Introduction to Functional Programming in Haskell

Outline

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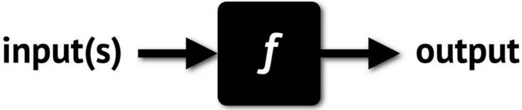
What is a (pure) function?



A function is **pure** if:

- it always returns the same output for the same inputs
- it doesn't do anything else — no "side effects"

In Haskell: whenever we say "function" we mean a **pure function**!

What are and aren't functions?

Always functions:

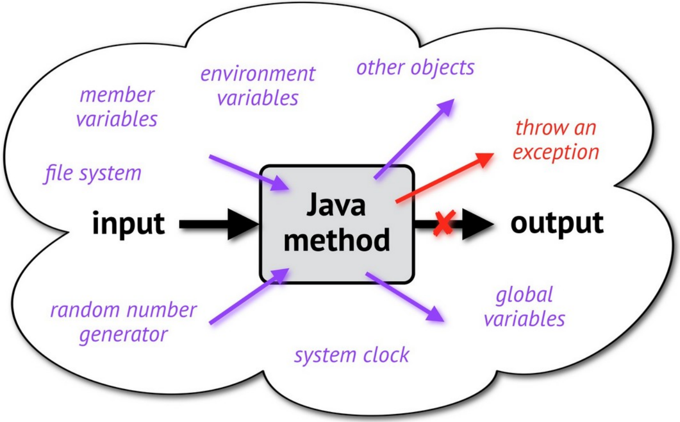
- mathematical functions $f(x) = x^2 + 2x + 3$
- encryption and compression algorithms

Usually not functions:

- C, Python, JavaScript, . . . "functions" (procedures)
- Java, C#, Ruby, . . . methods

Haskell only allows you to write (pure) functions!

Why procedures/methods aren’t functions



- output depends on environment
- may perform arbitrary side effects

Getting into the Haskell mindset

Haske

```
sum :: [Int] -> Int
 sum []     = 0
sum (x:xs) = x +
sum xs
```

Jav

```
int sum(List<Int> xs) {
   int s = 0;
  for (int x : xs) {
       s = s + x;
}
return s;
}
```

In Haskell, "=" means *is* not *change to*!

Getting into the Haskell mindset

Quicksort in

```
qsort :: Ord a => [a] -> [a]
 qsort []     = []
qsort (x:xs) = qsort (filter
(<= x) xs)
        ++ x : qsort (filter
        (>  x) xs)
```

Quicksort in

```
void qsort(int low, int high) {
   int i = low, j = high;
int pivot = numbers[low +
  (high-low)/2];

while (i <= j) {
    while (numbers[i] < pivot) {
       i++;
  }
    while (numbers[j] > pivot) {
       j--;
  }
  if (i <= j) {
      swap(i, j);
       i++;
      j--;
    }
  }
  if (low < j)
    qsort(low, j);
    qsort(i, high);
}

void swap(int i, int j) {
  int temp = numbers[i];
  numbers[i] = numbers[j];
   numbers[j] = temp;
}
```

Referential transparency

a.k.a. **referent**

An expression can be replaced by its **value**
without changing the overall program behavior

```
    length [1,2,3] + 4
⇒               3 + 4
```

what if `length` was a Java method?

Corollary: an expression can be replaced by **any expression**
with the same value without changing program behavior

Supports **equational reasoning**

Equational reasoning

Computation is just substitution!

```
sum :: [Int] -> Int
 sum []      = 0
sum (x:xs) = x +
sum xs
```

equatio
ns

```
     sum [2,3,4]
⇒    sum (2:(3:(4:[])))
⇒    2 + sum (3:(4:[]))
⇒    2 + 3 + sum (4:[])
⇒    2 + 3 + 4 + sum []
⇒    2 + 3 + 4 + 0
⇒    9
```

Describing computations

Function definition: a list of **equations** that relate inputs to output

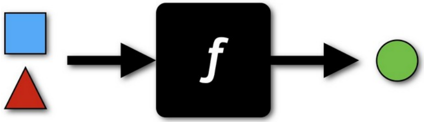
- matched top-to-bottom
- applied left-to-right

imperative view:
how do I rearrange the elements in the list?
C

functional view:
how is a list related to its reversal? ✓

```
reverse :: [a] -> [a]
 reverse []     = []
reverse (x:xs) =
reverse xs ++ [x]
```

