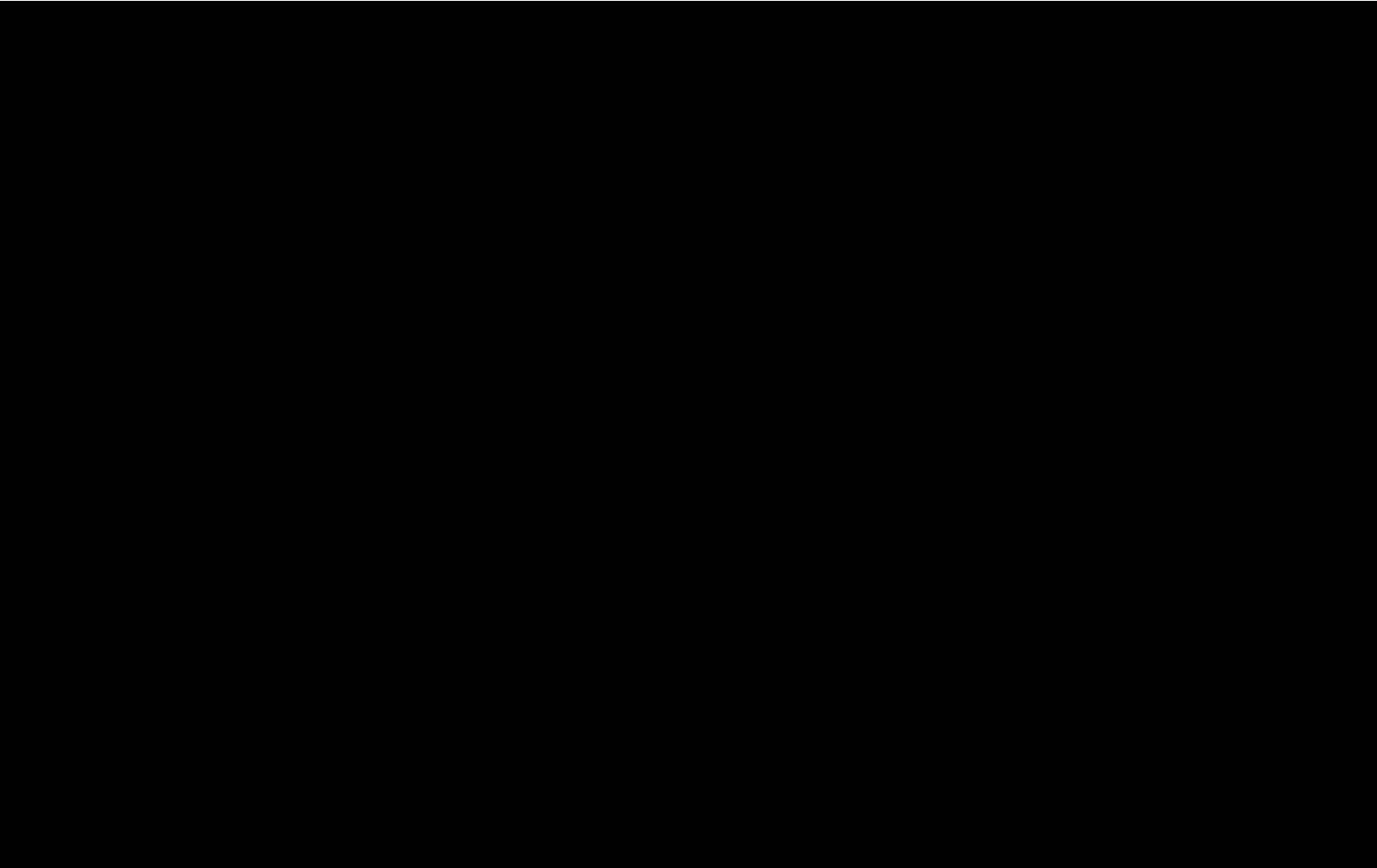


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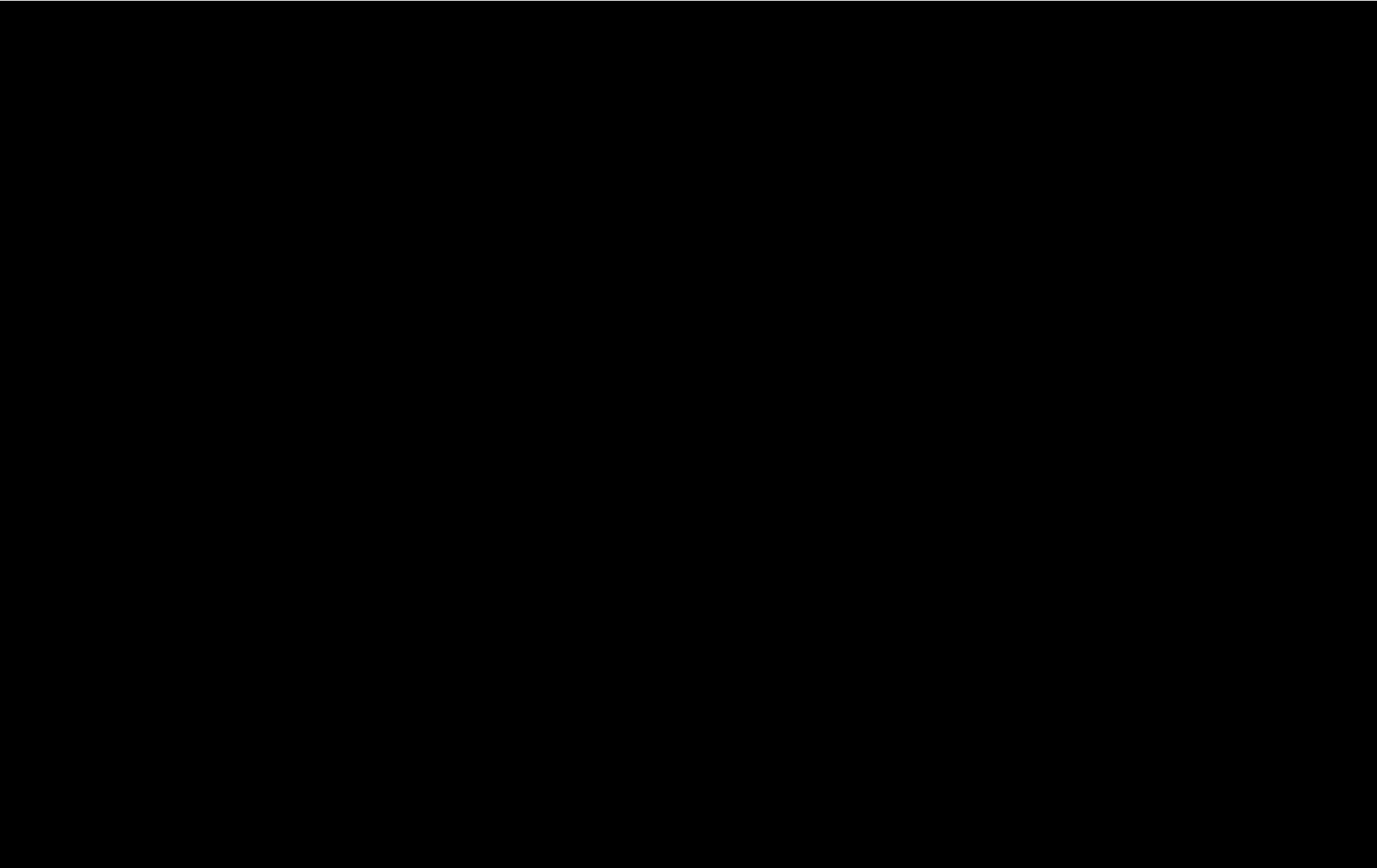
Video of Demo Min vs. Exp (Min)



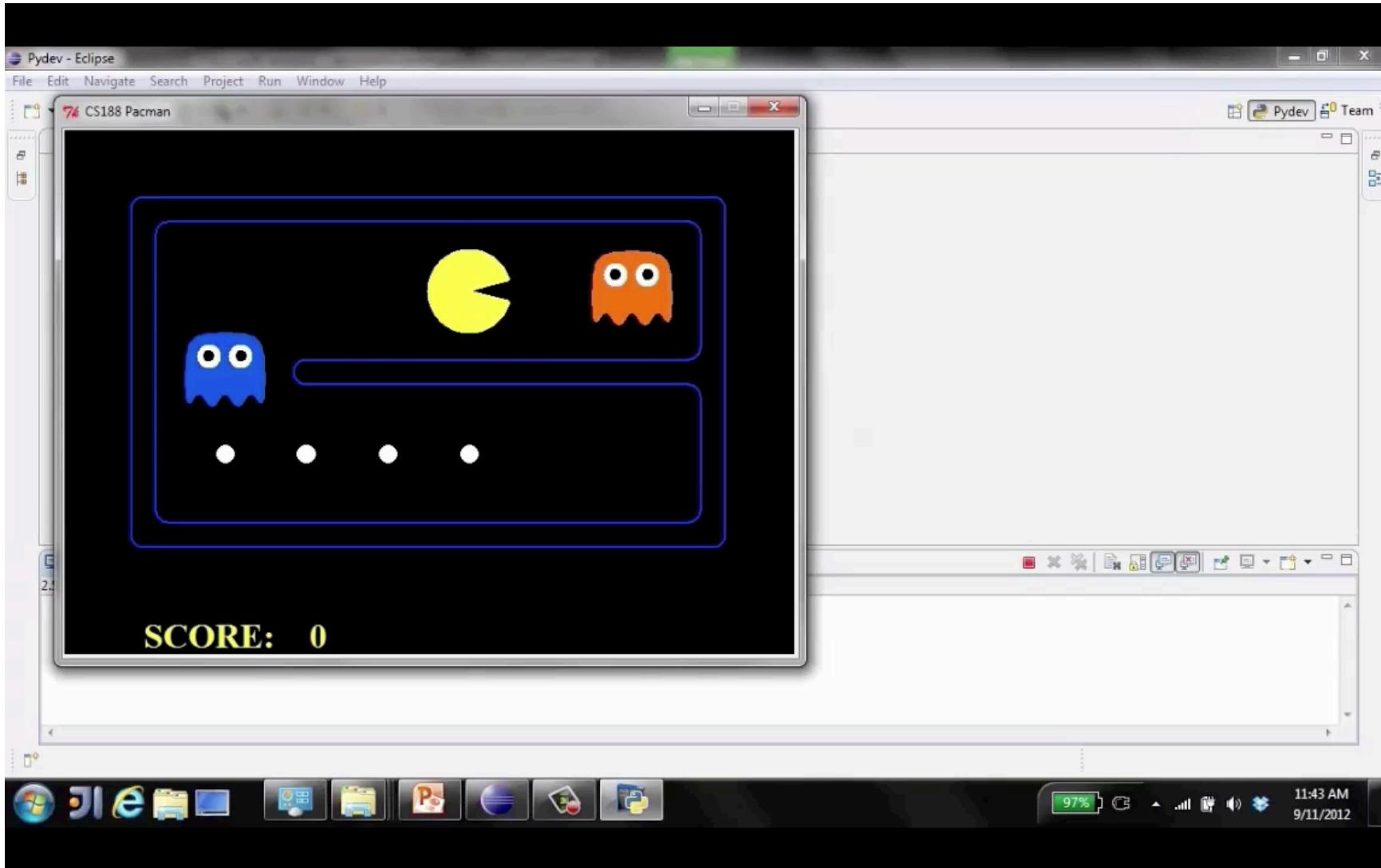
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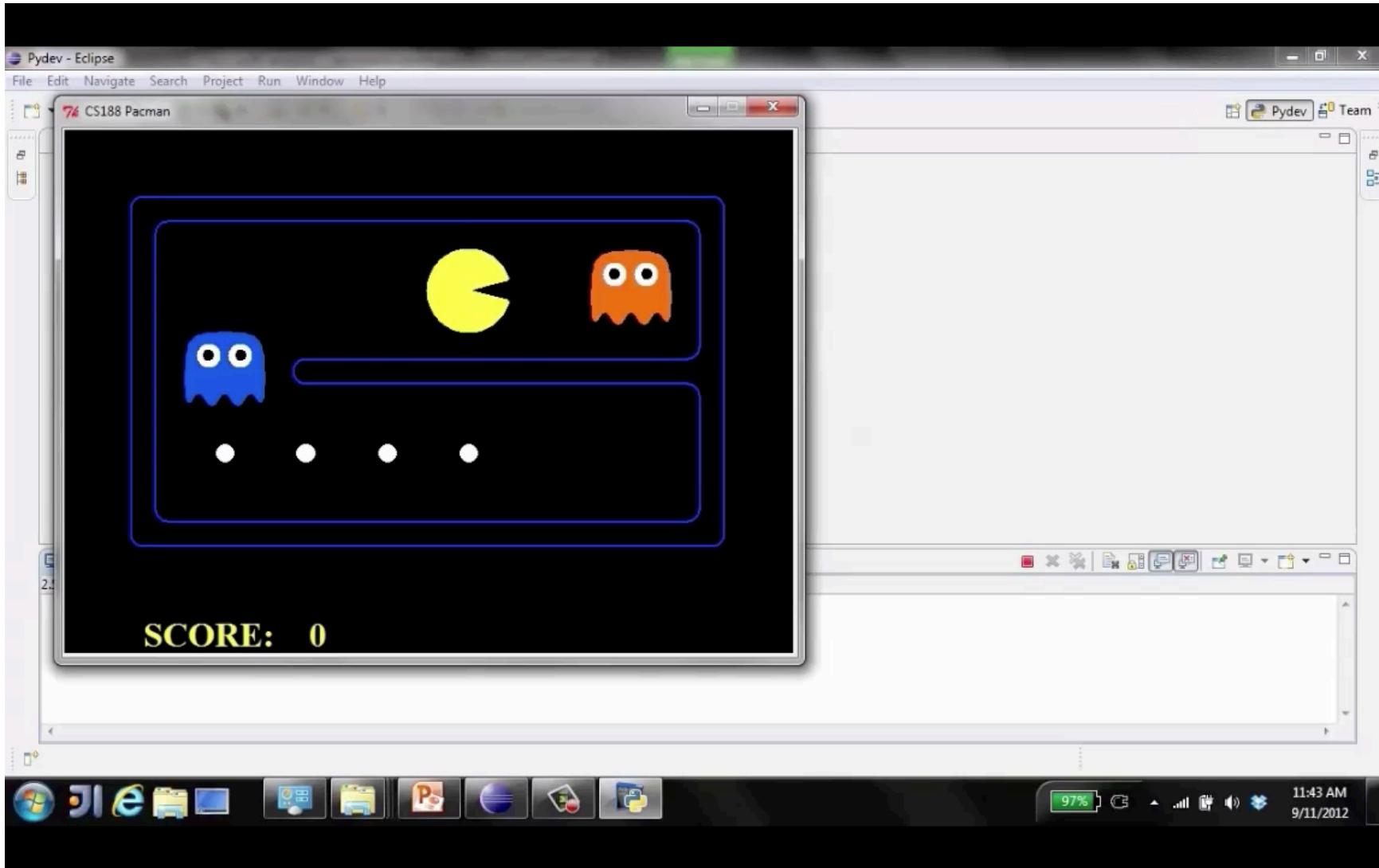
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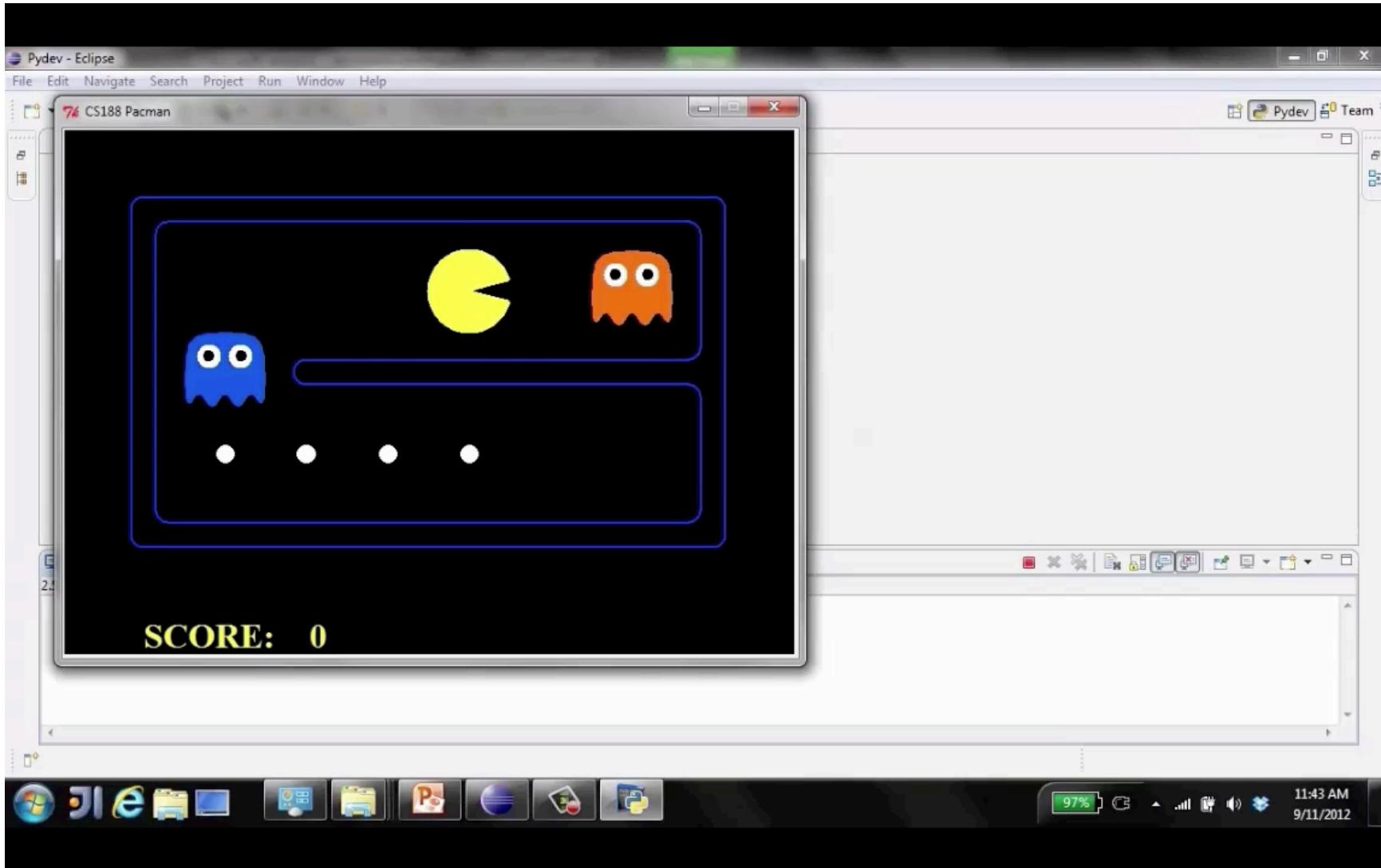
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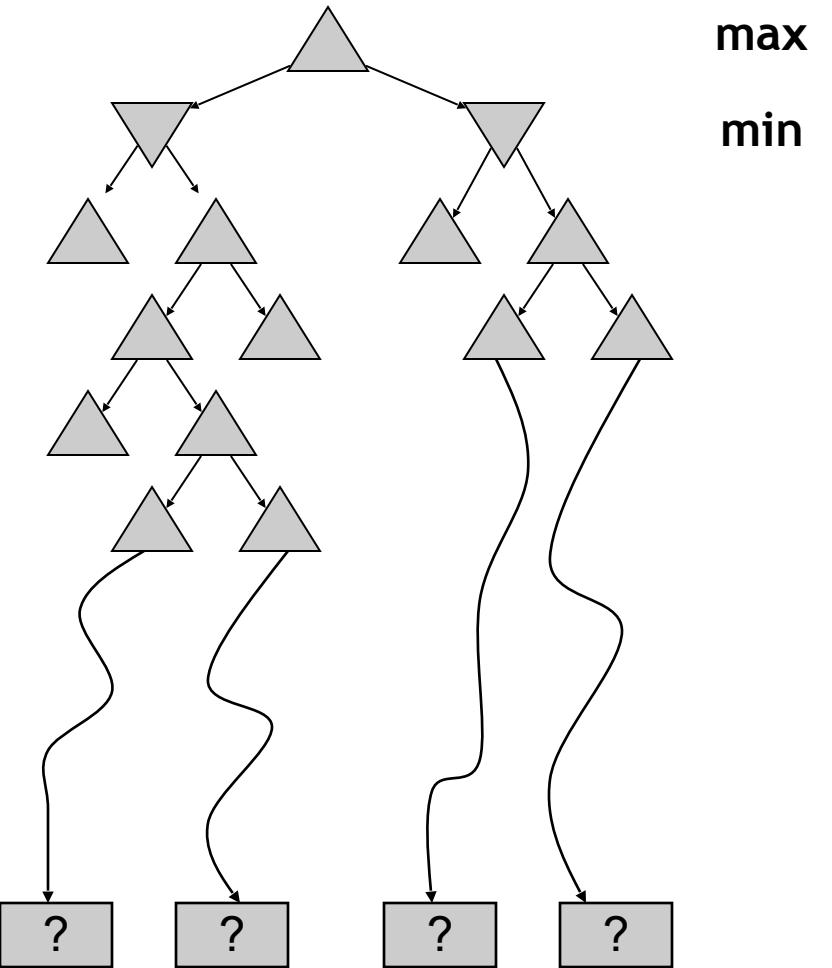


Resource Limits



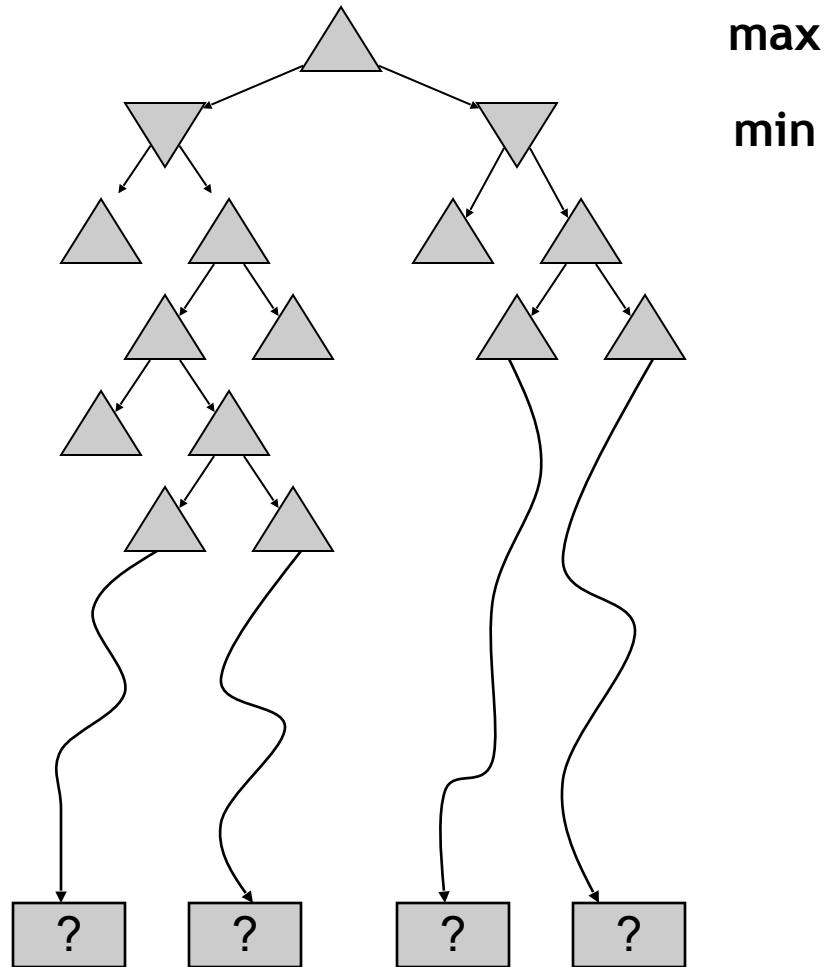
Resource Limits

- Problem: In realistic games, cannot search to leaves!



