Formula specification

Token0 Must be smaller address than Token1

Token1 Must be greater address than Token0

x is Long token0 at uint256

y is Long token1 at uint256

z is Short per second at uint256 as UQ160.96

d is duration at uint96 (maturity must be such that d is less than max of uint96)

$$(x+y)z = L^2$$

$$\frac{z}{x+y} = I$$

L is total liquidity balance owned by all LP at uint160 (stored as variable)

$\sqrt{I}$ is square root of marginal interest rate per second at uint160 as UQ64.96 (stored as variable)

$$x+y = \frac{L2^{96}}{\sqrt{I}}$$

$$z = \frac{L\sqrt{I}}{2^{96}}$$

Deleverage transactions

Case 1.1 Given $\Delta x + \Delta y$ to get $\sqrt{I} - \Delta\sqrt{I}$ then get Δz_d

$$x + y + \Delta x + \Delta y = \frac{L2^{96}}{\sqrt{I} - \Delta\sqrt{I}}$$

$$\sqrt{I} - \Delta\sqrt{I} = \frac{L2^{96}}{\frac{L2^{96}}{\sqrt{I}} + \Delta x + \Delta y} = \frac{L2^{96}\sqrt{I}}{L2^{96} + (\Delta x + \Delta y)\sqrt{I}}$$

$$\Delta z_d = \frac{\Delta\sqrt{I}dL}{2^{192}}$$

Code

$\text{long amount} = \Delta x + \Delta y$

Short is cost for the pool, thus the delta rate is cost. We round down delta rate, thus round up new rate.

$$\sqrt{I} - \Delta\sqrt{I} = \left\lceil \frac{L2^{96}}{\left\lfloor \frac{L2^{96}}{\sqrt{I}} \right\rfloor + \Delta x + \Delta y} \right\rceil = \left\lceil \frac{L2^{96}\sqrt{I}}{L2^{96} + (\Delta x + \Delta y)\sqrt{I}} \right\rceil$$

Short is cost for the pool, so round down.

$$\Delta z_d = \left\lfloor \frac{\Delta\sqrt{I} dL}{2^{192}} \right\rfloor$$

$\text{short amount with fees} = \Delta z_d$

Fees is gain for LP, so round up

$$\text{fees} = \left\lceil \frac{\Delta z_d f}{2^{16}} \right\rceil$$

$\text{short amount} = \text{short amount with fees} - \text{fees}$

Case 1.2 Given Δz_d to get $\sqrt{I} - \Delta\sqrt{I}$ then get $\Delta x + \Delta y$

$$z - \frac{\Delta z_d 2^{96}}{d} = \frac{L\left(\sqrt{I} - \Delta\sqrt{I}\right)}{2^{96}}$$

$$\Delta\sqrt{I} = \frac{\Delta z_d 2^{192}}{dL}$$

$$\Delta x + \Delta y = L2^{96}\left(\frac{1}{\sqrt{I} - \Delta\sqrt{I}} - \frac{1}{\sqrt{I}}\right) = \frac{L2^{96}\Delta\sqrt{I}}{\sqrt{I}\left(\sqrt{I} - \Delta\sqrt{I}\right)}$$

Code

$\text{short amount with fees} = \text{short amount} + \text{fees}$

Fees is gain for LP, so round up.

$$\text{fees} = \left\lceil \frac{short\ amount\ f}{2^{16} - f} \right\rceil$$

$$\text{short amount with fees} = \text{short amount} + \text{fees} = \Delta z_d$$

Long is gain for the pool, thus the delta rate is gain. We round up delta rate.

