1 Exercise 2

1.1 a) and b)

This section answers Question a) and b).
Rewrite the function: $\frac{1}{1+\exp(-z)} = (1+\exp(-z))^{-1}$
We can now calculate $(1+exp(-z))^{-1}$‘:
$(1+\exp(-z))^{-1}$‘ $= -1 \cdot (1+\exp(-z))^{-2} \cdot \exp(-z) \cdot (-1)$
=> $\frac{\exp(-z)}{(1+\exp(-z))^2} = \frac{1}{1+\exp(-z)} \cdot \frac{\exp(-z)}{1+\exp(-z)} = \sigma(z) \cdot \frac{\exp(-z)}{1+\exp(-z)}$
=> $\sigma(z) \cdot \frac{1+\exp(-z)-1}{1+\exp(-z)} = \sigma(z) \cdot (\frac{1+\exp(-z)}{1+\exp(-z)} - \frac{1}{1+\exp(-z)}) = \sigma(z) \cdot (1-\sigma(z))$ qed.

Ex 2 c)

$$f(z) = -\log(\sigma(-z))$$

$$f(z)' = -1 \cdot \frac{1}{\sigma(-z)} \cdot (\sigma(-z) \cdot (1 - \sigma(-z)) \cdot -1$$

$$= \quad +1 \cdot \frac{1-\sigma(-z)}{1}$$

$$= \sigma(-z) \quad 1$$

$$= \frac{1}{1-e^z} - 1 = \frac{1}{1-e^z} - \frac{1-e^z}{1-e^z} = \frac{\cancel{1-e^z}}{1+e^z} \quad \frac{-e^z}{1-e^z}$$

$$= e^z$$

$$f(z)' = -1 \cdot \frac{1}{\sigma(-z)} \cdot \sigma(-z) \cdot (1 - \sigma(-z)) \cdot (-1)$$

$$= 1 - \sigma(-z)$$

$$= 1 - \frac{1}{1+e^z}$$

$$= \frac{1+e^z}{1+e^z} - \frac{1}{1+e^z} = \frac{e^z}{1+e^z}$$

$$\sigma(-z) = \frac{1}{1+e^{-(-z)}} = \frac{1}{1+e^z}$$

$$\cancel{\log(x) = \frac{1}{x}}$$

$$\cancel{\sigma(z)' = \sigma(z) \cdot (1 - \sigma(z))}$$

$$\cancel{= \frac{1}{1+e^{-z}} \left(1 - \frac{1}{1+e^{-z}}\right)}$$

$$f''(z) = \left(\frac{e^z}{1+e^z}\right)'$$

$$= e^z \cdot \frac{1}{1+e^z}$$

$$= e^z \cdot (1+e^z)^{-1}$$

$$= e^z \cdot (1+e^z)^{-1} + e^z \cdot (-1) \cdot (1+e^z)^{-2} \cdot e^z$$

$$= \frac{e^z}{1+e^z} - \frac{e^{2z}}{(1+e^z)^2}$$

$$=$$

d) - Jupyter

f)

$$\cancel{c_1(x) = (\sigma(x) - 1)(\sigma(x) - 1)}$$

$$\cancel{= (\sigma(x) - 1)' \cdot (\sigma(x) - 1) \cdot 2}$$

$$\cancel{= 1 \cdot \sigma(x)(1 - \sigma(x)) \cdot 2(\sigma(x) - 1) \cdot 2}$$

$$\cancel{= 2\,\sigma(x)(1 - \sigma(x)) \cdot (\sigma(x) - 1)}$$

Ex 2 g)

$$c_2(x) = -y \cdot \log(\sigma(\omega x)) + (1-y) \cdot \log(1-\sigma(\omega x))$$

$$-y \cdot \log(\sigma(\omega x))' = -y \cdot \frac{1}{\sigma(\omega x)} \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$= -y \cdot (1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$(1-y) \cdot \log(1-\sigma(\omega x))' = (1-y) \cdot \frac{1}{1-\sigma(\omega x)} (-1) \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$= -(1-y) \cdot \sigma(\omega x) \cdot \cancel{x}\omega$$

$$\Rightarrow c_2'(x) = -y(1-\sigma(\omega x)) \cdot \omega - (1-y) \cdot \sigma(\omega x) \cdot \omega$$

$$c_2''(x) = -y \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot (-1) \cdot \omega - (1-y) \cdot \sigma(\omega x) \cdot (1-\sigma(\omega x)) \cdot \omega$$

Ex 2f)

$$c_1(x)' = (\sigma(x)-1)\cdot(\sigma(x)-1))'$$

$$\xi(\sigma(x)-1)': \quad (\sigma(x)-1)' = \sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1) \quad \cdot 2$$

$$= 2\sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1)$$

$$c_1(x)'' = 2\cdot\sigma(x)(1-\sigma(x))(1-\sigma(x))\cdot(\sigma(x)-1) +$$

$$(-1)\cdot 2\ \sigma(x)\sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1) +$$

$$2\cdot\sigma(x)\cdot(1-\sigma(x))\cdot\sigma(x)(1-\sigma(x))$$

Ex 2g)

$$\cancel{-y\log(\omega x)+(1-y)\cdot\log(\omega x)}$$

$$\cancel{(-y\log\sigma(\omega x))' = -y\cdot\frac{1}{\sigma(\omega x)}}$$