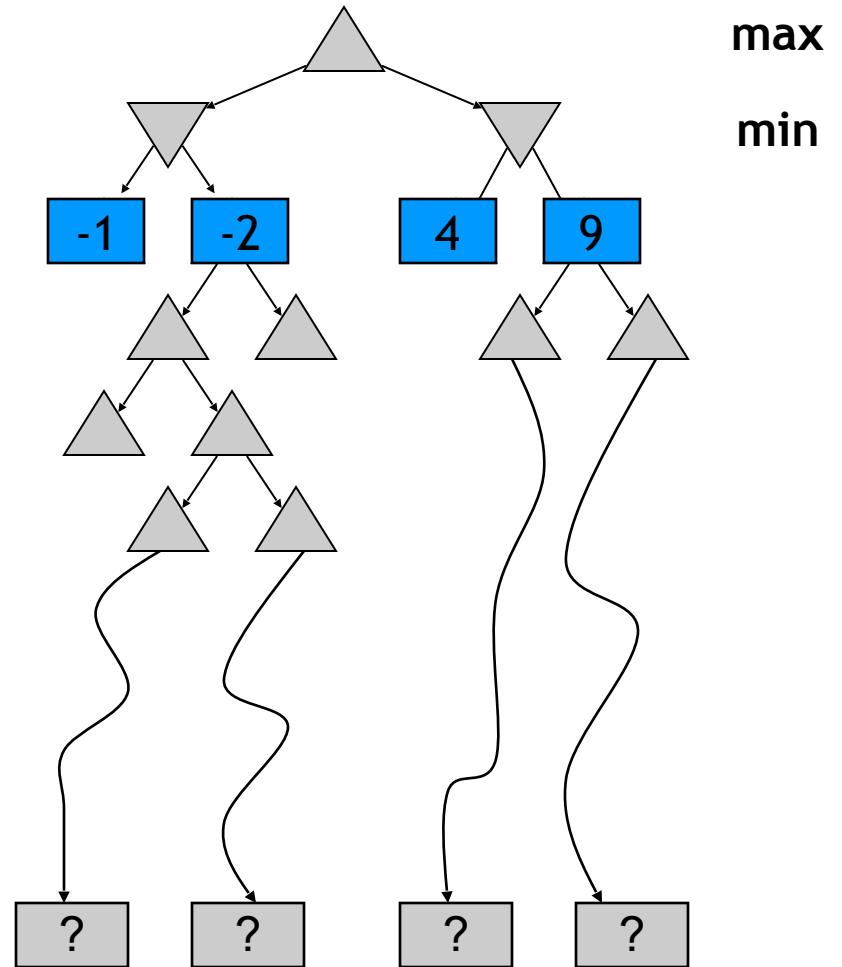
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- Problem: In realistic games, cannot search to leaves!
- Solution: Depth-limited search
 - Instead, search only to a limited depth in the tree
 - Replace terminal utilities with an evaluation function for non-terminal positions



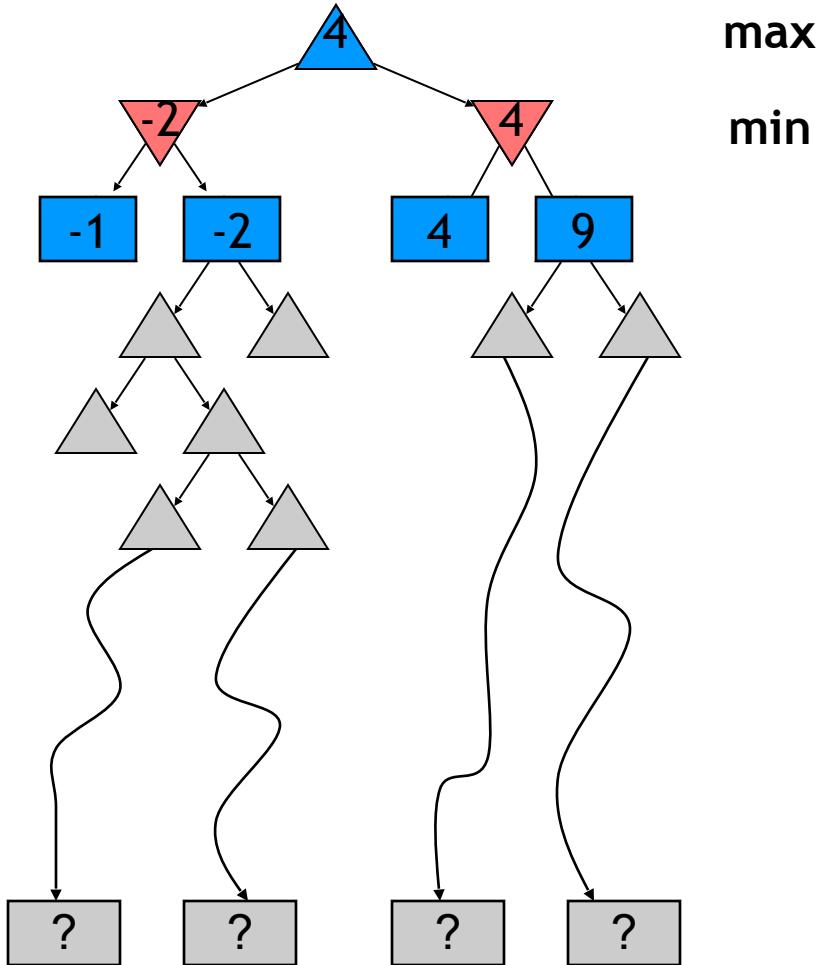
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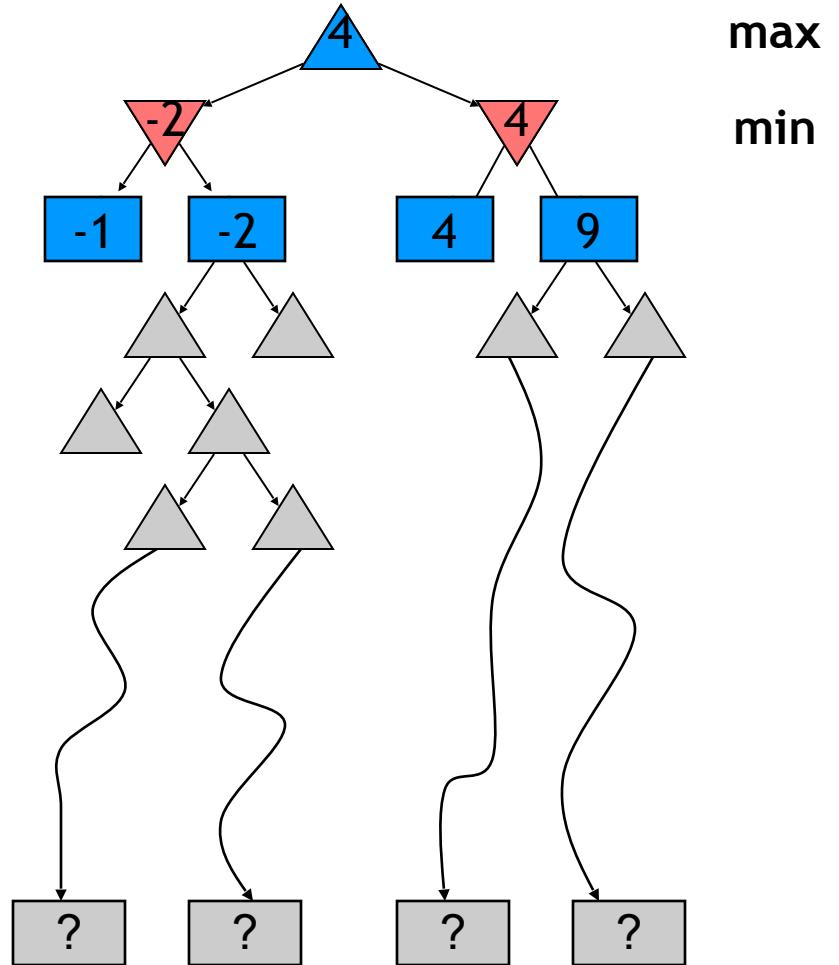
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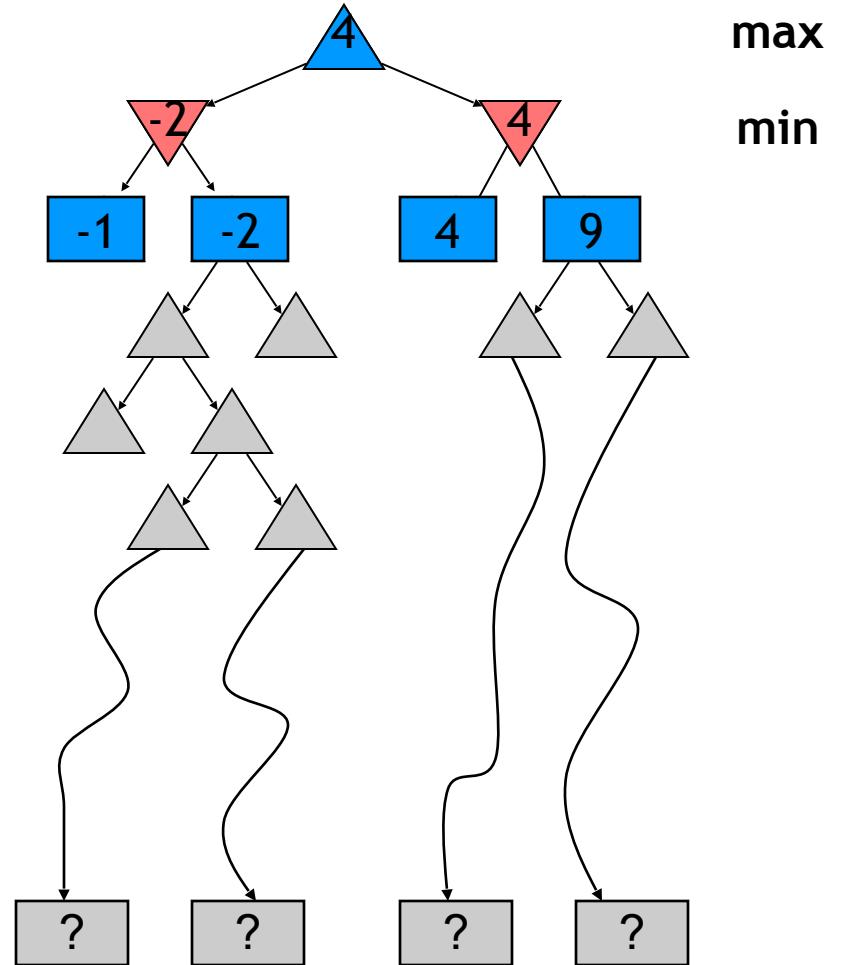
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- Example:
 - Suppose we have 100 seconds, can explore 10K nodes / sec
 - So can check 1M nodes per move
 - α - β reaches about depth 8 - decent chess program



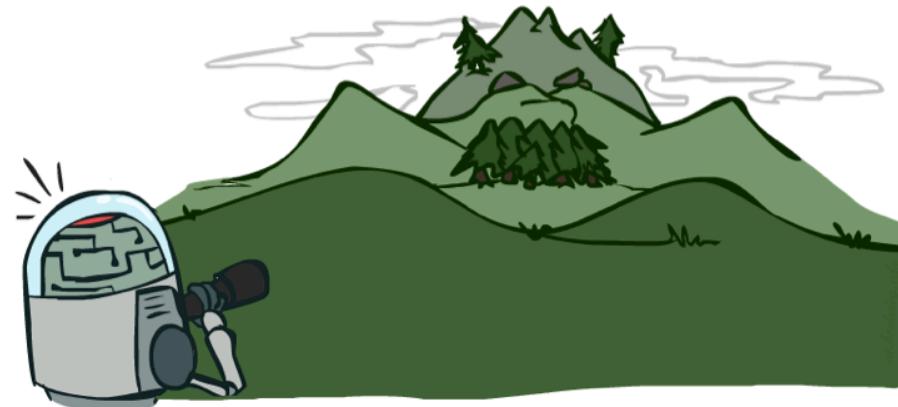
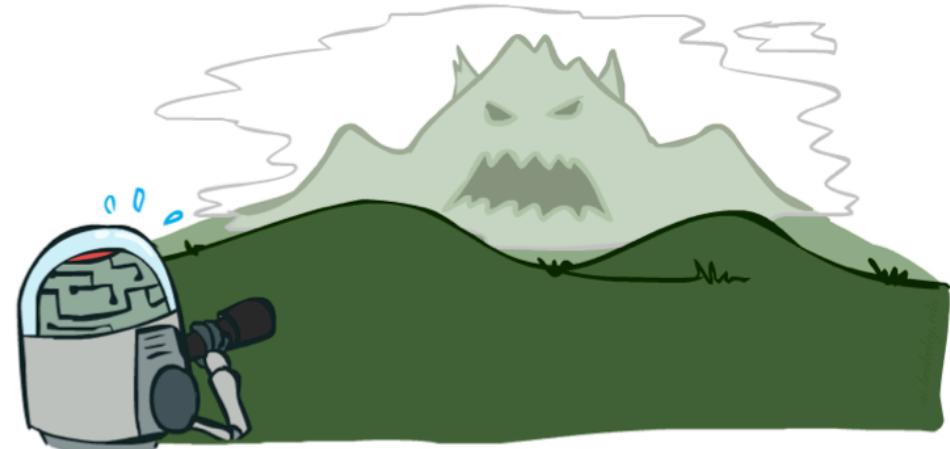
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- Example:
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 - $\alpha\beta$ reaches about depth 8 - decent chess program
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Use **iterative deepening** for an anytime algorithm



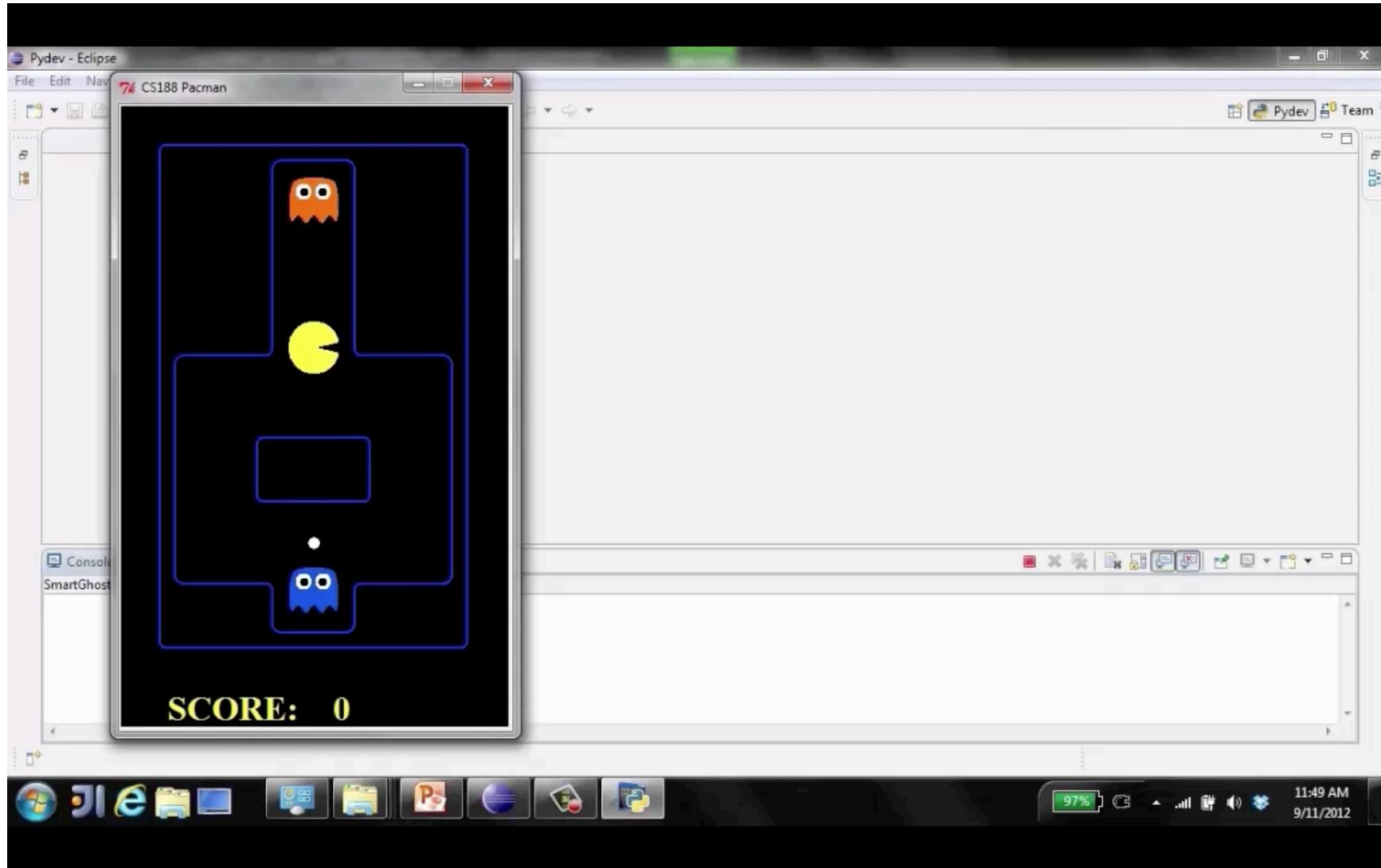
Depth Matters

- Evaluation functions are always imperfect
- The deeper in the tree the evaluation function is buried, the less the quality of the evaluation function matters
- An important example of the tradeoff between complexity of features and complexity of computation

