Chapter 4:

Exercise 4.1

1. $y = x^3 - 12x^2 + 36x$

A. Domain is $(-\infty, \infty)$

B. For x-intercept, put $y = 0$, so $x^3 - 12x^2 + 36x = 0$

i.e. $x(x^2 - 12x + 36) = 0$

i.e. $x(x - 6)(x - 6) = 0$

$\therefore$ x intercepts $x = 0, 6$

For y-intercept, put $x = 0$, so $y = 0$

C. Symmetry: Here $f(-x) = -x^3 - 12x^2 - 36x = -(x^3 + 12x^2 + 36x)$

$\therefore f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

So given function neither even nor odd.

D. Asymptote: Here $\lim_{x\to\infty} f(x)$ = not exist, so has no horizontal asymptote. Also has no vertical asymptote (Domain is $\Re$)

E. Interval of increasing and decreasing

$f(x) = 3x^2 - 24x + 36$

Let $f'(x) = 0$

$\Rightarrow x^2 - 8x + 12 = 0$

$\Rightarrow (x - 2)(x - 6) = 0$

$\therefore x = 2, 6$

Interval	$(-\infty, 2)$	$(2, 6)$	$(6, \infty)$
Sign of $f'(x)$	+ve	−ve	+ve
Nature of $f(x)$	Increasing	Decreasing	Increasing

F. Maxima occur at $x = 2$ i.e. at point $(2, f(2)) = (2, 32)$

Minima occur at $x = 6$ i.e. at point $(6, f(6)) = (6, 0)$

G. Concavity and point of intersection

$f''(x) = 6x - 24$

For $f''(x) = 0$

$\Rightarrow x = 4$

Interval	$(-\infty, 4)$	$(4, \infty)$
Sign of $f''(x)$	−ve	+ve
Nature of $f(x)$	Concave down	Concave up

Point of inflection is at $x = 4$ i.e. at $(4, f(u)) = (4, 16)$

Summarizing table on E and G

Interval	$(-\infty, 2)$	$(2, 4)$	$(4, 6)$	$(6, \infty)$
Nature of $f(x)$	Increasing	Decreasing	Decreasing	Increasing
Nature of $f(x)$	Concave down	Concave down	Concave up	Concave up

Chapter 4:

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i.e. $x(x^2 - 12x + 36) = 0$

i.e. $x(x-6)(x-6) = 0$

$\therefore$ x intercepts $x = 0, 6$

For y-intercept, put $x = 0$, so $y = 0$

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$\therefore f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

So given function neither even nor odd.

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Sign of $f'(x)$	+ve	−ve	+ve
Nature of $f(x)$	Increasing	Decreasing	Increasing

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G. Concavity and point of intersection

$f''(x) = 6x - 24$

For $f''(x) = 0$

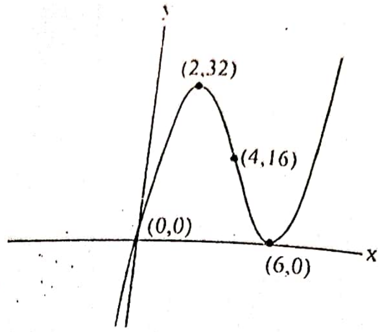
$\Rightarrow x = 4$

Interval	$(-\infty, 4)$	$(4, \infty)$
Sign of $f''(x)$	−ve	+ve
Nature of $f(x)$	Concave down	Concave up

Point of inflection is at $x = 4$ i.e. at $(4, f(u)) = (4, 16)$

Summarizing table on E and G

Interval	$(-\infty, 2)$	$(2, 4)$	$(4, 6)$	$(6, \infty)$
Nature of $f(x)$	Increasing	Decreasing	Decreasing	Increasing
Nature of $f(x)$	Concave down	Concave down	Concave up	Concave up



2. $f(x) = 2 + 3x^2 - x^3$

A. Domain is $(-\infty, \infty)$

B. Intercepts: For x-intercept

Put $y = 0$, $2 + 3x^2 - x^3$

C. Asymptote: $\lim_{a \to \infty} f(x)$ = not exist, so there is not horizontal Asymptote. There has no vertical asymptote (domain . $\mathfrak{R}$).

D. Symmetry: Here $f(-x) \neq f(x)$ and

$f(-x) \neq -f(x)$

So it is neither even nor odd function.

