# Personal notes

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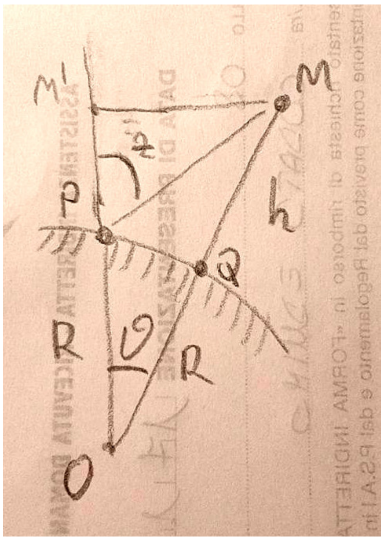
## 1 Equivalent for meteors of Ragazzoni 2020

### 1.1 Geometry/visibility

A meteor has an height $h$ above the point $Q$ on the (spherical) surface of the Earth. The




angular separation between $Q$ and the observing site $P$ is $\theta$. $O$ is the center of the Earth, $M$ the position of the meteor and $M'$ its projection over the vertical of $P$. We have

$$\begin{aligned} MM' &= (R+h)\sin\theta \\ PM' &= (R+h)\cos\theta - R \end{aligned} \tag{1}$$

between the observed angular zenit distance $z$ and the other parameters is

$$\tan z = \frac{(R+h)\sin\theta}{(R+h)\cos\theta - R} \tag{2}$$

To be meaningful, the denominator at the right hand side must be positive, i.e.

$$\cos\theta \geq \frac{R}{R+h} \tag{3}$$

which set the visibility limit in terms of phase angle between the meteor and the observed, wrt the Earth. Note when $\cos\theta \to R/(R+h)$ then $\tan z \to \pi/2$ i.e. the meteor is observed at the horizon.

**Low altitude approximation.** The formula above becomes

$$\tan z \approx \frac{\sin\theta}{\cos\theta - 1} = \tan\left(\frac{\theta}{2}\right) \rightarrow \ z \approx \theta/2 \tag{4}$$

in fact if $(R+h) \approx R$ we can act as if earth surface and the spherical surface at the height of the meteor were the same one. Then the $z$ is a circumference angle while $\theta$ is a center angle insisting on the same arc $\widehat{PQ}$ so they will be one half of the other. The visibility limit becomes

$$\cos\theta \geq \left(1 + \frac{h}{R}\right)^{-1} \approx 1 - \frac{h}{R} \tag{5}$$

Ablation typically starts at $80-90\,\mathrm{km}$ [Ceplecha+98] so assuming $(h+R) \approx R$ is legit at first approximation.

# 2 Motion of meteors

## 2.1 Intrinsic motion

**Shape of the trajectory.** Before entering the atmosphere the motion of a meteoroid consist on its intrinsic solar orbital motion + acceleration due to Earth attraction. Typical velocity ranges from 11.2 km/s (pure Earth attraction) to 72.8 km/s (solar + Earth attraction).

During the ablation the trajectory is a straight line, because we neglect

- the effect of gravitational attraction of the Earth, which is much smaller than the air drag exerted by the atmosphere;
- the effect of the curvature of the planet. This is because during the burn meteors covers hundreds of km $\ll R$.

In general the motion along this straight line is not uniform as the meteoroid is being slew down by air drag.

**Equation of motion.** Motion on the trajectory is described by the set of differential equation [Cepleca+98]

$$\begin{aligned} a &= \frac{\mathrm{d}v}{\mathrm{d}t} = -\Gamma A \rho_m^{-2/3} \rho m^{-1/3} v^2 \\ \dot{m} &= \frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{\Lambda A}{2\xi} \rho_m^{-2/3} m^{2/3} v^3 \end{aligned} \tag{6}$$

where

- $\Gamma$ is the drag coefficient (fraction of momentum transferred to the air from the body, ranges from 0 for no exchange of momentum, to 2 for a perfectly elastic impact),
- $\Lambda$ the heat transfer coefficient (fraction of kinetic energy converted into heat),
- $\xi$ the energy for the ablation of a unit mass,
- $A = Sm^{-2/3}\rho_m^{2/3} = S/V^{2/3}$ the shape factor ($S$ the cross-section, for cube $A = 1$, for a sphere $A \approx 1.2$, getting more elongated toward the direction of motion makes $A$ grow),

- $\rho_m$ the bulk density,
- $m$ the mass and
- $v$ the velocity.