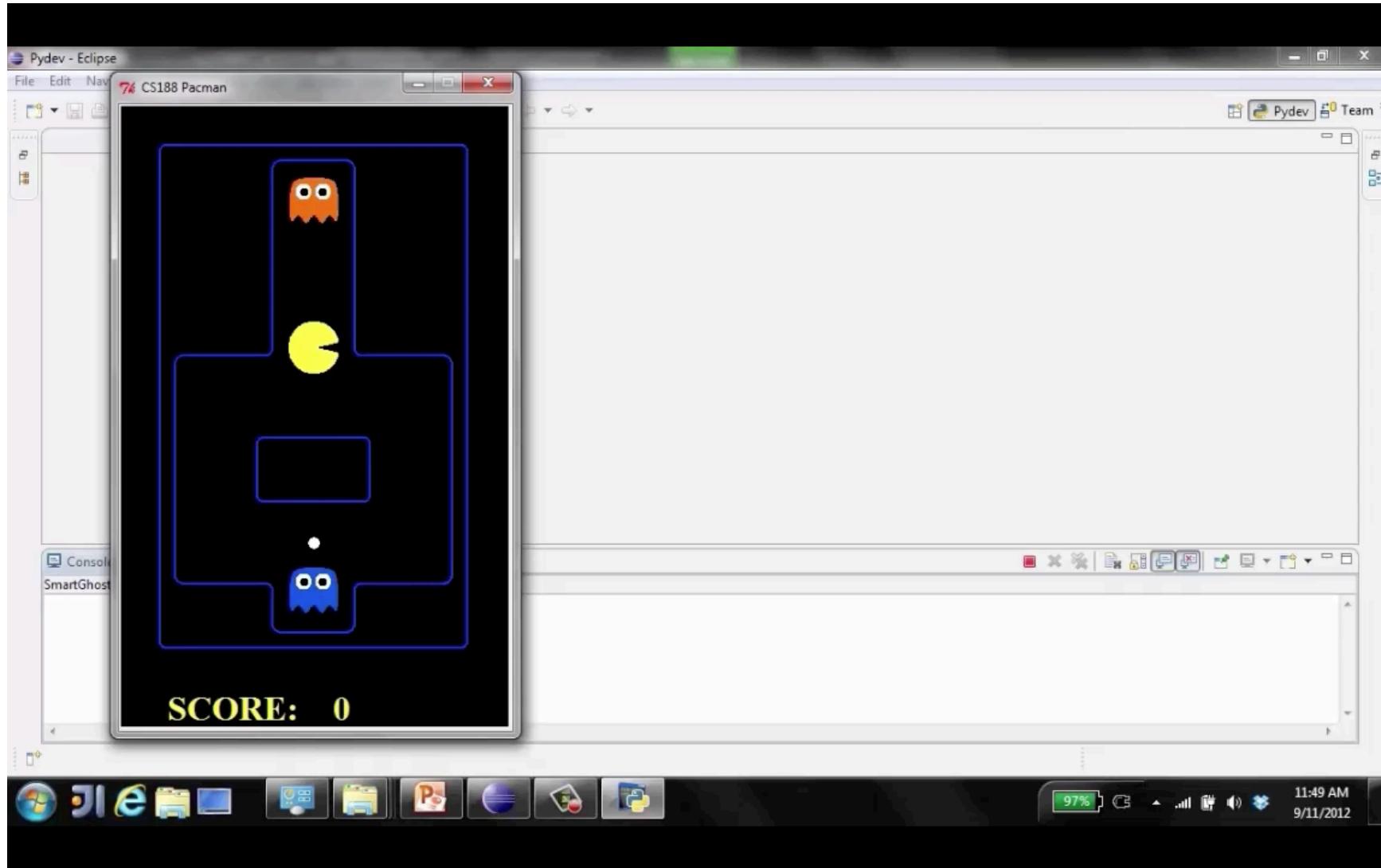


[Demo: depth limited (L6D4, L6D5)]

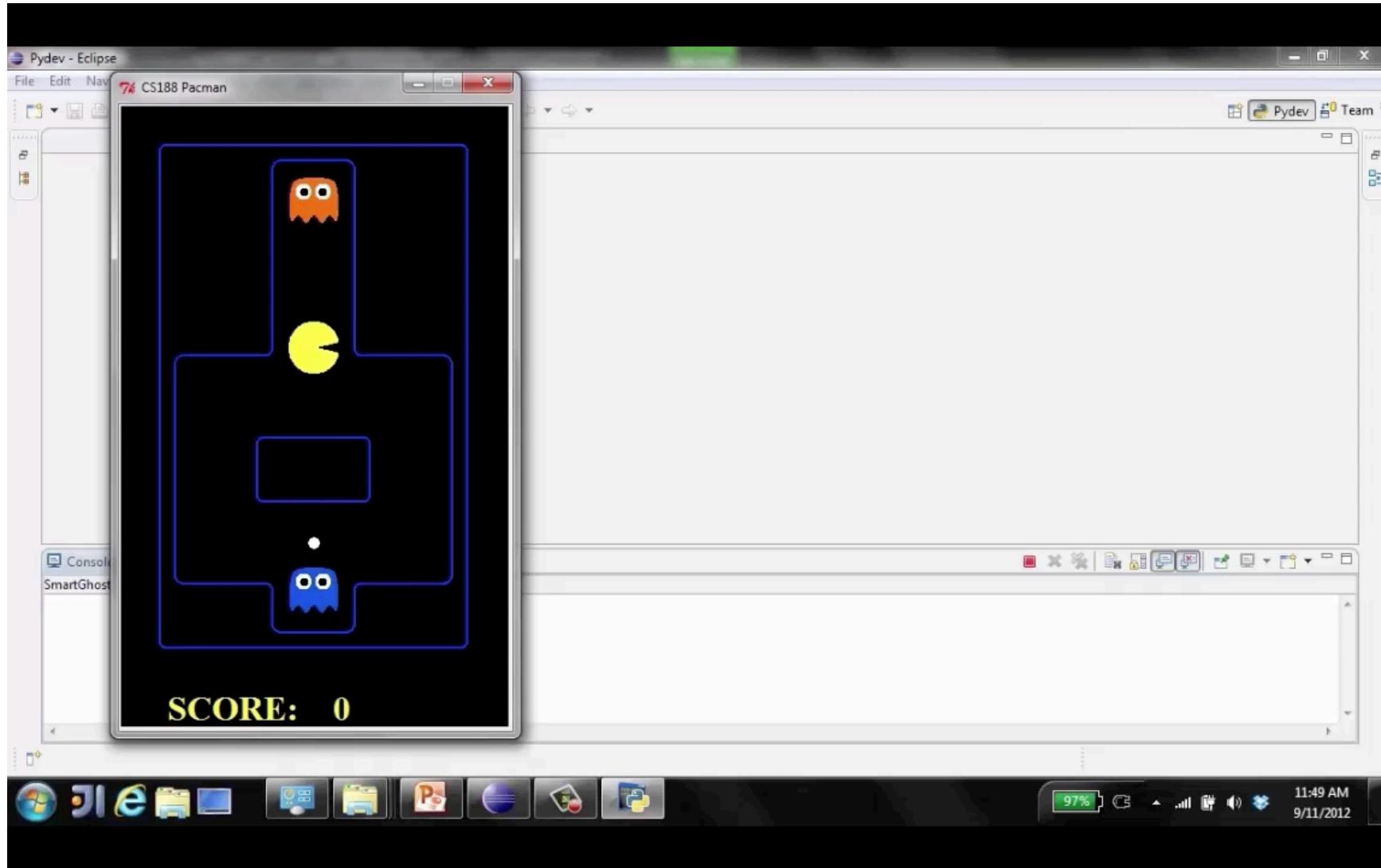
Video of Demo Limited Depth (2)



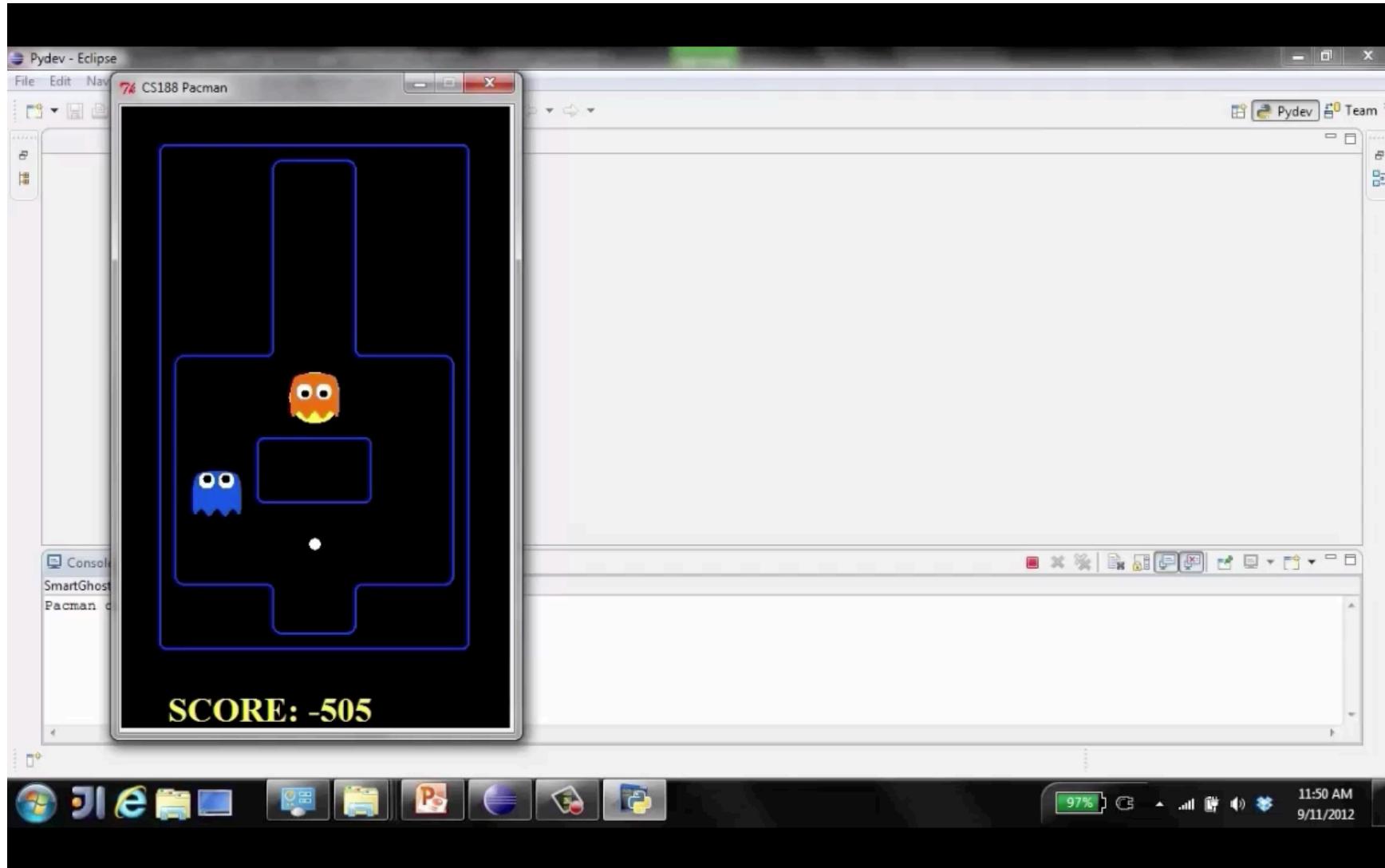
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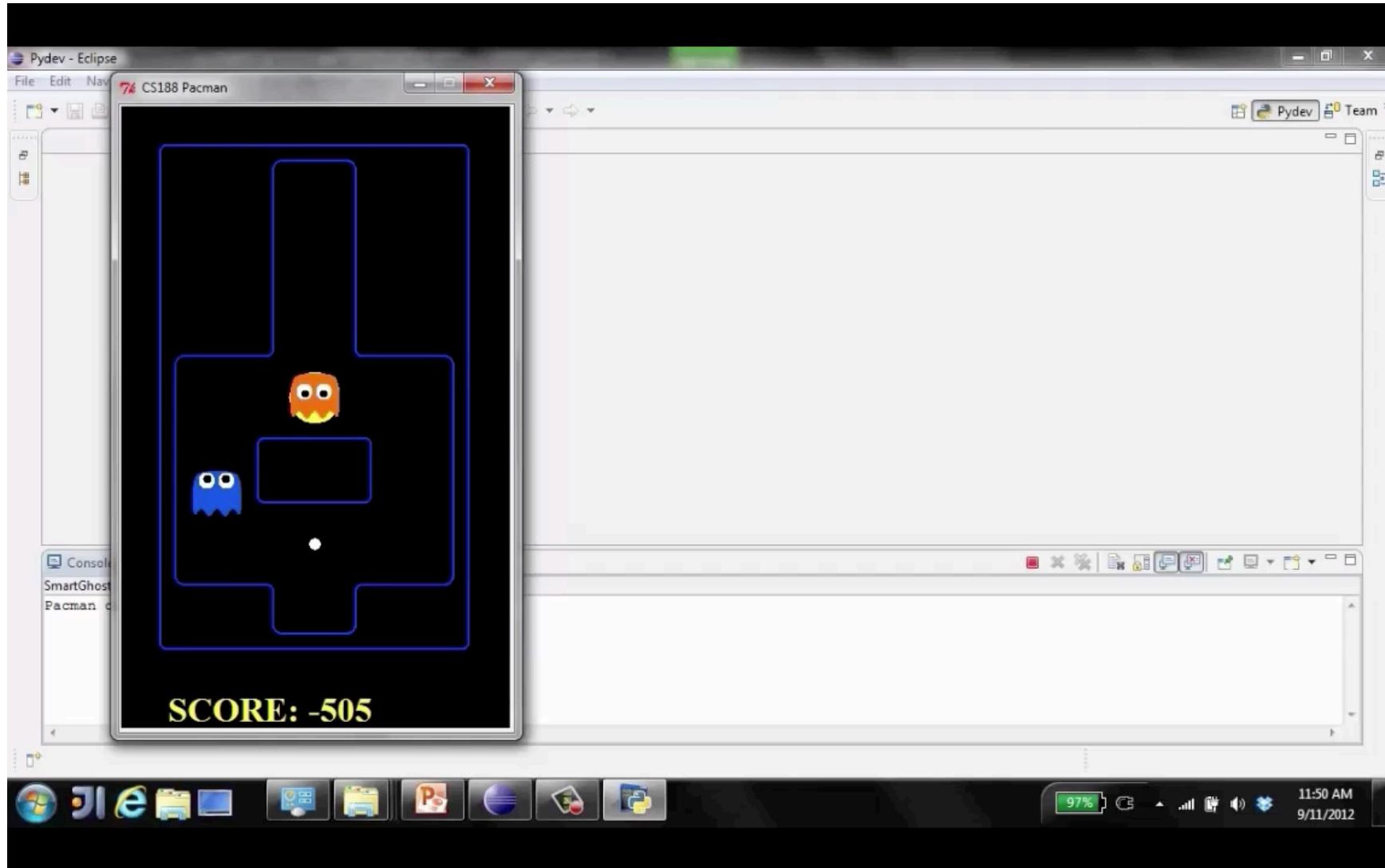
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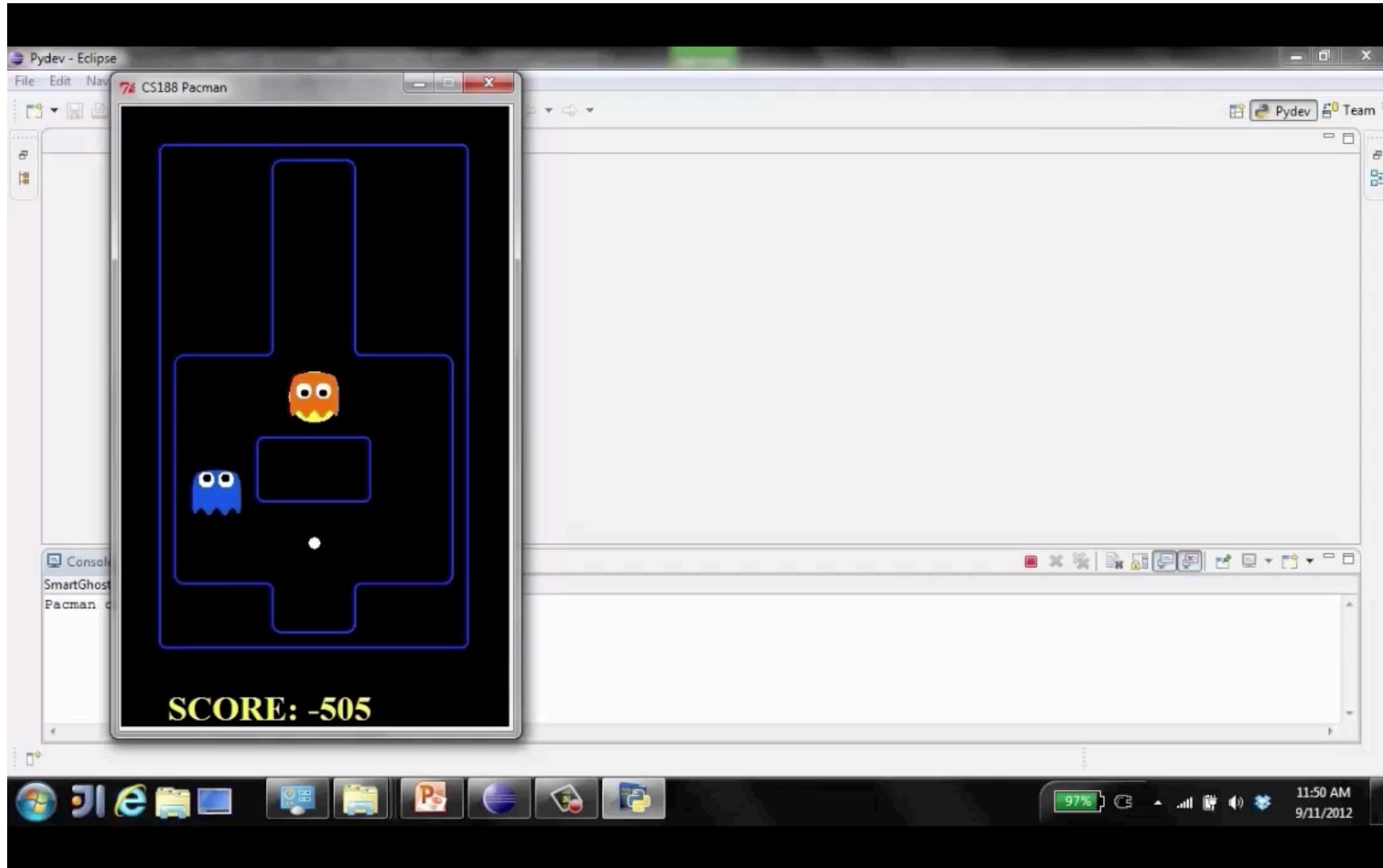
Video of Demo Limited Depth (10)



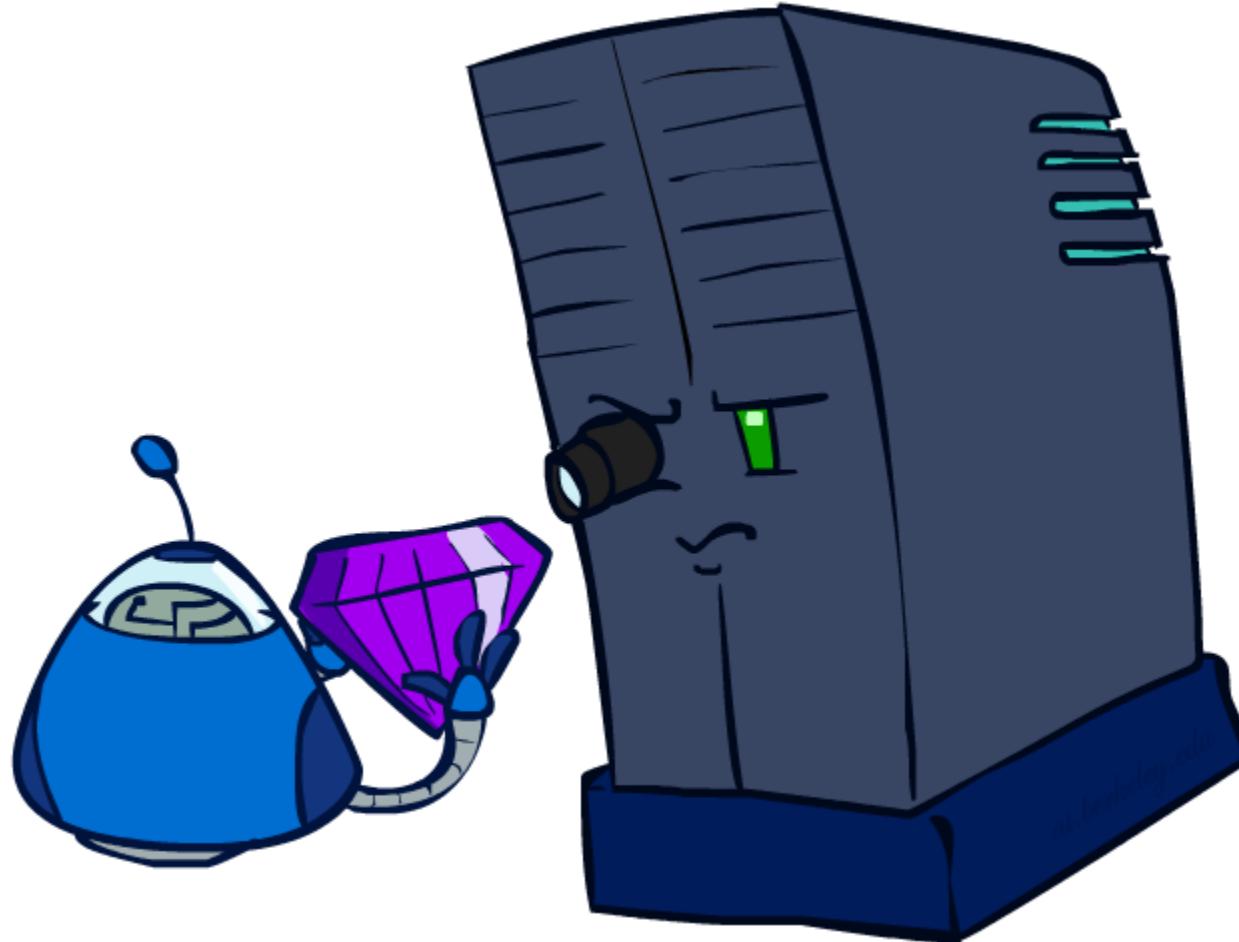
Video of Demo Limited Depth (10)