E. Interval of increasing and decreasing

Here, $f'(x) = 6x - 3x^2$

Let $f'(x) = 0$

$\Rightarrow 6x - 3x^2 = 0$

$3x(2 - x) = 0$

$\therefore x = 0, 2$

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Sign of f'(x)	Decreasing	Increasing	Decreasing
Nature of f(x)	-ve	+ve	-ve

F. Minima occurs at $x = 0$, i.e. $(0, f(0)) = (0, 2)$

Maxima occurs at $x = 2$, i.e. $(2, f(2)) = (2, 6)$

G. Concavity and point of inflection

$f''(x) = 6 - 6x$

Let $f''(x) = 0$

$\Rightarrow 6 - 6x = 0$

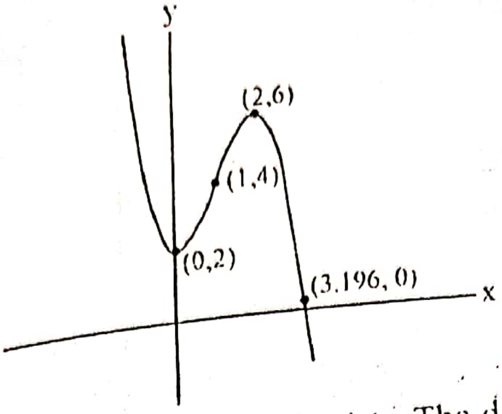
$\Rightarrow x = 1$

Interval	$(-\infty, 1)$	$(1, \infty)$
Sign of f''(x)	+ve	-ve
Nature of f(x)	Concave up	Concave down

Point of inflection is at $x = 1$, i.e. at $(1, f(1)) = (1, 4)$

H. Summarizing table E and G

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, \infty)$
Nature of f(x)	Decreasing Concave up	Increasing Concave up	Decreasing Concave down	Decreasing Concave down



3. $y = x^4 - 4x^3 + 10$
f is continuous since $f'(x) = 4x^3 - 12x^2$ exists. The domain of f is $(-\infty, \infty)$, and the domain of f' is also $(-\infty, \infty)$. Thus, the critical points of f occur only at the zeros of f'. Since
$f'(x) = 4x^3 - 12x^2 = 4x^2 (x - 3)$
the first derivative is zero at $x = 0$ and $x = 3$.

Intervals	$x < 0$	$0 < x < 3$	$3 < x$
Sign of f'	Negative	Negative	Positive
Behavior of f	Decreasing	Decreasing	Increasing

a. Using the First Derivative Test for local extrema and the table above, we see that there is no extremum at $x = 0$ and a local minimum at $x = 3$.

b. Using the table above, we see that f is decreasing on $(-\infty, 0]$ and $[0, 3]$, and increasing on $[3, \infty)$.

c. $f''(x) = 12x^2 - 24x = 12x (x - 2)$ is zero at $x = 0$ and $x = 2$.

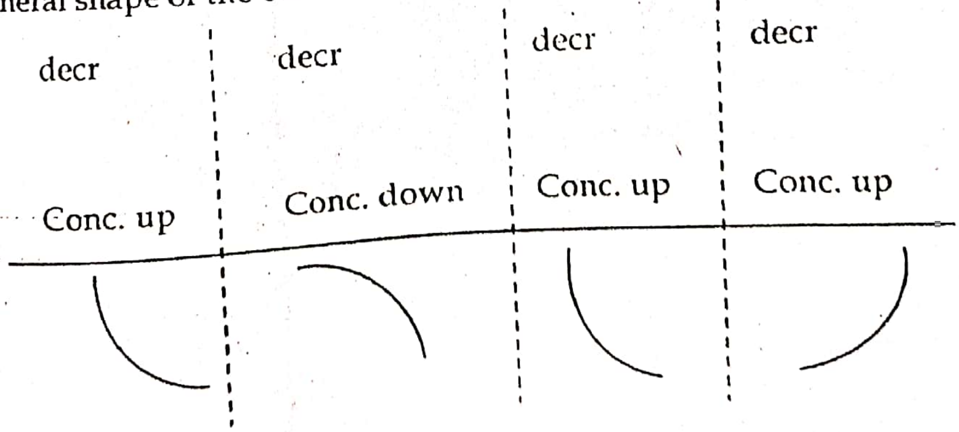
Intervals	$x < 0$	$0 < x < 2$	$2 < x$
Sign of f'	Positive	Negative	Positive
Behavior of f	Concave up	Concave down	Concave up

We see that f is concave up on the intervals $(-\infty, 0)$ and $(2, \infty)$, and concave down on 0, 2.

d. Summarizing the information in the two tables above, we obtain

$x < 0$	$0 < x < 2$	$2 < x < 3$	$3 < x$
Decreasing	Decreasing	Decreasing	Increasing
Concave up	Concave down	Concave up	Concave up

The general shape of the curve is



4. $y = x - 3x^{1/3}$

A: Domain is $(-\infty, \infty)$

B: Intercept: For x-intercept, put y = 0

$$x - 3x^{1/3} = 0$$
$$x^{1/3}(x^{2/3} - 3) = 0$$

i.e.
$$x = 0,\ x^{2/3} = 3$$
$$x = \pm 5.196$$

For y intercept, put x = 0, y = 0

C. Symmetry:

$$f(-x) = -x - 3(-x)^{1/3}$$
$$= -x - 3x - 1\,(x)^{1/3}$$
$$= -x + 3x^{1/3}$$
$$= -(x - 3x^{1/3})$$
$$= -f(x)$$

∴ This is odd function.