Usually independent parameters are gathered in the two terms [Cepleca+98]

$$\begin{aligned} \sigma &= \frac{\Lambda}{2\xi\Gamma} \\ K &= \Lambda A \rho_m^{-2/3} \end{aligned} \tag{7}$$

respectively the ablation and the shape-density coefficients. Motion equation become [adapted from Cepleca+98]

$$\begin{aligned} a &= -K\rho m^{-1/3} v^2 \\ \dot{m} &= -K\sigma m^{2/3} v^3 \end{aligned} \tag{8}$$

**Dependence on the height.** Note that $\rho = \rho(h) = \rho[h(t)]$, i.e. we need a further relation to express the variation of atmospheric density as a function of the height. A good approximation of the atmospheric density profile is

$$\rho = \rho_0 e^{-h/H} \tag{9}$$

where the scale height $H$ is

$$H = \frac{P_0}{g\rho_0} \sim 8\,\mathrm{km} \tag{10}$$

with $P$ the pressure. The pedix represents the value at the sea level, for $h = 0$.

We introduce the third equation of motion

$$v_h = \frac{\mathrm{d}h}{\mathrm{d}t} = \cos(z_0)\, v \tag{11}$$

where $z_0$ is the zenit angle of the meteor radiant (i.e. inclination of the trajector wrt the local vertical).

**Final equations.** The final set of ODE that solve the motion of a meteor is

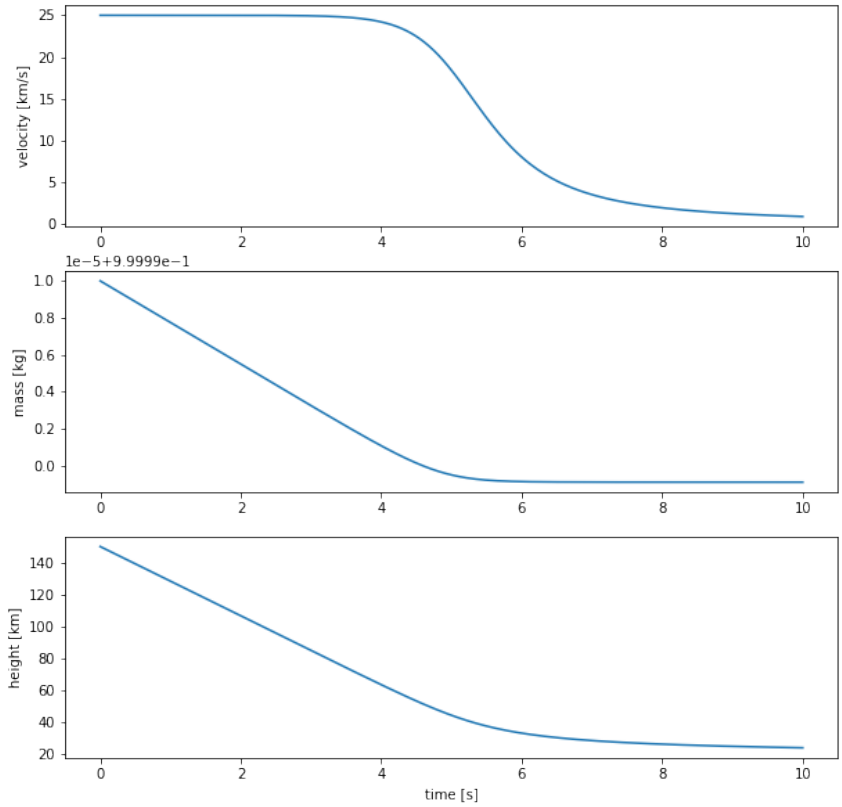
$$\begin{aligned} a &= -K\rho_0 e^{-h/H} m^{-1/3} v^2 \\ \dot{m} &= -K\sigma m^{2/3} v^3 \\ v_h &= \cos z_0\, v \end{aligned} \tag{12}$$

with $K$ and $\sigma$ that comes from the properties and composition of the body, $z_0$ from the pre-atmospheric motion while $\rho_0$ and $H$ are two constants dependent on the atmospheric model.

