
CIS 471 / 571 (Fall 2020): Introduction to Artificial Intelligence

Assignment Project Exam Help

Lecture 9: MDPs (Part 2)

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Source: <http://ai.berkeley.edu/home.html>



Announcement

- Project 3: Reinforcement Learning
 - Deadline: Nov 10th, 2020

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- Homework 3: MDPs and Reinforcement Learning
 - Will be posted tomorrow
 - Deadline: Nov 09th, 2020

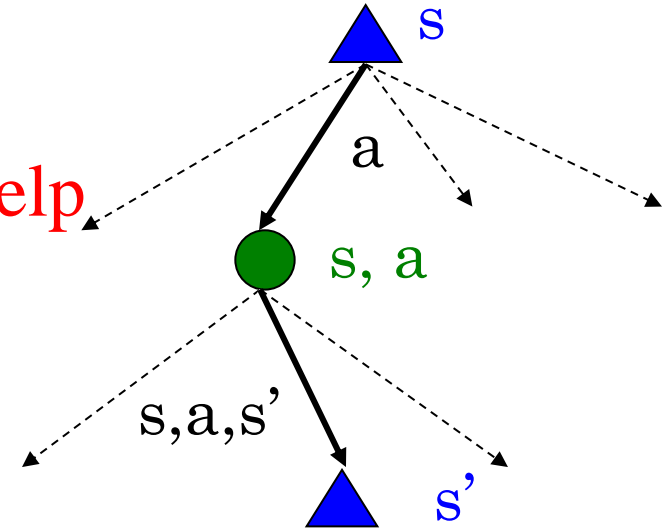
Recap: MDPs

- Markov decision processes:

- States S
- Actions A
- Transitions $P(s' | s, a)$ (or $T(s, a, s')$)
- Rewards $R(s, a, s')$ (and discount γ)
- Start state s_0

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- Quantities:

- Policy = map of states to actions
- Utility = sum of discounted rewards
- Values = expected future utility from a state (max node)
- Q-Values = expected future utility from a q-state (chance node)



Optimal Quantities

- The value (utility) of a state s :

$V^*(s)$ = expected utility starting in s
and acting optimally

- The value (utility) of a q -state (s,a) :

$Q^*(s,a)$ = expected utility starting
out having taken action a from
state s and (thereafter) acting
optimally

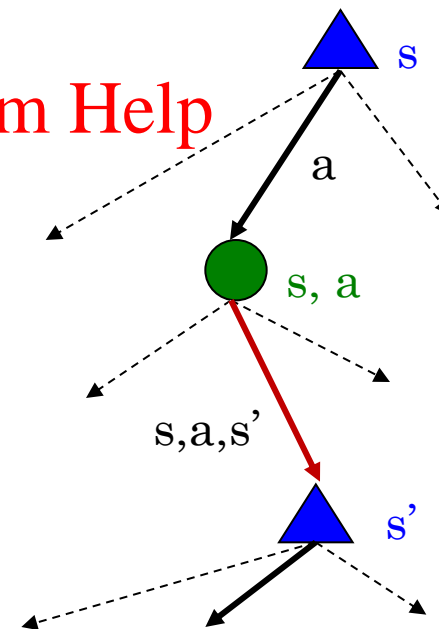
- The optimal policy:

$\pi^*(s)$ = optimal action from state s

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s is a
state

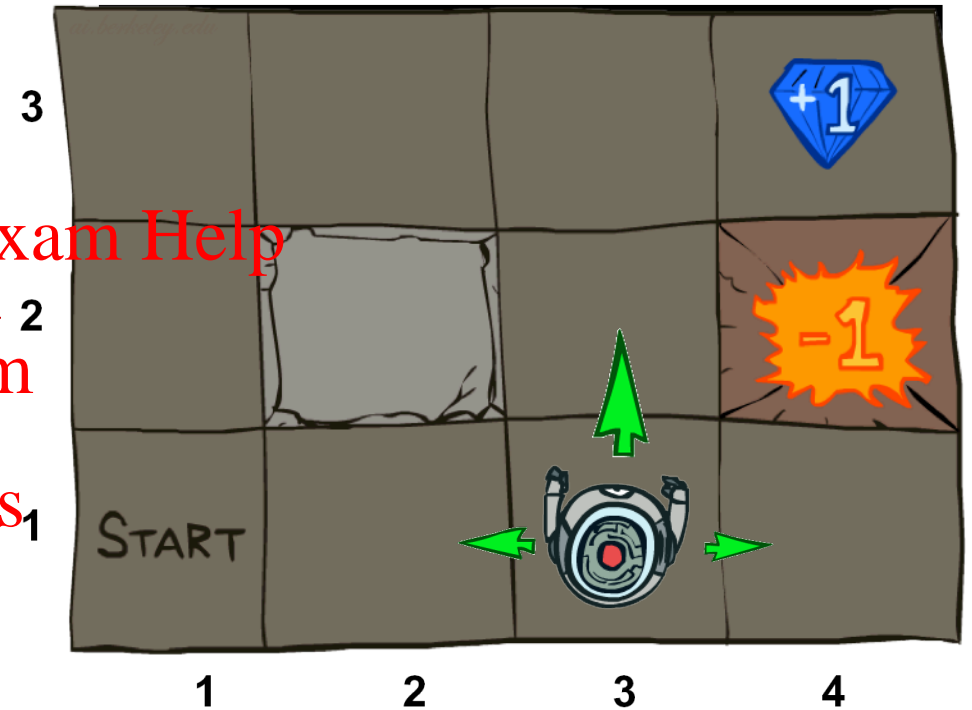
(s, a) is a
 q -state

(s, a, s') is a
transition

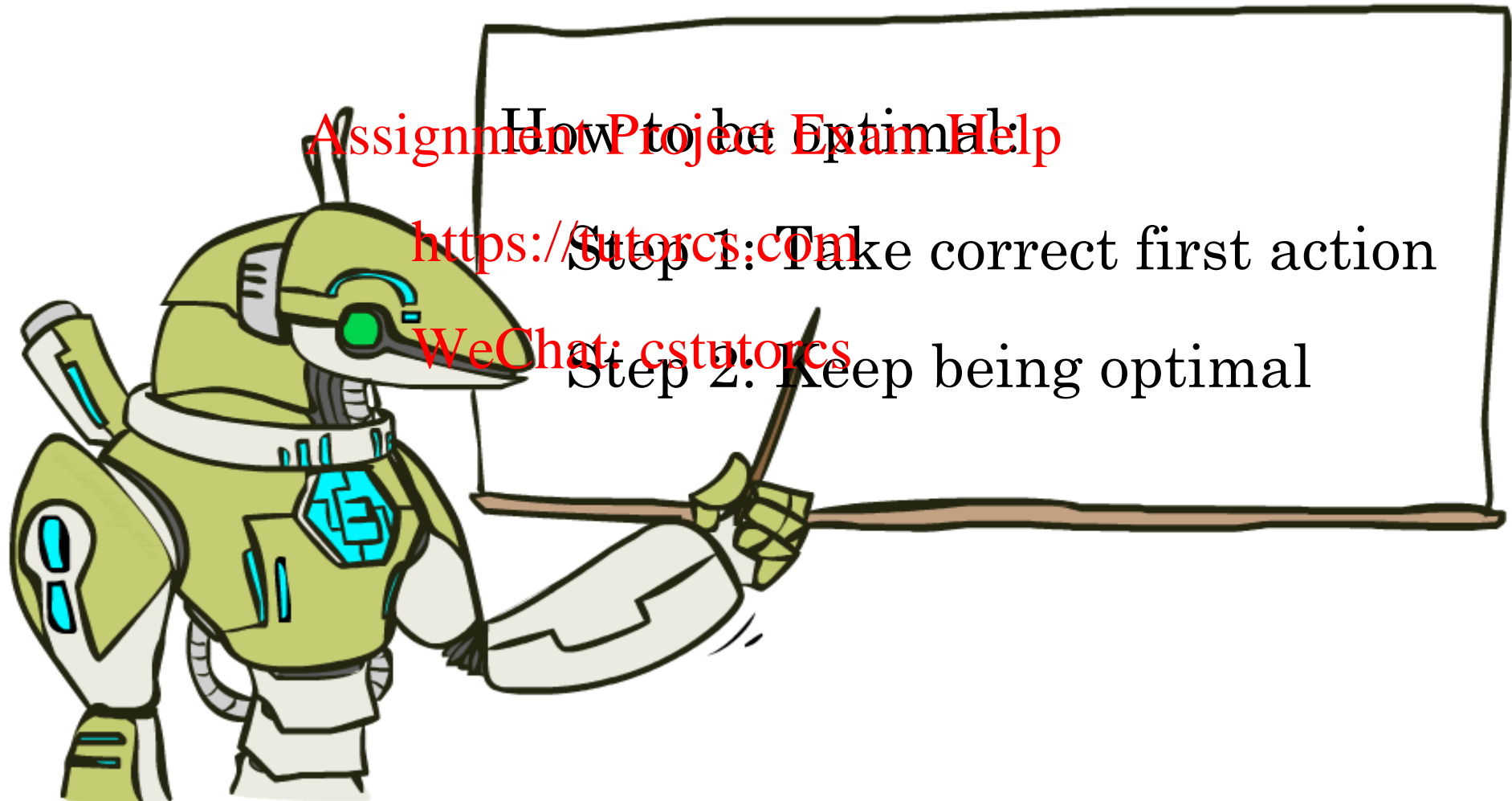


Example: Grid World

- A maze-like problem
 - The agent lives in a grid
 - Walls block the agent's path
- Noisy movement: actions do not always go as planned
 - 80% of the time, the action North takes the agent North
 - 10% of the time, North takes the agent West, 10% East
 - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
 - Small “living” reward each step (can be negative)
 - Big rewards come at the end (good or bad)
- Goal: maximize sum of (discounted) rewards