D. Asymptote:

Here $\lim\limits_{x\to\infty} f(x) = \infty$. So has no horizontal asymptote.

Also has no vertical asymptote (domain is $\Re$).

E. Interval of increasing and decreasing

Here $f'(x) = 1 - \dfrac{1}{x^{2/3}} = \dfrac{x^{2/3} - 1}{x^{2/3}}$

For $f'(x) = 0$

$\Rightarrow$ $x^{2/3} = 1$

$x^2 = (1)^{1/3}$

$x^2 = 1$

$x = \pm 1$

For $f'(x) = \infty \Rightarrow x = 0$

Thus, x = 0, 1, – 1

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of f'(x)	Increasing	Decreasing	Increasing	Increasing
Nature of f(x)	Positive	Negative	Negative	Positive

Maxima at x = –1, i.e. at (–1, f(–1)) = (–1, 2)

Minima at x = 1, i.e. at (1, f(1)) = (1, –2)

F. Concavity and point of inflection.

$$f''(x) = \frac{2}{3x^{5/3}}$$

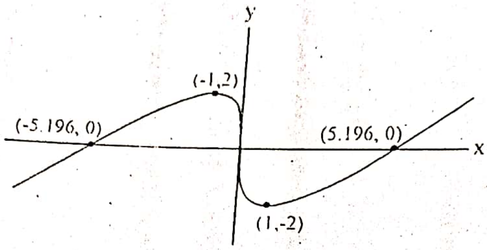
Let $f''(x) = 0$

$\Rightarrow$ $x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of $f''(x)$	Negative	Positive
Nature of $f(x)$	Concave down	Concave up

G. Summarizing the table E and G, we get-

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $f'(x)$	Increasing	Decreasing	Decreasing	Increasing
$f(x)$	Concave down	Concave down	Concave up	Concave up



5. $y = \frac{5}{2}x^{2/3} - x^{5/3}$

A: Domain is $(-\infty, \infty)$

B: Intercepts: For x-intercept, put $y = 0$

$$\frac{5}{2}x^{2/3} - x^{5/3} = 0$$

$$\frac{5}{2}x^{2/3} - x^{5/3}\, x^{2/3}\, x^{-2/3} = 0$$

$$x^{2/3}\left[\frac{5}{2} - x\right] = 0$$

$\therefore$ $x = 0$ and $x = 5/2 = 2.50$

$\therefore$ Curve meet x-axis at $x = 0$ and $x = 5/2$

For y-intercepts, $x = 0$, $y = 0$

C: Symmetric:

Here, $f(-x) = \frac{5}{2}(-x)^{2/3} - (-x)^{5/3}$

$$= \frac{5}{2}x^{2/3} + x^{5/2} \neq f(x) \text{ and}$$

$$\neq -f(x)$$

So curve is neither even nor odd.

D: Asymptote: Here $\lim_{a\to\infty} f(x) = \infty$. So there is no horizontal asymptote/ Here, has no vertical asymptote (Domain is $\Re$)

E: Interval of increasing and decreasing

$$f'(x) = \frac{5}{2} \times \frac{2}{3}x^{-1/3} - \frac{5}{3}x^{2/3}$$

$$= \frac{5}{3}x^{-1/3} - \frac{5}{3}x^{2/3}$$

$$= \frac{5}{3}x^{-1/3} - \frac{5}{3}x^{-1/3}x^{1/3}x^{2/3}$$

$= \frac{5}{3}x^{-1/3}(1 - x)$

$= \frac{-5(x-1)}{3x^{1/3}}$

Let $f'(x) = 0 \Rightarrow x = 1$ and

$f'(x) = \infty \Rightarrow x = 0$

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Sign of $f'(x)$	Negative	Positive	Negative
Nature of $f(x)$	Decreasing	Increasing	Decreasing