Note we assumed, more or less implicitly:

- straight-line trajectory (see above);
- constant inclination $z_0$ (direction of the vertical does not changes significantly below the short meteor path);
- $\sigma$ and $K$ constant. It is assumed that the shape and properties does not change significantly during ablation.

**Solution.** An example for $\rho_m = 3000\,\mathrm{kg/cm^3}$, $\Lambda = 1$, $A = 1$, $\sigma = 0.03\,\mathrm{s^2/km^2}$ and $z_0 = 30°$. Initial condition velocity 25 km/s, mass 1.0 kg and 150 km of altitude. Integration over 10 s, 300 time-steps.




## 2.2 Apparent motion

Due to the short path and low altitude approximation, the motion can be naturally expressed using a rectangular coordinate system in the neighborhood of the meteor, i.e. assuming locally the Earth to be flat right below the meteor.

An object at height $h$ can actually be observed up to a distance $\approx \sqrt{2Rh}$ (in low altitude approximation) from its projection on the ground. In the case of meteors with height up to a hundred of km, they can be observed up to $10^3$ km, which is no longer negligible wrt the curvature.

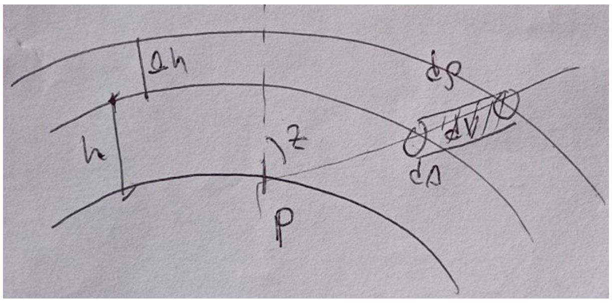
If $\theta$ is the angular separation between the vertical of the meteor and the observer the apparent inclination of the meteor is $z = z_0 - \theta$

# 3 Meteor background – simplified

**Procedure.** A tentative approach.

1. For [Cook+78] the meteor flux through the atmosphere respect

$$\log \Phi = -a + bM = -17.89 + 0.534M \tag{13}$$




where $\Phi$ is the number flux (meteor/cm$^2$/s) with an absolute magnitude lower (i.e. brighter) than $M$.

2. Assume all the meteors burn in a shell of thickness $\Delta h$ at average height $h$.

3. From a point $P$ is observed at direction $z$, wrt the vertical, through the shell. The volume of the shell column of unit area d$A$ observed from $P$ is

$$\mathrm{d}V = \mathrm{d}A\mathrm{d}\rho \approx \mathrm{d}A\frac{\Delta h}{\cos z} \tag{14}$$

4. We pass from the cumulative flux $\Phi$ to the differential count $\varphi$ (number of meteor per surface and time unit with a magnitude range between $M$ and $M + \mathrm{d}M$)

$$\varphi(M) = \frac{\mathrm{d}\Phi}{\mathrm{d}M} = \frac{\mathrm{d}}{\mathrm{d}M}10^{-a+bM} = 10^{-a}\ln 10 b 10^{bM} \tag{15}$$

The flux per volume unit $\psi$ will have the same expression but different units.

5. The total number of object of magnitude $M$ seen under a unit area d$A$ from $P$ is

$$\mathrm{d}N = \psi(M)\mathrm{d}V = \psi\mathrm{d}A\frac{\Delta h}{\cos z} \tag{16}$$

6. The intensity of a single object with magnitude $M$ is

$$I = 10^{-0.4M} \tag{17}$$

so the total intensity from the volume d$V$ is

$$I_{\mathrm{tot}} = \int_{M_{\mathrm{min}}}^{M_{\mathrm{max}}} \mathrm{d}NI\mathrm{d}M = \mathrm{d}A\frac{\Delta h}{\cos z}\frac{b}{b-0.4}10^{-a+(b-0.4)M_{\mathrm{max}}} \tag{18}$$

we can set $M_{\mathrm{min}} \to -\infty$ and have no problems with the boundaries since $b > 0.4$ and the exponential will tend to 0, when evaluated at $-\infty$.

