DeLearn Week 2 PW – Christian Renold, Group 13

Ex. 2 a) $\sigma(z) = \frac{1}{1+e^{-z}} = (1+e^{-z})^{-1}$

$\sigma'(z) = -1 \cdot (1+e^{-z})^{-2} \cdot -e^{-z}$

b) $= \frac{(-1) \cdot -e^{-z}}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})(1+e^{-z})}$

$= \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} = \frac{1}{1+e^{-z}} \cdot \frac{1+e^{-z}-1}{1+e^{-z}}$

$= \frac{1}{1+e^{-z}} \cdot \frac{1+e^{-z}}{1+e^{-z}} - \frac{1}{1+e^{-z}}$

$= \frac{1}{1+e^{-z}} \cdot \left(1 - \frac{1}{1+e^{-z}}\right)$

$\sigma'(z) = \underline{\underline{\sigma(z) \cdot (1 - \sigma(z))}}$

Ex. 2 c) $\beta(z) = -\log(\sigma(-z))$

$= -\log\left(\frac{1}{1+e^{z}}\right)$

$\frac{d\beta}{dz} = \frac{1}{\frac{1}{1+e^{z}}} \cdot \sigma(-z) \cdot (1 - \sigma(-z))$

$= (1+e^{z}) \cdot \frac{1}{1+e^{z}} \cdot \left(1 - \frac{1}{1+e^{z}}\right)$

$= 1 - \frac{1}{1+e^{z}}$

$= \frac{1+e^{z}-1}{1+e^{z}}$

$\beta'(z) = \frac{e^{z}}{1+e^{z}}$

$\beta''(z) = \left(\frac{e^{z}}{1+e^{z}}\right)' = \left(e^{z} \cdot (1+e^{z})^{-1}\right)'$

$= e^{z} \cdot (1+e^{z})^{-1} + e^{z} \cdot e^{z} \cdot -1(1+e^{z})^{-2}$

$= \frac{e^{z}}{1+e^{z}} + \frac{e^{z} \cdot e^{z} \cdot -1}{(1+e^{z})^{2}}$

$= \frac{(1+e^{z})e^{z}}{(1+e^{z})^{2}} + \frac{e^{2z} \cdot (-1)}{(1+e^{z})^{2}}$

$= \frac{e^{2z} + e^{z} - e^{2z}}{(1+e^{z})^{2}}$

$\beta'' = \frac{e^{z}}{(1+e^{z})^{2}}$