The Bellman Equations



The Bellman Equations

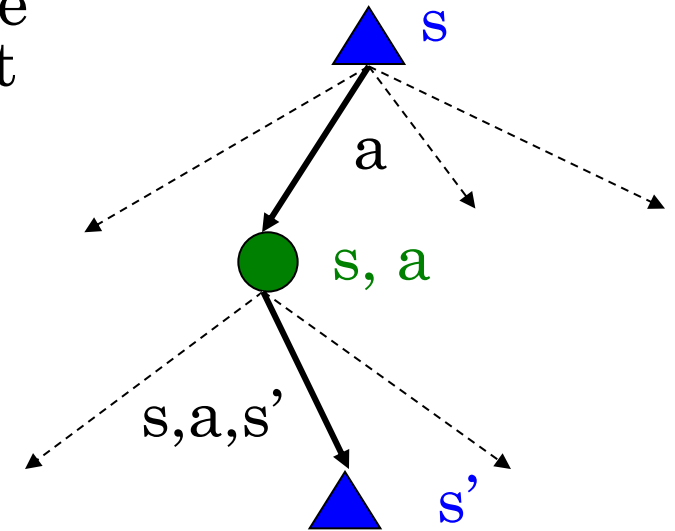
- Definition of “optimal utility” via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s, a)$$

$$Q^*(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

$$V^*(s) = \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

- These are the Bellman equations, and they characterize optimal values in a way we'll use over and over



Value Iteration

- Bellman equations **characterize** the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

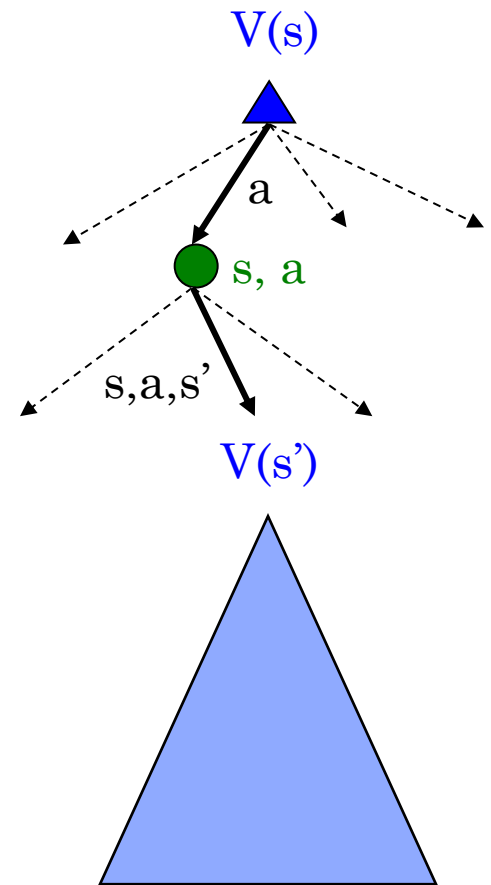
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- Value iteration **computes** them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

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- Value iteration is just a fixed point solution method
 - ... though the V_k vectors are also interpretable as time-limited values
- Theorem: will converge to unique optimal values
 - Basic idea: approximations get refined towards optimal values
 - Policy may converge long before values do



Problems with Value Iteration

- Value iteration repeats the Bellman updates:

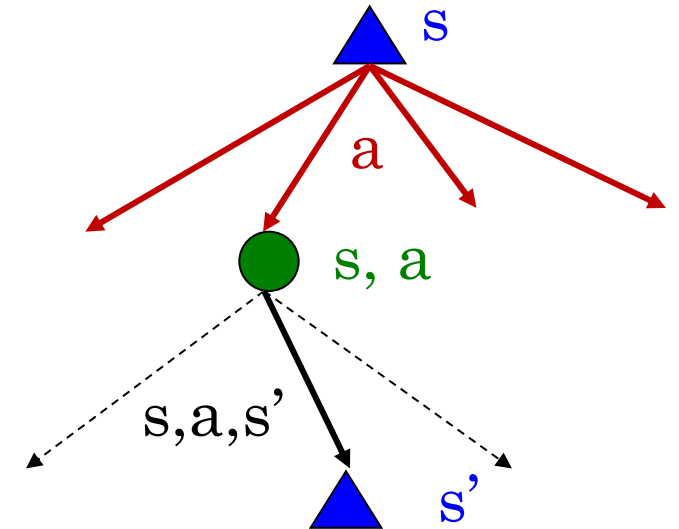
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

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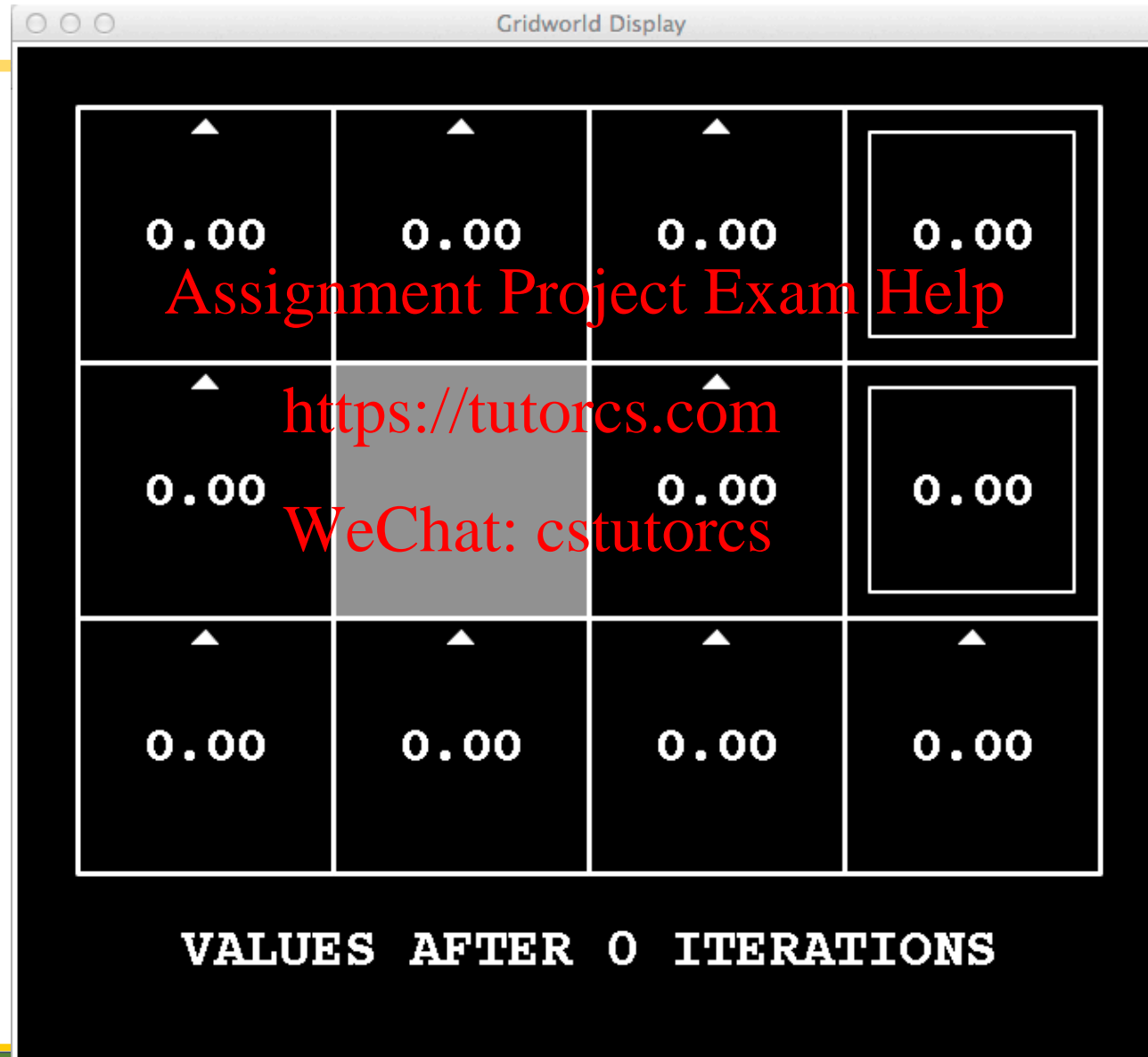
- Problem 1: It's slow – $O(S^2A)$ per iteration

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- Problem 2: The “max” at each state rarely changes
- Problem 3: The policy often converges long before the values