$$\Delta\sqrt{I} = \left\lceil \frac{\Delta z_d 2^{192}}{dL} \right\rceil$$

Long is gain for the pool, so round up

$$\Delta x + \Delta y = \left\lceil \frac{L 2^{96} \Delta\sqrt{I}}{\sqrt{I}\left(\sqrt{I} - \Delta\sqrt{I}\right)} \right\rceil = \left\lceil \frac{\left\lceil \frac{L 2^{96} \Delta\sqrt{I}}{\sqrt{I}} \right\rceil}{\sqrt{I} - \Delta\sqrt{I}} \right\rceil$$

$$\text{long amount} = \Delta x + \Delta y$$

Case 1.3 Given $\Delta x + \Delta y + \Delta z_d \frac{(2^{16} - f)}{2^{16}}$ to get Δz_d then get $\sqrt{I} - \Delta\sqrt{I}$ and $\Delta x + \Delta y$

$$\Delta x + \Delta y + \Delta z_d \frac{(2^{16} - f)}{2^{16}} = \frac{L 2^{96} \Delta\sqrt{I}}{\sqrt{I}\left(\sqrt{I} - \Delta\sqrt{I}\right)} + \Delta z_d \frac{(2^{16} - f)}{2^{16}}$$

$$A = \Delta x + \Delta y + \Delta z_d \frac{(2^{16} - f)}{2^{16}}$$

$$\Delta\sqrt{I} = \frac{\Delta z_d 2^{192}}{dL}$$

$$\text{A} = \frac{L 2^{96} \left(\frac{\Delta z_d 2^{192}}{dL}\right)}{\sqrt{I}\left(\sqrt{I} - \frac{\Delta z_d 2^{192}}{dL}\right)} + \Delta z_d \frac{(2^{16} - f)}{2^{16}}$$

$$A\left(\sqrt{I} - \frac{\Delta z_d 2^{192}}{dL}\right) = \frac{L 2^{96} \left(\frac{\Delta z_d 2^{192}}{dL}\right)}{\sqrt{I}} + \Delta z_d \left(\frac{(2^{16} - f)}{2^{16}}\right)\left(\sqrt{I} - \frac{\Delta z_d 2^{192}}{dL}\right)$$

$$A\sqrt{I}-\frac{A2^{192}}{dL}\Delta z_d=\frac{2^{288}}{d\sqrt{I}}\Delta z_d+\frac{(2^{16}-f)\sqrt{I}}{2^{16}}\Delta z_d-\frac{2^{176}(2^{16}-f)}{dL}(\Delta z_d)^2$$

$$\frac{2^{176}(2^{16}-f)}{dL}(\Delta z_d)^2-\left(\frac{A2^{192}}{dL}+\frac{2^{288}}{d\sqrt{I}}+\frac{(2^{16}-f)\sqrt{I}}{2^{16}}\right)\Delta z_d+A\sqrt{I}=0$$

$$(\Delta z_d)^2-\left(\frac{A2^{16}}{2^{16}-f}+\frac{L2^{112}}{\sqrt{I}(2^{16}-f)}+\frac{dL\sqrt{I}}{2^{192}}\right)\Delta z_d+\frac{AdL\sqrt{I}}{2^{176}(2^{16}-f)}=0$$

$$a=1$$

$$b=-\left(\frac{A2^{16}}{2^{16}-f}+\frac{L2^{112}}{\sqrt{I}(2^{16}-f)}+\frac{dL\sqrt{I}}{2^{192}}\right)$$

$$c=\frac{AdL\sqrt{I}}{2^{176}(2^{16}-f)}$$

$$-b=\frac{A2^{16}}{2^{16}-f}+\frac{L2^{112}}{\sqrt{I}(2^{16}-f)}+\frac{dL\sqrt{I}}{2^{192}}$$

$$4ac=\frac{AdL\sqrt{I}}{2^{174}(2^{16}-f)}$$

$$\Delta z_d=\frac{-b-\sqrt{(-b)^2-4ac}}{2}$$

$$\Delta\sqrt{I}=\frac{\Delta z_d 2^{192}}{dL}$$

$$\Delta x+\Delta y=A-\Delta z_d\frac{(2^{16}-f)}{2^{16}}$$

Code

$\text{sum amount}=A$

Short is cost for the pool, thus negative b is cost, we round down.

$$-b=\left\lfloor\frac{dL\sqrt{I}}{2^{192}}\right\rfloor+\left\lfloor\frac{L2^{112}}{\sqrt{I}(2^{16}-f)}\right\rfloor+\left\lfloor\frac{A2^{16}}{2^{16}-f}\right\rfloor$$

Short is cost for the pool, thus 4ac is cost, we round down.