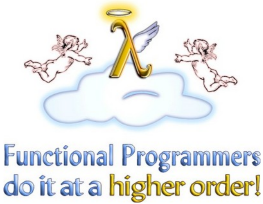
First-order functions

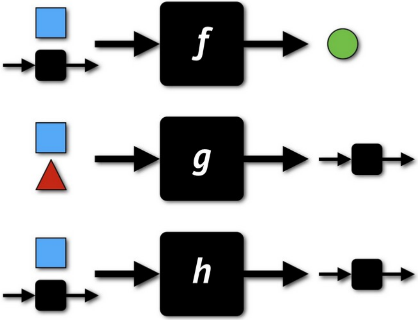


Examples

- `cos :: Float -> Float`
- `even :: Int -> Bool`
- `length :: [a] -> Int`

Higher-order functions





Examples

- `map :: (a -> b) -> [a] -> [b]`
- `filter :: (a -> Bool) -> [a] -> [a]`
- `(.) :: (b -> c) -> (a -> b) -> a -> c`

Higher-order functions as control structures

map: *loop for doing something to each element in a list*

```
map :: (a -> b) -> [a] -> [b]
 map f []     = []
map f (x:xs) = f x : map f xs
```

```
map f [2,3,4,5] = [f 2, f 3, f 4, f 5]
```

```
map even [2,3,4,5]
= [even 2, even 3, even 4, even 5]
= [True,False,True,False]
```

fold: *loop for aggregating elements in a list*

```
foldr :: (a->b->b) -> b -> [a] -> b
 foldr f y []     = y
foldr f y (x:xs) = f x (foldr f y
xs)
```

```
foldr f y [2,3,4] = f 2 (f 3 (f 4 y))
```

```
foldr (+) 0 [2,3,4]
= (+) 2 ((+) 3 ((+) 4 0))
= 2 + (3 + (4 + 0))
= 9
```

Function composition

Can create new functions by **composing** existing functions

- *apply the second function, then apply the first*

Function

```
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \x -> f (g x)
```

```
(f . g) x = f (g x)
```

Types of existing

```
not   :: Bool ->
Bool   succ :: Int ->
Int   even :: Int ->
Bool   head :: [a] ->
a
tail :: [a] -> [a]
```

Definitions of new

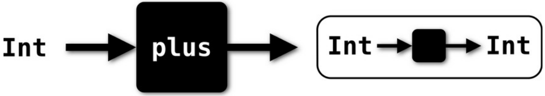
```
plus2   = succ .
succ   odd      =
not . even   second =
head . tail   drop2
        = tail .
tail
```

Curying / partial application

In Haskell, functions that take multiple arguments are
implicitly higher order

Haskell Curry

```
plus :: Int -> Int -> Int
```




```
increment :: Int -> Int
 increment = plus 1
```

Currie `plus 2 3`

```
plus :: Int -> Int -> Int
```

Uncurrie `plus (2,3)`

```
plus :: (Int,Int) -> Int
```

a pair of ints

Lazy evaluation

In Haskell, expressions are reduced:

- only when needed
- at most once

```
nats :: [Int]
nats = 1 : map (+1) nats

fact :: Int -> Int
fact n = product (take n nats)
```

```
min3 :: [Int] -> [Int]
 min3 = take 3 . sort
```

Supports:

- infinite data structures
- separation of concerns

What is the running time of this function?

John Hughes, *Why Functional Programming Matters*, 1989

Good Haskell style

Why it matters:

- layout is significant!
- eliminate misconceptions
- we care about *elegance*

Easy stuff:

- **use spaces!** (tabs cause layout errors)
- align patterns and guards

See style guides on course web page

Formatting function applications

Function application:

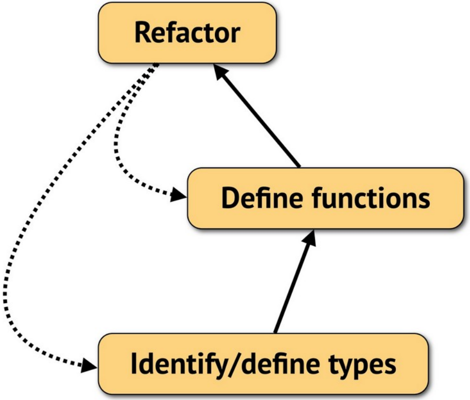
- is *just a space*
- associates to the left
- binds most strongly