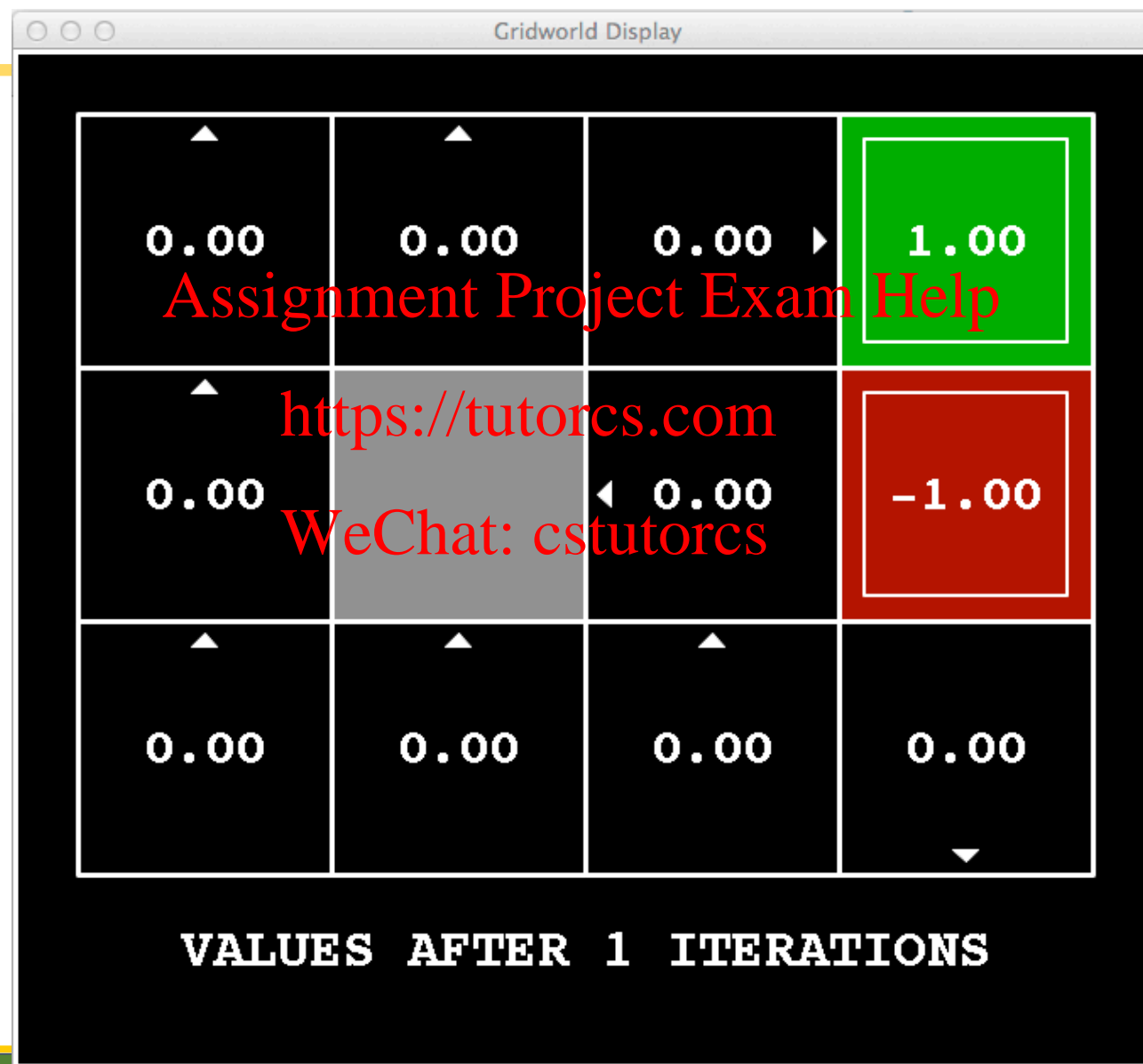
$k=0$



Noise = 0.2
Discount = 0.9
Living reward = 0



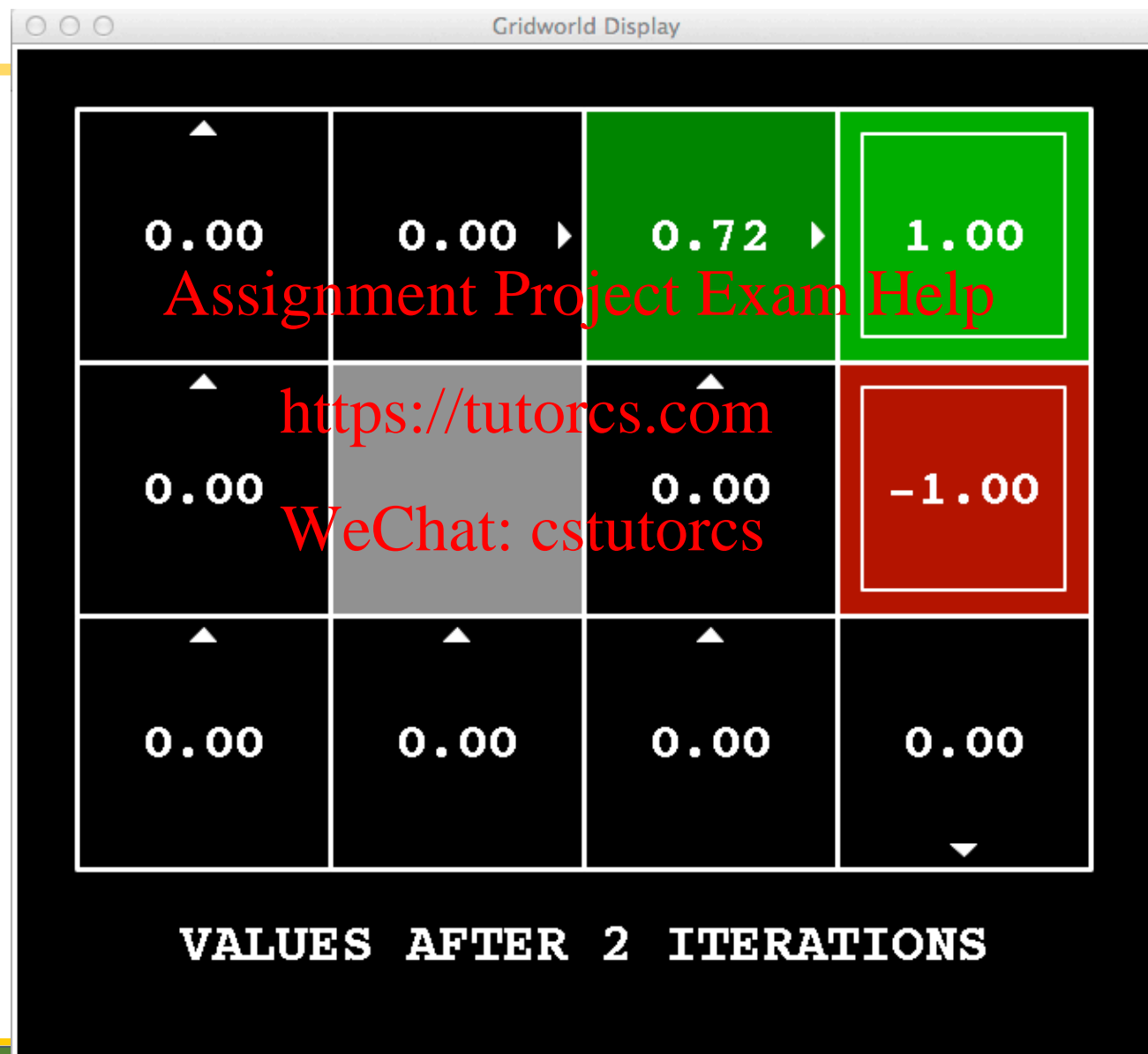
$k=1$



Noise = 0.2
Discount = 0.9
Living reward = 0



$k=2$



Noise = 0.2
Discount = 0.9
Living reward = 0



$k=3$



Noise = 0.2
Discount = 0.9
Living reward = 0



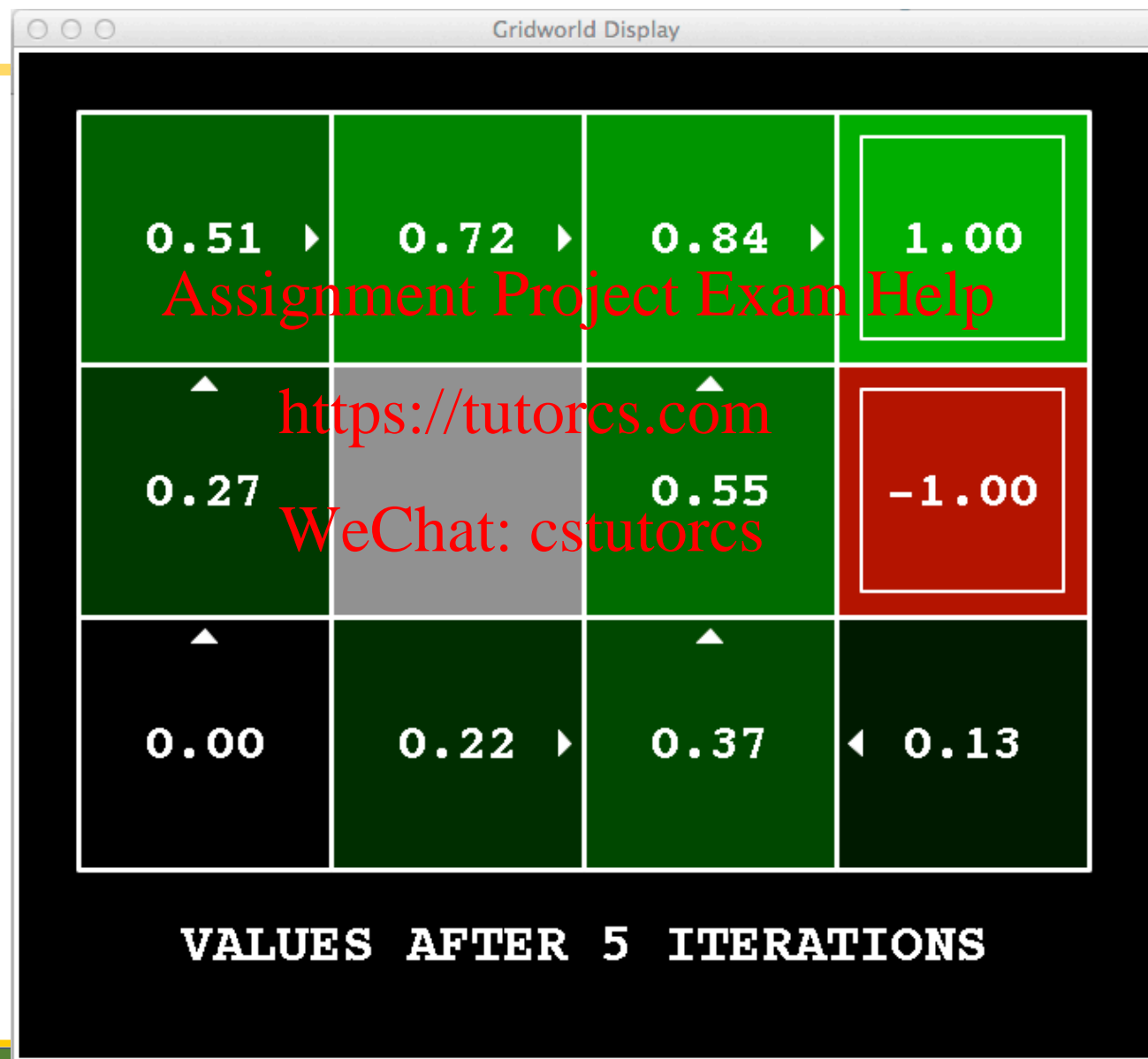
$k=4$



Noise = 0.2
Discount = 0.9
Living reward = 0



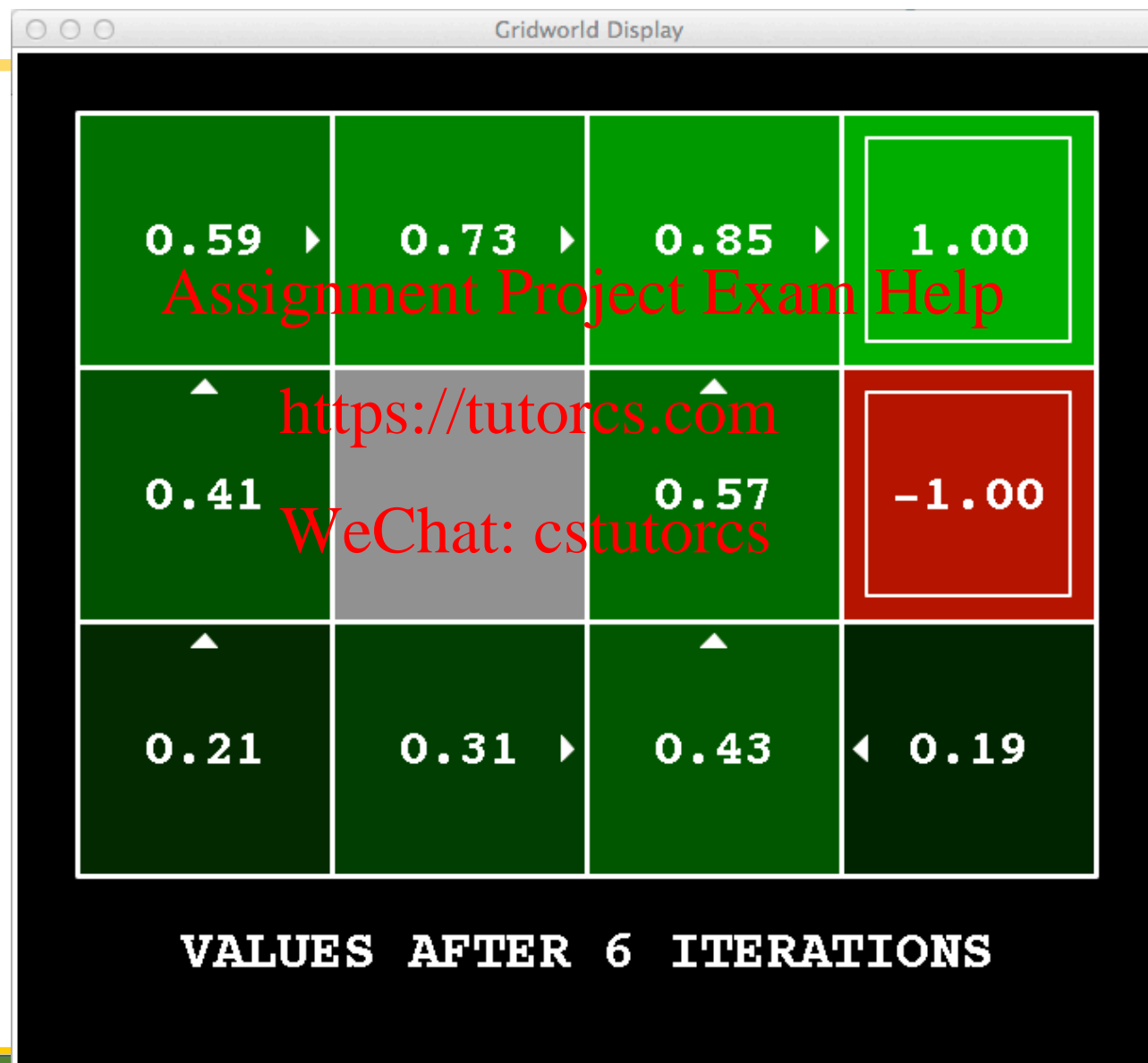
$k=5$



Noise = 0.2
Discount = 0.9