7. According to [Blaauw+16] the relation between absolute magnitude and mass is

$$M = -8.75\log(v_\infty) - 2.25\log m + 11.59 \tag{19}$$

with velocity in m/s and mass in grams. In particular the maximum magnitude (faintest meteor) is the one with smallest mass and lowest pre-atmospheric velocity.

A conservative approach may consist on considering $v_{\infty,\mathrm{min}} = 11.2\,\mathrm{km/s}$ (escape velocity) and as $m_{\mathrm{min}}$ the one associated to the smallest meteoroids. Conventionally the boundary between dust and meteoroid is for a size of $30\,\mu\text{m} = 3 \times 10^{-5}$ m, the boundary under which the body is small enough to dissipate the heat due to atmospheric friction without vaporizing. The equivalent volume, assuming a rocky density of $3\,\text{g/cm}^3$ is

$$m_{\text{min}} \approx (3 \times 10^{-5})^3 \cdot 3000 \sim 10^{-7}\,\text{g} \tag{20}$$

The final value for the maximum magnitude is (conservatively) $M_{\text{max}} = 18.37$ mag.

8. The total magnitude observed from the column will be

$$M_{\text{tot}} = -2.5 \log I_{\text{tot}} \tag{21}$$

9. An observer at 100 km from the column will see $M = m$ by definition. At this distance it will observe the surface area $\mathrm{d}A$ under the solid angle

$$\mathrm{d}\Omega \cong \frac{\mathrm{d}A}{\rho^2} 206265^2 = 7.037 \times 10^6\,\mathrm{d}A \tag{22}$$

with $\mathrm{d}A$ in km$^2$ units and $\mathrm{d}\Omega$ in arcsec$^2$.

10. By definition the surface brightness at 100 km is

$$S = M + 2.5 \log \Omega \tag{23}$$

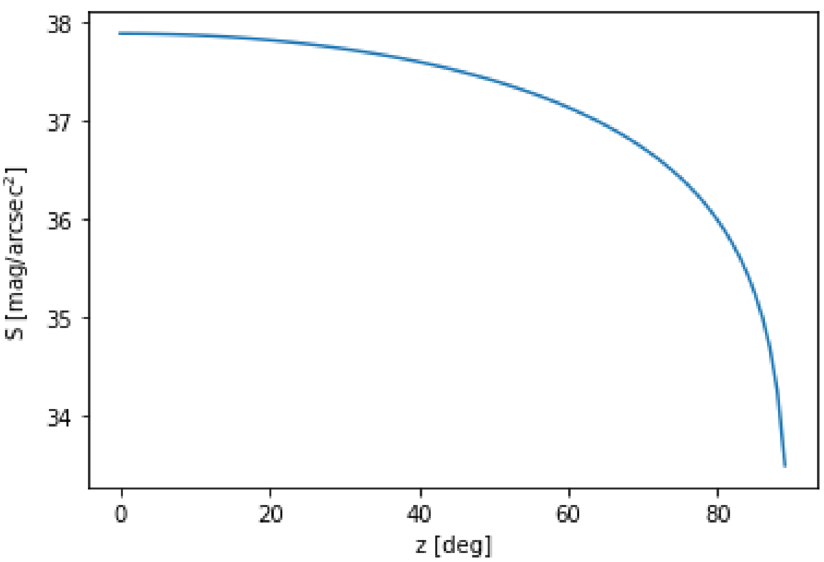
But since both the intensity and the solid angle scale with $1/\rho^2$ at the end $S$ will be independent on the distance, i.e. we can apply compute $S$ once.

In our case

$$S = -2.5 \log \Delta h + 2.5 \log \cos z + 2.5 \log \left( \frac{b}{b - 0.4} \right) + 2.5a - 2.5(b - 0.4) M_{\text{max}} - 4.69 \tag{24}$$

**Result.** If we assume $a$ and $b$ from [Cook+78] a thickness of the shell of $\Delta h = 20$ km and $M_{\text{max}}$ as above we have $S$ as a function of the zenit angle $z$

$$S(z) = 37.82 + 2.5 \log(\cos z) \tag{25}$$


38
37
36
35
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S [mag/arcsec$^2$]
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