In The Name of God

Statistical Inference

HW#3

Spring 1400

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Problem 1.

1

(a) True

because we reject if p.value $< \alpha$

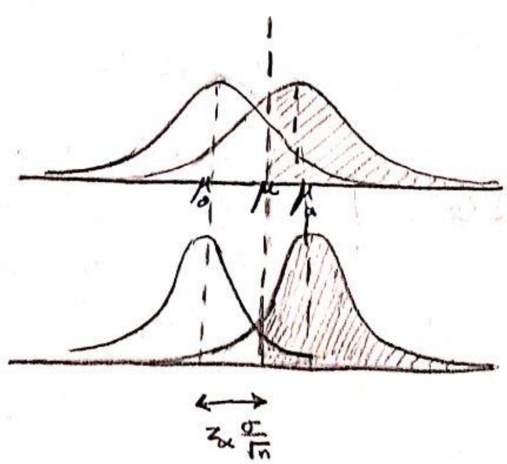
(b) False - If we decrease α then it will make it harder to reject H_0 $\rightarrow$ it will decrease probability of type 1 error

(c) False - We can only say that we can't reject H_0 so we don't know if it is actually true or not.

(d) True

(e) True

(f) False - when sample size is decreased, the true distribution and hypothesized (false) distribution of test statistic become more distinct, because they have less variance and hence they are narrower.



larger sample size $\downarrow$ ($\frac{\sigma}{\sqrt{n}}$) less variance $\downarrow$ more distinct distributions $\downarrow$ more power

(g) False, If the significance level of the test is decreased, then the H_0 will be rejected with lower probability no matter H_0 is actually true or false.

So by definition of power, power will also decrease.

(h) False, We can't evaluate the probability of trueness or falseness of H_0, we can only assume them and calculate conditional probabilities based on the assumptions.

(i) False, Power of test means the probability that H_0 is rejected correctly.

(j) False, when the test statistic falls into rejection rejion of the test, H_0 is rejected. If H_0 is actually false then there is no error. If H_0 is actually true, then type I error has occured.

(K) False, when shift from μ_0 and μ_a increases, the true and false (hypothesized) distributions are more distinct and hence $\underbrace{\text{type II error}}_{\beta}$ decreases and thus power $= 1-\beta$ increases.

(l) False, when σ decreases $\rightarrow$ the false and actual distributions are more distinct (less overlap) so β decreases and power $(1-\beta)$ increases.

Problem 2.

②

(a) Type I error : deciding that mean weight of potato chips is less than right value (H_0 is false), when actually mean weight is correct (H_0 is true)

(b) type II error : deciding that mean weight of potato chips is just right (H_0 is true), when actually mean weight is less (H_0 is false)

(c) type II error – because in this situation the customers get less chips than what they have paid for, and supplier decides to do nothing.

(d) type I error – because in this situation the supplier decides to put more chips in bags when it is not needed and it will hurt the supplier financially.

Problem 3.

Central Limit Theorem conditions are satisfied:

- Randomness ---> Independence,
- n = 50 > 30

a) $\bar{x} \pm \frac{\sigma}{\sqrt{n}} Z_{0.015} = 115 \pm \frac{10}{\sqrt{50}} \times 2.17 = (111.931, 118.069)$

b) "We are 97% confident that the found interval captures the true mean of lifetime of batteries, i.e I'm 97% confident that true average lift time is between 111.931 and 118.069"

c) I. True, because sample mean is 115 and 115 is between 112.23 and 117.77 so we are 100% confident.

II. False, this interval only tells us that if we take multiple samples of batteries, 95% of the samples will capture the true mean of the population.

III. False, factory's batteries' true average is fixed, it is not random variable, so it is either whitin this interval or not

IV. True

↳ If the mentioned sample is not from the market but from the factory product line ⟶ $\boxed{\text{False}}$

Problem 4.