Living reward = 0



$k=6$



Noise = 0.2
Discount = 0.9
Living reward = 0



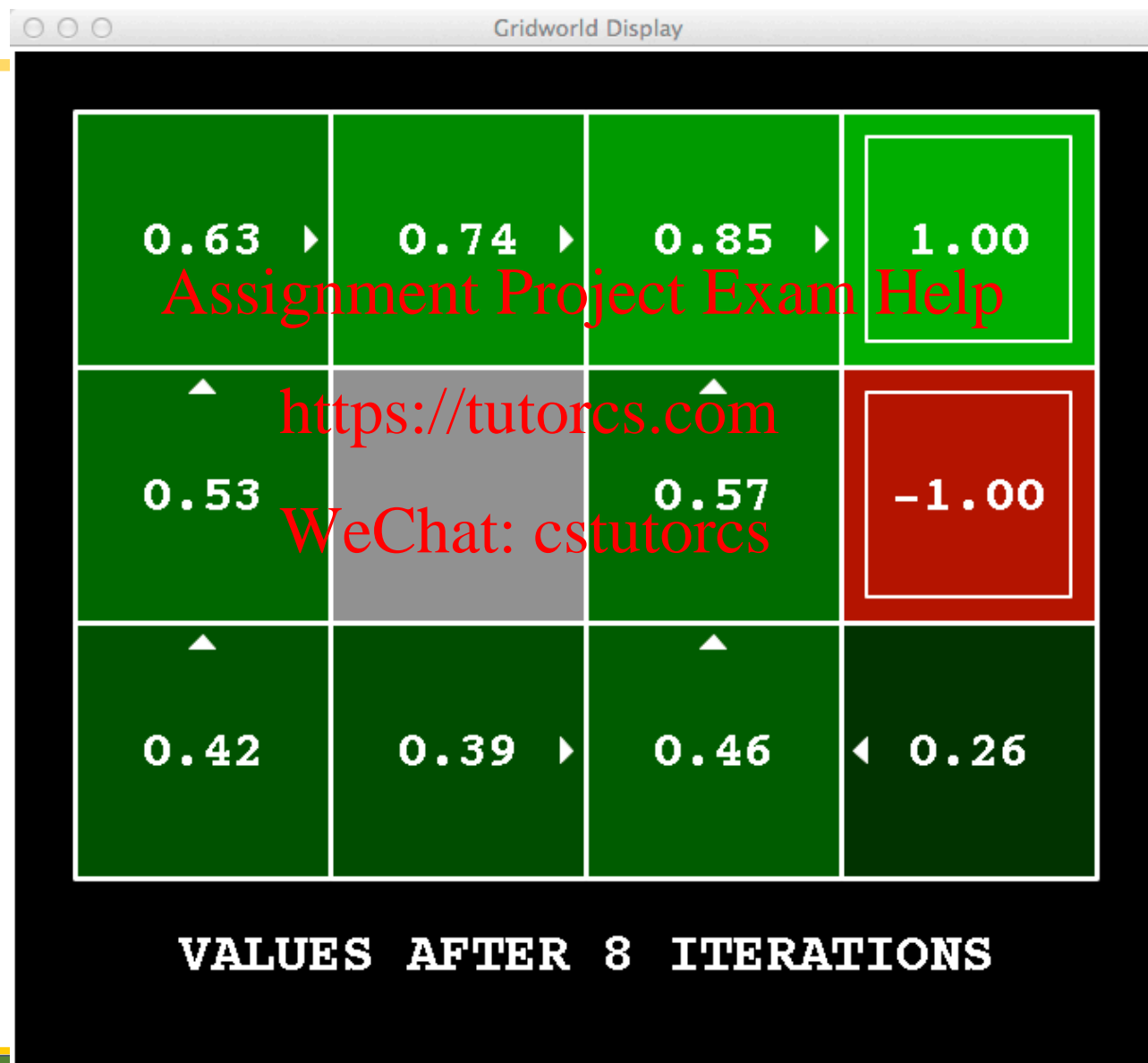
$k=7$



Noise = 0.2
Discount = 0.9
Living reward = 0



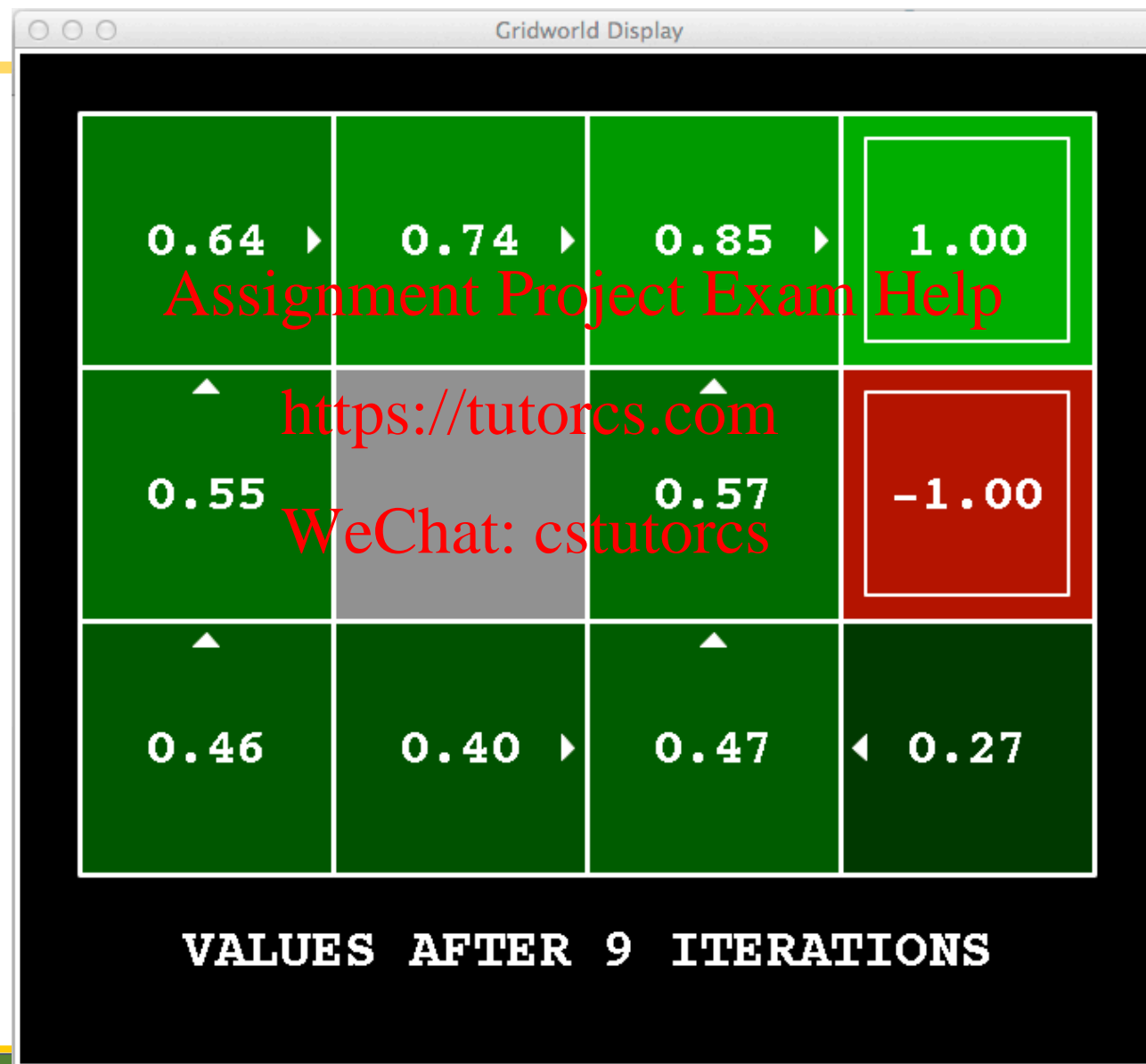
$k=8$



Noise = 0.2
Discount = 0.9
Living reward = 0



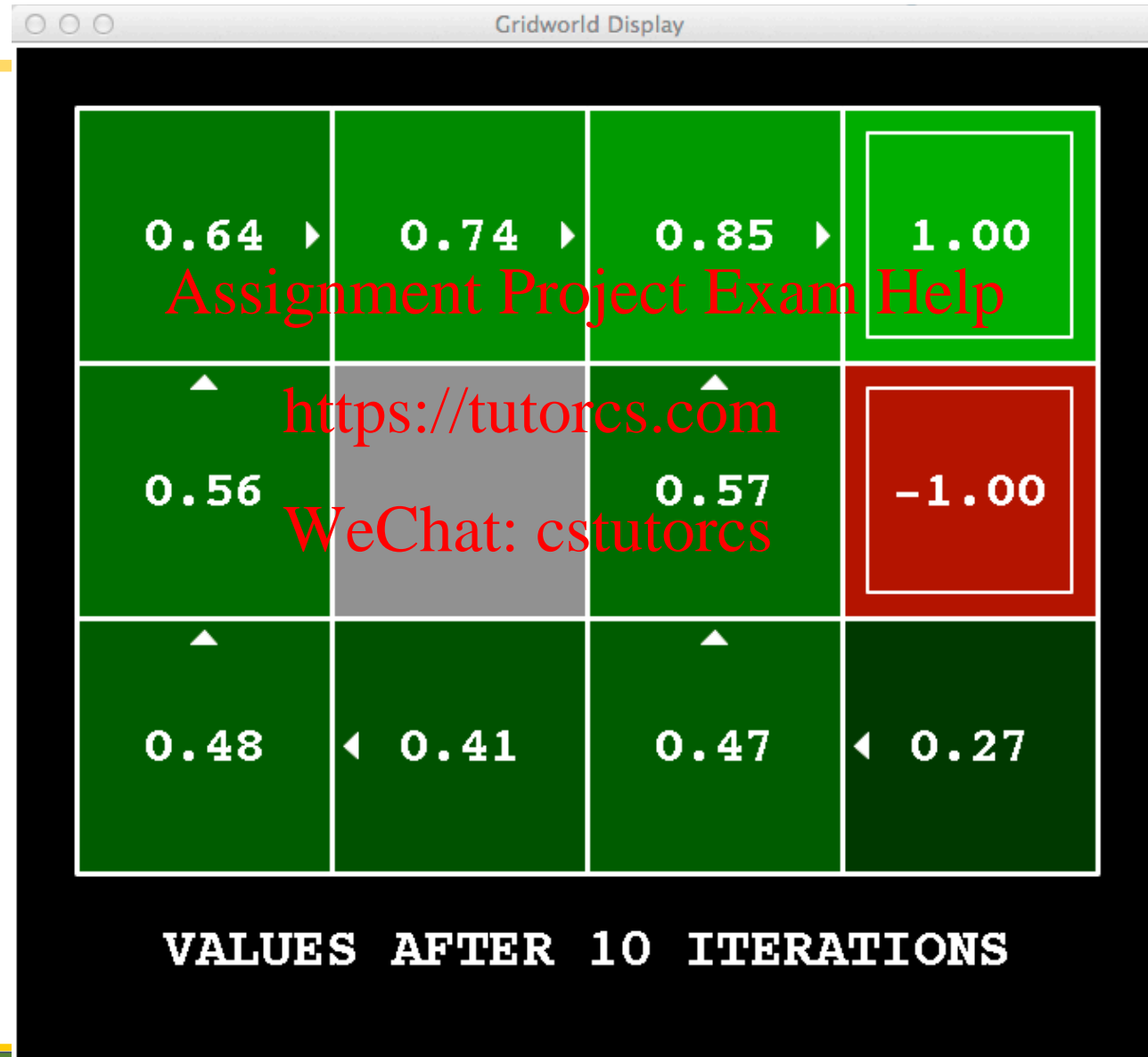
$k=9$



Noise = 0.2
Discount = 0.9
Living reward = 0



$k=10$



Noise = 0.2
Discount = 0.9
Living reward = 0



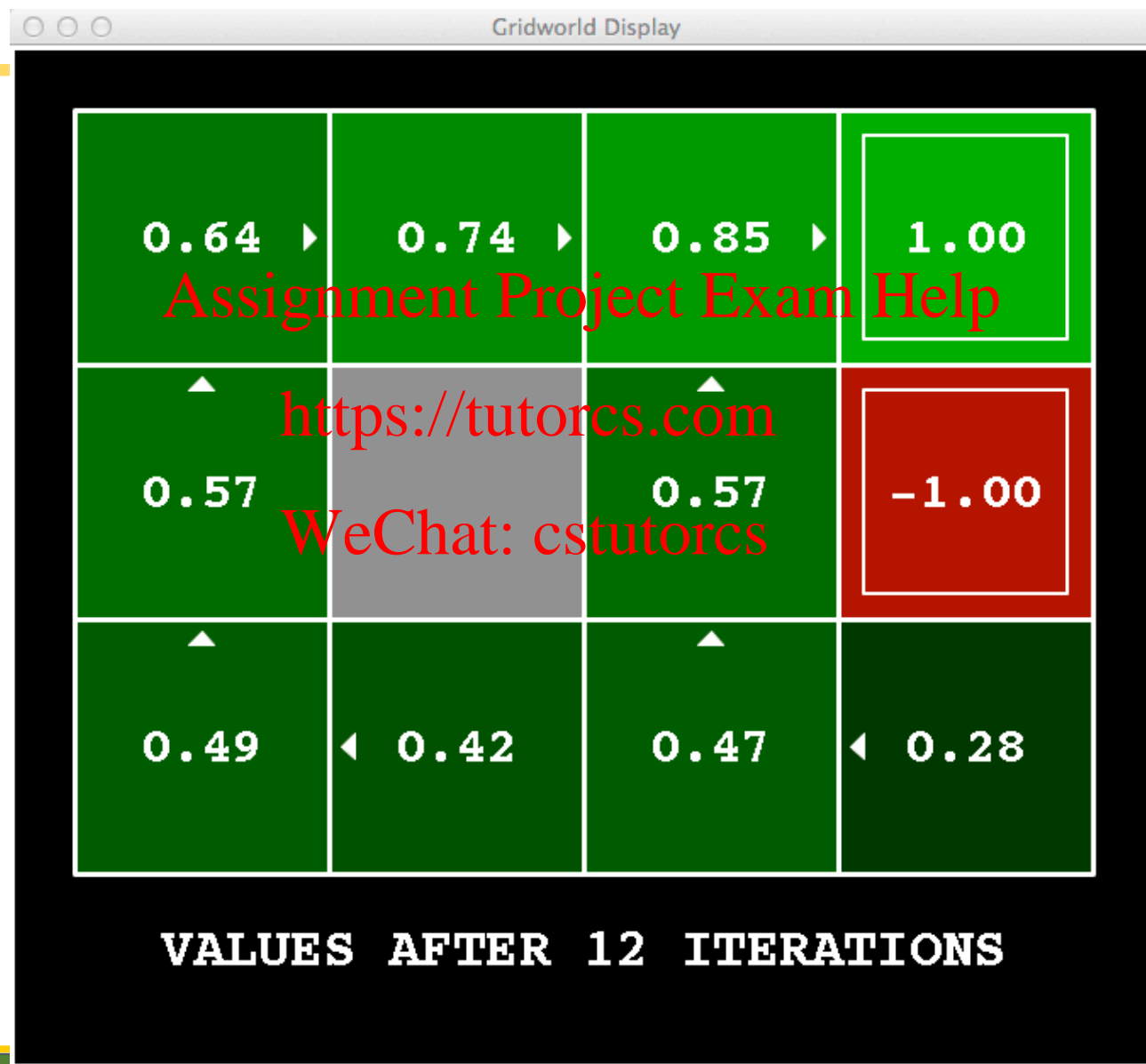
