Markov decision processes

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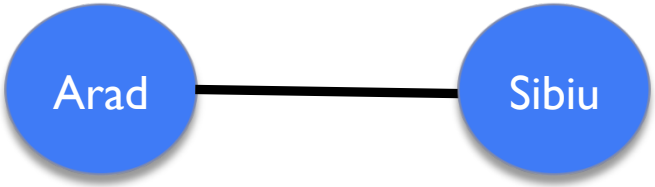
Outline

- Modeling stochastic worlds
  - Stochastic versus deterministic worlds
  - Policies versus plans
  - Markov decision processes versus classical search
- A simple game
  - Modeling game as MDP
  - Solving for optimal policy using the principle of maximizing expected utility
- The value function and the optimal policy
  - V, Q and Bellman's equations
- Algorithms for solving MDPs
  - Policy iteration
    - Properties and proof of convergence
  - Value iteration
  - Discounting
  - Proof of convergence of value iteration

Non-deterministic worlds

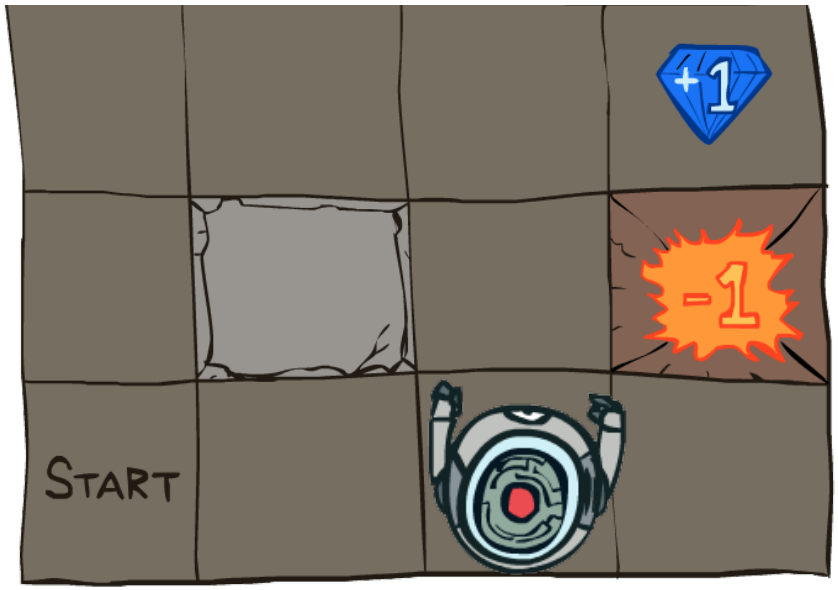
- So far, we have looked at deterministic worlds – performing an action in a state moves an agent to a new state. We model this graph-theoretically as an edge between two states.



- But, in the physical world, we need to account for the fact that not all intended actions succeed, and that an action can have unintended consequences. That is, the real world is non-deterministic!

Example: grid world (deterministic)

- Environment is a grid
  - Bounded by walls
  - Has internal obstacles
  - Has pit of Tartarus
  - Has the Elysian Fields
  - Agent knows where these are.
- Agent effectors are perfect
  - Agent moves in intended direction 100% of the time.
  - Available directions: NSEW
- Agent sensors are perfect
  - It knows where it is perfectly at all times

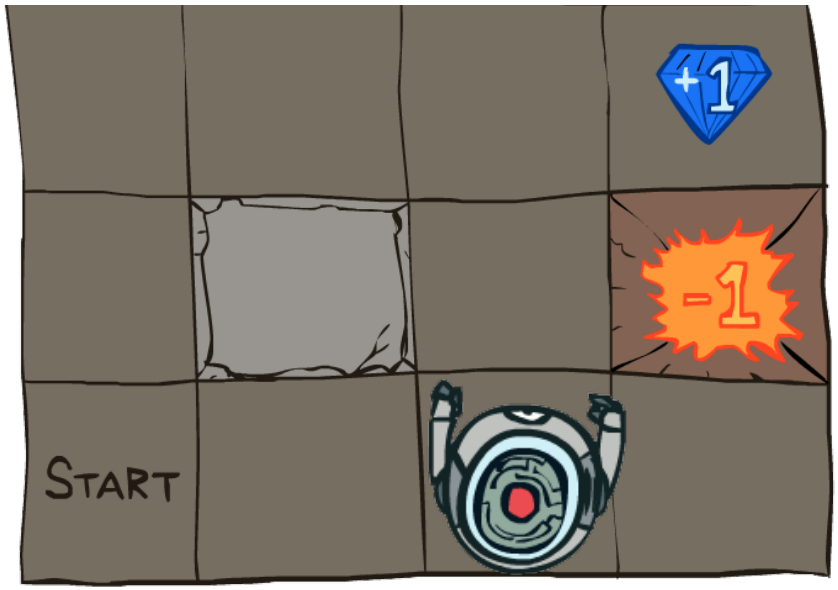


Goal: get to the Elysian field as quickly as possible

What should the agent do, according to A*?

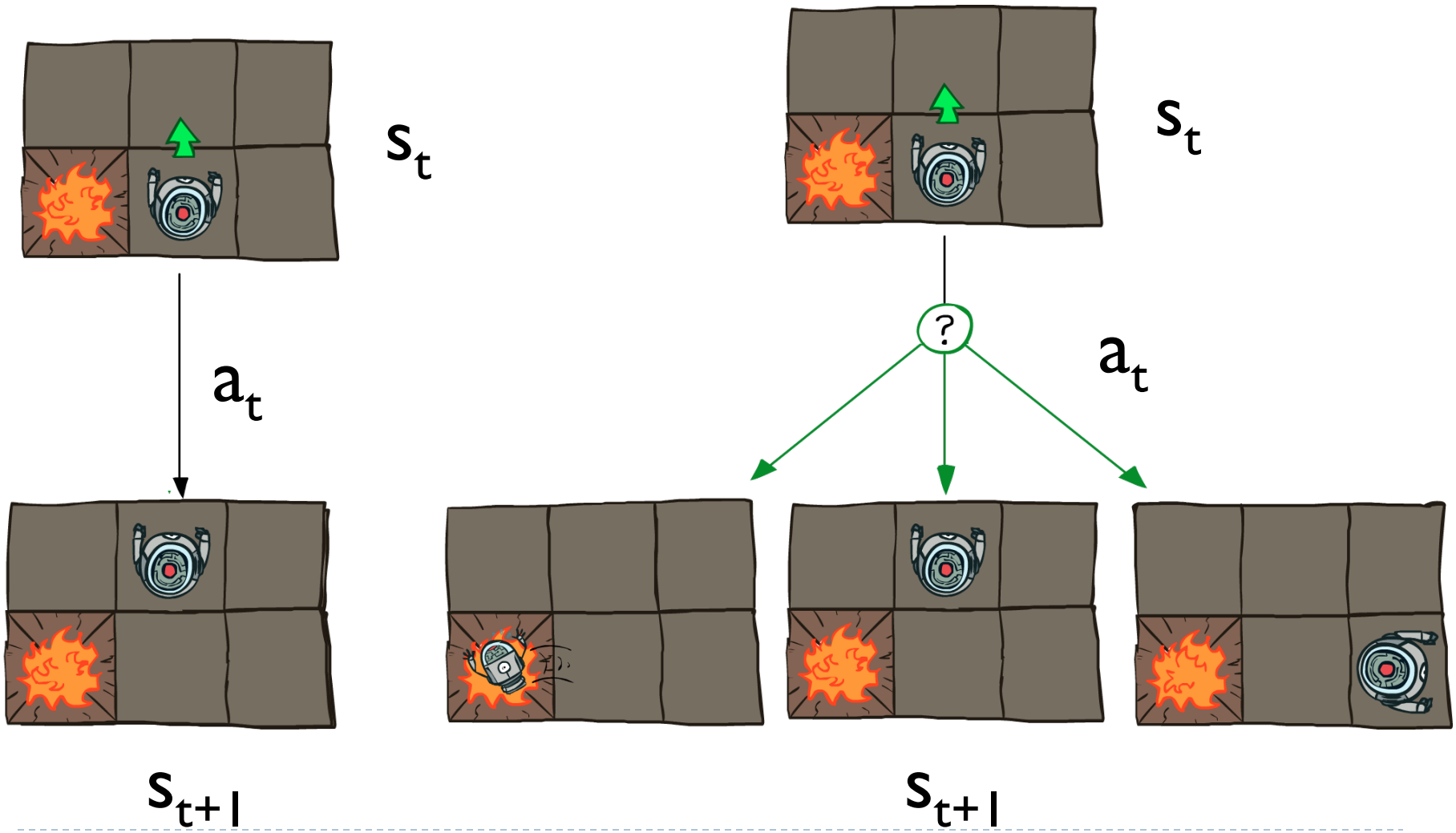
Example: grid world (stochastic)

- Environment is a grid
  - Bounded by walls
  - Has internal obstacles
  - Has pit of Tartarus
  - Has an Elysian Fields
  - Agent knows where these are.
- Agent effectors are noisy
  - 80% of time agent moves in intended direction
  - 10% of time agent moves -90 degrees from intended direction.
  - 10% of the time agent moves +90 degrees from intended direction
- Agent sensors are perfect