Central Limit Theorem conditions are satisfied:

- Randomness ---> Independence,
- There are at least 10 successes and 10 failures in the sample(np , n(1-p) >= 10)

a) we can estimate population σ with sample $s = \sqrt{\hat{p}(1-\hat{p})}$

$$\hat{p} = \frac{4781}{10^5} \rightarrow \overset{SE}{\frac{s}{\sqrt{n}}} = \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{10^5}} = \sqrt{\frac{\frac{4781}{10^5} \times \frac{95219}{10^5}}{10^5}} \approx 67 \times 10^{-5}$$

$\rightarrow$ 95% confidence interval $= (\hat{p} - Z_{90\%} SE \, , \hat{p} + Z_{90\%} \cdot SE)$

$= (0.04781 - 1.64 \times 67 \times 10^{-5} \, , 0.04781 + 1.64 \times 67 \times 10^{-5})$

$= (0.04671 \, ; 0.04890)$

b) This in not plausible, because 0.05 doesn't fall into 90% confidence interval

Problem 5.

Central Limit Theorem conditions are satisfied:

- Randomness ---> Independence,
- N1 = 30, N2 = 32 ----> N1, N2 >= 30

a.

code:

```r
#here we create samples

men <- c(128.35, 160.34, 133.74, 138.12, 91.00, 97.43, 128.58, 148.78, 150.65, 110.96,
         135.7, 118.77, 147.1, 107.2, 122.46, 129.36, 158.14, 102.72, 136.59, 146.02, 105.88,
         111.24, 131.22, 124.6, 137.85, 136.46, 145.31, 166.71, 158.66, 108.63, 103.11, 149.29)
women <- c(116.62, 137.15, 106.07, 172.58, 151.33, 98.73, 136.11, 149.9, 140.8, 98.58,
         158.4, 97.97, 117.99, 126.53, 128.67, 126.57, 124.3, 120.39, 150.08, 143.05, 130.18,
         108.04, 136.39, 124.94, 136.86, 143.03, 128.58, 142.51, 151.68, 120.94)

#this function calculates variance of sample:
miu_var <- function(sample) {
  return(var(sample) / length(sample))
}

#this function prints the interval:
print_interval <- function(lower, upper) {
  cat("95% confidence interval is: (")
  cat(lower_bound, upper_bound)
  cat(')')
}

#here we find std of the difference of the means:
sigma <- sqrt(miu_var(men) + miu_var(women))

#finding point estimate:
point_estimate <- mean(men) - mean(women)

#finding the right and left decision boundaries:
lower_bound <- point_estimate - sigma * qnorm(0.975)
upper_bound <- point_estimate + sigma * qnorm(0.975)
print_interval(lower_bound, upper_bound)

################### Output ############################

# 95% confidence interval is: (-10.07359 9.094549)
```

output:

95% confidence interval is: (-10.07359   9.094549)

Interpretation:

Here I give two interpretations which are equivalent:

1. In repeated sampling, this method of finding 95% confidence intervals, produces intervals that capture the true difference of the means of the two groups in 95% of the samples.

2. We are 95% confident that this interval captures the true difference of the means of the two groups, i.e., We are 95% confident that the true difference of the means of the two groups is between -10.07359 and 9.094549.

b.

because 0 doesn't fall into the interval, we can't reject H0 so we don't have sufficient evidence to conclude that there is a difference between the means of the two groups.

Problem 6.

⑥ a) $H_0: \mu_0 = 28$

$H_A: \mu_A = \mu < 28$

$\alpha = 0.05$

$\bar{y} = 25.9 \quad s = 5.6$

p-value = $P(\bar{y} < 25.9 \mid H_0)$

$$Z = \frac{\bar{y} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{25.9 - 28}{\frac{5.6}{\sqrt{50}}} \approx -2.65$$

$\rightarrow$ p-value $= P(z < -2.65) \approx 0.004$

$\Rightarrow$ p-value $< \alpha \rightarrow$ Reject $H_0 \rightarrow$ Accept H_A

$\downarrow$

$\mu < 28$ ✓

b) $P(\beta) = P(\text{accept } H_0 \mid H_0 \text{ is false}, \mu = 27)$

$$= P\left(\frac{\bar{Y} - \mu_0}{\frac{5.6}{\sqrt{50}}} < \underset{-1.64}{Z_{0.05}} \;\middle|\; \bar{Y} \sim N\left(27, \frac{5.6}{\sqrt{50}}\right)\right)$$

$$= P(\bar{Y} \geq \underbrace{-1.64 \times 0.79 + 28}_{26.7} \mid \bar{y} \sim N(27, 0.79)) = P\left(\frac{\bar{y} - 27}{0.79} > \frac{\overset{-0.3}{\cancel{26.7 - 27}}}{0.79}\right)$$

$$= 1 - \text{pnorm}\left(\frac{-0.3}{0.79}\right) = 1 - 0.352 = 0.648$$

c) No, because in part a we rejected H_0 in favor of H_A and hence by definition we could have possibly made type I error, not type II error; type II error only occurs when we accept H_0 wrongly