F: So Maxima at $x = 1$ i.e. at $(1, 3/2)$

Minima at $x = 0$ i.e. at $(0, 0)$

G: Concavity and point of inflection

$f''(x) = \frac{5}{3} \times \frac{-1}{3}x^{-4/3} - \frac{10}{9}x^{-1/3}$

$= -\frac{5}{9}x^{-4/3} - \frac{10}{9}x^{-1/3}$

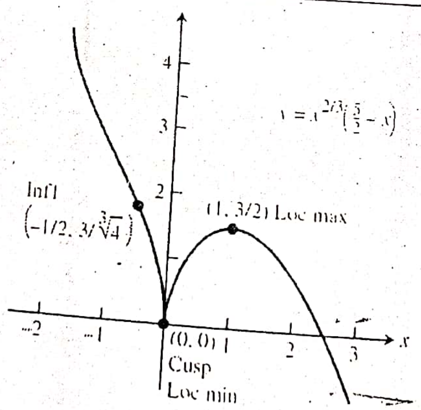
$= -\frac{5}{9}x^{-4/3} - \frac{10}{9}x^{-1/3}x^{-4/3}x^{4/3}$

$= -\frac{5}{9}x^{-4/3}(1 + 2x)$

Let $f''(x) = 0 \Rightarrow x = -\frac{1}{2}$

$f''(x) = \infty \Rightarrow x = 0$

Interval	$(-\infty, -1/2)$	$(-1/2, 0)$	$(0, \infty)$
Sign of $f''(x)$	Positive	Negative	Negative
Nature of $f(x)$	Concave up	Concave down	Concave down



6. $y = x^{5/3} - 5x^{2/3}$

A: Domain is set of all real numbers i.e. $(-\infty, \infty)$

B: Intercepts: y intercept is $y = 0$ (Put $x = 0$, weight $y = 0$)

x intercept is $x = 0, 5$ (Put $y = 0$, we get $x = 0, 5$)

C: Symmetry: sina $f(-x) = (-x)^{5/3} - 5(-x)^{2/3}$

$= -x^{5/3} - x^{2/3}$

$= -(x^{5/3} + x^{2/3})$

$\therefore \quad f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

Thus, given function is neither even nor odd.

D: Asymptotes: since $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to -\infty} f(x) = -\infty$

Thus, has no horizontal asymptotes. Also has no vertical asymptotes (domain is ℜ)

E: Interval of increasing and decreasing

$$f'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$$

$$= \frac{5(x-2)}{3x^{1/3}}$$

Let $f'(x) = 0$ then $x = 2$ and

$f'(x) = \infty$ then $x = 0$

Intervals	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
Sign of $f'(x)$	Positive	Negative	Positive
Nature of $f(x)$	Increasing	Decreasing	Increasing

F: Maxima at $x = 0$ i.e. at $(0, f(0)) = (0, 0)$

Minima at $x = 2$ i.e. at $(2, f(2)) = (2, -4.76)$

G: Concavity and point of inflection

$$f''(x) = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3}$$

$$= \frac{10(x+1)}{9x^{4/3}}$$

Let $f''(x) = 0 \Rightarrow x = -1$

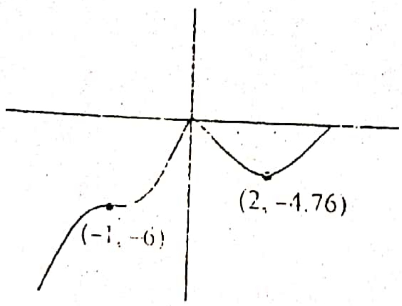
$f''(x) = \infty \Rightarrow x = 0$

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
Sign of $f''(x)$	Negative	Positive	Positive
Nature of $f(x)$	Concave down	Concave up	Concave up

Point of inflection is at $x = -1$ i.e. at $(-1, f(-1)) = (-1, -6)$

H: Summarizing the table at E and G

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, 2)$	$(2, \infty)$
Nature of $f(x)$	Increasing Concave down	increasing Concave up	Decreasing Concave up	Increasing Concave up



7. $y = \frac{x}{x-1}$

A: Domain set of all real number except 1 i.e. $(-\infty, 1) \cup (1, \infty)$.

B: Intercepts: For x-intercept, put $y = 0$, $x = 0$

for y-intercept, put $x = 0$, $y = 0$

∴ Curve meet the both axises at $(0, 0)$

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C: Symmetry: $f(-x) = \frac{+x}{x+1} \neq f(x)$ and

$\neq -f(x)$

$\therefore$ Function is neither even nor odd.

D: Asymptote: Here $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x}{x-1} = \lim_{x \to -\infty} \frac{1}{1 - \frac{1}{x}} = 1$

and $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x}{x-1} = \lim_{x \to \infty} \frac{1}{1 - \frac{1}{x}} = 1$

$\therefore$ $y = 1$ is horizontal asymptote.

