

# In The Name of God

## Statistical Inference HW#3

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### Problem 1

- a. False, because standard deviation of original sample is influenced by skewness but the standard deviation of the bootstrap distribution is not, because bootstrap distribution is closer to normal.
- b. False, in order to have better representativeness, it is better to use sample size larger than the size of original sample. We often do a resample of 10000 when the size of the original sample may be very small.
- c. False, it is created by resampling(with replacement) from the sample data, not from the population.
- d. True
- e. True
- f. True
- g. True
- h. True
- i. False, if beta is larger ---> alpha is smaller and thus the power will be smaller(because we will reject less often) ---> type II error will be larger so we must choose smaller beta.

## Problem 2

Checking Conditions:

- Randomness  $\rightarrow$  Independence,
- $n = 25 < 30 \rightarrow$  t-test

a)

$n = 25 < 30 \rightarrow$  we use t-test

$$H_0: \mu = 24.3$$

$$H_A: \mu \neq 24.3$$

$$\bar{x} = 25.02$$

$$s = 1.34$$

$$SE = \frac{s}{\sqrt{n}} = \frac{1.341}{\sqrt{25}} = \frac{1.341}{5} = 0.26$$

$$T = \frac{\bar{x} - \mu}{SE} = \frac{25.02 - 24.3}{0.26} = 2.71$$

$$p\text{-value} = 2 \times pt(2.71, df = 25 - 1, \text{lower.tail} = \text{False})$$

$$\Rightarrow p\text{-value} \approx \boxed{0.0121 < 0.05} \rightarrow \text{reject } H_0$$

b)

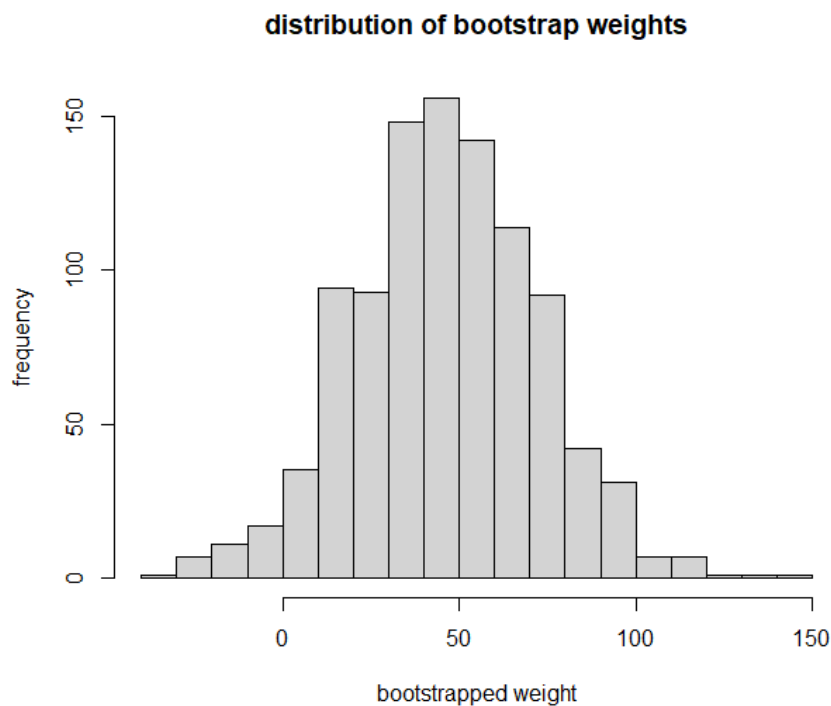
$$\text{interval: } \bar{x} \pm SE \times t_{24, 0.975}^* = 25.02 \pm 0.26 \times 1.95$$

$$\rightarrow \text{interval} = (24.47413, 25.58187)$$

## Problem 3

a.

```
1 #a
2 num_reps = 1000
3 data = read.csv('Chick.csv')
4 s1 <- data[data$feed == 'casein', 'weight']
5 s2 <- data[data$feed == 'meatmeal', 'weight']
6
7 n1 <- length(s1)
8 n2 <- length(s2)
9
10 boot_dist = c()
11 boot_stats = c()
12 for(i in (1:num_reps)) {
13   smp1 = sample(s1, n1, rep=T)
14   smp2 = sample(s2, n2, rep=T)
15   diff_of_means = mean(smp1)-mean(smp2)
16   smp_SE = sqrt(var(smp1)/n1 + var(smp2)/n2)
17   boot_dist = c(boot_dist, diff_of_means)
18   boot_stats = c(boot_stats, diff_of_means / smp_SE)
19 }
20
21 hist(boot_dist, breaks = 20)
```