`f(x)`	`f x`
`(f x) y`	`f x y`
`(f x) + (g y)`	`f x + g y`

Use parentheses only to *override* this behavior:

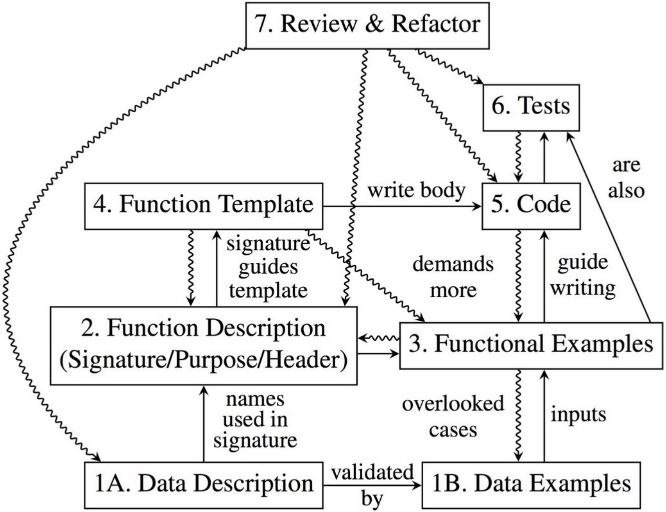
- `f (g x)`
- `f (x + y)`

FP workflow (simple)



"obsessive compulsive refactoring disorder"

FP workflow (detailed)



Norman Ramsey, *On Teaching "How to Design Programs"*, ICFP'14

Algebraic data types

Data type

- introduces new **type** of value
- enumerates ways to **construct** values of this type

Some example data

```
data Bool = True | False

data Nat  = Zero | Succ Nat

data Tree = Node Int Tree Tree
          | Leaf Int
```

Definitions consists of . . .

- a **type name**
- a list of **data constructors** with **argument types**

Definition is **inductive**

- the arguments may **recursively** include the type being defined
- the constructors are the **only way** to build values of this type

Anatomy of a data type definition

type name

```
data Expr = Lit Int
          | Plus Expr Expr
```

cases

data constructor

types of arguments

Example: 2 + 3 + 4 `Plus (Lit 2) (Plus (Lit 3) (Lit 4))`

FP data types vs. OO classes

Haske

```
data Tree = Node Int Tree Tree
          | Leaf
```

Jav

```
abstract class Tree { ... }
 class Node extends Tree {
  int label;
  Tree left, right;
  ...
}
class Leaf extends Tree
{ ... }
```

- separation of type- and value-level
- set of cases closed
- set of operations open

- merger of type- and value-level
- set of cases open
- set of operations closed

Extensibility of cases vs. operations = the "expression problem"

Type parameters

type parameter

(*Like generics in Java*)

```
data List a = Nil
            | Cons a (List a)
```

reference to type parameter

recursive reference to type

Specialized lists

```
type IntList = List Int
type CharList = List Char
type RaggedMatrix a =
List (List a)
```

What is a type class?

1. an **interface** that is supported by many different types
2. a **set of types** that have a common behavior

```
class Eq a where
  (==) :: a -> a -> Bool
```

types whose values can be compared for equality

```
class Show a where
  show :: a -> String
```

types whose values can be shown as strings

```
class Num a where
  (+) :: a -> a -> a
(*) :: a -> a -> a
  negate :: a -> a
  ...
```

types whose values can be manipulated like numbers

Type constraints

```
class Eq a where
  (==) :: a -> a -> Bool
```

List elements can be of any type

```
length :: [a] -> Int
 length []     = 0
length (_:xs) = 1 +
length xs
```

List elements must support equality!

```
elem :: Eq a => a -> [a] -> Bool
 elem _ []     = False
elem y (x:xs) = x == y || elem y
xs
```

Tools for defining functions

Recursion and other functions

```
sum :: [Int] -> Int
sum xs = if null xs then 0
         else head xs + sum (tail xs)
```