Because of rational function $\lim_{x \to 1^+} \frac{x}{x-1} = \infty$ and

$\lim_{x \to 1^-} \frac{x}{x-1} = -\infty$

Thus, $x = 1$ is vertical asymptote

E: Interval of increasing and decreasing

Here $f'(x) = \frac{(x-1)-x}{(x-1)^2} = \frac{-1}{(x-1)^2}$

Let $f'(x) = \infty$ then $x = 1$

Interval	$(-\infty, 1)$	$(1, \infty)$
Sign of $f'(x)$	Negative	Negative
Nature of $f(x)$	Decreasing	Decreasing

F: There is no maxima and minima

G: Concavity and point of infiection

Here $f''(x) = \frac{2}{(x-1)^3}$

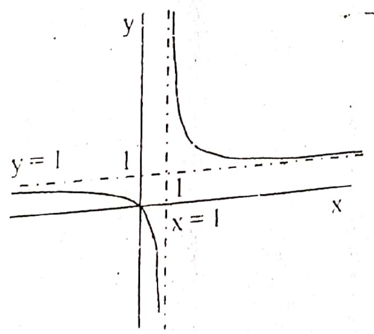
Let $f''(x) = \infty$ then $x = 1$

Interval	$(-\infty, 1)$	$(1, \infty)$
Sign $f''(x)$	Positive	Positive
Nature of $f(x)$	Concave down	Concave up

There is no point of inflection.

H: Summarizing table of E and G

Interval	$(-\infty, 1)$	$(1, \infty)$
Nature of $f(x)$	Decreasing	Decreasing
	Concave down	Concave up



8. $y = \frac{x^2}{x^2+9}$

A: Domain is $(-\infty, \infty)$

B: Intercepts: x intercept is $x = 0$

y intercept is $y = 0$

C: Symmetry: Here, $f(-x) = \frac{x^2}{x^2+9} = f(x)$. Given function is even function. So, symmetrical about y-axis.

D: Asymptote: For horizontal asymptote,

$$\lim_{x\to\pm\infty} \frac{x^2}{x^2+9} = \lim_{x\to\pm\infty} \frac{1}{1+\frac{9}{x^2}} = 1$$

Thus, $y = 1$, is horizontal asymptote.

There is no vertical asymptote because domain is $\Re$.

E: Interval of increasing and decreasing

$$f'(x) = \frac{(x^2+9)\,2x - x^2 \times 2x}{(x^2+9)} = \frac{18x}{(x^2+9)^2}$$

Let $f'(x) = 0$ then $x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of f'(x)	Negative	Positive
Nature of f(x)	Decreasing	Increasing

F: Minima occur at $x = 0$, i.e. at $(0, f(0)) = (0, 0)$

G: Concavity and point of inflection

$$f''(x) = \frac{(x^2+9)^2 \cdot 18 - 18x\,(x^2+9)\cdot 2x}{(x^2+9)^4} = \frac{54\,(x^2-3)}{(x^2+9)^3}$$

Let $f''(x) = 0$ then $x = \pm\sqrt{3} = \pm 1.732$

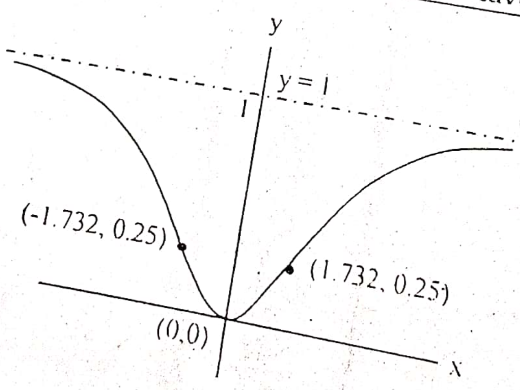
Interval	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, \sqrt{3})$	$(-\sqrt{3}, \infty)$
Sign of f''(x)	Negative	Positive	Negative
Nature of f(x)	Concave down	Concave up	Concave down

Point of inflection are at $x = -\sqrt{3}$ and $x = \sqrt{3}$ i.e.

at $(-\sqrt{3}, f(-\sqrt{3}))$, $(\sqrt{3}, f(\sqrt{3})) = (-\sqrt{3}, 0.25)$, $(\sqrt{3}, 0.25)$

H: Summarizing the table on E and G

Interval	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, \infty)$
Nature of f(x)	Decreasing Concave down	Decreasing Concave up	Increasing Concave up	Increasing Concave down



9. $y = \frac{x^2}{x^2 + 3}$.