**Result:** We can see from the distribution that it is close to normal and is roughly symmetric.

b.

```

23 #b
24 #calculating the pvalue with original sample
25 p.value.original = t.test(s1, s2)$p.value
26 cat("p-value with original sample =", p.value.original, '\n')
27
28 #calculating the pvalue with bootstrap samples
29 orig.statistic <- t.test(s1, s2)$statistic
30 p.value.bootstrap = mean(boot_stats - mean(boot_stats) > orig.statistic)
31 cat("p-value with bootstrap samples =", p.value.bootstrap, '\n')
32
22:1 (Top Level)

```

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**Console**   **Terminal** ×   **Jobs** ×

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```

> source('E:/University/fall-2020/Spring 00/SI/hws/hw4/3.R')
p-value with original sample = 0.09866222
p-value with bootstrap samples = 0.064

```

**Result:** As we can see here, the p-values are close. P-value with bootstrap samples is a bit less than the p-value of the original.

C.

```
33 #c
34 cat("confidence interval with original sample =", t.test(s1, s2, conf.int=95)$conf.int, '\n')
35 cat("confidence interval with bootstrap sample =", quantile(boot_dist, c(.025,.975)), '\n')
36
37
```

39:1 (Top Level) ↕ R Scr

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confidence interval with original sample = -9.504377 102.8529  
confidence interval with bootstrap sample = -6.242992 97.73731

**Result:** We can see here that the confidence interval with the bootstrap samples is smaller than the confidence interval with the original sample. This means that the result of the bootstrapping is more accurate.

## Problem 4

Checking Conditions:

- Randomness  $\rightarrow$  Independence,
- $n_1, n_2 < 30 \rightarrow$  t-test

$$n_1 = 10 \quad n_2 = 13 \quad n_{1,2} < 30 \rightarrow \text{t-test}$$

$$df = n_1 + n_2 - 2 = 21$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_A: \mu_1 - \mu_2 \neq 0$$

$$\bar{x}_1 - \bar{x}_2 = 7.34$$

$$s_2 = 6.84$$

$$s_1 = 5.31$$

$$SE = \sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1 + n_2 - 2} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$$
$$= 2.62$$

$$T = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SE}$$
$$= \frac{7.34}{2.62} = 2.8$$

$$t_{21, 0.025} = 2.07$$

$$\rightarrow T > t_{21, 0.025}$$

$\rightarrow$  reject  $H_0$

$\downarrow$   
there is difference in the means  
of two groups

If we didn't use pooled version of SE:

$$df = \min(13-1, 10-1) = 9$$

$$SE = 2.53$$

$$T = 2.9$$

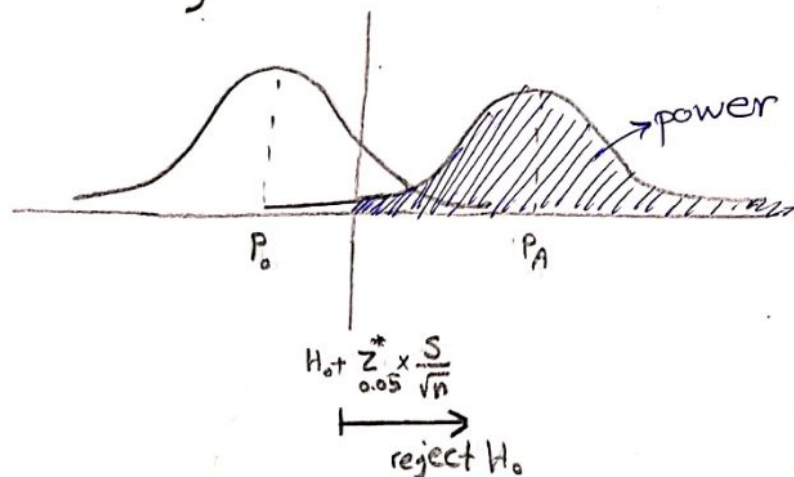
$$T_{9, 0.025} = 2.26$$

$$T < T_{9, 0.025} \Rightarrow \text{reject } H_0$$

