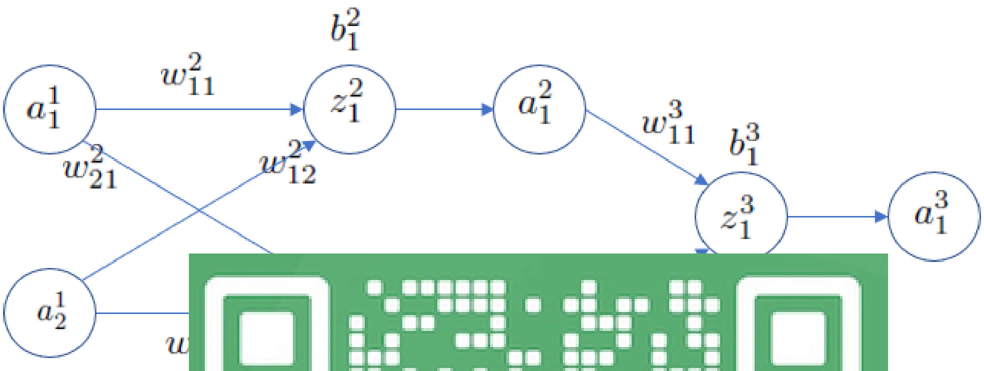
UNIVERSITY OF
BIRMINGHAM

Question 1

Let us consider solving regression problems with a neural network. In particular, we consider a neural network of the following structure:



As illustrated in the le... variables in the neural network.

$$\mathbf{z}^2 = \begin{pmatrix} z_1^2 \\ z_2^2 \end{pmatrix} = \quad \ldots \quad z_1^2) \\ z_2^2) \end{pmatrix},$$

where σ is the activati... s use $\sigma(x) = x^2$ in this neural network. ... n z_1^3, a_1^3 and $\mathbf{a}^2$.

(a) Compute the nu... ing this neural network. Please ... **[3 marks]**

(b) Suppose

$$\mathbf{W}^2 = \begin{pmatrix} \ldots \\ w_{21}^2 & w_{22}^2 \end{pmatrix} \ldots \begin{pmatrix} \ldots \\ -1 & 1 \end{pmatrix}, \quad \ldots, 1),$$

$$\mathbf{b}^2 = \begin{pmatrix} b_1^2 \\ b_2^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad b_1^3 = -3.$$

Consider the training example

$$\mathbf{x} = \begin{pmatrix} a_1^1 \\ a_2^1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad y = 1.$$

Let us consider the square loss function $C_{\mathbf{x},y}(\mathbf{W}, \mathbf{b}) = \frac{1}{2}(a_1^3 - y)^2$, where $\mathbf{W} = \{\mathbf{W}^2, \mathbf{W}^3\}, \mathbf{b} = \{\mathbf{b}^2, b_1^3\}$. Use the forward propagation algorithm to compute $\mathbf{a}^2$, a_1^3 and the loss $C_{\mathbf{x},y}(\mathbf{W}, \mathbf{b})$ for using the neural network to do prediction on the above example $(\mathbf{x}, y)$. Please write down your step-by-step calculations. **[7 marks]**

(c) Let us consider the neural network with the above $\mathbf{W}^2, \mathbf{W}^3, \mathbf{b}^2, b_1^3$ and the above training example $\mathbf{x}, y$. Use the back propagation algorithm to compute the gradients. For simplicity, we only require you to compute the explicit number of

$$\frac{\partial C_{\mathrm{x},y}(\mathbf{W},\mathbf{b})}{\partial z_1^3}, \quad \frac{\partial C_{\mathrm{x},y}(\mathbf{W},\mathbf{b})}{\partial z_1^2}, \quad \frac{\partial C_{\mathrm{x},y}(\mathbf{W},\mathbf{b})}{\partial z_2^2}, \quad \frac{\partial C_{\mathrm{x},y}(\mathbf{W},\mathbf{b})}{\partial \omega_{11}^3}, \quad \frac{\partial C_{\mathrm{x},y}(\mathbf{W},\mathbf{b})}{\partial \omega_{12}^3}.$$

Please write down your step-by-step calculations. **[10 marks]**

Question 2

Given the weights (w_1, w_2, w_3) and the biases (b_2, b_3), we have the following recurrent neural network (RNN) which takes in an input vector x_t and a hidden state vector h_{t-1} and returns an output vector y_t:

(1)

where $\mathbf{g}$ and $\mathbf{f}$ are act … ph depicts such a RNN.