A: Domain is set of all real numbers i.e. $(-\infty, \infty)$

B: Intercepts: x-intercept, $x = 0$ (Put $y = 0$)

y-intercept, $y = 0$ (put $x = 0$)

C: Symmetry: Here, $f(-x) = f(x)$. Thus, given function is even so symmetric about y-axis.

D: Asymptote: Here $\lim_{x \to \pm\infty} \frac{x^2}{x^2 + 3} = 1$

Thus, $y = 1$ is horizontal asymptote.

There is no vertical asymptote, because domain is $\Re$.

E: Interval of increasing and decreasing

$$f'(x) = \frac{(x^2 = 3)\, 2x - x^2\,(2x)}{(x^2 + 3)^2} = \frac{6x}{(x^2 + 3)^2}$$

Let $f'(x) = 0 \Rightarrow x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of $f'(x)$	Negative	Positive
Nature of $f(x)$	Decreasing	Increasing

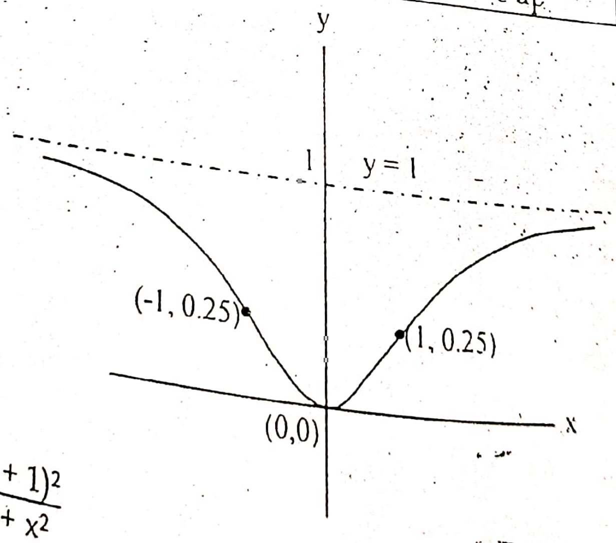
F: There is minima at point $x = 0$ i.e. at $(0, f(0)) = (0, 0)$

G: Concavity and point of inflection

$$f''(x) = \frac{(x^2 + 3)^2 . 6 - (6x)^2\,(x^2 + 3)\,(2x)}{(x^2 + 3)^4} = \frac{-18x^2 + 18}{(x^2 + 3)^3}$$

Let $f''(x) = 0 \Rightarrow x = \pm 1$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of $f''(x)$	Negative	Positive	Negative
Nature of $f(x)$	Concave down	Concave up	Concave down



10. $f(x) = \frac{(x + 1)^2}{1 + x^2}$

A: The domain of f is $(-\infty, \infty)$ and there are no symmetries about either axis or the origin.

B: Find f' and f''

$$f(x) = \frac{(x + 1)^2}{1 + x^2}$$

$$f'(x) = \frac{(1 + x^2) . 2(x + 1) - (x + 1)^2 . 2x}{(1 + x^2)^2} = \frac{2\,(1 - x^2)}{(1 + x^2)^2}$$

$$f''(x) = \frac{(1+x^2)^2 \cdot 2(-2x) - 2(1-x^2)[2(1+x^2) \cdot 2x]}{(1+x^2)^4} = \frac{4x(x^2-3)}{(1+x^2)^3}$$

C: Behavior at critical points. The critical points occur only at $x = \pm 1$ where $f'(x) = 0$. Since f' exists everywhere over the domain of f. At $x = -1$, $f''(-1) = 1 > 0$ yielding a relative minimum by the second derivative test. At $x = 1$, $f''(1) = -1 < 0$ yielding a relative maximum by the second derivative test. We will see in step 6 that both are absolute extrema as well.

D: Increasing and decreasing. We see that on the interval $(-\infty, -1)$ the derivative $f'(x) < 0$, and the curve is decreasing. On the interval $(-1, 1)$, $f'(x) > 0$ and the curve is increasing; it is decreasing on $(1, \infty)$ where $f'(x) < 0$ again.

E: Inflection points. Notice that the denominator of the second derivative is always positive. The second derivative f" is zero when $x = -\sqrt{3}$, 0, and $\sqrt{3}$. The second derivative changes sign at each of these points: negative on $(-\infty, -\sqrt{3})$, positive on $(-\sqrt{3}, 0)$; negative on $(0, \sqrt{3})$, and positive again on $(\sqrt{3}, \infty)$. Thus each point is a point of inflection. The curve is concave down on the interval $(-\infty, -\sqrt{3})$, concave up on $(-\sqrt{3}, 0)$, concave down on $(0, \sqrt{3})$, and concave up again on $(\sqrt{3}, \infty)$.

