MATH4/68091 Statistical Computing
Coursework 1

Deadline: 11:00am Monday 28 October 2024

Please submit via Blackboard. Late submissions may be penalised according to the Department's procedures. Note also that I do not have the authority to grant extensions to the deadline.

By submitting the coursework you declare that you are its sole author. In particular, you should not collaborate with your peers.

Your submitted solutions should all be in one "pdf" document. You are strongly advised to produce this document using LaTeX, but you will not be penalised if you use other software. Do not include screenshots of the R conso [illegible] a typewriter type font. In LaTeX this can be done [illegible]

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\begin{verbatim}
R code
\end{verbatim}
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Plots should be saved to [illegible] it with suitable LaTeX commands. Examples of [illegible] ints.pdf", available on Blackboard and on my we [illegible] the facilities provided by it.

For each part of the [illegible] you completed what is required, show your wo [illegible] aphical results, where applicable. Aim to be con [illegible]

The total marks for t [illegible] dual parts have equal weights.

1. (8 marks)

The standard double e [illegible] has pdf

$$f(x) = \frac{1}{2} \exp -|x|, \qquad \text{for } -\infty < x < \infty.$$

(a) Find the cdf of the standard double exponential distribution.

(b) Describe in detail the steps of the inverse PIT (inverse cdf) method to obtain a random sample of size n from $f(x)$.

(c) Write an R function implementing your procedure from part (b) for generating random samples from $f(x)$.

(d) Run your function from part (c) to generate a random sample of size $n = 5000$ from $f(x)$. Construct a histogram of the generated data, superimpose the pdf $f(x)$ and comment on the goodness-of-fit.

2. (12 marks)

Suppose that we want to simulate random data from the standard Normal distribution, N(0,1), whose pdf is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \qquad \text{for } -\infty < x < \infty.$$

It is proposed to develop a rejection sampling algorithm, where the proposal distribution is the standard double exponential distribution from Q1 with pdf

$$g(x) = \frac{1}{2} e^{-|x|}, \qquad \text{for } -\infty < x < \infty.$$

(a) Define the constant K by

$$K = \sup_x \left(\frac{f(x)}{g(x)} \right).$$

Show that $K = \sqrt{\frac{2e}{\pi}}$.

(b) Produce on the same plot graphs of the functions $f(x)$ and $Kg(x)$ using the value of K determined in part (a).

(c) Describe a rejection [illegible] uses as proposal distribution the standard [illegible]

(d) Theoretically, how e[illegible] in part (c)?

(e) Write an R function [illegible] t (c). It should return a random sample fr[illegible] ion should provide an empirical estimate o[illegible]

Note: For simulatio[illegible] u can use your function from Q1. Alternativ[illegible] age.

(f) Run your function [illegible] the standard Normal distribution. Graph[illegible] being sampled from a standard Normal di[illegible]

Note: Don't go ove[illegible] table interpretation is sufficient for full ma[illegible] tence, especially when the answer is affirmative. Optionally, feel free to provide additional plot(s) (with interpretation) and maybe results of a test. This is completely risk-free, in that these will not affect your mark but potentially will inform me about concepts for discussion.