Goal: get to the Elysian field as quickly as possible

Deterministic vs stochastic action models



Example: grid world

- Environment is a grid
  - Bounded by walls
  - Has internal obstacles
  - Has pit(s) of Tarturus
  - Has an Elysian Fields
  - Agent knows where these are.
- Agent effectors are noisy
  - 80% of time agent moves in intended direction
  - 10% of time agent moves -90 degrees from intended direction.
  - 10% of the time agent moves +90 degrees from intended direction
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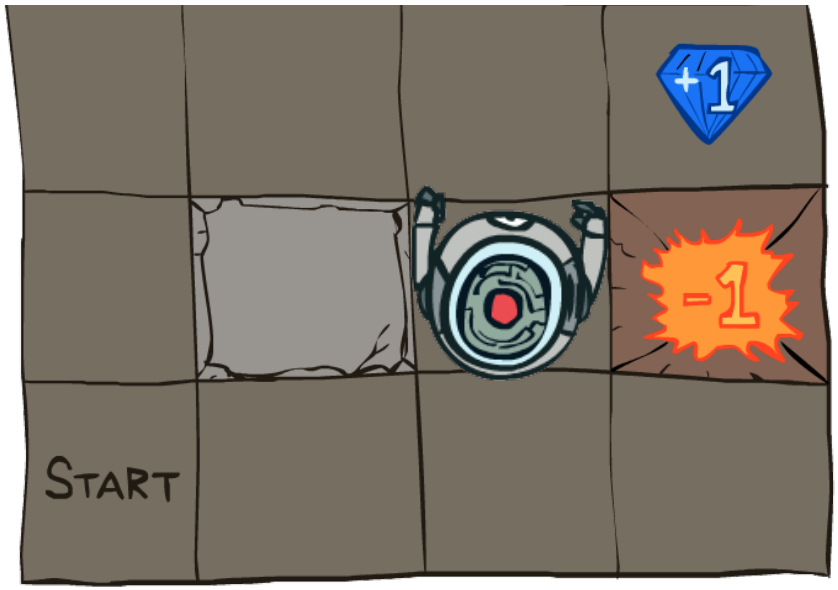
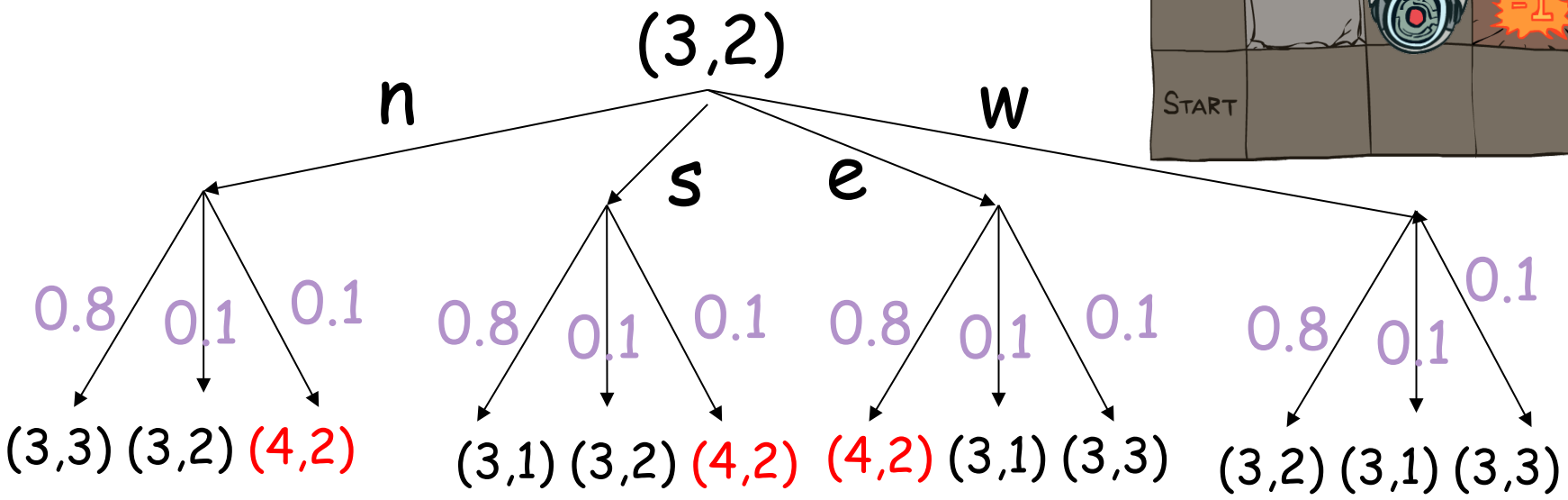


Goal: get to the Elysian field as quickly as possible

Agent is at (3,2).
Elysian field is at (4,3).
What should agent do?

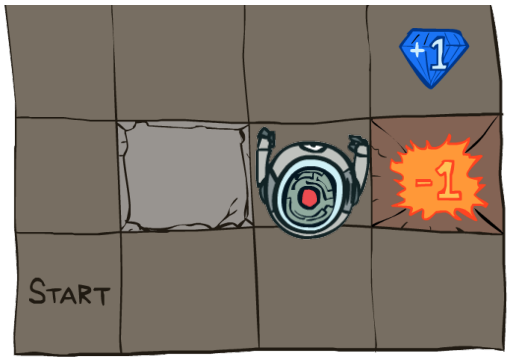
A planning agent

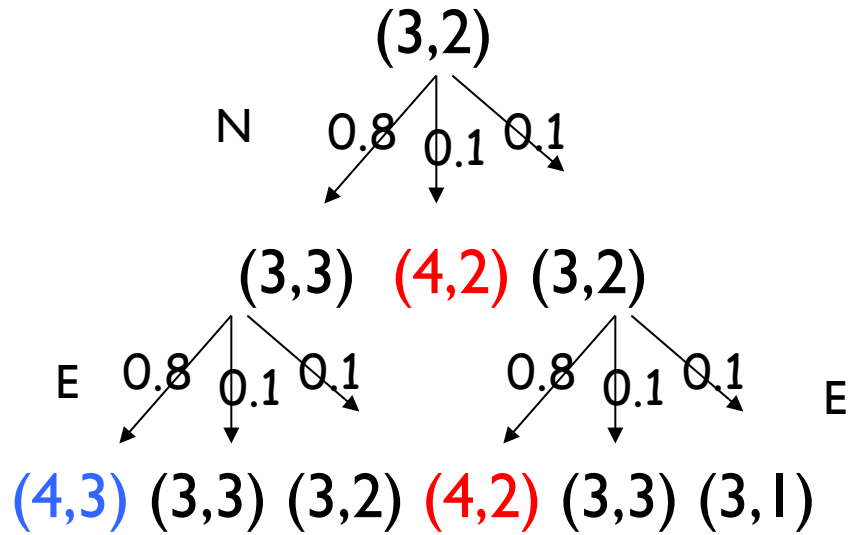
- Should the agent make a plan (a sequence of actions)?



Probabilistic forward projection

- Consider the plan [N,E]

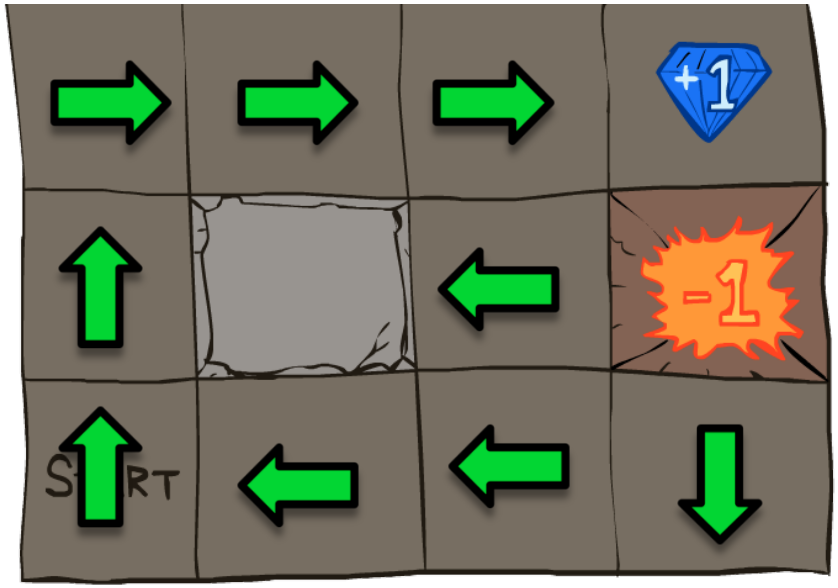




- Is this plan guaranteed to reach (4,3)?
- Is there a plan that will reach (4,3) with probability 1?

Policies versus plans

- A plan is a fixed length sequence of actions from a start state to a goal state.
  - In deterministic worlds, an optimal plan takes an agent from a specified start to a specified goal state.
- A policy is a mapping from states to actions.
  - In non-deterministic worlds, an agent follows an optimal policy which is defined as one that maximizes expected utility if followed.