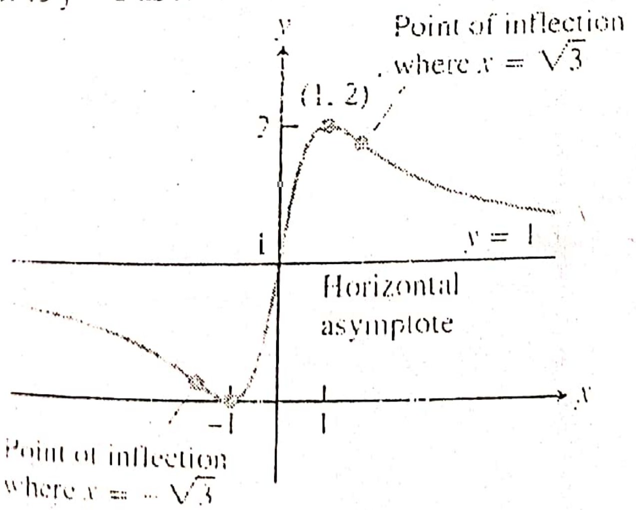
F: Asymptotes. Expanding the numerator of f(x) and then dividing both numerator and denominator by x^2 gives

$$f(x) = \frac{(x+1)^2}{1+x^2} = \frac{x^2+2x+1}{1+x^2} = \frac{1+(2/x)+(1/x^2)}{(1/x^2)+1}$$

We see that $f(x) \to 1^+$ as $x \to \infty$ and that $f(x) \to 1^-$ as $x \to -\infty$. Thus, the line $y = 1$ is a horizontal asymptote.

Since f decreases on $(-\infty, -1)$ and then increases on $(-1, 1)$, we know that $f(-1) = 0$ is a local minimum. Although f decreases on $(1, \infty)$, it never crosses the horizontal asymptote $y = 1$ on that interval (it approaches the asymptote from above). So the graph never becomes negative, and $f(-1) = 0$ is an absolute minimum as well. Likewise, $f(1) = 2$ is an absolute maximum because the graph never crosses the asymptote $y = 1$ on the interval $(-\infty, -1)$, approaching it from below. Therefore, there are no vertical asymptotes (the range of f is $0 \le y \le 2$).

F: The graph of f is sketched in figure. Notice how the graph is concave down as it approaches the horizontal asymptote $y = 1$ as $x \to -\infty$, and concave up in its approach to $y = 1$ as $x \to \infty$.



11. $y = \sqrt{x^2 + x - 2}$

A: Domain: $x^2 + x - 2 \geq 0$

i.e. $(x + 2)(x - 1) \geq 0$

$\therefore$ $x \leq -2, x \geq 1$

Thus, domain is $(-\infty, -2] \cup [1, \infty)$

B: Intercepts: For x-intercepts, put $y = 0$

So $x = -2$ and $x = 1$

Curve meet x axis at $x = -2$ and $x = 1$

For y-intercept put $x = 0$

$y = \sqrt{-2}$ = no real point

C: Symmetry: $f(-x) = \sqrt{x^2 - x - 2} \neq f(x)$

$\neq -f(x)$

Thus neither even nor odd function.

D: Asymptote: $\lim\limits_{x \to \infty} f(x) = \infty$, so no horizontal asymptote. Also there has no vertical asymptote.

E: Interval of increasing and decreasing

Here, $f'(x) = \dfrac{2x + 1}{2\sqrt{x^2 + x - 1}}$

Let $f'(x) = 0$ then $x = -\dfrac{1}{2}$ (outside the domain)

$f'(x) = \infty$ then $x = -2, 1$

Interval	$(-\infty, -2)$	$(1, \infty)$
Sign of $f'(x)$	Negative	Positive
Nature of $f(x)$	Decreasing	Increasing

Here interval $(-2, 1)$ is no need to take because this interval is outside of domain.

F: There is no maxima and minima.

G: Concavity and point of inflection

$f''(x) = \dfrac{-9}{4(x^2 + x - 2)^{3/2}}$

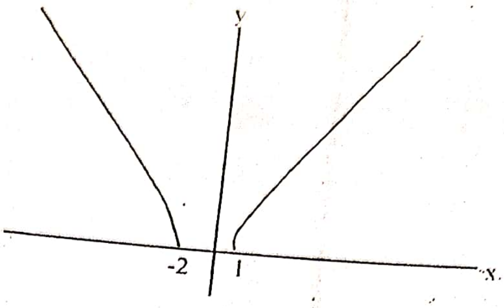
Here $f''(x) = \infty$ then $x = -2, 1$

Interval	$(-\infty, -2)$	$(1, \infty)$
Sign of $f''(x)$	Negative	Negative
Nature of $f(x)$	Concave down	Concave down

There is no point of inflection.