$$4ac = \left\lfloor \frac{AdL\sqrt{I}}{2^{174}(2^{16} - f)} \right\rfloor = \left\lfloor \frac{dL\sqrt{I}}{2^{174}(2^{16} - f)} \right\rfloor A$$

Short is cost for the pool, so round down.

$$\Delta z_d = \left\lfloor \frac{-b - \sqrt{(-b)^2 - 4ac}}{2} \right\rfloor$$

$$\text{short amount with fees} = \Delta z_d$$

Same reason as case 1.2, round up.

$$\Delta\sqrt{I} = \left\lceil \frac{\Delta z_d 2^{192}}{dL} \right\rceil$$

Fees is gain for LP, so round up

$$\text{fees} = \left\lceil \frac{\Delta z_d f}{2^{16}} \right\rceil$$

$$\text{short amount} = \text{short amount with fees} - \text{fees}$$

$$\text{long amount} = \text{sum amount} - \text{short amount}$$

Leverage transactions

Case 2.1 Given $\Delta x + \Delta y$ to get $\sqrt{I} + \Delta\sqrt{I}$ then get Δz_d

$$x + y - (\Delta x + \Delta y) = \frac{L2^{96}}{\sqrt{I} + \Delta\sqrt{I}}$$

$$\sqrt{I} + \Delta\sqrt{I} = \frac{L2^{96}}{\frac{L2^{96}}{\sqrt{I}} - (\Delta x + \Delta y)} = \frac{L2^{96}\sqrt{I}}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}}$$

$$\Delta z_d = \frac{\Delta\sqrt{I} dL}{2^{192}}$$

Code

$$\text{long amount with fees} = \text{long amount} + \text{fees}$$

Fees is gain for LP, so round up.

$$\text{fees} = \left\lceil \frac{long\ amount\ f}{2^{16} - f} \right\rceil$$

$$\text{long amount with fees} = \text{long amount} + \text{fees} = \Delta x + \Delta y$$

Short is gain for the pool, thus delta rate is gain. Round up delta rate, thus round up new rate.

$$\sqrt{I} + \Delta\sqrt{I} = \left\lceil \frac{L2^{96}\sqrt{I}}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}} \right\rceil$$

Short is gain for the pool, round up.

$$\Delta z_d = \left\lceil \frac{\Delta\sqrt{I} dL}{2^{192}} \right\rceil$$

Case 2.2 Given Δz_d to get $\sqrt{I} + \Delta\sqrt{I}$ then get $\Delta x + \Delta y$

$$z + \frac{\Delta z_d 2^{96}}{d} = \frac{L\left(\sqrt{I} + \Delta\sqrt{I}\right)}{2^{96}}$$

$$\Delta\sqrt{I} = \frac{\Delta z_d 2^{192}}{dL}$$

$$\Delta x + \Delta y = L2^{96}\left(\frac{1}{\sqrt{I}} - \frac{1}{\sqrt{I} + \Delta\sqrt{I}}\right) = \frac{L2^{96}\Delta\sqrt{I}}{\sqrt{I}\left(\sqrt{I} + \Delta\sqrt{I}\right)}$$

Code

$$\text{short amount} = \Delta z_d$$

Long is cost for the pool, delta rate is cost, round down.

$$\Delta\sqrt{I} = \left\lfloor \frac{\Delta z_d 2^{192}}{dL} \right\rfloor$$

Long is cost for the pool, round down.

$$\Delta x + \Delta y = \left\lfloor \frac{L2^{96}\Delta\sqrt{I}}{\sqrt{I}\left(\sqrt{I} + \Delta\sqrt{I}\right)} \right\rfloor$$

$$\text{long amount with fees} = \Delta x + \Delta y$$

Fees is gain for LP, so round up

$$\text{fees} = \left\lceil \frac{(\Delta x + \Delta y)f}{2^{16}} \right\rceil$$

$$\text{long amount} = \text{long amount with fees} - \text{fees}$$

Case 2.3 Given $(\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}} + \Delta z_d$ to get $\Delta x + \Delta y$, then get $\sqrt{I} + \Delta\sqrt{I}$ and Δz_d

$$(\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}} + \Delta z_d = (\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}} + \frac{\Delta\sqrt{I}dL}{2^{192}}$$