Examples of decision making in stochastic worlds

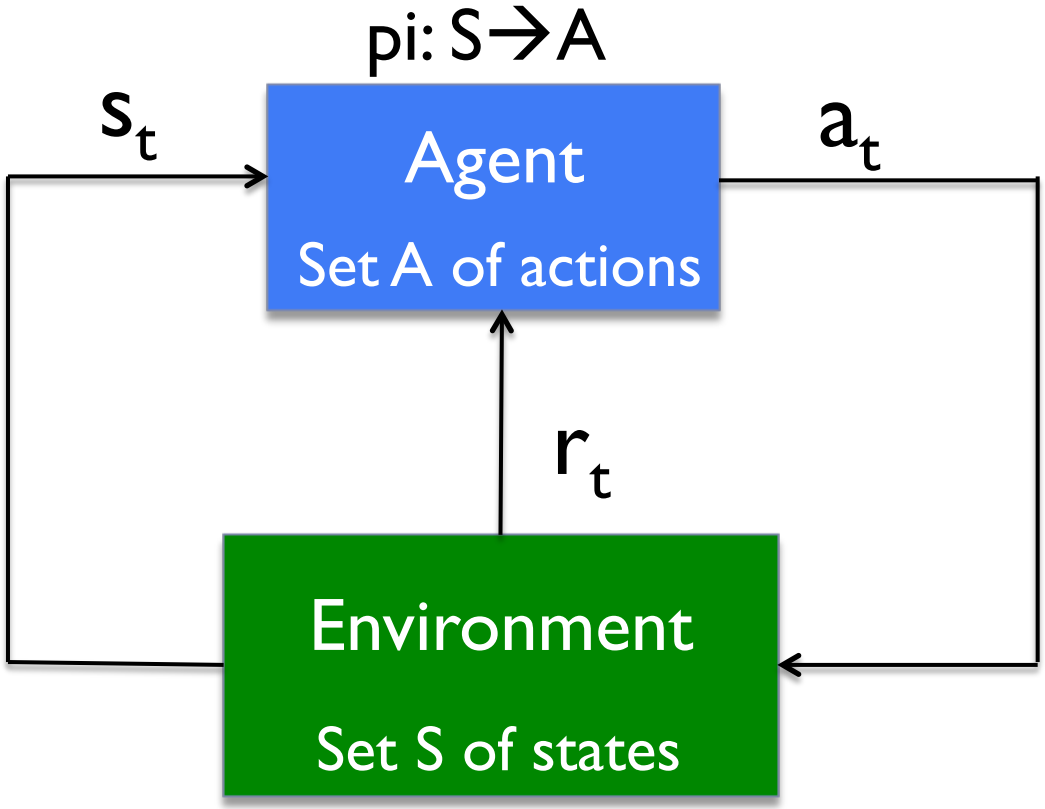
- Robotics: calculate how to get to a goal state when actuators are imperfect.
- Crop planning: calculate what to plant to optimize profits when weather and customer demand are not known with certainty.
- Product planning: calculate what mix of products to make in a factory but state of machines and customer demand for products are unknown. (Mars candy factory example)
- Life: how to optimize happiness when available actions as well as effects of actions/decisions are uncertain.

Markov decision processes (MDP)

- An MDP consists of
  - A set of states S, a subset of which are terminal states (isEnd(s)=True).
  - Actions(s): possible actions from state s
    - No actions from terminal states
  - A transition function T(s,a,s') where s,s' in S, a in Actions(s)
    - T(s,a,s') = probability of transitioning to state s' if action a is taken in state s
  - A reward function r(s,a,s') where s,s' in S, a in Actions(s)
    - r(s,a,s') = reward for taking action a in state s and ending up in state s'
    - Can be sometimes of the form r(s,a) or r(s)

- The grid world problem introduced before is an example of an MDP.
- What's Markov about an MDP is the state transition model T(s,a,s')

Markov decision process

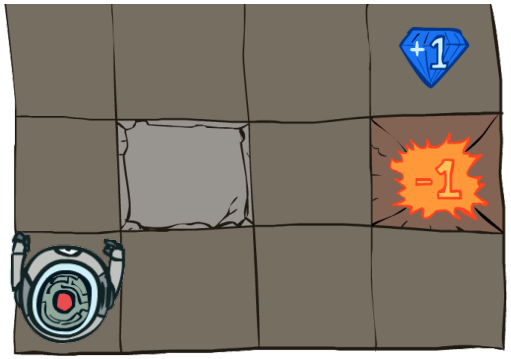


r(s,a,s') = reward for transitioning to s' from s via a

T(s,a,s') = probability of transitioning to s' from s via a

Transition function for robot navigation

- The next state is stochastically determined by the current state and the current action.
- $T(s_{t+1}=(x_2,y_2)|s_t=(x_1,y_1),a_t=a)$
  - $T(s_{t+1}=(1,2)|s_t=(1,1), a_t=N) = 0.8$
  - $T(s_{t+1}=(1,1)|s_t=(1,1), a_t=N) = 0.1$
  - $T(s_{t+1}=(2,1)|s_t=(1,1), a_t=N) = 0.1$



- This state transition model is a first-order Markov model, because the robot's position at time t+1 is a function of position at time t and action at time t.

Reward function for robot navigation

- The agent receives rewards at each time step.
  - r(s,a,s') = -0.01, where s,s' in S and s' is not a terminal state, a in Actions(s)
    - Small cost for each step
  - r(s,a,(4,3)) = +1, where s in S and a in Actions(s) and T(s,a,(4,3)) > 0
  - r(s,a,(4,2)) = -1, where s in S and a in Actions(s) and T(s,a,(4,2)) > 0
    - Big reward/cost at the end (Elysian fields or the pits of Tartarus)

MDPs and classical search

- An MDP consists of
  - A set of states S, a subset of which are terminal states (isEnd(s)=True).
  - Actions(s): possible actions from state s
    - No actions from terminal states
  - A transition function T(s,a,s') where s,s' in S, a in Actions(s)
    - T(s,a,s') = probability of transitioning to state s' if action a is taken in state s
  - A reward function r(s,a,s') where s,s' in S, a in Actions(s)
    - r(s,a,s') = reward for taking action a in state s and ending up in state s'
    - Can be sometimes of the form r(s,a) or r(s)
  - Solution is a **policy** mapping S to Actions(s) with highest expected sum of rewards.

- A classical search problem consists of
  - A set of states S, a subset of which are terminal states (isEnd(s)=True) and a start state.
  - Actions(s): possible actions from state s
    - No actions from terminal states
  - A successor function succ(s,a) where s in S, a in Actions(s)
    - succ(s,a) = state that results when action a is taken in state s
  - A cost function cost(s,a) where s in S, a in Actions(s)
    - cost(s,a) = cost of taking action a in state s
  - Solution is a **sequence of actions** from start to an end state with minimal cost

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A simple game

- For each round r = 1,2,…
  - You choose to stay or quit.
  - If you quit, you get $10 and we end the game.
  - If you stay, you get $4 and then I roll a 6-sided dice.
    - If the dice roll is 1 or 2, we end the game.
    - Otherwise, continue to the next round.
- What should you do at each round?
  - quit or stay?

Simple game MDP

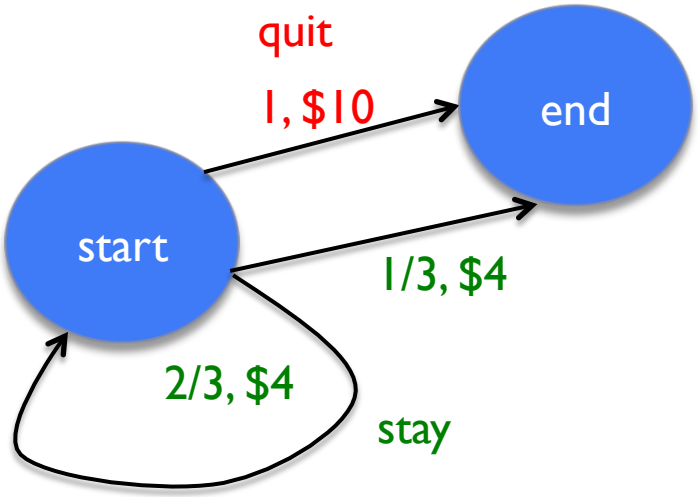
- States = {start,end}, end is terminal state
- Actions = {stay, quit}
  - Actions(start) = {stay,quit}
  - Actions(end) = {}
- Transitions T(s,a,s') and rewards r(s,a)

s	a	s'	T(s,a,s')
start	quit	end	1
start	stay	end	1/3
start	stay	start	2/3

s	a	r(s,a)
start	quit	10
start	stay	4

$$\sum_{s' \in States} T(s,a,s') = 1$$

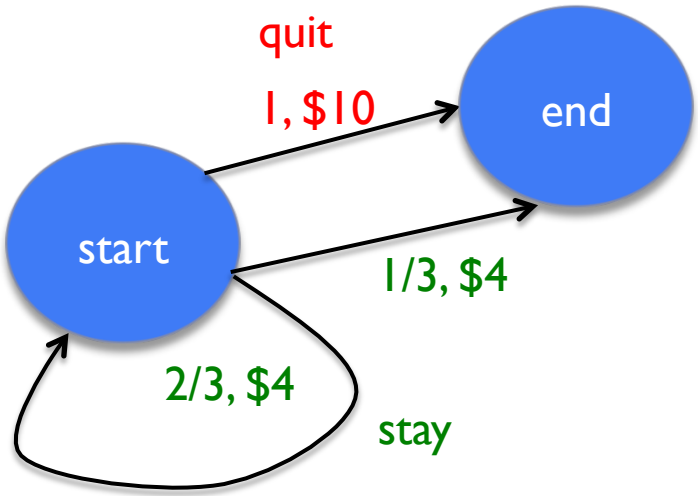
Maximizing expected utility



- Maximize expected utility!
  - Calculate expected utility of all available policies, and choose the one with the highest expected utility.
- Expected utility of "quit"
  - 10
- Expected utility of "stay"
  - 1/3 * 4 + 2/3 * 1/3 * 8 + 2/3*2/3*1/3 * 12 + … = 12

Solution for simple game

- A policy is a mapping from non-terminal states in the MDP to actions.
- The optimal policy is to choose "stay" in start state.