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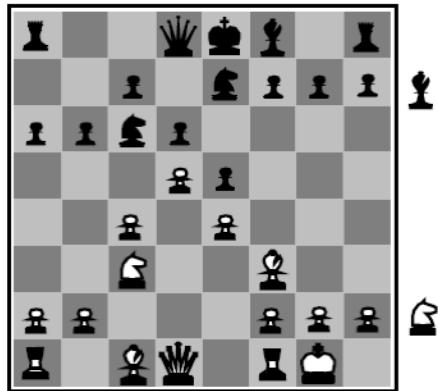
Evaluation Functions



ms

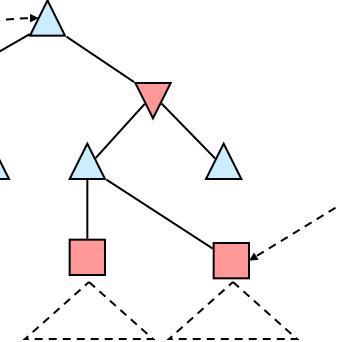
Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



Black to move

White slightly better



White to move

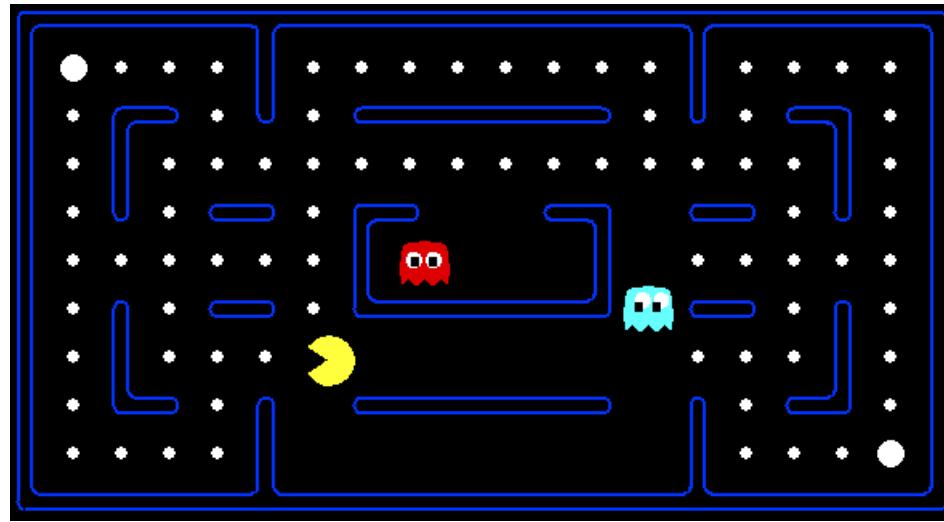
Black winning

- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

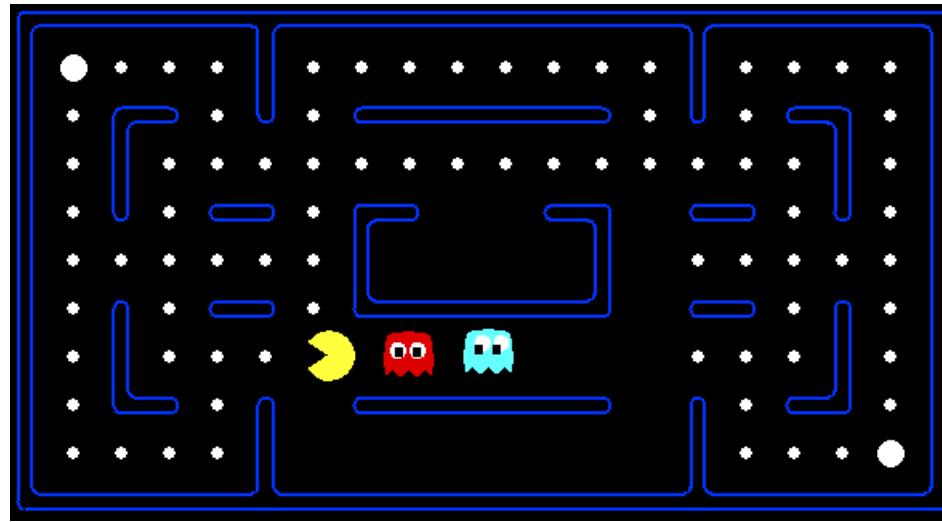
- e.g. $f_1(s) = (\text{num white queens} - \text{num black queens})$, etc.

Evaluation for Pacman



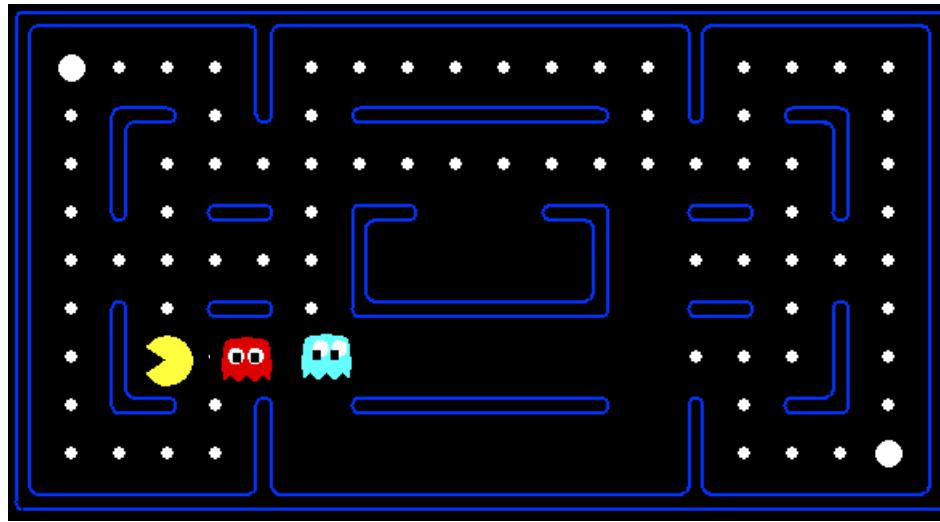
[Demo: thrashing d=2, thrashing d=2 (fixed evaluation function), smart ghosts coordinate

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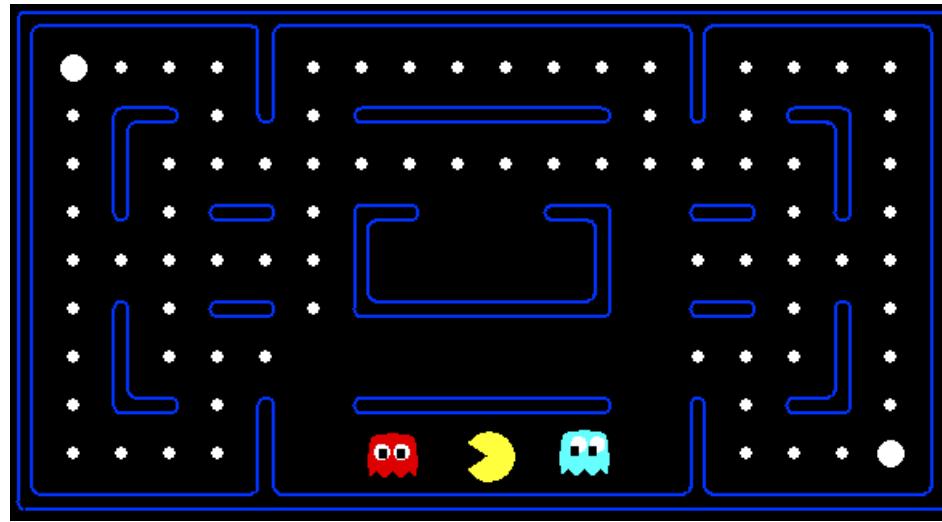
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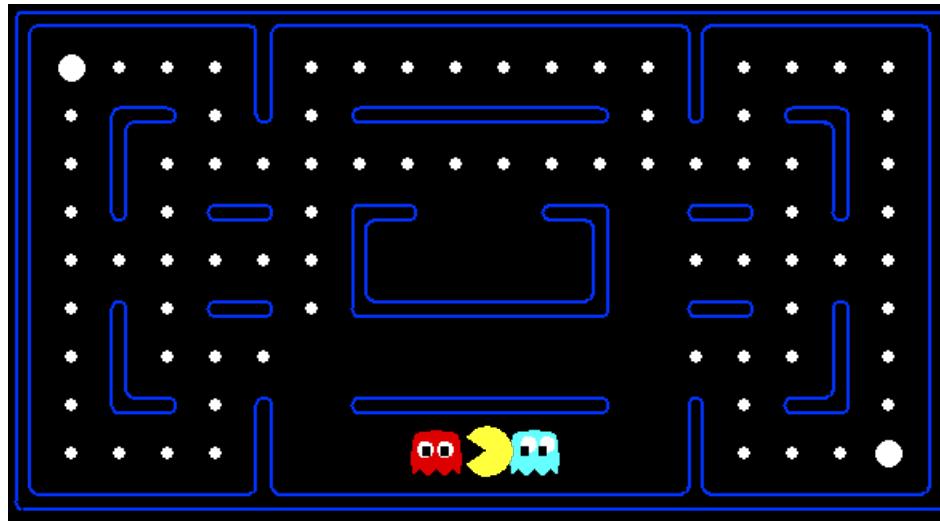
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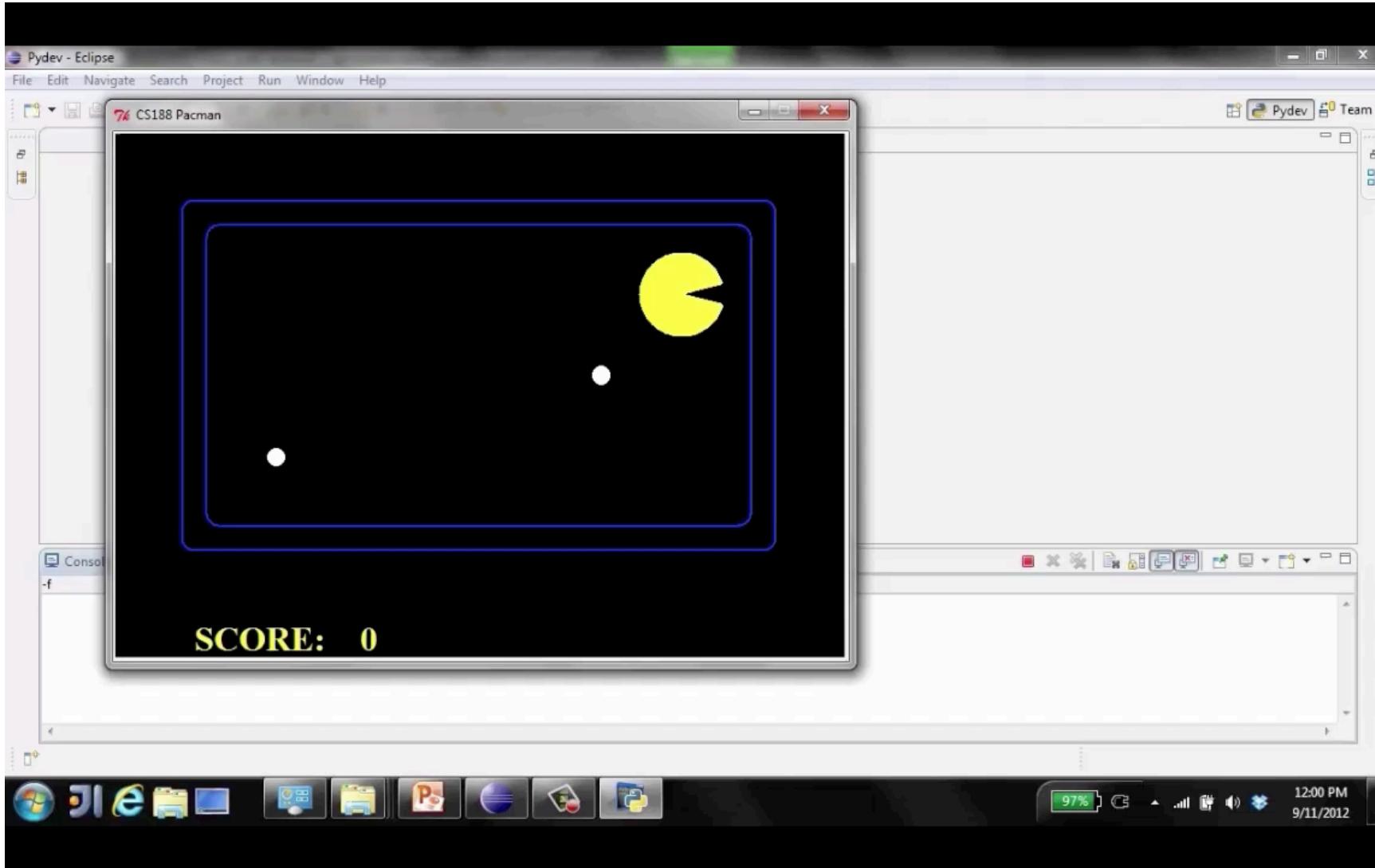
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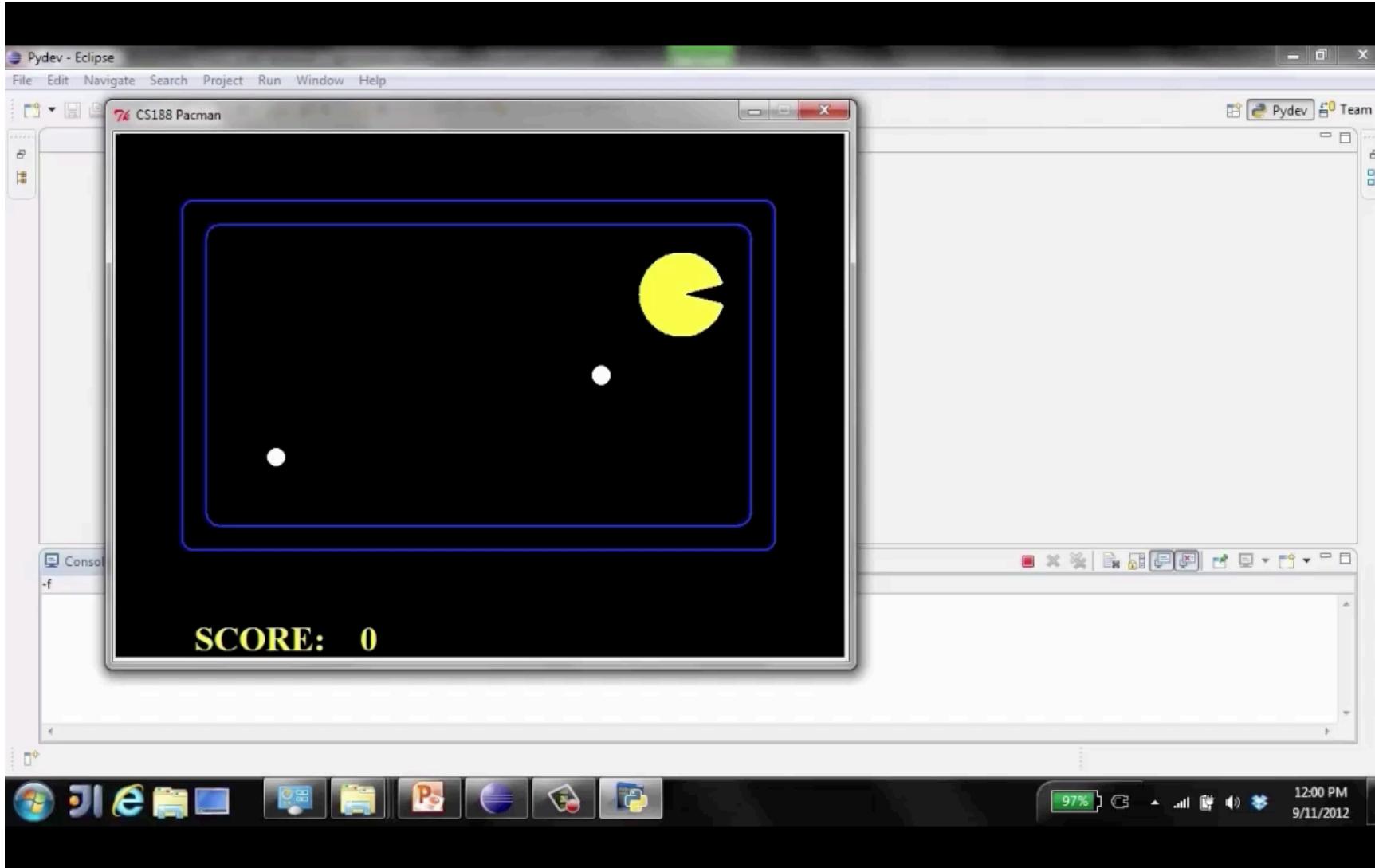


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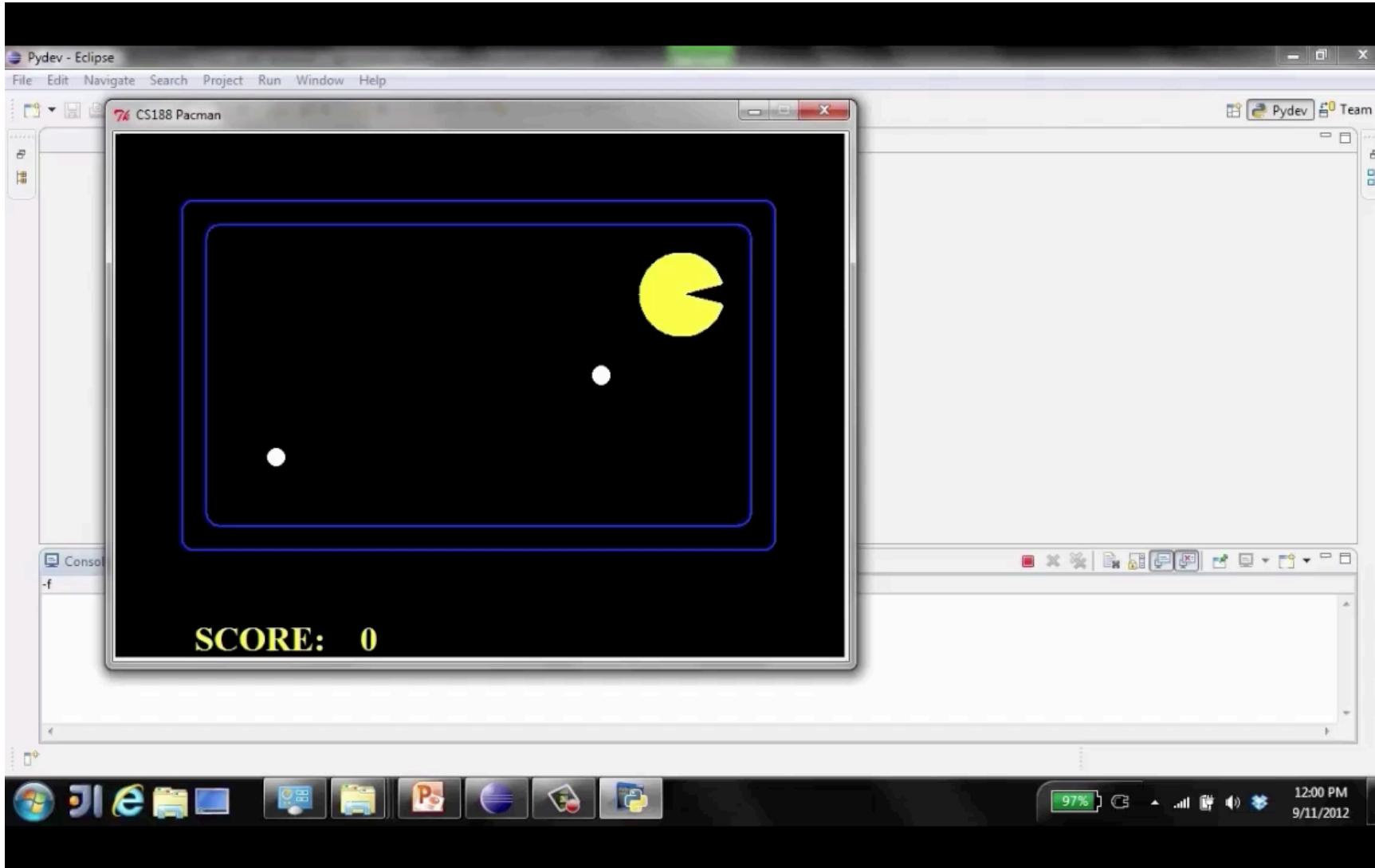
Video of Demo Thrashing (d=2)



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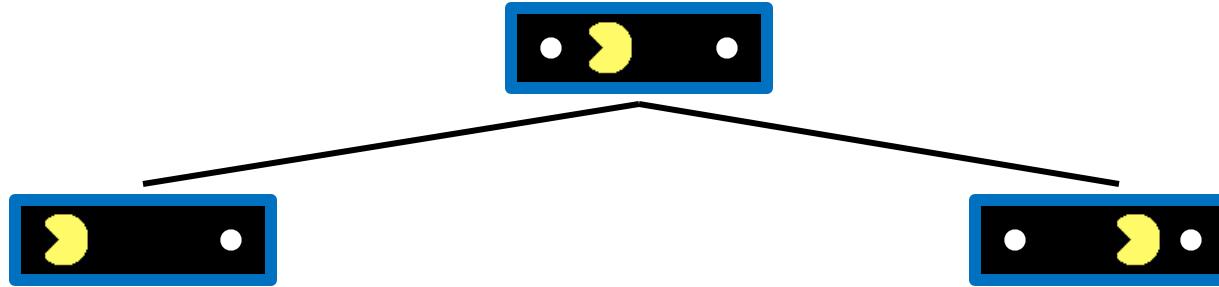


Why Pacman Starves



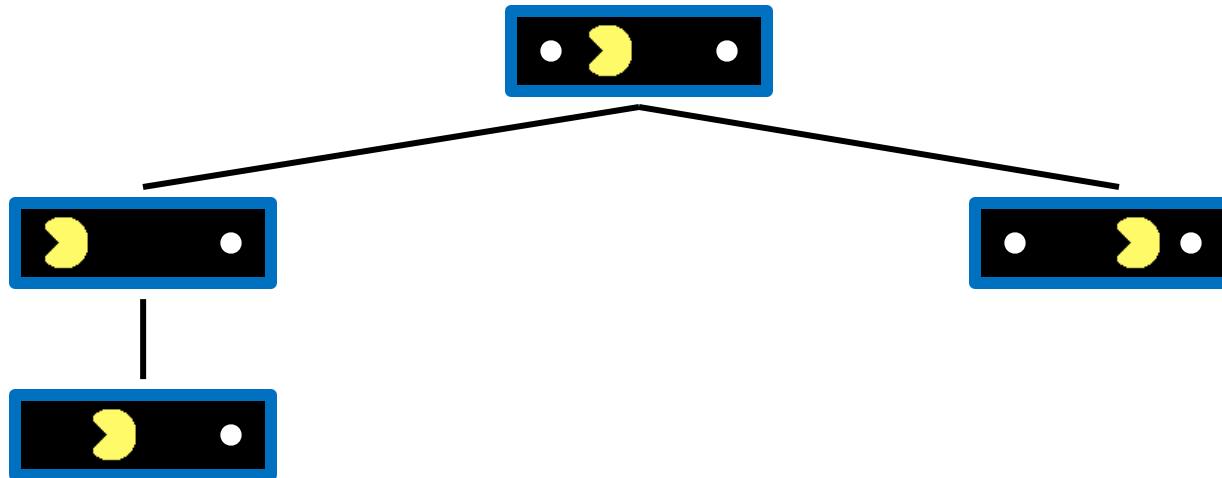
- A danger of replanning agents!
 - He knows his score will go up by eating the dot now (west, east)
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 - There are no point-scoring opportunities after eating the dot (within the horizon, two here)
 - Therefore, waiting seems just as good as eating: he may go east, then back west in the next round of replanning!