H: Summarizing the tables on E and G

Interval	$(-\infty, -2)$	$(1, \infty)$
Nature of $f(x)$	Decreasing	Increasing
	Concave down	Concave down



12. $y = xe^x$

A: The domain is $\Re$.

B: The x- and y-intercepts are both 0.

C: Symmetry: None

D: Because both x and e^x become large as $x \to \infty$, we have $\lim_{x\to\infty} xe^x = \infty$.

As $x \to -\infty$, however, $e^x \to 0$ and so we have an indeterminate product that requires the use of l'Hospital's Rule:

$$\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{e^{-x}} = \lim_{x\to-\infty} \frac{1}{-e^{-x}} = \lim_{x\to-\infty} (-e^x) = 0$$

Thus the x-axis is a horizontal asymptote.

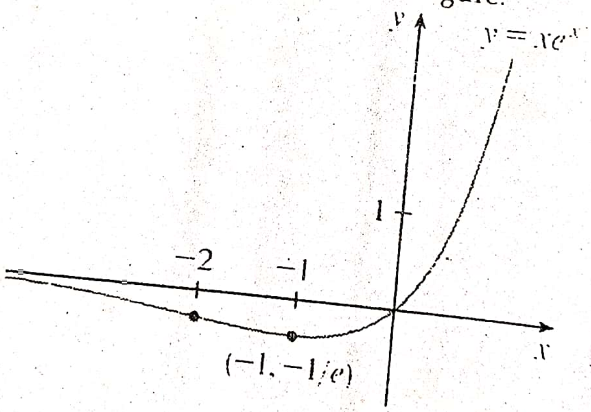
E: $f'(x) = xe^x + e^x = (x + 1)\, e^x$

Since e^x is always positive, we see that $f'(x) > 0$ when $x + 1 > 0$, and $f'(x) < 0$ when $x + 1 < 0$. So f is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$.

F: $f''(x) = (x + 1)\, e^x + e^x = (x + 2)\, e^x$

Since $f''(x) > 0$ if $x > -2$ and $f''(x) < 0$ if $x < -2$, f is concave upward on $(-2, \infty)$ and concave downward on $(-\infty, -2)$. The inflection point is $(-2, -2e^{-2})$.

H: We use this information to sketch the curve figure.



13. $y = (1 - x)\, e^x$

A: Domain is set of all real numbers i.e. $(-\infty, \infty)$

B: Intercepts: For x-intercept put $y = 0$ then $e^x (1 - x) = 0$

$\Rightarrow e^x = 0$ has no solution

and $(1 - x) = 0$

$\Rightarrow x = 1$

For y intercept put $x = 0$, then $y = 1$

Thus, this curve meet x-axis at $x = 1$ and y axis at $y = 1$

C: Symmetry: Here $f(-x) = (1 + x)\, e^{-x} \neq f(x)$
and $\neq -f(x)$

So neither even nor odd function.

D: Asymptotes: Here $\lim_{x \to -\infty} (1 - x)\, e^{x} = \lim_{x \to -\infty} \frac{1 - x}{e^{-x}}$

$= \lim_{x \to -\infty} \frac{-1}{-e^{-x}}$

$= \lim_{x \to -\infty} \frac{1}{e^{-x}}$

$= 0$

$\therefore$ $y = 0$ is horizontal asymptote.

(Here, $\lim_{x \to \infty} (1 - x)\, e^{x} = -\infty$)

There is no vertical asymptote.

uct that

E: Interval of increasing and decreasing.

Here $f'(x) = e^{x}(1 - x)\,(-1)\, e^{x}$

$= -x\, e^{x}$

Let $f'(x) = 0$

$\Rightarrow$ $x = 0$

Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of $f'(x)$	Positive	Negative
Nature of $f(x)$	Increasing	Decreasing

$) < 0$

F: Maxima is occur at $x = 0$ i.e. at $(0, f(0)) = (0, 1)$

G: $f''(x) = (-x)\, e^{x} + e^{x}(-1)$

$= -e^{x}(x + 1)$

$, \infty)$

Let $f''(x) = 0$

$\Rightarrow$ $x = -1$

Interval	$(-\infty, -1)$	$(-1, \infty)$
Sign of $f'(x)$	Positive	Negative
Nature of $f(x)$	Concave up	Concave down

The point of inflection is $x = -1$ i.e. at $(-1, f(-1)) = (-1, 2/e) = (-1, 0.736)$.

H: Summarizing the tables on E and G

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
Nature of $f(x)$	Increasing	Increasing	Decreasing
	Concave up	Concave down	Concave down