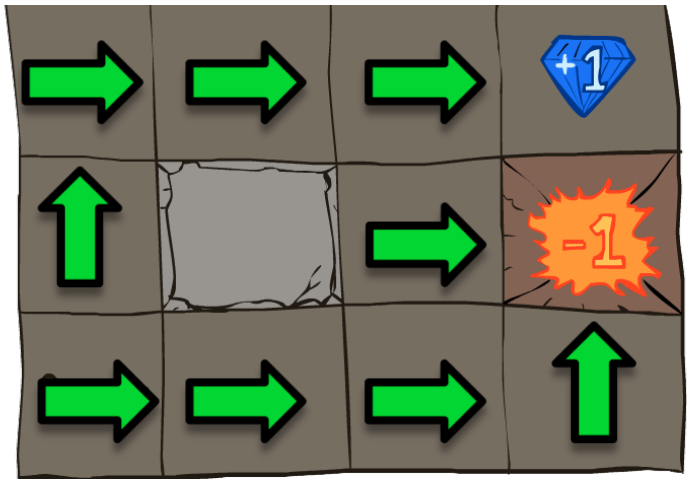


$\pi^*(\text{start}) = \text{stay}$

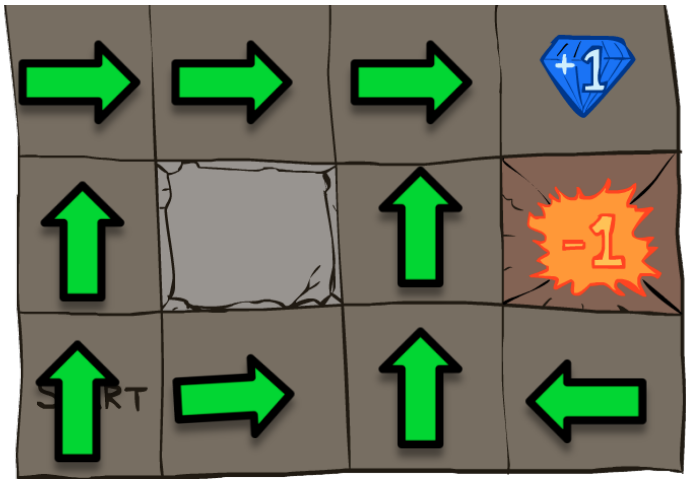
How to calculate optimal policies

- Model the decision problem as an MDP
  - Define the states S and actions Actions(s) and identify the terminal states.
  - Construct the transition function T(s,a,s'), where s,s' in S and a in Actions(s).
  - Construct the reward function r(s,a,s') where s,s' in S and a in Actions(s)
- Design an algorithm that solves the MDP, i.e., constructs an optimal policy given a set of states, actions, the transition function and the reward function.

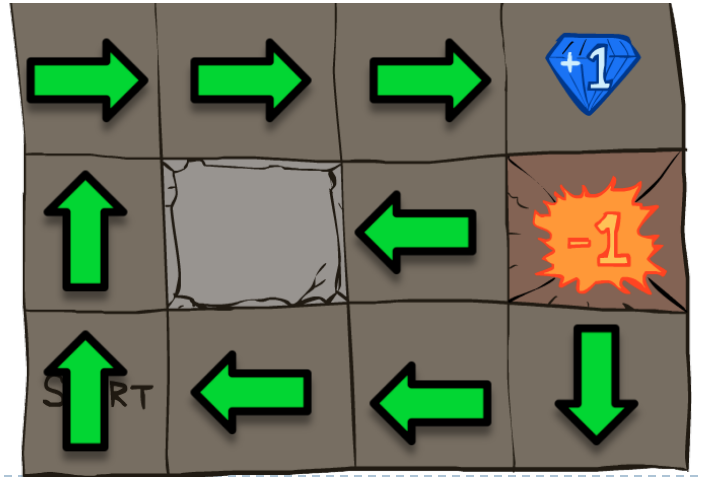
Optimal policies for robot navigation



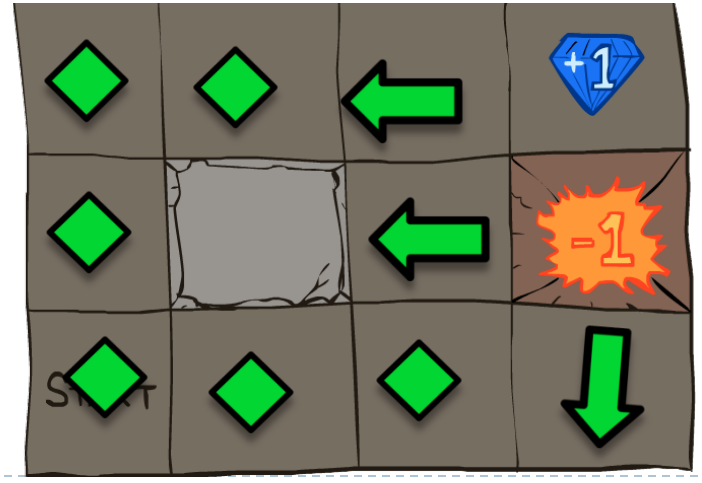
r(s) < -1.6284



-0.4278 < r(s) < -0.0850



-0.0221 < r(s) < 0



r(s) > 0

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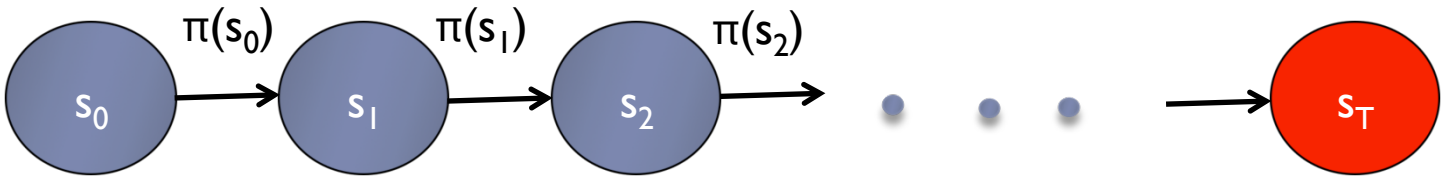
Evaluating a policy π: V_π

Following a policy yields a path through the set of states S, culminating in a terminal state.