$k=11$



Noise = 0.2
Discount = 0.9
Living reward = 0



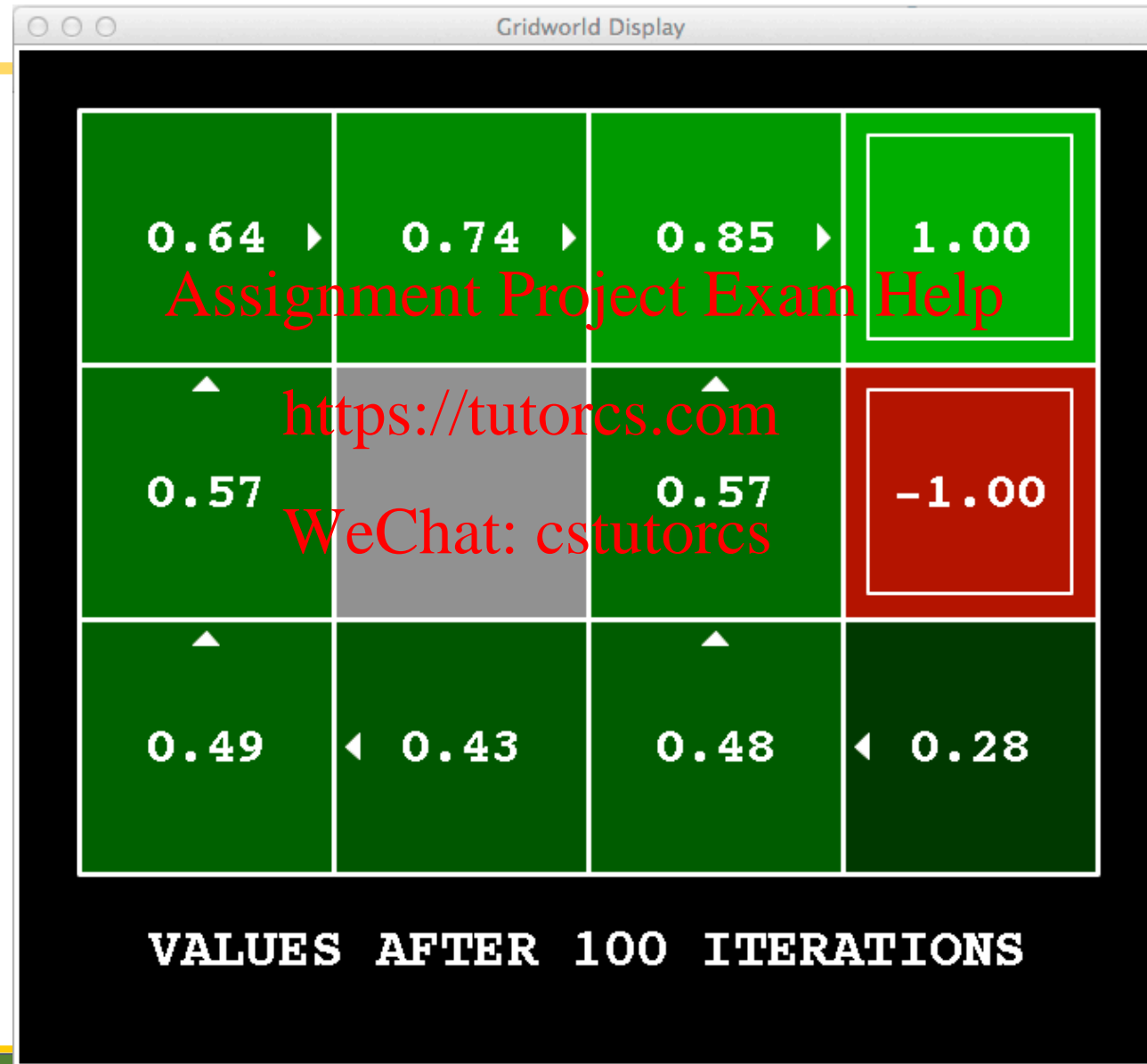
$k=12$



Noise = 0.2
Discount = 0.9
Living reward = 0



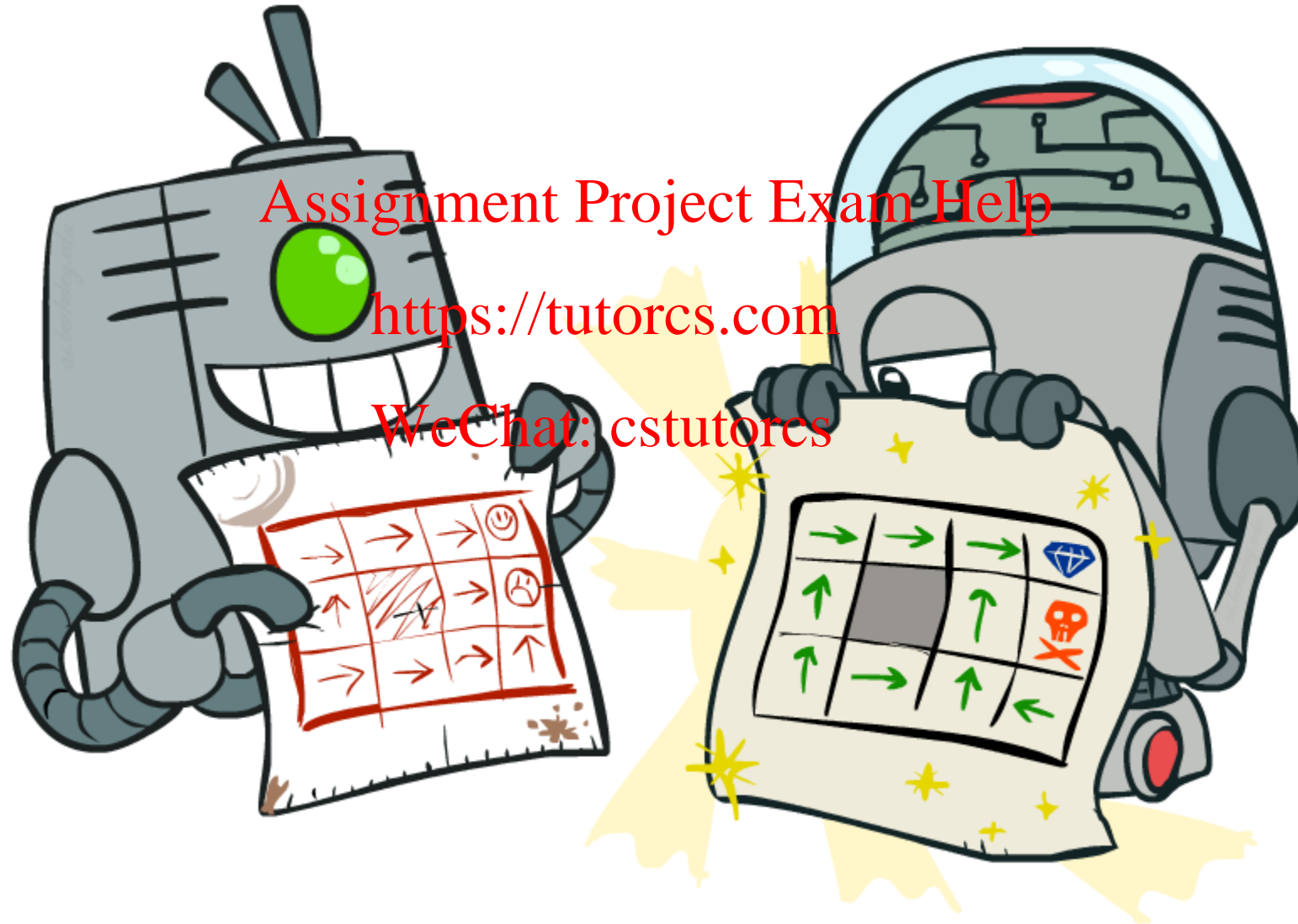
$k=100$



Noise = 0.2
Discount = 0.9
Living reward = 0



Policy Methods

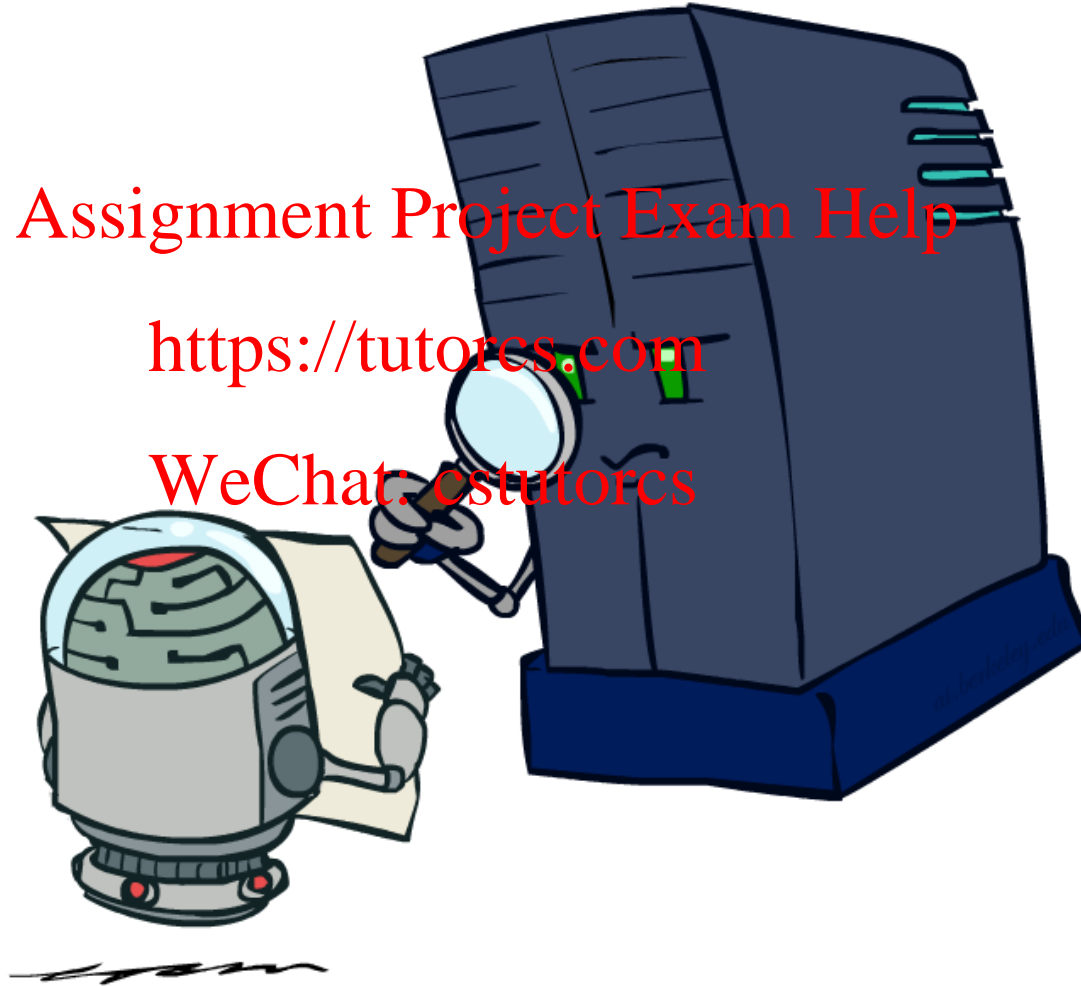


Policy Evaluation

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Fixed Policies

Do the optimal action

Do what π says to do



- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state
 - ... though the tree's value would depend on which policy we fixed



Utilities for a Fixed Policy

- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy

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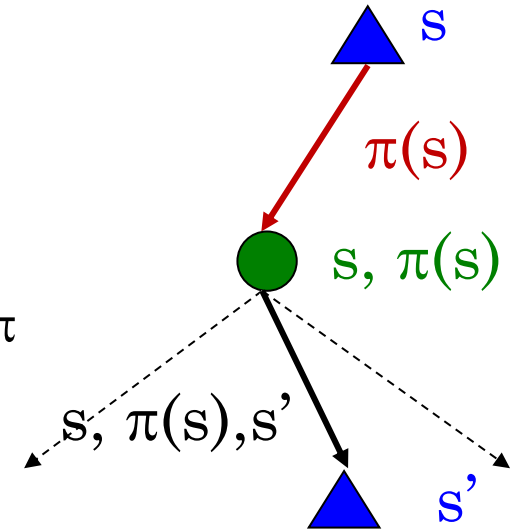
- Define the utility of a state s , under a fixed policy π :

$V^\pi(s)$ = expected total discounted rewards starting in s and following π

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- Recursive relation (one-step look-ahead / Bellman equation):

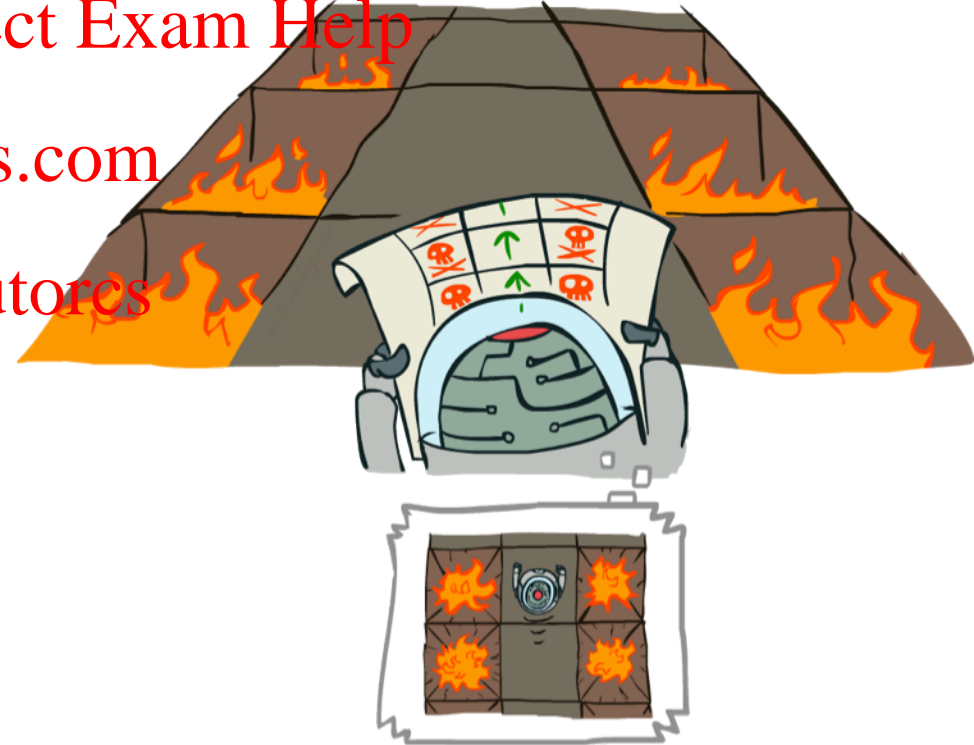
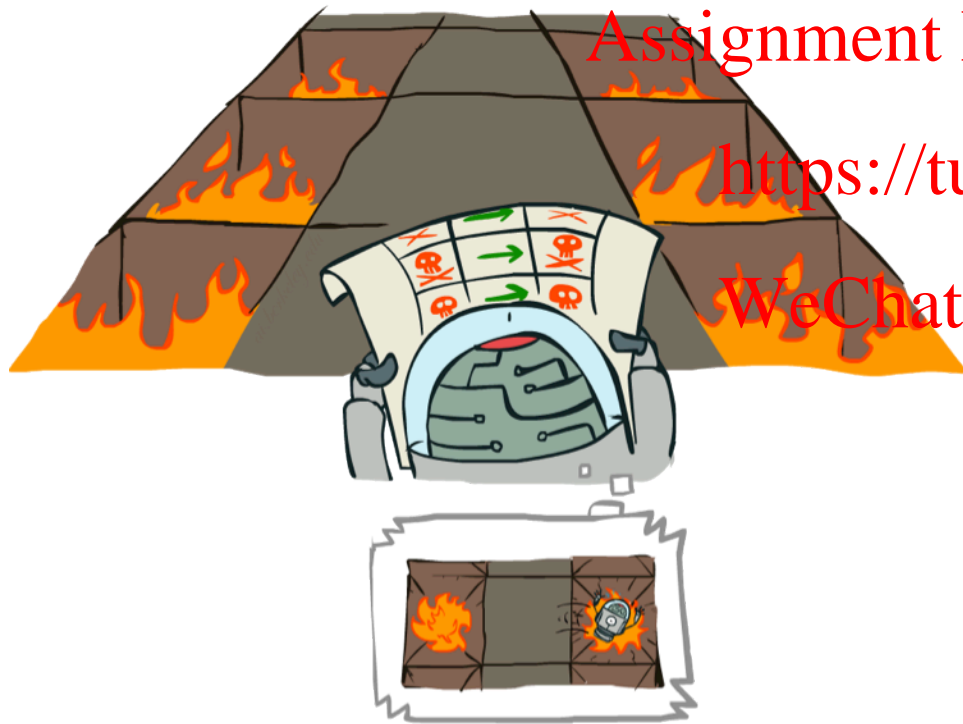
$$V^\pi(s) = \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V^\pi(s')]$$



Example: Policy Evaluation

Always Go Right

Always Go Forward



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Example: Policy Evaluation

Always Go Right

-10.00	100.00	-10.00
-10.00	1.09 ▶	-10.00
-10.00	-7.88 ▶	-10.00
-10.00	-8.69 ▶	-10.00