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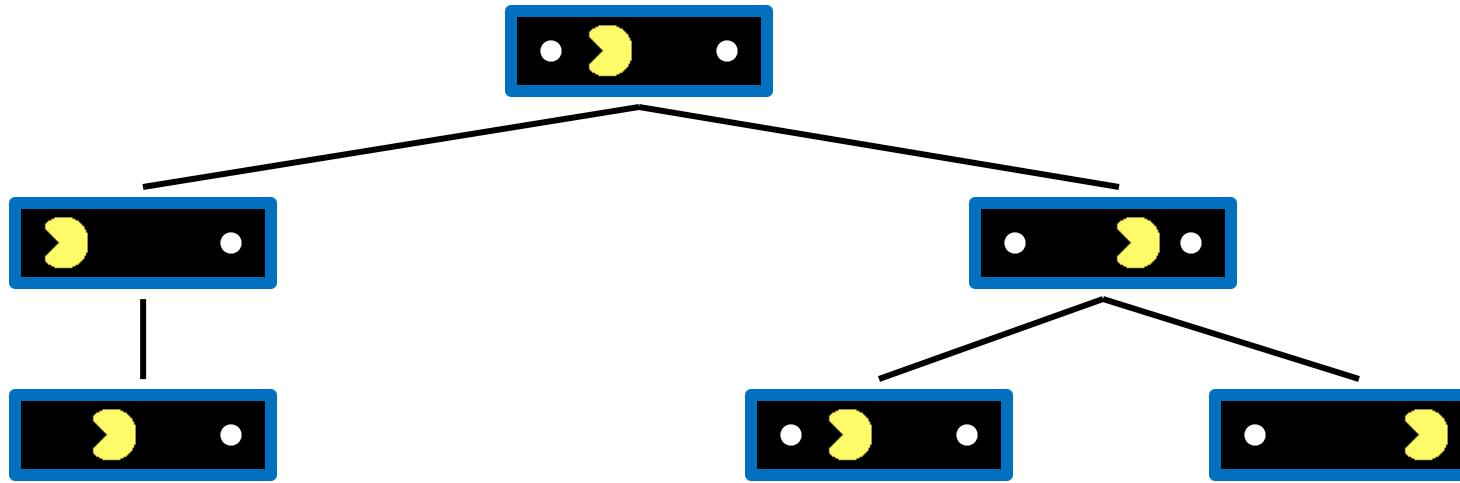
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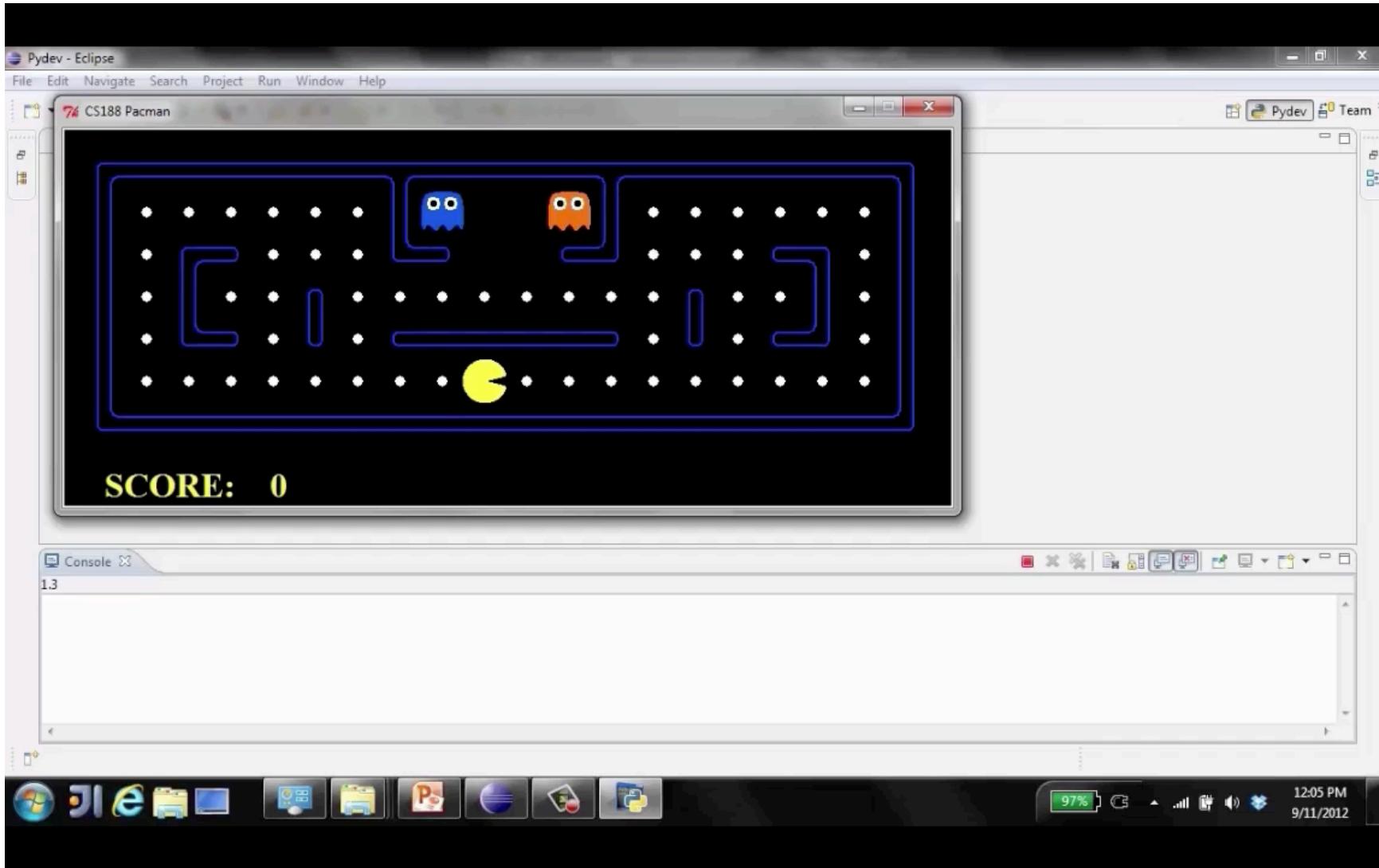
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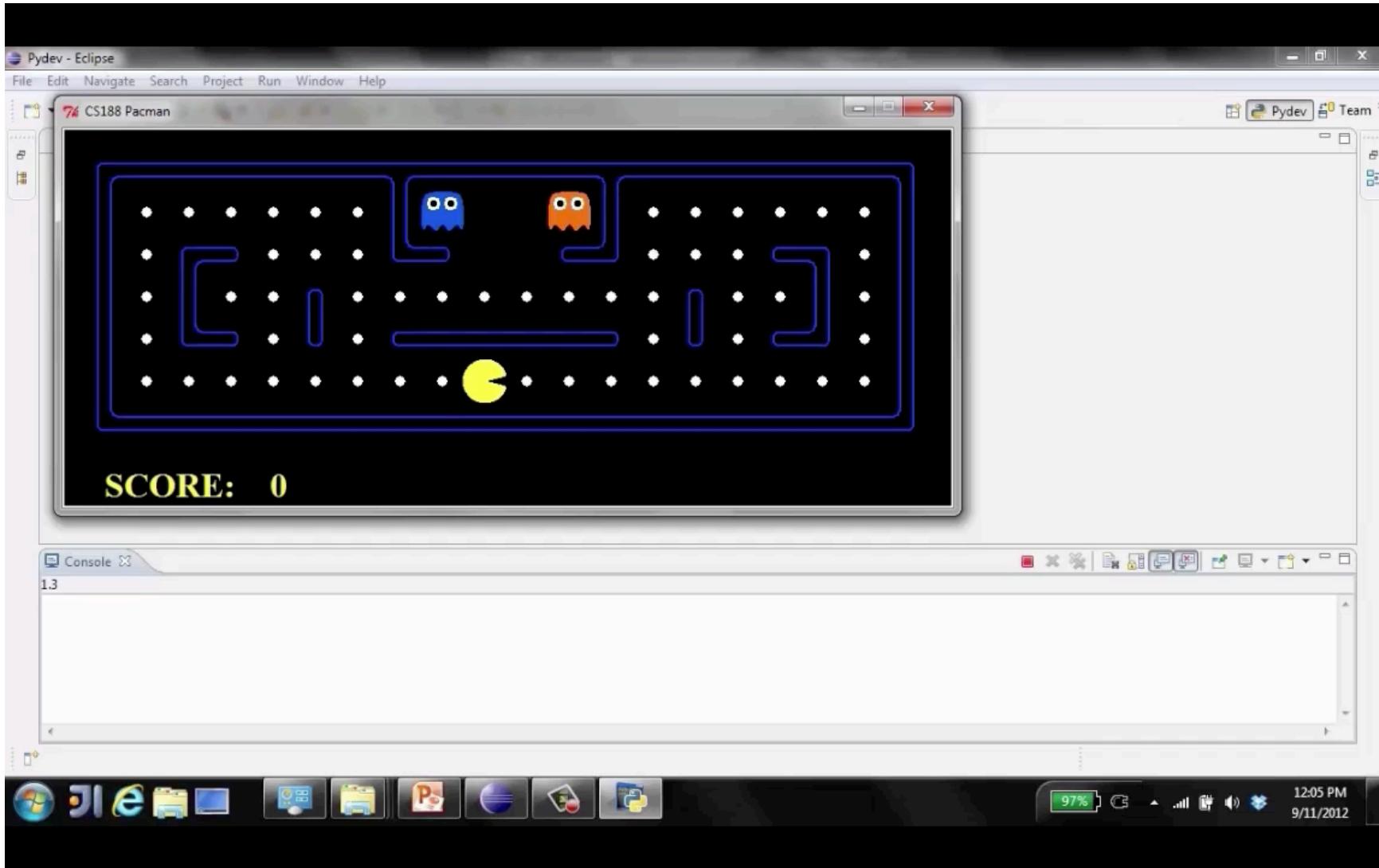
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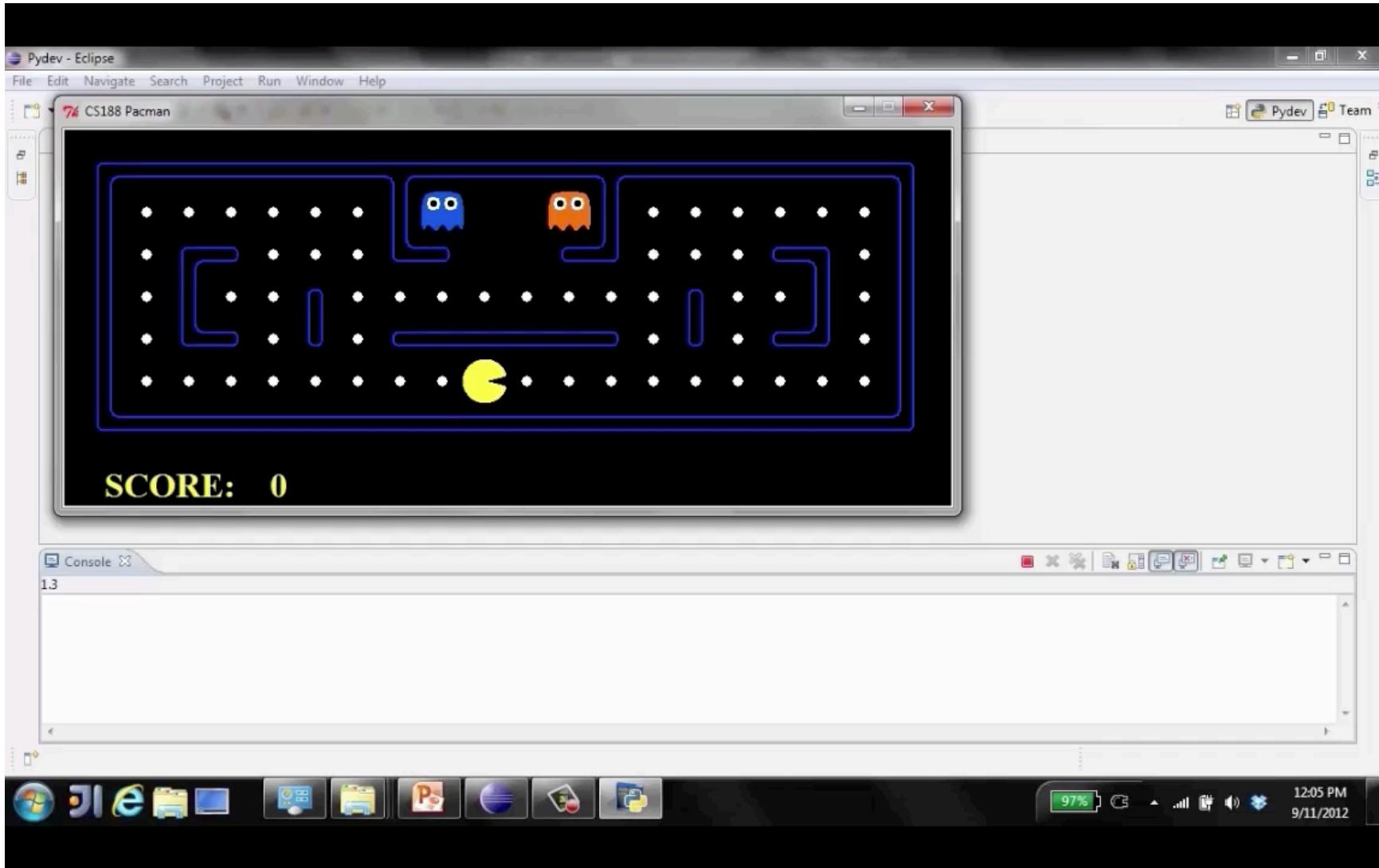
Video of Demo Smart Ghosts (Coordination)



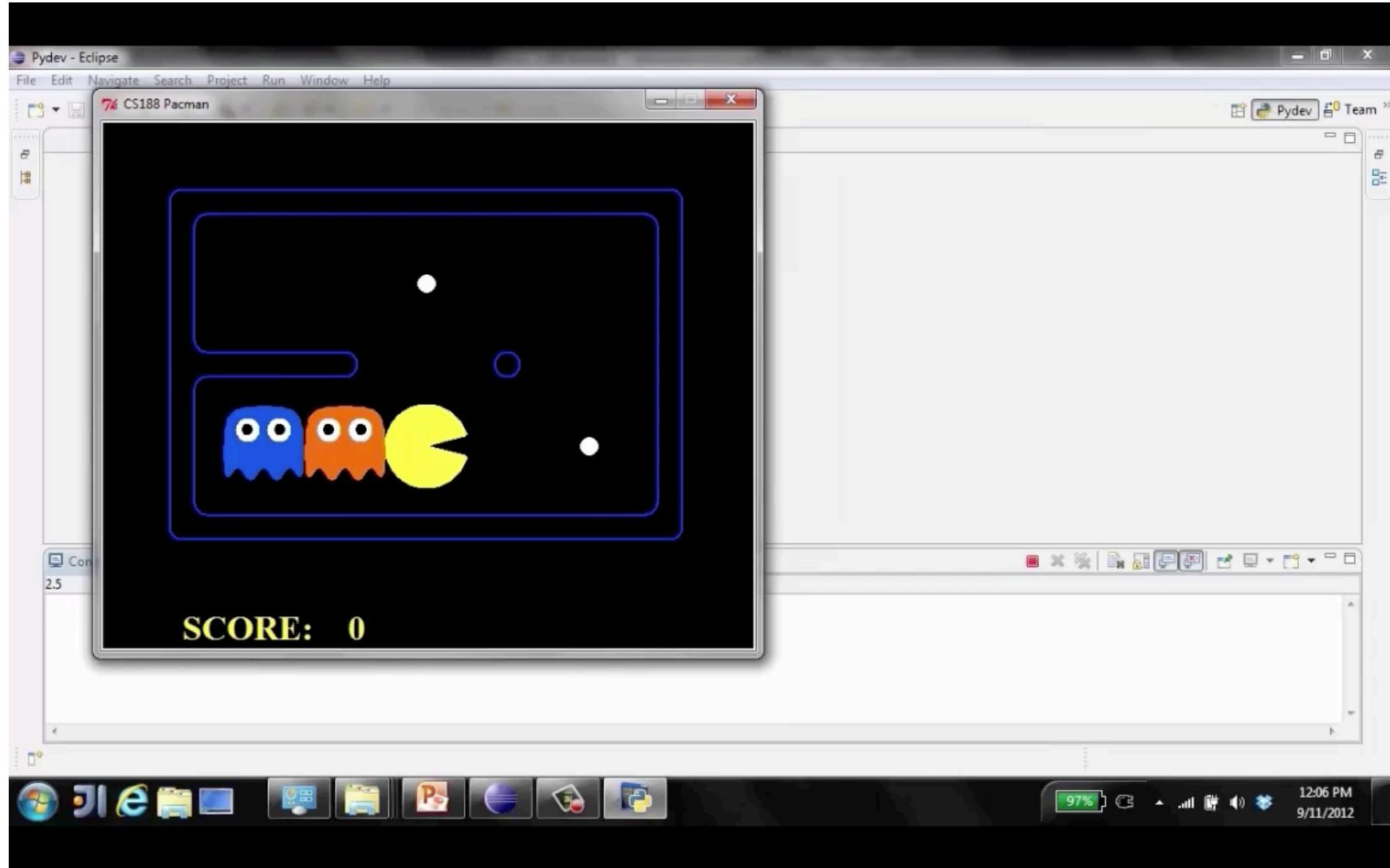
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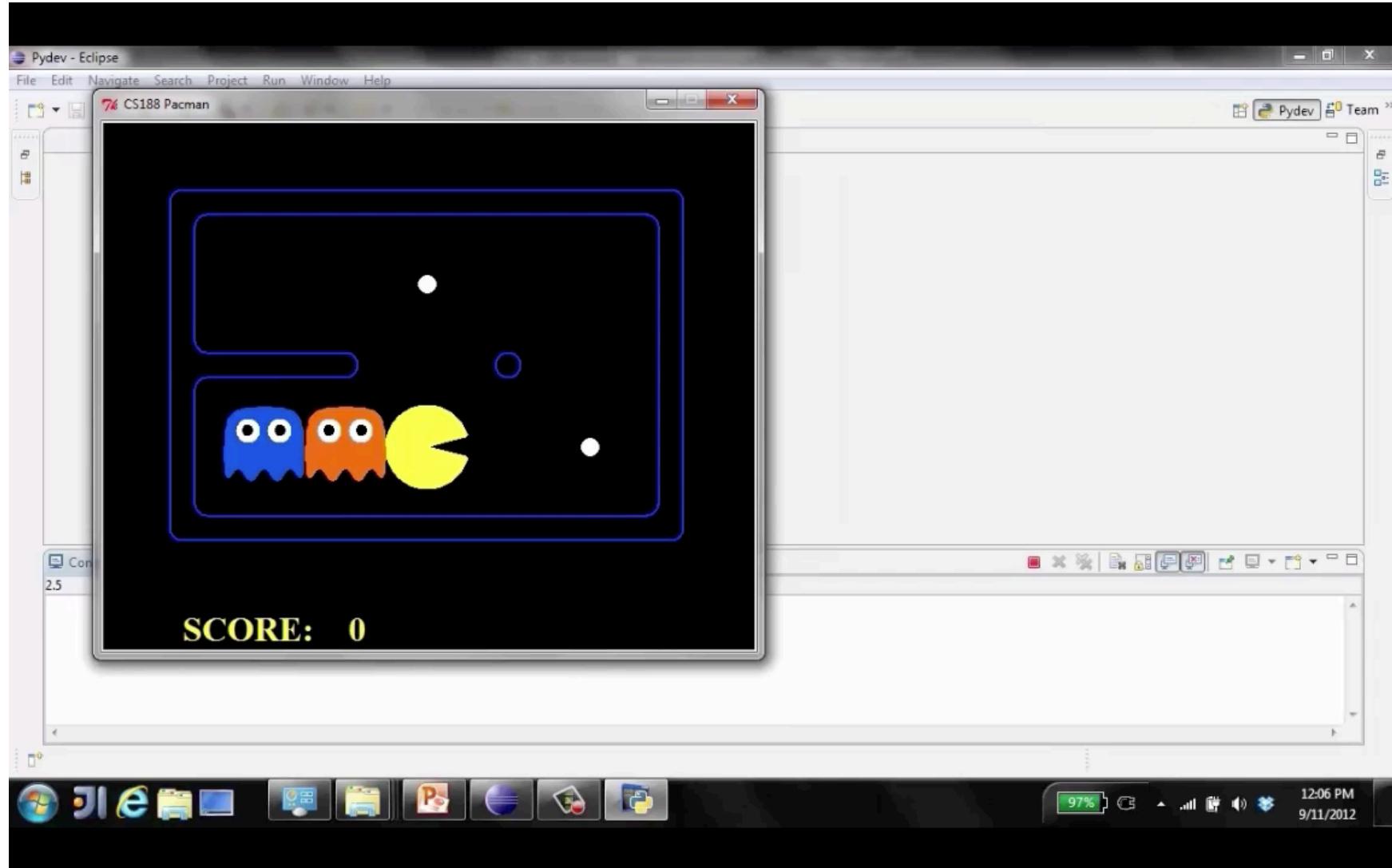
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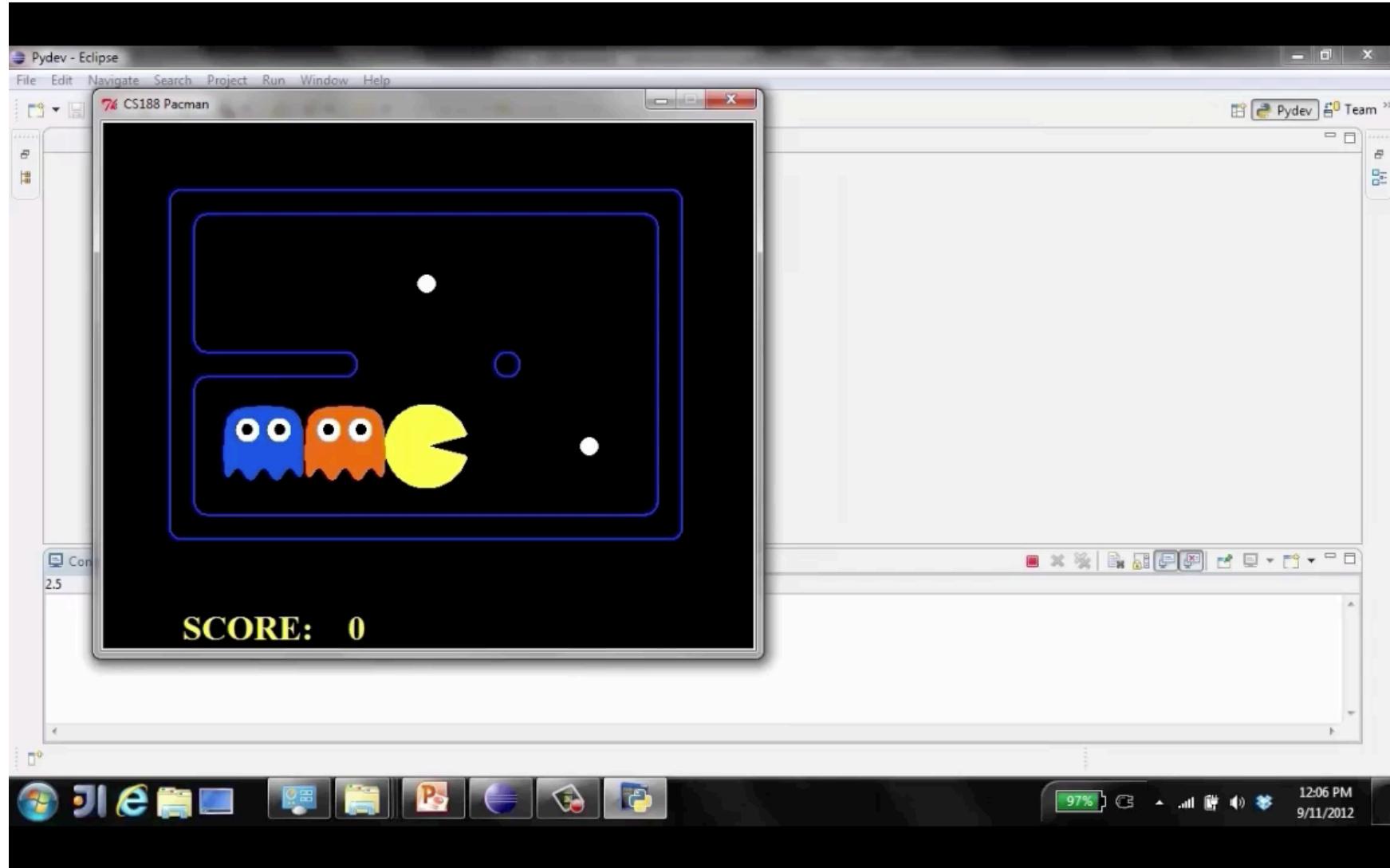
Video of Demo Smart Ghosts (Coordination) - Zoomed In



