DeLearn Week 2 PW – Christian Renold, Group 13

Ex. 2 a) $\sigma(z) = \frac{1}{1+e^{-z}} = \left(1+e^{-z}\right)^{-1}$

$$\sigma'(z) = -1 \cdot \left(1+e^{-z}\right)^{-2} \cdot -e^{-z}$$

b)

$$= \frac{(-1) \cdot -e^{-z}}{\left(1+e^{-z}\right)^2} = \frac{e^{-z}}{(1+e^{-z})(1+e^{-z})}$$

$$= \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} = \frac{1}{1+e^{-z}} \cdot \frac{1+e^{-z}-1}{1+e^{-z}}$$

$$= \frac{1}{1+e^{-z}} \cdot \frac{1+e^{-z}}{1+e^{-z}} - \frac{1}{1+e^{-z}}$$

$$= \frac{1}{1+e^{-z}} \cdot \left(1 - \frac{1}{1+e^{-z}}\right)$$

$$\sigma'(z) = \underline{\underline{\sigma(z) \cdot (1 - \sigma(z))}}$$

Ex. 2 c) $\zeta(z) = -\log(\sigma(-z))$

$$= -\log\left(\frac{1}{1+e^z}\right)$$

$$\frac{d\zeta}{dz} = \frac{1}{\frac{1}{1+e^z}} \cdot \sigma(-z) \cdot (1 - \sigma(-z))$$

$$= (1+e^z) \cdot \cancel{\frac{1}{1+e^z}} \cdot \left(1 - \frac{1}{1+e^z}\right)$$

$$= 1 - \frac{1}{1+e^z}$$

$$= \frac{\cancel{1} + e^z - \cancel{1}}{1+e^z}$$

$$\zeta'(z) = \underline{\underline{\frac{e^z}{1+e^z}}}$$

$$\zeta''(z) = \left(\frac{e^z}{1+e^z}\right)' = \left(e^z \cdot (1+e^z)^{-1}\right)'$$

$$= e^z \cdot (1+e^z)^{-1} + e^z \cdot e^z \cdot -1(1+e^z)^{-2}$$

$$= \frac{e^z}{1+e^z} + \frac{e^z \cdot e^z \cdot -1}{(1+e^z)^2}$$

$$= \frac{(1+e^z)e^z}{(1+e^z)^2} + \frac{e^{2z} \cdot (-1)}{(1+e^z)^2}$$

$$= \frac{\cancel{e^{2z}} + e^z - \cancel{e^{2z}}}{(1+e^z)^2}$$

$$\zeta'' = \underline{\underline{\frac{e^z}{(1+e^z)^2}}}$$

Ex 2f)

$c_1(x)' = (\sigma(x)-1)\cdot(\sigma(x)-1))'$

$\&(\sigma(x)-1)': \quad (\sigma(x)-1)' = \sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1) \quad \cdot 2$

$= 2\sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1)$

$c_1(x)'' = 2\cdot\sigma(x)(1-\sigma(x))(1-\sigma(x))\cdot(\sigma(x)-1) +$

$(-1)\cdot 2\ \sigma(x)\sigma(x)\cdot(1-\sigma(x))\cdot(\sigma(x)-1) +$

$2\cdot\sigma(x)\cdot(1-\sigma(x))\cdot\sigma(x)(1-\sigma(x))$

Ex 2g)

$\cancel{-y\log(\omega x)+(1-y)\cdot\log(\omega x)}$

$\cancel{(-y\log\sigma(\omega x))' = -y\cdot\frac{1}{\sigma(\omega x)}}$

Ex 2 g)

$$c_2(x) = -y \cdot \log(\sigma(\omega x)) + (1-y) \cdot \log(1-\sigma(\omega x))$$

$$-y \cdot \log(\sigma(\omega x))' = -y \cdot \frac{1}{\sigma(\omega x)} \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$= -y \cdot (1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$(1-y) \cdot \log(1-\sigma(\omega x))' = (1-y) \cdot \frac{1}{1-\sigma(\omega x)} (-1) \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot \cancel{x}\omega$$

$$= -(1-y) \cdot \sigma(\omega x) \cdot \cancel{x}\omega$$

$$\Rightarrow c_2'(x) = -y(1-\sigma(\omega x)) \cdot \omega - (1-y) \cdot \sigma(\omega x) \cdot \omega$$

$$c_2''(x) = -y \cdot \sigma(\omega x)(1-\sigma(\omega x)) \cdot (-1) \cdot \omega - (1-y) \cdot \sigma(\omega x) \cdot (1-\sigma(\omega x)) \cdot \omega$$