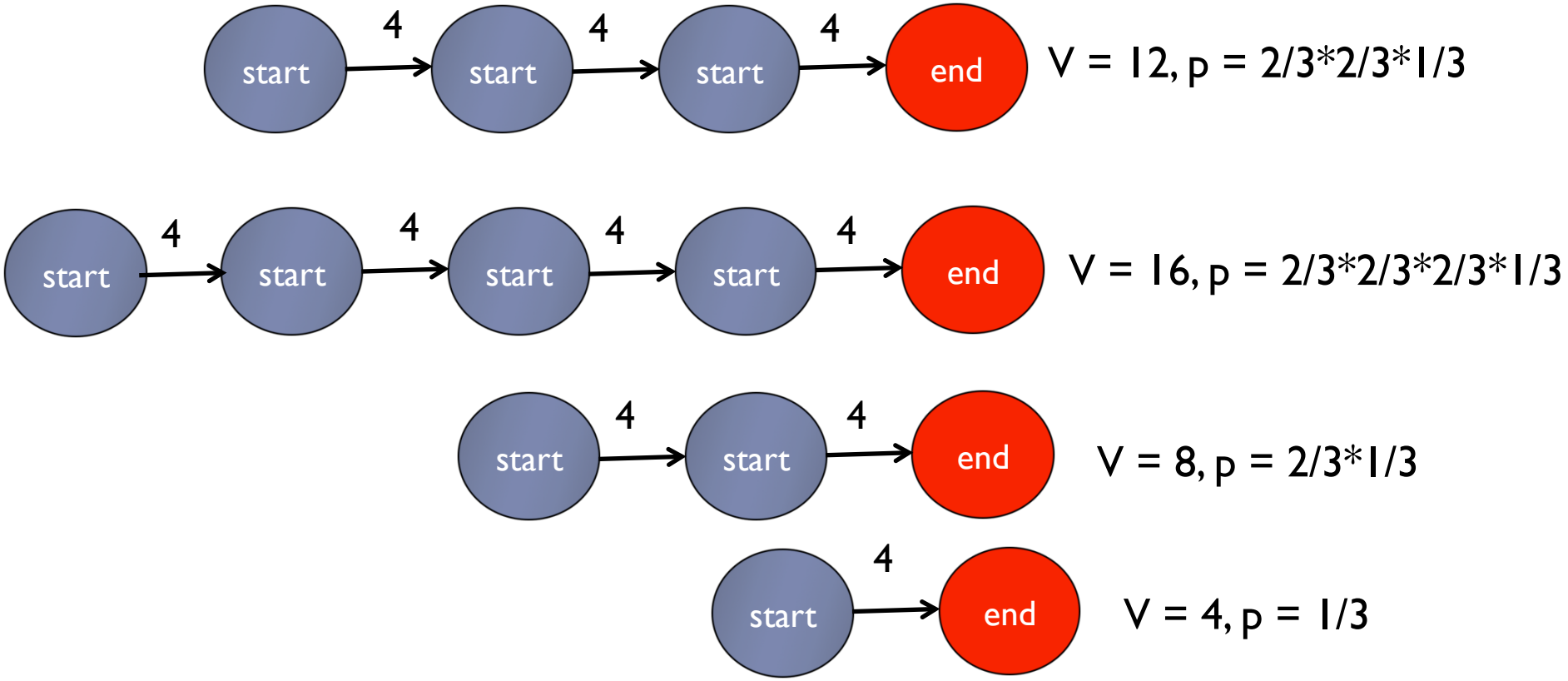


$$V_\pi(s_0 s_1 \ldots s_T) = \sum_{t=0}^{T} r(s_t, \pi(s_t), s_{t+1})$$

Following a policy yields a state sequence (with some associated probability).
The value of a state sequence is the sum of the rewards over each transition in the state sequence.

r(s,a,s') = -0.01 in non-terminal states s'
r(s,a,s') = +1 if s'= (4,3)
r(s,a,s') = -1 if s'= (4,2)

Evaluating the policy "stay" in the simple game



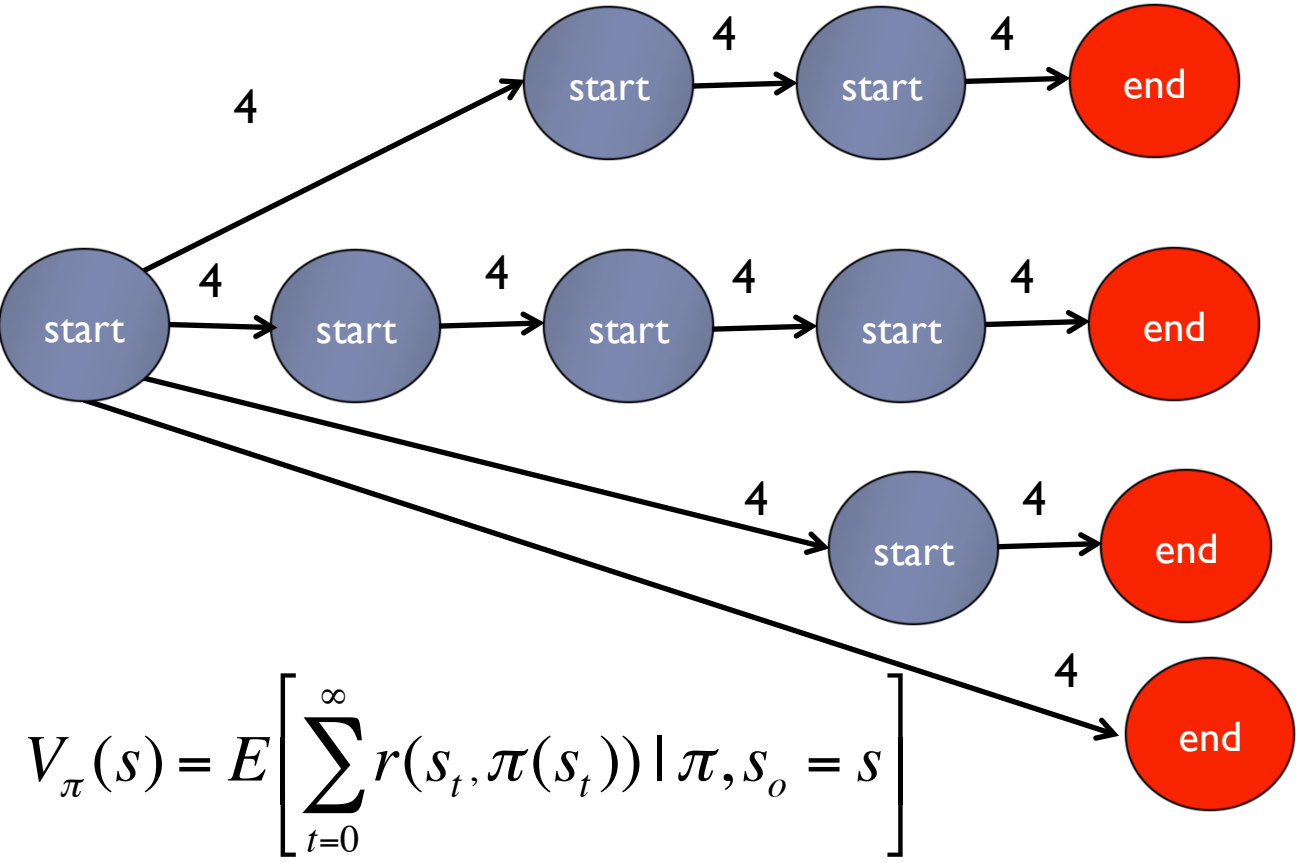
The value function V_π

- Expected utility of a state s under policy π

$$V_\pi(s) = E\left[\sum_{t=0}^{\infty} r(s_t, \pi(s_t)) \mid \pi, s_o = s\right]$$

- Expectation is calculated over all sequences of states generated by the policy, that start with state s. It is the average sum of rewards along every path in the state space starting from s, generated by the policy, weighted by the probability of that path.

The value function $V_\pi(s)$ for the simple game

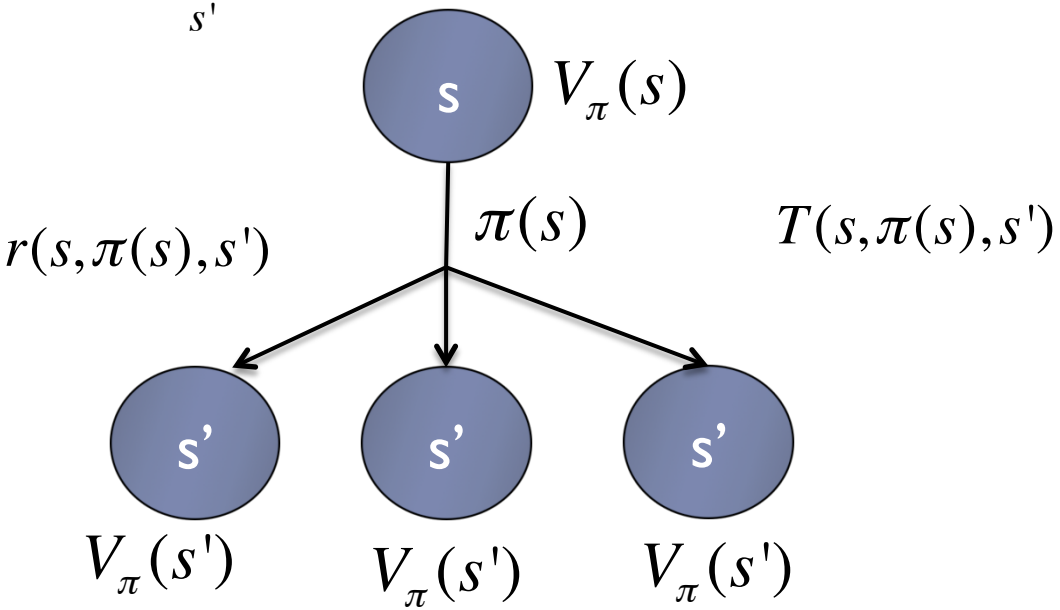


$$V_\pi(s) = E\left[\sum_{t=0}^{\infty} r(s_t, \pi(s_t)) \mid \pi, s_o = s\right]$$

Recursive computation of V_π

- The value of a state s under the policy π is the expected reward associated with taking action π(s) plus the utility of following policy π from a successor state s'.

$$V_\pi(s) = \sum_{s'} T(s,\pi(s),s')[r(s,\pi(s),s') + V_\pi(s')]$$



The Q value of a policy π

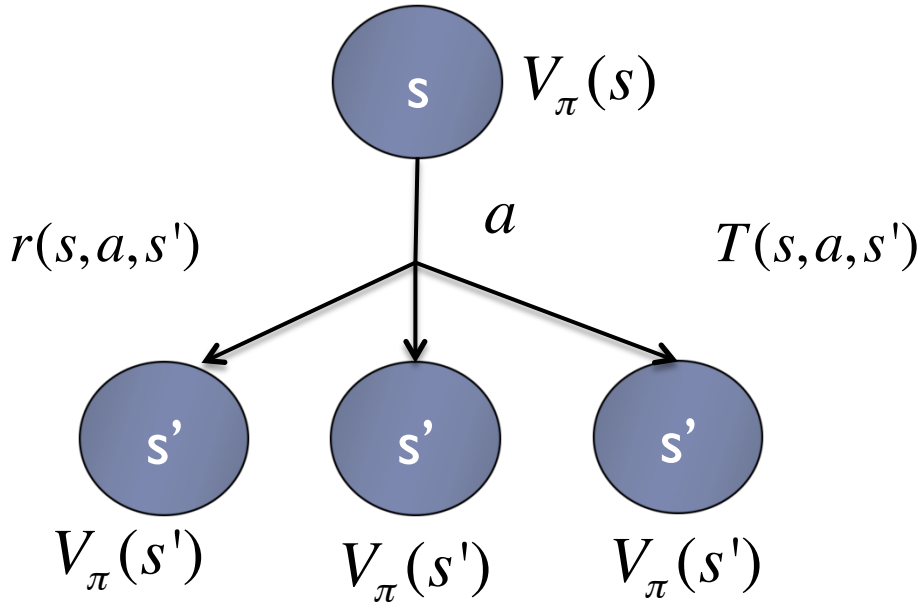
- $Q_\pi(s,a)$ is the expected value of taking action a from state s, and then following policy π thereafter.
- $V_\pi(s)$ is the expected value of following policy π from state s.