Pattern

```
sum :: [Int] -> Int
 sum []      = 0
sum (x:xs) = x +
sum xs
```

(1) case analysis

(2) decomposition

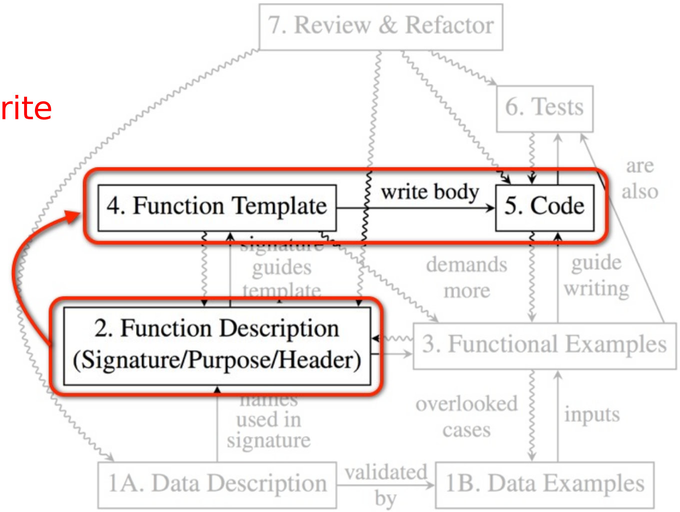
Higher-order

```
sum :: [Int] -> Int
 sum = foldr (+) 0
```

no recursion or variables needed!

What is type-directed programming?

Use the **type** of a function to help write its **body**



Type-directed programming

Basic goal: transform values of **argument types** into **result type**

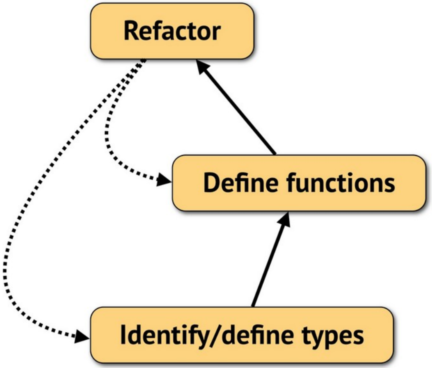
If argument type is

- **atomic type** (e.g. `Int`, `Char`)
  - apply functions to it
- **algebraic data type**
  - use pattern matching
    - case analysis
    - decompose into parts
- **function type**
  - apply it to something

If result type is

- **atomic type**
  - output of another function
- **algebraic data type**
  - build with data constructor
  - function composition
- **function type**
  - partial application
  - build with lambda abstraction

Refactoring in the FP workflow



"obsessive compulsive refactoring disorder"

Motivations:

- separate concerns
- promote reuse
- promote understandability
- gain insights

Refactoring relations

Semantics-preserving **laws**

can prove with equational reasoning + induction

- Eta reduction:

```
\x -> f x
≡
f
```

- Map–map fusion:

```
map f . map g   ≡   map (f . g)
```

- Fold–map fusion:

```
foldr f b . map g   ≡   foldr (f . g) b
```

"Algebra of computer programs"

John Backus, *Can Programming be Liberated from the von Neumann Style?*, ACM Turing Award Lecture, 1978

Strategy: systematic generalization

```
commas :: [String] -> [String]
 commas []     = []
commas [x]     = [x]
commas (x:xs) = x : ", " :
commas xs
```

```
commas :: [String] -> [String]
 commas = intersperse ", "
```

Introduce parameters for constants

```
seps :: String -> [String] -> [String]
 seps _ []     = []
seps _ [x]     = [x]
seps s (x:xs) = x : s : seps s xs
```

Broaden the types

```
intersperse :: a -> [a] -> [a]
 intersperse _ []

= []  intersperse _ [x]

= [x]
```

Strategy: abstract repeated templates

abstract (v): extract and make reusable (as a function)

```
showResult :: Maybe Float -> String
 showResult Nothing = "ERROR"
showResult (Just v) = show v

moveCommand :: Maybe Dir -> Command
 moveCommand Nothing = Stay
moveCommand (Just d) = Move d

safeAdd :: Int -> Maybe Int -> Int
 safeAdd x Nothing = x
safeAdd x (Just y) = x + y
```

Repeated structure:

- pattern match
- default value if `Nothing`
- apply function to contents if `Just`

Strategy: abstract repeated templates

Describe repeated structure in function

```
maybe :: b -> (a -> b) -> Maybe a -> b
 maybe b _ Nothing = b
maybe _ f (Just a) = f a
```

Reuse in implementations

```
showResult   = maybe "ERROR"
show  moveCommand = maybe Stay
Move  safeAdd x             =
maybe x (x+)
```