$$B = (\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}} + \Delta z_d$$

$$\Delta\sqrt{I} = \frac{L2^{96}\sqrt{I}}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}} - \sqrt{I} = \frac{L2^{96}\sqrt{I} - L2^{96}\sqrt{I} + (\Delta x + \Delta y)I}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}} = \frac{(\Delta x + \Delta y)I}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}}$$

$$B = (\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}} + \frac{dL}{2^{192}}\left(\frac{(\Delta x + \Delta y)I}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}}\right)$$

$$B\left(2^{96} - (\Delta x + \Delta y)\sqrt{I}\right) = (\Delta x + \Delta y)\frac{(2^{16}-f)}{2^{16}}\left(L2^{96} - (\Delta x + \Delta y)\sqrt{I}\right) + \frac{dL}{2^{192}}(\Delta x + \Delta y)I$$

$$B2^{96} - B\sqrt{I}(\Delta x + \Delta y) = L2^{80}(2^{16}-f)(\Delta x + \Delta y) - \frac{(2^{16}-f)\sqrt{I}}{2^{16}}(\Delta x + \Delta y)^2 + \frac{dLI}{2^{192}}(\Delta x + \Delta y)$$

$$\frac{(2^{16}-f)\sqrt{I}}{2^{16}}(\Delta x + \Delta y)^2 - \left(B\sqrt{I} + L2^{80}(2^{16}-f) + \frac{dLI}{2^{192}}\right)(\Delta x + \Delta y) + B2^{96} = 0$$

$$(\Delta x + \Delta y)^2 - \left(\frac{B2^{16}}{2^{16}-f} + \frac{L2^{96}}{\sqrt{I}} + \frac{dL\sqrt{I}}{2^{176}(2^{16}-f)}\right)(\Delta x + \Delta y) + \frac{B2^{112}}{(2^{16}-f)\sqrt{I}} = 0$$

$$a = 1$$

$$b = -\left(\frac{B2^{16}}{2^{16} - f} + \frac{L2^{96}}{\sqrt{I}} + \frac{dL\sqrt{I}}{2^{176}(2^{16} - f)}\right)$$

$$c = \frac{B2^{112}}{(2^{16} - f)\sqrt{I}}$$

$$-b = \frac{B2^{16}}{2^{16} - f} + \frac{L2^{96}}{\sqrt{I}} + \frac{dL\sqrt{I}}{2^{176}(2^{16} - f)}$$

$$4ac = \frac{B2^{114}}{(2^{16} - f)\sqrt{I}}$$

$$\Delta x + \Delta y = \frac{-b - \sqrt{(-b)^2 - 4ac}}{2}$$

$$\sqrt{I} + \Delta\sqrt{I} = \frac{L2^{96}\sqrt{I}}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}}$$

$$\Delta z_d = B - (\Delta x + \Delta y)\frac{(2^{16} - f)}{2^{16}}$$

Code

$$\text{sum amount} = B$$

Long is cost to the pool, round down.

$$-b = \left\lfloor\frac{B2^{16}}{2^{16} - f}\right\rfloor + \left\lfloor\frac{L2^{96}}{\sqrt{I}}\right\rfloor + \left\lfloor\frac{dL\sqrt{I}}{2^{176}(2^{16} - f)}\right\rfloor$$

Long is cost to the pool, round down.

$$4ac = \left\lfloor\frac{B2^{114}}{(2^{16} - f)\sqrt{I}}\right\rfloor$$

Long is cost to the pool, round down.

$$\Delta x + \Delta y = \left\lfloor\frac{-b - \sqrt{(-b)^2 - 4ac}}{2}\right\rfloor$$

$\text{long amount with fees} = \Delta x + \Delta y$

Same reason as case 2.1, round up.

$$\sqrt{I} + \Delta\sqrt{I} = \left\lceil \frac{L2^{96}\sqrt{I}}{L2^{96} - (\Delta x + \Delta y)\sqrt{I}} \right\rceil$$