Always Go Forward

-10.00	100.00	-10.00
-10.00	70.20 ▲	-10.00
-10.00	48.74 ▲	-10.00
-10.00	33.30 ▲	-10.00

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Policy Evaluation

- How do we calculate the V 's for a fixed policy π ?
- Idea 1: Turn recursive Bellman equations into updates (like value iteration)

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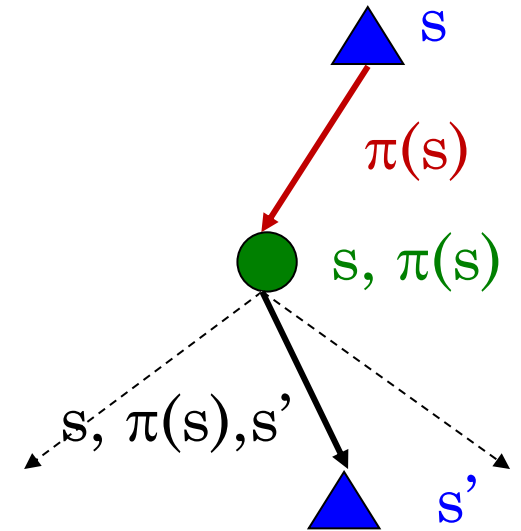
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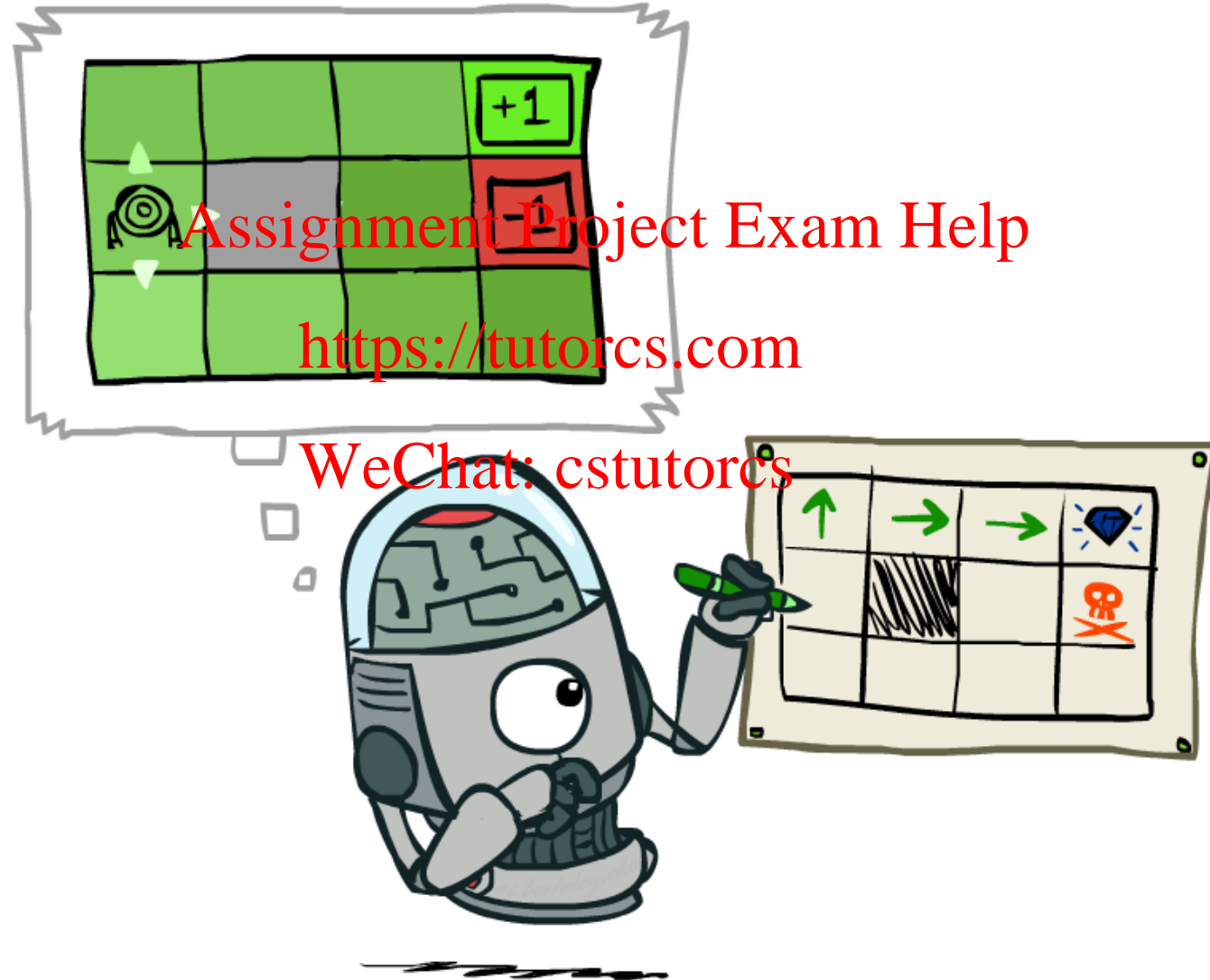
$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

- Efficiency: $O(S^2)$ per iteration
- Idea 2: Without the maxes, the Bellman equations are just a linear system
 - Solve with Matlab (or your favorite linear system solver)



Policy Extraction



Computing Actions from Values

- Let's imagine we have the optimal values $V^*(s)$

- How should we act?
 - It's not obvious!

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- We need to do a mini-expectimax (one step)



$$\pi^*(s) = \arg \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^*(s')]$$

- This is called **policy extraction**, since it gets the policy implied by the values



Computing Actions from Q-Values

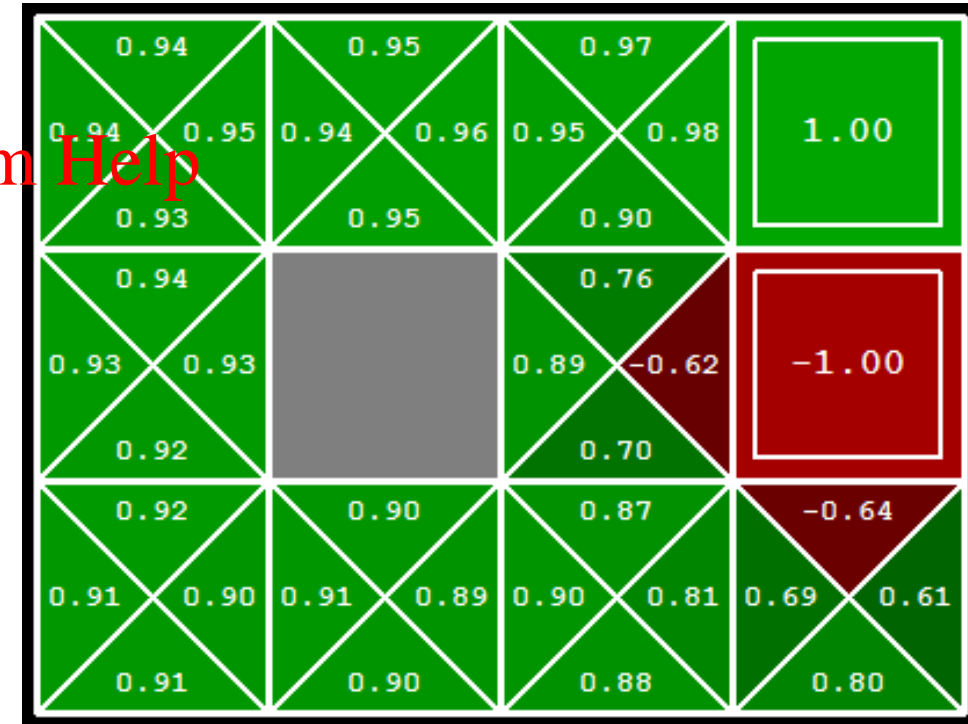
- Let's imagine we have the optimal q-values:

- How should we act?

- Completely trivial to decide!

$$\pi^*(s) = \arg \max_a Q^*(s, a)$$

- Important lesson: actions are easier to select from q-values than values!



Policy Iteration



Policy Iteration

- Alternative approach for optimal values:
 - **Step 1: Policy evaluation:** calculate utilities for some fixed policy (not optimal utilities!) until convergence
 - **Step 2: Policy improvement:** update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
 - Repeat steps until policy converges
- This is **policy iteration**
 - It's still optimal!
 - Can converge (much) faster under some conditions

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Policy Iteration

- Evaluation: For fixed current policy π , find values with policy evaluation:
 - Iterate until values converge:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') [R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s')]$$

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- Improvement: For fixed values, get a better policy using policy extraction
 - One-step look-ahead:

$$\pi_{i+1}(s) = \arg \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V^{\pi_i}(s')]$$



Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)
- In value iteration:
 - Every iteration updates both the values and (implicitly) the policy
 - We don't track the policy, but taking the max over actions implicitly recomputes it

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- In policy iteration:
 - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
 - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
 - The new policy will be better (or we're done)
- Both are dynamic programs for solving MDPs



Summary: MDP Algorithms

- So you want to....
 - Compute optimal values: use value iteration or policy iteration
 - Compute values for a particular policy: use policy evaluation
 - Turn your values into a policy: use policy extraction (one-step lookahead)
 - These all look the same
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- They basically are – they are all variations of Bellman updates
 - They all use one-step look-ahead expectimax fragments
 - They differ only in whether we plug in a fixed policy or max over actions



Example: Racing

- Discount: $\gamma = 0.1$
- Initial policy
 - $\pi_0(\text{Cool}) = \text{Slow}$
 - $\pi_0(\text{Warm}) = \text{Slow}$
 - $\pi_0(\text{Overheated}) = \emptyset$