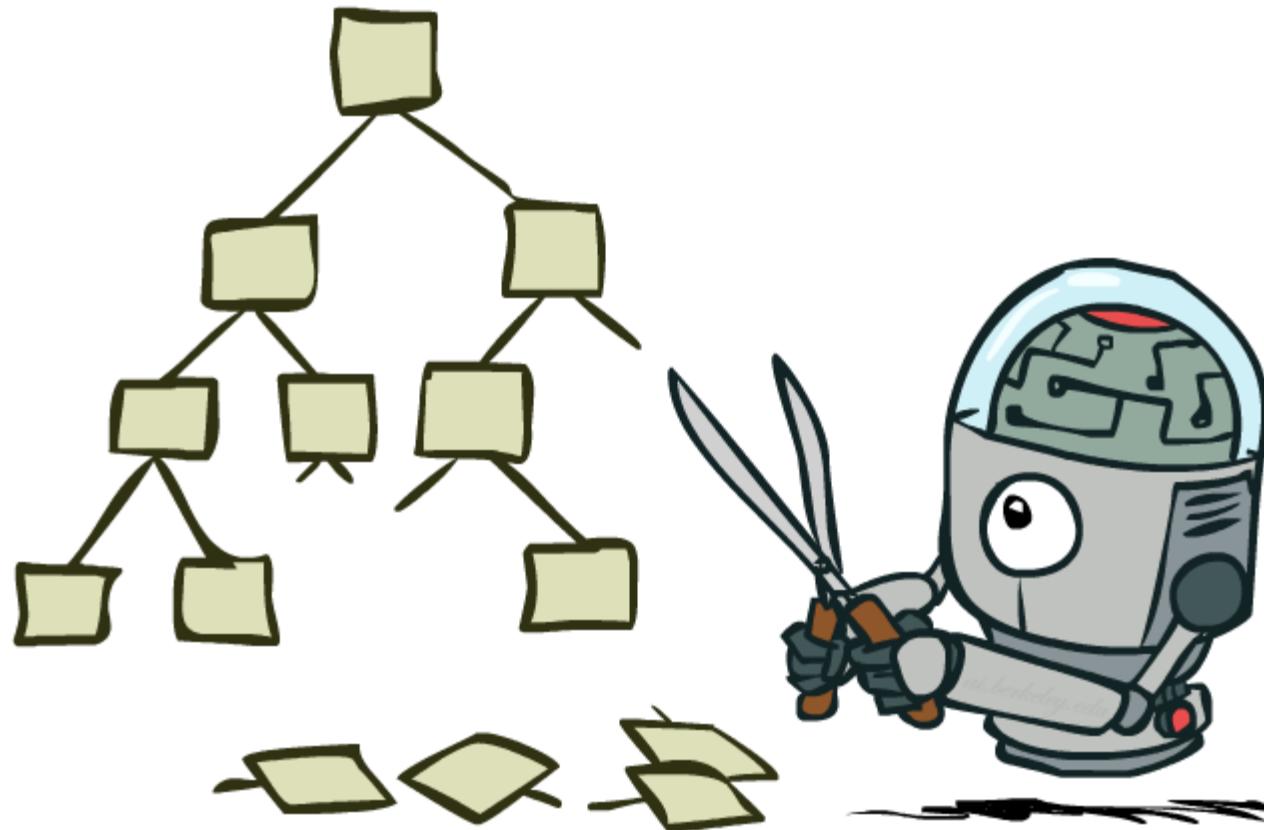
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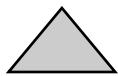
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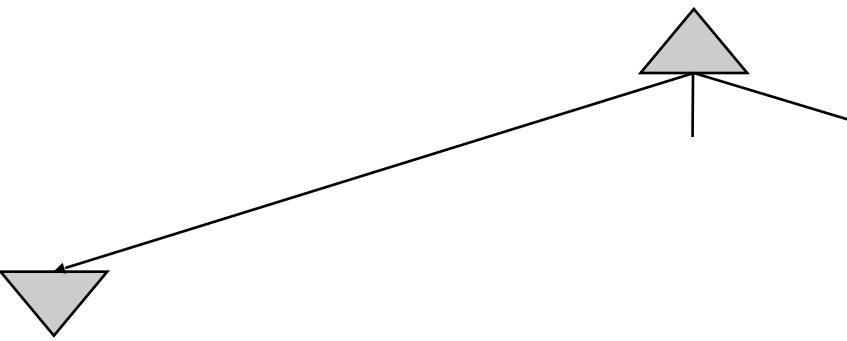
Game Tree Pruning



