

1 Exercise 2

1.1 a) and b)

This section answers Question a) and b).

Rewrite the function: $\frac{1}{1+\exp(-z)} = (1 + \exp(-z))^{-1}$

We can now calculate $(1 + \exp(-z))^{-1}$:

$$(1 + \exp(-z))^{-1} = -1 \cdot (1 + \exp(-z))^{-2} \cdot \exp(-z) \cdot (-1)$$

$$\Rightarrow \frac{\exp(-z)}{(1+\exp(-z))^2} = \frac{1}{1+\exp(-z)} \cdot \frac{\exp(-z)}{1+\exp(-z)} = \sigma(z) \cdot \frac{\exp(-z)}{1+\exp(-z)}$$

$$\Rightarrow \sigma(z) \cdot \frac{1+\exp(-z)-1}{1+\exp(-z)} = \sigma(z) \cdot \left(\frac{1+\exp(-z)}{1+\exp(-z)} - \frac{1}{1+\exp(-z)} \right) = \sigma(z) \cdot (1 - \sigma(z)) \text{ qed.}$$