## Problem 5

$H_0: p_{\text{yes}} = 0.3$   
 $H_A: p_{\text{yes}} > 0.3$

$\rightarrow$  power = probability of rejecting  $H_0$  if  $H_A$  is true



$$\begin{aligned}
 \rightarrow \text{power} &= P\left(\bar{x} > P_0 + 1.64 \times \frac{S}{\sqrt{n}}\right) \\
 &= P\left(\frac{\bar{x} - P_A}{\frac{S}{\sqrt{n}}} > \frac{P_0 + 1.64 \frac{S}{\sqrt{n}} - P_A}{\frac{S}{\sqrt{n}}}\right) \\
 &= P\left(Z > 1.64 - \frac{P_A - P_0}{\frac{S}{\sqrt{n}}}\right)
 \end{aligned}$$

we must maximize  $\frac{P_A - P_0}{\frac{S}{\sqrt{n}}} \rightarrow \begin{cases} P_A - P_0 \uparrow \\ n \uparrow \end{cases}$

$\rightarrow n = 200$   
 $p = 50\% \rightarrow \textcircled{d} \checkmark$

The larger  
 the sample  
 size and  
 actual mean  
 $\downarrow$   
 the larger  
 the power



## Problem 6

Checking Conditions:

- Randomness  $\rightarrow$  Independence,
- $n > 30 \rightarrow$  z-test

a.

$n = 55 \rightarrow$  z test

$$\begin{array}{l|l} H_0: \mu = 7.5 & z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{7.22 - 7.5}{\frac{1.6}{\sqrt{55}}} = \frac{-0.28}{\frac{1.6}{\sqrt{55}}} = -1.29 \\ H_A: \mu < 7.5 & \\ \bar{x} = 7.22 & \text{pvalue} = 0.09 \rightarrow \text{pvalue} > 0.05 \\ \alpha = 0.05 & \\ s = 1.6 & \rightarrow \text{we can not reject } H_0 \end{array}$$

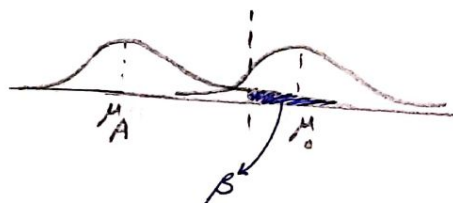
b.

$$p(\text{type II error}) = P(\text{not reject } H_0 \mid \mu_A = 7.4)$$

$$= P\left(x > \mu_0 - Z_{0.05}^* \frac{\sigma}{\sqrt{n}}\right)$$

$$= P\left(z > \frac{\mu_0 - 1.64 \times \frac{1.6}{\sqrt{55}} - \mu_A}{\frac{1.6}{\sqrt{55}}}\right)$$

$$= P\left(z > \frac{7.5 - 7.4}{\frac{1.6}{\sqrt{55}}} - 1.64\right) = \underline{\underline{0.8803}}$$



## Problem 7

a.

$H_0 : \mu_1 = \mu_2 = \mu_3 \Rightarrow$  The average score is the same across all three groups

$H_A$  : The average score differ between at least one pair of groups

b.

$$\begin{aligned}
 SST &= \sum_{i=1}^{18} (y_i - \bar{y})^2 = 972311.1 \\
 SSG &= \sum_{j=1}^K n_j (\bar{y}_j - \bar{y})^2 = 152477.8 \rightarrow MSG = \frac{SSG}{2} = 76238.9 \\
 SSE &= SST - SSG = 819833.3 \rightarrow MSE = \frac{SSE}{15} = 54655.55 \\
 F &= \frac{MSG}{MSE} = 1.394898 \\
 &\rightarrow Pr(>F) \approx 0.278
 \end{aligned}$$

		Df	Sum Sq	Mean Sq	F value	Pr(>F)
Group	Class	3-1 = 2	152477.8	76238.9	1.394898	0.2782289
Error	Residuals	17-2 = 15	819833.3	54655.55		
	Total	18-1 = 17	972311.1			

c.

Because p-value is large(greater than 0.05) we fail to reject  $H_0$

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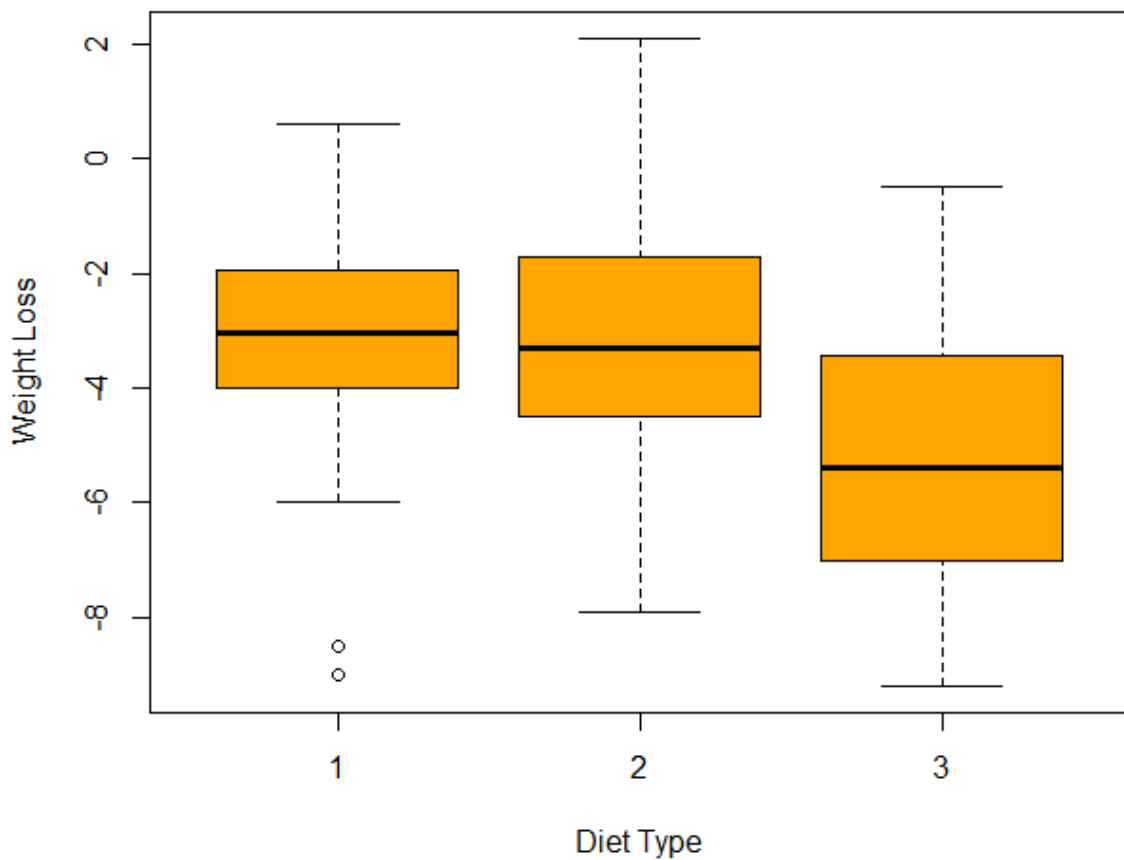
## Problem 8

a.