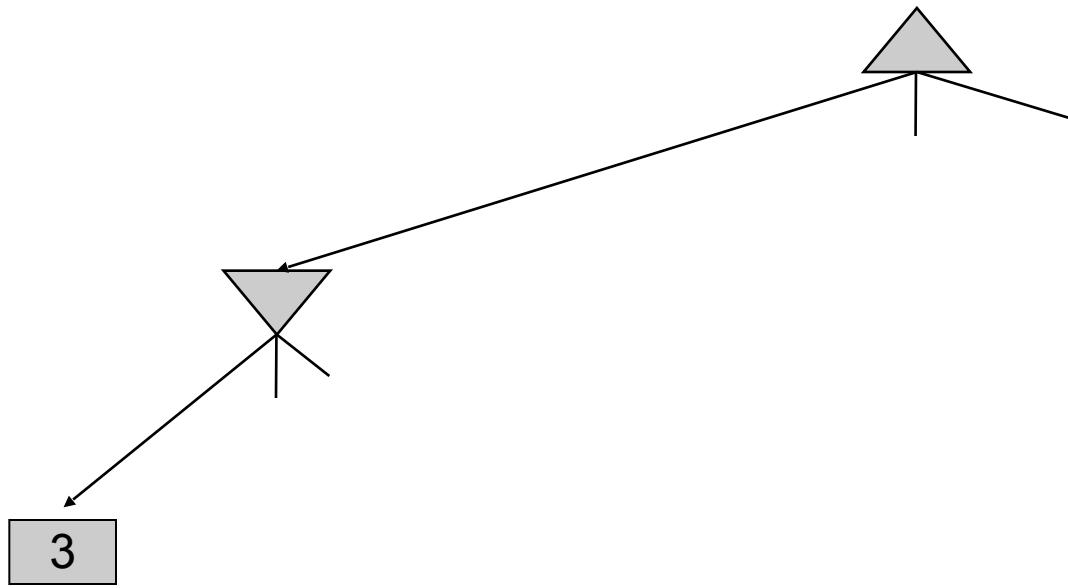
Minimax Example



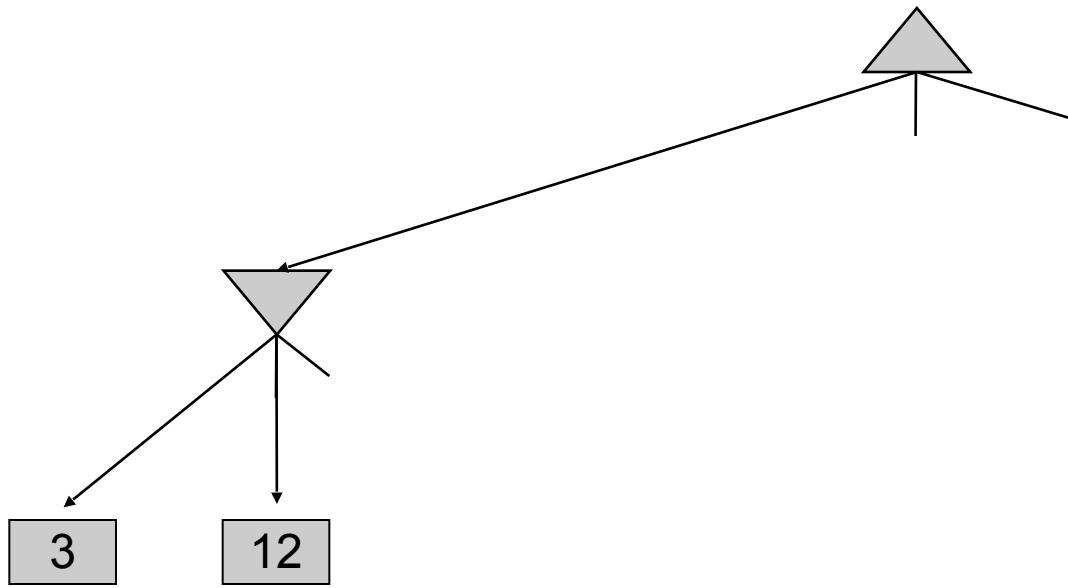
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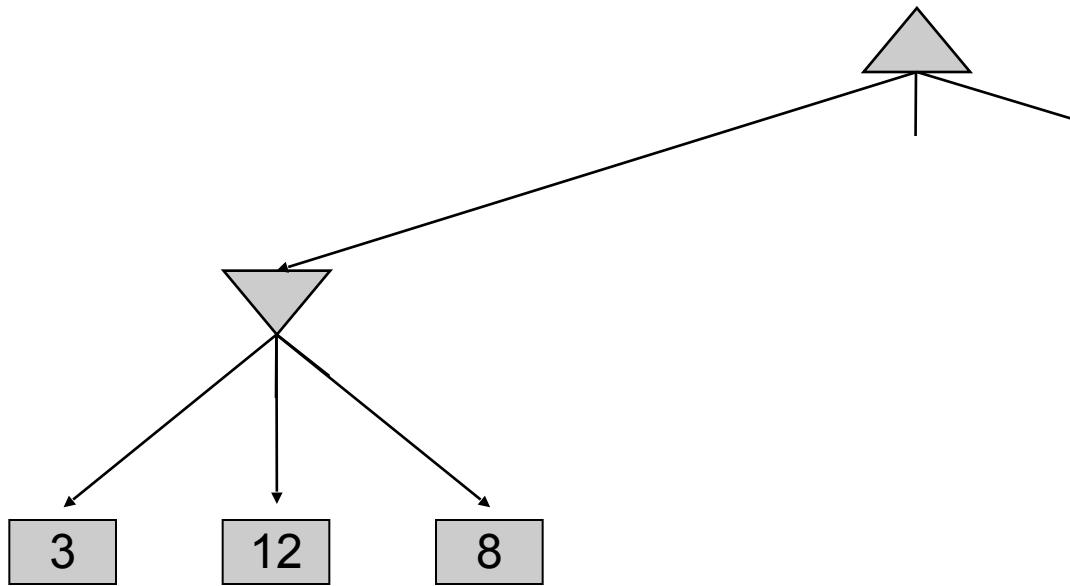
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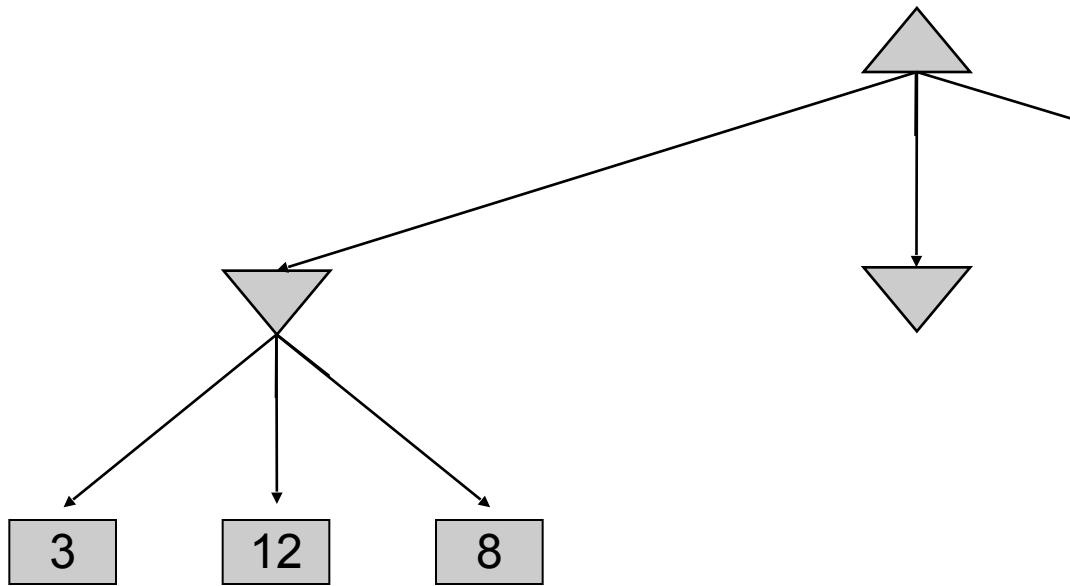
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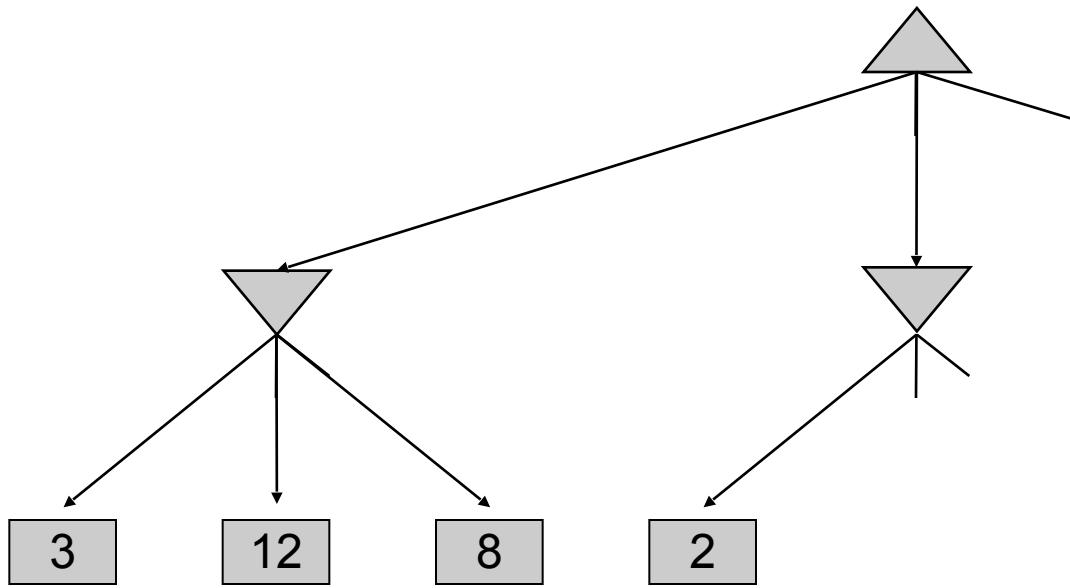
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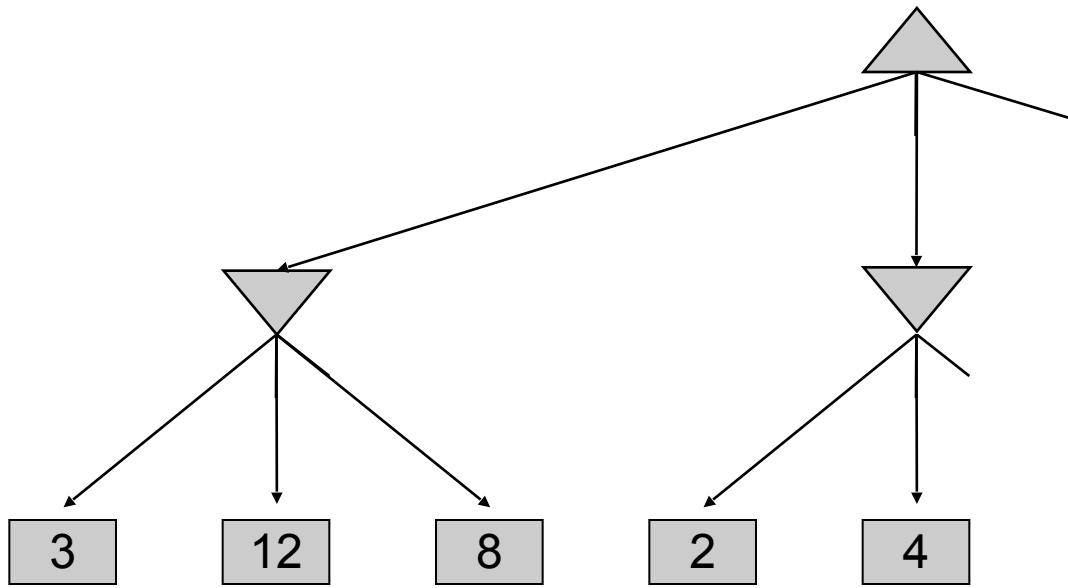
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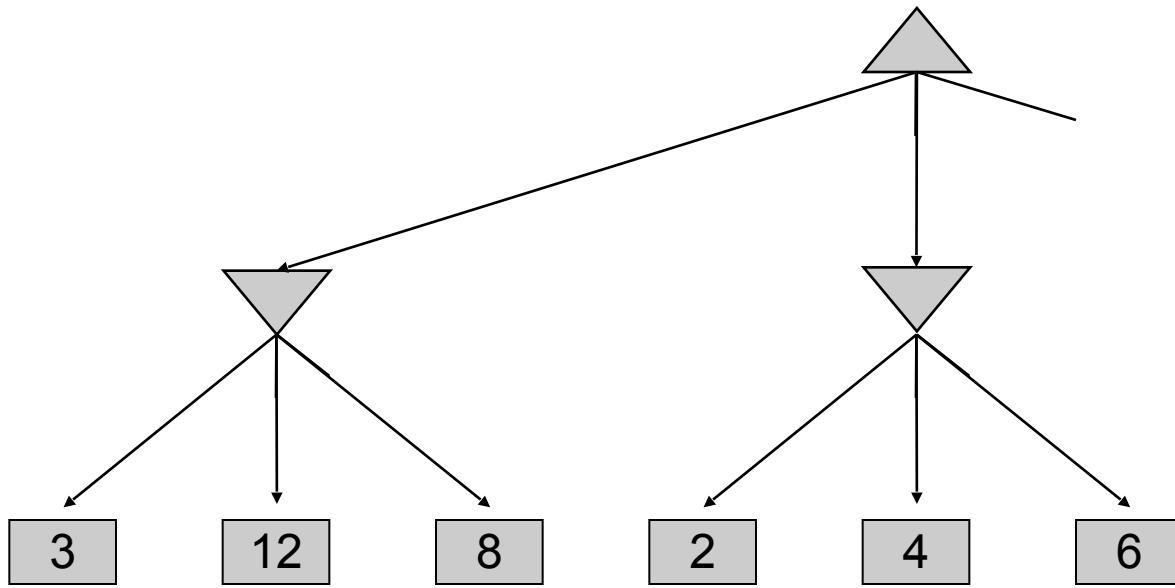
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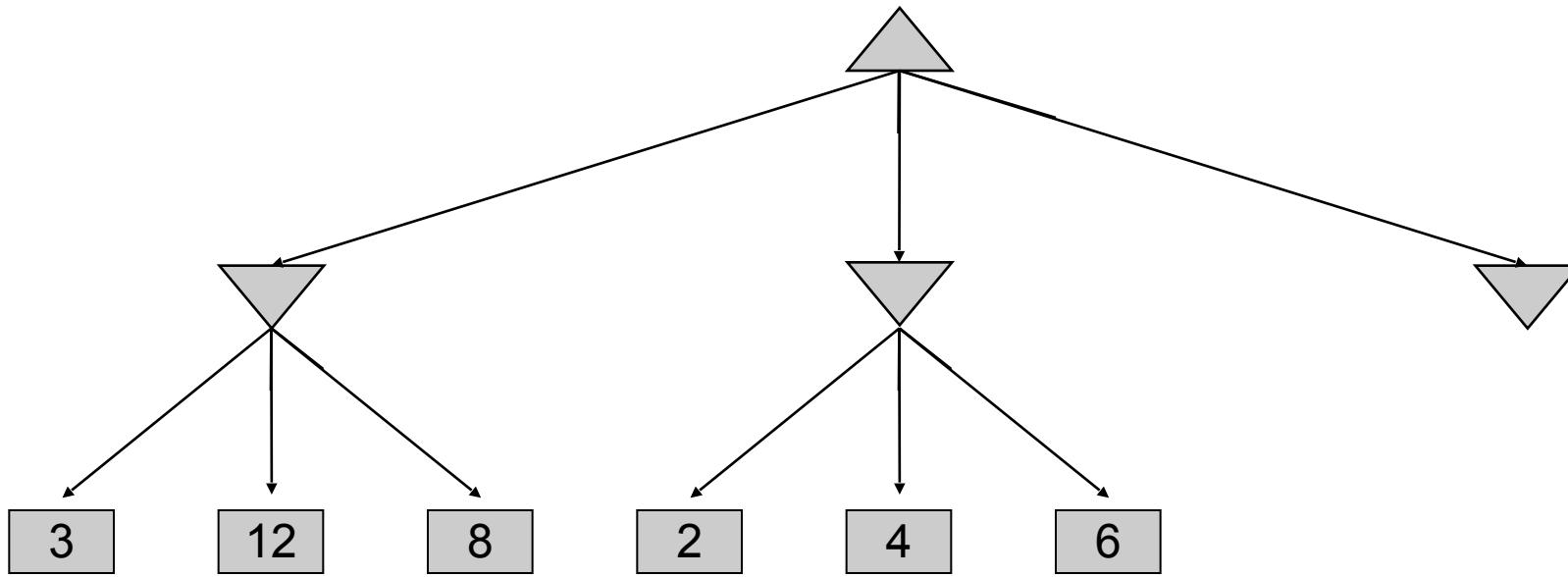
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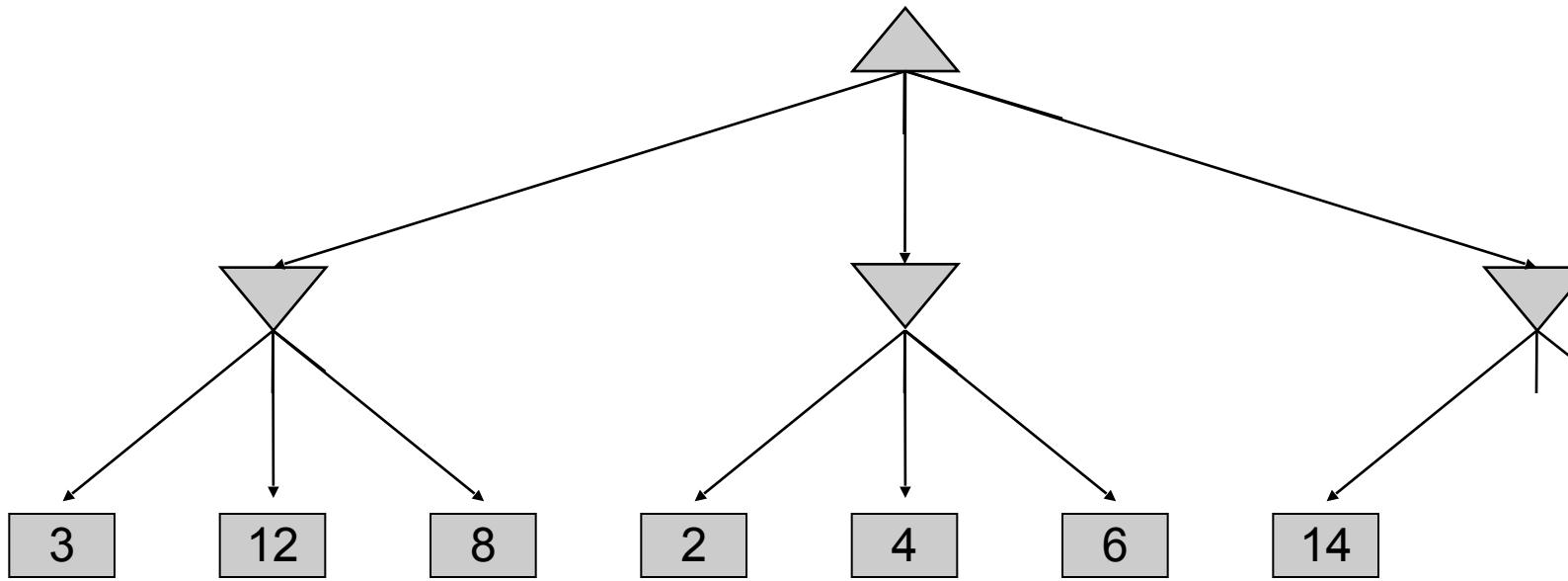
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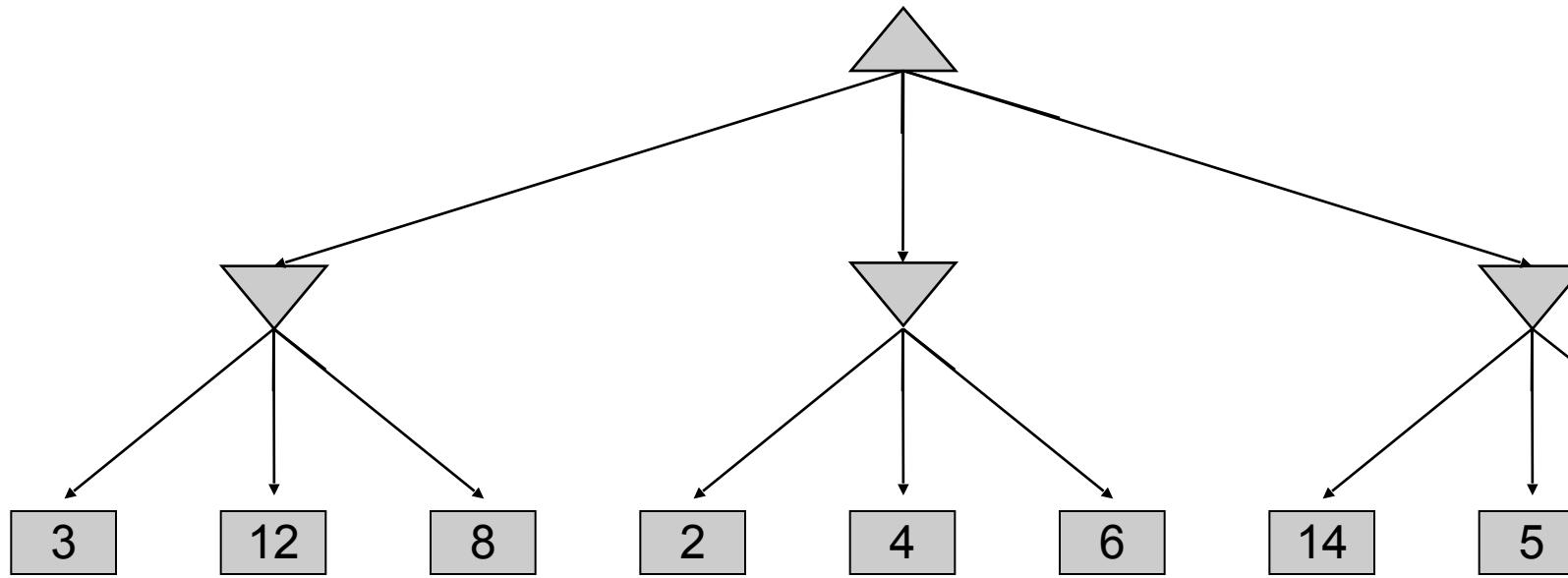
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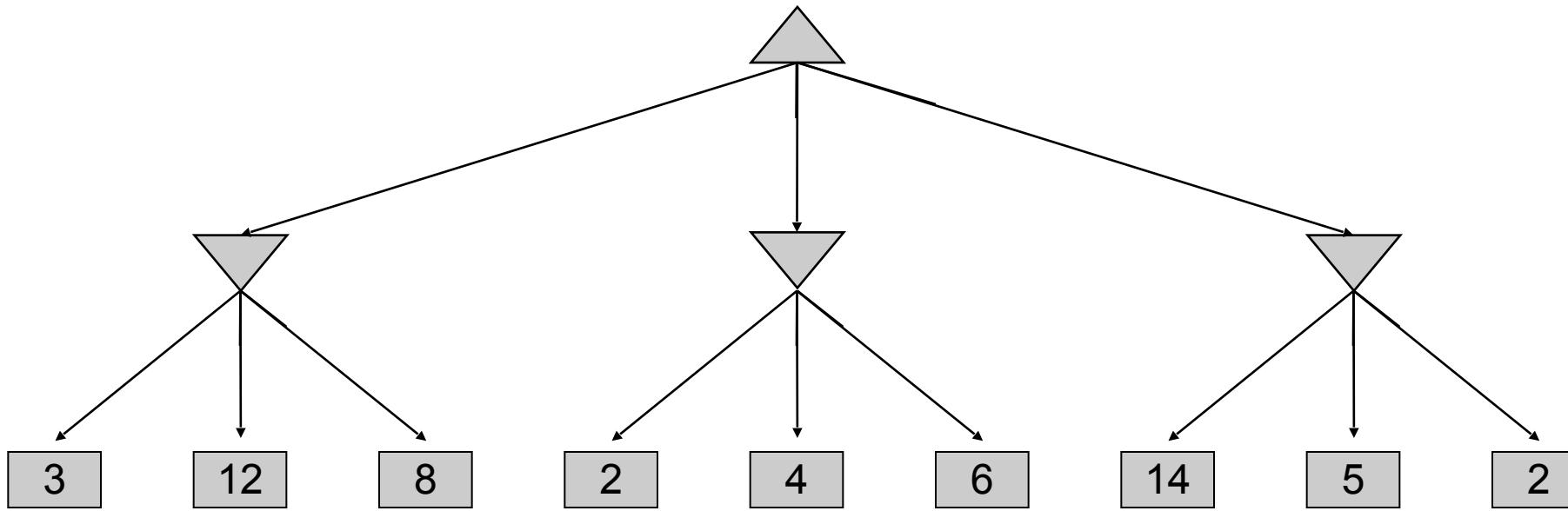
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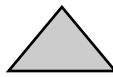
Minimax Example