$$V_\pi(s) = \begin{cases} 0 \text{ if s is a terminal state} \\ Q_\pi(s, \pi(s)) \text{ otherwise} \end{cases}$$

Recursive computation of Q_π

- $Q_\pi(s,a)$ is the expected value of taking action a from state s, and then following policy π thereafter.

$$Q_\pi(s,a) = \sum_{s'} T(s,a,s')[r(s,a,s') + V_\pi(s')]$$



$$V_\pi(s) = \sum_{s'} T(s,\pi(s),s')[r(s,\pi(s),s') + V_\pi(s')]$$

Computing V and Q for the simple game

- States = {start,end}, end is terminal state
- Actions = {stay, quit}
  - Actions(start) = {stay,quit}
  - Actions(end) = {}
- Transitions T(s,a,s') and rewards r(s,a)

Let π(start) = stay.

$$V_\pi(start) = Q_\pi(start, stay)$$

$$V_\pi(start) = \frac{1}{3}(4+0) + \frac{2}{3}(4 + V_\pi(start))$$

$$V_\pi(start) = 12$$

s	a	s'	T(s,a,s')
start	quit	end	1
start	stay	end	1/3
start	stay	start	2/3

s	a	r(s,a)
start	quit	10
start	stay	4

System of linear equations which can be solved in closed form.

Policy evaluation

- Calculation of V_π for a given policy π
- Approach 1: set up system of linear equations and solve for V_π from the recurrences.
- Approach 2: (iterative improvement) repeatedly apply recurrences on V till convergence
  - Initialize $V_\pi^{(0)}(s)$ to be 0 for all states s.
  - For i = 1,2,….T
    - for each state s

$$V_\pi^i(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[r(s,\pi(s),s') + V_\pi^{i-1}(s')]$$

Convergence of iterative method

- Maintain V_π for i and for i-1. Check that the highest difference between the two vectors is less than or equal to a specific epsilon.

$$\max_s \left| V_\pi^i(s) - V_\pi^{i-1}(s) \right| \leq \varepsilon$$

- Each iteration of policy evaluation takes time $O(|S|^2)$
- Complexity of policy evaluation is $O(T|S|^2)$

Optimal policy

- The optimal policy π^* has the highest expected utility among all policies. The associated value function is V*.

$$V^*(s) = V_{\pi^*}(s) = \max_{\pi} V_{\pi}(s)$$

- We can use the recurrence on V to write the recurrence on V*:

$$V_{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[r(s, \pi(s), s') + V_{\pi}(s')]$$

$$V^*(s) = \max_{a \in Actions(s)} \sum_{s'} T(s, a, s')[r(s, a, s') + V^*(s')]$$

Bellman's equation

Relationship between V* and Q*

- Given V*, we can compute Q*

$$Q^*(s,a) = Q_{\pi^*}(s,a) = \sum_{s'} T(s,a,s')[r(s,a,s') + V^*(s')]$$

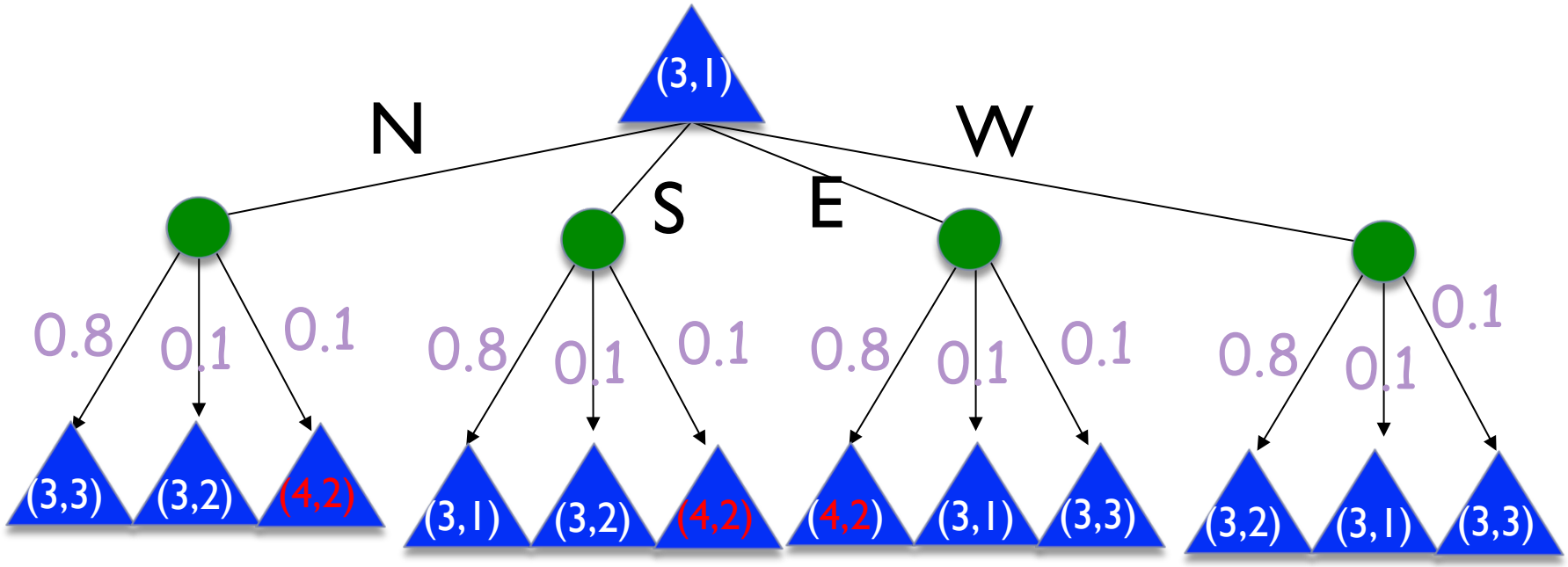
- Given Q*, we can compute V*

$$V^*(s) = \max_{a \in Actions(s)} Q^*(s,a)$$

Calculating π* from V* (policy extraction)

Principle of maximizing expected utility

$$\pi^*(s) = \underset{a \in Actions(s)}{\operatorname{argmax}} \sum_{s'} T(s,a,s')[r(s,a,s') + V^*(s')] = \underset{a \in Actions(s)}{\operatorname{argmax}} Q^*(s,a)$$



Outline

- Modeling stochastic worlds
  - Stochastic versus deterministic worlds
  - Policies versus plans
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Policy improvement

- Once we calculate V_π we can try to improve the policy to something better.
- Algorithm:
  - Compute $Q_\pi(s,a)$ from $V_\pi(s)$

$$Q_\pi(s,a) = \sum_{s'} T(s,a,s')[r(s,a,s') + V_\pi(s')]$$

  - Update π

$$\pi_{new}(s) = \arg\max_{a \in Actions(s)} Q(s,a)$$

    - Take the action prescribed by the computed Q(s,a) – the action that has the highest expected value from state s.

Example of policy improvement

- Suppose π(start) = quit in the simple game.
- V_π(start) = 10 and V_π(end) = 0 under this policy

$$Q_\pi(s,a) = \sum_{s'} T(s,a,s')[r(s,a,s') + V_\pi(s')]$$

- Q_π(start,quit) = 10
- Q_π(start,stay) = 1/3*[4+0] + 2/3[4+10] = 10.67
- Policy improvement
  - $\pi_{new(}$(start) = stay

