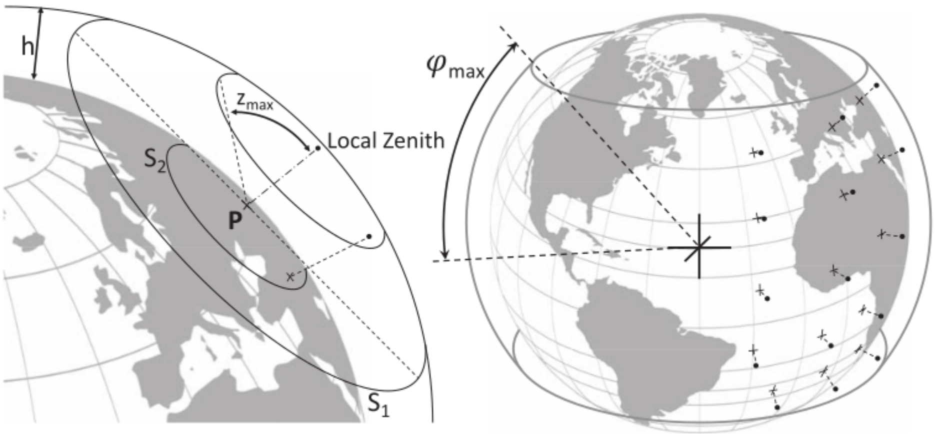
Personal notes

March 7, 2022

1 Derivations of Ragazzoni 2020

1.1 Geometry

Satellite orbital plane has inclination $\phi_{\max}$ wrt equator, i.e. $\phi_{\max}$ is the highest latitude on the surface where the satellite passes at the zenit.



The height above the surface is h and the Earth radius R.

The surface wrapped by the satellite around the earth is a spherical zone area = $2\pi \times$ radius $\times$ height of area

$$S = 2\pi(R+h) \times 2(R+h)\sin\phi_{\max} = 4\pi(R+h)^2 \sin\phi_{\max} \tag{1}$$

Satellites above the horizon. An observer in a point P well inside the area wrapped by the satellite will see a limited of the total surface, i.e. the spherical cusp given by the intersection of the satellite are with the horizon plane in P

$$S_1 = 2\pi(R+h)h \tag{2}$$

For a population of satellites that share the same $\phi_{\max}$ an h the fraction of satellites above the horizon is

$$\eta_1 = \frac{S_1}{S} = \frac{1}{2\sin\phi_{\max}}\frac{h}{R+h} \tag{3}$$

In the limit of low-altitude orbit $h \ll R$ and

$$\eta_1 \approx \frac{1}{2\sin\phi_{\max}}\frac{h}{R} \tag{4}$$

Satellites close to the zenit. In the typical case we are interested on satellite with a zenital angle $z < z_{\max}$. The relative fraction of satellite close to the zenit in low altitude regime is only

$$\eta_2 \approx \frac{1}{2 \sin \phi_{\max}} \frac{h^2}{R^2} \tan^2 z_{\max} \tag{5}$$

i.e. drastically reduced.

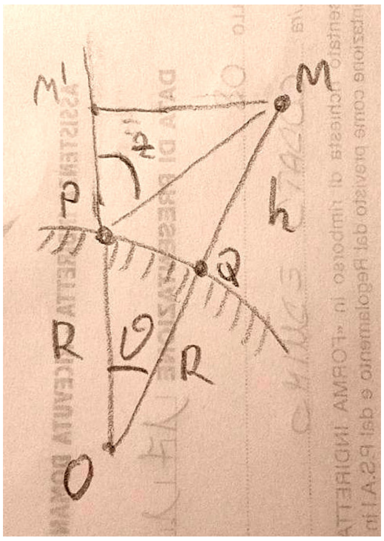
Satellite visibility. To be visible a satellite must be illuminated by the Sun. Relation between h and the satellite-earth-sun phase angle Ψ is

$$\cos \Psi = \frac{R}{R+h} \tag{6}$$

2 Equivalent for meteors

2.1 Geometry/visibility

A meteor has an height h above the point Q on the (spherical) surface of the Earth. The



angular separation between Q and the observing site P is θ. O is the center of the Earth, M the position of the meteor and M' its projection over the vertical of P. We have

$$\begin{aligned} MM' &= (R+h)\sin\theta \\ PM' &= (R+h)\cos\theta - R \end{aligned} \tag{7}$$

between the observed angular zenit distance z and the other parameters is

$$\tan z = \frac{(R+h)\sin\theta}{(R+h)\cos\theta - R} \tag{8}$$

To be meaningful, the denominator at the right hand side must be positive, i.e.

$$\cos\theta \geq \frac{R}{R+h} \tag{9}$$

which set the visibility limit in terms of phase angle between the meteor and the observed, wrt the Earth. Note when $\cos\theta \to R/(R+h)$ then $\tan z \to \pi/2$ i.e. the meteor is observed at the horizon.

Low altitude approximation. The formula above becomes

$$\tan z \approx \frac{\sin\theta}{\cos\theta - 1} = \tan\left(\frac{\theta}{2}\right) \rightarrow \ z \approx \theta/2 \tag{10}$$

in fact if $(R+h) \approx R$ we can act as if earth surface and the spherical surface at the height of the meteor were the same one. Then the z is a circumference angle while θ is a center angle insisting on the same arc $\overparen{PQ}$ so they will be one half of the other. The visibility limit becomes

$$\cos\theta \geq \left(1 + \frac{h}{R}\right)^{-1} \approx 1 - \frac{h}{R} \tag{11}$$

Ablation typically starts at $80-90\,\text{km}$ [Ceplecha+98] so assuming $(h+R) \approx R$ is legit at first approximation.

3 Kinematics and coordinate conversion