(a) Write down clea … pdated) hidden state vector h_t shown in Figure 1. **[3 marks]**

(b) When $t = 3$ (starting from 1), please show how information is propagated through time by drawing an unfolded feedforward neural network that corresponds to the RNN in Figure 1. Please make sure that hidden states, inputs and outputs as well as network weights and biases are annotated on your network. **[4 marks]**

(c) Assume x_t, h_{t-1}, h_t and y_t are all scalars in Equation (1), and the activation functions are a linear unit and a binary threshold unit, respectively defined as:

$$\mathbf{g}(x) = x,$$
$$\mathbf{f}(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}.$$

When $t = 3$ (starting from 1), please calculate the values of the outputs (y_1, y_2, y_3) given $(w_1 = 1, w_2 = -3, w_3 = 5)$, $(b_2 = 1, b_3 = 3)$, $(x_1 = 5, x_2 = 3, x_3 = 1)$ and $h_0 = 0$. Please show your calculations in detail. **[3 marks]**

(d) Again let us assume x_t, h_{t-1}, h_t and y_t are all scalars with $h_0 = 0$ and the activation functions the same as above. Compute (w_1, w_2, w_3) and (b_2, b_3) such that the network outputs 0 initially, but when it receives an input of 1, it outputs 1 for all subsequent time steps. For example, if the input is 00001000100, the output will be 00001111111. Please justify your answer.

Note: here we want a solution that satisfies (1) the hidden state h_t is zero until the input x_t becomes 1, at which point the hidden state changes to 1 forever, and (2) the output always predicts the same as the hidden state, i.e. $y_t = h_t$. **[10 marks]**

Question 3

Note: Each item belo ... *xt. This is only an informal indication,* ... *nfluence marks.*

(a) Consider the Var ... t predicts mean $\mu_\phi(x)$ and stand ... ian that is the conditional $p_\phi(z$ ... d-dimensional input vector x is

(2)

where $\mathcal{L}_{rec}(x) =$ ... nstruction loss, $\mathcal{L}_{reg}(x) = \frac{1}{2}\sum_{j=1}^{v}$ ... regularizer, z is a v-dimensional ... e non-trainable scalars for weigh ... vector h.

(i) If you train ... and $\lambda_{reg} > 0$), what values ... r means $\mu_\phi(x)$ and standa ... **[4 marks]**

(ii) Assume that z is 2 dimensional (i.e. $v = 2$). Assume that for an input data point x_1 the encoder outputs vectors $\mu_\phi(x_1) = (0.5, 0.1)$ and $\sigma_\phi(x_1) = (0.1, 0.3)$. Calculate the value of $\mathcal{L}_{reg}(x_1)$. Show the steps of the calculation. (Note: For simplicity, use $\log_e 0.1 \approx -2.3$ and $\log_e 0.3 \approx -1.2$) **[4 marks]**

(iii) Assume you are given an implementation of the above VAE with a bottleneck (i.e. $v < d$). You are asked to train the VAE so that it will be as good as possible for the task of compressing data (via bottleneck) and uncompressing them with fidelity. Generation of fake data or other applications are not of interest. What values would you choose for λ_{rec} and λ_{reg}? For each, specify either *equal to* 0 or *greater than* 0. Explain why. **[5 marks]**

(b) Consider a Generative Adversarial Network (GAN) that consists of a Generator G that takes input noise vector z and outputs $G(z)$, and Discriminator D that given input x it outputs $D(x)$. We assume that $D(x) = 1$ means that D predicts with certainty that input x is a real data point, and $D(x) = 0$ means D predicts with certainty that x is a fake, generated sample. Figure 2 shows two loss functions that could be used for training G. Which of the two loss functions in Figure 2 is more appropriate for training G in practice? Explain why, based on the gradients for lowest and highest values of $D(G(z))$ and how they would influence training. **[7 marks]**

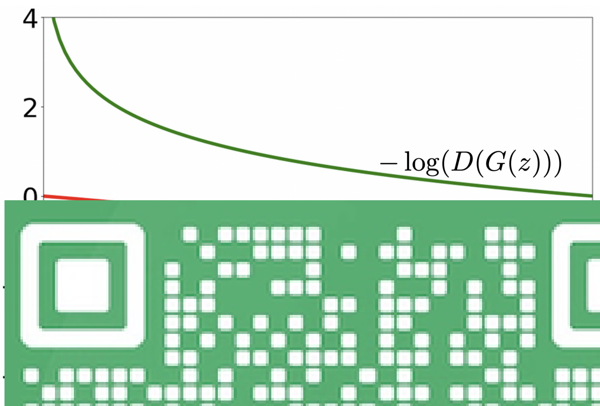


Figure 2: Two loss fu[illegible] Generator G of a GAN. Shown as a fu[illegible]hat a generated sample $G(z)$ is real. [illegible] certainty that $G(z)$ is real, whereas [illegible]$G(z)$ is fake.