Policy improvement properties

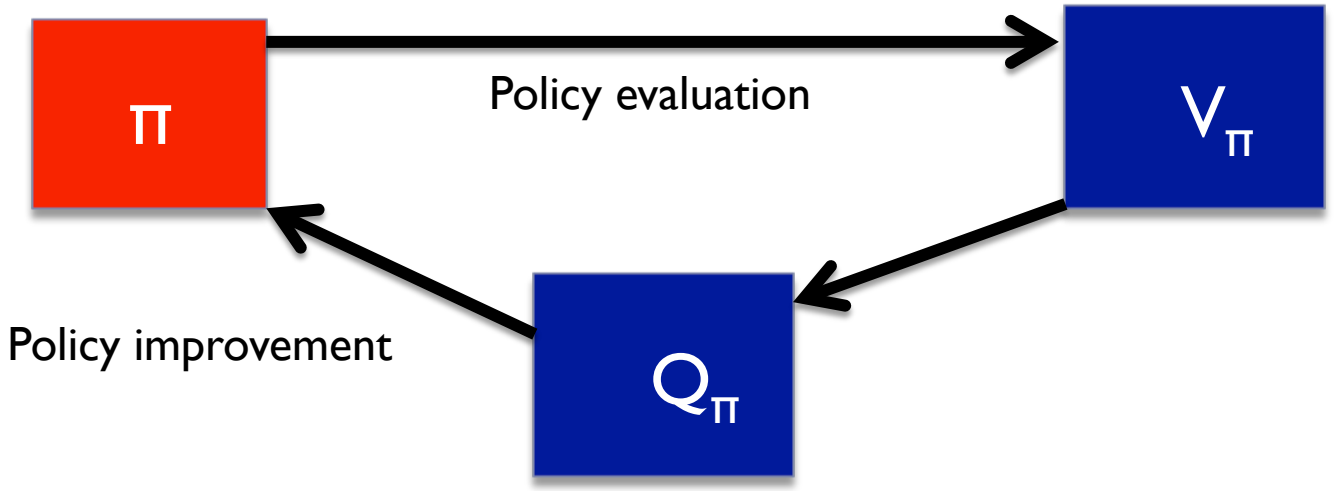
- The value of the new policy is at least as large as the old one.

$$V_{\pi_{new}}(s) \geq V_{\pi}(s) \text{ for all states } s$$

- Complexity of policy improvement: $O(|S|^2+|A||S|)$

Policy iteration

- Iterative process
  - Start with a (random) policy π
  - For i = 1,2,…,T
    - Policy evaluation: Compute V_π corresponding to policy π
    - Policy improvement: Update policy based on Q_π calculated from V_π



Policy iteration algorithm

- Start with a random policy π
- Loop
  - Compute value of policy π

$$V_\pi(s) = \sum_{s'} T(s, \pi(s), s')[r(s, \pi(s), s') + V_\pi(s')]$$

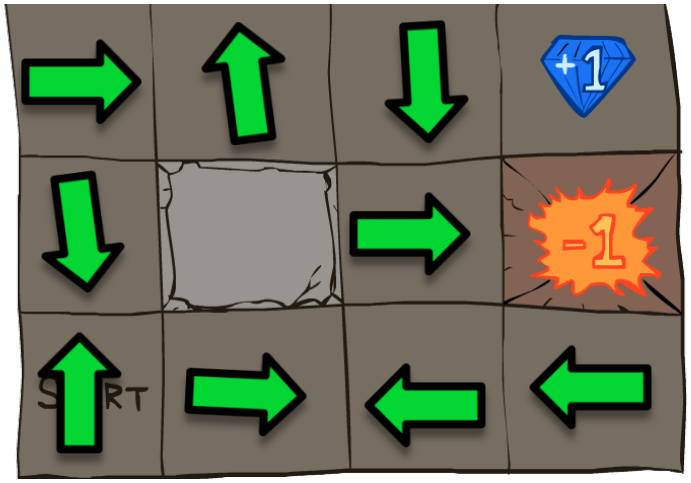
  - Improve the policy at each state, if possible

$$\pi'(s) = \arg\max_a \sum_{s'} T(s, a, s')[r(s, a, s') + V_\pi(s')]$$

- Until no change to policy occurs

Example of policy iteration

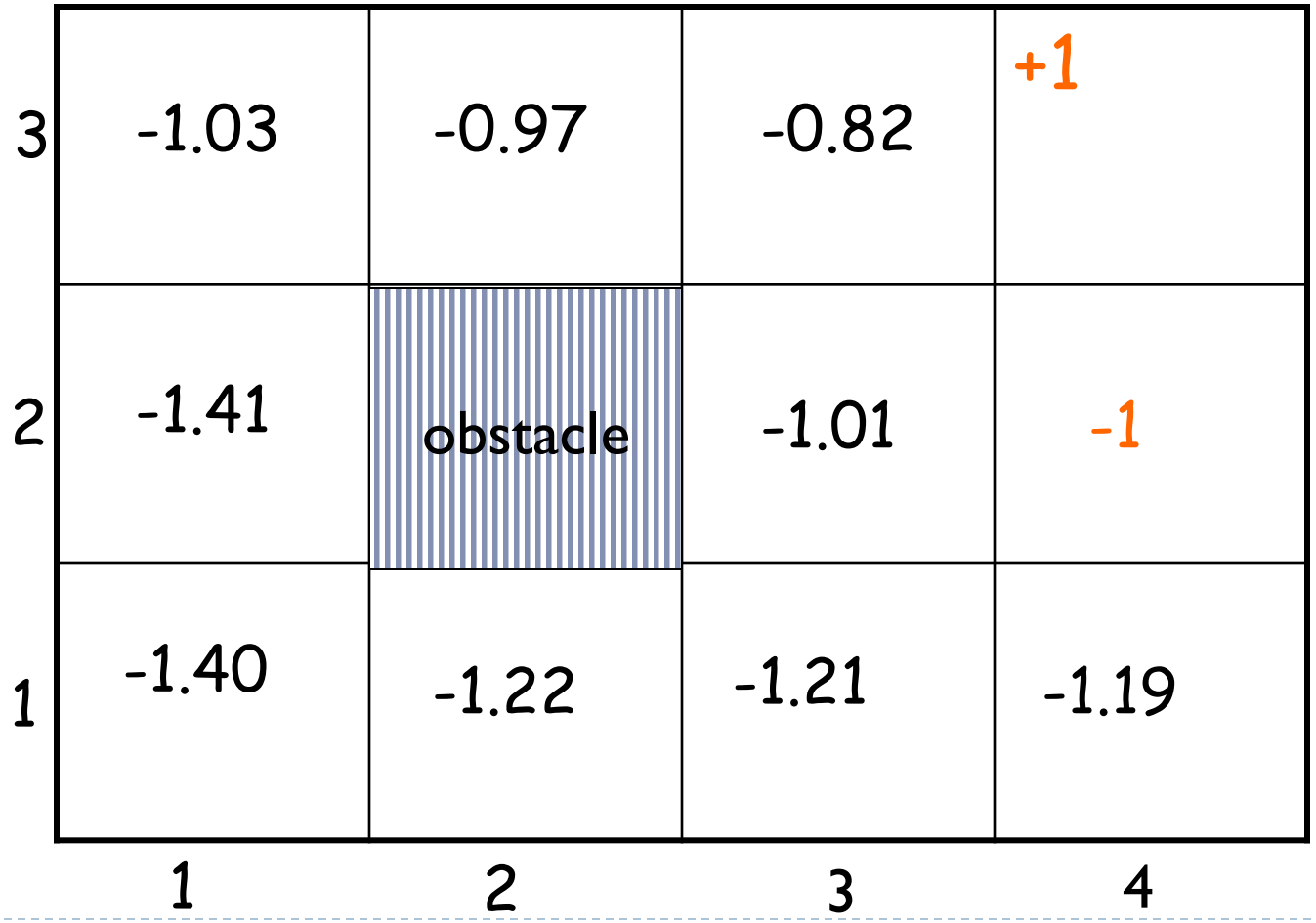
- Step 0: Make a random initial policy



- Step 1: Evaluate policy
  - V(1,1)=-0.01 + 0.8*V(2,1)+0.1*V(1,1)+0.1*V(2,1)
  - …
  - V(3,3) = -0.01 + 0.8*V(2,3)+0.1*1+0.1*V(3,2)
  - 9 linear equations in nine unknowns (Gaussian elimination) or by iterative refinement of V.

Example of policy iteration

Value of policy from step 0



Example of policy iteration

- Step 2: Policy improvement
  - For every state calculate best action based on V computed in previous step

$$Q(s,a) = \sum_{s'} T(s,a,s')[r(s,a,s') + V(s')]$$

$$\pi(s) = \underset{a \in Actions(s)}{\operatorname{argmax}}\, Q(s,a)$$

- If there are any changes in policy, go back to policy evaluation (Step 1)

Convergence of policy iteration

- Algorithm terminates when the policy improvement step yields no change.
- There are only finitely many policies for a finite state space, and each iteration can be shown to yield a better policy.
- So policy iteration must terminate.
- Can be shown to terminate on the optimal policy because V has a single fixed point (Howard, 1980).

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Computing policy from Bellman's equations directly

- We can write equations for all 9 non-terminal states in the grid world problem.

$$V(1,1) = -0.01 + \max\{0.8V(1,2) + 0.1V(2,1) + 0.1V(1,1),$$
$$0.9V(1,1) + 0.1V(1,2),$$
$$0.9V(1,1) + 0.1V(2,1),$$
$$0.8V(2,1) + 0.1V(1,2) + 0.1V(1,1)\}$$

- Unique solutions exist to this system of equations (Bellman, 1957).