Fees is gain for LP, so round up

$$\text{fees} = \left\lceil \frac{(\Delta x + \Delta y)f}{2^{16}} \right\rceil$$

$\text{long amount} = \text{long amount with fees} - \text{fees}$

$\text{short amount} = \text{sum amount} - \text{long amount}$

Mint transactions

Case 3.1 Given ΔL to get $\Delta x + \Delta y$ and Δz_d

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

$$\Delta z_d = \frac{\Delta L d\sqrt{I}}{2^{192}}$$

Code

Long is gain for the pool, round up

$$\Delta x + \Delta y = \left\lceil \frac{\Delta L 2^{96}}{\sqrt{I}} \right\rceil$$

Short is gain for the pool, round up

$$\Delta z_d = \left\lceil \frac{\Delta L d\sqrt{I}}{2^{192}} \right\rceil$$

Case 3.2 Given $\Delta x + \Delta y$ to get ΔL then get Δz_d

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

$$\Delta L = \frac{(\Delta x + \Delta y)\sqrt{I}}{2^{96}}$$

$$\Delta z_d = \frac{\Delta L d \sqrt{I}}{2^{192}}$$

Code

Liquidity minted is cost for the pool, round down

$$\Delta L = \left\lfloor \frac{(\Delta x + \Delta y)\sqrt{I}}{2^{96}} \right\rfloor$$

Short is gain for the pool, round up

$$\Delta z_d = \left\lceil \frac{\Delta L d \sqrt{I}}{2^{192}} \right\rceil$$

Case 3.3 Given Δz_d to get ΔL then get $\Delta x + \Delta y$

$$\Delta z_d = \frac{\Delta L d \sqrt{I}}{2^{192}}$$

$$\Delta L = \frac{\Delta z_d 2^{192}}{d\sqrt{I}}$$

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

Code

Liquidity minted is cost for the pool, round down

$$\Delta L = \left\lfloor \frac{\Delta z_d 2^{192}}{d\sqrt{I}} \right\rfloor$$

Long is gain for the pool, round up

$$\Delta x + \Delta y = \left\lceil \frac{\Delta L 2^{96}}{\sqrt{I}} \right\rceil$$

Case 3.4 Given larger to get ΔL then get $\Delta x + \Delta y$ and Δz_d

Just a combination of case 3.2 and 3.3

Burn transactions

Case 4.1 Given ΔL to get $\Delta x + \Delta y$ and Δz_d

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

$$\Delta z_d = \frac{\Delta L d \sqrt{I}}{2^{192}}$$

Code

Long is cost for the pool, round down

$$\Delta x + \Delta y = \left\lfloor \frac{\Delta L 2^{96}}{\sqrt{I}} \right\rfloor$$

Short is cost for the pool, round down

$$\Delta z_d = \left\lfloor \frac{\Delta L d \sqrt{I}}{2^{192}} \right\rfloor$$

Case 4.2 Given $\Delta x + \Delta y$ to get ΔL then get Δz_d

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

$$\Delta L = \frac{(\Delta x + \Delta y)\sqrt{I}}{2^{96}}$$

$$\Delta z_d = \frac{\Delta L d \sqrt{I}}{2^{192}}$$

Code

Liquidity burn is gain for the pool, round up

$$\Delta L = \left\lceil \frac{(\Delta x + \Delta y)\sqrt{I}}{2^{96}} \right\rceil$$

Short is cost for the pool, round down

$$\Delta z_d = \left\lfloor \frac{\Delta L d \sqrt{I}}{2^{192}} \right\rfloor$$

Case 4.3 Given Δz_d to get ΔL then get $\Delta x + \Delta y$

$$\Delta z_d = \frac{\Delta L d \sqrt{I}}{2^{192}}$$

$$\Delta L = \frac{\Delta z_d 2^{192}}{d\sqrt{I}}$$

$$\Delta x + \Delta y = \frac{\Delta L 2^{96}}{\sqrt{I}}$$

Code

Liquidity burn is gain for the pool, round up

$$\Delta L = \left\lceil \frac{\Delta z_d 2^{192}}{d\sqrt{I}} \right\rceil$$

Long is cost for the pool, round down

$$\Delta x + \Delta y = \left\lfloor \frac{\Delta L 2^{96}}{\sqrt{I}} \right\rfloor$$

Case 4.4 Given larger to get ΔL then get $\Delta x + \Delta y$ and Δz_d

Just a combination of case 4.2 and 4.3