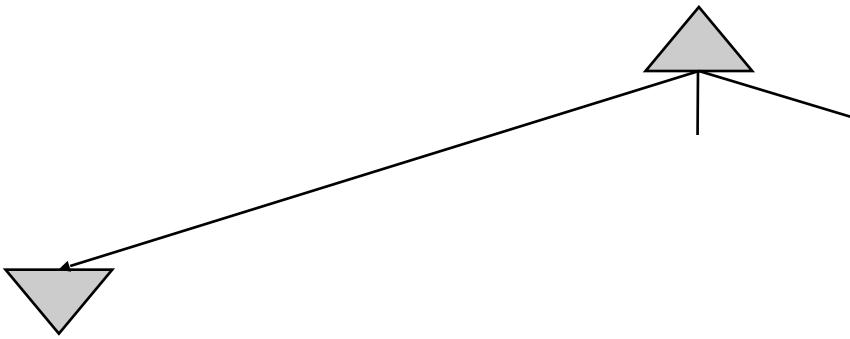
Minimax Example



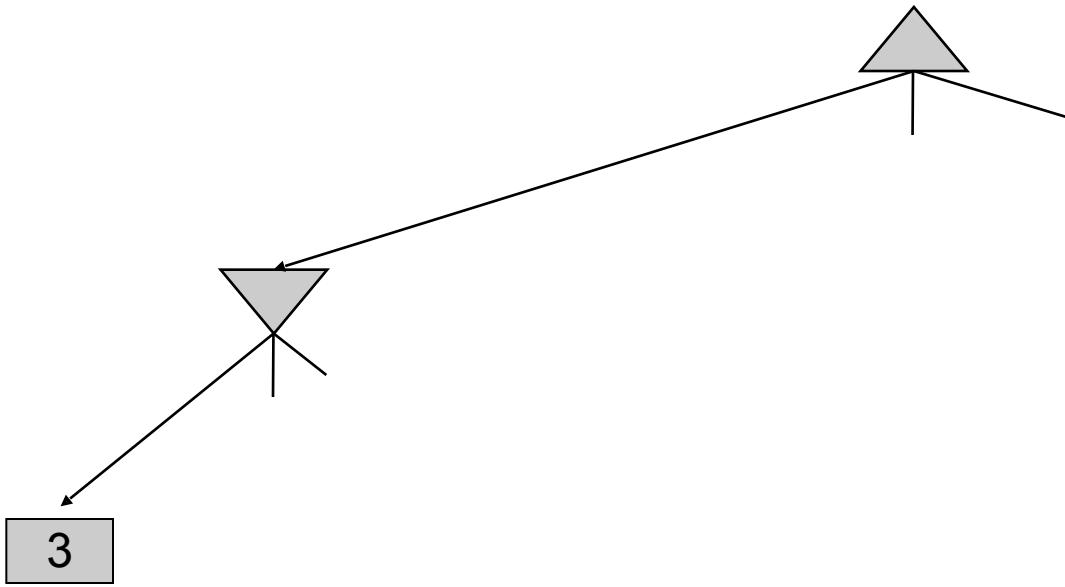
Minimax Pruning



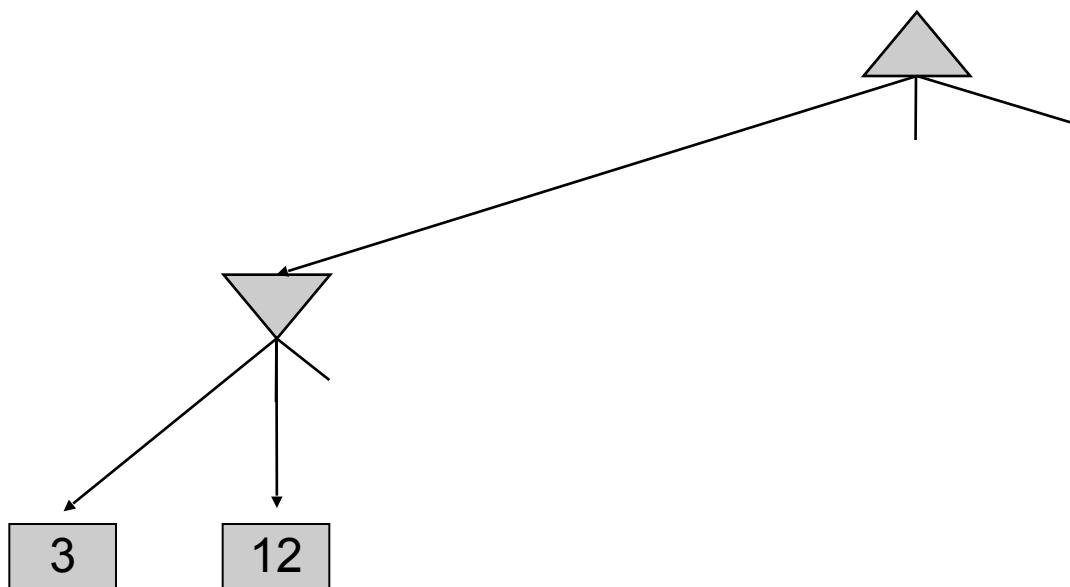
Minimax Pruning



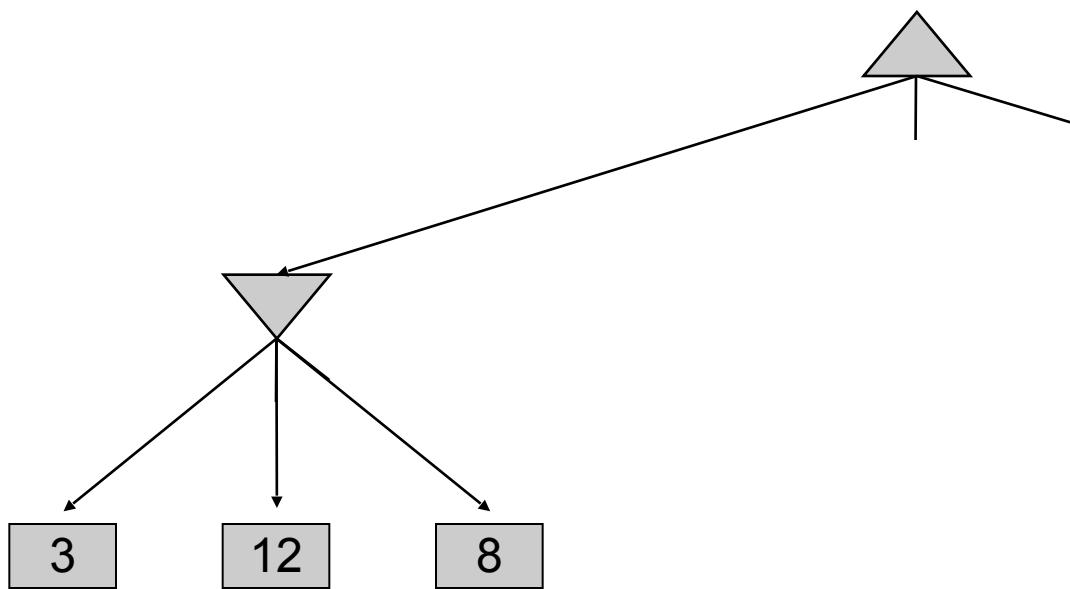
Minimax Pruning



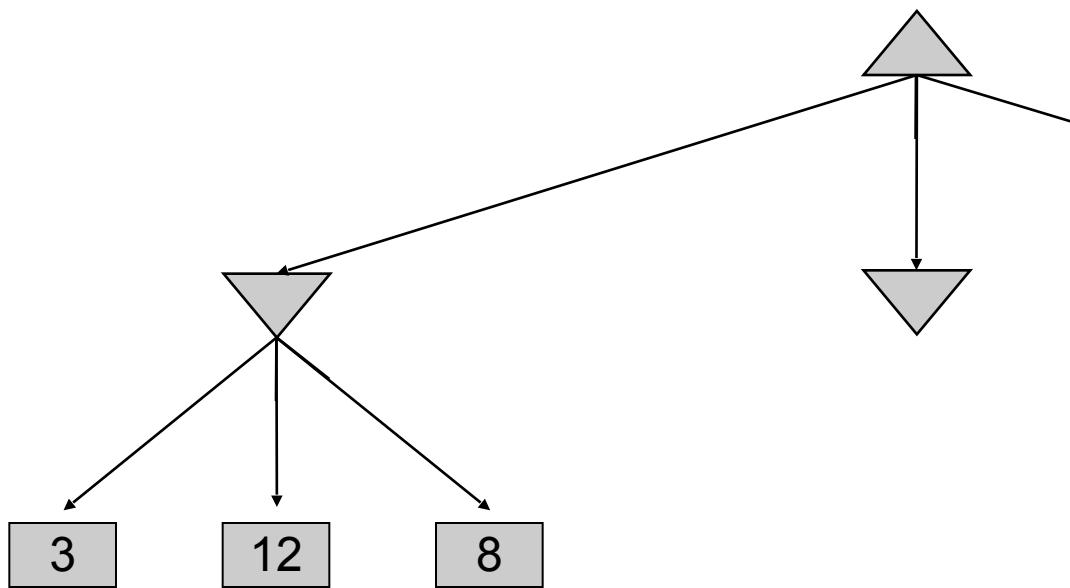
Minimax Pruning



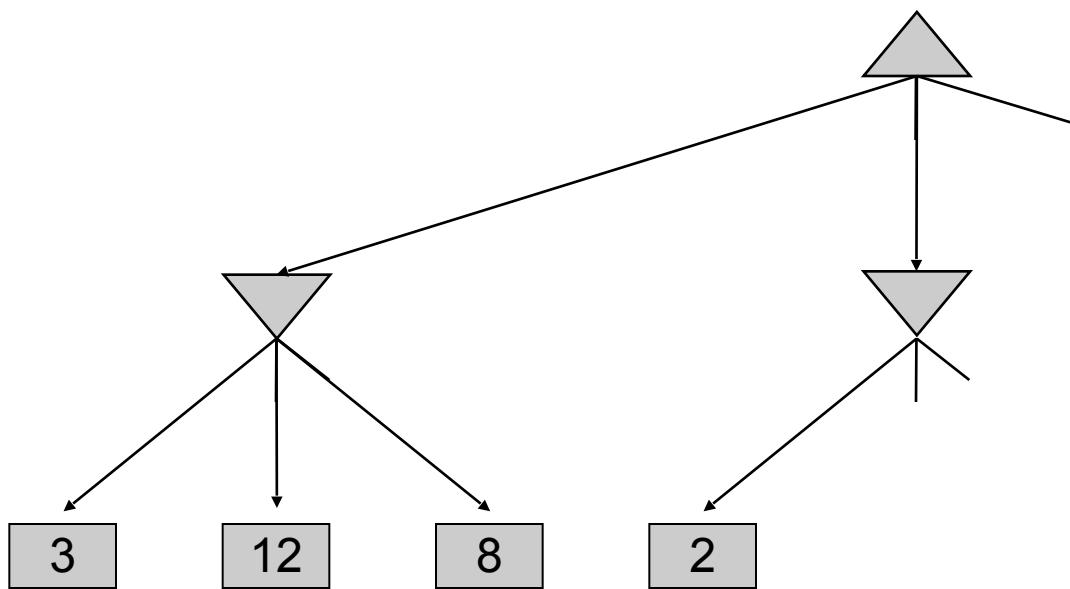
Minimax Pruning



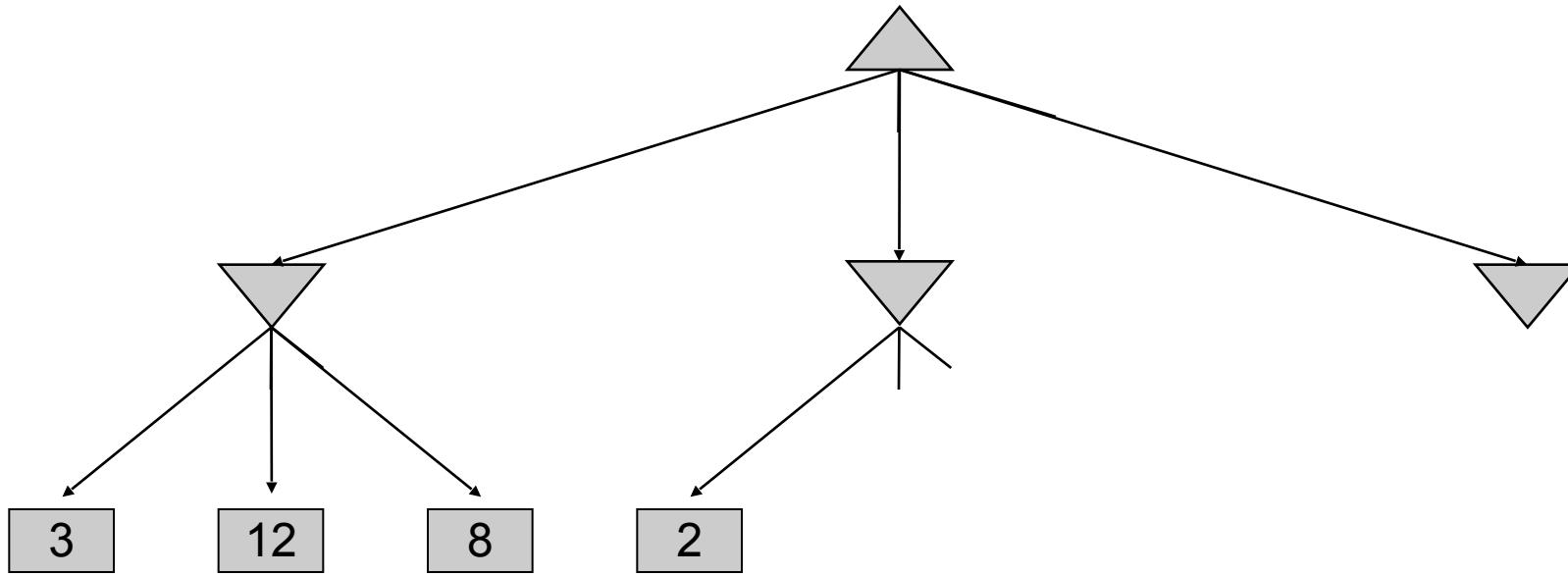
Minimax Pruning



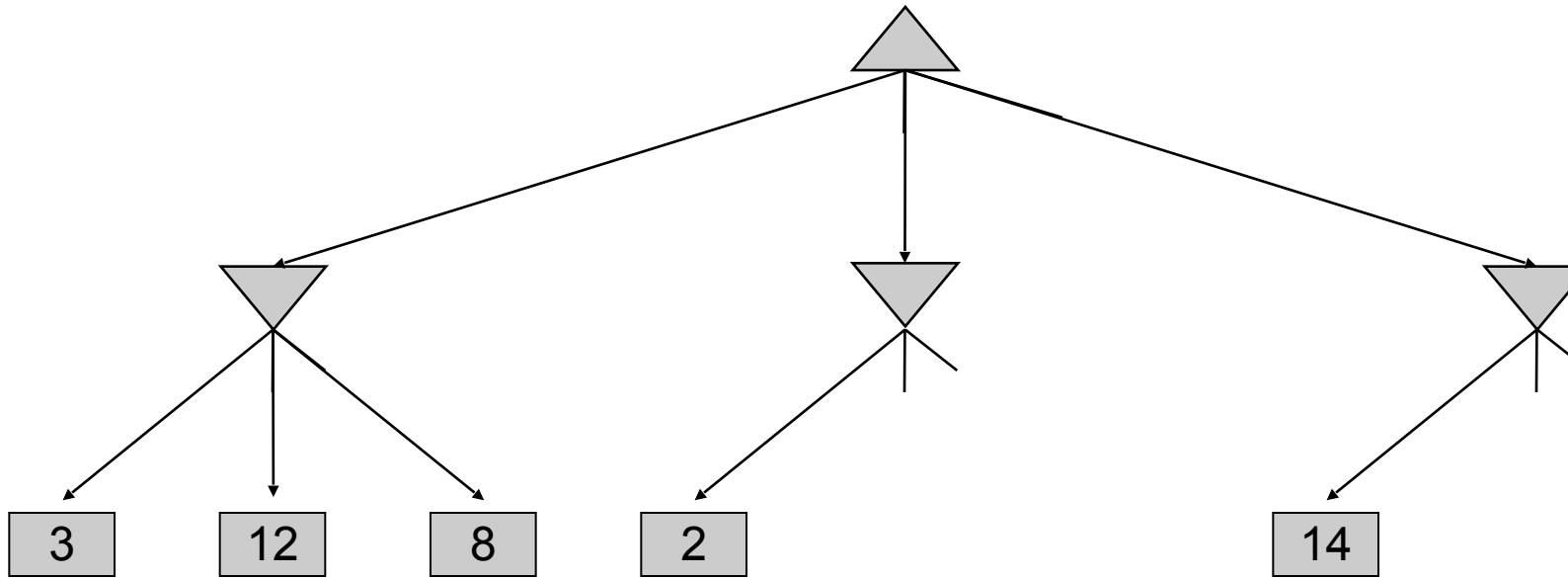
Minimax Pruning



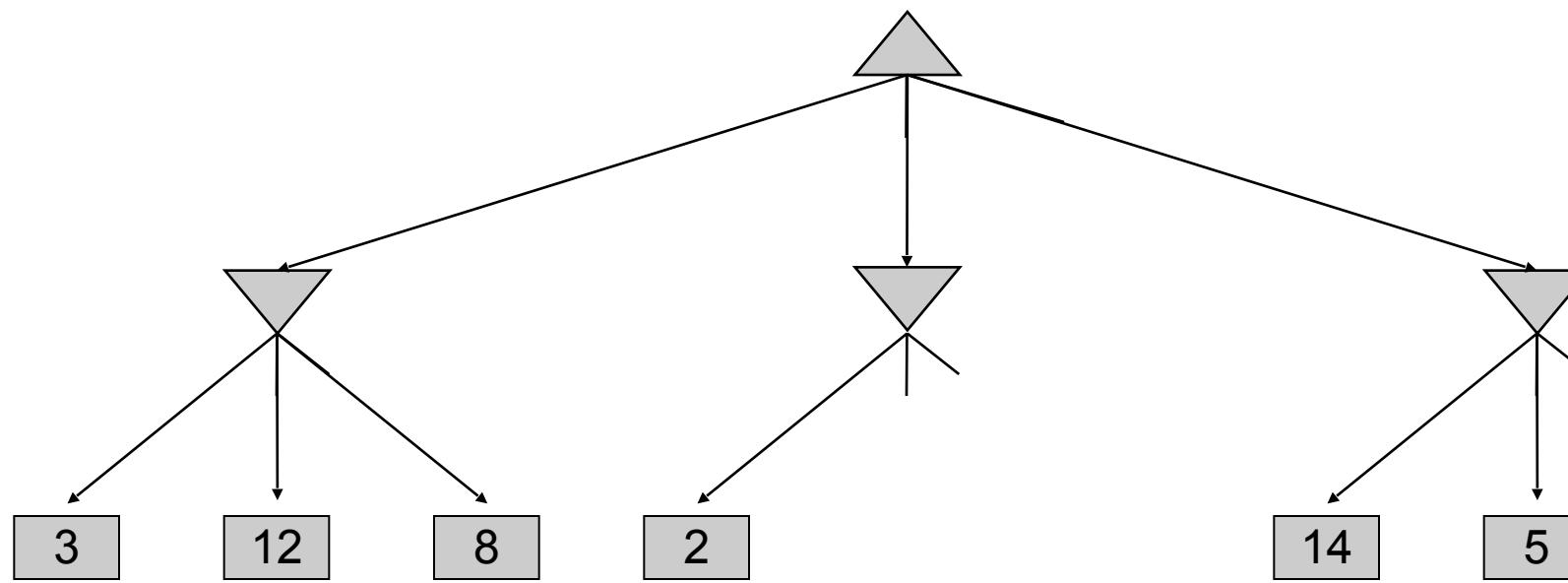
Minimax Pruning



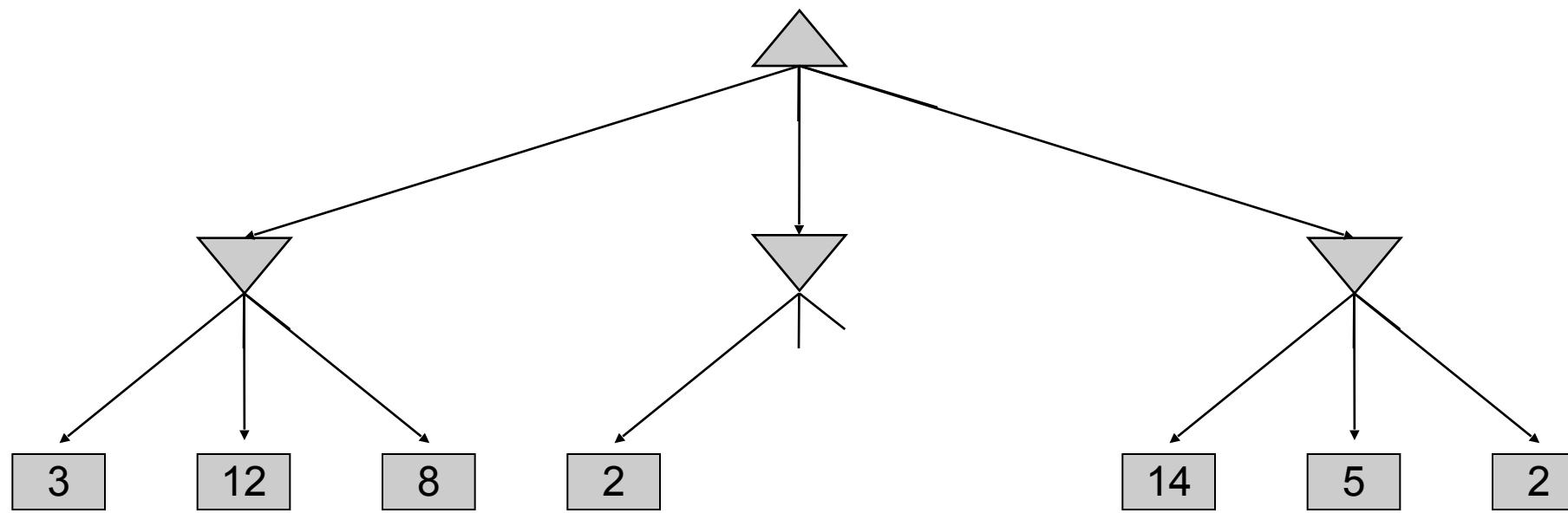
Minimax Pruning



Minimax Pruning



Minimax Pruning

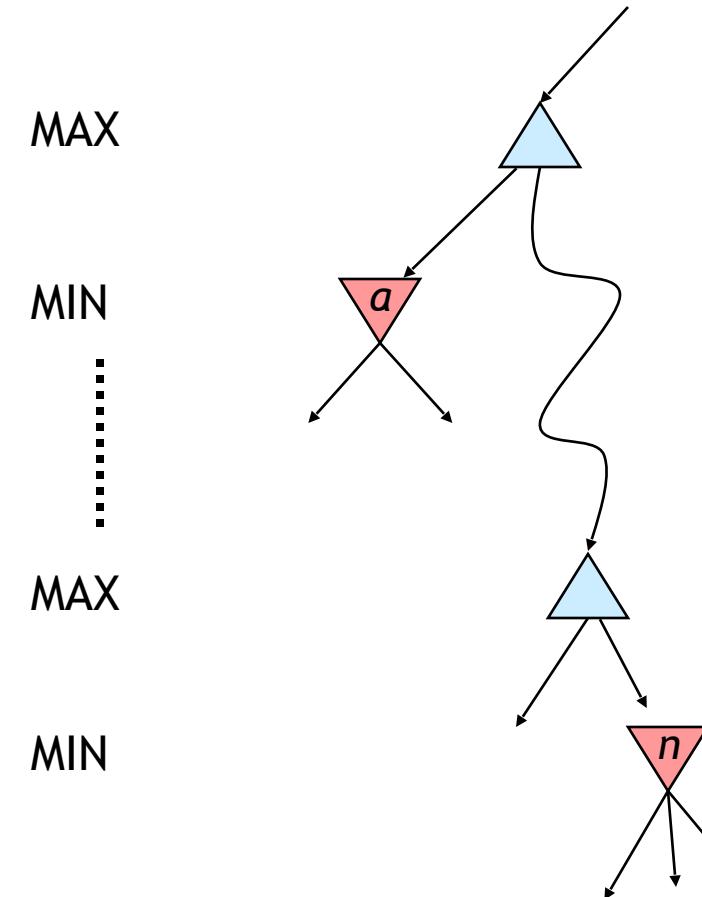


Alpha-Beta Pruning

- General configuration (MIN version)

- We're computing the MIN-VALUE at some node n
- We're looping over n 's children
- n 's estimate of the childrens' min is dropping
- Who cares about n 's value? MAX
- Let a be the best value that MAX can get at any choice point along the current path from the root
- If n becomes worse than a , MAX will avoid it, so we can stop considering n 's other children (it's already bad enough that it won't be played)

- MAX version is symmetric



Alpha-Beta Implementation

α : MAX's best option on path to root
 β : MIN's best option on path to root

```
def max-value(state, α, β):  
    initialize v = -∞  
    for each successor of state:  
        v = max(v, value(successor, α, β))  
        if v ≥ β return v  
        α = max(α, v)  
    return v
```

```
def min-value(state , α, β):  
    initialize v = +∞  
    for each successor of state:  
        v = min(v, value(successor, α, β))  
        if v ≤ α return v  
        β = min(β, v)  
    return v
```

Minimax Implementation (Recap)

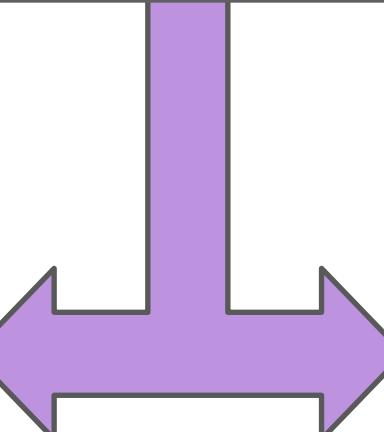
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

Minimax Implementation (Recap)

```
def value(state):
    if the state is a terminal state: return the state's utility
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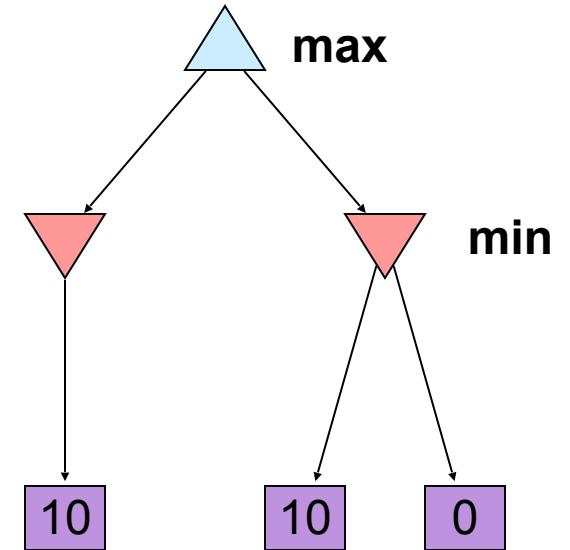
```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```



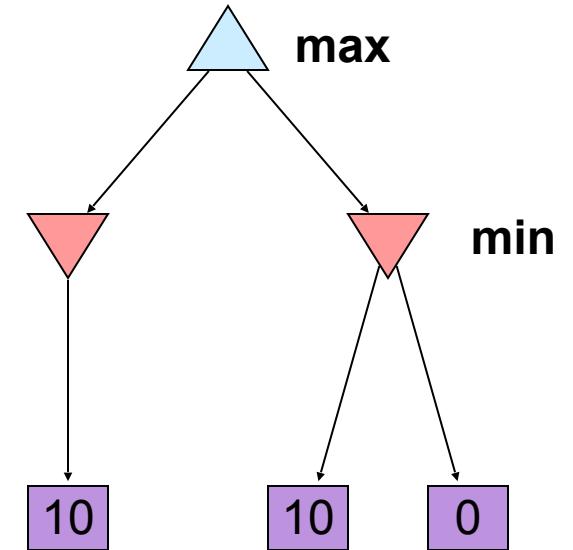
Alpha-Beta Pruning Properties

- This pruning has **no effect** on minimax value computed for the root!
- Values of intermediate nodes might be wrong
 - Important: children of the root may have the wrong value
 - So the most naïve version won't let you do action selection



Alpha-Beta Pruning Properties

- This pruning has **no effect** on minimax value computed for the root!
- Values of intermediate nodes might be wrong
 - Important: children of the root may have the wrong value
 - So the most naïve version won't let you do action selection
- Good child ordering improves effectiveness of pruning
- With “perfect ordering”:
 - Time complexity drops to $O(b^{m/2})$
 - Doubles solvable depth!
 - Full search of, e.g. chess, is still hopeless...
- This is a simple example of **metareasoning** (computing about what to compute)



Alpha-Beta Quiz