```
1 data = read.csv('Diet.csv')
2 data["weight_loss"] = data$weight6weeks - data$pre.weight
3
4 # a
5 boxplot(data$weight_loss ~ data$Diet,
6         col="orange",
7         main="Boxplot of the weight loss across the groups",
8         ylab="Weight Loss",
9         xlab="Diet Type")
10
```

Result:

**Boxplot of the weight loss across the groups**



**Result:** As we can see in the box plot, the means of groups 1 and 2 are very close to each other and they are different from the mean of group3

## 8. b,c

```
11 # b
12 anova.result <- aov(weight_loss ~ as.factor(Diet), data=data)
13
14 # c
15 summary(anova.result)
16
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
as.factor(Diet)	2	71.1	35.55	6.197	0.00323 **
Residuals	75	430.2	5.74		

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
> |

**Result:** As we can see, the F-value is equal to 0.00323 which is smaller than 0.05 so the null hypothesis is rejected. So as we expected, it indicates that there is some difference between some two groups.

d.

```
17 # d
18 TukeyHSD(anova.result)
19
```

17:1 (Top Level) ⚡

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```
> TukeyHSD(anova.result)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = weight_loss ~ as.factor(Diet), data = data)

$`as.factor(Diet)`
      diff      lwr      upr    p adj
2-1  0.2740741 -1.332467  1.8806155 0.9124737
3-1 -1.8481481 -3.454690 -0.2416067 0.0201413
3-2 -2.1222222 -3.680796 -0.5636481 0.0047819
```

## Result:

- Hypothesis for each two groups a and b is:
  - $H_0$ : The means of group a and b are equal
  - $H_A$ : There is a difference between means of group a and b
- Level of significance for all tests is  $\alpha = 0.05$ 
  - As we can see, the p-value of the comparison between groups 1-2 is large --->  $H_0$ : **Not rejected**
  - As we can see, the p-value of the comparison between groups 1-3 is less than 0.05 --->  $H_0$ : **rejected**
  - As we can see, the p-value of the comparison between groups 2-3 is less than 0.05 --->  $H_0$ : **rejected**

The results is in confirmation with the box plots---> we do have evidence for a significant difference between groups 1-3 and 2-3.

- As we can see in the pairwise comparison results, the estimated differences between means of groups 1-2 , 3-1, 3-2 is written in column “diff”

(We have been taught in the Statistical Inference class that alpha for pairwise comparison should be 0.05 divided by 3 but here it is 0.05)