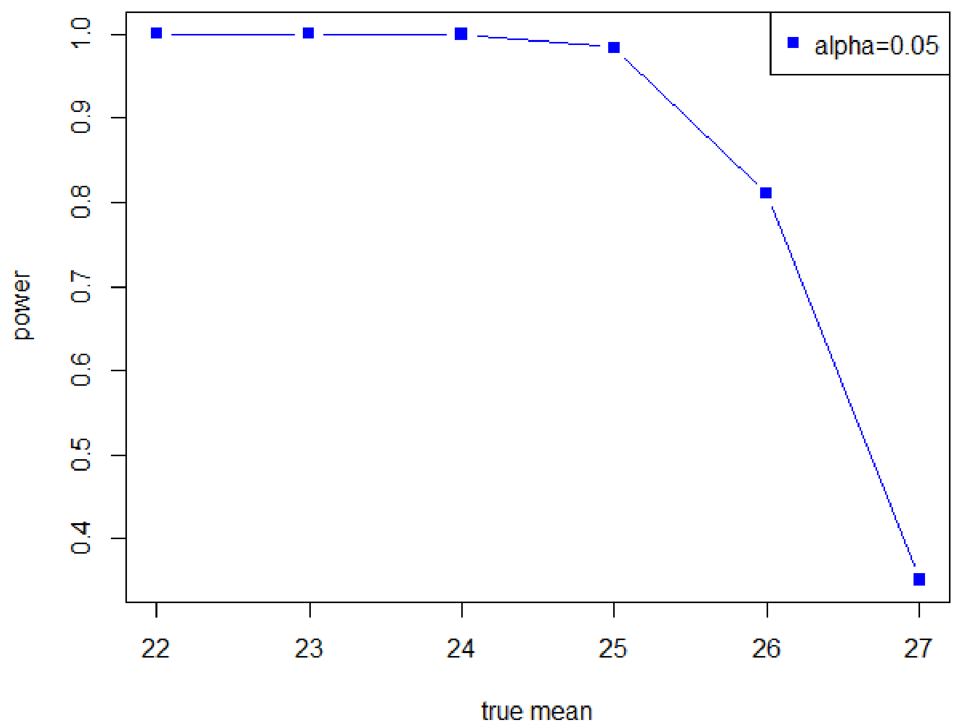
Problem 7.

Codes:

```
#################################################################
#a

miu_0 <- 28
miu_a <- c(22, 23, 24, 25, 26, 27)
s <- 5.6

calc_power <- function(alpha, n) {
  se <- s / sqrt(n)
  left_boundary <- miu_0 - qnorm(1-alpha)*se
  powers <- pnorm((left_boundary - miu_a) / se)
}

powers_a <- calc_power(0.05, 50)

plot(miu_a, powers_a, type = "b", col='blue', pch=15)

legend("topright", legend=c("alpha=0.05"),
       col=c("blue"), pch = 15)

title(xlab="true mean", ylab="power")
#################################################################
#b
powers_a <- calc_power(0.05, 50)
plot(miu_a, powers_a, type = "b", col='blue', pch=15)
powers_b <- calc_power(0.01, 50)
lines(miu_a, powers_b, col='red', lw=2, pch=17, type = "b")
legend("topright", legend=c("alpha=0.05", "alpha=0.01"),
        col=c("blue", "red"), pch = c(15,17))
title(xlab="true mean", ylab="power")
#################################################################
#c
powers_a <- calc_power(0.05, 50)
plot(miu_a, powers_a, type = "b", col='blue', pch=15)
powers_a <- calc_power(0.05, 20)
lines(miu_a, powers_a, col='green', lw=2, pch=19, type = "b")
legend("topright", legend=c("n=50", "n=20"), col=c("blue", "green"),
       pch = c(15, 19))
title(xlab="true mean", ylab="power")
```

a.

output:

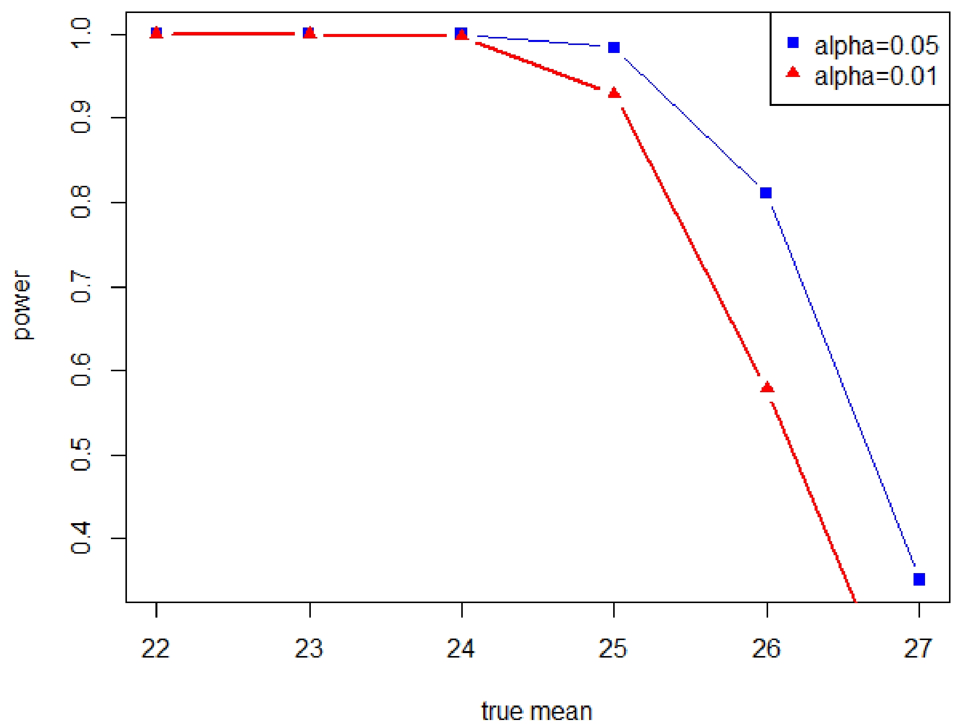


Interpretation:

The closer the true mean to the hypothesized mean(28) , the smaller the power. This is because when true mean is closer to the hypothesized mean, it is more difficult(has smaller probability) to reject H0 and hence by definition, power is smaller.

b.

output:

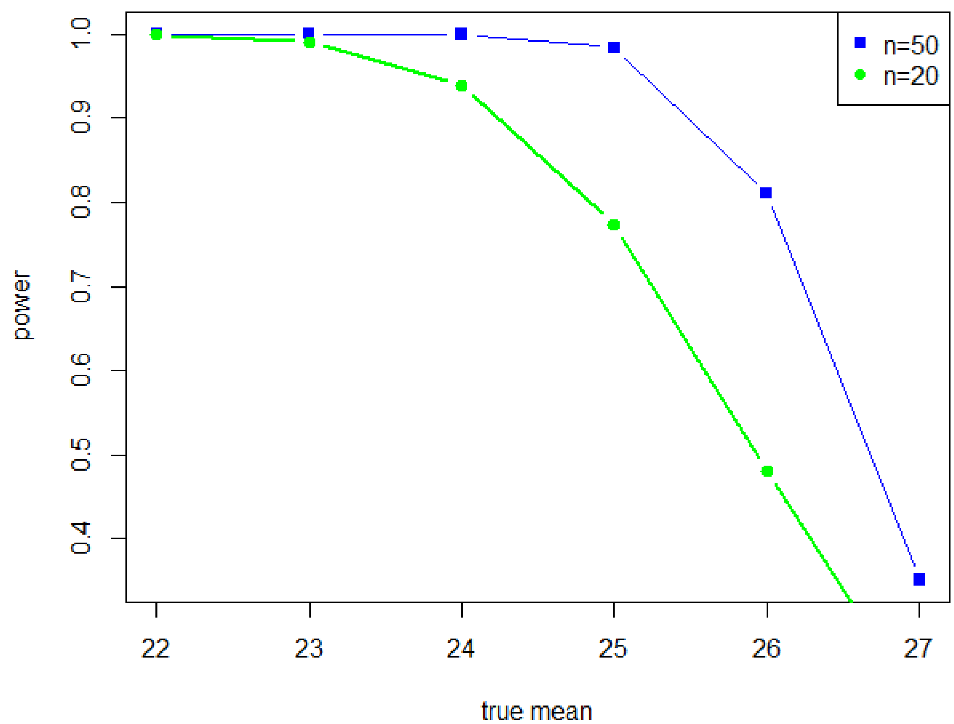


Interpretation:

When alpha is larger, power is also larger. This is because when alpha is larger, H0 is rejected with more probability, and hence by definition, power is larger.

c:

output:



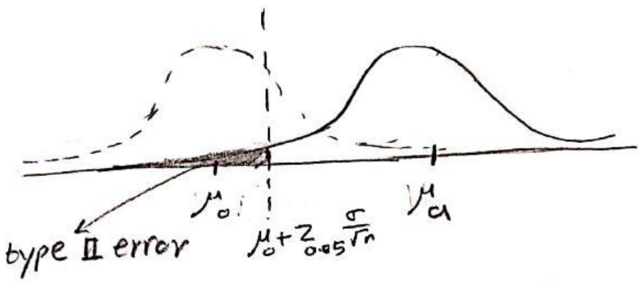
Interpretation:

Larger sample size ---> less variance ---> true and hypothesized distributions are more distinct ---> power is larger

Problem 8.

(8)

a)



$$P(\beta) \leq 0.025 \rightarrow P\left(Z \leq \frac{\mu_0 + Z_{0.05}\frac{\sigma}{\sqrt{n}} - \mu_a}{\frac{\sigma}{\sqrt{n}}}\right) \leq 0.025$$

$$\rightarrow pnorm\left(\frac{\mu_0 - \mu_a}{\frac{\sigma}{\sqrt{n}}} + 1.644\right) \leq 0.025$$

$$\rightarrow \frac{(525 - 550)\sqrt{n}}{80} + 1.64 \leq qnorm(0.025) = -1.959$$

$$\rightarrow \sqrt{n} \geqslant \frac{(1.64 + 1.96)25}{80} \rightarrow n \geqslant 133.06$$

$$\rightarrow \boxed{n \geqslant 134}$$

b)

$$\text{p-value} = P\left(Z > \frac{\bar{Y} - \mu_a}{\frac{\sigma}{\sqrt{n}}}\right) = 1 - pnorm\left(\frac{542 - 525}{\frac{76}{10}}\right)$$

$$= 1 - pnorm(2.24) \approx 0.012 < \alpha$$

$$\downarrow$$

$$\boxed{\text{reject } H_0}$$

we have sufficient evidence
to conclude the the mean has increased

Problem 9.

(9) first group : $\hat{p}_1 = \frac{81}{744} \approx 0.108$ $\quad \sigma_1^2 = \frac{\hat{p}_1(1-\hat{p}_1)}{n_1} \approx 0.00013$

second group : $\hat{p}_2 = \frac{298}{3055} \approx 0.097$ $\quad \sigma_2^2 = \frac{\hat{p}_2(1-\hat{p}_2)}{n_2} \approx 2.881$

using unpooled variance :

$$SE = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.012$$

$H_0 : \hat{p}_1 - \hat{p}_2 = 0$

$H_a = \hat{p}_1 - \hat{p}_2 > 0$

sample: $\hat{p}_1 - \hat{p}_2 = 0.011$

$$\text{p-value} = P\left(Z > \frac{\hat{p}_1 - \hat{p}_2 - 0}{SE}\right)$$

$$= P\left(Z > \frac{0.011}{0.012}\right) = P(Z > 0.897)$$

$$= 1 - \text{pnorm}(0.897) = \boxed{0.184 > 0.05}$$

we fail to reject H_0

using pooled variace

$$\hat{p}_{Pooled} = \frac{\overset{\hat{p}_1 n_1}{81} + \overset{\hat{p}_2 n_2}{298}}{\underset{n_1}{744} + \underset{n_2}{3055}} \approx 0.099$$

$$SE = \sqrt{\hat{p}_{pooled}(1-\hat{p}_{pooled})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{0.099 \times 0.900 \times \left(\frac{1}{744}+\frac{1}{3055}\right)}$$

$$\approx 0.012$$

Fail to reject H_0 ⟵ so it is the same as unpooled version