Refactoring data types

```
data Expr = Var Name
          | Add Expr Expr
          | Sub Expr Expr
          | Mul Expr Expr
```

```
vars :: Expr -> [Name]
 vars (Var x)   = [x]
vars (Add l r) = vars l ++ vars r
 vars (Sub l r) = vars l ++ vars
r   vars (Mul l r) = vars l ++
vars r

eval :: Env -> Expr -> Int
 eval m (Var x)   = get x m
eval m (Add l r) = eval m l + eval m r
 eval m (Sub l r) = eval m l - eval m
r   eval m (Mul l r) = eval m l * eval
m r
```

Refactoring data types

Factor out shared structure

```
data Expr = Var Name
          | BinOp Op Expr Expr

data Op = Add | Sub | Mul
```

```
vars :: Expr -> [Name]
vars (Var x)        =
[x]
vars (BinOp _ l r) =
vars l ++ vars r

eval :: Env -> Expr -> Int
 eval m (Var x) = get x m
eval m (BinOp o l r) = op o (eval m l) (eval m r)
   where
  op Add = (+)
    op Sub = (-)
      op Mul =
```

Type inference

How to perform type inference (bottom-up strategy)

For each literal, data constructor, and named function: write down the type

Repeat until you know the type of the whole expression:

1. find an application $e_1\ e_2$ where you know $e_1 :: T_1$ and $e_2 :: T_2$
2. T_1 should be a function type $T_1 = T_{arg}$ -> T_{res}
3. check that the argument type is what the function expects: $T_{arg} =^? T_2 \sim \sigma$
   - this step is called **type unification**
   - σ is an assignment of the type variables to make the two sides equal
4. write down $e_1\ e_2 :: \sigma T_{res}$ ($\sigma T_{res} = T_{res}$ with type variables

Type unification (1/2)

Find a **type variable assignment** (σ) that makes the two sides **equal**

T_1	$\stackrel{?}{=}$	T_2	$\rightsquigarrow$	σ
`a`	$\stackrel{?}{=}$	`Int`	$\rightsquigarrow$	$\{$`a` $=$ `Int`$\}$
`Bool`	$\stackrel{?}{=}$	`a`	$\rightsquigarrow$	$\{$`a` $=$ `Bool`$\}$
`a`	$\stackrel{?}{=}$	`b`	$\rightsquigarrow$	$\{$`a` $=$ `b`$\}$
`Int`	$\stackrel{?}{=}$	`Bool`	$\rightsquigarrow$	Fail!
`a`	$\stackrel{?}{=}$	`Int -> Bool`	$\rightsquigarrow$	$\{$`a` $=$ `Int -> Bool`$\}$
`a ->`	$\stackrel{?}{=}$	`Int -> Bool`	$\rightsquigarrow$	$\{$`a` $=$ `Int`$\}$

Type unification (2/2)

Find a **type variable assignment** (σ) that makes the two sides **equal**

T_1	$\stackrel{?}{=}$	T_2	$\rightsquigarrow$	σ
`a -> b`	$\stackrel{?}{=}$	`Int -> Bool`	$\rightsquigarrow$	{`a` = `Int`, `b` = `Bool`}
`Int ->`	$\stackrel{?}{=}$	`b -> Bool`	$\rightsquigarrow$	{`a` = `Bool`, `b` = `Int`}
`a` `a ->`		`Int`	$\rightsquigarrow$	Fail!
`b`	$\stackrel{?}{=}$	`Int -> Bool -> Char`	$\rightsquigarrow$	{`a` = `Int`, `b` = `Bool -> Char`}
`a ->`		`(Int -> Bool) -> Char`	$\rightsquigarrow$	{`a` = `Int -> Bool`, `b` = `Char`}
`b` `a`	$\stackrel{?}{=}$	`Int -> Bool -> Char`	$\rightsquigarrow$	Fail!
`-> b`				
`(a -> b) ->`	$\stackrel{?}{=}$			

Exercises

Give

```
data Maybe a = Nothing | Just a
 gt          :: Int -> Int ->
Bool
map  :: (a -> b) -> [a] -> [b]
(.)  :: (b -> c) -> (a -> b) -> a ->
c
not          ::
Bool -> Bool
even :: Int -> Bool
```

1. `Just`
2. `not even 3`
3. `not (even 3)`
4. `not . even`
5. `even . not`
6. `map (Just . even)`