Value iteration

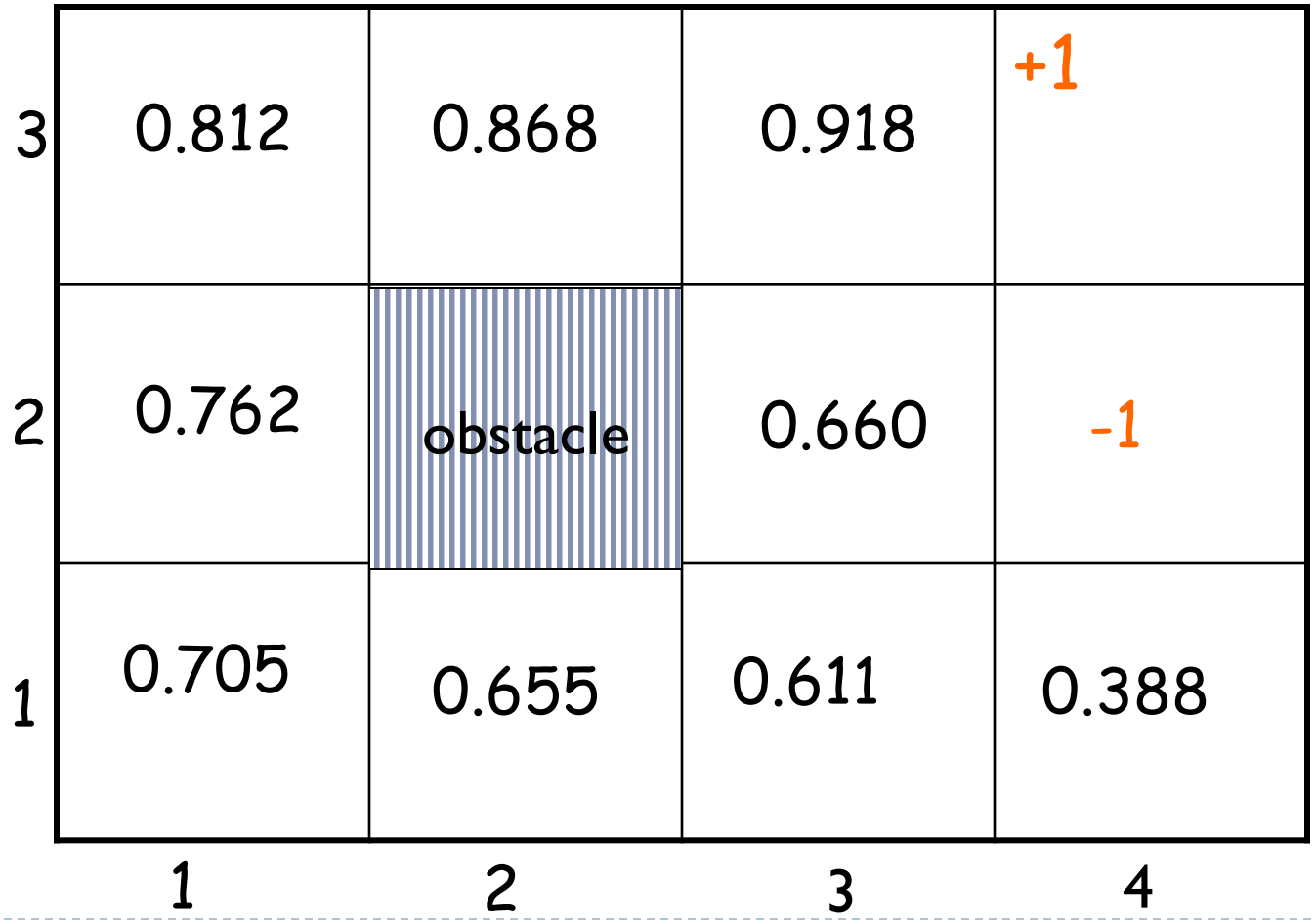
- Define $V^{(i)}(s)$ = optimal value of state s if we reach a terminal state in i more steps (maximum sum of rewards starting from state s and ending in i steps)
- Start with $V^{(0)}(s) = 0$ for all states s in S
- Calculate $V^{(i+1)}(s)$ from $V^{(i)}(s)$ using Bellman update

$$V^{(i+1)}(s) \leftarrow \max_{a \in Actions(s)} \sum_{s'} T(s,a,s')[r(s,a,s') + V^{(i)}(s')]$$

- Repeat until convergence (successive values of V are close enough)

Computation of V*

r(s) = -0.04



Value iteration and Bellman's equations

- Bellman's equations characterize the optimal values V*

$$V^*(s) = \max_a \sum_{s'} T(s,a,s')[r(s,a,s') + V^*(s')]$$

- Value iteration computes the optimal values V*

$$V^{(i+1)}(s) \leftarrow \max_a \sum_{s'} T(s,a,s')[r(s,a,s') + V^{(i)}(s')]$$

- Value iteration is a fixed point solution method.

Complexity of value iteration

- Each iteration is $O(S^2A)$. Why?
- We will prove convergence of value iteration and place a bound on the number of iterations needed for value iteration to converge.

Termination criteria for value iteration

- RMS error of the utility values.
- Policy loss: stop when policies on subsequent iterations are the same.
  - Practically, policy convergence occurs much before value convergence.

Optimizing value iteration

- It is slow: each iteration takes $O(S^2A)$ time
- Many optimizations to speed it up
  - Early termination: policy convergence
  - Selective updating: Updating V values of important states rather than all states in each iteration, by taking advantage of state space structure

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A problem with sum of rewards utility

- For infinite horizon problems, with no terminal states, the sum of rewards becomes infinity.

$$V(s_0 s_1 \ldots) = \sum_{t=0}^{\infty} r(s_t)$$

- There is no way to compare state histories and define an optimal policy!

Discounting

- Discounting makes the sum of rewards finite, so we can compare histories.

$$V(s_0 s_1 \ldots) = \sum_{t=0}^{\infty} \gamma^t r(s_t)$$

- Discount factor γ

$$[1,1,1,\ldots.] \rightarrow [1,\gamma,\gamma^2,\ldots.]$$

- Discounting is a good model of human and animal preferences over time.

Value iteration with discounting

- Define $V^{(i)}(s)$ = optimal value of state s if we reach a terminal state in i more steps
- Start with $V^{(0)}(s) = 0$ for all states s in S
- Calculate $V^{(i+1)}(s)$ from $V^{(i)}(s)$ using Bellman update

$$V^{(i+1)}(s) \leftarrow \max_{a \in Actions(s)} \sum_{s'} T(s,a,s')[r(s,a,s') + \gamma V^{(i)}(s')]$$

- Repeat until convergence (successive values of V_t are close enough) (converged $V_i = V^*$)

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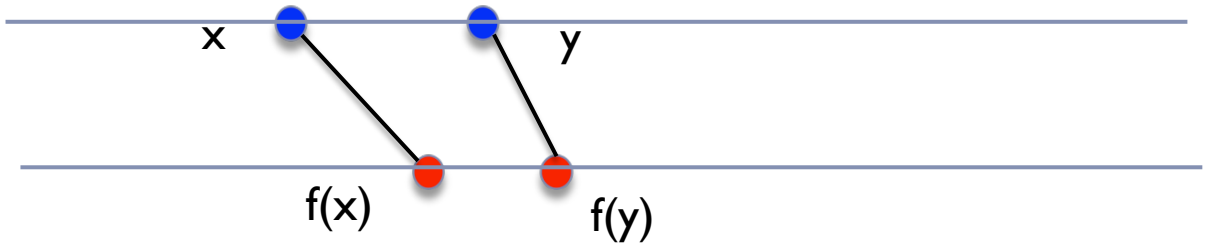
Contraction mapping

- A contraction is a function f: R → R such that

$$| f(x) - f(y) | \leq c | x - y | \text{ where } 0 \leq c < 1$$

  - where |x-y| denotes the distance between x,y in R.



Contraction mapping principle

- Consider a contraction mapping f:R→R on the reals with contraction coefficient γ
- Take the sequence $\{x_1, x_2, \ldots,\}$
  - where $x_{i+1}=f(x_i)$
- The sequence has a unique fixed point a and

$$|a - x_n| \leq \gamma^n |a - x_1|$$

  - We can calculate the number n of applications of the contraction mapping to get to within a specified ε of the fixed point a.

Application to value iteration convergence

- We construct the sequence $V_0, V_1, \ldots, V_n$
  - where $V_{i+1} = BV_i$
  - B is the Bellman update (which is a contraction by factor γ)
- The fixed point of the sequence is V*

$$\left|V^* - V_n\right| \leq \gamma^n \left|V^* - V_0\right|$$

($-r_{max}, r_{max}$) is the highest range in the reward function

- We will take the max norm of a vector as the distance function |.|
- Then, we want $|V^* - V_n| <= \varepsilon$
- We know V_0 = all zeros and the biggest distance between V* and V_0 can only be $\frac{2r_{\max}}{(1-\gamma)}$

Number of iterations

- If V* were the true utility vector, then

$$\left\| BV - BV^* \right\| \leq \gamma \left\| V - V^* \right\|$$

- To get to the desired error bound ε,

$$\gamma^N 2r_{\max} / (1-\gamma) \leq \varepsilon$$

Types of MDPs

- Finite horizon MDPs: A decision problem in which the agent has to maximize its expected utility over a specific time period.
  - The optimal policy for finite horizon problems could be non-stationary (be a function of the length of the horizon).
  - We do not study finite horizon MDPs in this class.
- Indefinite and infinite horizon MDPs (i.e., no fixed deadline)
  - Optimal policy is stationary; the optimal action in a given state is a function of that state and not the time at which the agent is at that state.
  - We have covered this family of MDPs in this class.